Lecture2: Neutrino Masses

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Neutrino mass

- From neutrino oscillations:

$$\Delta m_{21}^2 = 7.50 \times 10^{-5}\mathrm{eV}^2 \ ,$$
$$|\Delta m_{31}^2| = 2.55 \times 10^{-3}\mathrm{eV}^2(\mathrm{NO}),\ 2.45 \times 10^{-3}\mathrm{eV}^2(\mathrm{IO}) \ .$$

de Salas, Forero, Gariazzo, Martnez-Mirav, Mena, Ternes, Trtola, Valle, JHEP02(2021)071

- From cosmology:

$$\sum m_\nu \leq 0.12\mathrm{eV}$$

Planck Collab., Aghanim, et. al., A&A 641, A6 (2020)

- From β-decay:

$$m_\beta = \sqrt{|U_{ei}^2| m_i^2} \leq 1.1\mathrm{eV} \quad (0.2 \text{ eV expected})$$

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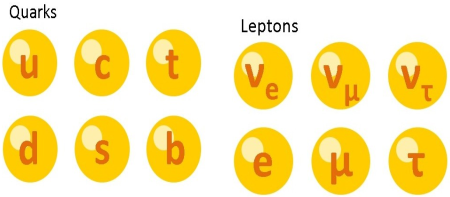
- **What is the dynamical origin associated with neutrinos mass generation ?**

The Standard Model:



- The SM fermions under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group, C denotes color, L implies left handed chirality, and Y is the hypercharge

Lepton doublet L_L**(1,2,-1)**	**Quark doublet** Q_L**(3,2,** $\frac{1}{3}$**)**	**Lepton singlet** ℓ_R**(1,1,-2)**	**Up Quark** U_R**(3,1,** $\frac{4}{3}$**)**	**Down Quark** D_R**(3,1,-** $\frac{2}{3}$**)**
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	e_R	u_R	d_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	μ_R	c_R	s_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	τ_R	t_R	b_R

- Here, $Q = T_3 + Y/2$

Helicity:

- Helicity is the projection of the spin along the direction of momentum:

$$h = \frac{\vec{S}.\vec{p}}{|\vec{p}|} = \frac{1}{2}\frac{\vec{\Sigma}.\vec{p}}{|\vec{p}|} ,$$

- Here,

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} ,$$

where $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ are 2×2 Pauli matrices

- The helicity operator commute with the Dirac Hamiltonian:

$$[H, \vec{\Sigma}.\vec{p}] = 0; \qquad H = \vec{\alpha}.\vec{p} + \beta m$$

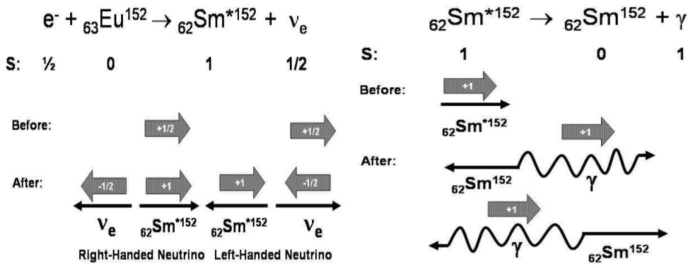
"Helicity is a good quantum number"

- The eigenvalues of h are $\pm$ 1. Also, h $= +1$ (-1) is called right (left)-handed, where spin and momentum vector parallel (antiparallel)

For Dirac, Majorana and Weyl fermions: P. B. Pal, arXiv: 1006.1718

Goldhaber-Experiment:

In 1957, Goldhaber, Grodzins and Sunyar experiment to measure the Neutrino-Helicity



- ^{152}Eu is a spin 0 state the sum of the $^{152}Sm^{*}$ and ν spins must be equal to the spin of the electron
- Hence, the polarization of the ν (left/right handed) is always the same as $^{152}Sm^{*}$
- $^{152}Sm^{*}$ decays to ^{152}Sm spin-0 state and γ
- Thus, γs emitted at the direction of $^{152}Sm^{*}$ have the same helicity as νs

Measured polarisation of photons $\Rightarrow H(\nu) = -1.0 \pm 0.3 \Rightarrow$ neutrinos are left-handed

Chirality or Handedness:

- The chirality/handedness operator in the Pauli-Dirac representation:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

- Here, γ^0, and γ^i's are defined as:

$$\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

- Properties γ^5: $\{\gamma^5, \gamma^\mu\} = 0, (\gamma^5)^2 = 1, (\gamma^5)^\dagger = \gamma^5$, and eigenvalues ± 1.
- Define: $\gamma^5\psi_R = +\psi_R, \;\; \gamma^5\psi_L = -\psi_L$, where $\psi_R(\psi_L)$ is the right (left)-handed field.
- Its possible to write $\psi = \psi_R + \psi_L$, where

$$\psi_R = \frac{1+\gamma^5}{2}\psi = P_R\psi, \qquad \psi_L = \frac{1-\gamma^5}{2}\psi = P_L\psi$$

  with P_R, P_L are the chirality projection matrices.
- Properties of P_R, P_L:

$$P_R + P_L = 1, \quad (P_R)^2 = P_R, \quad (P_L)^2 = P_L, \quad P_R P_L = P_L P_R = 0$$

Dirac Equation:

- The Dirac Lagrangian,

$$\mathcal{L} = \overline{\psi}(x)(i \overleftrightarrow{\not\partial} - m)\psi(x),$$

where $\overline{\psi} = \psi^\dagger \gamma^0$, $\not\partial = \gamma^\mu \partial_\mu$ and $\overleftrightarrow{\partial} = (\overrightarrow{\partial} - \overleftarrow{\partial})/2$

- The Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \overline{\psi})} \right) - \frac{\partial \mathcal{L}}{\partial \overline{\psi}} = 0$$

- The Dirac equation:

$$(i\not\partial - m)\psi(x) = 0 \ ,$$

where

$$\psi(x) = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}} \sum_s \left[a(p,s)u(p,s)e^{-ip.x} + b^\dagger(p,s)v(p,s)e^{ip.x} \right] \ ,$$

with $b^\dagger(a)$ is creation (annihilation) operator and $u, \ v$ are spionrs.

Cont...

- The Dirac Lagrangian:

$$\begin{aligned}\mathcal{L} &= \overline{\psi}(i\overleftrightarrow{\not\partial} - m)\psi \ , \\ &= (\overline{\psi_R} + \overline{\psi_L})(i\overleftrightarrow{\not\partial} - m)(\psi_R + \psi_L) \ , \\ &= \overline{\psi_R} i\overleftrightarrow{\not\partial}\,\overline{\psi_R} + \overline{\psi_L} i\overleftrightarrow{\not\partial}\,\psi_L - m(\overline{\psi_R}\psi_L + \overline{\psi_L}\psi_R)\end{aligned}$$

- ψ_R, and ψ_L have independent kinetic terms but coupled by the mass term
- Non-zero Dirac mass needs both ψ_L and ψ_R
- In massless limit, we have: $i\not\partial\,\overline{\psi_{R,L}} = 0$

$\boxed{* \text{ Also called Weyl equation}}$

- Using Dirac equation for $m = 0$ i.e., $i\not\partial\,\overline{\psi} = 0$, one can show:

$$\frac{\vec{\Sigma}.\vec{p}}{|\vec{p}|}\psi = \gamma^5\psi$$

$$\Rightarrow \frac{\vec{\Sigma}.\vec{p}}{|\vec{p}|}\psi_R = +\psi_R, \quad \frac{\vec{\Sigma}.\vec{p}}{|\vec{p}|}\psi_L = -\psi_L$$

- Chirality coincides with helicity in the massless limit

Fermion masses:

- In the SM, the mass of fermions arises as a result of the Higgs mechanism.
- The Higgs-lepton Yukawa Lagrangian: $\mathcal{L} \supset y_e \overline{\ell_{eL}} \Phi \ell_{eR} + h.c.$
  - $\mathcal{L}$ should be gauge singlet under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where C, L, and Y denote color, left-handed chirality and weak hypercharge
  - Here $\ell_{eL} \sim (1, 2, -1)$, $\Phi \sim (1, 2, 1)$, and $\ell_{eR} \sim (1, 1, -2)$
  - $SU(2)$ doublets: $\ell_{eL} = (\nu_{eL}, e_L)^T$ and $\Phi = (\phi^+, \phi^0)^T$
  - Note: Multiplication of two doublets $2 \otimes 2 \Rightarrow 1 \oplus 3$

$$
\begin{array}{lccc}
 & \overline{\ell_{eL}} & \Phi & \ell_{eR} \\
C \rightarrow & 1 & 1 & 1 \\
L \rightarrow & \multicolumn{2}{c}{\underbrace{2 \quad 2}_{1}} & 1 \\
Y \rightarrow & 1 & 1 & -2
\end{array}
$$

- Important: No such term for 'neutrino' in SM as there is no ν_R

Cont...

- In the unitary gauge, Φ can be written as

$$\Phi = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \upsilon + H(x) \end{bmatrix},$$

where, υ is the vacuum expectation value or *vev* that arises from the spontaneous breaking of the SM symmetry

- The Higgs-lepton Yukawa Lagrangian:

$$\mathcal{L}_Y = \left(\frac{\upsilon + H}{\sqrt{2}}\right) y_e \ \overline{\ell_{eL}} \ \ell_{eR} + H.c.$$

- The mass terms for electron:

$$\mathcal{L}_m = m_e \overline{\ell_{eL}} \ell_{eR},$$

where $\boxed{m_e = \frac{y_e \upsilon}{\sqrt{2}}}$

- For top quark: $m_t \approx \frac{\upsilon}{\sqrt{2}} \approx 170$ GeV $\Rightarrow y_t \sim \mathcal{O}(1)$
- For electron: $m_e \approx 0.5\ \mathrm{MeV}$, $\frac{\upsilon}{\sqrt{2}} \approx 170$ GeV $\Rightarrow y_e \sim \mathcal{O}(10^{-6})$

Neutrino mass:

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- Add right-handed neutrinos in the SM
- The Higgs-lepton Yukawa Lagrangian: $\mathcal{L} \supset y_\nu \overline{\ell_{eL}} \tilde{\Phi} \nu_{eR} + h.c.$
  where $\tilde{\Phi} = i\sigma^2 \Phi^*$

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- The Higgs-lepton Yukawa Lagrangian: $\mathcal{L} \supset y_\nu \overline{\ell_{eL}} \tilde{\Phi} \nu_{eR} + h.c.$
  where $\tilde{\Phi} = i\sigma^2 \Phi^*$
- Dirac neutrino mass:

$$y_\nu \, \bar{\ell}_L \, \tilde{\Phi} \, \nu_R$$

$$\begin{array}{lccc} C \to & 1 & 1 & 1 \\ L \to & 2 & 2 & 1 \\ Y \to & 1 & -1 & 0 \end{array}$$

$$y_\nu \overline{(\nu \;\; e)}_L \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \nu_R$$

$$\sim m_\nu \overline{\nu_L} \nu_R$$

$$\Rightarrow m_\nu = \frac{y_\nu v}{\sqrt{2}}$$

$$m_\nu = 0.1\,\text{eV}, \; \frac{v}{\sqrt{2}} \sim 174\,\text{GeV} \Rightarrow y_\nu \sim 10^{-13}$$

- There is no explanation for the extremely small values of y_ν
- This doesn't mean that the possibility of Dirac neutrino is ruled out

Majorana Neutrino:

So, what's the way out?

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- **E. Majorana proposal:** $\nu = \nu^C$?

"E. Majorana"

Charge-conjugation

- Under charge conjugation the spinor fields transform:

$$\nu = \eta_M \nu^c = \mathbf{C}\nu\mathbf{C}^{-1} = \mathcal{C}\,\overline{\nu}^T$$

- By definition particle-antiparticle creation and annihilation operator are related as,

$$b = \eta_c^\star d \qquad\qquad b^\dagger = \eta_c^\star d^\dagger$$

- Thus, Majorana field operator can be written as,

$$\nu(x) = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}} \sum_s (b(p,s)u(p,s)e^{-ip.x} + \eta_c^\star b^\dagger(p,s)v(p,s)e^{ip.x})$$

Cont...

Properties of Charge-conjugation operator

- In the Dirac representation:

$$\mathcal{C} = i\gamma^2\gamma^0 = -i\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}$$

- Also,

$$\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu\ , \quad \mathcal{C}(\gamma^5)^T\mathcal{C}^{-1} = \gamma^5\ ,$$
$$\mathcal{C}^\dagger = \mathcal{C}^{-1}\ , \quad \mathcal{C}^T = -\mathcal{C}$$

- Now, we write:

$$\begin{aligned} \nu &= \nu_L + \nu_R = \nu_L + \nu_L^C\ , \\ &= \nu_L + \mathcal{C}\ \overline{\nu_L}^T\ , \\ &= \nu^C \end{aligned}$$

To check $\mathcal{C}\ \overline{\nu_L}^T$ is right-handed, $P_L(\mathcal{C}\ \overline{\nu_L}^T) = 0$

- **Now, one has $\nu_L^C = \mathcal{C}\ \overline{\nu_L}^T$, and $\overline{\nu_L^C} = -\nu_L^T\mathcal{C}^\dagger$**

Cont...

- Dirac mass term:

$$\mathcal{L}^D = -m\,\overline{\nu}\nu\ , \qquad \nu = \nu_R + \nu_L$$
$$= -m\,\overline{\nu}_R\nu_L + h.c.$$

- Notice, only $\overline{\nu}_R\nu_L$, and $\overline{\nu}_L\nu_R$ survives, whereas $\overline{\nu}_L\nu_L = 0 = \overline{\nu}_R\nu_R$
- As the ν_L^C has the correct properties to replace ν_R, the Majorana mass term can be written as

$$\mathcal{L}^M = -\frac{1}{2}m\,\overline{\nu_L^C}\nu_L + h.c.$$
$$= \frac{1}{2}m\,\nu_L^T\mathcal{C}^\dagger\nu_L + h.c.$$

- The Majorana mass term in matrix form:

$$\mathcal{L}^M_{mass} = -\frac{1}{2}(\overline{\nu^C}_L M\nu_L) + h.c.$$
$$= \frac{1}{2}\sum_{\alpha,\beta=e,\mu,\tau}(\nu^T_{\alpha L}C^\dagger M_{\alpha\beta}\nu_{\beta L}) + h.c.$$

- Using anti-commutation property of neutrino field one can show:

$$M_{\beta\alpha} = M_{\alpha\beta}\ ,$$

i.e., Majorana mass matrix is symmetric

Cont...

- The Majorana Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\left[\overline{\nu_L} i \overleftrightarrow{\not\partial}\, \overline{\nu_L} + \overline{\nu_L^c} i \overleftrightarrow{\not\partial}\, \nu_L^c - m(\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c)\right]$$

- Lepton number (L):
  under global U(1) gauge transformation $\nu \to e^{i\varphi}\nu$,

$$\mathcal{L}^D = -m\,\bar{\nu}\nu, \quad \Delta L = 0\;, \qquad \text{conserved lepton \#}$$

$$\mathcal{L}^M = \frac{1}{2} m\,\nu^T \mathcal{C}^\dagger \nu, \quad \Delta L = \pm 2\;, \qquad \text{violated lepton \#}$$

- Also, hypercharge $Y = -1$ for ν_L. Thus $\nu^T \mathcal{C}^\dagger \nu \Rightarrow Y = -2$
- Since the SM does not contain any particle with $Y = -2$, Majorana neutrino mass term is not possible in the SM
- In the minimally extended SM i.e. SM + 3 ν_R, it is possible to have Majorana neutrino mass term

$$\mathcal{L}^M = \frac{1}{2} m\,\nu_R^T \mathcal{C}^\dagger \nu_R; \quad \Delta L = \pm 2, Y = 0$$

Seesaw mechanism:

- The Dirac-Majorana Lagrangian has the form,

$$\mathcal{L}^{D+M}_{mass} = \mathcal{L}^{D}_{mass} + \mathcal{L}^{M}_{mass,R}$$

- It can be written as,

$$\begin{aligned}
-\mathcal{L}^{D+M}_{mass} &= \overline{\nu}_R m_D \nu_L + \frac{1}{2}\overline{\nu}_R M_R \nu^c_R + h.c. \\
&= \frac{1}{2}\overline{\nu^c_L} m_D^T \nu^c_R + \frac{1}{2}\overline{\nu}_R m_D \nu_L + \frac{1}{2}\overline{\nu}_R M_R \nu^c_R + h.c. \\
&= \frac{1}{2}\overline{\nu}^c M \nu + h.c.
\end{aligned}$$

where,

$$M = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}, \quad \nu = \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} \text{ and } \overline{\nu^c} = \begin{pmatrix} \overline{\nu^c_L} & \overline{\nu_R} \end{pmatrix}$$

Cont...

- M can be diagonalized:

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

where, $U = O\rho$ with

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}$$

- One can write,

$$O^T M O = \begin{pmatrix} m'_1 & 0 \\ 0 & m'_2 \end{pmatrix}$$

$$\Rightarrow m'_{2,1} = \frac{M_R}{2} \pm \frac{\sqrt{M_R^2 + 4m_D^2}}{2}$$

- If $m_D << M_R$,

$$m'_{2,1} = \frac{M_R}{2} \pm \frac{M_R}{2}\sqrt{1 + \frac{4m_D^2}{M_R^2}}$$

Cont...

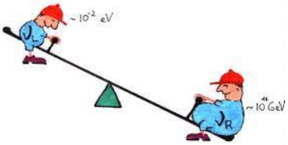
- Therefore,

$$m_2' \simeq M_R \quad \text{and} \quad m_1' \simeq \frac{m_D^2}{M_R} \quad (using \;\; correct \;\; phase)$$

- ν_2 is very heavy for large value of M_R
- ν_1 is very light for the small value of the ratio $\frac{m_D^2}{M_R}$

This is the famous 'See-saw mechanism'

[Minkowski77, Yanagida79, GelMann/Slansky/Ramond79, Mohapatra/Senjanovic80, Schecter/Valle80]



- Let $m_D \simeq m_t \simeq 170\text{GeV}, M_R \simeq 10^{15}$ GeV $\Rightarrow$ $m_1 = \frac{m_D^2}{M_R} \simeq 10^{-2} eV$
- The mixing angle

$$\tan 2\theta = \frac{2m_D}{M_R} << 1$$

$\Rightarrow$ ν_1 is composed of active ν_L and ν_2 is composed of sterile ν_R

The effective theory:

- The SM can be treated as an effective field theory, that is the low energy limit of a more fundamental theory
- The dominant effective operator is the Weinberg operator $\mathcal{L}_5 \sim l_L l_L \phi\phi$ (dim-5) that can generate a Majorana ν mass

Weinberg, PRL43(1979)

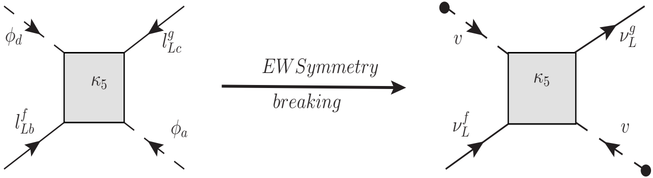
$d \rightarrow \quad 3/2 \quad 3/2 \quad 1 \quad 1$

$\mathcal{L}_5 \supset \frac{1}{\Lambda} l_L l_L \phi \phi$

$Y \rightarrow \quad -1 \quad -1 \quad 1 \quad 1$

After SSB, $\langle \phi^0 \rangle = v/\sqrt{2}$

$\mathcal{L}_5 \approx m_\nu \nu_L \nu_L$



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- With ℓ_L and ϕ, 3-ways to construct 'lepton number violating' terms:

$$\left(\overline{\ell_L^C}\varepsilon\phi\right)\left(\phi^T\varepsilon\ell_L\right),$$
$$\left(\overline{\ell_L^C}\sigma^i\varepsilon\phi\right)\left(\phi^T\sigma^i\varepsilon\ell_L\right),$$
$$\left(\overline{\ell_L^C}\sigma^i\varepsilon\ell_L\right)\left(\phi^T\sigma^i\varepsilon\phi\right)$$

where σ^i are Pauli matrices and $\varepsilon = i\sigma^2$.

- One can show,

$$\left(\overline{\ell_L^C}\sigma^i\varepsilon\phi\right)\left(\phi^T\sigma^i\varepsilon\ell_L\right) = -\left(\overline{\ell_L^C}\varepsilon\phi\right)\left(\phi^T\varepsilon\ell_L\right)$$
$$\left(\overline{\ell_L^C}\sigma^i\varepsilon\ell_L\right)\left(\phi^T\sigma^i\varepsilon\phi\right) = 2\left(\overline{\ell_L^C}\varepsilon\phi\right)\left(\phi^T\varepsilon\ell_L\right)$$

here following relation has been used.

$$(\sigma^i)_{ab}(\sigma^i)_{cd} = 2\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd}\ ,$$

and a, b, c, d are $SU(2)$ indices

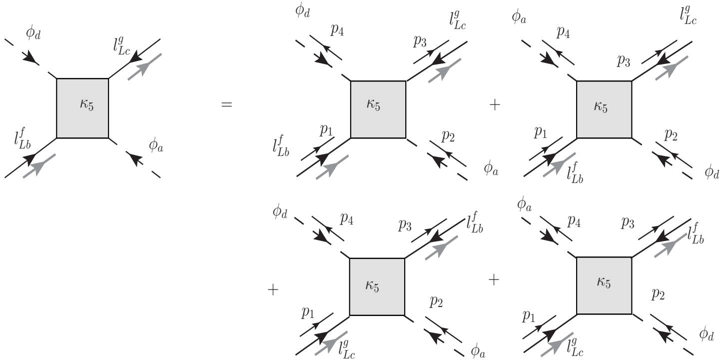
Cont...

- $\mathcal{L}_5$ with proper $SU(2)$ indices:

$$\mathcal{L}_5 = \frac{1}{4}\kappa_{gf}\left(\overline{l^C_{Lc}}^g \varepsilon_{cd}\phi_d\right)\left(l^f_{Lb}\varepsilon_{ba}\phi_a\right)$$

  where, $f, g \in \{1, 2, 3\}$

- Feynman diagram for effective operator:



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- Note: Feynman rules for fermion number violating interactions, Denner, Eck, Hahn, Kublbeck, Nucl. Phys. B387 (1992), 467-484

Dirac: $\psi\bar{\psi}$, Majorana: $\chi\chi$, $\bar{\chi}\bar{\chi}$

- The matrix element:

$$
\begin{aligned}
i\mathcal{M} &= \langle p_4^{\beta} p_3^{\alpha} | \frac{1}{4} \kappa_{gf} \overline{\ell^{C}_{Lc}}^{g} \varepsilon_{cd} \phi_d \ell^{f}_{Lb} \varepsilon_{ba} \phi_a | p_1^{\nu} p_2^{\mu} \rangle \\
&= i \frac{1}{2} \kappa_{gf} \left(\varepsilon_{cd} \varepsilon_{ba} + \varepsilon_{ca} \varepsilon_{bd} \right) P_L
\end{aligned}
$$

Seesaw Mechanism:

- $\ell_L \sim (1, 2, -1)$ and $\phi \sim (1, 2, 1)$ lead to four SM gauge singlet terms
- $\ell_L \ell_L \Rightarrow$ a singlet and a triplet components:
  $(1, 2, -1) \otimes (1, 2, -1) \sim (1, 1, -2) \oplus (1, 3, -2)$
- $\phi\phi \Rightarrow$ a singlet and a triplet components: $(1, 2, 1) \otimes (1, 2, 1) \sim (1, 1, 2) \oplus (1, 3, 2)$
- Thus, for $\ell_L \ell_L \phi\phi$:

  1. each $\ell - \phi$ forms a fermion singlet and which can arise from the tree level exchange of a right handed fermion singlet, called the 'type-I seesaw mechanism'
  2. each $\ell - \ell$ and $\phi - \phi$ forms scalar triplet and this can arises by the tree level exchange of heavy Higgs triplet giving rise to the 'type-II seesaw mechanism'
  3. each $\ell - \phi$ forms a fermion triplet and this arises by the tree level exchange of right-handed fermion triplet giving rise to 'type-III seesaw mechanism'
  4. each $\ell - \ell$ and $\phi - \phi$ pair forms a scalar singlet but this will lead to terms like $\overline{\nu_L^C} e_L$ and which does not generate neutrino mass

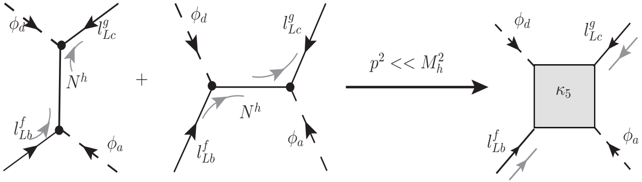
type-I seesaw:

- Extend the SM by right-handed singlet fermions N_R
- The modified SM Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} + \frac{1}{2}\overline{N}^f i\gamma^\mu \partial_\mu N^f - \frac{1}{2}\overline{N}^g M_{gf} N^f - \left((Y_\nu)_{gf}\overline{N}^g \tilde{\phi}^\dagger l_L^f + h.c. \right) \ ,$$

where, $N^f = N_R^f + (N_R^f)^C$

- The Feynman diagram:



- The amplitude:

$$\begin{aligned} A &= \left\{ -i(Y_\nu^T)_{gh}\varepsilon_{cd}P_L \right\} \frac{i(\not{p} + M_h)}{p^2 - M_h^2} \left\{ -i(Y_\nu)_{hf}(\varepsilon^T)_{ab}P_L \right\} \\ &+ \left\{ -i(Y_\nu^T)_{gh}\varepsilon_{ca}P_L \right\} \frac{i(\not{p} + M_h)}{p^2 - M_h^2} \left\{ -i(Y_\nu)_{hf}(\varepsilon^T)_{db}P_L \right\} \\ &= 2i(Y_\nu^T M^{-1} Y_\nu)_{gf} \frac{1}{2} \left\{ \varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd} \right\} P_L \end{aligned}$$

J. Kersten, Diploma Thesis (2001), Ray, arXiv: 1005.1938

Cont...

- Remember:

$$A = i\frac{1}{2}\kappa_{gf}\left(\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd}\right)P_L$$

- We get: $\boxed{\kappa = 2Y_\nu^T M^{-1} Y_\nu}$
- The small neutrino mass:

$$\begin{aligned}\mathcal{L}_5 &= \frac{1}{4}\kappa\overline{\nu_L^C}\nu_L\phi^0\phi^0 \\ &= -\frac{1}{2}m_\nu\overline{\nu_L^C}\nu_L\end{aligned}$$

where,

$$\begin{aligned}m_\nu &= -\frac{1}{2}\kappa\phi^0\phi^0 \ , \\ &= -\frac{1}{2}2Y_\nu^T M^{-1} Y_\nu \frac{v^2}{2} \ , \\ &= -\frac{v^2}{2}Y_\nu^T M^{-1} Y_\nu\end{aligned}$$

type-II seesaw:

- each $\ell - \ell$ and $\phi - \phi$ forms scalar triplet
- $\ell - \ell$ forms (1,3,-2), one needs Higgs triplet field Δ with (1,3,2)

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\Delta}$$

where,

$$\mathcal{L}_{\Delta} = \mathcal{L}_{\Delta,kin} + \mathcal{L}_{\Delta,\phi} + \mathcal{L}_{\Delta,yuk}$$

- Here

$$-\mathcal{L}_{\Delta,yuk} = \frac{1}{\sqrt{2}}\left((Y_{\Delta})_{gf}\ell_L^{Tg} C\varepsilon \ell_L^f\right)\Delta + h.c.$$

- The Higgs triplet Δ transform in the adjoint representation of $SU(2)_L$ as,

$$\Delta = \frac{\sigma^i \Delta^i}{\sqrt{2}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix},$$

where $\Delta^{++} \equiv (\Delta^1 - i\Delta^2)/\sqrt{2}$, $\Delta^0 \equiv (\Delta^1 + i\Delta^2)/\sqrt{2}, \Delta^+ \equiv \Delta^3$ and σ^i are the Pauli matrices.

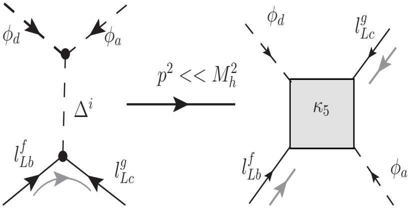
Ray, arXiv: 1005.1938, Akhmedov, arXiv: 0001264

Cont...

- The term between ϕ, and Δ

$$\sim \left[\frac{\Lambda_6}{\sqrt{2}}\phi^T \varepsilon \Delta^\dagger \phi + h.c.\right]$$

- The Feynman diagram:



- The amplitude:

$$A = \left[-i(Y_\Delta)_{gf}(\varepsilon\sigma^i)_{cb}P_L\right]\left[\frac{i}{p^2 - M_\Delta^2}\right]\left[-i\Lambda_6(\varepsilon\sigma^i)_{ad}\right]$$

$$= -2\frac{i\Lambda_6}{M_\Delta^2}(Y_\Delta)_{gf}\frac{1}{2}\left[\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd}\right]P_L$$

$$\kappa = -\frac{2\Lambda_6 Y_\Delta}{M_\Delta^2}$$

Cont...

- After EW breaking, Δ will get a *vev* given by $\langle\Delta_0\rangle \sim \dfrac{\Lambda_6 v^2}{2\sqrt{2}M_\Delta^2}$
- The neutrino mass term:

$$
\begin{aligned}
-\mathcal{L}_{\Delta,yuk} &= \frac{1}{\sqrt{2}} Y_\Delta \nu_L^T C \langle\Delta^0\rangle \nu_L + h.c. \\
&= \frac{1}{2} m_\nu \nu_L^T C \nu_L + h.c.
\end{aligned}
$$

where,

$$
\begin{aligned}
m_\nu &= \frac{2}{\sqrt{2}} Y_\Delta \Delta^0 \ , \\
&= \sqrt{2} Y_\Delta \frac{\Lambda_6 v^2}{2\sqrt{2}M_\Delta^2} \ , \\
&= \frac{v^2}{2} \frac{\Lambda_6 Y_\Delta}{M_\Delta^2}
\end{aligned}
$$

type-III seesaw:

- each $\ell - \phi$ forms a fermion triplet (1,3,0)
- One adds right-handed fermion triplet Σ_R (1,3,0) in the SM
- This triplet can be represented as

$$\Sigma_R = \begin{pmatrix} \Sigma_R^0/\sqrt{2} & \Sigma_R^+ \\ \Sigma_R^- & -\Sigma_R^0/\sqrt{2} \end{pmatrix} \equiv \frac{\Sigma_R^i \sigma^i}{\sqrt{2}}$$

  where $\Sigma_R^\pm = \frac{\Sigma_R^1 \mp i\Sigma_R^2}{\sqrt{2}}$, $\Sigma_R^0 = \Sigma_R^3$, and $\Sigma \equiv \Sigma_R + \Sigma_R^C$
- The Lagrangian:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_\Sigma$$

  where,

$$\mathcal{L}_\Sigma = \mathcal{L}_{\Sigma,kin} + \mathcal{L}_{\Sigma,mass} + \mathcal{L}_{\Sigma,Yuk}$$

- Also,

$$\mathcal{L}_{\Sigma,kin} = \frac{1}{2} Tr\left[\overline{\Sigma} i \not{D} \Sigma\right]; \quad D_\mu \Sigma = \partial_\mu \Sigma + i g_2 [W_\mu, \Sigma]$$

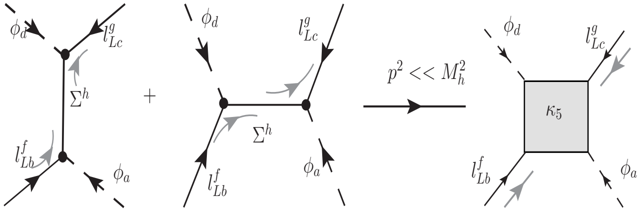
$$-\mathcal{L}_{\Sigma,mass} = \frac{1}{2} Tr\left[\overline{\Sigma} M_\Sigma \Sigma\right] + h.c.$$

$$-\mathcal{L}_{\Sigma,Yuk} = \tilde{\phi}^\dagger \overline{\Sigma} \sqrt{2} Y_\Sigma L_L + \overline{L_L} \sqrt{2} Y_\Sigma^\dagger \Sigma \tilde{\phi}.$$

Ray, arXiv: 1005.1938, Chakrabortty, Dighe, Goswami, Ray, arXiv: 0812.2776

Cont...

- The Feynman diagram:



- The amplitude:

$$
\begin{aligned}
A = & \left[-i(Y_\Sigma^T)_{gh}(\varepsilon^T\sigma^i)_{cd}P_L\right]\frac{i(\not{p}+M_h)}{p^2-M_h^2}\left[-i(Y_\Sigma)_{hf}(\varepsilon^T\sigma^i)_{ab}P_L\right] \\
& + \left[-i(Y_\Sigma^T)_{gh}(\varepsilon^T\sigma^i)_{ca}P_L\right]\frac{i(\not{p}+M_h)}{p^2-M_h^2}\left[-i(Y_\Sigma)_{hf}(\varepsilon^T\sigma^i)_{db}P_L\right] \\
& = 2i(Y_\Sigma^T M_\Sigma^{-1}Y_\Sigma)_{gf}P_L\frac{1}{2}\left[\varepsilon_{ab}\varepsilon_{dc}+\varepsilon_{db}\varepsilon_{ac}\right]
\end{aligned}
$$

$$\kappa = 2(Y_\Sigma^T M_\Sigma^{-1} Y_\Sigma)$$

Cont...

- The mass terms:

$$-\mathcal{L}_{mass} = \overline{\Psi} M_\Sigma \Psi + \frac{1}{2}\left[\overline{\Sigma_R^0} M_\Sigma \Sigma_R^{0C} + h.c.\right]$$
$$+\left[\phi^0 \overline{\Sigma_R^0} Y_\Sigma \nu_L + \sqrt{2}\phi^0 \overline{\Psi} Y_\Sigma \ell_L + \phi^0 \overline{\ell_L}\sqrt{2} Y_\Sigma^\dagger \Psi + h.c\right],$$

where, $\Psi = \Sigma_R^{+C} + \Sigma_R^-$

- After electroweak symmetry breaking, (taking $\langle\phi\rangle = (0, v/\sqrt{2})^T$)

$$\begin{aligned}
-\mathcal{L}_{mass} &= \frac{1}{2}\overline{\Sigma_R^0} M_\Sigma \Sigma_R^{0C} + \overline{\Sigma_R^0}\frac{vY_\Sigma}{\sqrt{2}}\nu_L + h.c \\
&= \frac{1}{2}\overline{\Sigma_R^0} M_\Sigma \Sigma_R^{0C} + \overline{\Sigma_R^0} m_D^T \nu_L + h.c. \\
&= \frac{1}{2}\left[\overline{\Sigma_R^0} m_D^T \nu_L + \overline{\nu_L^C} m_D \Sigma_R^{0C} + \overline{\Sigma_R^0} M_\Sigma \Sigma_R^{0C}\right] + h.c. \\
&= \frac{1}{2}\begin{pmatrix} \overline{\nu_L^C} & \overline{\Sigma_R^0} \end{pmatrix}\begin{pmatrix} o & m_D \\ m_D^T & M_\Sigma \end{pmatrix}\begin{pmatrix} \nu_L \\ \Sigma_R^{0C} \end{pmatrix} + h.c. \\
&= \frac{1}{2}\overline{\nu^C} M \nu + h.c.
\end{aligned}$$

Popular variants of the type-I seesaw:

- **In the (ν_L^c, N_R, S) basis:**

$$\mathcal{M} = \begin{pmatrix} m_L & m_D^T & m_{DS}^T \\ m_D & M_R & m_{RS}^T \\ m_{DS} & m_{RS} & M_S \end{pmatrix}.$$

- **Double seesaw:**

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & m_{RS}^T \\ 0 & m_{RS} & M_S \end{pmatrix},$$

with the conditions $m_D, m_{RS} \ll M_S$ and $m_D \ll m_{RS}^2/M_S$.

- To block-diagonalize $\mathcal{M}$, we define:

$$\mathbb{M}_D := \begin{pmatrix} m_D \\ 0 \end{pmatrix}, \quad \mathbb{M}_R := \begin{pmatrix} 0 & m_{RS}^T \\ m_{RS} & M_S \end{pmatrix}, \quad \text{and } \mathcal{M} = \begin{pmatrix} 0 & \mathbb{M}_D^T \\ \mathbb{M}_D & \mathbb{M}_R \end{pmatrix}$$

- The mass of the lightest neutrinos:

$$\tilde{m}_\nu = m_D^T\, m_{RS}^{-1}\, M_S\, (m_{RS}^T)^{-1}\, m_D + \mathcal{O}\left(M_S \frac{m_D^4}{m_{RS}^4} \left(1 + \frac{M_S^2}{m_{RS}^2} \right) \right),$$

- Correct order of magnitude of m_ν:

$$m_\nu^0 \approx \left(\frac{m_D}{10^2\,\mathrm{GeV}} \right)^2 \left(\frac{10^{16}\,\mathrm{GeV}}{m_{RS}} \right)^2 \left(\frac{M_S}{10^{19}\,\mathrm{GeV}} \right) \mathrm{eV}$$

Mohapatra, PRL56(1986)561, Mohapatra, Valle, PRD34(1986)1642, Hettmansperger, Lindner, Rodejohann, JHEP04(2011)123

Cont...

- **Inverse seesaw:**

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & m_{RS}^T \\ 0 & m_{RS} & M_S \end{pmatrix} ,$$

$\mathcal{M}$ is same as 'double seesaw', but with the condition $M_S \ll m_D \ll m_{RS}$

- The mass of the lightest neutrinos:

$$\tilde{m}_\nu = m_D^T \, m_{RS}^{-1} \, M_S \, (m_{RS}^T)^{-1} \, m_D + \mathcal{O}\left(M_S \frac{m_D^4}{m_{RS}^4} \left(1 + \frac{M_S^2}{m_{RS}^2} \right) \right) ,$$

- Correct order of magnitude of m_ν:

$$m_\nu^0 \approx \left(\frac{m_D}{10^2 \mathrm{GeV}} \right)^2 \left(\frac{\mathrm{TeV}}{m_{RS}} \right)^2 \left(\frac{M_S}{0.1 \mathrm{keV}} \right) \mathrm{eV}$$

Akhmedov, Lindner, Schnapka, Valle, PRD53(1996)2752, Barr,PRL92(2004)101601

Cont...

- **Linear seesaw:**

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & m_{DS}^T \\ m_D & 0 & m_{RS}^T \\ m_{DS} & m_{RS} & M_S \end{pmatrix},$$

  with the conditions that m_{RS} is much larger than m_D and m_{DS}.

- To block-diagonalize $\mathcal{M}$, we define:

$$\mathbb{M}_D := \begin{pmatrix} m_D \\ m_{DS} \end{pmatrix}, \quad \mathbb{M}_R := \begin{pmatrix} 0 & m_{RS}^T \\ m_{RS} & M_S \end{pmatrix}, \text{ and } \mathcal{M} = \begin{pmatrix} 0 & \mathbb{M}_D^T \\ \mathbb{M}_D & \mathbb{M}_R \end{pmatrix}$$

- The mass of the lightest neutrinos:

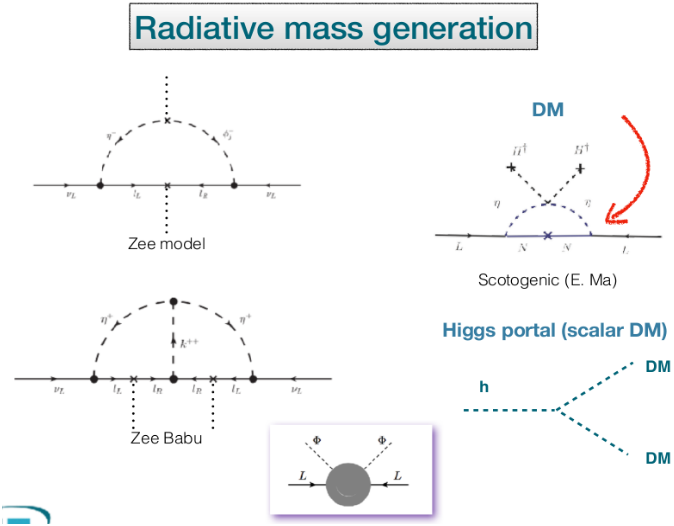
$$m_\nu^0 = m_D^T \, m_{RS}^{-1} \, M_S \, (m_{RS}^T)^{-1} \, m_D - \left[m_D^T \, m_{RS}^{-1} \, m_{DS} + m_{DS}^T \, (m_{RS}^T)^{-1} \, m_D \right],$$

- Correct order of magnitude of m_ν:

$$m_\nu^0 \approx \left(\frac{m_D}{10^2 \mathrm{GeV}} \right) \left(\frac{m_{DS}}{10^2 \mathrm{GeV}} \right) \left(\frac{10^{13} \mathrm{GeV}}{m_{RS}} \right) \mathrm{eV}$$

Malinsky, Romao, Valle, PRL95 (05)

Popular radiative seesaw models:



Credits: Peinado, UC Riverside'19

Neutrino Mixings:

- The charged current weak interaction Lagrangian:

$$-\mathcal{L}_I^{CC} = \frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L}\gamma^\rho l_{\alpha L} W^\rho + H.c.$$
$$= \frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1,2,3} U^*_{\alpha k}\overline{\nu}_{kL}\gamma^\rho l_{kL} W^\rho + H.c.$$

- The mass and flavor states are related as

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k}|\nu_k\rangle$$

where U is the PMNS mixing matrix and parameterized as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric, K2K, MINOS, T2K, etc.	Reactor Accelerator	Solar KamLAND

Cont...

- A $N \times N$ unitary mixing matrix contains N^2 independent parameters:

$$\text{mixing angles} \Rightarrow \frac{N(N-1)}{2}, \quad \text{phases} \Rightarrow \frac{N(N+1)}{2}$$

- Note: not all phases are physical observables

$$j^{\mu}_{W,Q} = 2\, \overline{q^U_L}\, \gamma^{\mu}\, V\, q^D_L \tag{4.13}$$

(the quark mixing matrix has no effect on the quark weak neutral current, because of the GIM mechanism, see eqn (3.178)). Apart from the weak charged current, the Lagrangian is invariant under global phase transformations of the quark fields of the type

$$q^U_{\alpha} \to e^{i\psi^U_{\alpha}}\, q^U_{\alpha}\,, \qquad q^D_k \to e^{i\psi^D_k}\, q^D_k\,, \tag{4.14}$$

with $\alpha = u, c, t$ and $k = d, s, b$. Performing this transformation, the quark charged current in eqn (4.13) becomes

$$j^{\mu}_{W,Q} = 2 \sum_{\alpha=u,c,t} \sum_{k=d,s,b} \overline{q^U_{\alpha L}}\, \gamma^{\mu}\, e^{-i\psi^U_{\alpha}}\, V_{\alpha k}\, e^{i\psi^D_k}\, q^D_{kL}\,, \tag{4.15}$$

which can be written as

$$j^{\mu}_{W,Q} = 2\, \underbrace{e^{-i(\psi^U_c - \psi^D_s)}}_{1} \sum_{\alpha=u,c,t} \sum_{k=d,s,b} \overline{q^U_{\alpha L}}\, \gamma^{\mu}\, \underbrace{e^{-i(\psi^U_{\alpha} - \psi^U_c)}}_{N-1=2}\, V_{\alpha k}\, \underbrace{e^{i(\psi^D_k - \psi^D_s)}}_{N-1=2}\, q^D_{kL}\,, \tag{4.16}$$

where we have factorized an arbitrary phase $e^{-i(\psi^U_c - \psi^D_s)}$ and we have indicated the number of independent phases in each term. From this expression, it is clear that there are

$$1 + (N-1) + (N-1) = 2\,N - 1 = 5 \tag{4.17}$$

Credits: Giunti and Kim

Cont...

- # of physical phases:

$$\#_\delta = \frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$$

- Total # of physical parameters:

$$\#_\theta = \frac{N(N-1)}{2} + \frac{(N-1)(N-2)}{2} = (N-1)^2$$

- **For 2-flavor neutrino oscillations:** $N = 2 \Rightarrow \#_\delta = 0, \#_\theta = 1$
- **For 3-flavor neutrino oscillations:** $N = 3 \Rightarrow \#_\delta = 1, \#_\theta = 4$

Wrap-up Comments:

- **An attempt has been made to explain smallness of neutrino mass**
- Noticed that Dirac mass term $\sim \overline{\nu_L}\nu_R$ is not possible for neutrino in the SM
- For Majorana mass term $\sim \nu_L^T \nu_L$ has $Y = -2$, since the SM does not have any fields with Y = 2, so in the SM no Majorana mass term
- **The lowest dimensional operator is the dim-5 operator** $ll\phi\phi$ **which could generate a Majorana neutrino mass**
- Later, type-I, II, and III 'seesaw mechanisms' have been discussed to address neutrino mass

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thank you