Lecture1: Neutrino Oscillations

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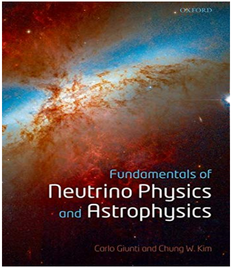
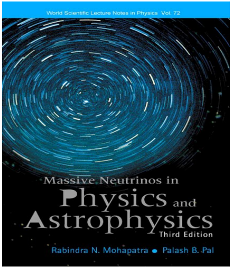
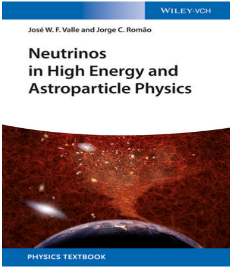
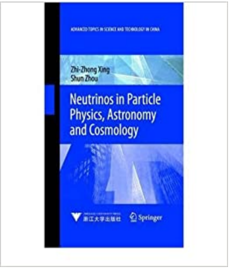
Books:





Giunti and Kim, **Mohapatra and Pal**





Valle and Romao, **Xing and Zhou**

Research in Fundamental Physics:

Convention:

- **In particle physics:** $c = \hbar = 1$
- **Unit of mass and energy (E) are eV/MeV/GeV.**
- **Unit of time and length are 1/E.**

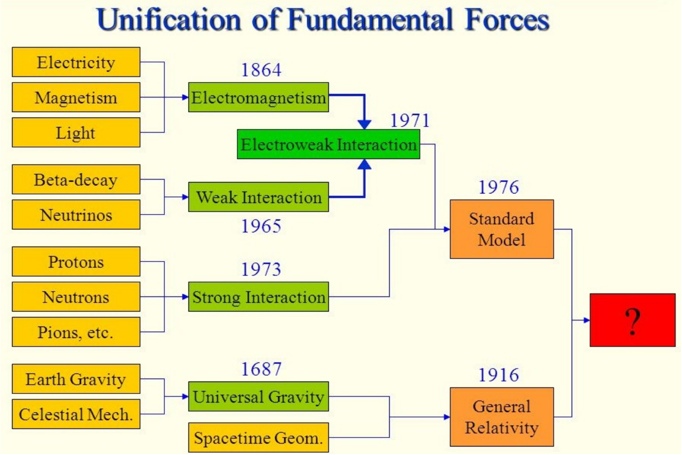
Fundamental Interactions:

Properties of the Interactions

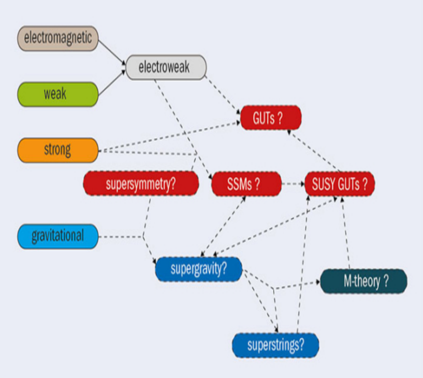
The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction (Electroweak)	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

Fundamental Interactions:



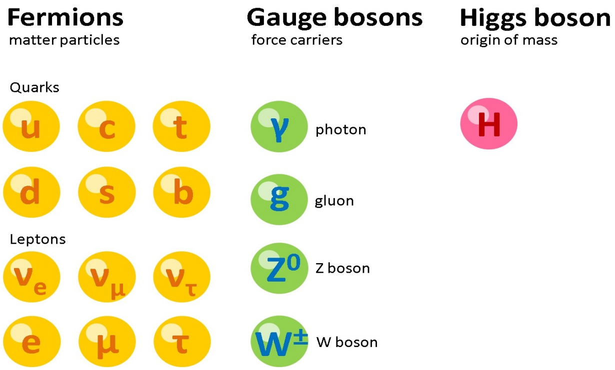
3-Million Dollar Prize:



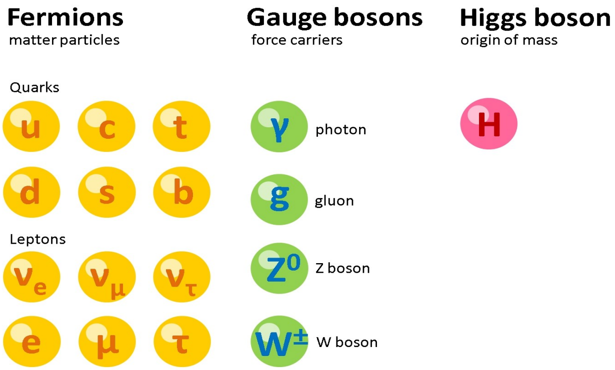
P. V. Nieuwenhuizen, S. Ferrara, D. Freedman

Supergravity: A Special Breakthrough Prize in Fundamental Physics 2019

Zoo of Particles: The Standard Model



Zoo of Particles: The Standard Model

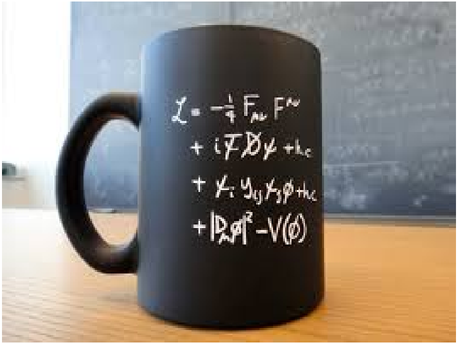


- On 4th July 2012, LHC (Large Hadron Collider) had announced "the discovery of Higgs boson".
- It was theorized by **Englert-Brout-Higgs-Guralnik-Hagen-Kibble** . (Nobel Prize 2013 to Englert and Higgs)

SM Lagrangian: Based on the unitary product group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\overline{\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa} + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + igMs_w\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0\right) .
\end{gathered}
$$

SM in T-cup:


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

The SM:

- **Successes:**
  - The prediction of the W, Z bosons, the gluons, the top and the charm quark.
  - Precise agreement with measurements of the fine structure constant $\alpha \approx 1/137.035999070$.
  - The prediction of the Higgs boson.
  - etc...

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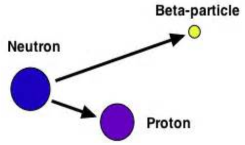
Focusing on Neutrinos

Neutrino:

- In 1914, J. Chadwick first demonstrated the observed β-decay,

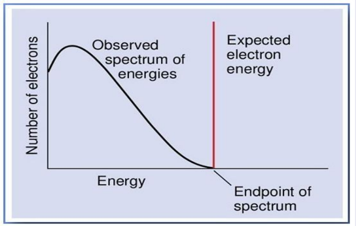


- Conservation of energy $\Rightarrow$ the energy of electron ,

$$E_e = (m_n^2 + m_e^2 - m_p^2)c^2/2m_n \tag{1}$$

- Experiment showed a continuous energy spectrum.

Beta decay Spectrum:



- Emission of β particle $\Rightarrow$ an integral change in spin.
- Violation of energy and angular momentum conservation have been assumed in nuclear β-decay.

Cont...

- In 1930, W. Pauli came with an idea known as Neutrino Hypothesis to explain the electron energy in β-decay.

"Wolfgang Pauli"

Cont...

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"Wolfgang Pauli"

- **In 1932, Fermi presented the fundamental β-decay through weak interaction,**

$$n \rightarrow p + e^- + \overline{\nu}$$

- **Neutrinos are weakly interacting and almost massless.**
- **Electrically neutral and spin $1/2$ fermions.**

Pauli's Remedy:

Physikalisches Institut
Der Eidg. Technischen Hochshule
Zurich

Zurich 4 dec. 1930
Gloariastr.

Dear Radioactive Ladies and Gentlemen

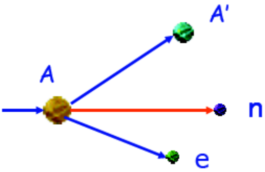
As the bearer pf these lines will explain to you in more detail – and I beg you to listen to him with benevolence – I have considered, in connection with the 'wrong' statistics of ^{14}N and ^{6}Li as well as with the continuous β spectrum, a way out for saving the 'law of change' of statistics and the conservation of energy: i.e. the possibility **that inside the nuclei there are particles electrically neutral, that I will call neutrons, which have spin ½** and follow the exclusion principle and that in addition differ from photons because they do not move with the velocity of light. The mass of neutrons should be of the same order of magnitude of that of the electrons and anyhow not greater than 0.01 protonic masses. **The continuous β spectrum would then be understandable, assuming that in the β decay together with the electron, in all cases, also a neutron is emitted, in such a way that the sum of the energy of the neutron and of the electron remains constant.**
The question is now to see which forces act on the neutrons. The most probable model appears to me to be, for wave mechanical reasons (the detail can be given to you by the bearer of these lines), for the neutron at rest to be a magnetic dipole pf a certain moment μ. The experimental data certainly require for the ionizing power of such a neutron to be not greater than that of a gamma ray and therefore μ should not be greater than $e \times 10^{-13}$ cm. **I do not consider advisable, for the moment, to publish something about these ideas** and first I apply to with confidence, dear Radioactives, with the question: what do you think about the possibility of providing the experimental proof of such a neutron, if it would possess a penetrating power equal or ten times greater of that of gamma rays?
I admit that my solution may appear to you not very probable, because it the neutron would exist, they would have been observed long since. But only who dares wins, and the gravity of the situation in regard to the continuous β spectrum is enlightened by the opinion of my predecessor in the chair Mr. Debye, who long since told me in Brussels: 'Oh, the best thing to do is not to talk about, like for new taxes'. For this reason one should consider seriously any way towards safety. Thus, dear Radioactives, consider and judge. **Unfortunately I cannot come personally to Tubingen, because I am necessary here for a ball that will take place in Zurich the night from 6 to 7 December.**

With many greetings to you as well as to Mr. Back.

Your devoted servant,

W. Pauli

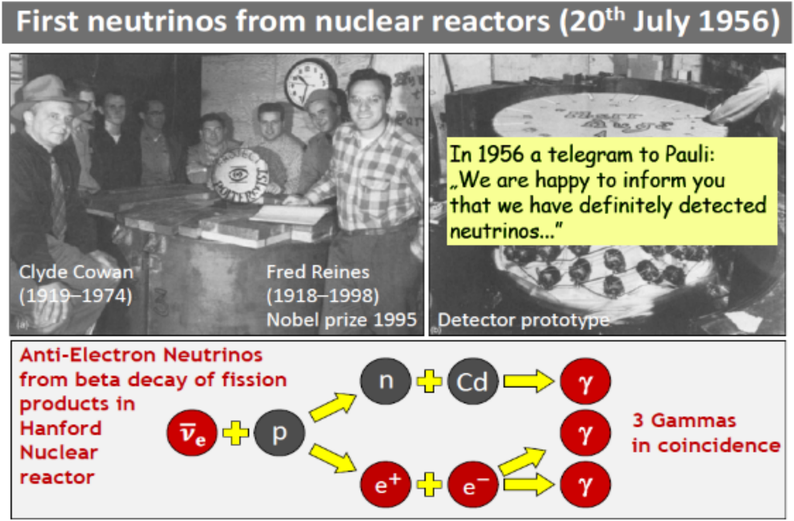
Dec 1930: A Desperate Remedy



"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do." W.Pauli

From neutrinos to cosmic sources, DK&ER

Cont...

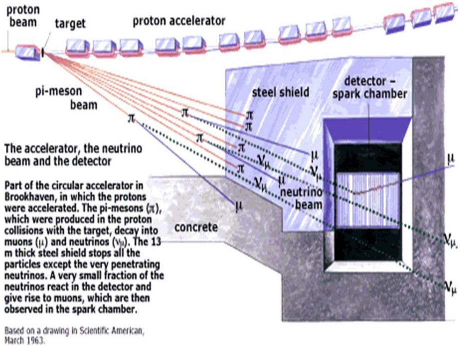


Note: the first ν was discovered 26 years after it was first proposed.

Cont...

- **In 1960, Pontecorvo suggested, ν produced in $\pi^+ \to \mu^+ + \nu$ may be different from ν in β-decay.**

Leon M. Lederman   Melvin Schwartz   Jack Steinberger



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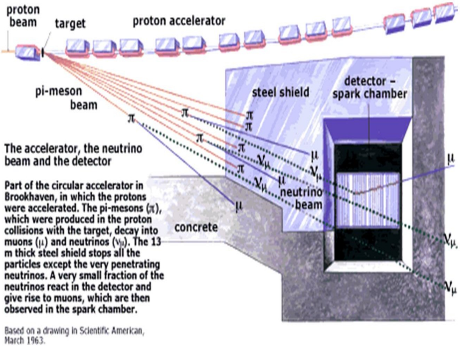

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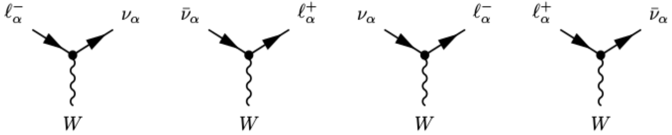
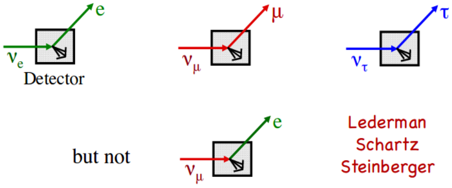
- In July 2000, the DONUT collaboration at FNAL announced the discovery of third type of neutrino called 'tau neutrino' (ν_τ).

Important property of neutrinos

- **Charged-current Lagrangian:**

$$-\mathcal{L}_I^{CC} = \frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \overline{\nu}_{\alpha L} \gamma^{\rho} \ell_{\alpha L} W^{\rho} + H.c.$$

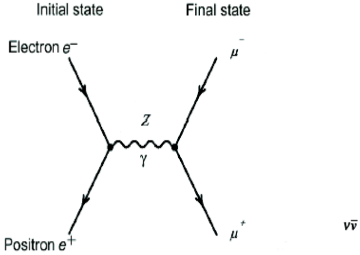


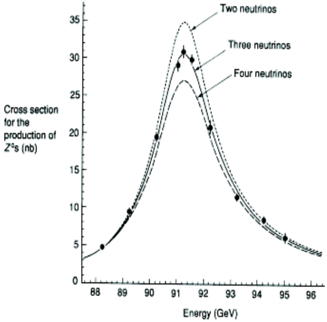


credits : B. Kayser, LAPP Annecy'17

- **A given flavor of neutrino interact with the detector and produce same flavor of charged-lepton**

How Many Neutrinos?





$$Z^0 \to q\bar{q}(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b})$$

$$Z^0 \to l\bar{l}\,(e^- e^+, \mu^- \mu^+, \tau^- \tau^+)$$

$$Z^0 \to \nu\bar{\nu}(\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau)$$

$\Gamma_Z, \Gamma_l, \Gamma_h$ - measured

Γ_ν - calculated

Total width: Γ~ decay probability (~1/lifetime)

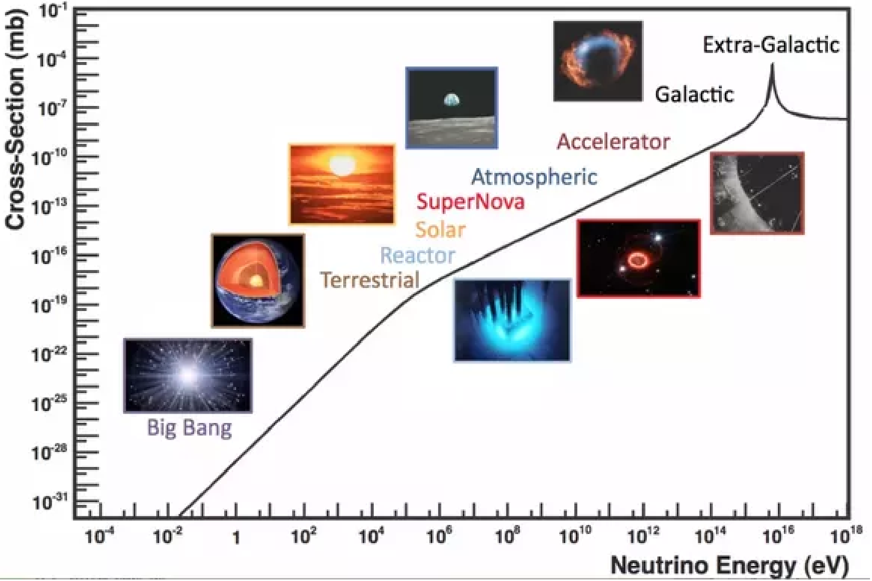
Partial widths: Γ_i~ branching rate (channel i)

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_l + N_\nu \Gamma_\nu$$

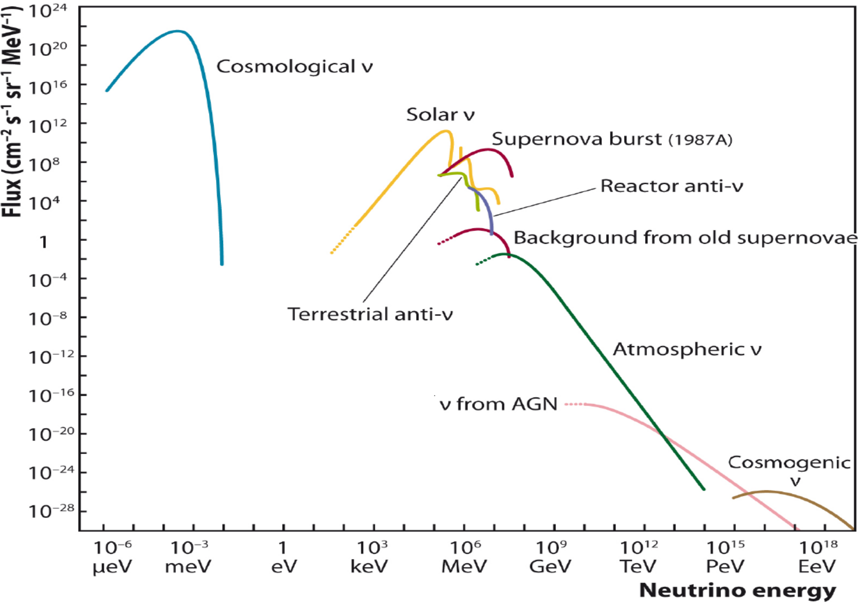
$$N_\nu = 2.99 \pm 0.02$$

From neutrinos to cosmic sources, DK&ER

Sources of Neutrinos :



Neutrinos Flux :

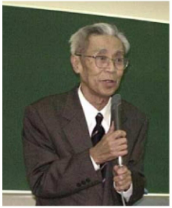


Neutrino oscillation:

- **In 1957, B.Pontecorvo 1st suggested the idea of ν-masses, mixing and oscillations from the analogy of $K^0 \rightleftharpoons \overline{K}^0$ oscillation.**
- **ν-flavor transitions was 1st considered by Maki, Nakagawa and Sakata in 1962.**

B. Pontecorvo
1913 - 1993

S. Sakata
1911 – 1970



Z. Maki
1929 – 2005

M. Nakagawa
1932 – 2001

- Neutrino oscillation $\Rightarrow$ **Transition from one flavor to another**

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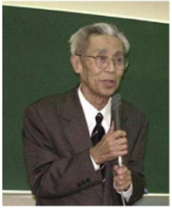

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1929 – 2005

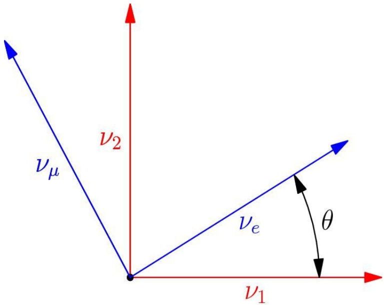
M. Nakagawa
1932 – 2001

- Neutrino oscillation $\Rightarrow$ **Transition from one flavor to another**

time $= 0$; **time** $= t$;

ν_e; $\longrightarrow$ **distance** $= L$; $\longrightarrow$ ν_e, ν_μ, ν_τ;

- For 2-flavor:



- Mathematically,

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix},$$

Flavor e.s. **Mass e.s.**

where θ is the mixing angle.

Cont...

- **Neutrino mass eigenstates $|\nu_k\rangle$ are eigenstates of the Hamiltonian:**
$$H|\nu_k\rangle = E_k|\nu_k\rangle$$
with energy eigenvalues $E_k = E + m_k^2/2E$ & $E_k - E_j \approx \frac{\Delta m_{kj}^2}{2E}$
- **The Schrödinger equation,**
$$i\frac{d}{dt}|\nu_k(t)\rangle = H|\nu_k(t)\rangle$$
- **Evolution of plane waves with time,**
$$|\nu_k(t)\rangle = e^{-iE_k t}|\nu_k\rangle$$
- **The time evolution of a flavor state $|\nu_\alpha(t)\rangle$ is given by,**
$$|\nu_\alpha(t)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t}|\nu_k\rangle$$
- After some time a state produced as ν_e evolve in vacuum into,
$$|\nu_e(t)\rangle = \cos\theta e^{-iE_1 t}|\nu_1(0)\rangle + \sin\theta e^{-iE_2 t}|\nu_2(0)\rangle$$
- **For an eigenstate $|\nu_e\rangle$, probability $\Rightarrow |\langle\nu_e|\nu_e\rangle|^2$**

- **Neutrino oscillation probability**,

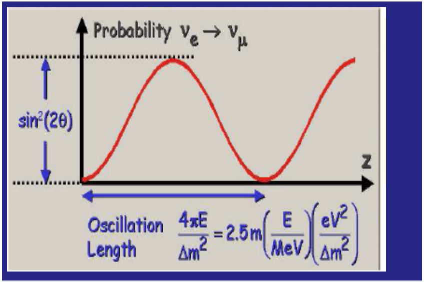
$$P_{\nu_e \longrightarrow \nu_e} = |\langle \nu_e(t)|\nu_e\rangle|^2 = 1 - \sin^2 2\theta \sin^2(\frac{\Delta m^2 L}{4E}) \quad HW?$$

$$P_{\nu_e \longrightarrow \nu_\mu} = 1 - P_{\nu_e \longrightarrow \nu_e} = \sin^2 2\theta \sin^2(\frac{1.27\Delta m^2 L}{E})$$

$$P_{\nu_e \longrightarrow \nu_\mu} = \sin^2 2\theta \sin^2(\frac{\pi L}{\lambda})$$

where $\lambda = \frac{4\pi E}{\Delta m^2} = 2.5m\left(\frac{E}{MeV}\right)\left(\frac{eV^2}{\Delta m^2}\right) \longrightarrow$ Oscillation Wavelength.

- **Neutrino Oscillation requires**, $\theta \neq 0, \Delta m^2 \neq 0$.



- Oscillation Wavelength $\lambda = 2.5m(E/MeV)(eV^2/\Delta m^2)$
- $\lambda >> L, \sin^2(\pi L/\lambda) \to 0$
- $\lambda << L, \sin^2(\pi L/\lambda) \to 1/2$
- $\lambda \sim 2L, \sin^2(\pi L/\lambda) \sim 1 \to \Delta m^2 \sim E/L$

Cont...

Type of experiment	L	E	Δm^2 sensitivity
Reactor SBL	$\sim 10\,\mathrm{m}$	$\sim 1\,\mathrm{MeV}$	$\sim 0.1\,\mathrm{eV}^2$
Accelerator SBL (Pion DIF)	$\sim 1\,\mathrm{km}$	$\gtrsim 1\,\mathrm{GeV}$	$\gtrsim 1\,\mathrm{eV}^2$
Accelerator SBL (Muon DAR)	$\sim 10\,\mathrm{m}$	$\sim 10\,\mathrm{MeV}$	$\sim 1\,\mathrm{eV}^2$
Accelerator SBL (Beam Dump)	$\sim 1\,\mathrm{km}$	$\sim 10^2\,\mathrm{GeV}$	$\sim 10^2\,\mathrm{eV}^2$
Reactor LBL	$\sim 1\,\mathrm{km}$	$\sim 1\,\mathrm{MeV}$	$\sim 10^{-3}\,\mathrm{eV}^2$
Accelerator LBL	$\sim 10^3\,\mathrm{km}$	$\gtrsim 1\,\mathrm{GeV}$	$\gtrsim 10^{-3}\,\mathrm{eV}^2$
ATM	20–$10^4\,\mathrm{km}$	0.5–$10^2\,\mathrm{GeV}$	$\sim 10^{-4}\,\mathrm{eV}^2$
Reactor VLB	$\sim 10^2\,\mathrm{km}$	$\sim 1\,\mathrm{MeV}$	$\sim 10^{-5}\,\mathrm{eV}^2$
Accelerator VLB	$\sim 10^4\,\mathrm{km}$	$\gtrsim 1\,\mathrm{GeV}$	$\gtrsim 10^{-4}\,\mathrm{eV}^2$
SOL	$\sim 10^{11}\,\mathrm{km}$	0.2–$15\,\mathrm{MeV}$	$\sim 10^{-12}\,\mathrm{eV}^2$

Interesting facts:

- **Mass of electron,** $m_e \approx 9.1 \times 10^{-31}$ **kg** ≈ 0.511 **MeV.**
- **Mass of neutrino,** $m_\nu \approx 0.01$ **eV.**

Interesting facts:

- **Mass of electron,** $m_e \approx 9.1 \times 10^{-31}$ **kg** ≈ 0.511 **MeV.**
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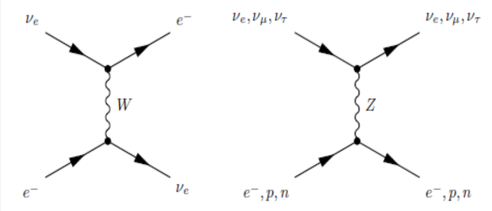
Do you know?

$\approx$ **100 billions of** ν**s pass through every square cm of your body per sec.**

Matter Potential:

- Wolfenstein in 1978 first pointed out that matter can drastically impact the neutrino oscillation
- In 1985 Mikheev and Smirnov discovered that it is possible to have resonant flavor transitions
- Charged current (CC) and Neutral current (NC) neutrino interaction

Feynman diagrams:



- Effective CC Hamiltonian:

$$\mathcal{H}_{eff}^{CC}(x) = \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right]\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]$$

- Using the Fierz transformation: (HW?)

$$\mathcal{H}_{eff}^{CC}(x) = \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_e(x)\gamma^{\mu}(1-\gamma^5)\nu_e(x)\right]\left[\bar{e}(x)\gamma_{\mu}(1-\gamma^5)e(x)\right]$$

- The interaction potential is the average of the effective Hamiltonian:

$$\langle H_{\text{eff}}^{CC}\rangle = \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_e\gamma^{\mu}(1-\gamma_5)\nu_e\right]\langle\bar{e}\gamma_{\mu}(1-\gamma_5)e\rangle.$$

In the non-relativistic limit:

$$\begin{aligned}\langle\bar{e}\gamma_0 e\rangle &= N_e,\\ \langle\bar{e}\gamma_{\mu}\gamma_5 e\rangle &\sim \text{spin},\\ \langle\bar{e}\gamma_i e\rangle &\sim \text{velocity},\end{aligned}$$

where, N_e is the electron number density of the medium.

- In the rest frame of unpolarized electrons only the first term contribute,

$$\begin{aligned}\langle H_{\text{eff}}^{CC}\rangle &= \sqrt{2}G_F N_e\ \ \bar{\nu}_{eL}\gamma^0\nu_{eL},\\ &= V_{CC}\ \ \bar{\nu}_{eL}\gamma^0\nu_{eL} = V_{CC}\ \ J_{\nu},\end{aligned}$$

where

$$\begin{aligned}V_{CC} &= \sqrt{2}G_F N_e \quad \text{for} \quad \nu\\ &= -\sqrt{2}G_F N_e \quad \text{for} \quad \bar{\nu}\end{aligned}$$

Cont...

- For NC:

$$\mathcal{H}_{eff}^{NC}(x) = \frac{G_F}{\sqrt{2}} \sum_\alpha \left[\bar{\nu}_\alpha(x)\gamma^\mu(1-\gamma^5)\nu_\alpha(x)\right] \sum_f \left[\bar{f}(x)\gamma_\mu(g_V^f - g_A^f\gamma^5)f(x)\right]$$

where,

$$g_V^e = -\frac{1}{2} + 2\sin^2\theta_w$$
$$g_V^p = \frac{1}{2} - 2\sin^2\theta_w, \quad g_V^n = -\frac{1}{2}$$

- Neutral-current potential V_{NC} is,

$$V_{NC} = V_Z^n + \underbrace{V_Z^p + V_Z^e}_{0}$$

- Total potential for neutrinos,

$$\boxed{V = V_{CC} + V_{NC} = \sqrt{2}G_F N_e - \frac{1}{2}\sqrt{2}G_F N_N}$$

Note: V_{NC} irrelevant for the flavor transitions as it generates a phase common.

Two flavor formalism in matter:

- The Schrodinger equation:

$$i\frac{d}{dt}|\nu_\alpha(t)\rangle = \mathcal{H}|\nu_\alpha(t)\rangle$$

where, $\boxed{\mathcal{H} = \underbrace{\mathcal{H}_0}_{vacuum} + \underbrace{\mathcal{H}_{\mathcal{I}}}_{matter}}$

- The Hamiltonian in the flavor basis:

$$\mathcal{H}_0 = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}.$$

- Total Hamiltonian:

$$\mathcal{H} = \frac{1}{4E}\begin{pmatrix} A - \Delta m^2 \cos 2\theta & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & -A + \Delta m^2 \cos 2\theta \end{pmatrix}, \quad \text{where} \quad A = 2EV_{CC}.$$

- Diagonalising $\mathcal{H}$,

$$E_{1,2} = \frac{1}{4E}\left[A \pm \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}\right].$$

Cont...

- Using $E_2 - E_1 = (m_2^2 - m_1^2)/2E$, the mass squared difference in matter,

$$(\Delta m^2)_M = \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}.$$

- The mixing angle "θ_M" in matter,

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A},$$

- The Mikheyev-Smirnov-Wolfenstein or MSW resonance condition:

$$\Delta m^2 \cos 2\theta = A$$

- A is +ve for νs, resonance occur: $\Delta m^2 > 0$ & $\theta < 45^\circ$ or $\Delta m^2 < 0$ & $\theta > 45^\circ$.
- For $\overline{\nu}$s A is -ve, resonance occur: $\Delta m^2 > 0$ & $\theta > 45^\circ$ or $\Delta m^2 < 0$ & $\theta < 45^\circ$.

Three flavor formalism in matter:

- The Hamiltonian in the flavor basis:

$$\mathcal{H}_{\mathcal{F}} = U^T \mathcal{H}_0 U^* + \mathcal{H}_{\mathcal{I}},$$
$$= \frac{1}{2E}(U\mathbb{M}^2 U^\dagger + \mathbb{A})$$

with

$$\mathbb{M}^2 = diag(0, \Delta_{21}, \Delta_{31}), \qquad \mathbb{A} = diag(A, 0, 0).$$

- Introducing, $\alpha = \Delta_{21}/\Delta_{31}$,

$$\mathcal{H}_{\mathcal{F}} = \frac{\Delta_{31}}{2E} U diag(0, \alpha, 1) U^\dagger + \frac{1}{2E} diag(A, 0, 0).$$

- The neutrino oscillation probabilities: $P_{\alpha\beta} = |S_{\beta\alpha}(t, t_0)|^2$, where $S(t, t_0)$ is the evolution matrix such that

$$|\nu(t)\rangle = S(t, t_0)|\nu(t_0)\rangle\,, \qquad S(t_0, t_0) = \mathbb{1}\,.$$

- To find $S(t, t_0)$, diagonalize $\mathcal{H}_{\mathcal{F}}$ as $\mathcal{H}_{\mathcal{F}} = U' \hat{H} U'^\dagger$, where U' is the leptonic mixing matrix, and $\hat{H} = \mathrm{diag}(E_1, E_2, E_3)$

Cont...

- The evolution matrix is then given by

$$S_{\beta\alpha}(t, t_0) = \sum_{i=1}^{3} (U'_{\alpha i})^* U'_{\beta i} \mathrm{e}^{-\mathrm{i} E_i L}, \qquad \alpha, \beta = e, \mu, \tau,$$

where we have identified $L \equiv t - t_0$

- Various approximate solutions of $\mathcal{H}_{\mathcal{F}}$:
  - one mass scale dominance (OMSD), $\alpha \to 0$
  - double expansion upto second order in $\alpha - s_{13}$
  - first order in α and exact dependence on s_{13}

Cervera, Donini, Gavela, Gomez Cadenas, Hernandez, Mena, Rigolin, NPB579(2000), Freund, PRD64(2001), Akhmedov, Johansson, Lindner, Ohlsson, Schwetz, JHEP04(2004)

Cont...

Within OMSD approximation

- **The appearance probability:**

$$P_{e\mu} = \sin^2\theta_{23}\sin^2 2\theta_{13}^M \sin^2(\Delta_{31}^M L/4E)$$

where,

$$\Delta_{31}^M = \sqrt{(\Delta_{31}\cos 2\theta_{13} - A)^2 + (\Delta_{31}\sin 2\theta_{13})^2}, \quad \tan 2\theta_{13}^M = \frac{\Delta_{31}\sin 2\theta_{13}}{\Delta_{31}\cos 2\theta_{13} - A}$$

- Note: 1-3 sector has resonance. Hence, the physics near the resonance region can be explained better using this approximation

Fogli, Lisi, Marrone, Scioscia, PRD59(199)

- The validity condition: $\Delta_{21} L/E \ll 1 \Rightarrow L/E \ll 10^4$ (km/GeV)
- The OMSD condition can be violated for $E \simeq 1$ GeV, and $L \geq 10^4$ km

Cont...

Series expansion up to second order in $\alpha - s_{13}$

- The Hamiltonian: $H \simeq \frac{\Delta m^2_{31}}{2E} O_{23} U_\delta \, M \, U_\delta^\dagger O_{23}^T$,
  where, $M \equiv O_{13} O_{12} \,\mathrm{diag}\,(0, \alpha, 1)\, O_{12}^T O_{13}^T + \mathrm{diag}\,(A, 0, 0)$
- Then, $M = M^{(0)} + M^{(1)} + M^{(2)}$, where $M^{(1)}$ ($M^{(2)}$) contains all terms of 1st (2nd) order in α and s_{13}
- One finds:

$$M^{(0)} = \mathrm{diag}(A, 0, 1) = \mathrm{diag}(\lambda_1^{(0)}, \lambda_2^{(0)}, \lambda_3^{(0)}), \quad M^{(1)} = \begin{pmatrix} \alpha s_{12}^2 & \alpha s_{12} c_{12} & s_{13} \\ \alpha s_{12} c_{12} & \alpha c_{12}^2 & 0 \\ s_{13} & 0 & 0 \end{pmatrix},$$

$$M^{(2)} = \begin{pmatrix} s_{13}^2 & 0 & -\alpha s_{13} s_{12}^2 \\ 0 & 0 & -\alpha s_{13} s_{12} c_{12} \\ -\alpha s_{13} s_{12}^2 & -\alpha s_{13} s_{12} c_{12} & -s_{13}^2 \end{pmatrix}.$$

- **The eigenvectors:** $v_i = v_i^{(0)} + v_i^{(1)} + v_i^{(2)}$
- The eigenvalues:

$$\lambda_i^{(1)} = M_{ii}^{(1)}, \quad \lambda_i^{(2)} = M_{ii}^{(2)} + \sum_{j \neq i} \frac{\left(M_{ij}^{(1)}\right)^2}{\lambda_i^{(0)} - \lambda_j^{(0)}},$$

Akhmedov, Johansson, Lindner, Ohlsson, Schwetz, JHEP04(2004)

Cont...

- The corrections to the eigenvectors:

$$v_i^{(1)} = \sum_{j\neq i} \frac{M_{ij}^{(1)}}{\lambda_i^{(0)} - \lambda_j^{(0)}} e_j \,,$$

$$v_i^{(2)} = \sum_{j\neq i} \frac{1}{\lambda_i^{(0)} - \lambda_j^{(0)}} \left[M_{ij}^{(2)} + \left(M^{(1)} v_i^{(1)} \right)_j - \lambda_i^{(1)} \left(v_i^{(1)} \right)_j \right] e_j \,.$$

- **The mixing matrix in matter:** $U' = O_{23} U_\delta W$ **with** $W = (v_1, v_2, v_3)$
- Appearance Channel:

$$P_{\mu e} = 4 s_{13}^2 s_{23}^2 \frac{\sin^2 (A-1)\Delta}{(A-1)^2}$$
$$+ \alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta + \delta_{cp}) \frac{\sin (A-1)\Delta}{(A-1)} \frac{\sin A\Delta}{A}$$
$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2 A\Delta}{A^2} \; ; \quad A = A_{CC}/\Delta m_{31}^2$$

- Disappearance Channel:

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \text{ subleading terms},$$

- For anti-neutrino replace: $\delta_{CP} \to -\delta_{CP}$ and $V \to -V$

Cont...

PMNS mixing matrix in 3-flavor

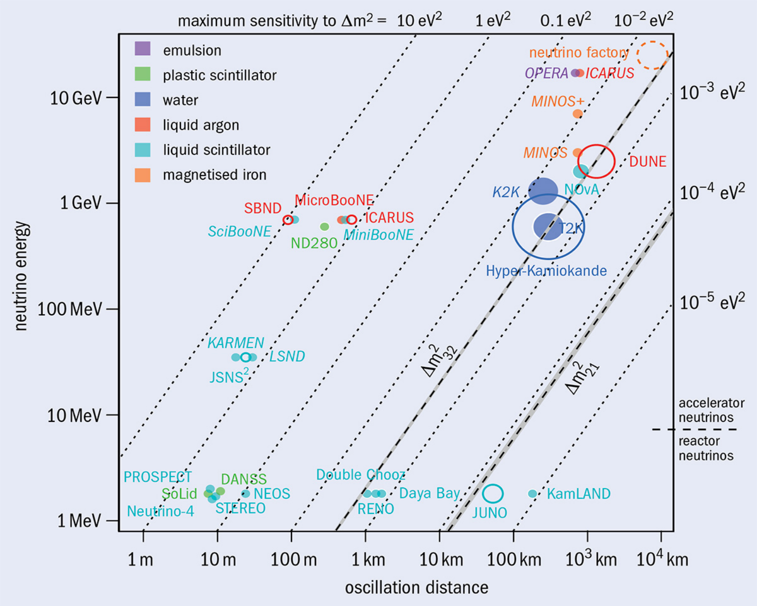
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric, K2K, MINOS, T2K, etc.	Reactor Accelerator	Solar KamLAND

The 6 parameters measurable in neutrino oscillations:

- ✳The atmospheric mass squared difference Δm^2_{31}
- ✳The solar mass squared difference $\Delta m^2_{21} = m^2_2 - m^2_1$
- ✳The atmospheric angle θ_{23}
- ✳The solar angle θ_{12}
- ✳The reactor angle θ_{13}
- ✳The CP violating phase δ

Cont...



maximum sensitivity to Δm² = 10 eV² 1 eV² 0.1 eV² 10⁻² eV²
10⁻³ eV²
10⁻⁴ eV²
10⁻⁵ eV²
emulsion
plastic scintillator
water
liquid argon
liquid scintillator
magnetised iron
10 GeV
1 GeV
100 MeV
10 MeV
1 MeV
neutrino energy
neutrino factory
OPERA
ICARUS
MINOS+
MINOS
DUNE
K2K
NOvA
T2K
Hyper-Kamiokande
SBND
MicroBooNE
ICARUS
SciBooNE
ND280
MiniBooNE
KARMEN
LSND
JSNS²
Δm²₃₂
Δm²₂₁
accelerator neutrinos
reactor neutrinos
PROSPECT
DANSS
SoLid
NEOS
Neutrino-4
STEREO
Double Chooz
Daya Bay
RENO
JUNO
KamLAND
1 m
10 m
100 m
1 km
10 km
100 km
10³ km
10⁴ km
oscillation distance

Current Status:

de Salas, Forero, Ternes, Valle, arXiv:1708.01186, PLB782 (2018)

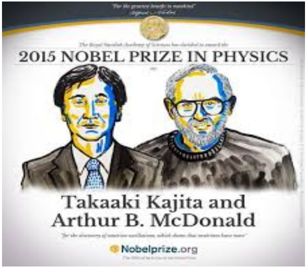
Parameter	Best fit $\pm$ 1σ	2σ range	3σ range
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.55^{+0.20}_{-0.16}$	7.20–7.94	7.05–8.14
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\mathrm{eV}^2]$ (NO)	2.50 ± 0.03	2.44–2.57	2.41–2.60
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\mathrm{eV}^2]$ (IO)	$2.42^{+0.03}_{-0.04}$	2.34–2.47	2.31–2.51
$\sin^2\theta_{12}/10^{-1}$	$3.20^{+0.20}_{-0.16}$	2.89–3.59	2.73–3.79
$\theta_{12}/^\circ$	$34.5^{+1.2}_{-1.0}$	32.5–36.8	31.5–38.0
$\sin^2\theta_{23}/10^{-1}$ (NO)	$5.47^{+0.20}_{-0.30}$	4.67–5.83	4.45–5.99
$\theta_{23}/^\circ$	$47.7^{+1.2}_{-1.7}$	43.1–49.8	41.8–50.7
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.51^{+0.18}_{-0.30}$	4.91–5.84	4.53–5.98
$\theta_{23}/^\circ$	$47.9^{+1.0}_{-1.7}$	44.5–48.9	42.3–50.7
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.160^{+0.083}_{-0.069}$	2.03–2.34	1.96–2.41
$\theta_{13}/^\circ$	$8.45^{+0.16}_{-0.14}$	8.2–8.8	8.0–8.9
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.220^{+0.074}_{-0.076}$	2.07–2.36	1.99–2.44
$\theta_{13}/^\circ$	$8.53^{+0.14}_{-0.15}$	8.3–8.8	8.1–9.0
δ/π (NO)	$1.32^{+0.21}_{-0.15}$	1.01–1.75	0.87–1.94
$\delta/^\circ$	238^{+38}_{-27}	182–315	157–349
δ/π (IO)	$1.56^{+0.13}_{-0.15}$	1.27–1.82	1.12–1.94
$\delta/^\circ$	281^{+23}_{-27}	229–328	202–349

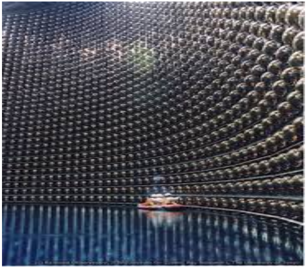
NO or IO ?

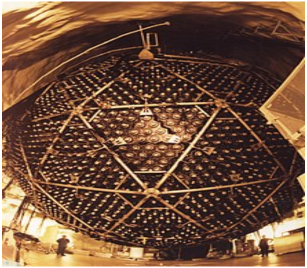
LO or HO ?

CPV

Most recent Nobel-Prize in Neutrino Physics



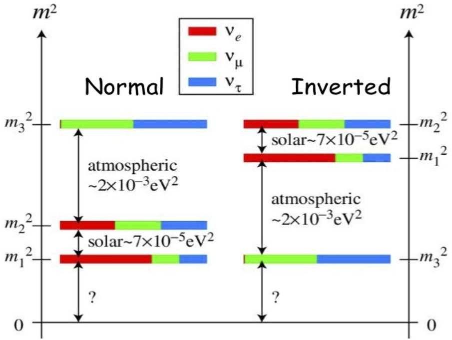




- **SK@Japan**
- **SNO@Canada**

Unknowns:

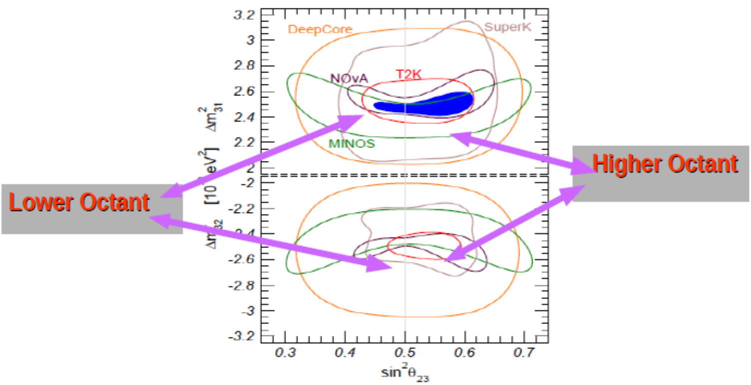
- Unknowns in Neutrino Physics.



Normal Hierarchy (NH) $\Rightarrow m_1 < m_2 < m_3$,
Inverted Hierarchy (IH) $\Rightarrow m_3 < m_1 \approx m_2$.

Cont...

- $\theta_{23} > 45^\circ$ $\Rightarrow$ Higher Octant or $\theta_{23} < 45^\circ$ $\Rightarrow$ Lower Octant.



NuFIT-4.0 (2018)

Cont...

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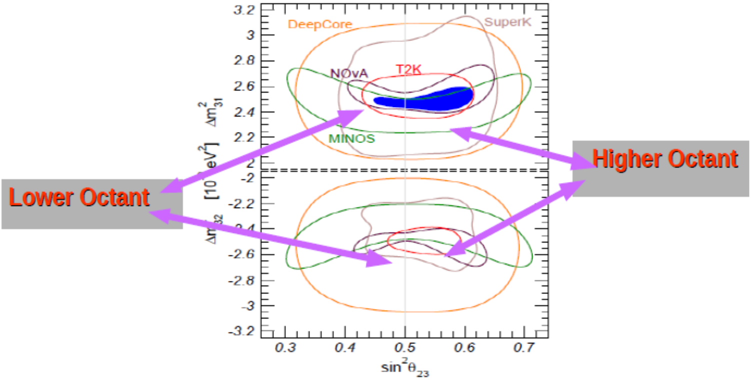


NuFIT-4.0 (2018)

- The Dirac CP phase δ_{CP}, where $\delta_{CP} \neq 0^o, \pm 180^o \Rightarrow$ *CP violation.*

Degeneracy:

Problem: Existence of parameter degeneracy

Degeneracy: Two different sets of neutrino oscillation parameters giving rise to same oscillation probability i.e.,

$$P_{\alpha\beta}(x) = P_{\alpha\beta}(y)$$

x, y: different sets of oscillation parameters i.e.,
$x = x(\theta_{ij}, \delta_{CP}, \Delta_{ij})$, $y = y(\theta'_{ij}, \delta'_{CP}, \Delta'_{ij})$

Conclusion:

Extraction of x will be confused with extraction of y

Degeneracy in the Disappearance Channel

$$P_{\mu\mu} \propto \sin^2 2\theta_{23} \sin^2 \Delta$$

- $P_{\mu\mu}(\Delta) \simeq P_{\mu\mu}(-\Delta)$: Intrinsic Hierarchy Degeneracy
- $P_{\mu\mu}(\theta_{23}) \simeq P_{\mu\mu}(\theta_{23} - \pi/2)$: Intrinsic Octant Degeneracy

Degeneracy in the Appearance Channel :

$$P_{\mu e} \propto s_{13}^2 s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2} + \alpha \cos(\Delta + \delta_{cp})$$

- No intrinsic degeneracy
- But can have:
  (i) $P_{\mu e}(\Delta, \delta_{CP}) = P_{\mu e}(-\Delta, \delta'_{CP})$, Minakata, Nunokawa, JHEP 0110 (2001).
  (ii) $P_{\mu e}(\theta_{13}, \delta_{CP}) = P_{\mu e}(\theta'_{13}, \delta'_{CP})$, Burguet-Castell ,Gavela, Gomez-Cadenas, Hernandez, Mena, NPB646 (2002).
- 8-fold degeneracy: $(\theta_{13}, \delta_{CP}), \Delta, (\theta_{23}) \simeq P_{\mu\mu}(\theta_{23} - \pi/2)$, Barger, Marfatia, Whisnant, PRD65 (2002)

Generalized Degeneracy

- For unknown hierarchy, octant and δ_{CP}:

$$P_{\mu e}(\theta_{23}, \Delta, \delta_{CP}) = P_{\mu e}(\theta'_{23}, -\Delta', \delta'_{CP})$$

$\Rightarrow$ generalized (hierarchy-θ_{23}-δ_{CP}) degeneracy

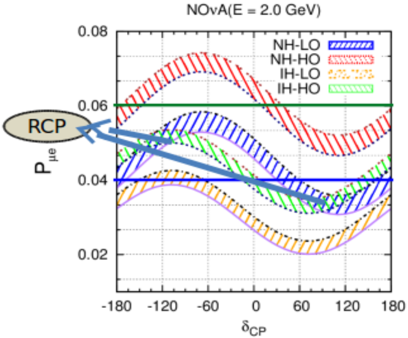
Ghosh, Ghoshal, Goswami, NN, Raut, PRD93 (2016)

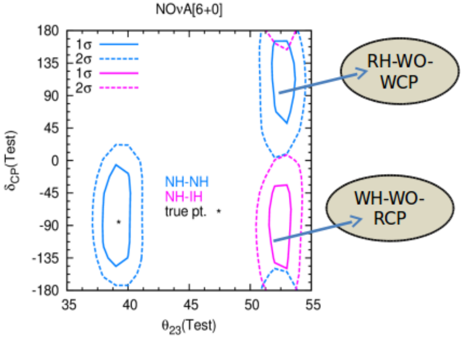
- Eight possibilities:

Solution with (right δ_{CP})	Solution with (wrong δ_{CP})
1) RH-RO-Rδ_{CP}	5) WH-WO-Wδ_{CP}
2) RH-WO-Rδ_{CP}	6) RH-RO-Wδ_{CP}
3) WH-RO-Rδ_{CP}	7) RH-WO-Wδ_{CP}
4) WH-WO-Rδ_{CP}	8) WH-RO-Wδ_{CP}

where, R=Right, W=Wrong, H= Hierarchy, O=Octant.

Degeneracies at NOνA are ,



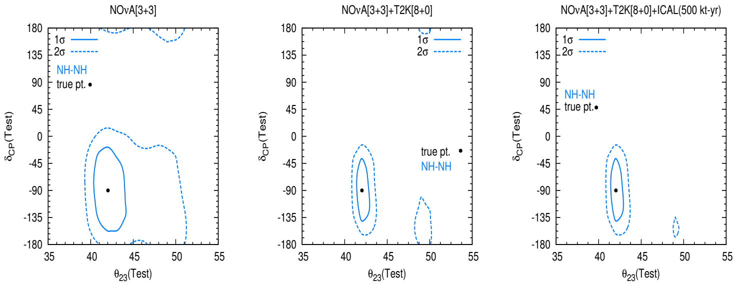


At Probability

- Overlapping region around $-120^o, +90^o \Rightarrow$ WH-WO-Rδ_{CP}.
- Same probability value for NH-LO and IH-LO $\Rightarrow$ WH-RO-Wδ_{CP}.
- Same probability value for NH-HO and NH-LO $\Rightarrow$ RH-WO-Wδ_{CP}.
- For NH-HO at $(48^o, -180^o)$ and $(48^o, 45^o)$ $\Rightarrow$ RH-RO-Wδ_{CP}.
- For NH-LO $(39^o, -180^o)$ and IH-HO $(51^o, 0^o)$ $\Rightarrow$ WH-WO-Wδ_{CP}.

Resolution:

- Resolution of degeneracies with NOνA, T2K, ICAL ,



- Addition of $\overline{\nu}$ removes WO-WH solution.
- Addition of more data provides better precision on θ_{23}.

Open Issues:

- **Why neutrino masses are so tiny?**

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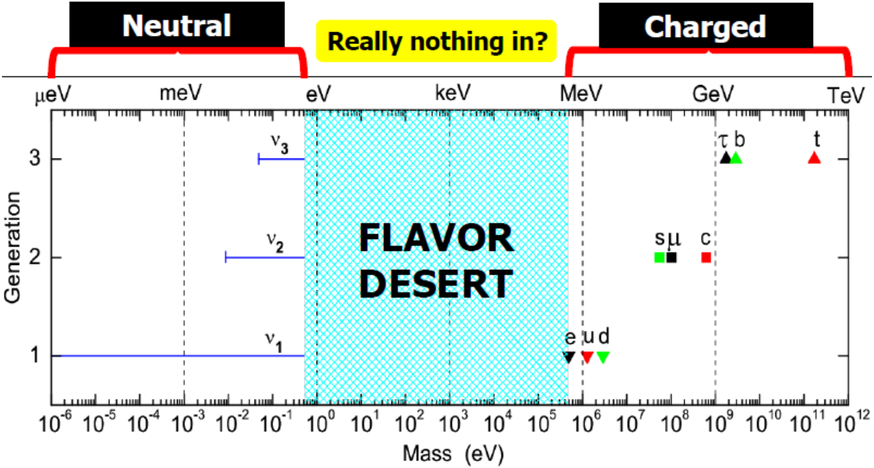

Cont...

- **Nature of neutrinos:**

Cont...

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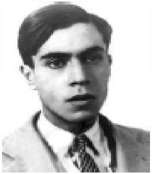
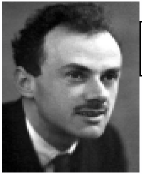
Neutrinos

$\Downarrow$



$\nu = \overline{\nu}$**?**



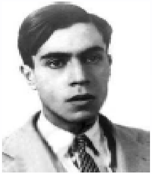
Dirac or Majorana?

- **Dirac mass,** $m_D \overline{\nu}_L \nu_R \Rightarrow$ **conserve lepton #.**

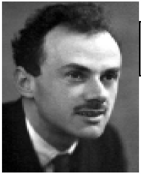
Cont...

- **Nature of neutrinos:**

Neutrinos



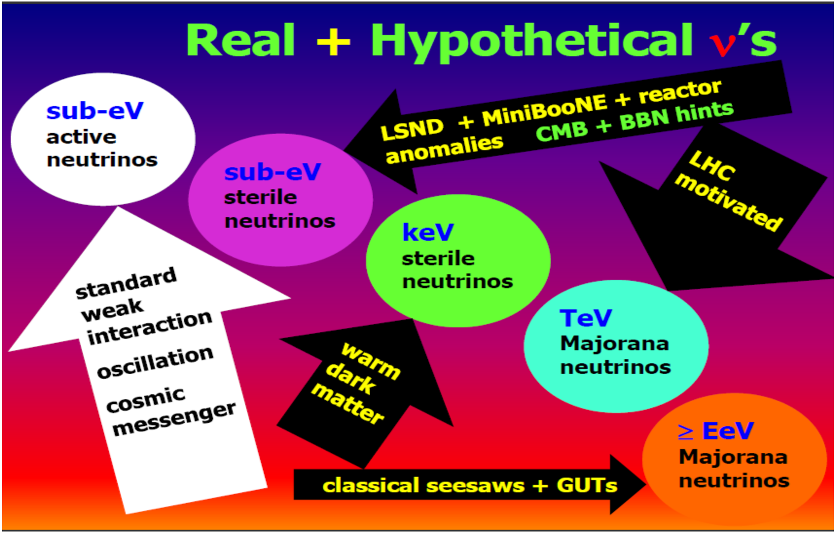
$\nu = \overline{\nu}$?



Dirac or Majorana?

- **Dirac mass,** $m_D \overline{\nu}_L \nu_R \Rightarrow$ **conserve lepton #.**
- **Majorana mass,** $m_M {\nu_L}^T C^{-1} \nu_L \Rightarrow$ **violate lepton # by 2 units.**

Neutrinos in BSM:

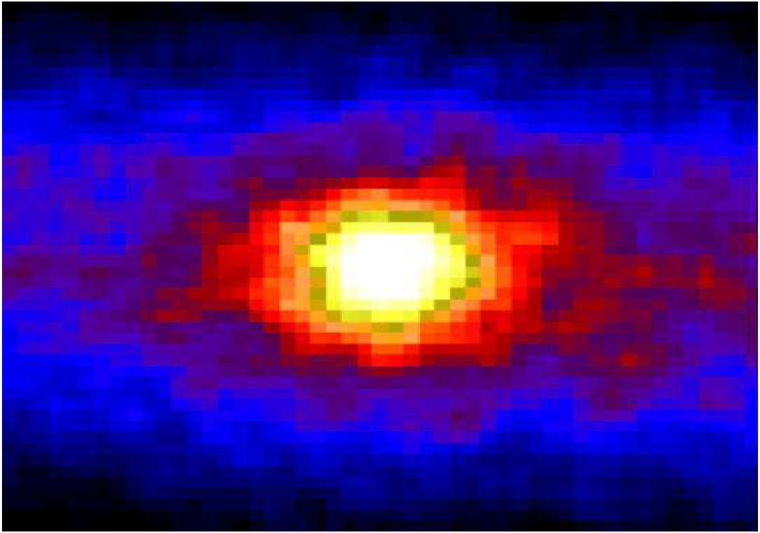


credits : Xing, Peking University'14

Cont...

- **Existence of other new physics effects on neutrino oscillations like non-unitarity, extra-dimensions, long range forces, etc.**
- **Does the neutrino possess a non-zero neutrino magnetic moment?**
- **Does the neutrino sector violate Lorentz or CPT symmetry?**
- **Formulation of Matter-antimatter Asymmetry of the Universe (BAU) through successful leptogenesis.**
- etc...

Sun in ν Light:



Wrap-up Comments:

- **An attempt has been made to give an overview of particle physics focusing on neutrinos**
- **Reminder-1: neutrino mass is approx. 100,000000 times lighter than electron mass**
- **Reminder-2: approx. 100,000000000 neutrinos pass every cm^2 of human body/sec**

Wrap-up Comments:

- **An attempt has been made to give an overview of particle physics focusing on neutrinos**
- **Reminder-1: neutrino mass is approx. 100,000000 times lighter than electron mass**
- **Reminder-2: approx. 100,000000000 neutrinos pass every cm^2 of human body/sec**

thank you

Back-up

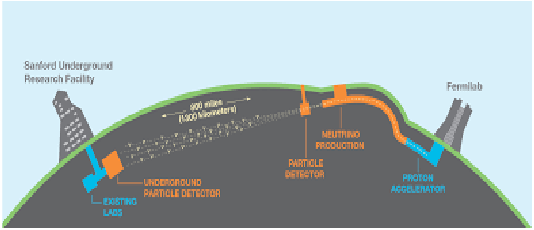
On-going Long Baseline Expts.:

T2K: Tokai to Kamioka, Japan

NOνA: NuMI Off-Axis ν_e Appearance, Fermilab, US

Up-coming Long Baseline Expts.:

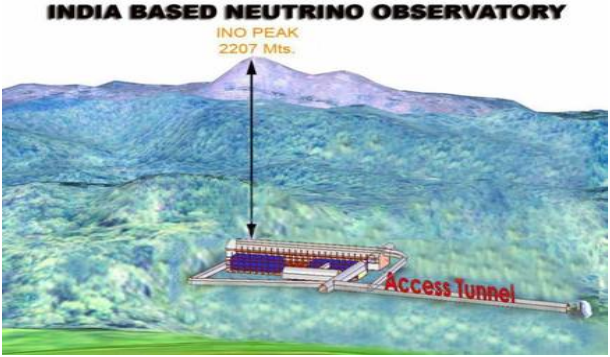
DUNE: Deep Underground Neutrino Experiment, US



T2HK: Tokai to Hyper Kamiokande is the upgraded version of T2K at Japan

Atmospheric ν expt.:

INO: India-based Neutrino Observatory, India



• Planned to construction in the Bodi West Hills Reserved Forest in the Theni district of Tamil Nadu.

• INO is the Iron-Calorimeter Detector which aims to probe the Earth matter effects on the propagation of atmospheric neutrinos.