Flavor Symmetry and Neutrino Physics

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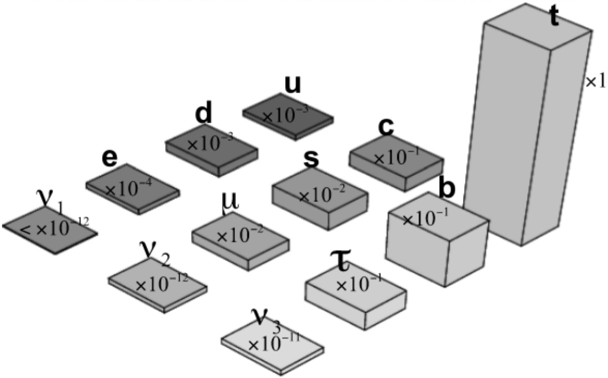
Mexico City

Neutrino Mass:

Tiny Neutrino mass: a long standing issue.



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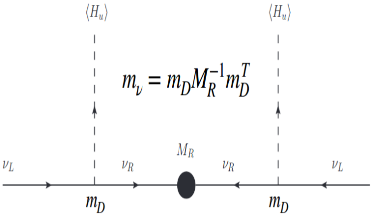
Type-I seesaw



[Minkowski'77, Yanagida'79, Gell-Mann/Slansky/Ramond'79, Mohapatra/Senjanovic'80, Schecter/Valle'80]

Cont...

- **The low energy Majorana neutrino mass matrix:**

$$m_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ * & m_{\mu\mu} & m_{\mu\tau} \\ * & * & m_{\tau\tau} \end{pmatrix};$$

- # of free parameters: 12
- The complex symmetric mass matrix m_ν can be diagonalized as:

$$m_\nu = V M_\nu^{diag} V^T; \qquad M_\nu^{diag} = Diag\{m_1, m_2, m_3\}$$

- **The neutrino mixing matrix V is parameterized as**

$V \equiv UP$,

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix};$$

  - **V is called as Pontecorvo-Maki-Nakagawa-Sakata matrix**

- **# of observables from neutrino oscillations experiments:**

$\Delta m^2_{21}, |\Delta m^2_{31}|, \theta_{13}, \theta_{12}, \theta_{23}$, **and** δ

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Mismatch in # of parameters

Neutrino Mixings:
The PMNS matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric, K2K, MINOS, T2K, etc.	Reactor Accelerator	Solar KamLAND

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- **2-additional Majorana phases.**

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Atmospheric, K2K, MINOS, T2K, etc.	Reactor Accelerator	Solar KamLAND

- **3-mixing angles, 1 CP-phase.**
- **2-additional Majorana phases.**

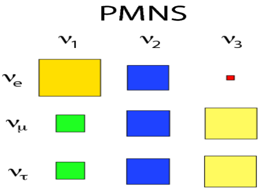
Is this the only parameterization?

$*$ There are total nine-ways to parameterize V

Fritzsch, Xing, PRD57 (1998) 594-597

Example: $R(\theta_{12})R(\theta_{23},\delta)$, $R^{-1}(\vartheta_{12})$, **and 8 more**

Cont...



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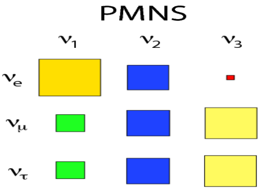

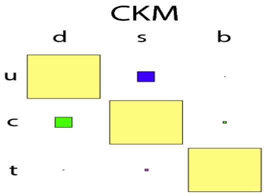

Why is this flavor structure?

[Xing, PR854 (2020) 1-147, arXiv: 1909.09610]

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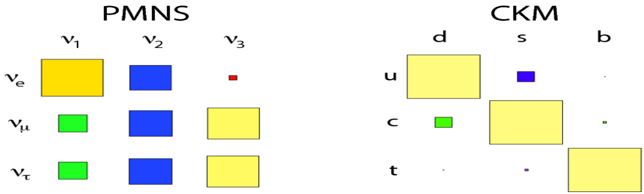

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Theoretical approaches

- texture zeros,
- flavor symmetries,
- seesaw mechanisms,
- radiative mechanisms,
- extra dimensions, etc...

$$m_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ * & m_{\mu\mu} & m_{\mu\tau} \\ * & * & m_{\tau\tau} \end{pmatrix}$$

Current Status:

parameter	best fit $\pm 1\sigma$	2σ range	3σ range
$\Delta m^2_{21}[10^{-5}\mathrm{eV}^2]$	$7.50^{+0.22}_{-0.20}$	7.12–7.93	6.94–8.14
$\lvert\Delta m^2_{31}\rvert[10^{-3}\mathrm{eV}^2]$ (NO)	$2.55^{+0.02}_{-0.03}$	2.49–2.60	2.47–2.63
$\lvert\Delta m^2_{31}\rvert[10^{-3}\mathrm{eV}^2]$ (IO)	$2.45^{+0.02}_{-0.03}$	2.39–2.50	2.37–2.53
$\sin^2\theta_{12}/10^{-1}$	3.18 ± 0.16	2.86–3.52	2.71–3.69
$\theta_{12}/^\circ$	34.3 ± 1.0	32.3–36.4	31.4–37.4
$\sin^2\theta_{23}/10^{-1}$ (NO)	5.74 ± 0.14	5.41–5.99	4.34–6.10
$\theta_{23}/^\circ$ (NO)	49.26 ± 0.79	47.37–50.71	41.20–51.33
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.78^{+0.10}_{-0.17}$	5.41–5.98	4.33–6.08
$\theta_{23}/^\circ$ (IO)	$49.46^{+0.60}_{-0.97}$	47.35–50.67	41.16–51.25
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.200^{+0.069}_{-0.062}$	2.069–2.337	2.000–2.405
$\theta_{13}/^\circ$ (NO)	$8.53^{+0.13}_{-0.12}$	8.27–8.79	8.13–8.92
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.225^{+0.064}_{-0.070}$	2.086–2.356	2.018–2.424
$\theta_{13}/^\circ$ (IO)	$8.58^{+0.12}_{-0.14}$	8.30–8.83	8.17–8.96
δ/π (NO)	$1.08^{+0.13}_{-0.12}$	0.84–1.42	0.71–1.99
$\delta/^\circ$ (NO)	194^{+24}_{-22}	152–255	128–359
δ/π (IO)	$1.58^{+0.15}_{-0.16}$	1.26–1.85	1.11–1.96
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- **Preference for NO at 2.5σ.**
- **Best-fit of θ_{23} favors HO for both NO and IO.**
- **$\theta_{13} = 0$ is excluded at more than 5σ.**

de Salas et. al. arXiv:2006.11237

$\mu - \tau$ symmetry:

First seed:

- Fukuyama, Nishiura proposed $\mu - \tau$ symmetry in the M_ν,

$$M_\nu = \begin{pmatrix} 0 & B & \pm B \\ B & C & D \\ \pm B & D & C \end{pmatrix}.$$

where $A, B, C \in \mathcal{R}$.

arXiv:hep-ph/9702253 & 1701.04985, (PTEP 2017)

- **Diagonalization of $M_\nu \Rightarrow \theta_{23} = \mp 45^\circ$ and $\theta_{13} = 0^\circ$**
- **$(M_\nu)_{11} = 0 \Rightarrow$ small θ_{12}, which had survived with the large mixing angle solution at that time**
- **Later, the KamLAND Collaboration, selected the larger part of solar neutrino angles**
- **An immediate generalization is to introduce parameter A in the (1,1)-entry for large θ_{12}**

Tri-Bi-Maximal Mixing:

- **Before Daya-Bay results, [PRL'13]:** M_ν looks as

$$\mathbf{M}_\nu = \begin{pmatrix} A & B & \pm B \\ B & C & D \\ \pm B & D & C \end{pmatrix} .$$

- M_ν is unchanged under

$$\nu_{\mathbf{e}} \leftrightarrow \nu_{\mathbf{e}}, \quad \nu_\mu \leftrightarrow \nu_\tau, \quad \nu_\tau \leftrightarrow \nu_\mu .$$

- M_ν can be diagonalized by

$$\mathbf{U_{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/3} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/3} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix} .$$

  - U_{TBM} **was 1st proposed by Harrison, Perkins & Scott** arXiv:hep-ph/0202074, PLB530 (2002)

- $U_{TBM} \Rightarrow$ $\sin\theta_{12} = \frac{1}{\sqrt{3}} \Rightarrow$ **'trimaximal mixing',**

  $\sin\theta_{23} = \frac{1}{\sqrt{2}} \Rightarrow$ **'bimaximal mixing' &** $\theta_{13} = 0^\circ$**.**

$\mu - \tau$ reflection symmetry

Originally proposed by Harrison & Scott, PLB547 (2002)

- M_ν is unchanged under:

$$\nu_e \leftrightarrow \nu_e^c, \quad \nu_\mu \leftrightarrow \nu_\tau^c \quad \nu_\tau \leftrightarrow \nu_\mu^c \ .$$

where,

$$M_\nu = \begin{pmatrix} D & A & A^* \\ A & B & C \\ A^* & C & B^* \end{pmatrix} \ \& \ M_\nu M_\nu^\dagger = \begin{pmatrix} z & w & w^* \\ w^* & x & y \\ w & y^* & x \end{pmatrix} .$$

where $\mathrm{C, D, x, z} \in \mathbb{R}$ & $\mathrm{A, B, w, y} \in \mathbb{C}$

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where $\mathrm{C, D, x, z} \in \mathbb{R}$ & $\mathrm{A, B, w, y} \in \mathbb{C}$

- M_ν respects, $X^T M_\nu X = M_\nu^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

- M_ν can be diagonalized by

$$U = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1^* & v_2^* & v_3^* \end{pmatrix} \Rightarrow |U_{\mu i}| = |U_{\tau i}|, i = 1, 2, 3$$

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- 2-important predictions: $\theta_{23} = \pi/4, s_{13} \cos\delta = 0 \Rightarrow \delta = \pm\pi/2$ **for** $\theta_{13} \neq 0$

Recent review: Xing and Zhao, RPP79 (2016)

For consistent model: Mohapatra and Nishi, JHEP08 (2015)

$\mu - \tau$ reflection symmetry and minimal seesaw:

- **Minimal seesaw: SM + 2 right-handed neutrinos**
  $\Rightarrow m_{light} = 0$ **(still allowed by latest data).**
- **Helps to address both the ν-mass & mixing patterns.**
- **We assume,**

$$\nu_L \to \mathbf{S}\nu_L^c, \qquad \mathbf{N_R} \to \mathbf{S}'\mathbf{N_R^c}$$

where $\nu_L^c = C\overline{\nu_L}^T$ and $N_R^c = C\overline{N_R}^T$ and

$$\mathbf{S} = \begin{pmatrix} 1 & 0 \\ 0 & S' \end{pmatrix}, \quad \mathbf{S}' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

- M_D, M_R obey,

$$M_D = SM_D^* S', \qquad M_R = S' M_R^* S'.$$

- One gets,

$$M_D = \begin{pmatrix} |b|e^{i\phi_b} & |b|e^{-i\phi_b} \\ |c|e^{i\phi_c} & |d|e^{i\phi_d} \\ |d|e^{-i\phi_d} & |c|e^{-i\phi_c} \end{pmatrix}, M_R = \begin{pmatrix} |m_{22}|e^{i\phi_m} & m_{23} \\ m_{23} & |m_{22}|e^{-i\phi_m} \end{pmatrix},$$

NN, Xing & Zhang, 1801.09931/hep-ph, EPJC 78 (2018)

Cont...

- In type-I seesaw,

$$-\mathbf{M}_\nu = \mathbf{M_D M_R^{-1} M_D^T} = \begin{pmatrix} A & B & B^* \\ B & C & D \\ B^* & D & C^* \end{pmatrix} .$$

- M_ν can be diagonalized by,

$$V = P_l \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu ,$$

where $P_l = diag(e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau})$ and $P_\nu = diag(1, e^{i\rho}, e^{i\sigma})$.

- **6-predictions:**

$$\phi_e = \mathbf{90}^\circ, \quad \phi_\mu \equiv -\phi_\tau = \phi, \quad \theta_{23} = \mathbf{45}^\circ, \quad \delta = \pm\mathbf{90}^\circ, \quad \rho,\ \sigma = \mathbf{0} \ \text{ or } \ \mathbf{90}^\circ.$$

- Also,

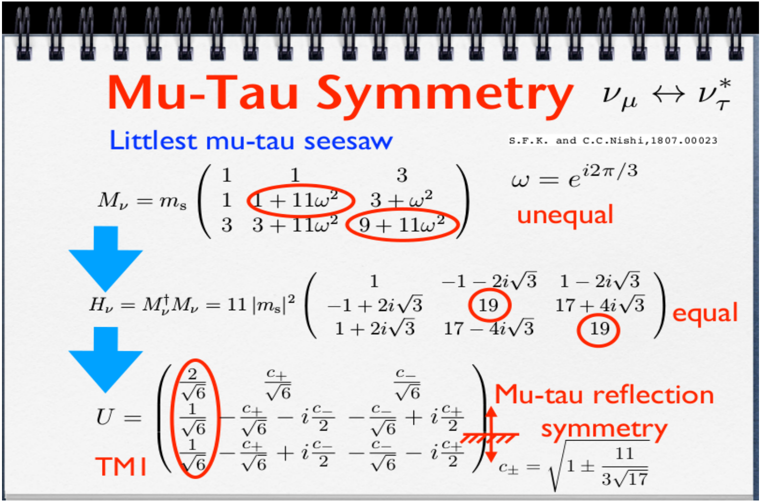
$$\tan\theta_{13} = \mp \frac{1}{\sqrt{2}} \frac{\mathrm{Im}\,(C')}{\mathrm{Im}\,(B')} \quad (C' = Ce^{-2i\phi}, B' = Be^{-i\phi}),$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}\cos 2\theta_{13}\mathrm{Im}\,(B')}{c_{13}\left[(\mathrm{Re}(C') - D)\cos 2\theta_{13} - (\mathrm{Re}(C') + D)s_{13}^2 + Ac_{13}^2\right]} \ ; \ \text{ for NH}$$

- Excellent agreement with the latest data.

A different thoughts:

$\mu - \tau$ **reflection symmetry:** M_ν violates but H_ν respects.



credits: King, IHEP, Beijing'19

Cont...

Reminder: Best-fit preferences $\theta_{23}, \delta \Rightarrow$

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LO or HO ?

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CPV

Looking for more realistic model

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CPV

Looking for more realistic model

- **Break $\mu - \tau$ reflection symmetry (RGE & Explicit).**
- **Generalized CP symmetry.**
- **Bi-large ansatze.**
- **Tetra-maximal mixing.**

Recent topics: Modular symmetries, littlest seesaw models, orbifold theory of flavor, tri-direct CP approaches etc...

Renormalization group running (RGE) effect:

- **RGE effect works as a bridge between the high-energy predictions and the low-energy measurements.**
- At the one-loop level, the energy dependence of M_ν is given by

$$16\pi^2 \frac{\mathrm{d}M_\nu}{\mathrm{d}t} = C\left(Y_l^\dagger Y_l\right)^T M_\nu + C M_\nu \left(Y_l^\dagger Y_l\right) + \alpha M_\nu \; ,$$

where, $t = ln(\mu/\mu_0)$ and μ is the renormalization scale

[**Chankowski, Pluciennik, PLB316(1993)**]

- With

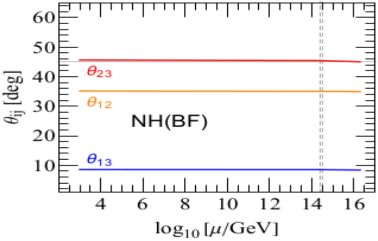
$$M_\nu(\Lambda_{\rm EW}) = I_\alpha I_\tau^\dagger M_\nu(\Lambda_{\mu\tau}) I_\tau^* \; ,$$

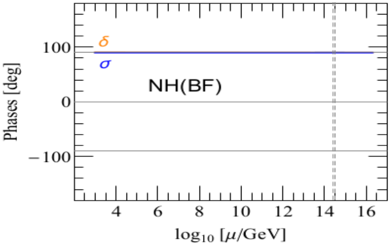
where one defines $I_\tau \simeq \mathrm{diag}\{1, 1, 1 - \Delta_\tau\}$ along with

$$\mathbf{I}_\alpha = \exp\left(\frac{\mathbf{1}}{\mathbf{16\pi^2}} \int_{\ln \mathbf{\Lambda}_{\mu\tau}}^{\ln \mathbf{\Lambda}_{\rm EW}} \alpha \; \mathrm{d}\mathbf{t}\right) , \qquad \mathbf{\Delta}_\tau = \frac{\mathbf{C}}{\mathbf{16\pi^2}} \int_{\ln \mathbf{\Lambda}_{\rm EW}}^{\ln \mathbf{\Lambda}_{\mu\tau}} \mathbf{y}_\tau^2 \; \mathrm{d}\mathbf{t} \; .$$

Cont...

- **Impact of RG running:**





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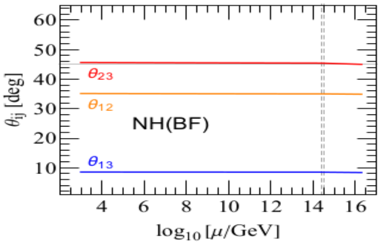
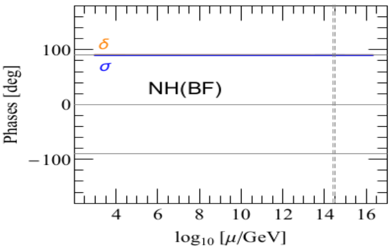
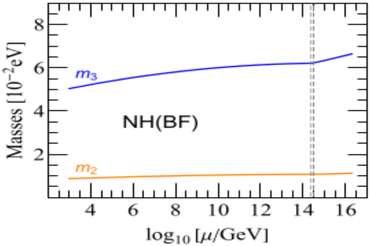






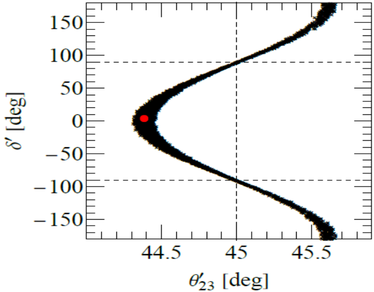
NN, Xing & Zhang, EPJC78 (2018).

Explicit Breaking

- Modify (12)-position of M_D,

$$\mathbf{S1}: \quad M'_D = \begin{pmatrix} b & b^*(1+\epsilon) \\ c & d \\ d^* & c^* \end{pmatrix},$$

"ϵ breaks $\mu - \tau$ Reflection Symmetry"

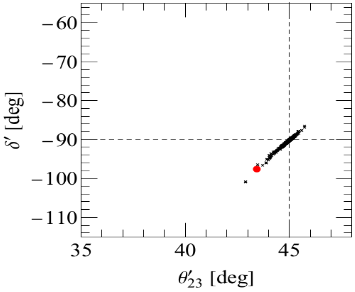


Cont...

- Modify (22)-position of M_R,

$$\textbf{S4}: \quad M'_R = \begin{pmatrix} m_{22} & m_{23} \\ m_{23} & m^*_{22}(1+\epsilon) \end{pmatrix} ,$$

"ϵ breaks $\mu - \tau$ Reflection Symmetry"



Summarizing all scenarios :

Breaking Scenarios	θ'_{23} [deg]	δ'_{CP} [deg]	$\Delta\theta'_{12}$ [deg]	$\Delta\theta'_{13}$ [deg]	$\sum m_\nu$ [eV]	m_{ee} [meV]
S1	$44.3 \to 45.7$	$-180 \to 180$	$-15 \to 10$	$-1 \to 9$	$0.0575 \to 0.061$	$1 \to 4.2$
S2	$35 \to 46$	$-100 \to -88$	$-18 \to 1$	$-0.1 \to 1.3$	$0.057 \to 0.061$	$3 \to 4.5$
	$40 \to 45$	$-90 \to -70$	$0 \to 9$	$0 \to 1.2$	–	–
S3	$37.5 \to 47$	$-98 \to -88$	$2 \to 7$	$-1.4 \to 0.2$	$0.057 \to 0.0615$	$3 \to 4.5$
	$46 \to 47$	$-94 \to -56$	$-20 \to 3$	$-1.7 \to 0.3$	–	–
S4	$43 \to 46$	$-100 \to -88$	$-0.2 \to 0.7$	$-3 \to 1$	$0.0575 \to 0.061$	$3.1 \to 4.4$
S5	$39 \to 46.5$	$-120 \to -84$	$-1 \to 2.6$	$-8 \to 8$	$0.057 \to 0.061$	$3 \to 4.5$

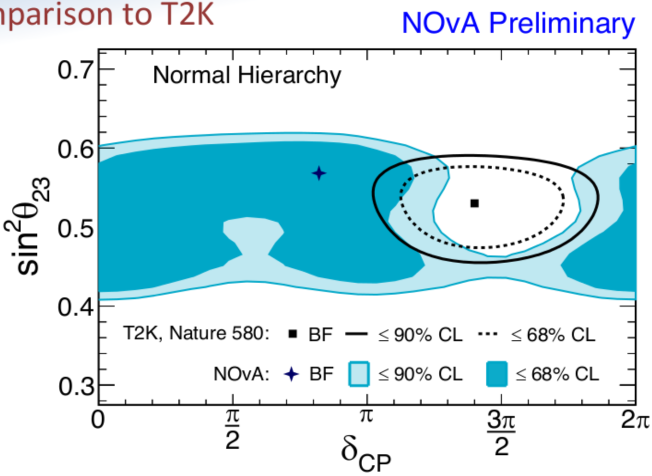
Scenarios $\Rightarrow$ $M_D(12) \to$ **S1**, $M_D(22) \to$ **S2**, $M_D(32) \to$ **S3**, $M_R(22) \to$ **S4**, $M_R(12) \to$ **S5**

NN, Xing & Zhang, EPJC78 (2018)

T2K & NOνA Tension:

* T2K : Tokai to Kamioka, 295 km, 0.76 GeV, 22.5 kT WC detector: SuperK
* NOνA : FNAL to Ash River, 810 km, 1.7 GeV, 14 kT TASD detector

Comparison to T2K



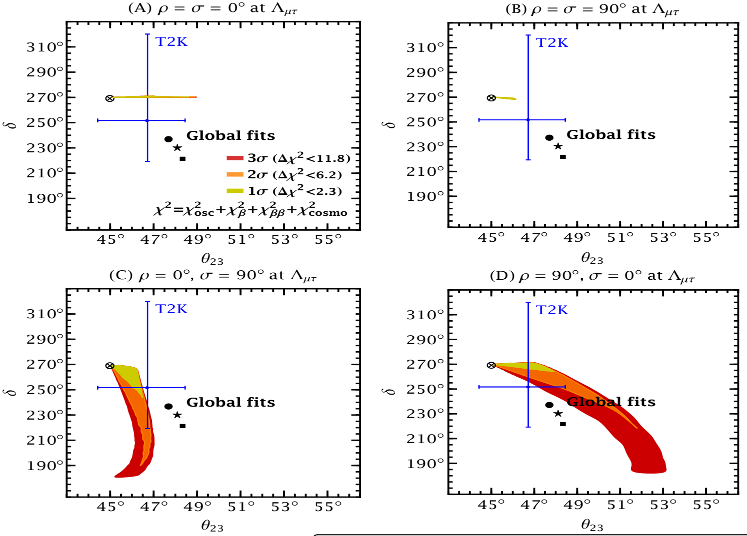
- Clear tension with T2K's preferred region.

Credits: Neutrino 2020

T2K 2020 results:

$\sin^2\theta_{23} = 0.53^{+0.03}_{-0.04}$, $\delta = -1.89^{+0.70}_{-0.58}$, for NO

T2K Collaboration Nature'2020



Huang, NN, EPJC80 (2020) 10, 914 arXiv:2004.12391

Implications:

- **Interesting to look for the consequences of "$\mu - \tau$ Reflection Symmetry" in long baseline neutrino oscillation experiments.**

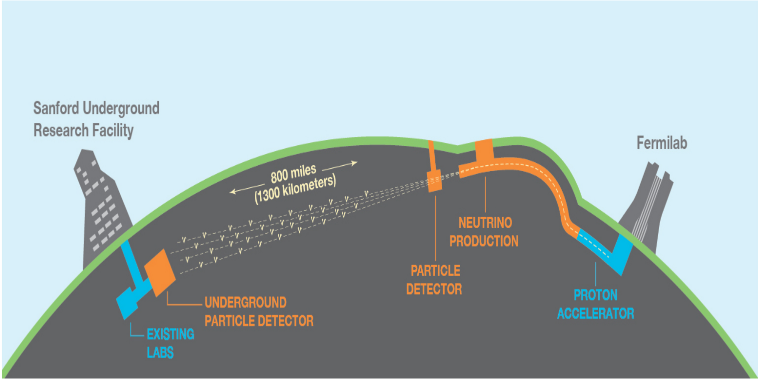
Implications:

- **Interesting to look for the consequences of "$\mu - \tau$ Reflection Symmetry" in long baseline neutrino oscillation experiments.**

- **We consider DUNE (Deep Underground Neutrino Experiment), a proposed long baseline experiment at Fermilab, USA**

- **DUNE will improve the precision of θ_{23} and play a key role to probe δ** [Acciarri et al.(DUNE), arXiv:1512.06148].

DUNE



- **DUNE : Neutrinos travel from Fermilab to Sanford Underground Research Facility (SURF), 1300 km, 2.3 GeV, 1.07 MW, 4$\times$10 kt-LArTPC detector**

[Alio et. al., (DUNE collab.), arXiv: 1601.09550].

- **Their first 2-modules are expected to be completed in 2024, with the beam operational in 2026**

arXiv: 0407333, 0701187

Framework:

- First scenario:

$$M_D = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & be^{-i\phi_b} \end{pmatrix}, M_R = diag(M_1, M_1)\ .$$

- Within type-I seesaw:

$$-M_\nu = M_D M_R^{-1} M_D^T,$$
$$= \frac{1}{M_1}\begin{pmatrix} 2a^2\cos 2\phi_a & abe^{i(\phi_a+\phi_b)} + ace^{-i(\phi_a-\phi_c)} & abe^{-i(\phi_a+\phi_b)} + ace^{i(\phi_a-\phi_c)} \\ - & b^2e^{2i\phi_b} + c^2e^{2i\phi_c} & 2bc\cos(\phi_b-\phi_c) \\ - & - & b^2e^{-2i\phi_b} + c^2e^{-2i\phi_c} \end{pmatrix}.$$

- $M_{ee} = M_{ee}^*\ , \quad M_{\mu\tau} = M_{\mu\tau}^*\ , \quad M_{e\mu} = M_{e\tau}^*\ , \quad M_{\mu\mu} = M_{\tau\tau}^*$

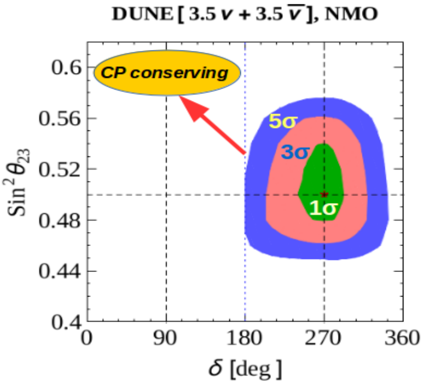
- Predicts non-zero θ_{13} with,

$$\theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ.$$

NN, PRD98 (2018), arXiv: 1805.05823

Cont...

- **DUNE's Potential:**



- CP-conservation hypothesis can be ruled out around 5σ

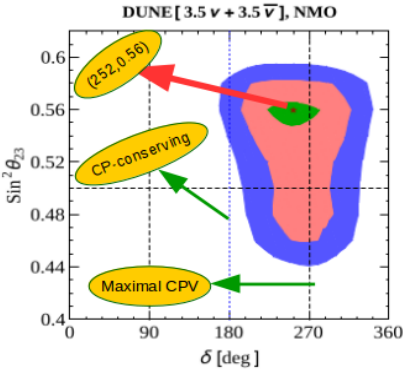
NN, arXiv: 1805.05823, PRD98 (2018)

Cont...

Break M_D:

$$\widehat{M}_D = \begin{pmatrix} ae^{i\phi_a} & ae^{-i\phi_a} \\ be^{i\phi_b} & ce^{i\phi_c} \\ ce^{-i\phi_c} & b(1+\epsilon)e^{-i\phi_b} \end{pmatrix} .$$

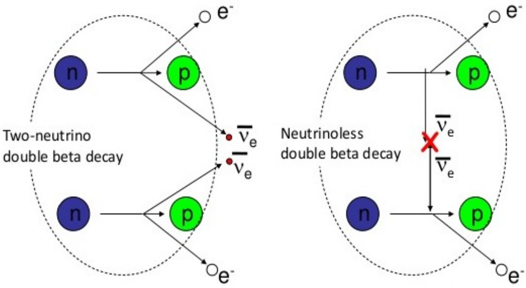
$$\widehat{M}_\nu \simeq M_\nu - \epsilon \frac{be^{-i\phi_b}}{M_1} \begin{pmatrix} 0 & 0 & ae^{-i\phi_a} \\ 0 & 0 & ce^{i\phi_c} \\ be^{-i\phi_a} & ce^{i\phi_c} & 2be^{-2i\phi_b} \end{pmatrix} + \mathcal{O}(\epsilon^2) .$$



- **Best fit: $(194^{+24}_{-22}, 49.26 \pm 0.79)$**
- **Predicted δ, θ_{23} are well within 1σ**
- **Maximal θ_{23} is ruled at $> 1\sigma$**

Impact on $0\nu\beta\beta$ decay:

- Whether $\nu = \overline{\nu}$ is yet unknown ?
- $0\nu\beta\beta$-decay is the only feasible process to address this issue.



- $0\nu\beta\beta \Rightarrow \Delta L = 2$; violate lepton number by 2 units.
- The half-life:

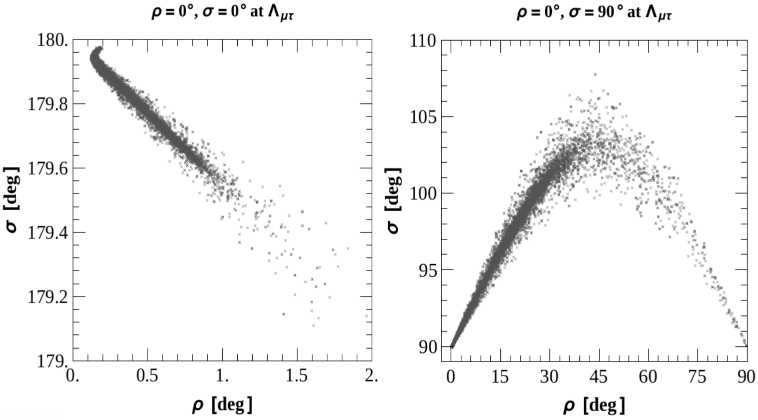
$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}|M_{0\nu}(A,Z)|^2|\langle m\rangle_{ee}|^2 \ ,$$

- The effective Majorana neutrino mass

$$| < m_{ee} > | = |m_1 c^2_{12} c^2_{13} e^{2i\rho} + m_2 s^2_{12} c^2_{13} e^{2i\sigma} + m_3 s^2_{13} e^{-2i\delta}| \ .$$

Cont....

- $\mu - \tau$ reflection symmetry $\Rightarrow$ $\boxed{\theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ, \quad \rho,\ \sigma = 0^\circ \ \text{ or } \ 90^\circ}$

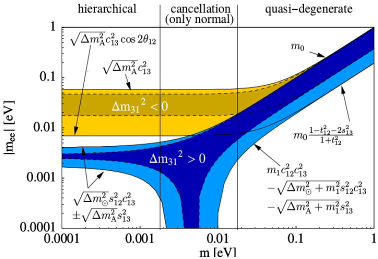


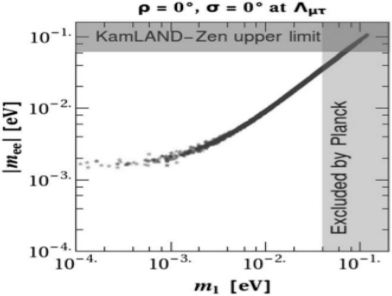
NN, PRD99 (2019)

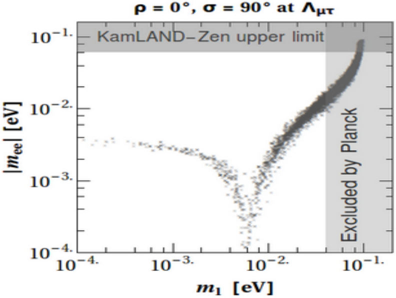
Cont...

- Under global-fit:

Rodejohann, arXiv:1206.2560 [hep-ph]







NN, PRD99 (2019)

Generalized CP (gCP) symmetry

- Reminder:

$$\psi \to X\psi \ ; \ \mu - \tau \ \textbf{permutation symmetry} \ ,$$

$$\psi \to X\psi^c \ ; \mu - \tau \ \textbf{reflection symmetry} \ ,$$

where

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

- In gCP, one assumes

$$\psi \xrightarrow{CP} iX_\psi \gamma^0 \psi^c$$

X_ψ are the generalized CP transformation matrices

[Ferugliao, Hagedorna, Ziegler, arXiv:1211.5560 Chen, Li, Ding, arXiv:1412.8352, Chen, Ding, Gonzalez-Canales, Valle, arXiv:1512.01551]

with

$$X_\psi^T m_\psi X_\psi = m_\psi^* \ , \quad (\text{Majorana fields})$$

$$X_\psi^\dagger M_\psi^2 X_\psi = M_\psi^{2*} \ , \quad (\text{Dirac fields}) \ .$$

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$$\boxed{X_\psi \;?}$$

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$\boxed{X_\psi \ ?}$ $\boxed{U_{PMNS} \ ?}$

Recent studies :Chen, Ding, Gonzalez-Canales, Valle, arXiv:1604.03510, Chen, Chulia, Ding, Srivastava, Valle, arXiv:1802.04275, Joshipura, Patel , arXiv : 1805.02002, Lu, Ding, arXiv: 1806.02301, Barreiros, Felipe, Joaquim, arXiv:1810.05454

Cont...

- **Steps to find U_{PMNS}:** **[Chen, Chulia, Ding, Srivastava, Valle, arXiv:1802.04275]**

$$U_\psi^T m_\psi U_\psi = diag(m_1, m_2, m_3)\ , \quad (\text{Majorana fields})$$
$$U_\psi^\dagger M_\psi^2 U_\psi = diag(m_1^2, m_2^2, m_3^2)\ , \quad (\text{Dirac fields})\ .$$

- U_ψ satisfies the following constraint

$$U_\psi^\dagger X_\psi U_\psi^* \equiv P = \begin{cases} \text{diag}(\pm 1, \pm 1, \pm 1), & \text{for Majorana fields,} \\ \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}), & \text{for Dirac fields}\,, \end{cases}$$

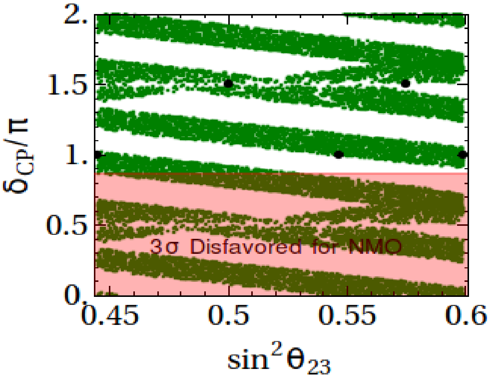
- **Unitary-symmetric matrix X_ψ can be decomposed as $X_\psi = \Sigma \cdot \Sigma^T$.**
- Subsequently, $P^{-\frac{1}{2}}\, U_\psi^\dagger\, \Sigma \;\equiv\; O_3\,, \Rightarrow U_\psi \;=\; \Sigma O_3^T\, P^{-\frac{1}{2}}\,,$
  where O_3,

$$O_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_1} & s_{\theta_1} \\ 0 & -s_{\theta_1} & c_{\theta_1} \end{pmatrix} \begin{pmatrix} c_{\theta_2} & 0 & s_{\theta_2} \\ 0 & 1 & 0 \\ -s_{\theta_2} & 0 & c_{\theta_2} \end{pmatrix} \begin{pmatrix} c_{\theta_3} & s_{\theta_3} & 0 \\ -s_{\theta_3} & c_{\theta_3} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- Maximum possible zeros in X_ψ: 4 ; **[Chen, Ding, Gonzalez-Canales, Valle, arXiv:1604.03510]**
- X_ψ : 11 possibilities, 8 are compatible with latest data.

Cont...

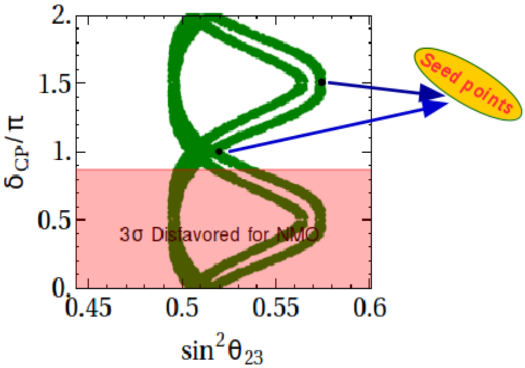
- No-Zeros in X : $\frac{1}{\sqrt{3}}\begin{pmatrix} e^{i\alpha} & e^{i(\frac{\alpha+\beta}{2}+\frac{2\pi}{3})} & e^{i(\frac{\alpha+\gamma}{2}+\frac{2\pi}{3})} \\ e^{i(\frac{\alpha+\beta}{2}+\frac{2\pi}{3})} & e^{i\beta} & e^{i(\frac{\beta+\gamma}{2}+\frac{2\pi}{3})} \\ e^{i(\frac{\alpha+\gamma}{2}+\frac{2\pi}{3})} & e^{i(\frac{\beta+\gamma}{2}+\frac{2\pi}{3})} & e^{i\gamma} \end{pmatrix}$
- This leads,



NN, Srivastava, Valle, arXiv:1811.07040, PRD99 (2019)

Cont...

- One-Zero in X : $\begin{pmatrix} e^{i\alpha} c_\Theta^2 & e^{i\gamma} c_\Theta s_\Theta & e^{i\beta} s_\Theta \\ e^{i\gamma} c_\Theta s_\Theta & e^{i(-\alpha+2\gamma)} s_\Theta^2 & -e^{i\alpha_1} c_\Theta \\ e^{i\beta} s_\Theta & -e^{i\alpha_1} c_\Theta & 0 \end{pmatrix}$
- This leads,



NN, Srivastava, Valle, arXiv:1811.07040, PRD99 (2019)

Cont...

- Two-Zeros in X :

$$\begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} c_\Theta & i e^{i(\beta+\gamma)/2} s_\Theta \\ 0 & i e^{i(\beta+\gamma)/2} s_\Theta & e^{i\gamma} c_\Theta \end{pmatrix}$$

- $\Rightarrow \sin^2\delta \sin^2 2\theta_{23} = \sin^2\Theta$, (Θ is the model parameter.)

  'Generalized $\mu - \tau$ reflection symmetry'

- $\Theta = \pm\pi/2 \Rightarrow$ 'exact $\mu - \tau$ reflection symmetry',

  i.e. $\boxed{\theta_{23} = 45^\circ, \delta = \pm 90^\circ}$

- $\Theta = 0 \Rightarrow$ **CP-conservation.**

- $\Theta \neq 0 \Rightarrow$ **deviations from the symmetry.**

Cont...

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- $\Theta = 0 \Rightarrow$ **CP-conservation.**

- $\Theta \neq 0 \Rightarrow$ **deviations from the symmetry.**

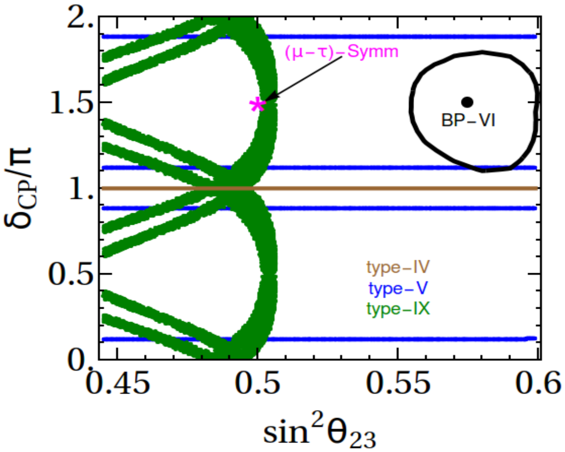
- Z_2 symmetric X :

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & e^{-i\alpha} s_\gamma \\ 0 & e^{i\alpha} s_\gamma & -c_\gamma \end{pmatrix} \Longrightarrow \textbf{TBM}$$

NPB'05 Grimus,Joshipura,Kaneko,Lavoura,Sawanaka,Tanimoto

Cont...

- DUNE's capability:



- **Precise measurement of θ_{23}, δ by DUNE can rule out various models.**

NN, Srivastava, Valle, arXiv:1811.07040, PRD99 (2019)

Bi-large ansatze

Motivation:

- Smallest leptonic-mixing angle $\simeq$ largest of the quark-mixing angle.
- Cabibbo angle (λ) may act as the universal seed for quark and lepton mixings.
- Bi-large patterns arise from the simplest GUTs model.

Boucenna, Morisi, Tortala, Valle: 1206.2555, Roy, Morisi, Singh, Valle: 1410.3658

- ν-mixing angles are related with λ,

$$\sin\theta_{12} = \sin\theta_{23} = \psi\lambda, \qquad \sin\theta_{13} = \lambda \quad \text{with } \lambda = 0.22453. \tag{1}$$

- ν-part of mixing matrix U_{BL} is given by

$$U_{BL} = \begin{bmatrix} c\sqrt{1-\lambda^2} & \psi\lambda\sqrt{1-\lambda^2} & \lambda \\ -c\psi\lambda(1+\lambda) & c^2 - \psi^2\lambda^3 & \psi\lambda\sqrt{1-\lambda^2} \\ -c^2\lambda + \psi^2\lambda^2 & -c\psi\lambda(1+\lambda) & c\sqrt{1-\lambda^2} \end{bmatrix}, \quad c \equiv \cos\sin^{-1}(\psi\lambda). \tag{2}$$

Ding, NN, Srivastava, Valle, arXiv:1904.05632, PLB796 (2019)

Cont...

- The $SO(10)$ GUT-motivated, CKM-type charged-lepton corrections,

$$U_{l_2} = \Phi^\dagger R_{12}^T(\,\theta_{12}^{CKM}\,)\Phi R_{23}^T(\,\theta_{23}^{CKM}\,) \simeq \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & -\lambda\, e^{i\phi} & A\lambda^3 e^{i\phi} \\ \lambda\, e^{-i\phi} & 1-\frac{1}{2}\lambda^2 & -A\lambda^2 \\ 0 & A\lambda^2 & 1 \end{bmatrix}$$

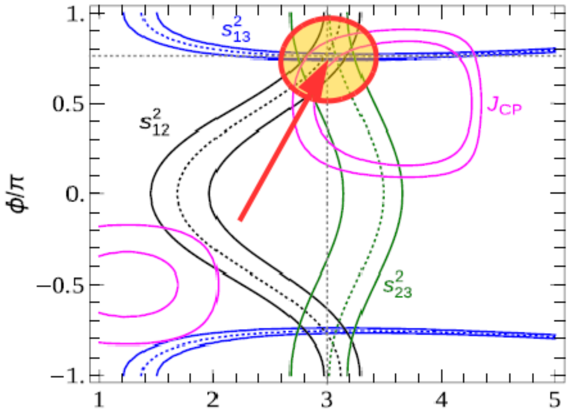
with $\sin\theta_{12}^{CKM} = \lambda$ and $\sin\theta_{23}^{CKM} = A\lambda^2$, where λ, A are the Wolfenstein parameters.

- The lepton mixing matrix is simply given by $U = U_{l_1}^\dagger U_{BL1} \Rightarrow$

$$\begin{aligned}
\sin^2\theta_{13} &\simeq \lambda^2 + 2\psi\lambda^3\cos\phi - \left(1-\psi^2\right)\lambda^4\,, \\
\sin^2\theta_{12} &\simeq \left(c^4 + 2c^2\psi\cos\phi + \psi^2\right)\lambda^2 + \left(c^4 - \psi^2\right)\lambda^4\,, \\
\sin^2\theta_{23} &\simeq \psi^2\lambda^2 + 2\,(Ac - \cos\phi)\,\psi\lambda^3 + \left(1 - \psi^2 - 2Ac\cos\phi + A^2c^2\right)\lambda^4\,, \\
J_{CP} &\simeq c^2\left[\psi + \left(\psi + Ac - \psi^3\right)\lambda\right]\lambda^3\sin\phi\,.
\end{aligned}$$

Cont...

- **Allowed parameter space for T4:**

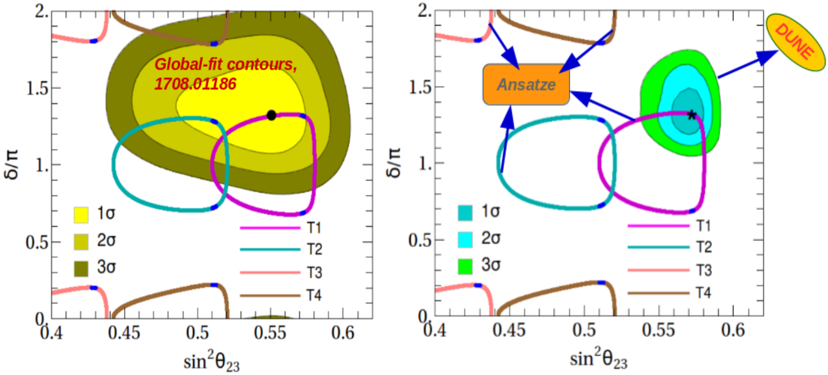


- **Predicts** $(\sin^2\theta_{23}, \delta) = (0.51, 1.78\pi)$

NN, Srivastava, Valle, arXiv:1811.07040, PRD99 (2019)

Cont...

- Left panel ⇒ Global-fit, right panel ⇒ DUNE analysis.



Ding, NN, Srivastava, Valle, arXiv:1904.05632, PLB796 (2019)

Tetra-maximal mixing

- It is expressed as

$$
\begin{aligned}
V_0 =& P_l \otimes O_{23}(\pi/4, \pi/2) \otimes O_{13}(\pi/4, 0) \otimes O_{12}(\pi/4, 0) \otimes O_{13}(\pi/4, \pi) \\
=& \frac{1}{2} \begin{pmatrix} 1 + \frac{1}{\sqrt{2}} & 1 & 1 - \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \left[1 + i(1 - \frac{1}{\sqrt{2}})\right] & 1 + i\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \left[1 - i(1 + \frac{1}{\sqrt{2}})\right] \\ -\frac{1}{\sqrt{2}} \left[1 - i(1 - \frac{1}{\sqrt{2}})\right] & 1 - i\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \left[1 + i(1 + \frac{1}{\sqrt{2}})\right] \end{pmatrix} .
\end{aligned}
$$

Proposed by Xing, PRD78 (2008)

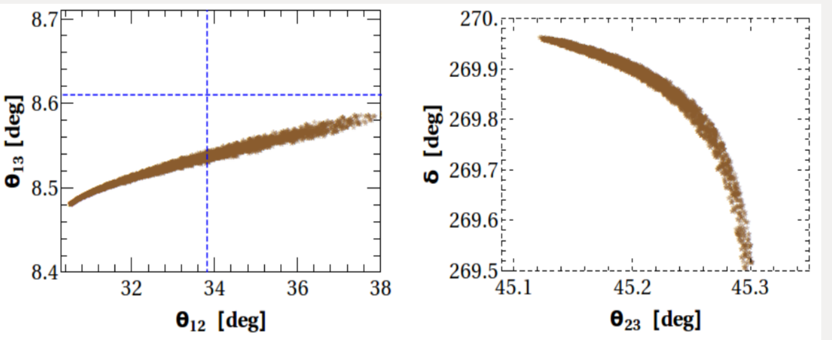
- This leads to

$$
\tan\theta_{12} = 2 - \sqrt{2} \; , \quad \tan\theta_{23} = 1 \; , \quad \sin\theta_{13} = \frac{1}{4}(2 - \sqrt{2}) \; ,
$$

where $\theta_{12} \approx 30.4^\circ$, $\theta_{13} \approx 8.4^\circ$, $\theta_{23} = 45^\circ$ and $-\delta = \rho = \sigma = 90^\circ$.

Breaking of tetra-maximal mixing

- Breaking due to RGE-running

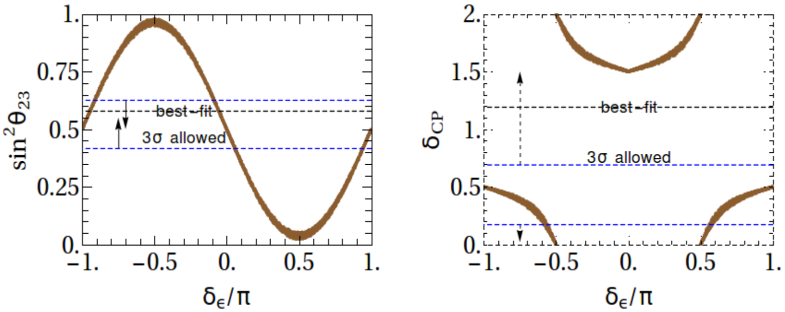


NN, NPB(956) 2020, arXiv: 1912.11517

Cont...

- Explicit breaking

$$
\begin{aligned}
V' &= P_l \otimes O_{23}\left(\frac{\pi}{4}, \frac{\pi}{2} + \delta_\epsilon\right) \otimes O_{13}\left(\frac{\pi}{4}, 0\right) \otimes O_{12}\left(\frac{\pi}{4} + \epsilon_{12}, 0\right) \otimes O_{13}\left(\frac{\pi}{4}, \pi\right) \\
&= V_0 + \frac{1}{4}\epsilon_{12}\begin{pmatrix} -\sqrt{2} & 2 & \sqrt{2} \\ -\sqrt{2} - i & -2 + i\sqrt{2} & \sqrt{2} + i \\ -\sqrt{2} + i & -2 - i\sqrt{2} & \sqrt{2} - i \end{pmatrix} + \frac{1}{4}\delta_\epsilon \begin{pmatrix} 0 & 0 & 0 \\ 1 - \sqrt{2} & \sqrt{2} & -1 - \sqrt{2} \\ i\sqrt{2} & i2 & i\sqrt{2} \end{pmatrix} \\
&+ \mathcal{O}(\epsilon_{13}^2, \delta_\epsilon^2, \epsilon_{13}\delta_\epsilon)
\end{aligned}
$$



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- We have made an attempt to address the theory behind neutrinos flavor mixing and their tiny masses.
- Our main focus was on $\mu - \tau$ reflection symmetry which leads to $\theta_{23} = \pi/4$, $\delta = \pm\pi/2$ and $\theta_{13} \neq 0$ along with $\rho, \sigma = 0, \pi/2$.
- Further, to explain realistic leptonic mixing patterns, we have discussed, (i) breaking of exact symmetry, (ii) generalized CP symmetries and (iii) bi-large ansatze.
- We have also presented the impact of these symmetries for DUNE as well as on $0\nu\beta\beta$-decay.
- Finally, present status of tetra-maximal mixing has been presented.

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thank you