Lecture 2: pQCD at hadron colliders, new perturbative methods, colllinear factorisation, parton densities and jets

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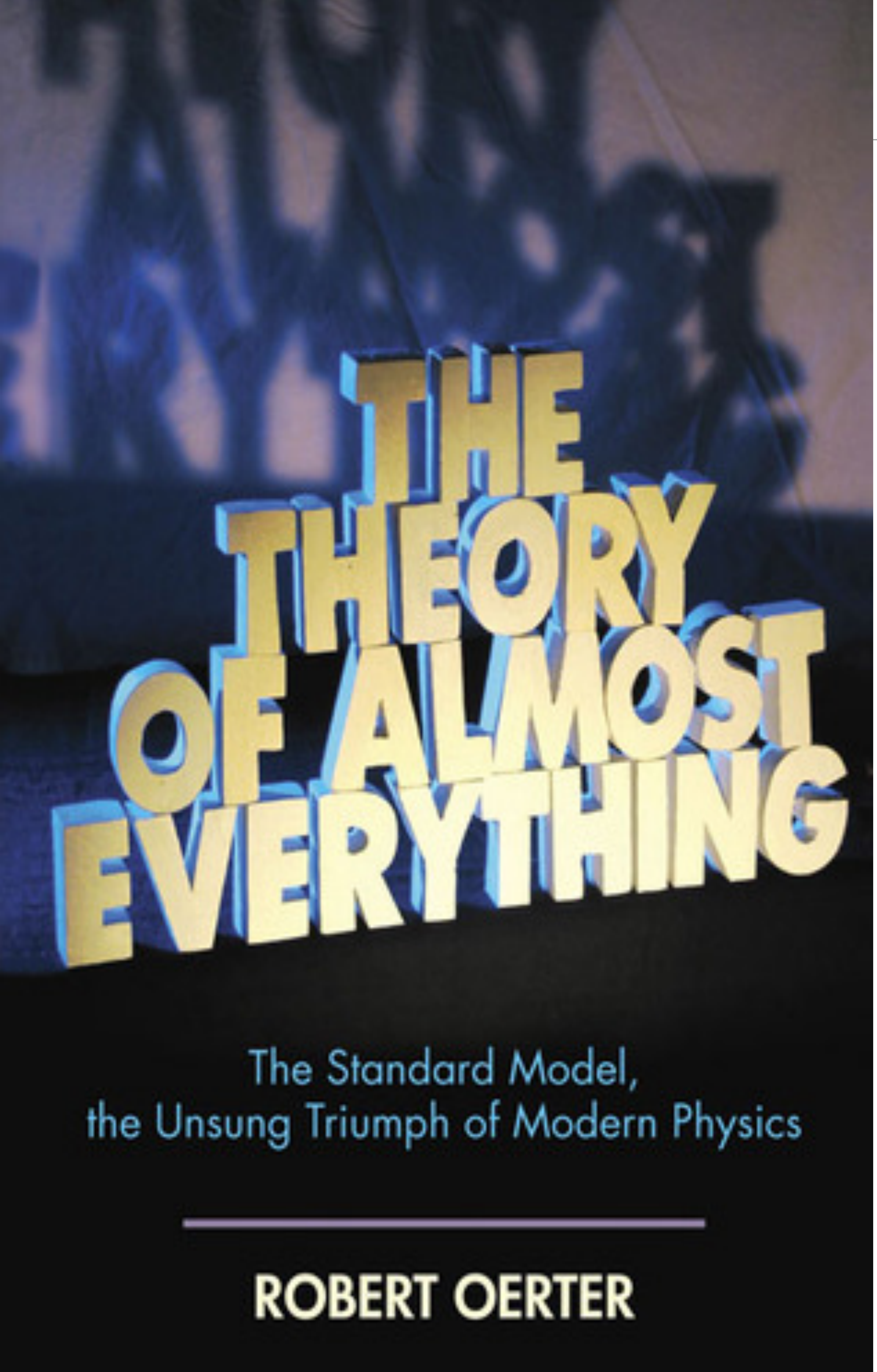
Strong interactions at colliders



- SM based in the simplest gauge symmetries: **SU(3)xSU(2)xU(1)**
- Also the **flavour sector very symmetric (GIM)**
- The **“natural”** theory at **“low” energies** (below the TeVs)
- We should expect that it will break at high energies: **departure scale undetermined | no theory guidance**

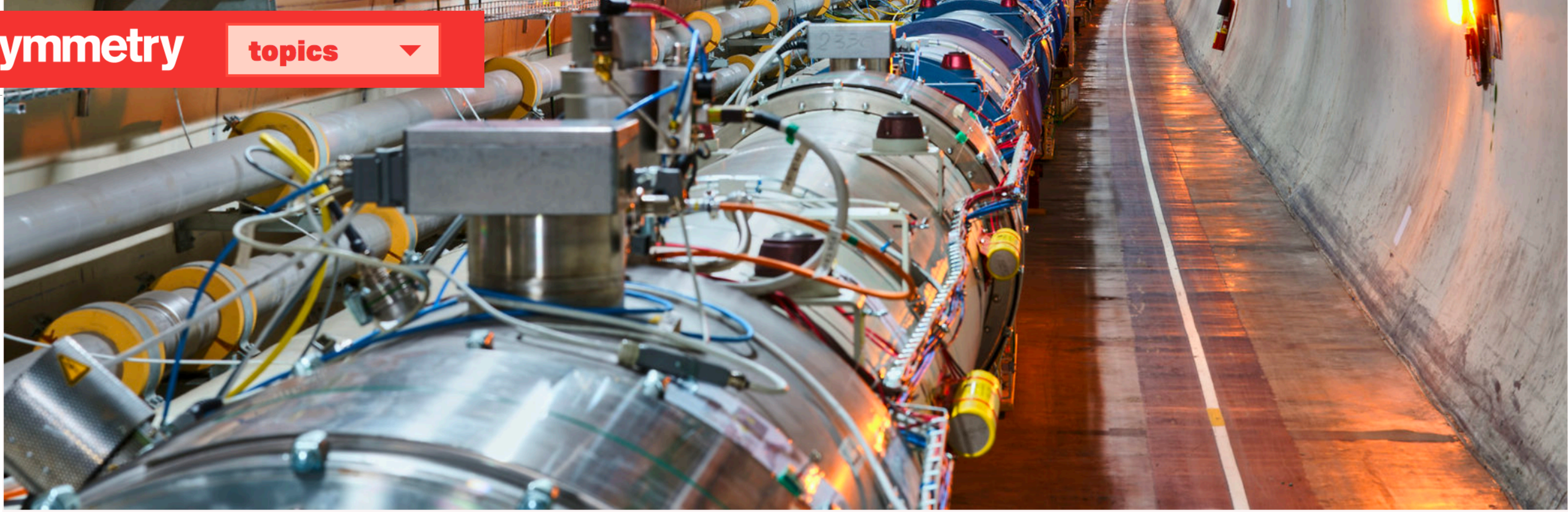
WHERE TO EXPECT A BSM SIGNAL?

- LHC results suggest that **new physics** will appear as a **gentle deviation** from the SM predictions / **rare events** suppressed in the SM

- Very unlikely to be visible in inclusive observables or total decay rates of known particles: the bulk of the contributions at “low energies”, the characteristic hard scale is “**low energy**”

- Higher chances at the tail of differential distributions (not necessarily a clear bump) **“high energy”** characteristic hard scale: more sensitive to **quantum corrections** / missing quantum corrections can fake BSM

“ Precise measurements of known particles and interactions are just as important as finding new particles symmetry topics



Maximilien Brice and Julien Marius Ordan, CERN

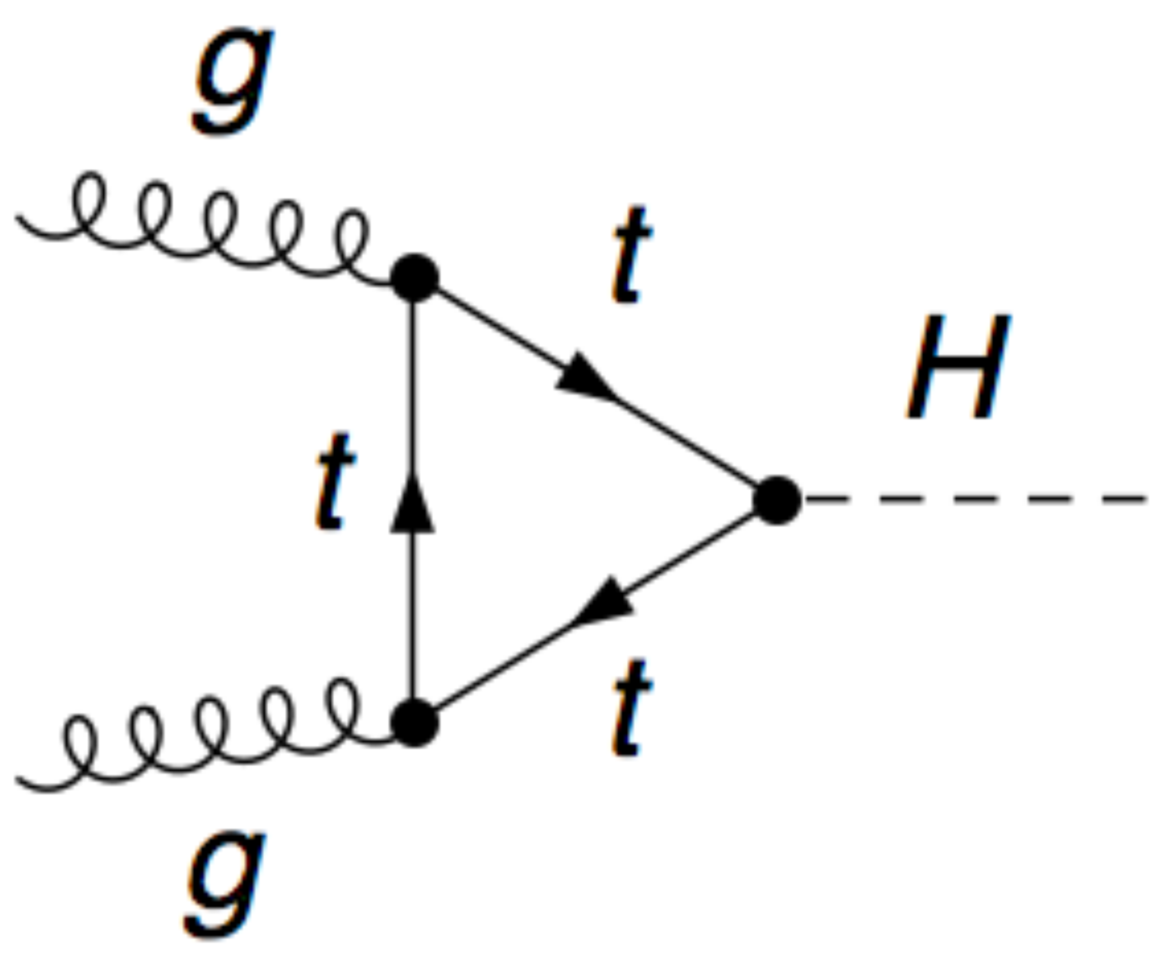
The unseen progress of the LHC

05/02/19 | By Sarah Charley

It's not always about what you discover.

“This work naturally pushes our search methods towards making more detailed and higher precision measurements that will help us constrain possible deviations by new physics,” Willocq says.

New channels open from LO to NLO at hadron colliders



Higgs boson production
is one loop at **LO**

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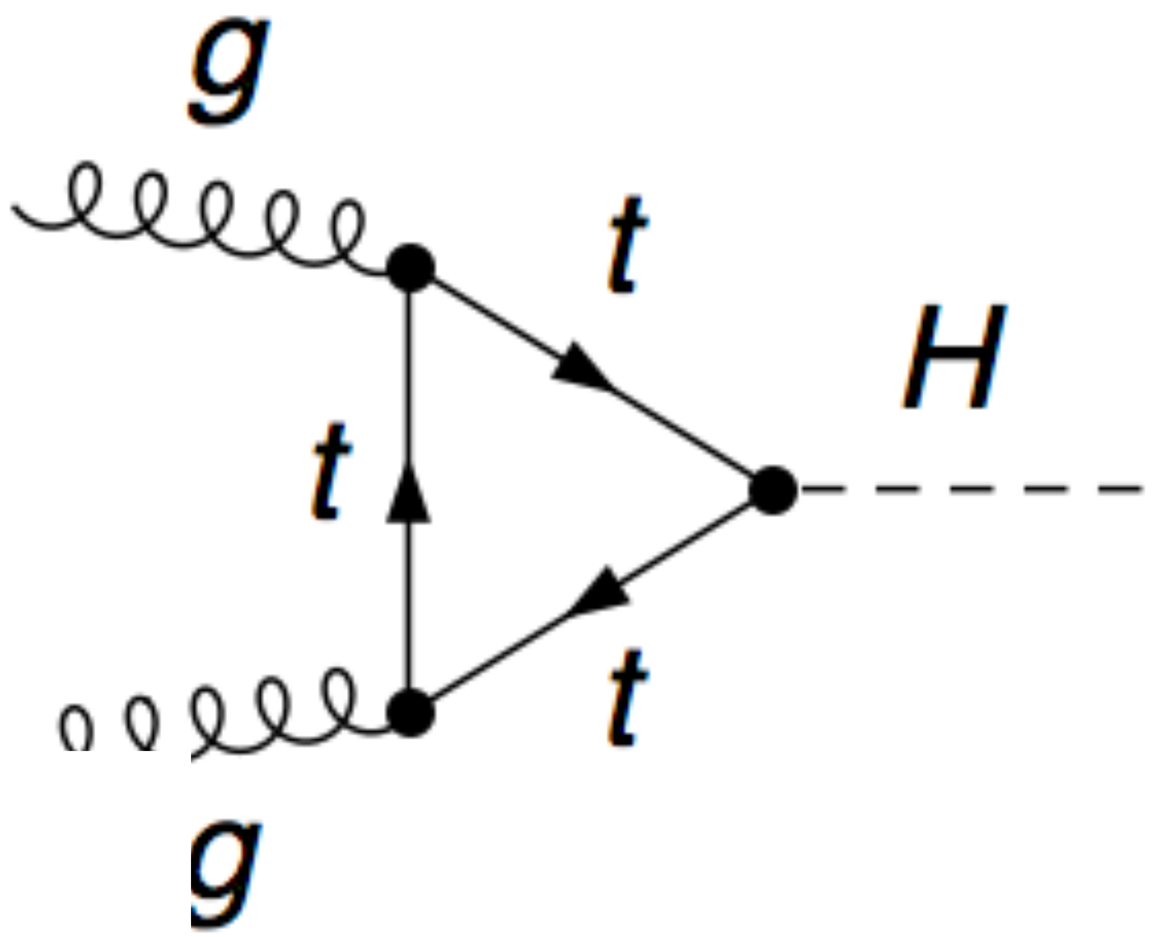

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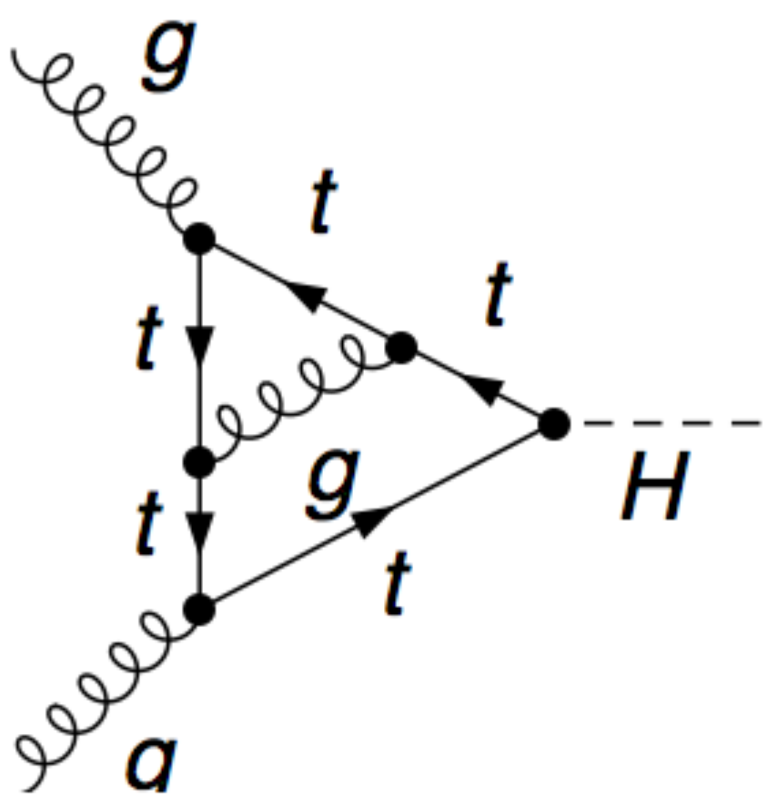

QCD correction to the **LO**

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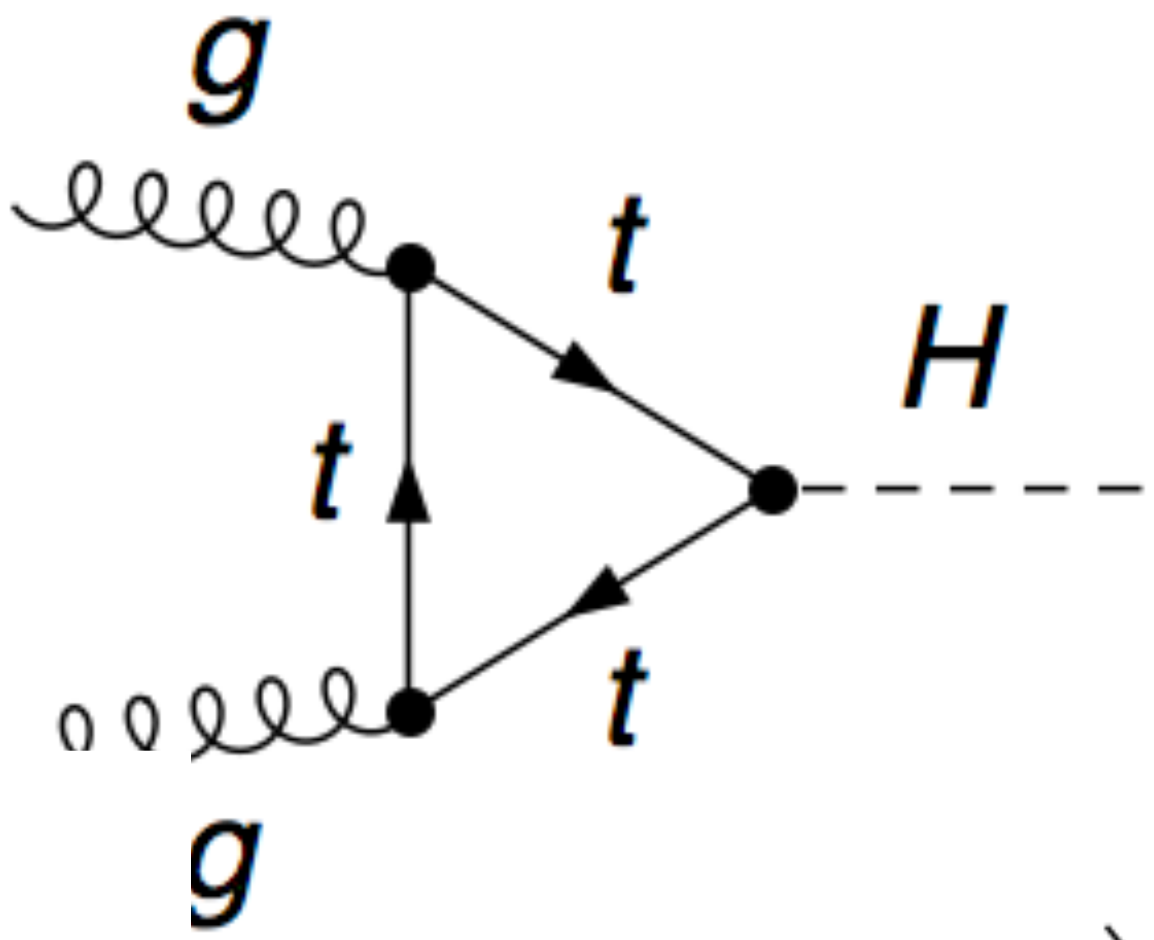

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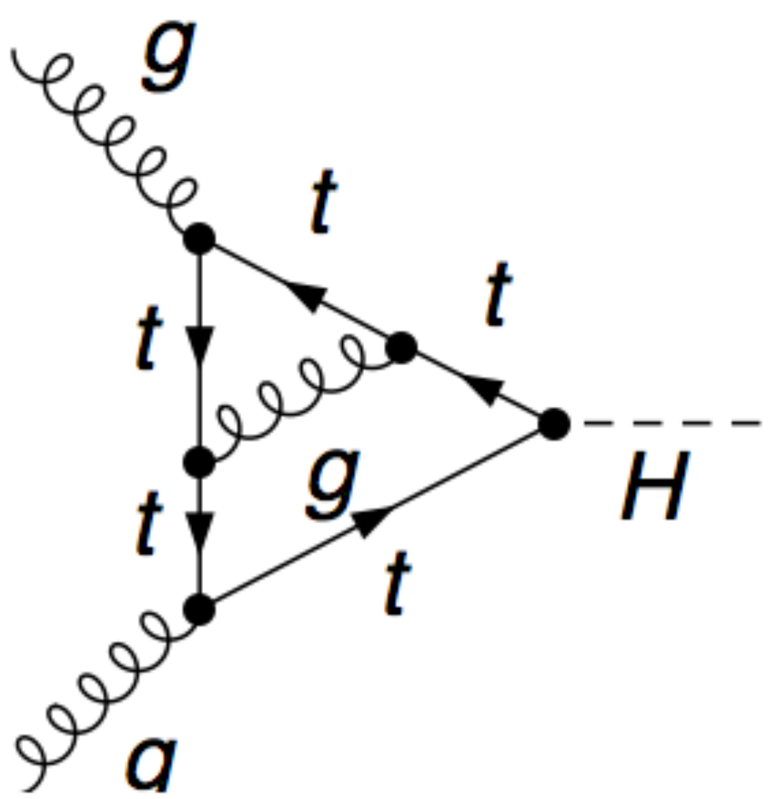

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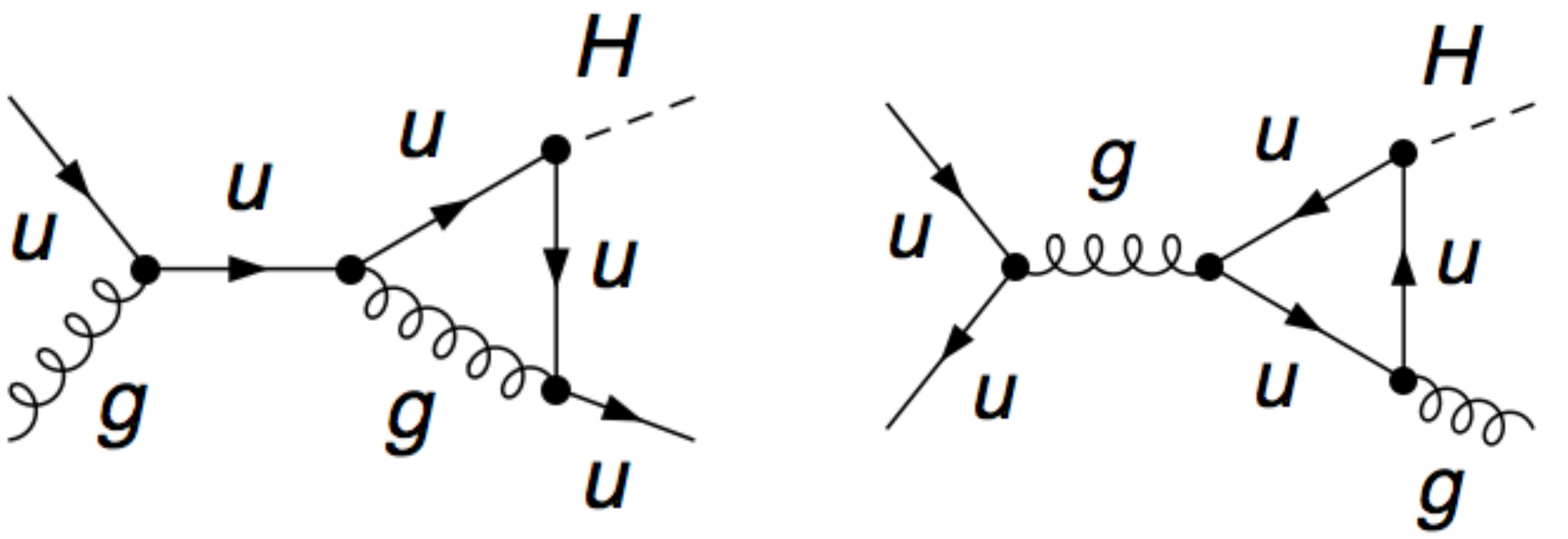

New channels at **NLO:** $qg(\bar{q}g)$ and $q\bar{q}$

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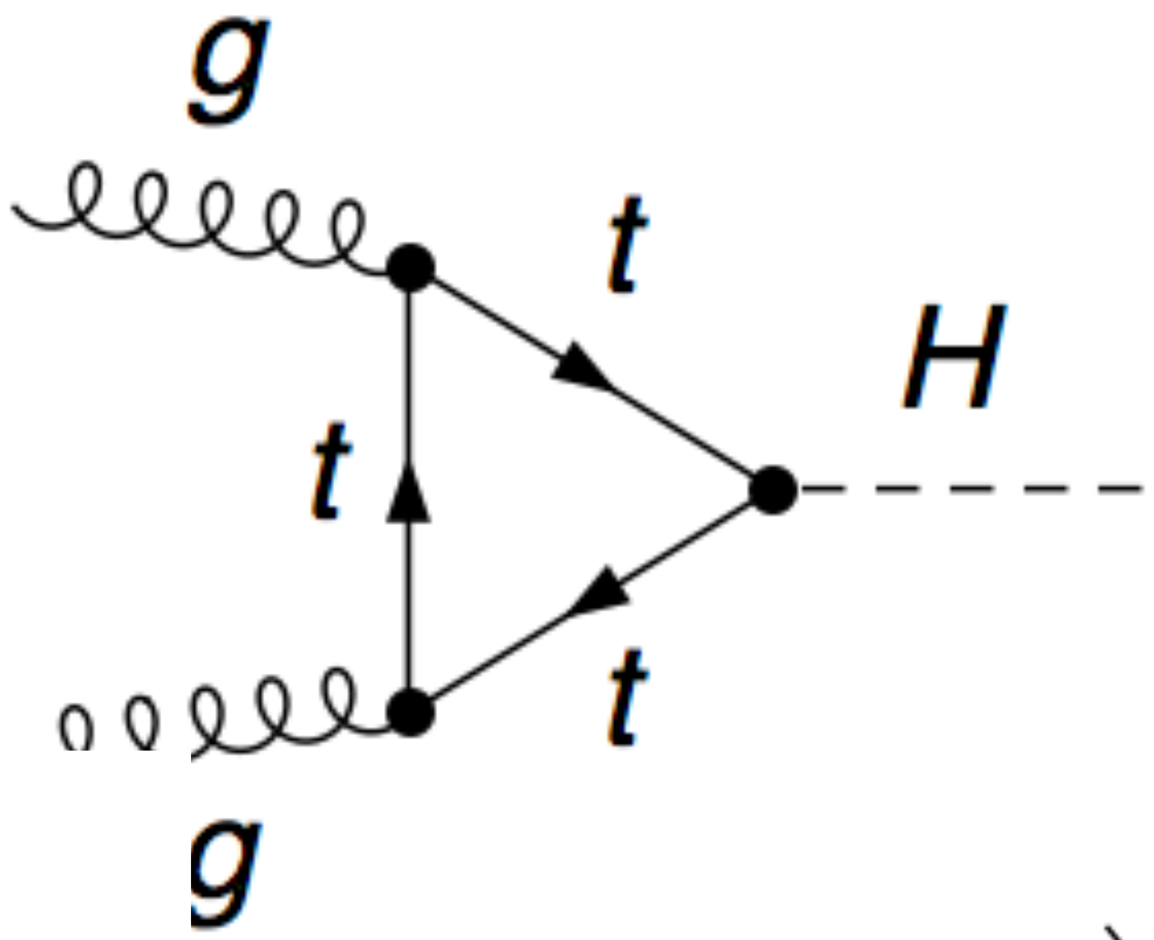

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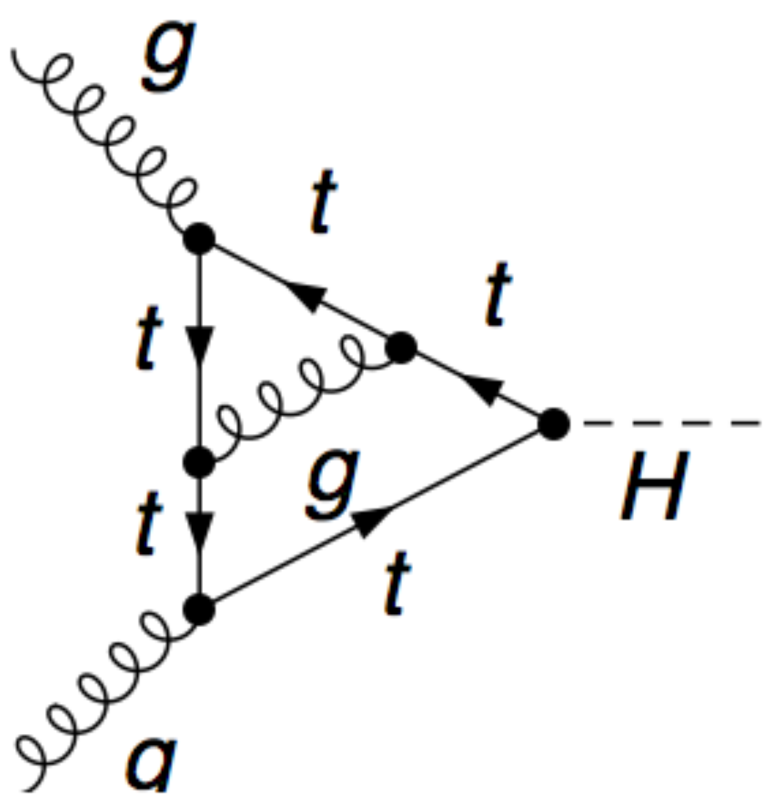

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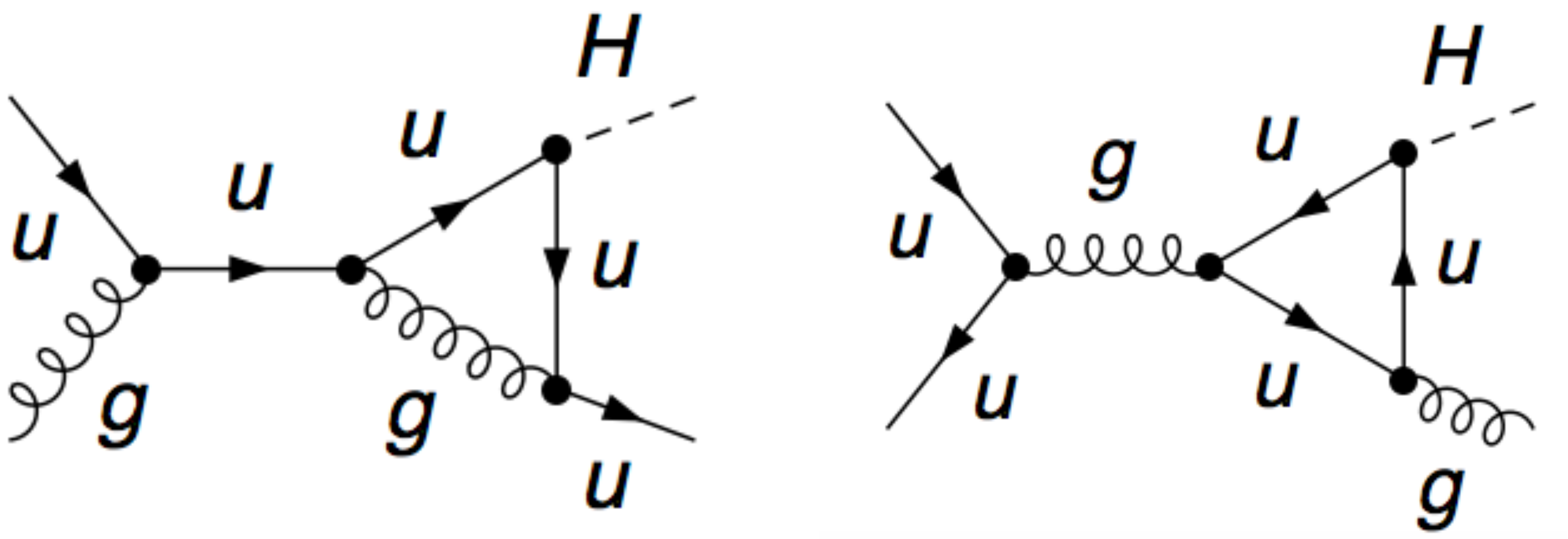
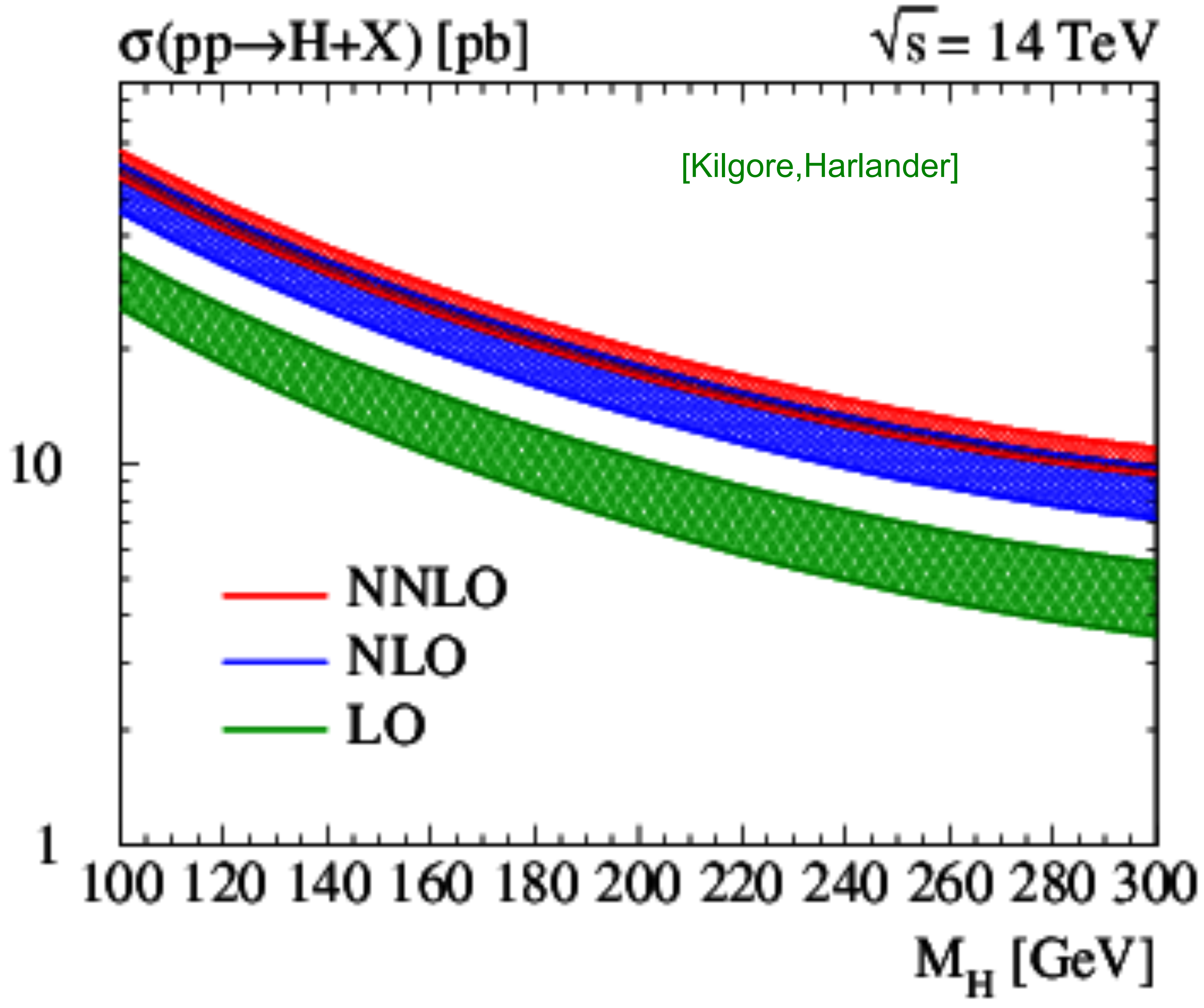


New channels at **NLO:** $qg(\bar{q}g)$ and $q\bar{q}$

- Only **NNLO** is a correction to all the channels that appear at the NLO

Perturbative progress at hadron-hadron colliders



higher perturbative orders **improve systematically** the precision of the theoretical predictions (estimated by **varying the renormalization / factorization scales**) for background and signal. Uncertainty bands are **expected to narrow and overlap** from one order to the next one

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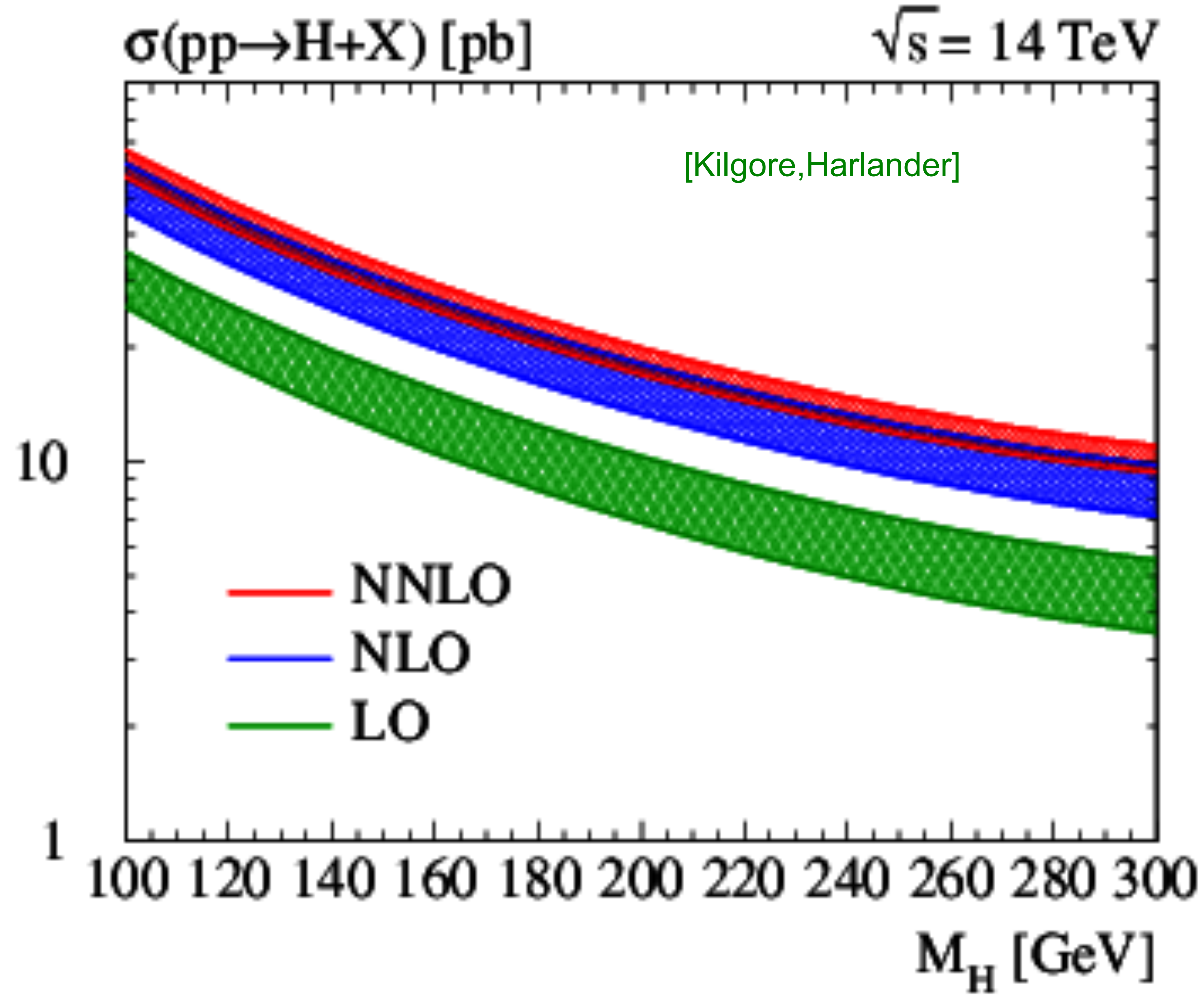

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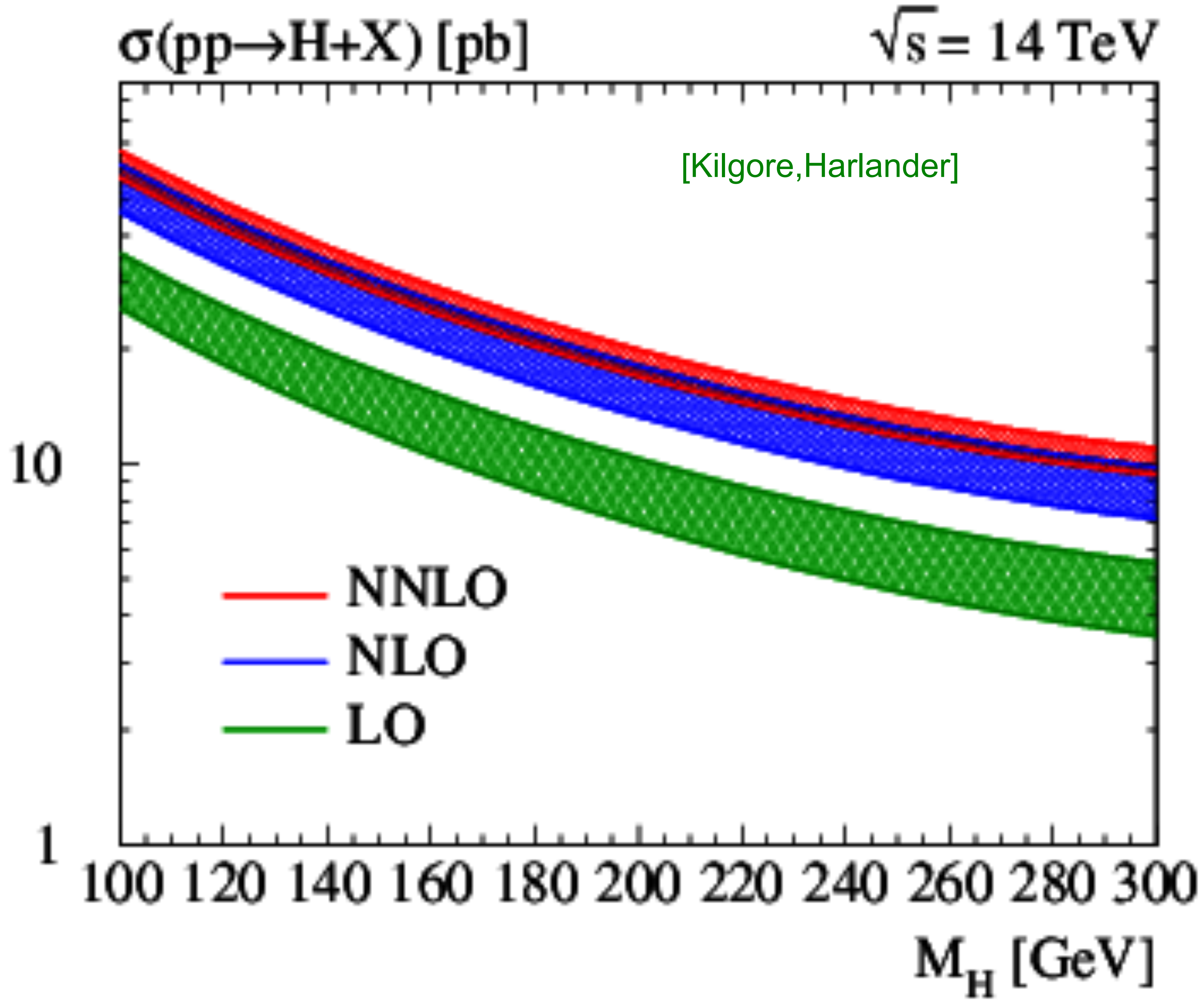

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- **NLO:** first reliable estimate of **central value** (because protons are not elementary)

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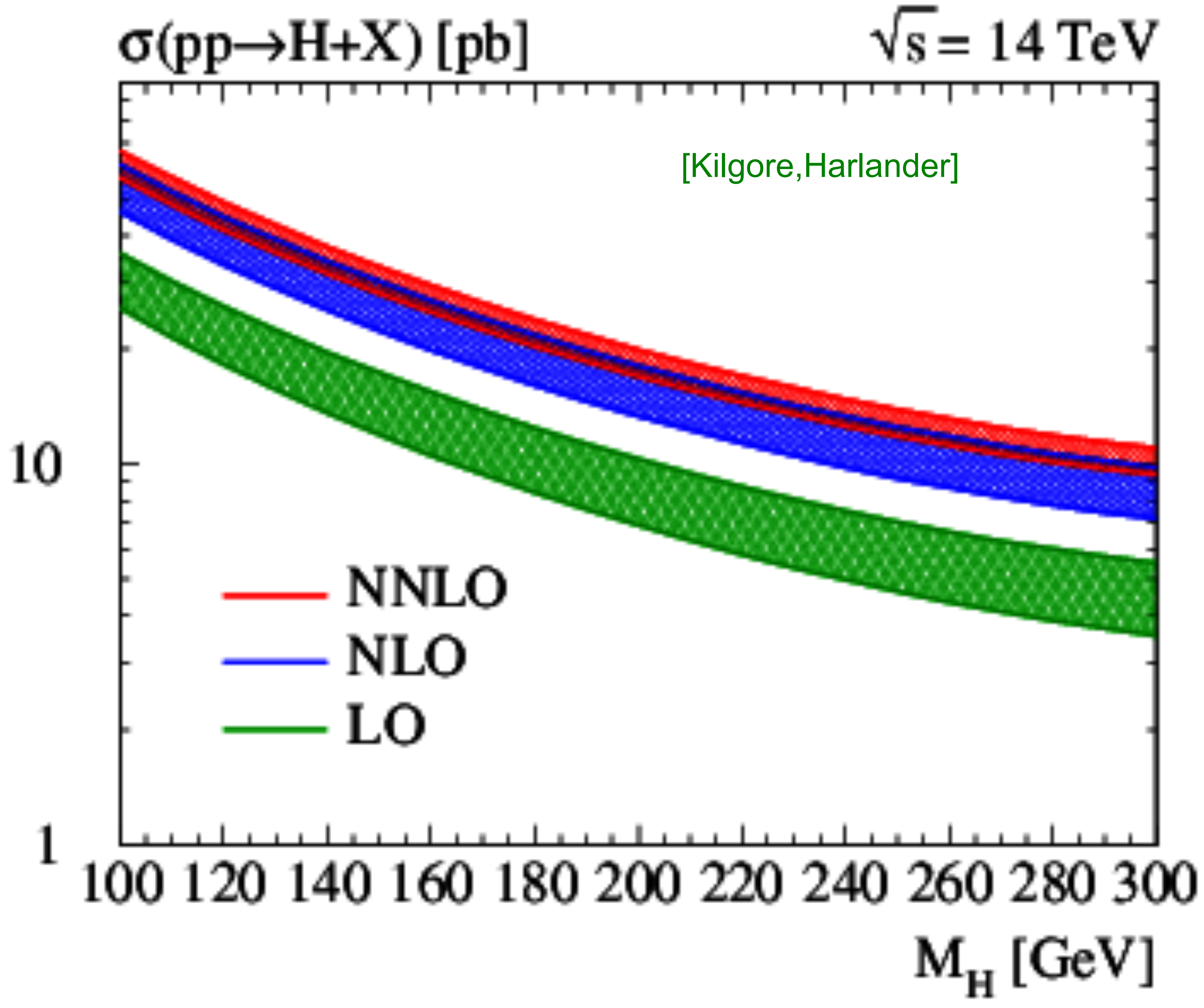

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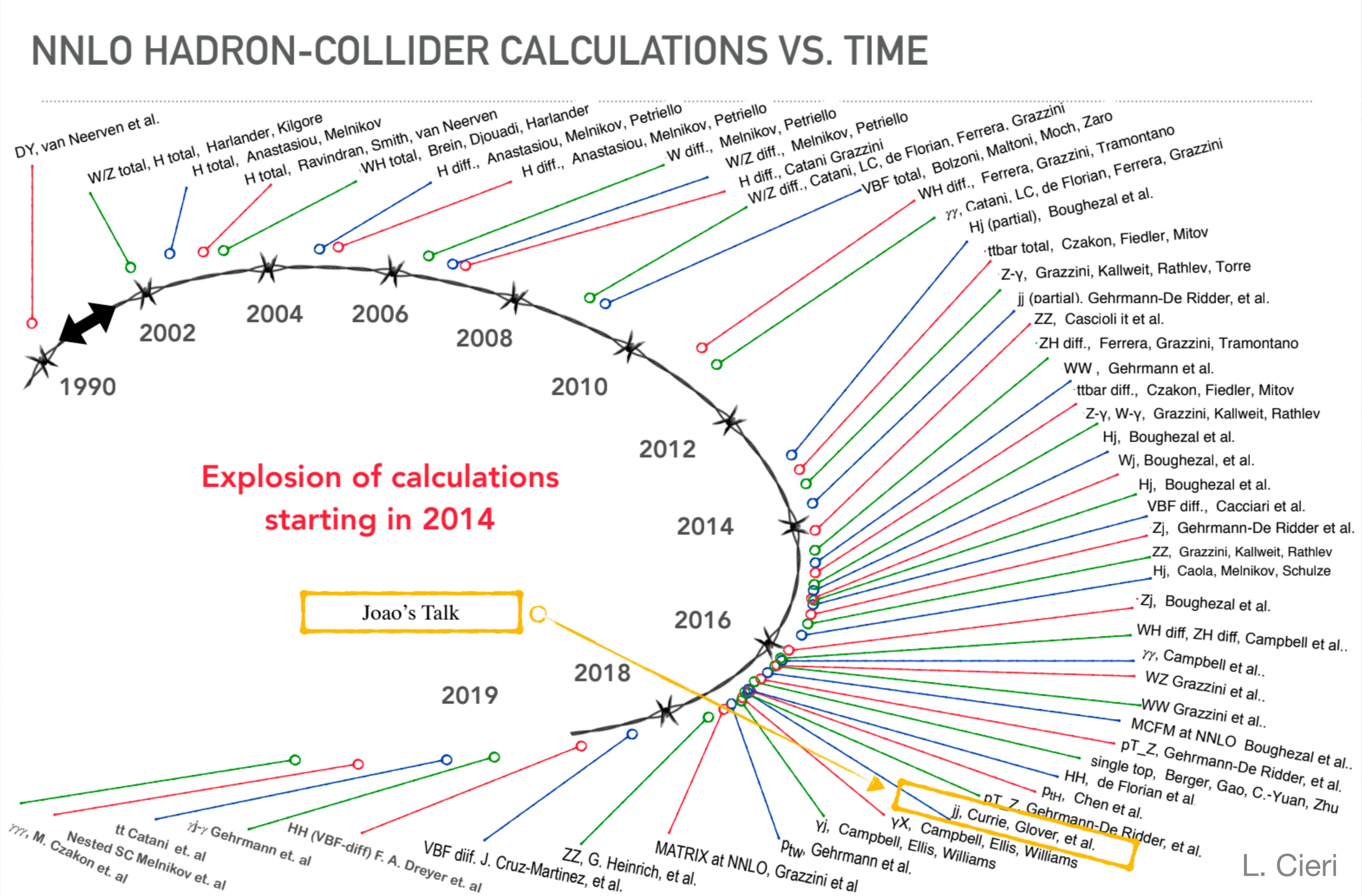
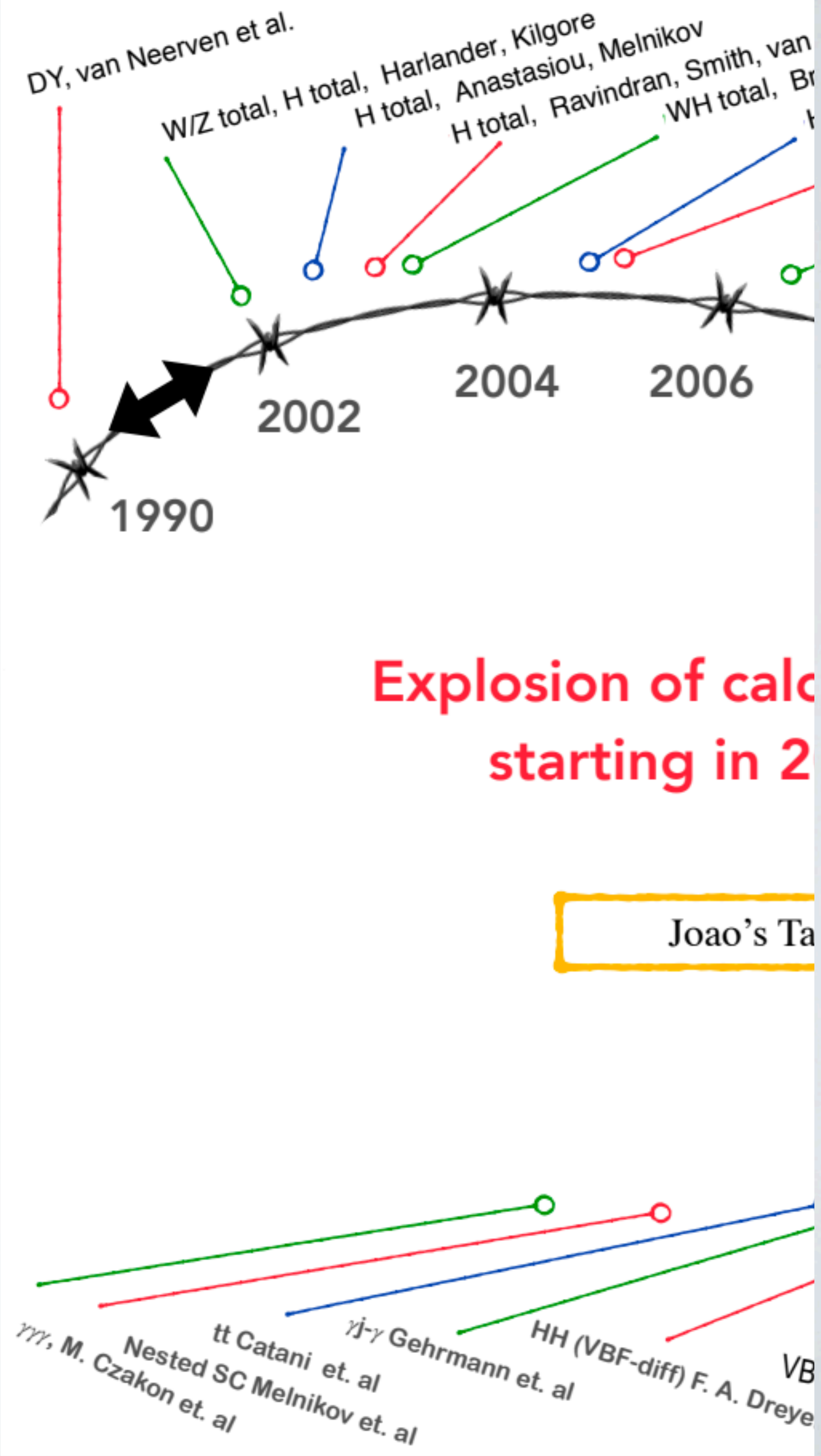
- **NLO:** first reliable estimate of **central value** (because protons are not elementary)
- **NNLO:** first **serious estimate** of the theoretical uncertainty

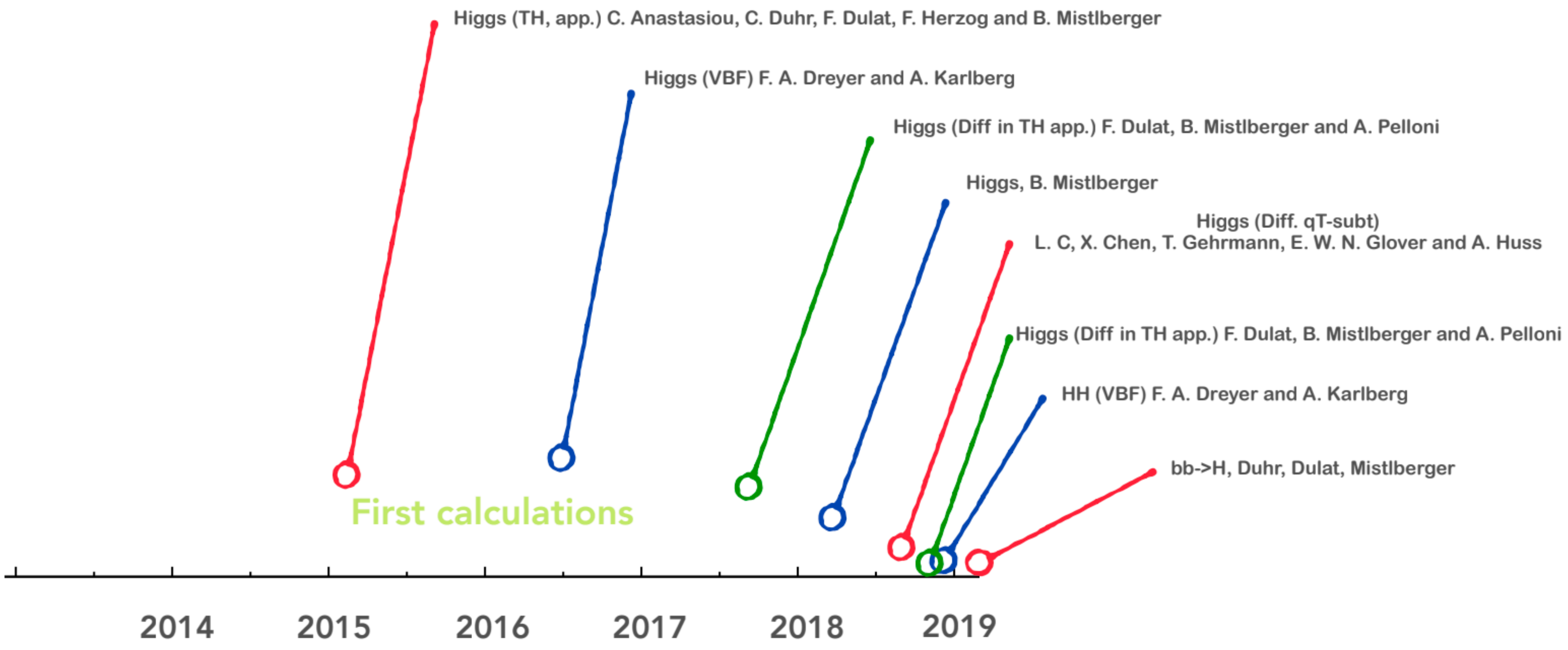
THE NNLO STANDARD



THE N^{3}LO ERA

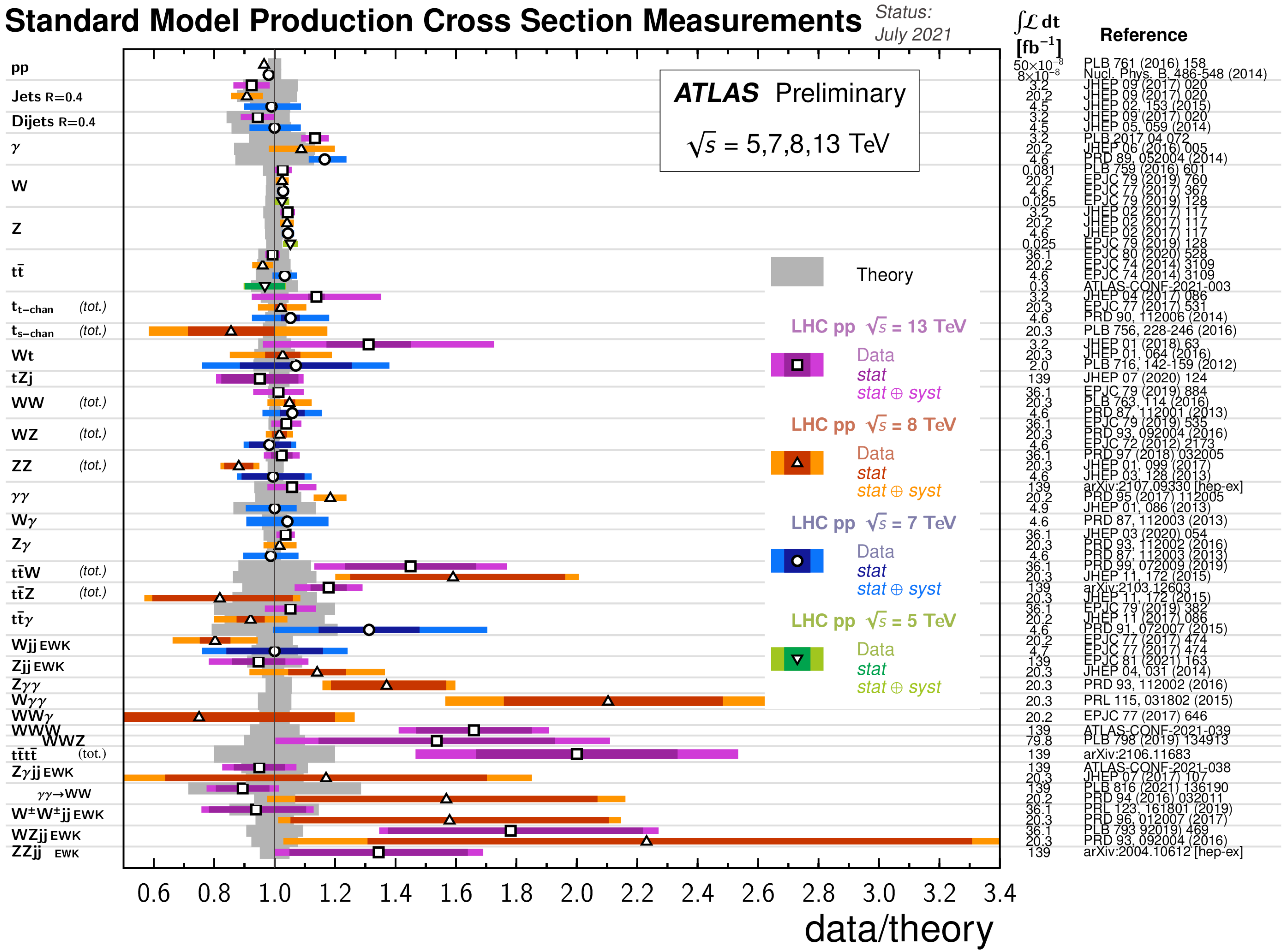
N^{3}LO HADRON-COLLIDER CALCULATIONS VS. TIME





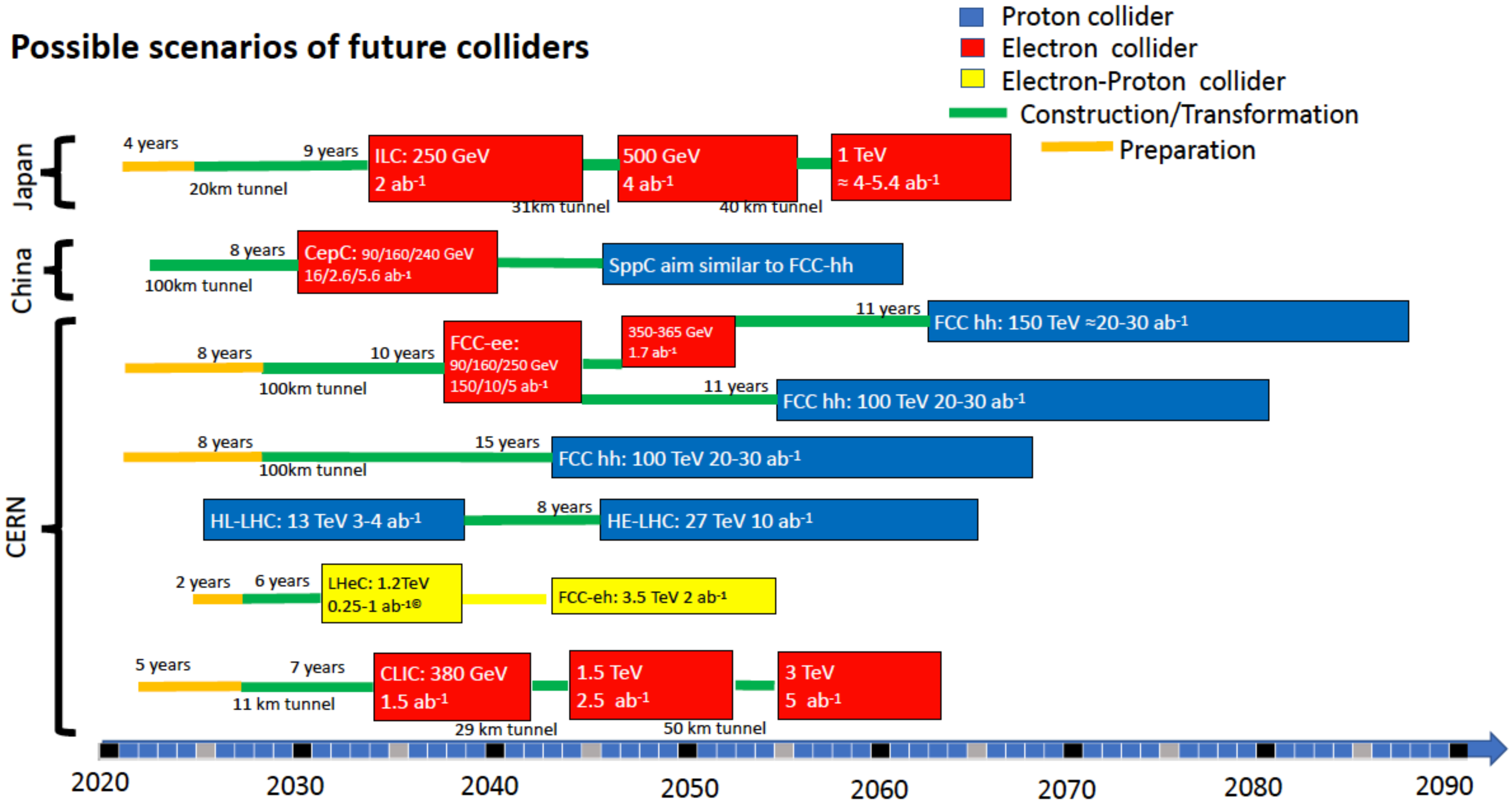
L. Cieri

DATA/THEORY | INCLUSIVE OBSERVABLES 5-20% THEORETICAL ACCURACY



G. Rodrigo, XIX MX School of Particles & Fields 2021

Possible scenarios of future colliders



13/05/2019 UB

Need for precision @ HL-LHC

- illustrated in the case of Higgs physics
- theory uncertainty (PDF + strong coupling + missing higher orders) dominates in 7/9 channels
- this is **with the assumption of reduction by x2 in today's uncertainties**
- depending on channel, it can be the uncertainties for the signal or the background that dominates.

Gavin Salam@ESPP2019

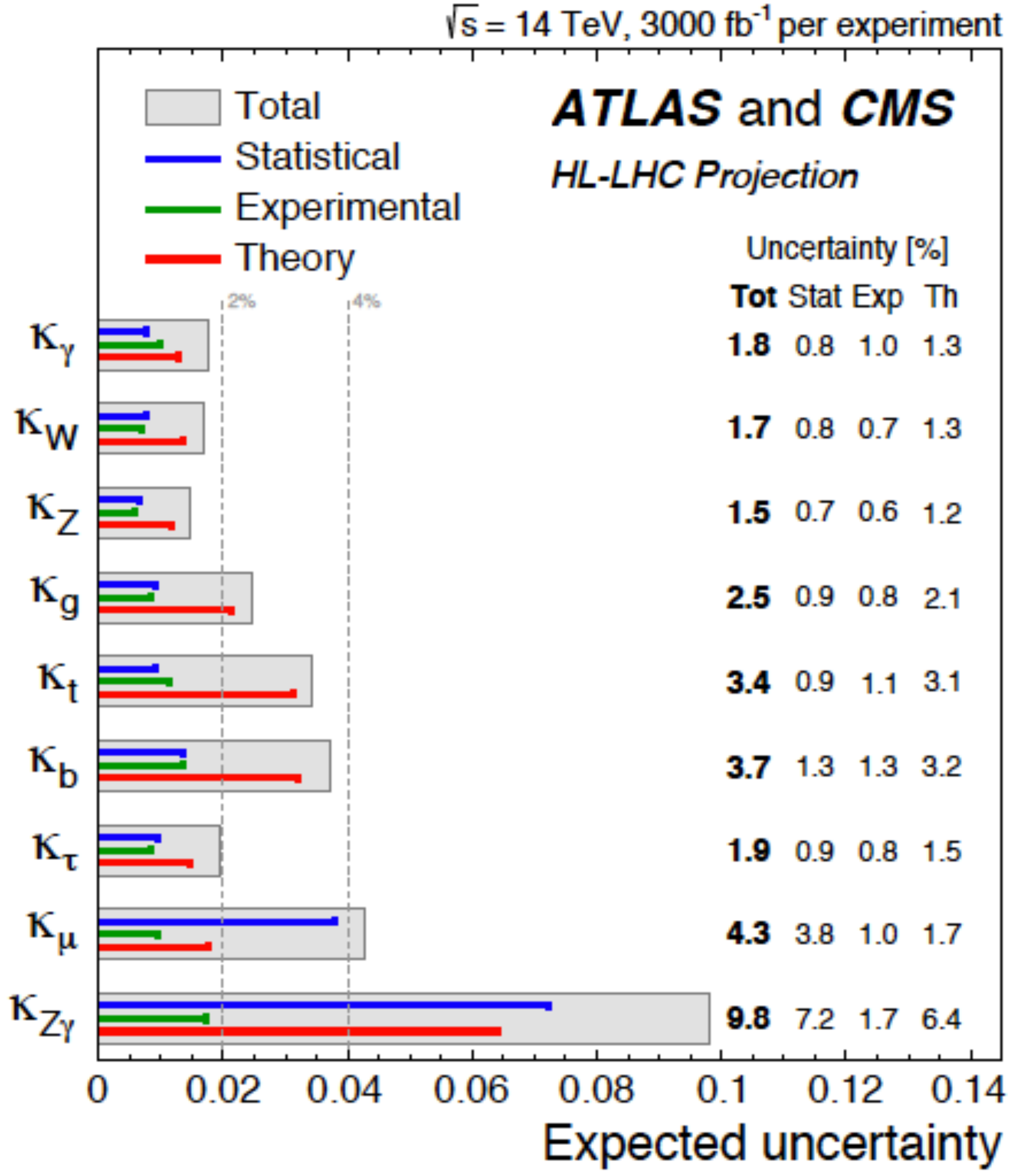
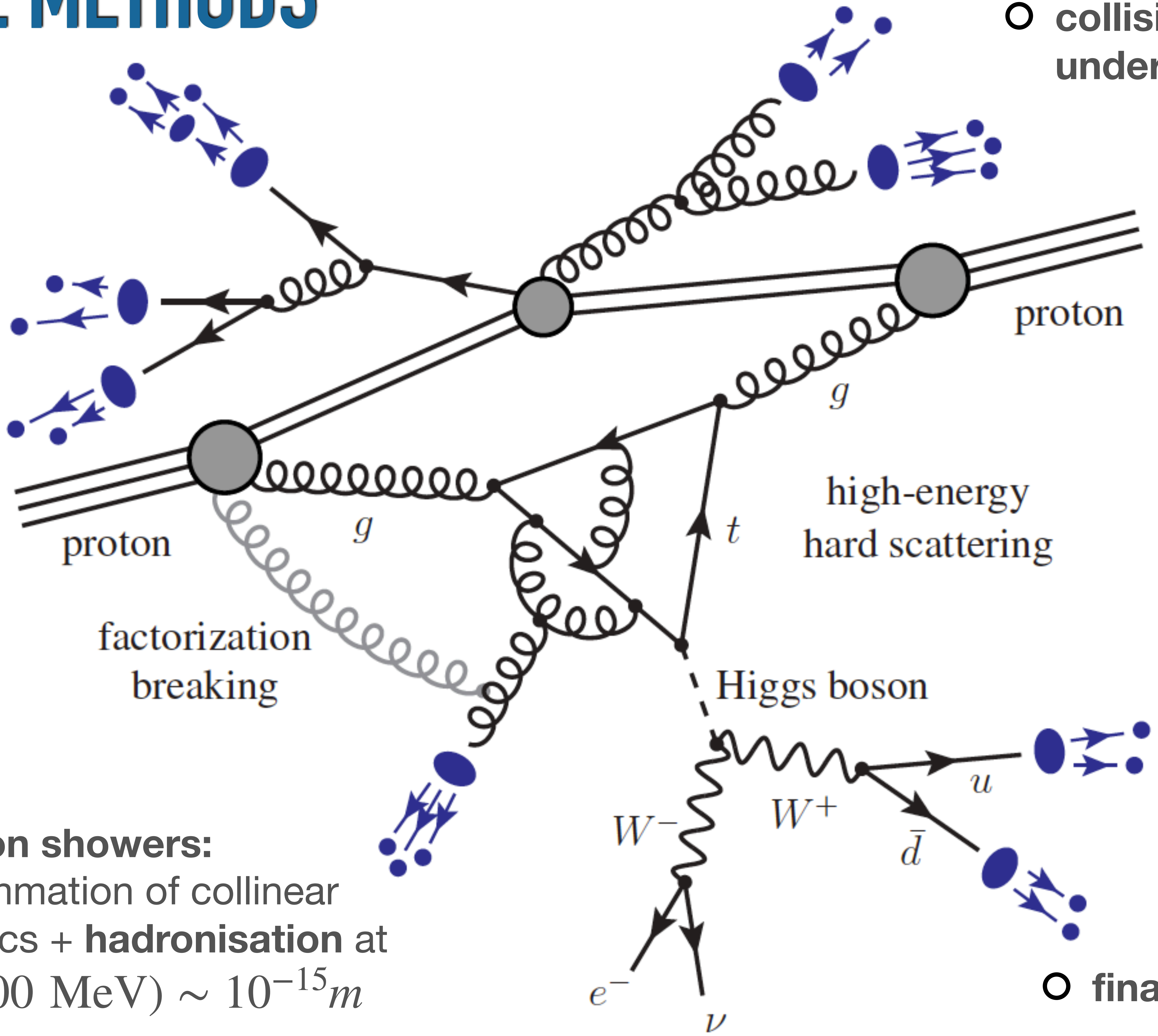


Figure 1. Projected uncertainties on κ_i, combining ATLAS and CMS: total (grey box), statistical (blue), experimental (green) and theory (red). From Ref. [2].

NEW PERTURBATIVE METHODS

factorisation into short distance (hard scattering = high energy) and long distance (initial and final state = low energy)



- **collision remnants / underlying event**
- High-energy **collision** at $1/\mathrm{TeV} \sim 10^{-19} m$
- **initial-state** parton densities $1/\mathrm{GeV} = 10^{-16} m$
- **Parton showers:** resummation of collinear physics + **hadronisation** at $1/(200\ \mathrm{MeV}) \sim 10^{-15} m$
- **final state:** e.g. jets
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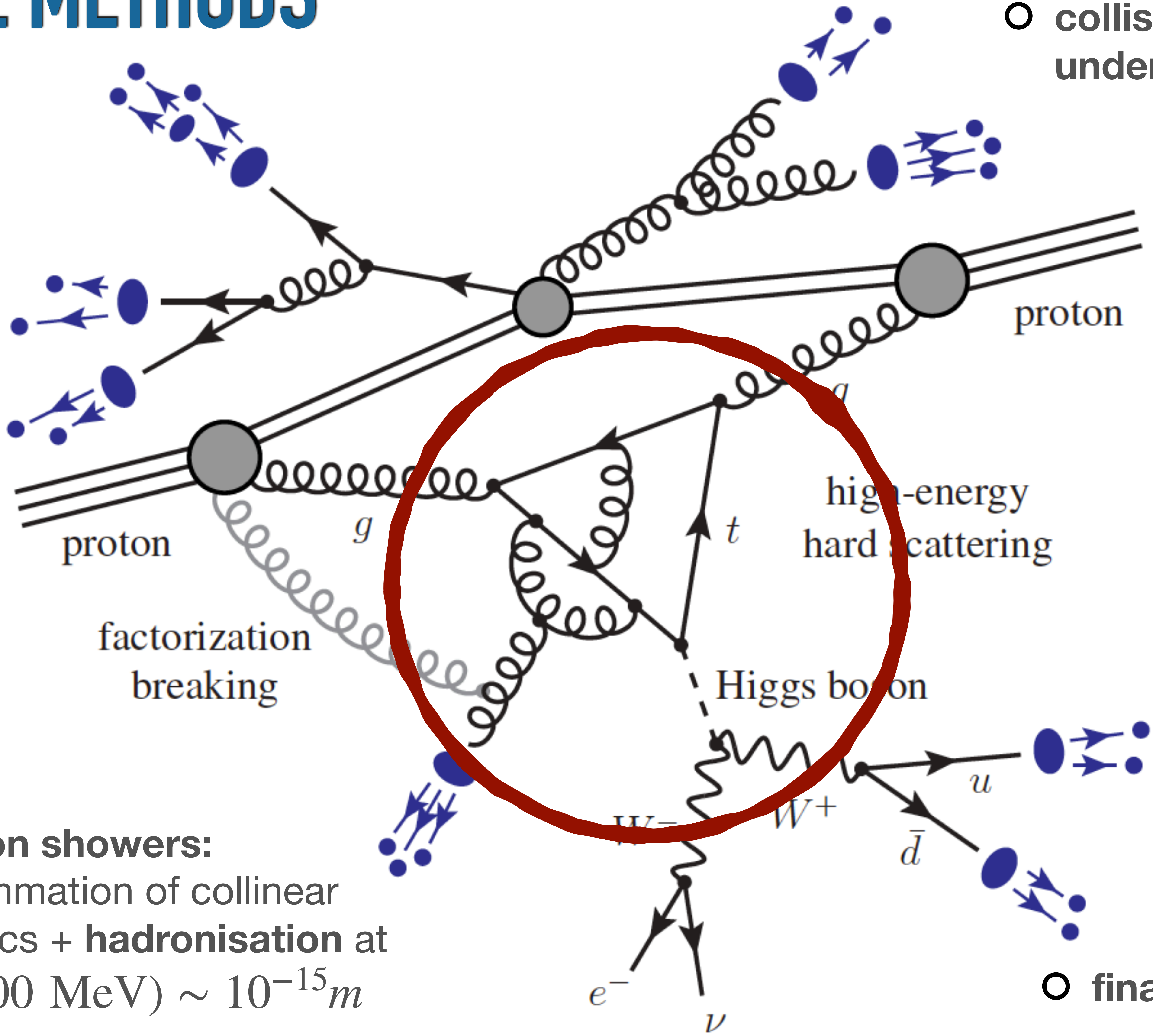

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pQFT without Feynman diagrams

One-loop amplitudes

- The classical paradigm for the calculation of one-loop diagrams was established in 1979

Reduction of tensor one-loop integrals to scalar integrals

G. Passarino, M. Veltman
One-loop corrections for e+e– annihilation into μ+μ– in the Weinberg model
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- Difficult for processes beyond 2→2
(Gramm determinants + large number of Feynman diagrams)
- At two-loops: reduction to Master Integrals (not unique) e.g. by **Integration-By-Parts** Identities [Chetyrkin, Tkachov 1981, Tarasov 1998, Laporta 2000]

Properties of the S-Matrix

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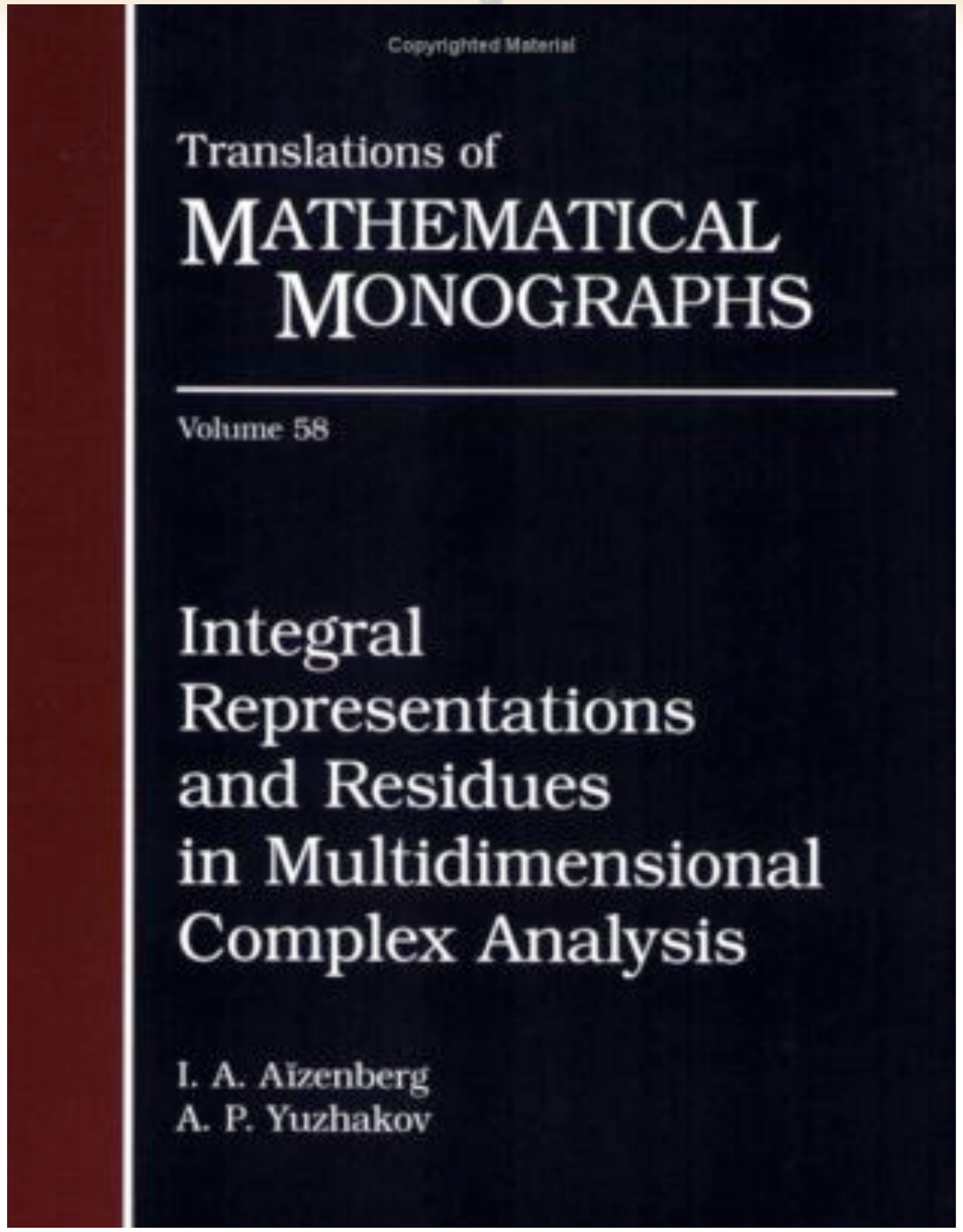
- **recycling**: using scattering amplitudes to calculate other scattering amplitudes



Here are the words of some enthusiast: “One of the most remarkable discoveries in elementary particle physics has been that of the existence of the **complex plane**”, “... the theory of **functions of complex variables** plays the role not of a mathematical tool, but of a fundamental description of nature inseparable from physics … .”

J. Schwinger, *Particles, Sources, and Fields*, Vol.1, p.36

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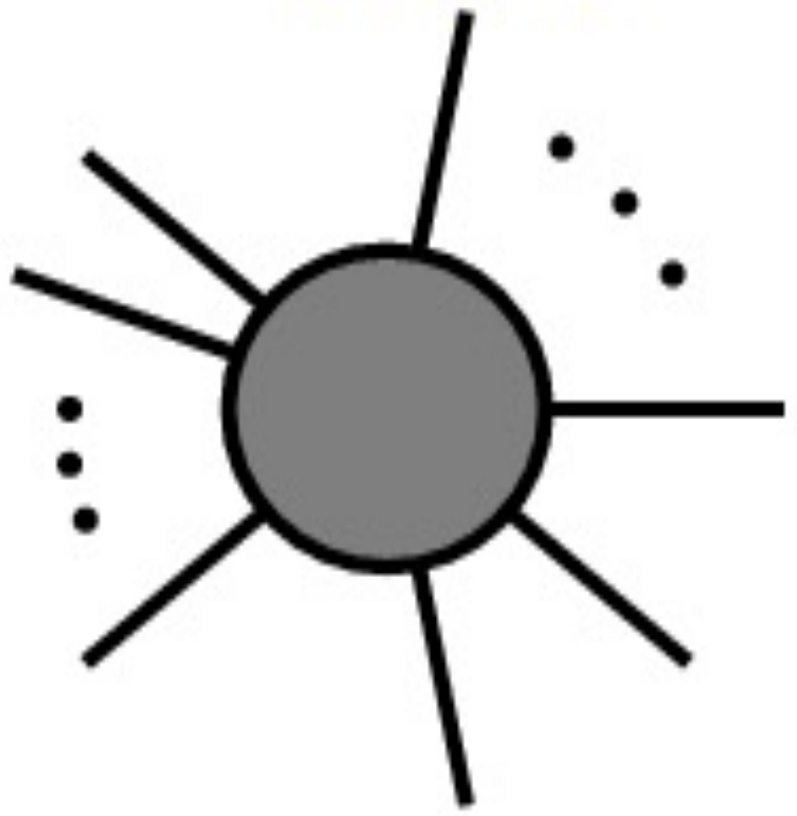
Helicity basis + colour decomposition



Expressions simplify by using “right variables” | e.g. for N-gluons at tree level

$$\mathcal{M}_N^{(0)}(\{p_i, h_i, a_i\}) = \sum_{P(1,\ldots,N)} \mathrm{Tr}(\mathbf{t}^{a_1}\mathbf{t}^{a_2}\cdots\mathbf{t}^{a_N})\, \mathcal{A}_N^{(0)}(\{p_i, h_i\})$$

sum over permutations

color ordered factor

colour ordered subamplitude:

- Depends on the momenta and helicities
- **gauge**-invariant
- fixed cyclic order of external legs

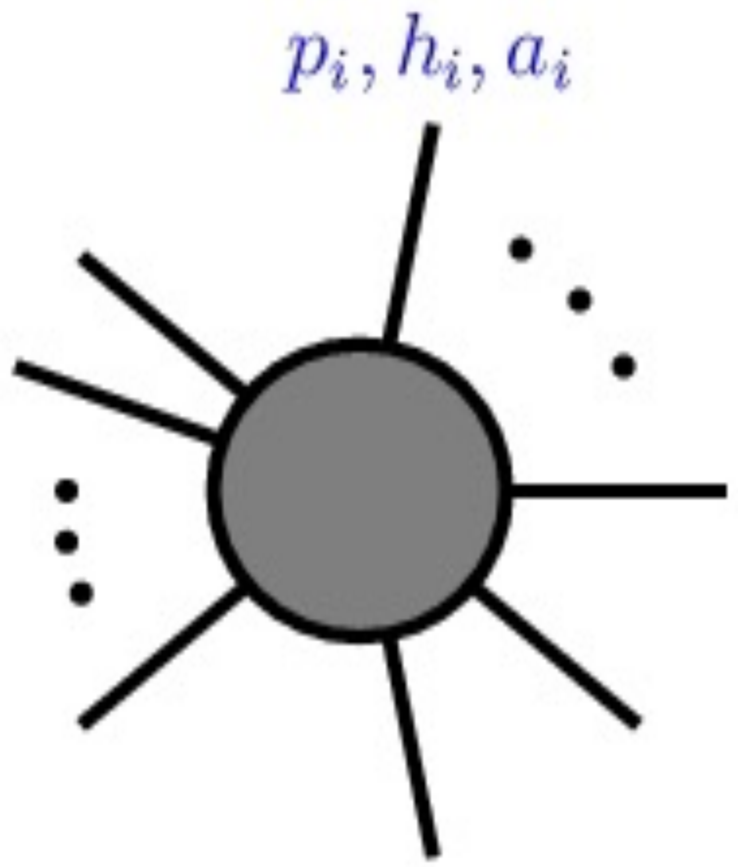
N-gluon amplitude

# gluons	# diagrams	# color ordered diagrams
4	4	3
5	25	10
6	220	36
7	2.485	133
8	34.300	501
9	559.405	1.991
10	10.525.900	7.225

[Cvitanovic, Lauwers, Scharbach, Berends, Giele, Mangano, Parke, Xu,Bern,Kosower, Lee, Nair]

Spinors

[Xu,Zhang,Chang, Berends, Kleiss, De Causmaeker, Gastmans, Wu,Gunion, Kunzst]



Four-dimensional spinors of definite helicity

$$|i^\pm\rangle = \frac{1}{2}(1 \pm \gamma_5)\, u(p_i) = v_\mp(p_i) \qquad \langle i^\pm | = \bar{u}_\pm(p_i) = \bar{v}_\mp(p_i)$$

$$p_i^2 = 0 \qquad p_i^{a\dot{a}} = p_i^\mu \sigma_\mu^{a\dot{a}} = \lambda_i^a \tilde{\lambda}_i^{\dot{a}}$$

spinor algebra and other useful identities

- holomorphic inner product:

$$\langle ij\rangle = \langle i^- | j^+\rangle = \varepsilon_{ab}\lambda_i^a\lambda_j^b = \sqrt{|s_{ij}|}\, e^{\imath\phi_{ij}} = -\langle ji\rangle$$

- anti-holomorphic inner product:

$$[ij] = \langle i^+ | j^-\rangle = \varepsilon_{\dot{a}\dot{b}}\tilde{\lambda}_i^{\dot{a}}\tilde{\lambda}_j^{\dot{b}} = -\langle ij\rangle^* = -[ji]$$

- sum over polarisations $\not{p}_i = |i\rangle[i| + |i]\langle i|$
- equation of motion $\not{p}_i |i^\pm\rangle = 0$
- other $\langle ij] = 0 = \langle ii\rangle \qquad [i|\gamma^\mu|j\rangle = \langle j|\gamma^\mu|i]$

  $s_{ij} = (p_i + p_j)^2 = \langle ij\rangle[ji]$

gluon polarisation vectors

$$\epsilon^2 = 0 = \epsilon^+ \cdot \epsilon^- \ , \quad k \cdot \epsilon^\pm(k) = 0$$

$$\epsilon_\mu^+(k,\xi) = \frac{\langle \xi|\gamma_\mu|k]}{\sqrt{2}\,\langle \xi k\rangle}$$

$$\epsilon_\mu^-(k,\xi) = \frac{[\xi|\gamma_\mu|k\rangle}{\sqrt{2}\,[k\xi]}$$

- equivalent to **axial gauge** $\xi = n$
- a clever choice of the gauge momentum can simplify calculations

MHV amplitudes

Multi-gluonic amplitudes at tree level: Amplitude for all gluons of positive helicity or one single gluon of negative helicity vanishes

- two negative helicities (**Maximal Helicity Violating** Amplitude) rather simple [Parke-Taylor, 1986]

$$\mathscr{A}_n^{(0)}(1^+, \ldots, i^\pm, \ldots, n^+) = 0$$

$$\mathscr{A}_n^{(0)}(1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle (n-1)n \rangle \langle n1 \rangle}$$

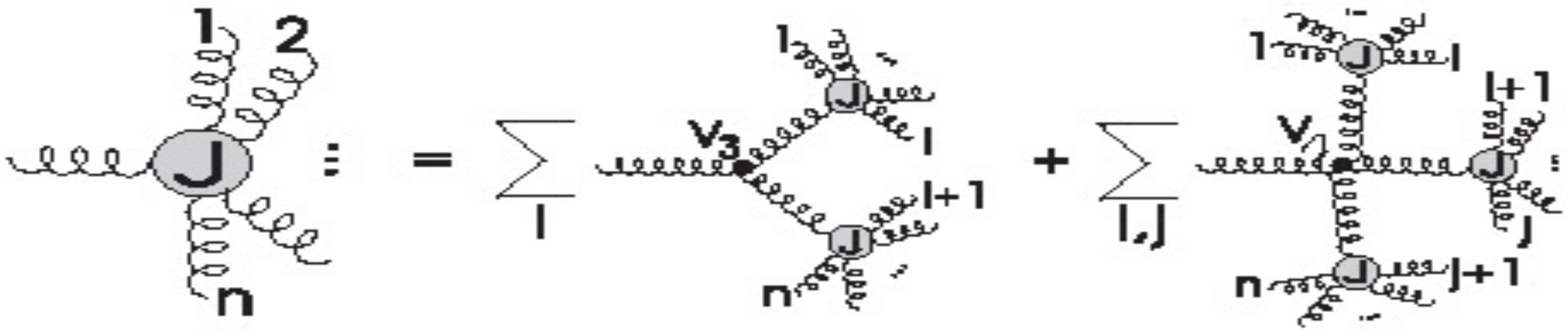
proven via recursion relations [Berends-Giele, Mangano-Parke-Xu,1988]

next-to-MHV $\mathscr{A}_n^{\mathrm{NMHV}}(1^+, \ldots, i^-, \ldots, j^-, \ldots, k^-, \ldots, n^+)$ does contain both $\langle ij \rangle$ and $[ij]$ [Kosower,1990]

Off-shell recursion relations

[Berends, Giele]

- Define **Off-shell** current: amplitude with one off-shell leg, building block for the off-shell current with higher multiplicity



- the gluonic current particularly simple for some helicity configurations

$$J^{\mu}(i^+, \ldots, j^+) = \frac{\langle \xi | \gamma^{\mu} \not{p}_{i,j} | \xi \rangle}{\sqrt{2}\langle \xi i \rangle \langle i (i+1) \rangle \cdots \langle j \xi \rangle}$$

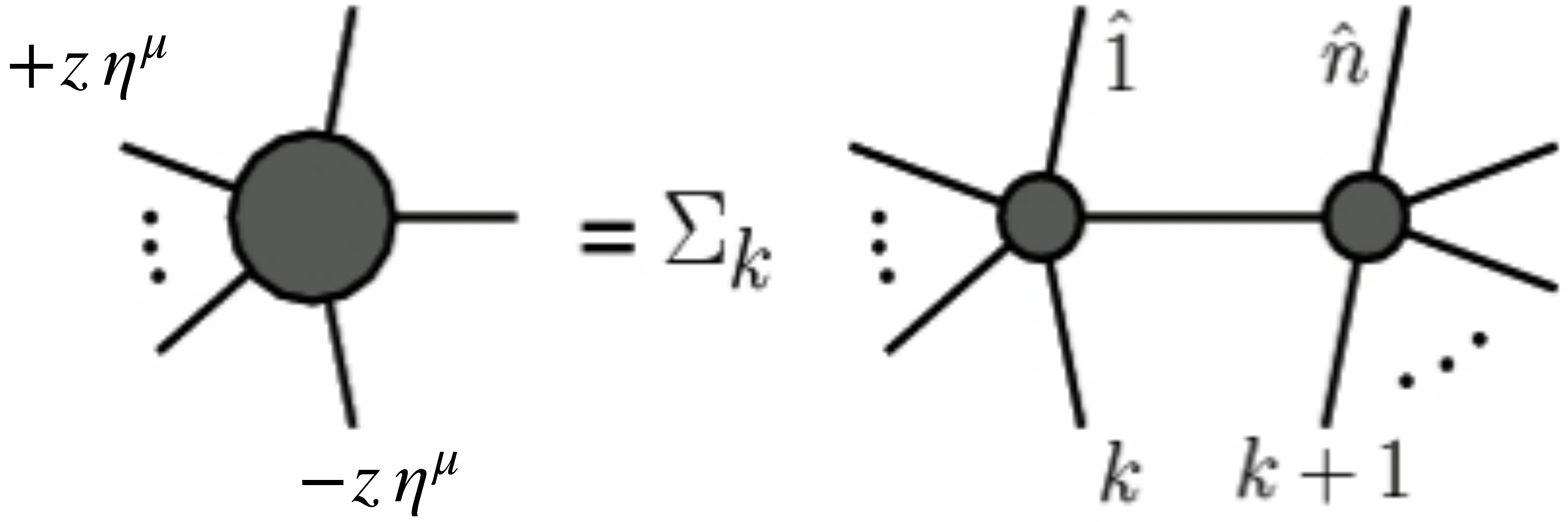
- on-shell amplitude by setting on-shell the off-shell leg

How to reconstruct a scattering amplitude from its singularities

Add $z\,\eta^\mu$ (z **complex**) to the four-momentum of one external particle and subtract it on another such that the shift leaves them on-shell

$$0 = \frac{1}{2\pi\imath}\oint_{C \text{ at } \infty} \frac{\mathscr{A}_n^{(0)}(z)}{z} = \mathscr{A}_n^{(0)}(z=0) - \sum_{z_i} \frac{\text{Res}_{z_i}\, \mathscr{A}_n^{(0)}(z)}{z_i}$$

has the correct residue at any multi-particle pole



$$\mathscr{A}_n^{(0)}(1,2,\ldots,n) = \sum \mathscr{A}_L^{(0)}(\hat{1},2,\ldots,-\hat{p}_{1,k})\,\frac{i}{s_{1,k}}\,\mathscr{A}_R^{(0)}(\hat{p}_{1,k},k+1,\ldots,\hat{n})$$

- Diagrammatic proof [Draggiotis, Kleiss, Lazopoulos, Papadopoulos]
- Compact analytical results, although colour dressed Berends-Giele (off-shell recursion) might be more efficient numerically [Duhr, Höche, Maltoni]

in practice

holomorphic shift ((-,+) is not a safe shift)

$$\hat{p}_i^\mu = p_i^\mu + \frac{z}{2}[i|\gamma^\mu|j\rangle \qquad |\hat{i}\rangle = |i\rangle + z|j\rangle \qquad |\hat{i}] = |i]$$

$$\hat{p}_j^\mu = p_j^\mu - \frac{z}{2}[i|\gamma^\mu|j\rangle \qquad |\hat{j}\rangle = |j\rangle \qquad |\hat{j}] = |j] - z|i]$$

anti-holomorphic shift (i ↔ j)

z determined by setting on-shell the intermediate momenta

$$\hat{p}_{1,k}^\mu = p_{1,k}^\mu + \frac{z}{2}[i|\gamma^\mu|j\rangle \ , \quad \hat{p}_{1,k}^2 = 0 \ , \quad z = -\frac{s_{1,k}}{[i|p_{1,k}|j\rangle}$$

☑ use only on-shell amplitudes

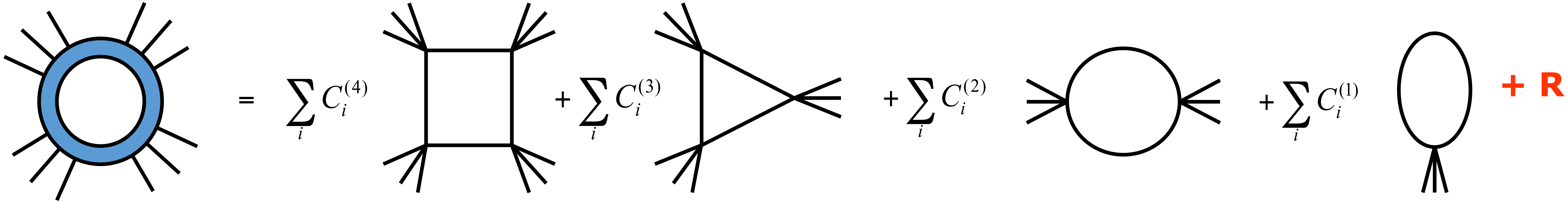
☑ rather compact expressions

☒ generates spurious poles at $[i|p_{1,k}|j\rangle$

while physical IR divergences at $s_{i,j} = (p_i + p_{i+1} + \cdots + p_j)^2$

Generalized Unitarity: the **one-loop basis**

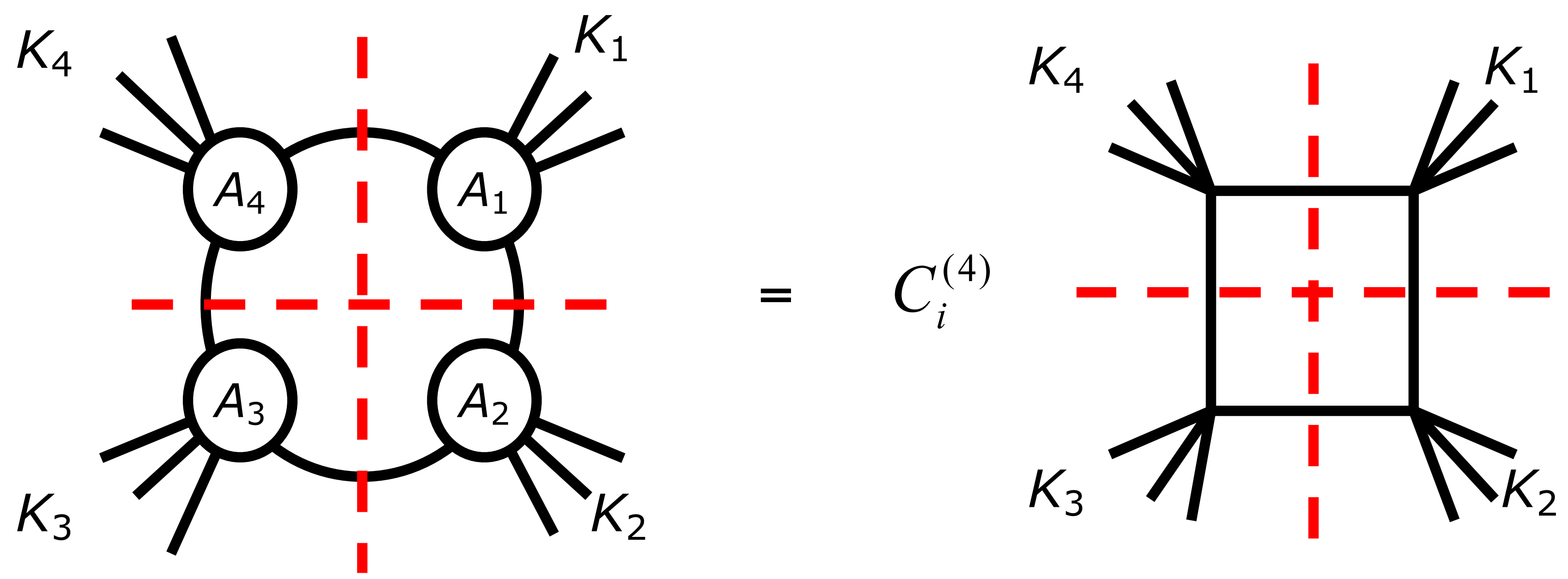
A dimensionally regulated n-point one-loop integral (scattering amplitude) is a linear combination of **scalar** boxes, triangles, bubbles and tadpoles with rational coefficients



- Pentagons and higher n-point functions can be reduced to lower point integrals and higher dimensional polygons that only contribute at O(ε) [Bern, Dixon, Kosower]
- The task is reduced to determining the coefficients: by applying **multiple cuts** at both sides of the equation [Brito, Cachazo, Feng]
- R is a finite piece that is entirely rational: can not be detected by four-dimensional cuts

Generalized Unitarity

Quadruple cut



The discontinuity across the leading singularity is unique

$$C_i^{(4)} = A_1 \times A_2 \times A_3 \times A_4$$

Four **on-shell** constrains ⇨ freeze the loop momenta

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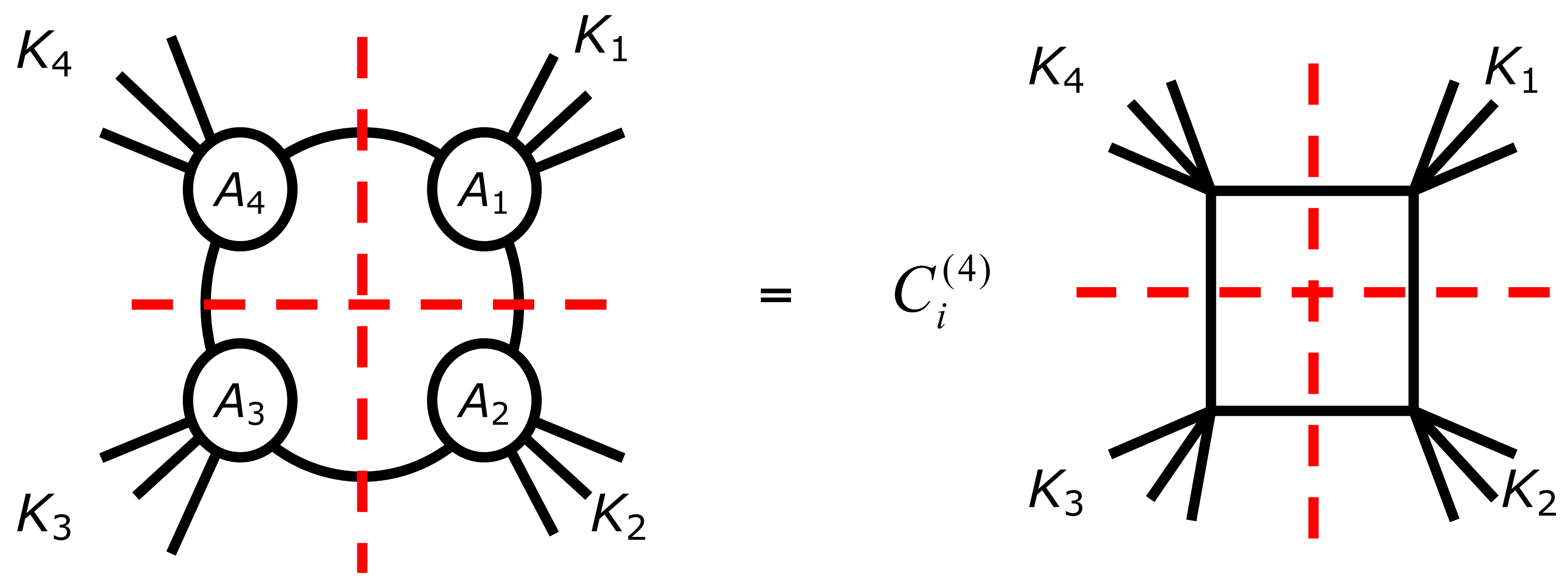
Quadruple cut



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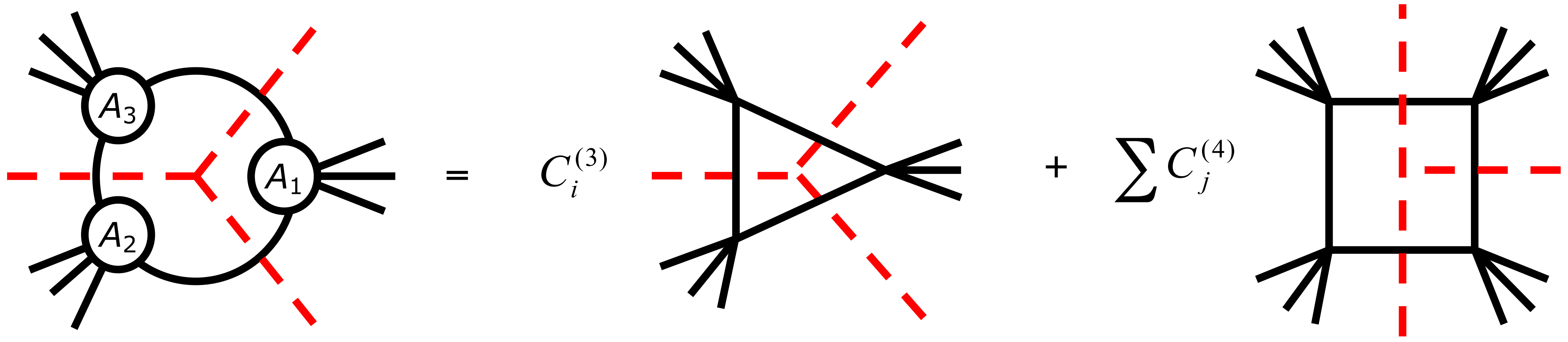
Triple cut



Only three **on-shell** constrains

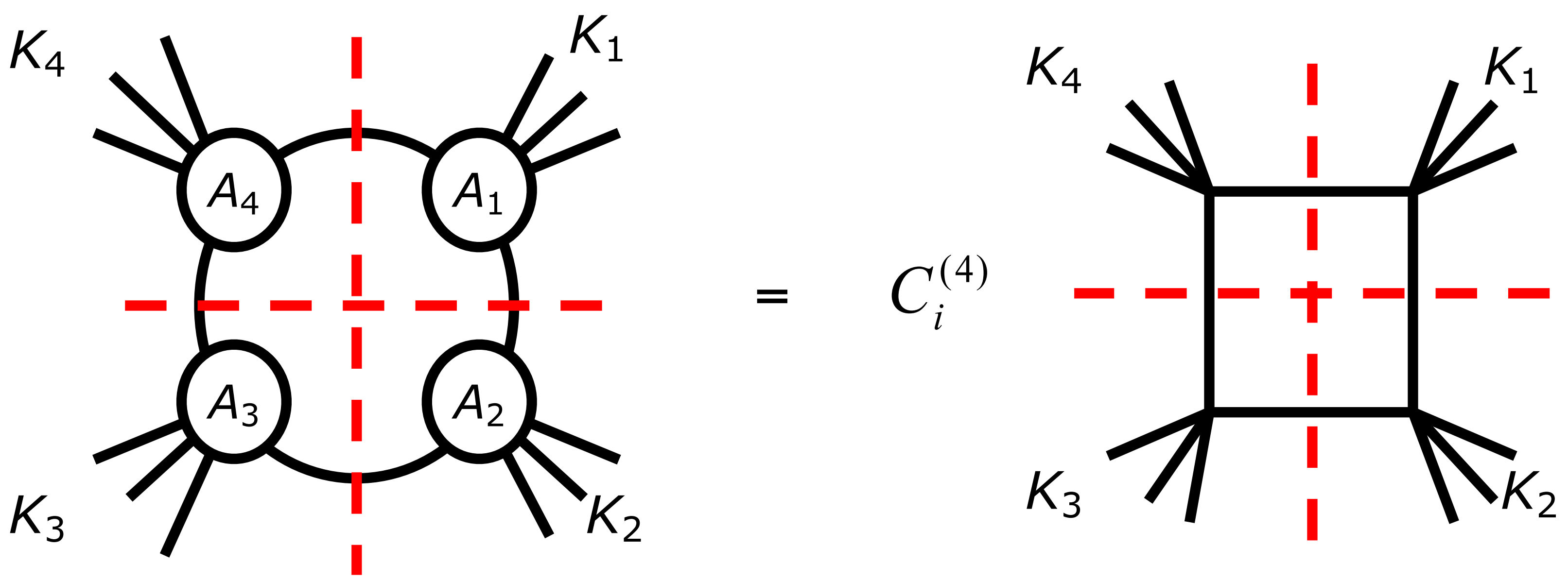
⇨ one free component of the loop momentum

And so on for double and single cuts

- OPP [Ossola, Pittau, Papadopoulos]: a systematic way to extract the coefficients

Generalized Unitarity

Quadruple cut

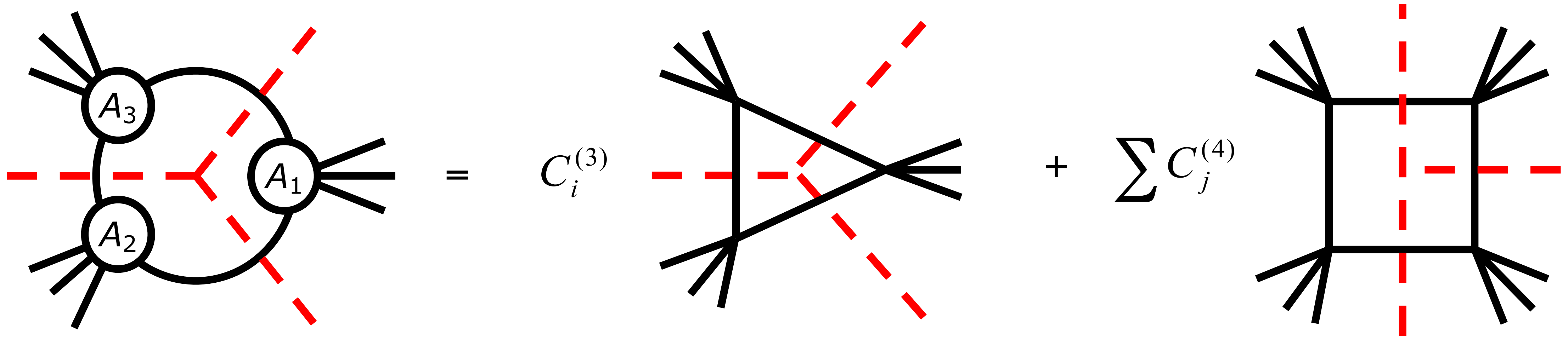


The discontinuity across the leading singularity is unique

$$C_i^{(4)} = A_1 \times A_2 \times A_3 \times A_4$$

Four **on-shell** constrains ⇨ freeze the loop momenta

Triple cut



Only three **on-shell** constrains

⇨ one free component of the loop momentum

And so on for double and single cuts

- OPP [Ossola, Pittau, Papadopoulos]: a systematic way to extract the coefficients

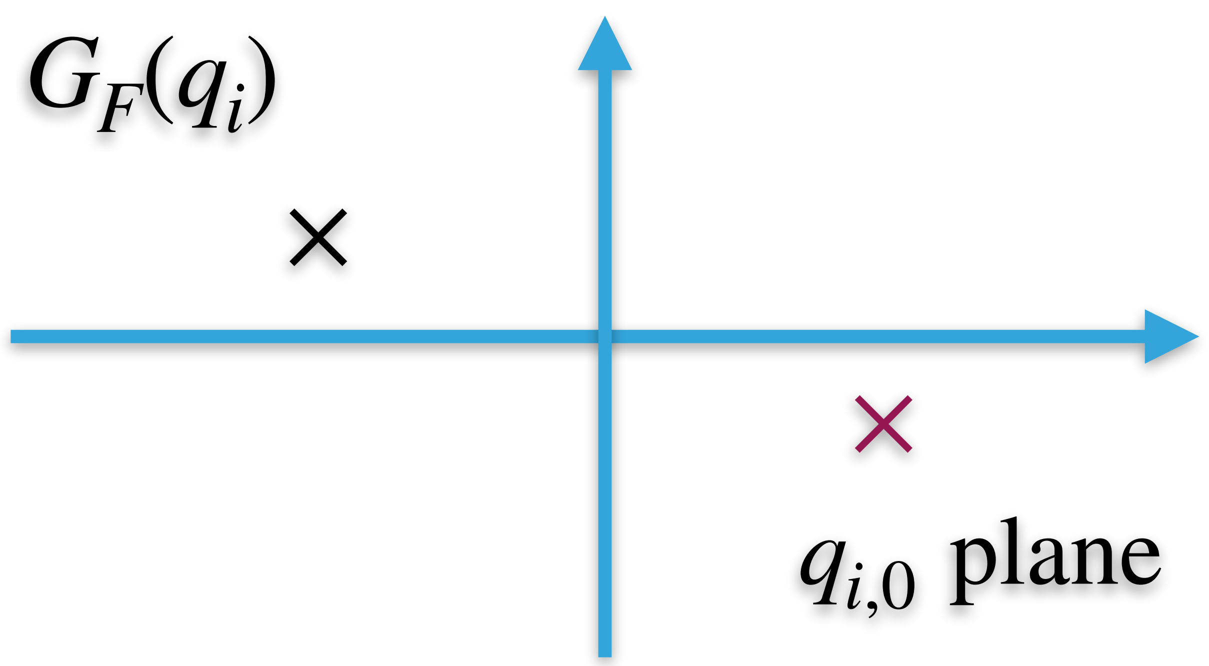
Rational terms

d-dimensional cuts, recursion relations (BCFW), Feynman rules …

[Catani et al. JHEP **0809**, 065]

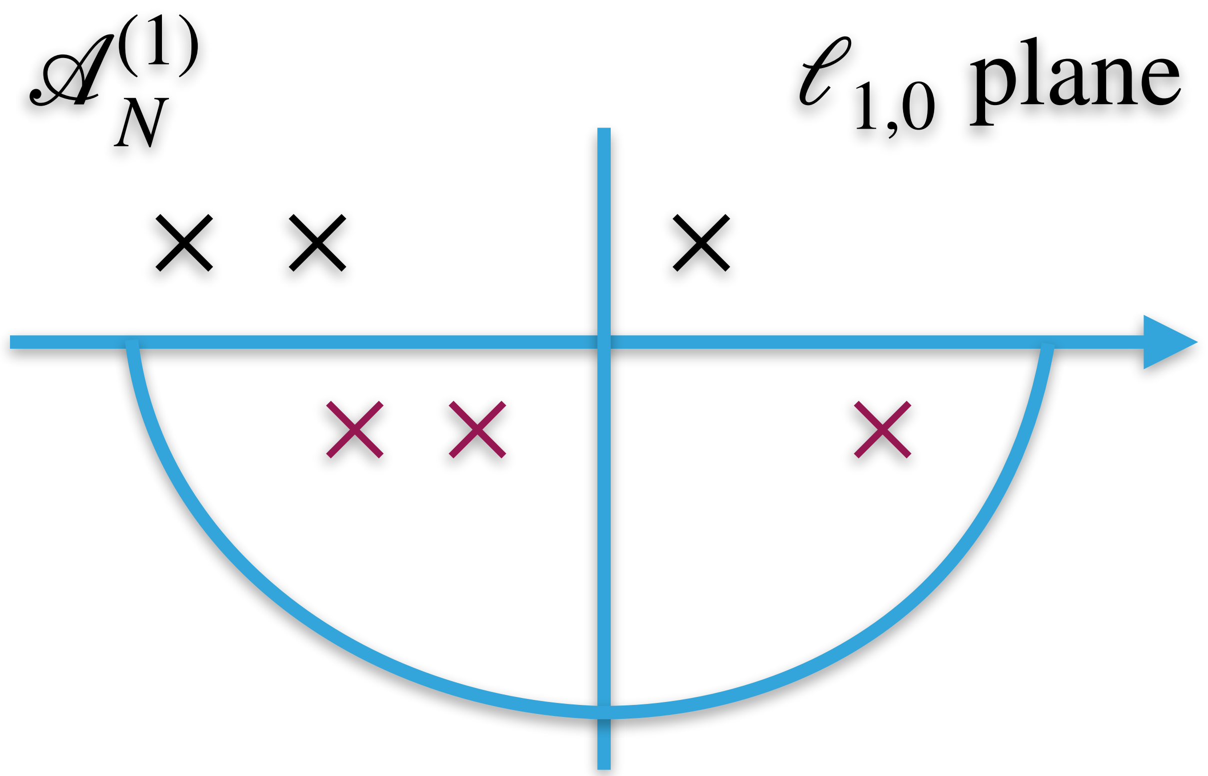
THE LOOP-TREE DUALITY (LTD)

[Aguilera-Verdugo, Drientcourt-Mangin, Hernández-Pinto, Plenter, Ramírez-Uribe, Rentería-Olivo, GR, Sborlini, Torres-Bobadilla]



Cauchy residue theorem

in the loop energy complex plane



Feynman Propagator +i0:
encodes **causality**, i.e. positive frequencies are propagated forward in time, and negative backward

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

selects residues with definite **positive energy and negative imaginary part**

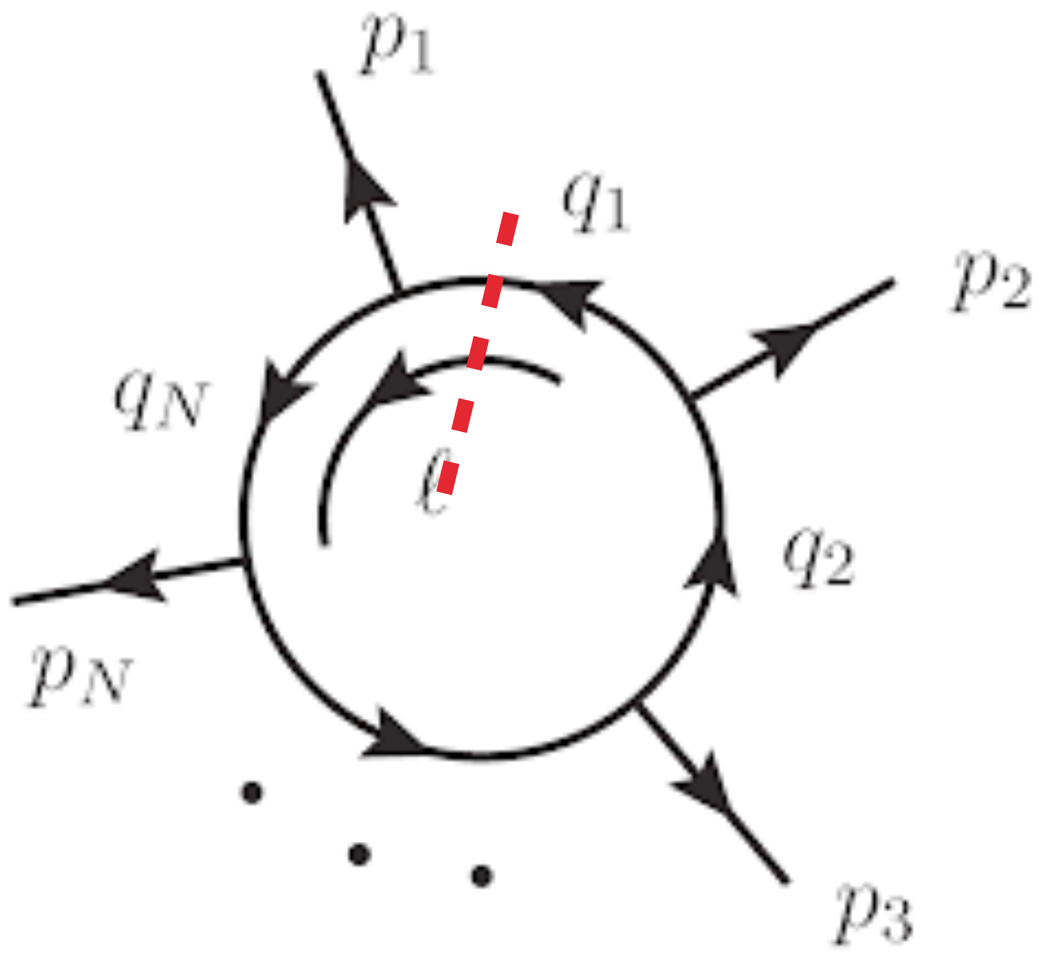
- in arbitrary coordinate systems: reduce the dimension of the integration domain by one unit
- from **Minkowski** to **Euclidean** (loop three-momenta)

[Catani et al. JHEP **0809**, 065]

THE LOOP-TREE DUALITY (LTD)

[Aguilera-Verdugo, Drientcourt-Mangin, Hernández-Pinto, Plenter, Ramírez-Uribe, Rentería-Olivo, GR, Sborlini, Torres-Bobadilla]

One-loop amplitudes in any relativistic, local and unitary QFT represented as a linear combination of N **single-cut phase-space/dual amplitudes** | **non-disjoint trees** (at higher orders: number of cuts equal to the number of loops)



$$\int_{\ell_1} \mathcal{N}(\ell_1) \prod G_F(q_i) = - \int_{\ell_1} \mathcal{N}(\ell_1) \otimes \sum \tilde{\delta}(q_i) \prod_{i \neq j} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = \imath 2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \dfrac{1}{q_j^2 - m_j^2 - \imath 0\, \eta k_{ji}}$ **dual propagator** $k_{ji} = q_j - q_i$ $\quad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}$

Σ [Catani et al. JHEP **0809**, 065]

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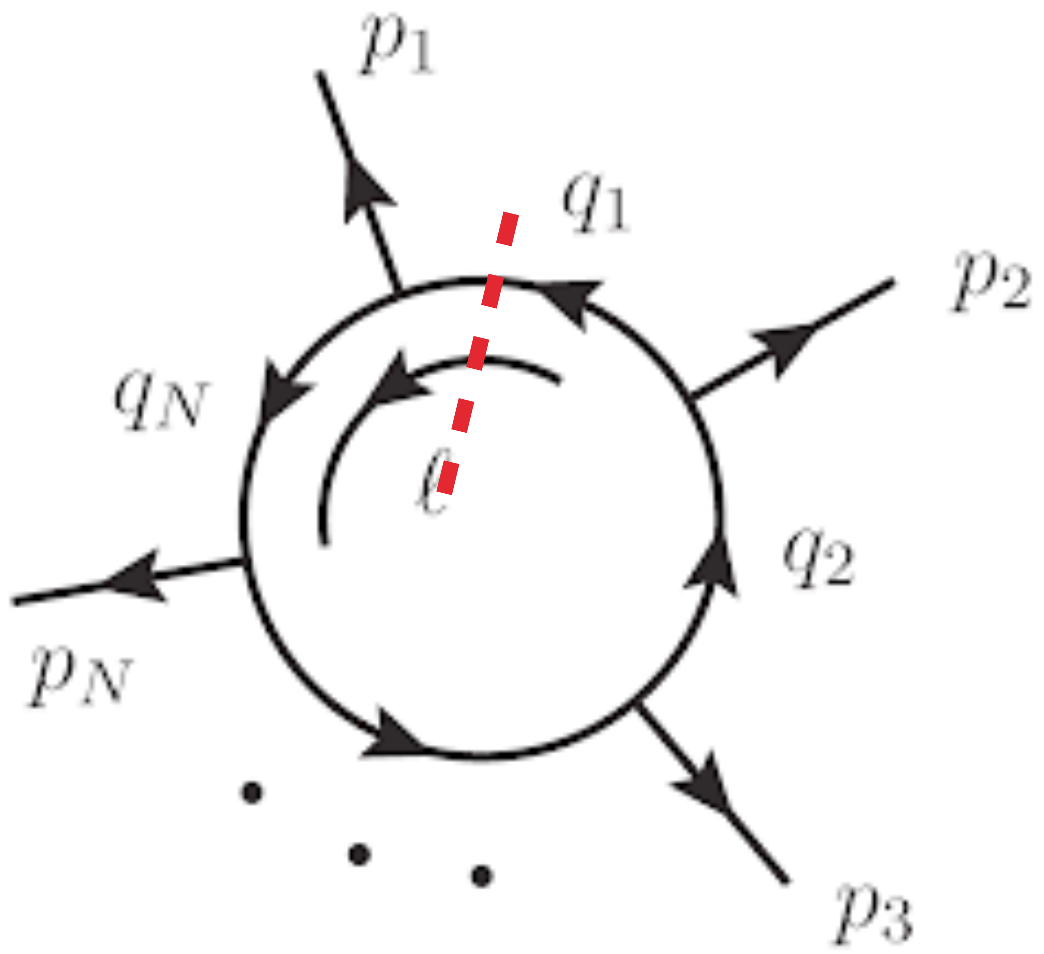

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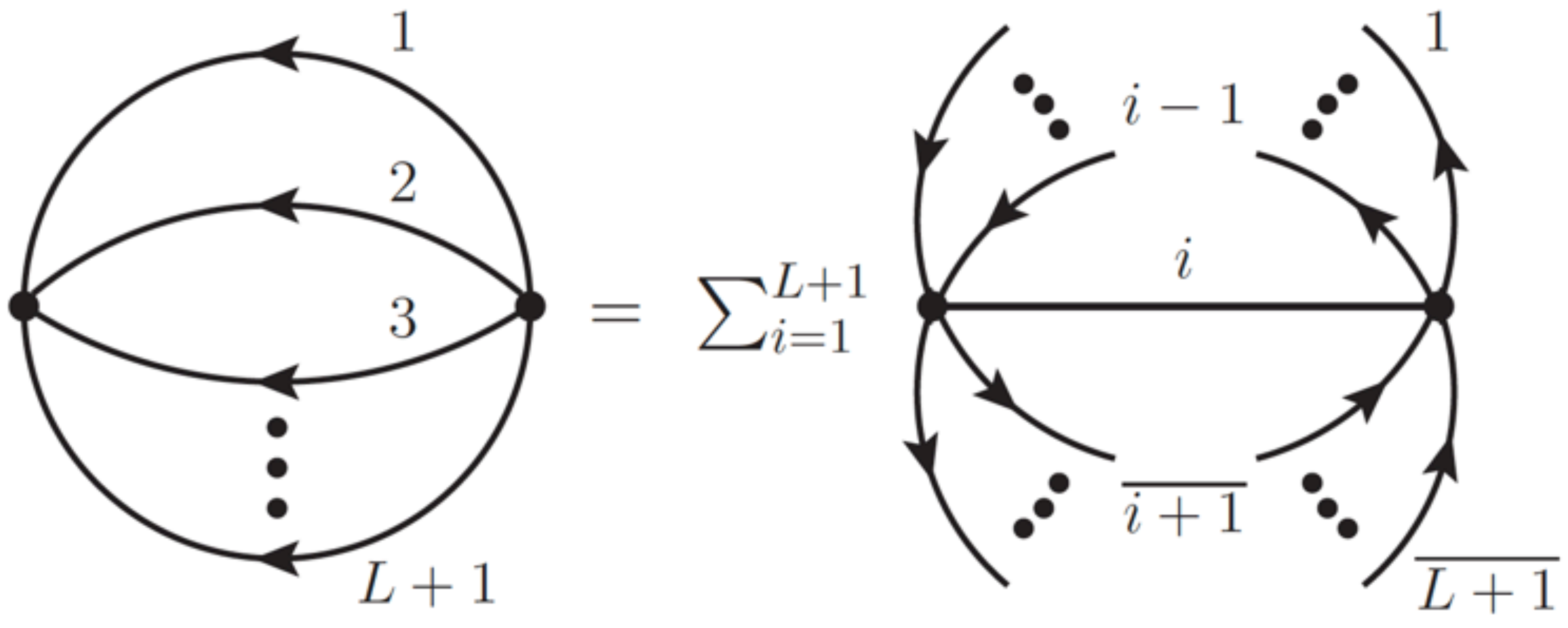
- LTD realised by **modifying the customary +i0 prescription** of the Feynman propagators (only the sign matters), it encodes in a compact way the effect of **multiple-cut** contributions that appear in the **Feynman's Tree Theorem**
- **Lorentz invariant**, best choice $\eta^\mu = (1,\mathbf{0})$: energy component integrated out, remaining integration in **Euclidean space**

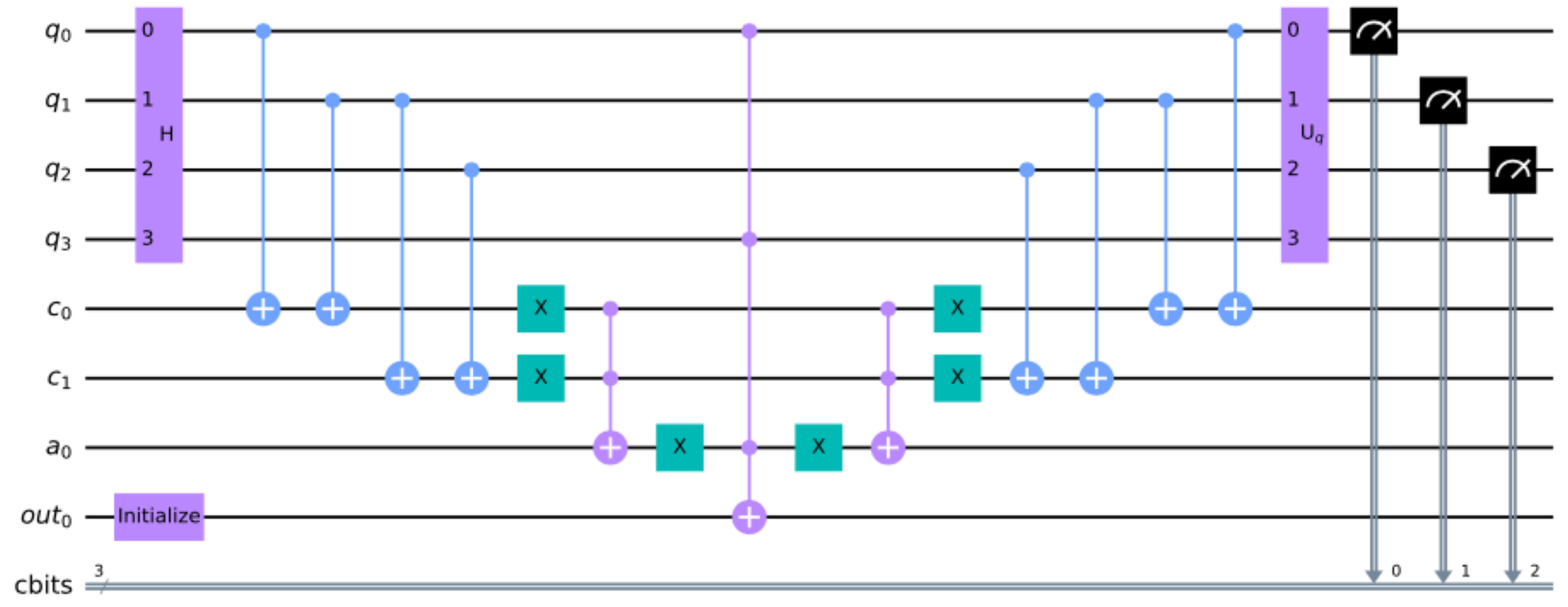
[Catani et al. JHEP **0809**, 065]

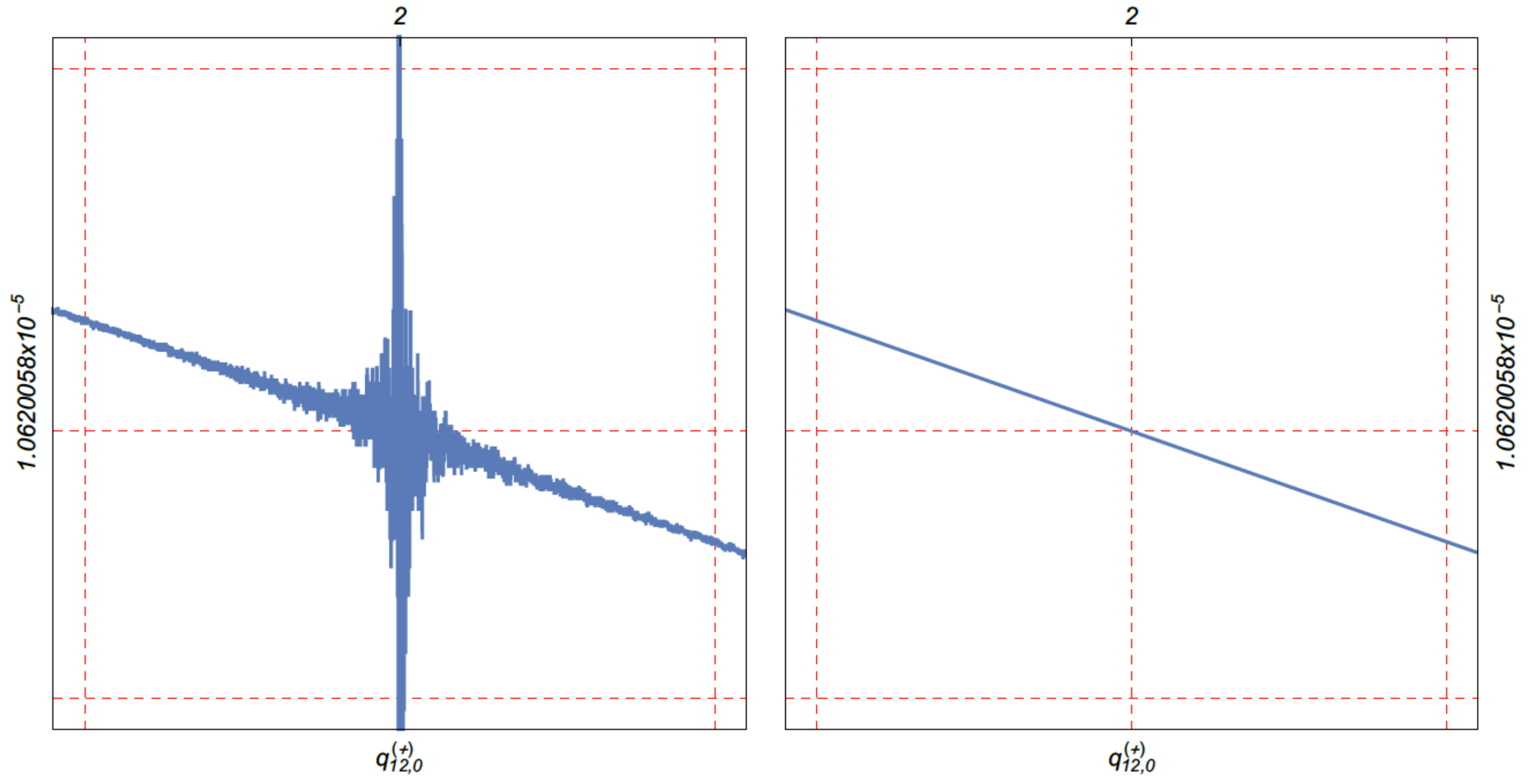
LTD AT HIGHER ORDERS + CAUSALITY

[Aguilera-Verdugo, Drientcourt-Mangin, Hernández-Pinto, Plenter, Ramírez-Uribe, Rentería-Olivo, GR, Sborlini, Torres-Bobadilla]



Qiskit





- The LTD representation is an integral in the loop tree-momenta

$$\mathscr{A}^{(L)}_{\mathrm{MLT}}(1,\ldots,n) = \int_{\vec{\ell}_1\ldots\vec{\ell}_L} \frac{1}{\prod 2q^{(+)}_{i,0}} \left(\frac{1}{\lambda^+_{1,n}} + \frac{1}{\lambda^-_{1,n}} \right), \qquad \lambda^\pm_{1,n} = \sum q^{(+)}_{i,0} \pm k_{1n,0}$$

- Independent of the initial momentum flow assignments
- **Manifestly free of non-causal singularities:** for all topologies and internal configurations

- **Integrand numerical instabilities** across a noncausal threshold
- **manifestly causal LTD** representation

Exercises:

1. Proof the **Fierz** and **Shouten** identities:

$$\langle 1 | \gamma^\mu | 2] \, [3 | \gamma_\mu | 4\rangle = 2 \langle 14\rangle \, [32]$$

$$\langle 12\rangle\langle 34\rangle + \langle 14\rangle\langle 23\rangle + \langle 13\rangle\langle 42\rangle = 0$$

Hint: multiply and divide by $\langle 23\rangle$ or $[23]$
and apply $\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_\mu = 4g^{\nu\sigma}$

2. Calculate the scattering amplitudes and squared amplitude for $e^+(p_1)\, e^-(p_2) \to q(p_3)\, \bar{q}(p_4)$ by using the helicity method, and compare with the traditional calculation

$$\mathcal{M}^{(0)}_{e^+e^-\to q\bar{q}} \sim \left[\bar{u}(p_3)\gamma^\mu v(p_4)\right] \left[\bar{v}(p_1)\gamma^\nu u(p_2)\right] d_{\mu\nu}(p_{12})$$

$$|\mathcal{M}^{(0)}|^2 = \mathrm{Tr}\left(\not{p}_1\, \gamma^\mu \not{p}_2\, \gamma^\sigma\right) \mathrm{Tr}\left(\not{p}_3\, \gamma^\nu \not{p}_4\, \gamma^\rho\right) d_{\mu\sigma}(p_{12})\, d_{\nu\rho}(p_{12})$$

How many independent helicity amplitudes there are?

Exercises:

3. Proof by induction that the Maximal Helicity Violating (MHV) amplitude for gluons is given by the expression

$$\mathcal{A}_n^{(0)}(1^+, \ldots, i^\pm, \ldots, n^+) = 0$$

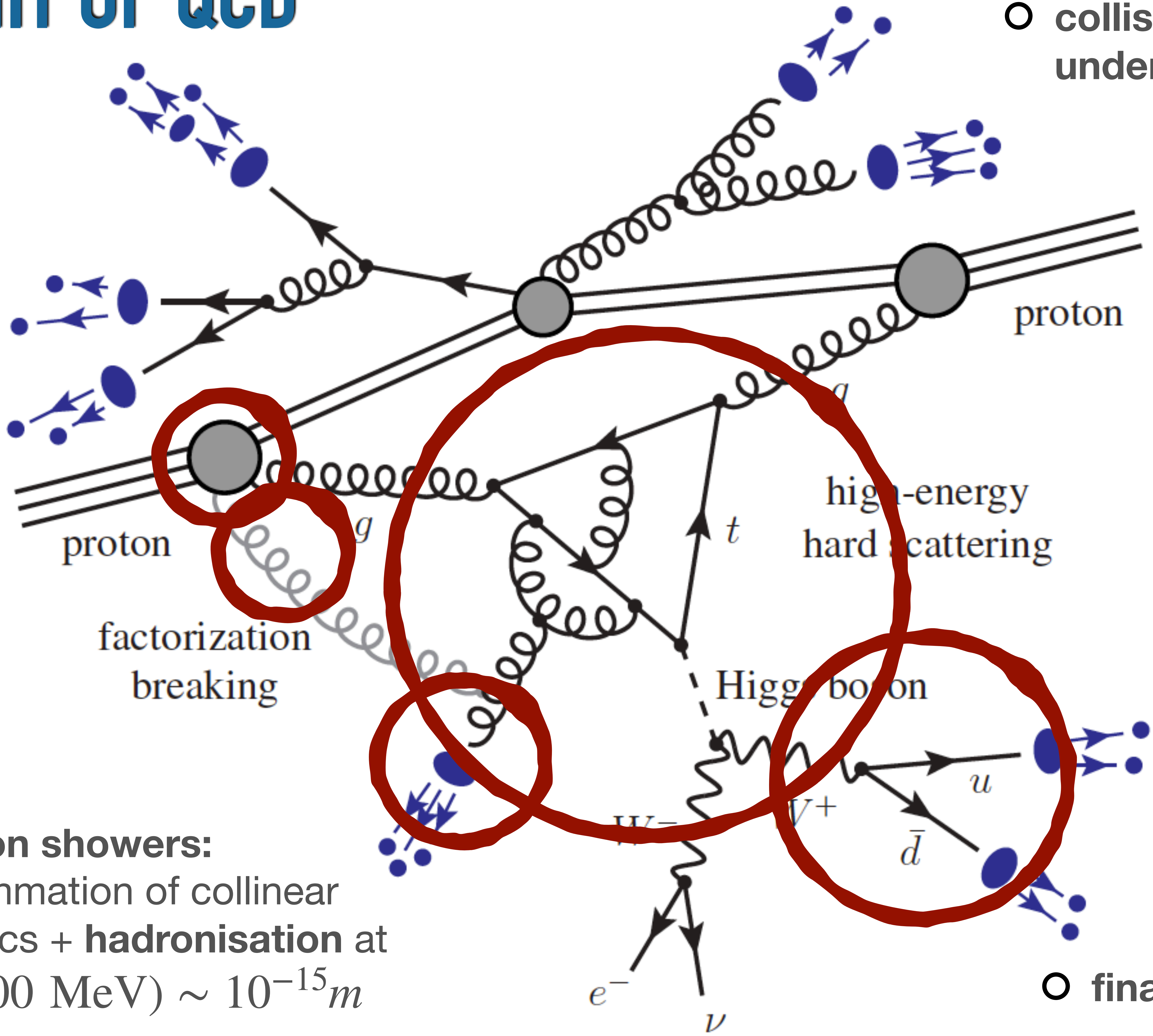
$$\mathcal{A}_n^{(0)}(1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle (n-1)n \rangle \langle n1 \rangle}$$

4. Calculate by using BCFW the six-gluon amplitude

$$\mathcal{A}_6(1^+,2^+,3^+,4^-,5^-,6^-) = \frac{i}{\langle 2 | 1+6 | 5]} \left(\frac{\langle 6 | 1+2 | 3]^3}{\langle 61 \rangle \langle 12 \rangle [34][45] s_{126}} + \frac{\langle 4 | 5+6 | 1]^3}{\langle 23 \rangle \langle 34 \rangle [56][61] s_{561}} \right)$$

THE COLLINEAR LIMIT OF QCD

factorisation into short distance (hard scattering = high energy) and long distance (initial and final state = low energy)



- collision remnants / underlying event
- **initial-state** parton densities $1/\text{GeV} = 10^{-16}m$
- High-energy **collision** at $1/\text{TeV} \sim 10^{-19}m$
- **Parton showers:** resummation of collinear physics + **hadronisation** at $1/(200\ \text{MeV}) \sim 10^{-15}m$
- **final state:** e.g. jets
- **final state:** e.g. leptons

Relevance of the collinear limit in QCD

- from **hard scattering amplitudes** to cross-sections: subtraction of IR singularities
- IR properties of amplitudes exploited to compute **logarithmic enhanced** perturbative terms: resummation of leading and subleading logs
- improve physics content of **Monte Carlo** event generators: parton showers
- **scale evolution** of PDF´s and fragmentation functions
- Factorization theorems: from e+e- and DIS to hadron colliders
- beyond QCD: hints on the structure of highly symmetric gauge theories (e.g. N=4 super-Yang-Mills)

Collinear factorisation theorem proven for sufficiently inclusive observables in the final state of the scattering of colorless hadrons [Collins, Soper, Sterman]

- Offen assumed that partonic scattering amplitudes factorize: fixed order and resummations
- Monte Carlo event generators are based on factorisation
- In neither of these cases factorization is guaranteed at higher orders.

pQCD for hard-scattering processes based on **universality**:

- the sole uncancelled IR divergences are due to partonic states whose momenta are collinear to the collider partons
- removed by redefinition of bare parton densities

Parton densities PDF

Hard scattering cross-section

$$\sigma = \sum_{a,b} \int dx_1\, dx_2\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab}(x_1 p_A, x_2 p_B; \mu_F, \mu_R) + \mathcal{O}\left(\frac{1}{Q}\right)$$

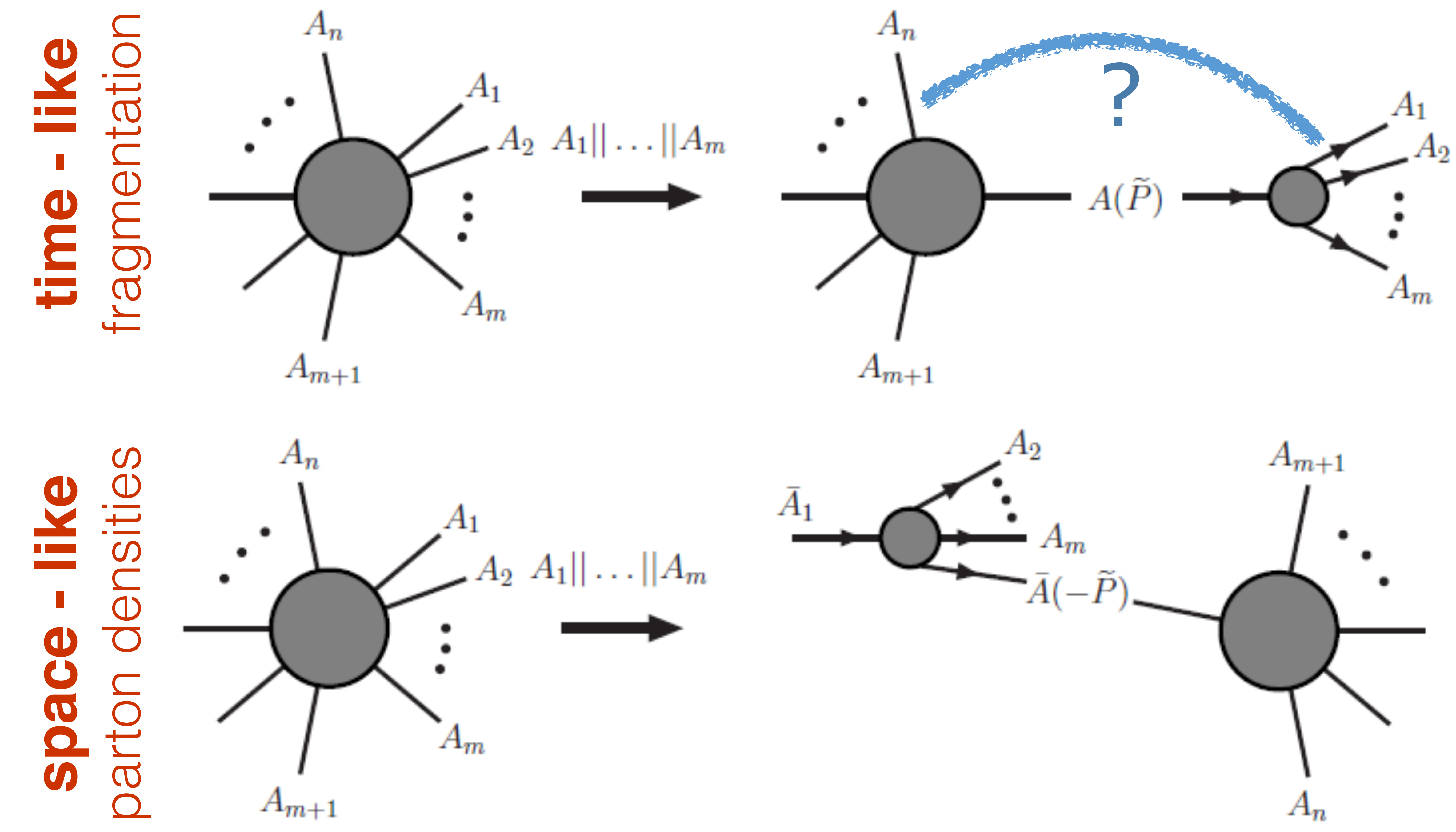
Factorization and renormalization scales

Higher twist

Partonic cms energy $\hat{s} = x_1\, x_2\, s$

Collinear factorisation at tree-level

- Momenta $p_1, \ldots, p_m$ of m **partons become collinear**
- Sub-energies $s_{ij} = (p_i + p_j)^2$ of the same order and vanish simultaneously
- leading singular behaviour $(\sqrt{s_{1,m}})^{1-m}$ with $p_{1,m} = p_1 + \ldots + p_m$



Collinear limit

- Most singular behaviour captured by **universal** (process independent) splitting amplitudes: the same for e+e-, DIS and hadron collisions
- The **splitting amplitude** depends on the collinear partons only
- Space-like and time-like related by crossing
- Process dependence in the **reduced matrix element**

$$|M^{(0)}(p_1, \ldots, p_n)\rangle = \boldsymbol{Sp}^{(0)}(p_1, \cdots, p_m; \tilde{P}) \, |\overline{M}^{(0)}(\tilde{P}; p_{m+1}, \ldots, p_n)\rangle + \mathcal{O}((\sqrt{s_{1,m}})^{3-m})$$

At two loops

$$|\mathcal{M}^{(2)}(p_1,\ldots,p_n)\rangle$$
$$\simeq \mathbf{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{\mathcal{M}}^{(2)}(\widetilde{P};p_{m+1},\ldots,p_n)\rangle$$
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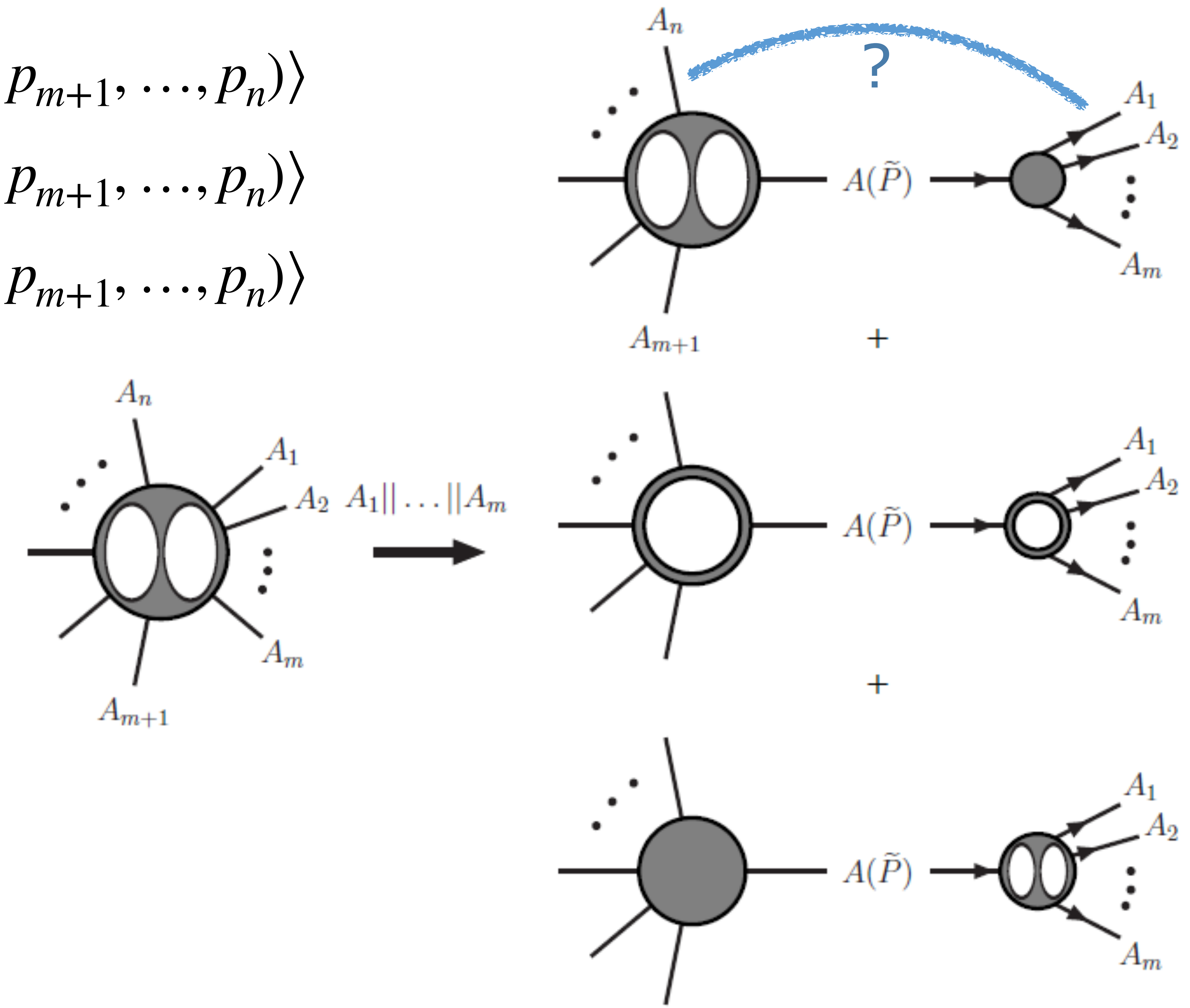

Qualitative interpretation: two collinear partons

- **Tree level:**
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  - Interactions separately spoil factorisation, but $\theta_{j1} \simeq \theta_{j2} \simeq \theta_{j\tilde{P}}$ and $\mathbf{T}_j \cdot (\mathbf{T}_1 + \mathbf{T}_2) = \mathbf{T}_j \cdot \mathbf{T}_{\tilde{P}}$: **colour coherence** restores factorisation, the parton j sees the two collinear partons as a single one.

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  - Both collinear partons in the final- or initial-state, otherwise colour coherence is limited by ***causality***

The collinear projection

- The **projection over the collinear limit** is obtained by setting the parent parton at on-shell momenta

$$\tilde{P}^\mu = p_{1,m}^\mu - \frac{s_{1,m}\, n^\mu}{2n \cdot \tilde{P}}$$

$\tilde{P}^\mu$: collinear direction

n^μ: describes how the collinear limit is approached $\tilde{P}^2 = 0, \; n^2 = 0$

$z_i = \dfrac{n \cdot p_i}{n \cdot \widetilde{P}}$: longitudinal momentu fraction $\sum z_i = 1$

- Factorisation holds in any arbitrary gauge, however, it is more evident in the **axial gauge** (physical polarisations): only diagrams where the parent parton emitted and absorbed collinear radiation

$$\frac{1}{\not{p}_{12}} = \frac{1}{s_{12}} \not{p}_{12} = \frac{1}{s_{12}} \left(\tilde{\not{P}} + \frac{s_{12}}{2n \cdot \tilde{P}} \not{n} \right) \simeq \frac{1}{s_{12}} u(\tilde{P}) \bar{u}(\tilde{P}) + \ldots$$

$$d_{\mu\nu}(k, n) = d_{\mu\nu}(\tilde{P}, n) + \ldots \simeq \epsilon_\mu(\tilde{P}) \epsilon_\nu^*(\tilde{P}) + \ldots$$

Splitting functions

The square of the splitting amplitude, summed over final-state colours and spins, and averaged over colours and spins of the parent parton, defines the m-parton (unpolarised) splitting function

$$\langle P \rangle = \left(\frac{s_{1,m}}{2\mu^{2\epsilon}} \right)^{m-1} \overline{|\mathbf{Sp}|^2}$$

which is a generalisation of the customary (i.e. with $m = 2$) **Altarelli-Parisi** splitting function

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- Probability to emit futher radiation with a given longitudinal momenta, from the leading singular behaviour
- Universal (process independent): the same fro e+e-, DIS or hadron collisions

Exercises:

1. Proof that $\mathbf{T}_j \cdot (\mathbf{T}_q + \mathbf{T}_{\bar{q}}) = \mathbf{T}_j \cdot \mathbf{T}_g$ and test other flavour combinations (colour coherence)
2. Calculate the splitting functions for the collinear processes $q \to qg$, $g \to q\bar{q}$ and $g \to gg$ by using the helicity method

Hint:

$$\mathbf{Sp}^{(0)}_{q \to q_1 g_2} = \mathbf{T}^{a_2} \frac{1}{s_{12}} \bar{u}(p_1) \not{\varepsilon}(p_2) v(\tilde{P})$$

$$P^{(0)}_{q \to q_1 g_2} = C_F \frac{1+z^2}{1-z} \qquad z = z_1 = \frac{n \cdot p_1}{n \cdot \tilde{P}} \qquad z_2 = 1 - z$$

Compare with $\mathcal{M}^{(0)}_{q\bar{q}g} \simeq (-\imath e_q)\,(\imath g_S)\,\mathbf{T}^a\,\bar{u}(p_1)\,\gamma^\mu\,v(p_2)\left(\frac{p_1 \cdot \varepsilon}{p_1 \cdot k} - \frac{p_2 \cdot \varepsilon}{p_2 \cdot k}\right)$

PARTON DENSITIES (PDF)

factorisation into short distance (hard scattering = high energy) and long distance (initial and final state = low energy)

- **collision remnants / underlying event**
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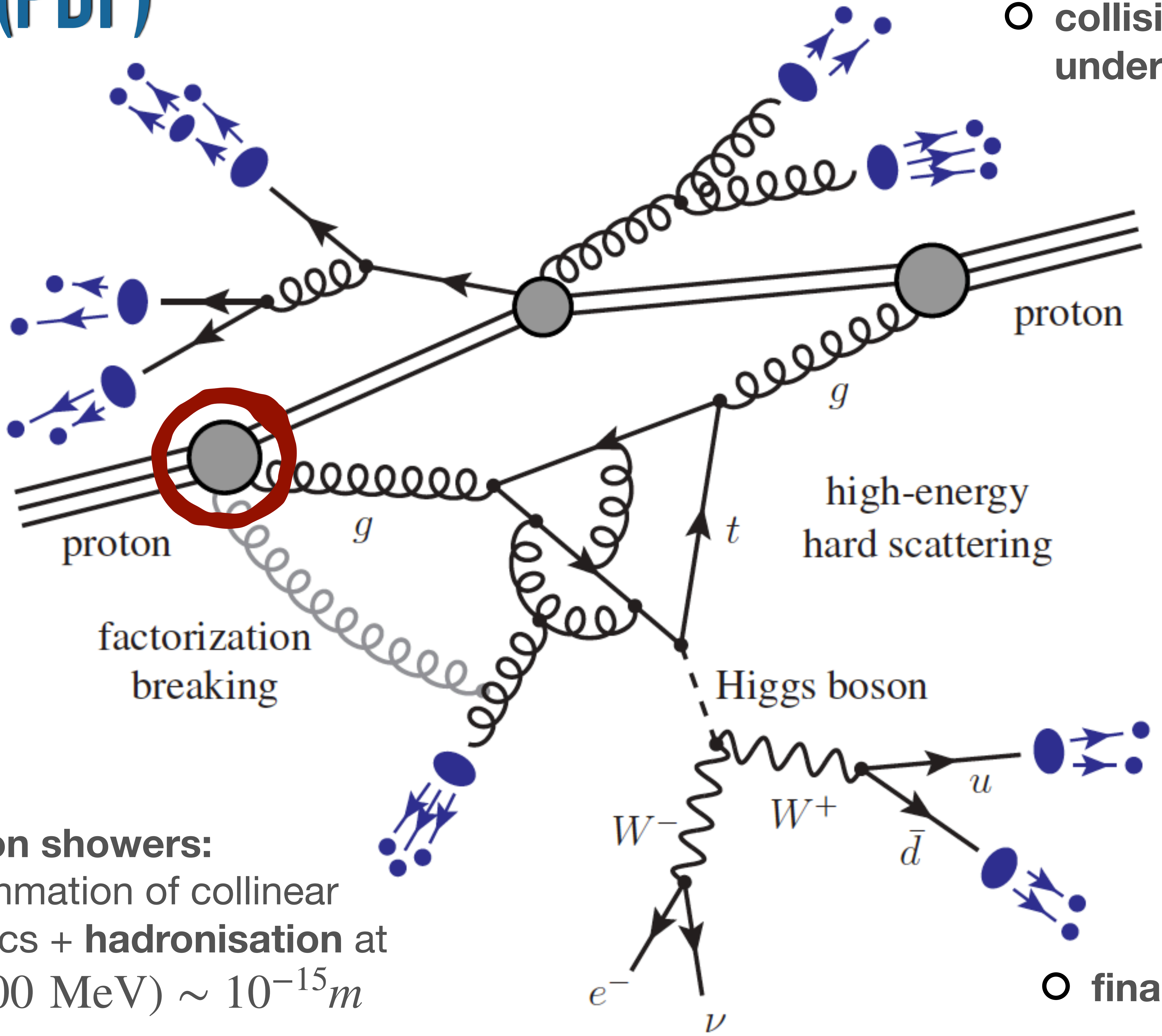
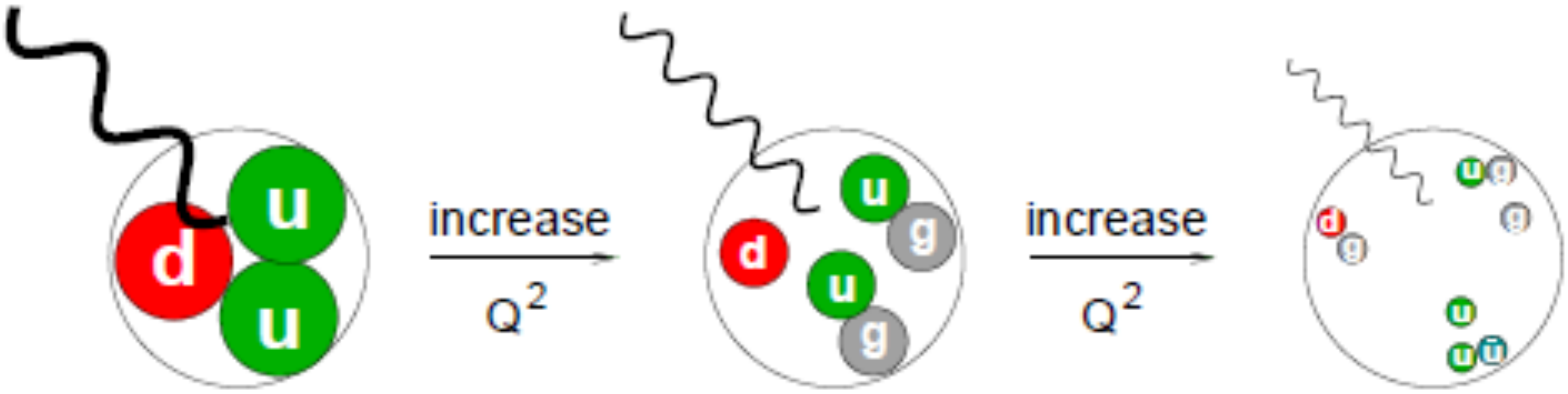


Looking inside the proton

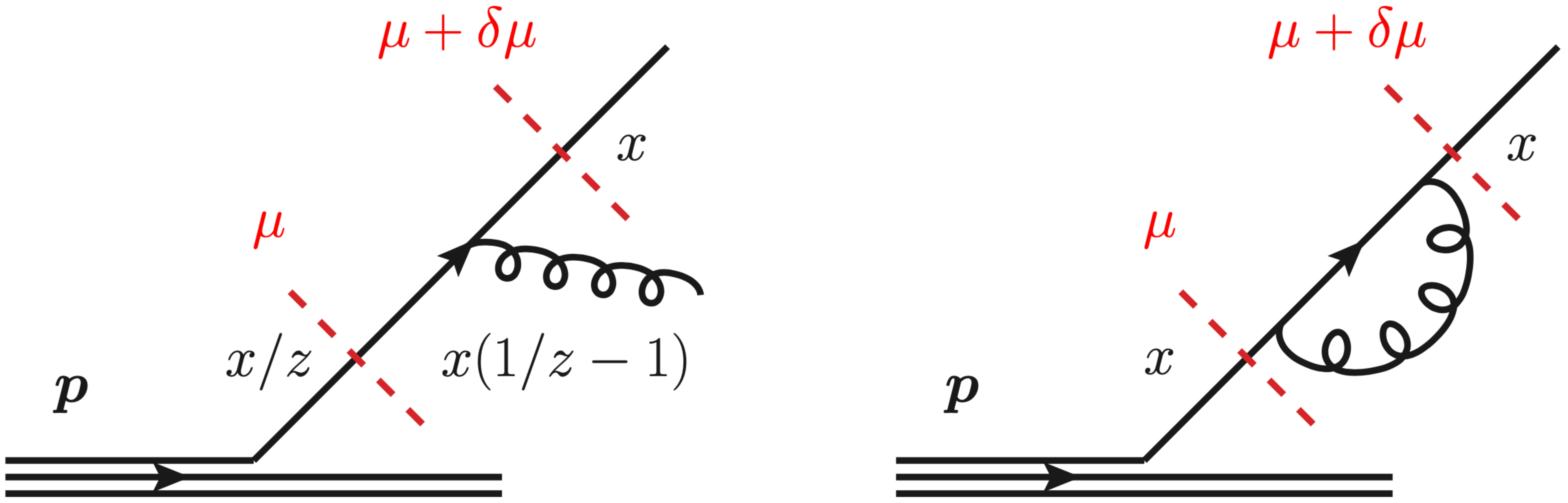


non-perturbative input

scale evolution determined by pQCD

Parton density (PDF): "*probability*" to find a parton of a given flavour carrying a longitudinal momentum fraction $x \in [0,1]$ of the momentum of the proton

DGLAP evolution [Dokshitzer–Gribov–Lipatov–Altarelli–Parisi 1972-1977]



$$\frac{\partial\, q(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dz}{z} P_{q\to qg}(z)\, q(x/z,\mu^2)$$

DGLAP flavour structure

The proton contains both quarks and gluons: DGLAP is a matrix in flavour space

$$\frac{\partial}{\partial \log \mu^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\to qg} & P_{g\to q\bar{q}} \\ P_{q\to gq} & P_{g\to gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

spanning over all flavours and anti-flavours

$$P_{q\to qg} = C_F \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right)$$

$$P_{q\to gq} = C_F \frac{1+(1-z)}{z}$$

$$P_{g\to q\bar{q}} = T_R \left[z^2 + (1-z)^2 \right]$$

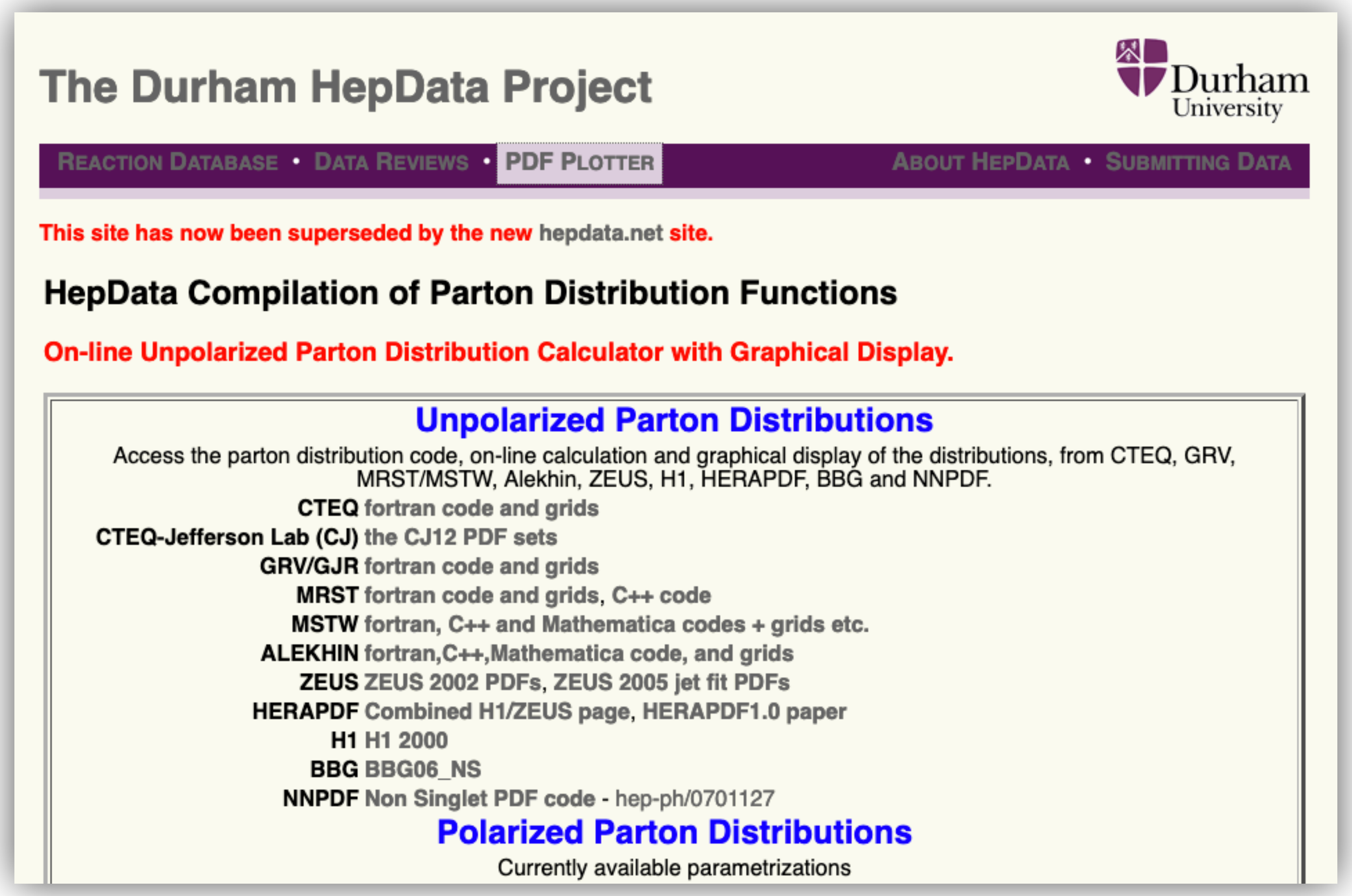
$$P_{g\to gg} = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + b_0\, \delta(1-z)$$

with the plus-prescription $z=1$ is soft: only soft configurations matches virtual with real corrections

$$\int_0^1 dz \, \frac{f(z)}{(1-z)_+} = \int_0^1 dz \, \frac{f(z)-f(1)}{1-z}$$

Parton densities

- Non-perturbative input determined from global fits to collider data, scale evolution from pQCD (NNLO)
- Vast choice: e.g. http://hepdata.cedar.ac.uk/pdfs



The Durham HepData Project

Durham University

REACTION DATABASE • DATA REVIEWS • PDF PLOTTER ABOUT HEPDATA • SUBMITTING DATA

This site has now been superseded by the new hepdata.net site.

HepData Compilation of Parton Distribution Functions

On-line Unpolarized Parton Distribution Calculator with Graphical Display.

Unpolarized Parton Distributions

Access the parton distribution code, on-line calculation and graphical display of the distributions, from CTEQ, GRV, MRST/MSTW, Alekhin, ZEUS, H1, HERAPDF, BBG and NNPDF.

CTEQ	fortran code and grids
CTEQ-Jefferson Lab (CJ)	the CJ12 PDF sets
GRV/GJR	fortran code and grids
MRST	fortran code and grids, C++ code
MSTW	fortran, C++ and Mathematica codes + grids etc.
ALEKHIN	fortran,C++,Mathematica code, and grids
ZEUS	ZEUS 2002 PDFs, ZEUS 2005 jet fit PDFs
HERAPDF	Combined H1/ZEUS page, HERAPDF1.0 paper
H1	H1 2000
BBG	BBG06_NS
NNPDF	Non Singlet PDF code - hep-ph/0701127

Polarized Parton Distributions

Currently available parametrizations

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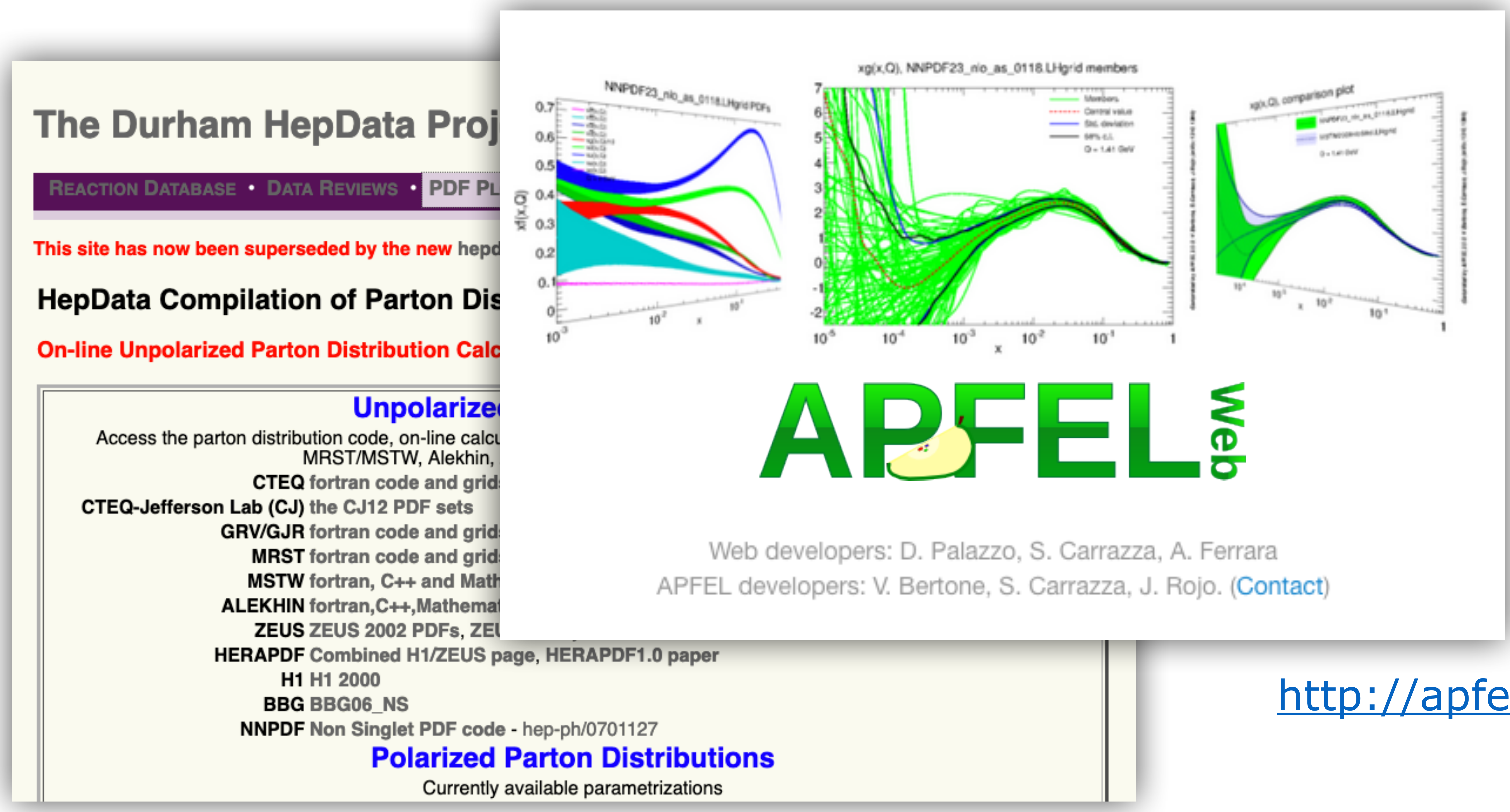
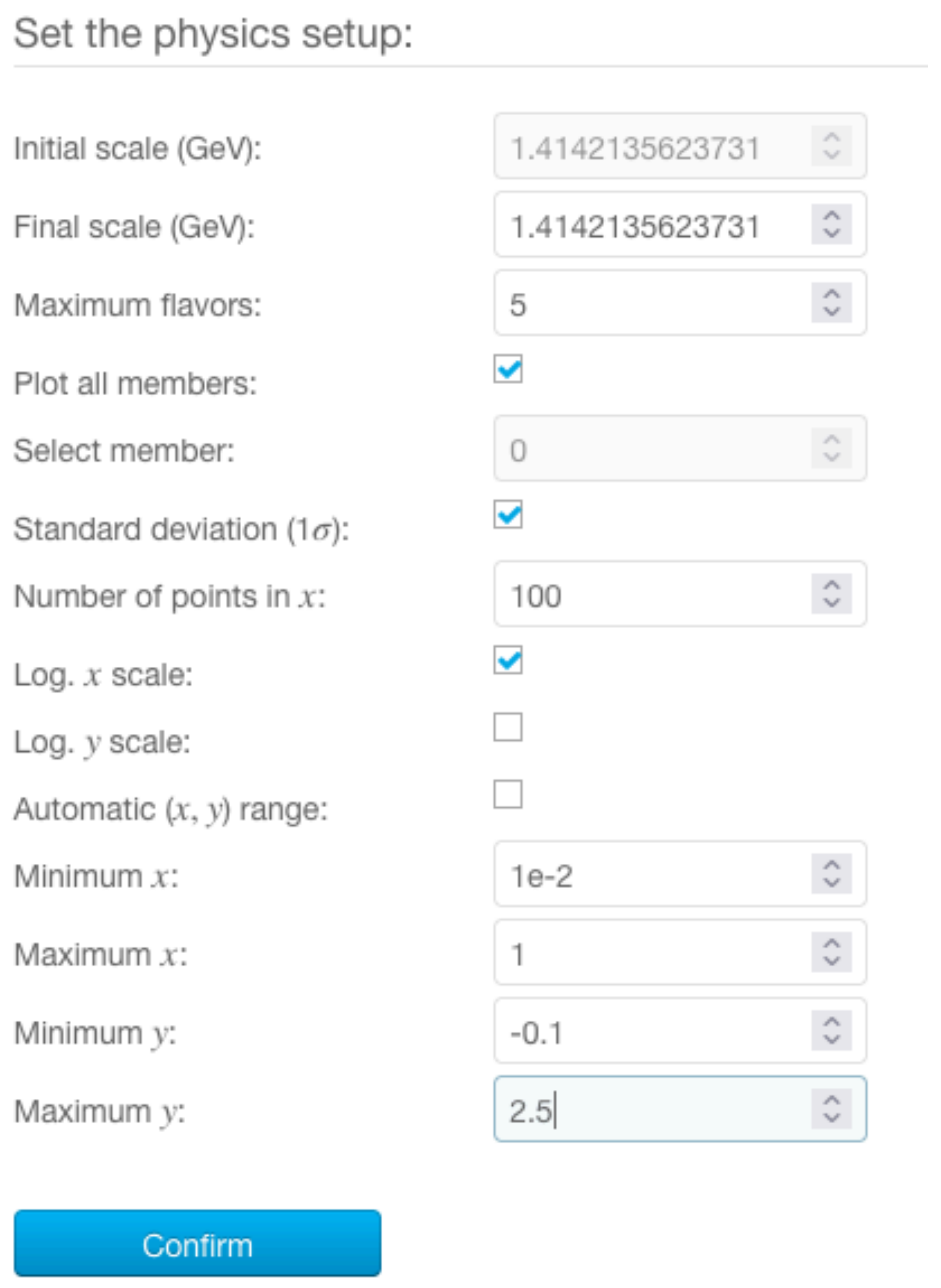
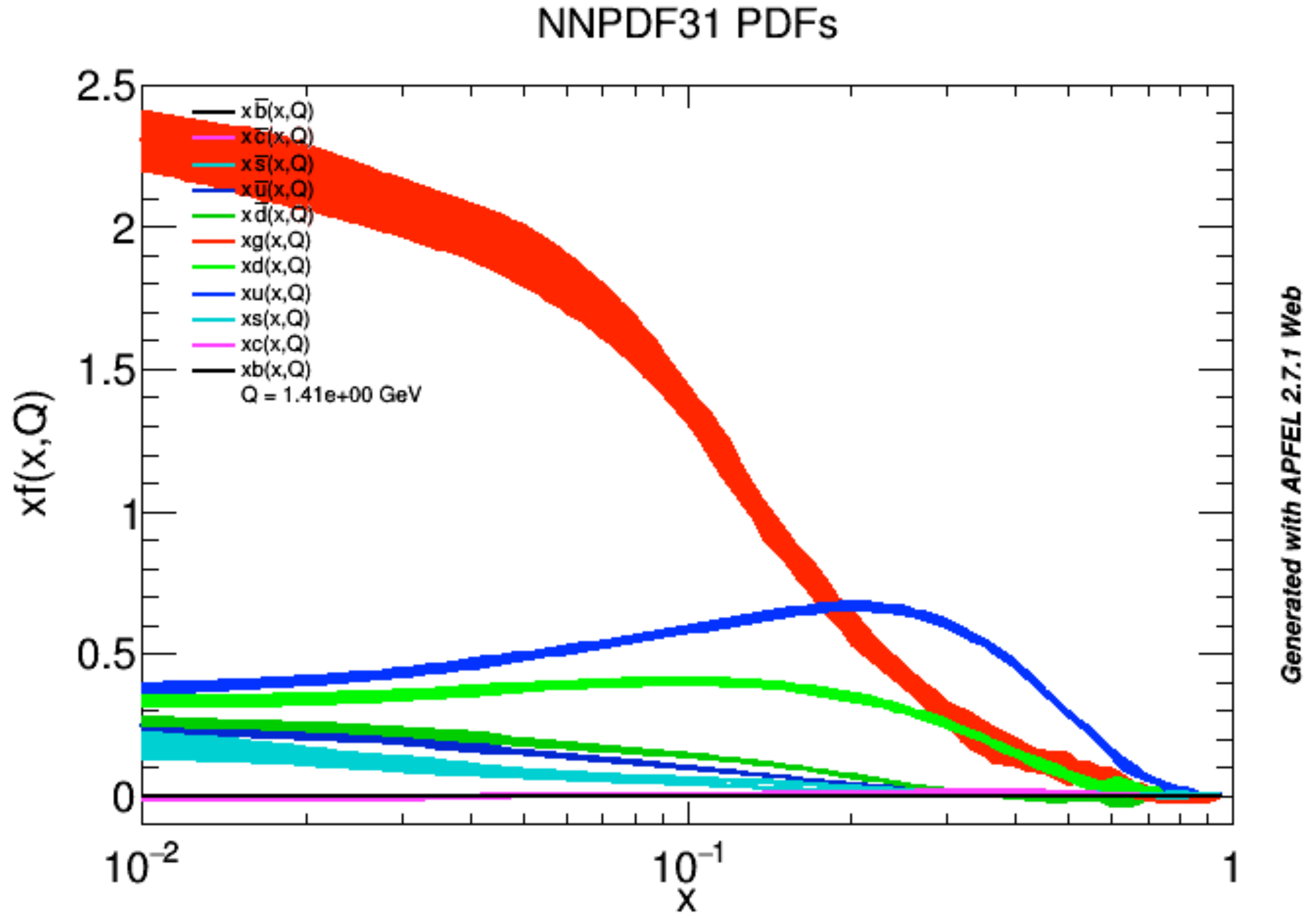
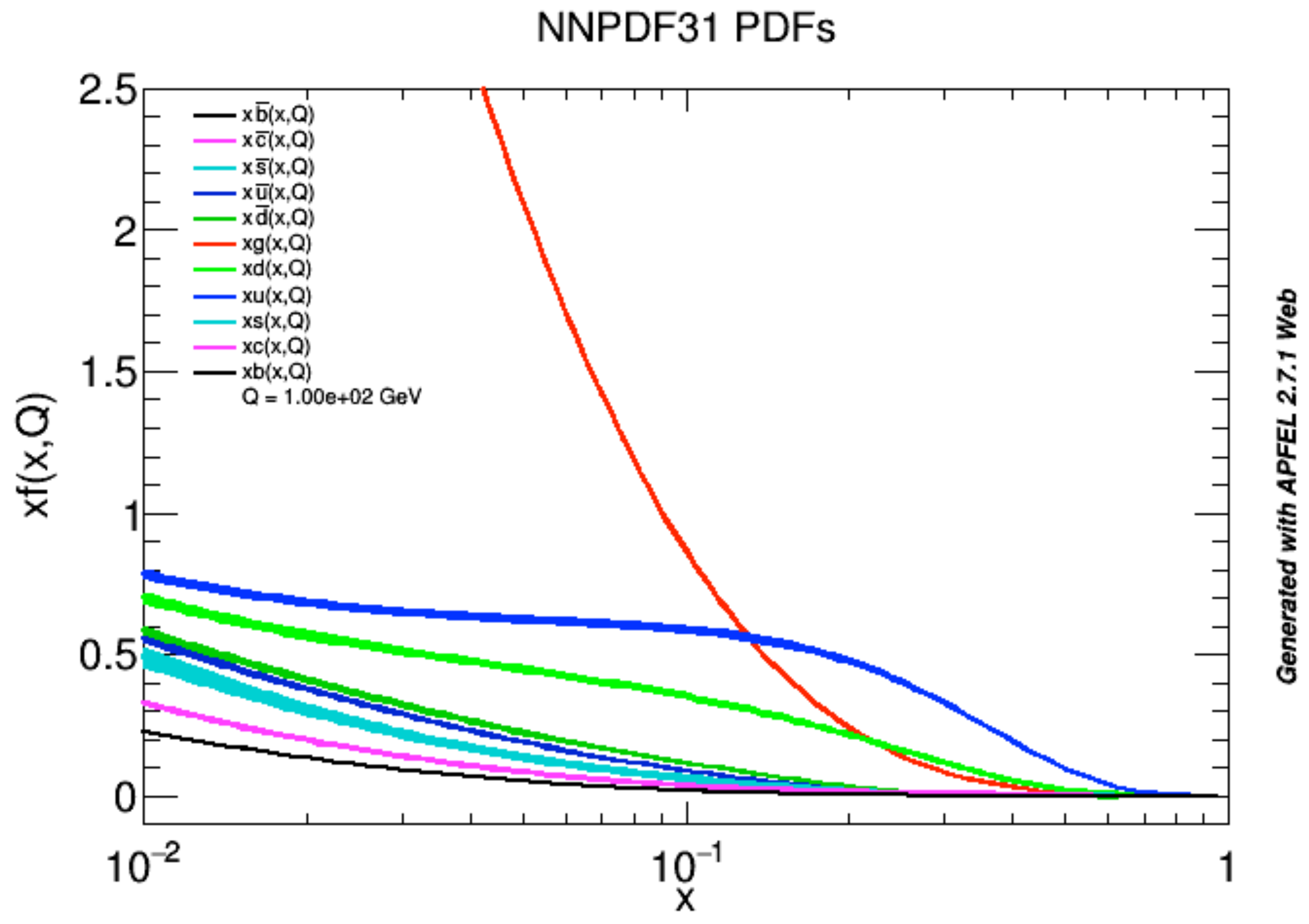


http://apfel.mi.infn.it/


Set the physics setup:
Initial scale (GeV): 1.4142135623731
Final scale (GeV): 1.4142135623731
Maximum flavors: 5
Plot all members:
Select member: 0
Standard deviation (1σ):
Number of points in x: 100
Log. x scale:
Log. y scale:
Automatic (x, y) range:
Minimum x: 1e-2
Maximum x: 1
Minimum y: -0.1
Maximum y: 2.5
Confirm


NNPDF31 PDFs
xf(x,Q)
x
Q = 1.41e+00 GeV
Generated with APFEL 2.7.1 Web


NNPDF31 PDFs
xf(x,Q)
x
Q = 1.00e+02 GeV
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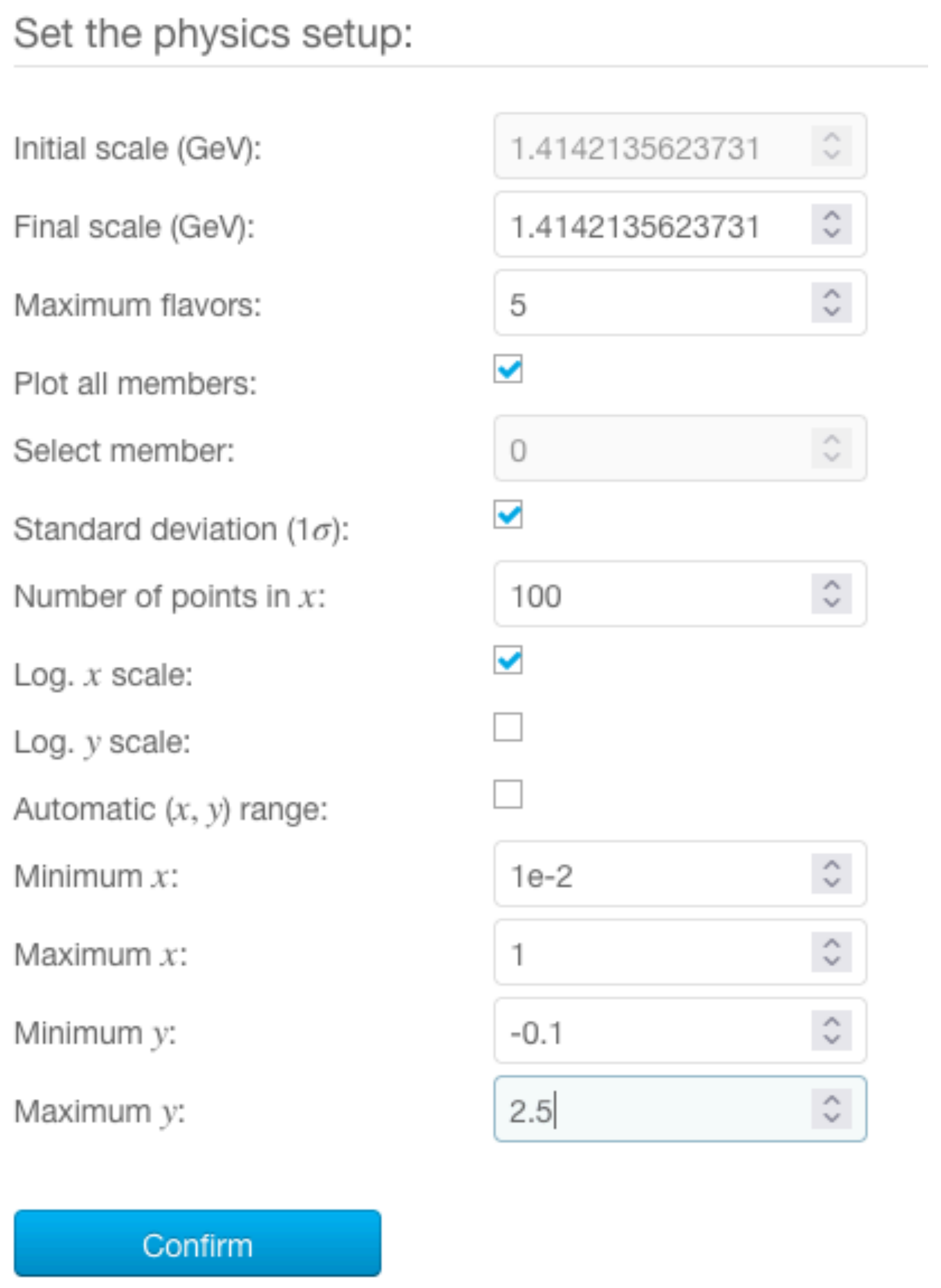
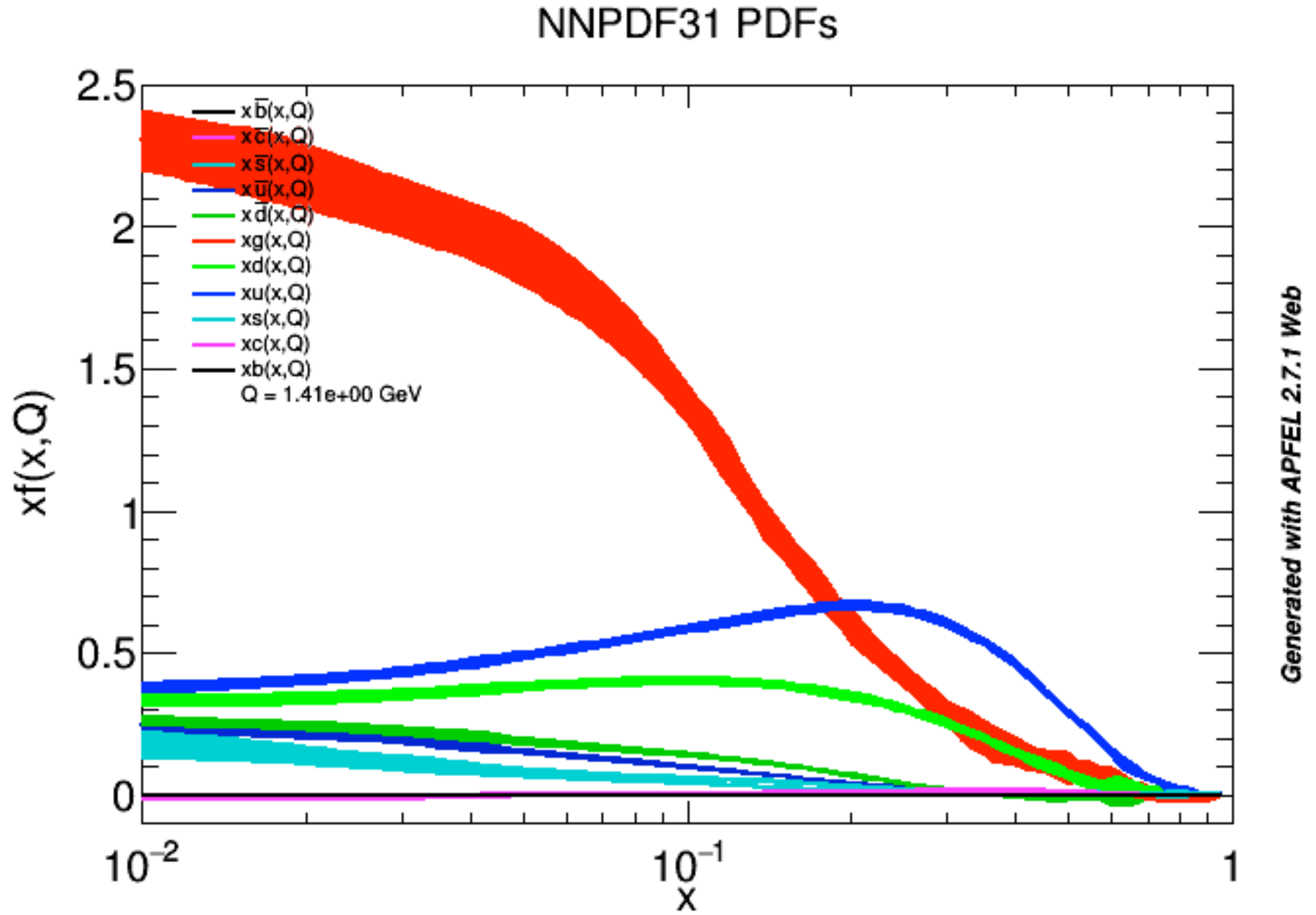
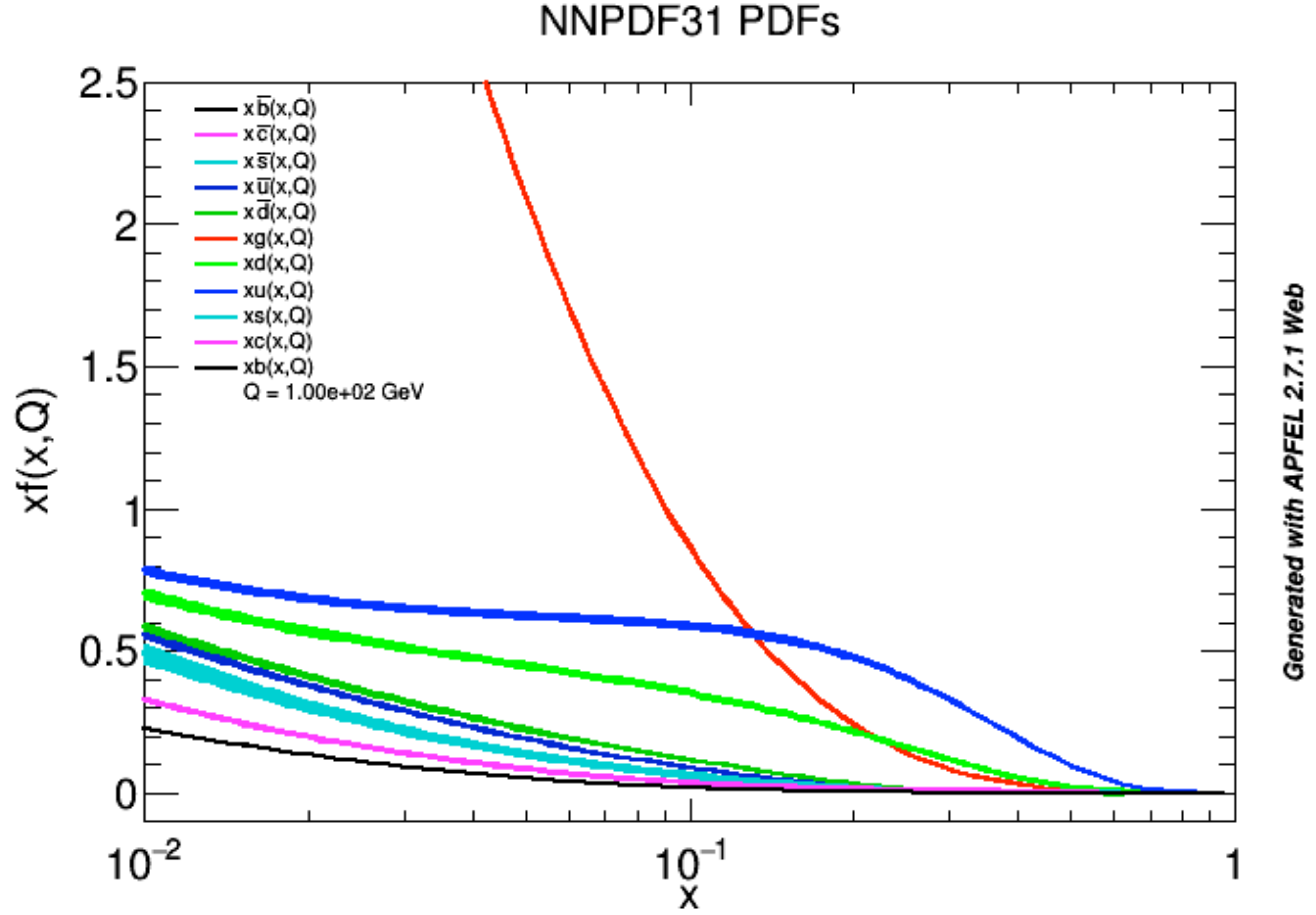
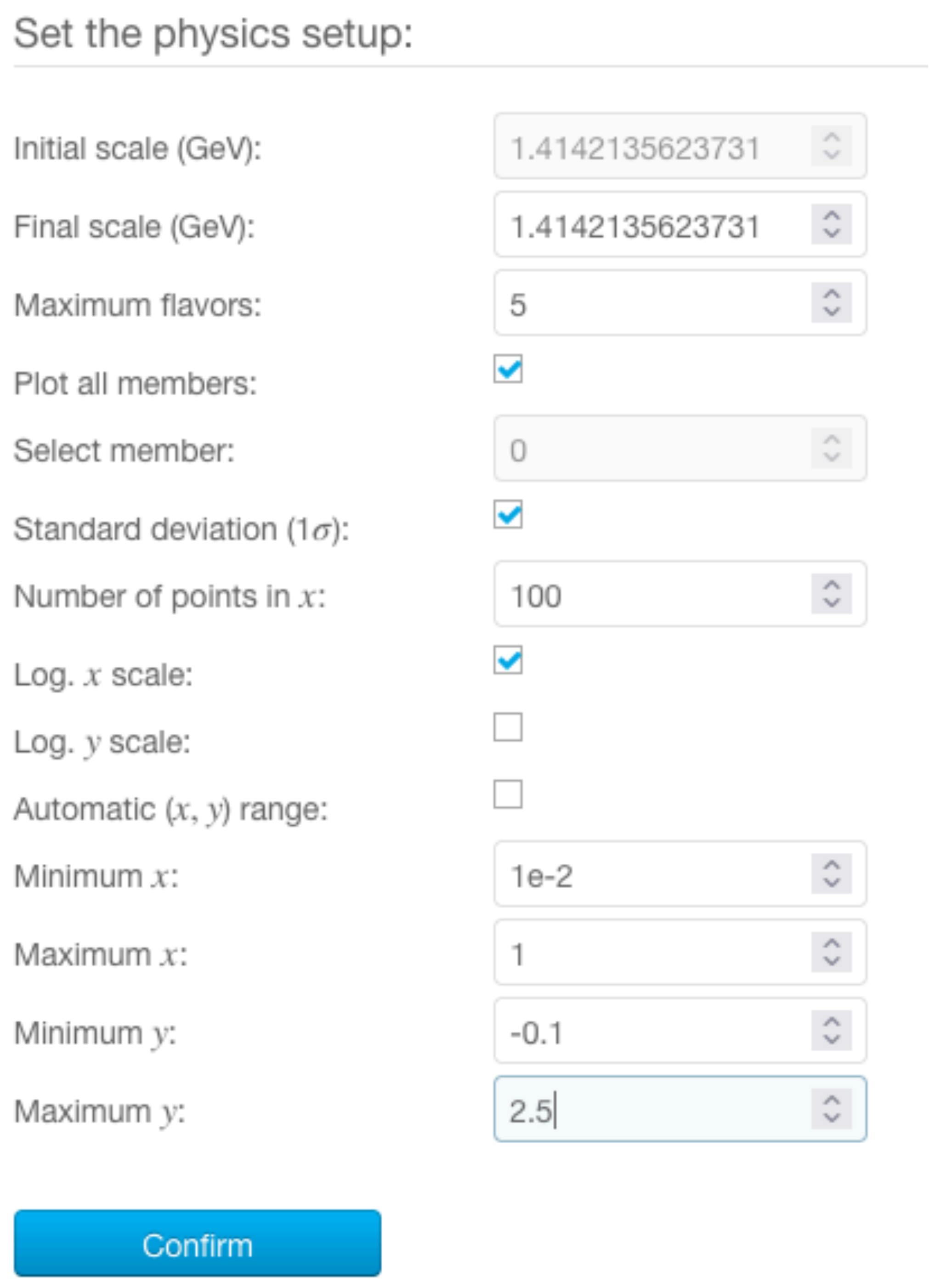
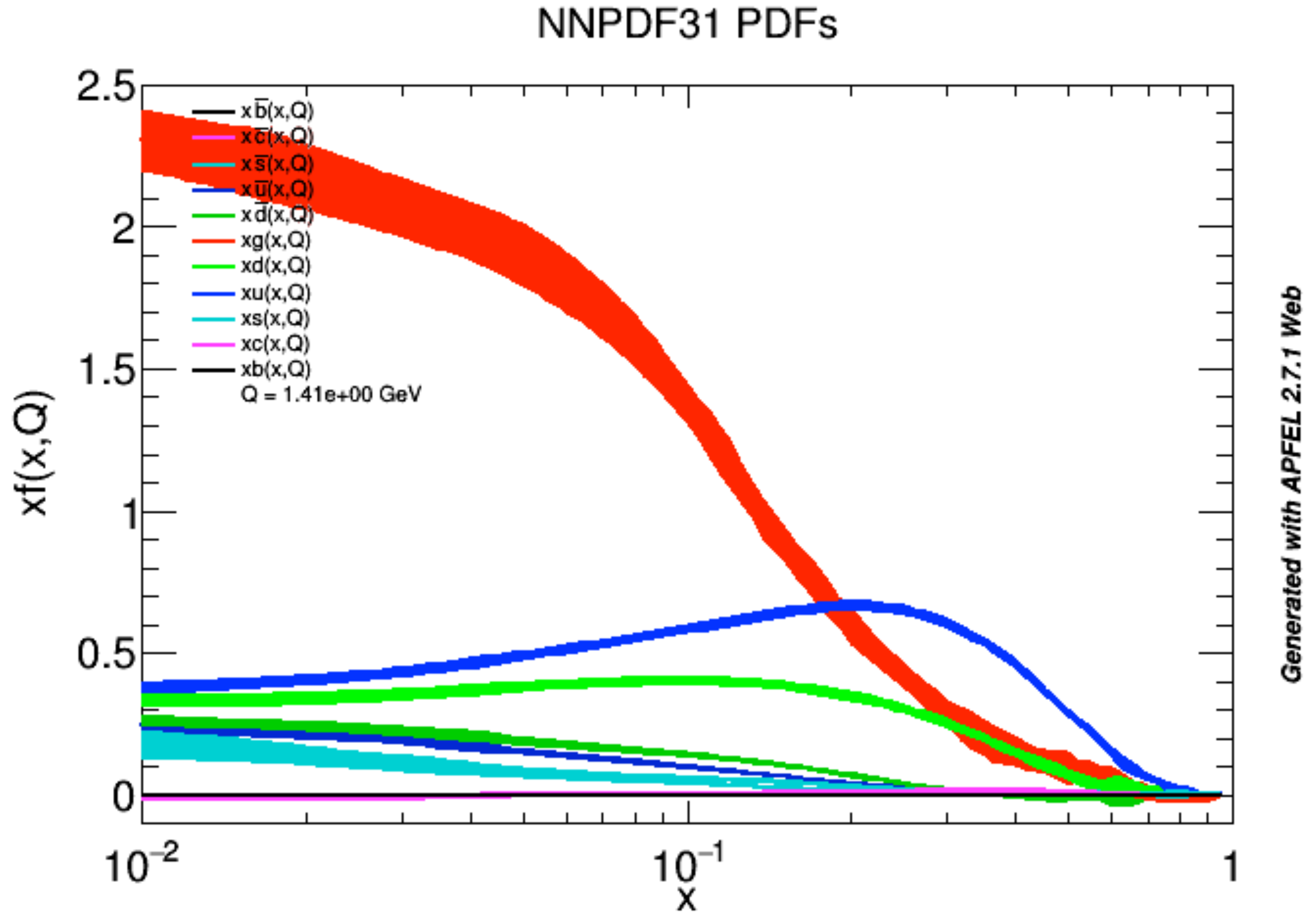


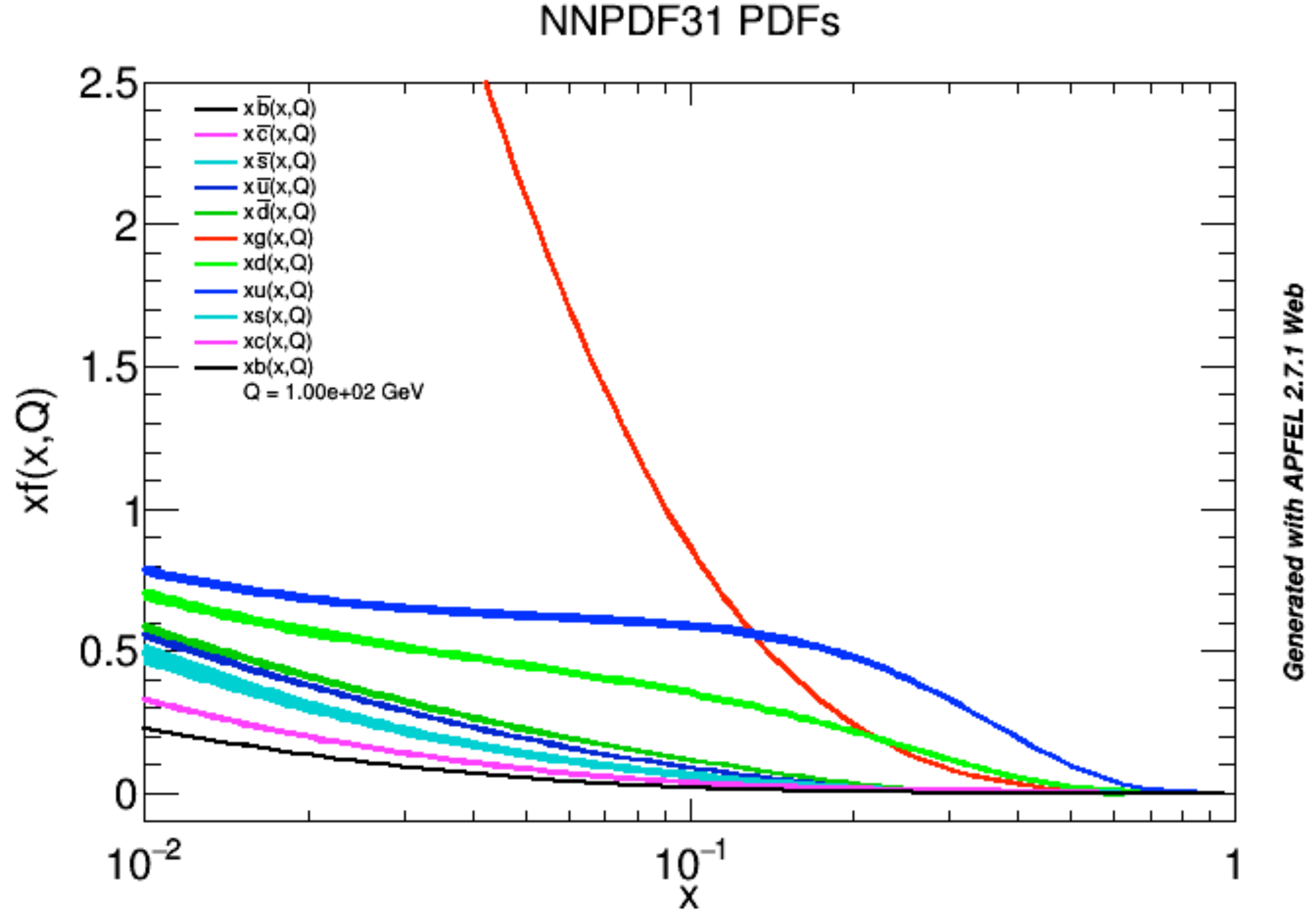




- Maximum of up and down at x=1/3: three quarks sharing the proton momentum







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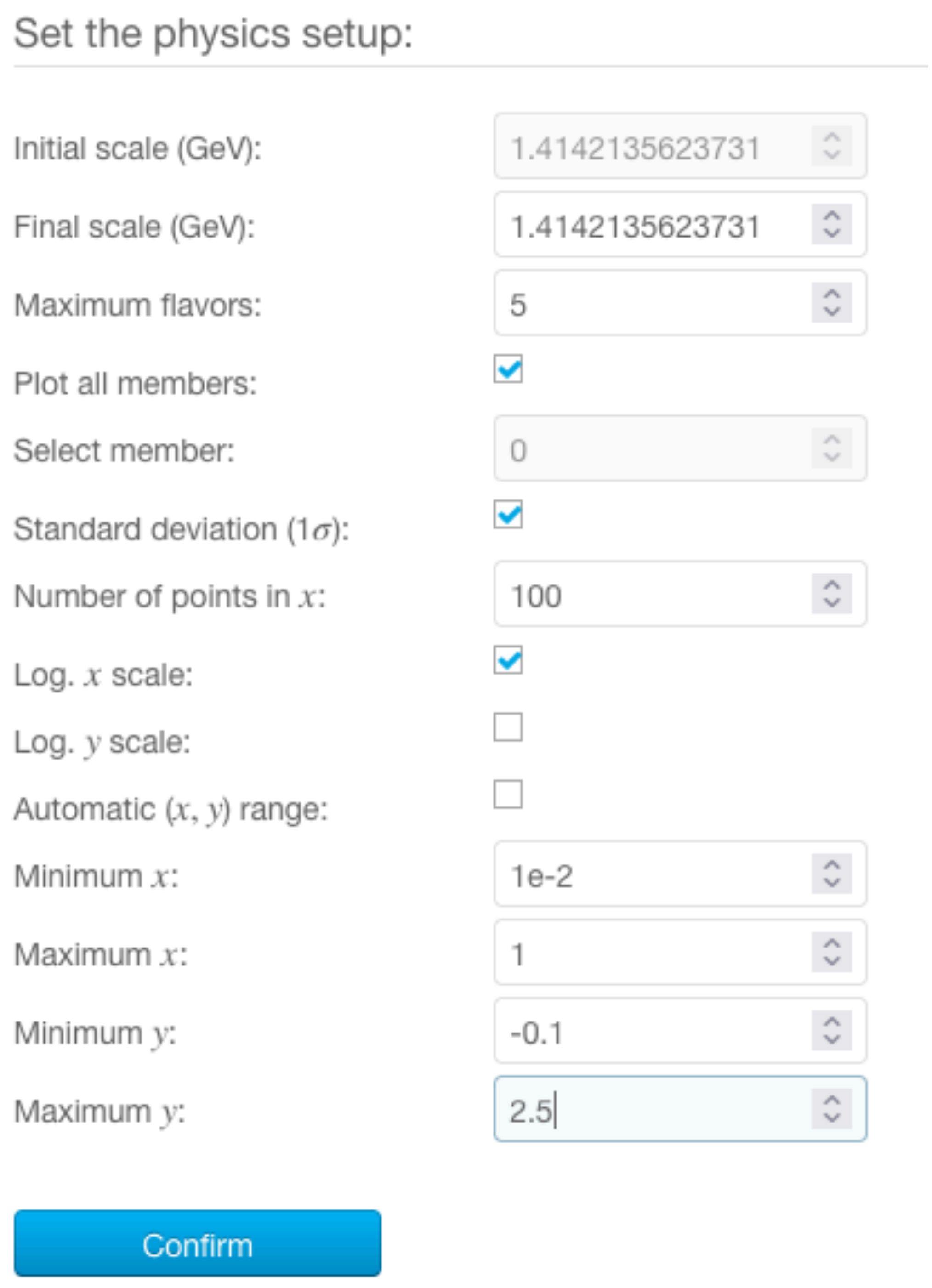
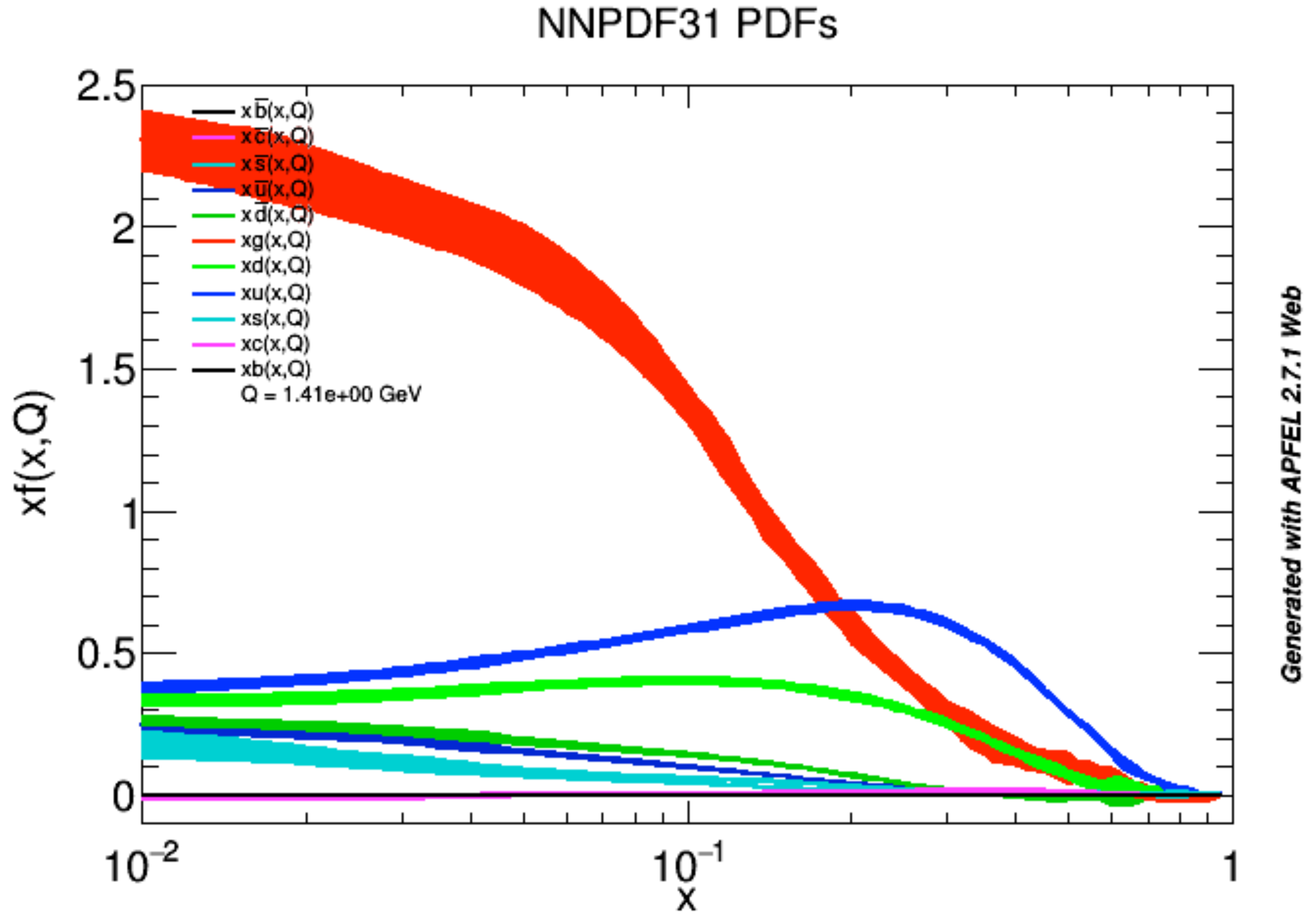
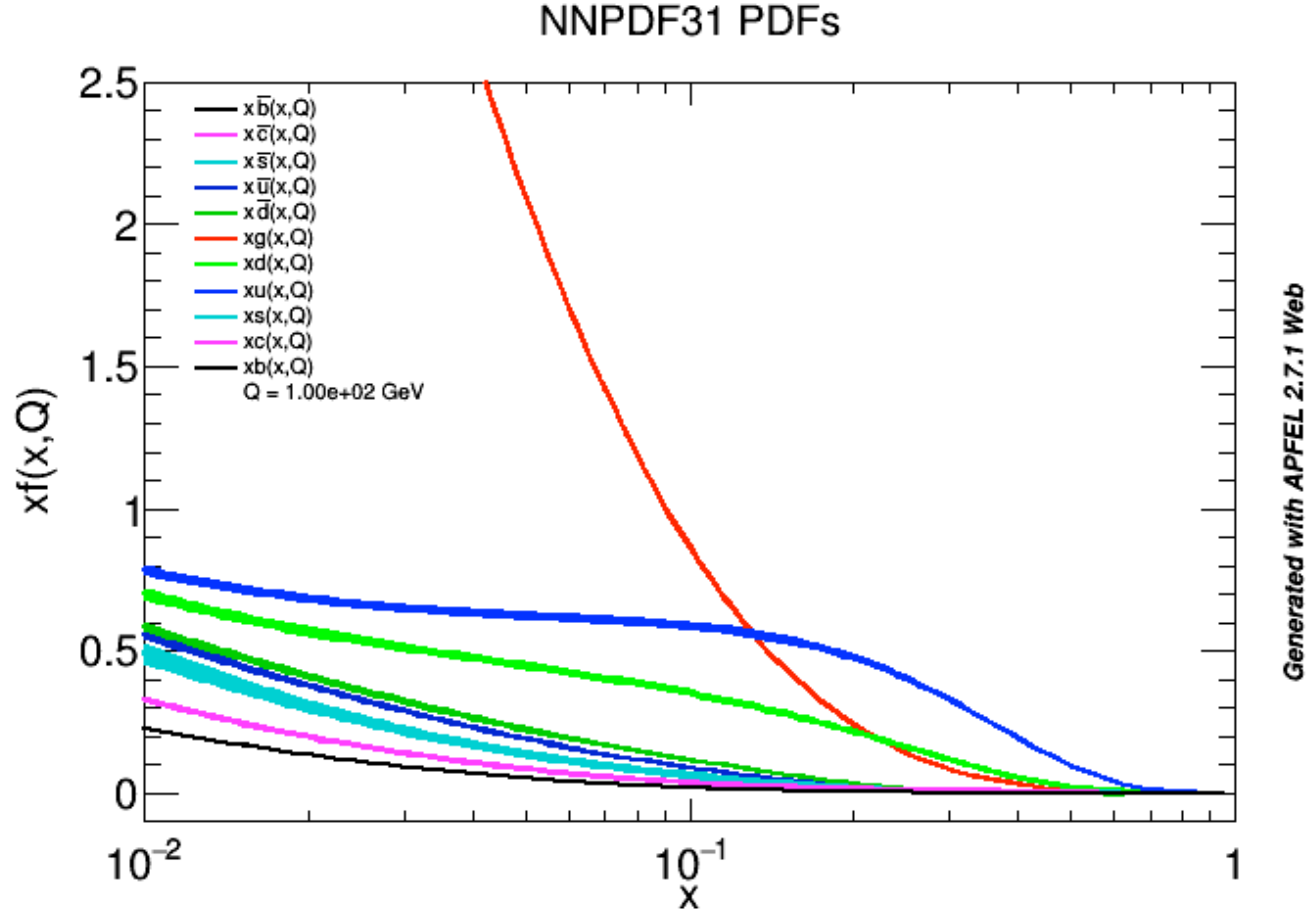
- up quark = 2 x down quark







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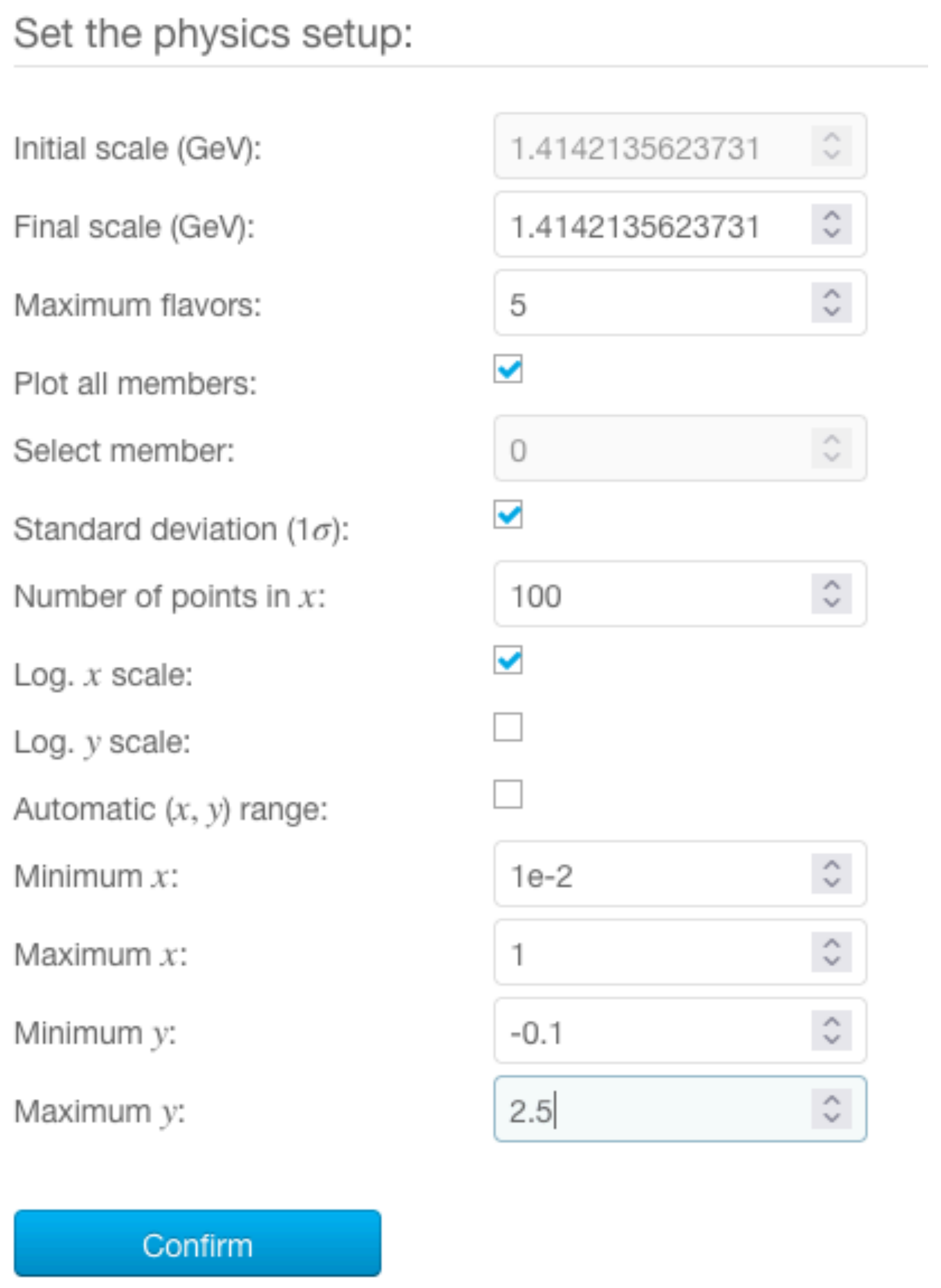
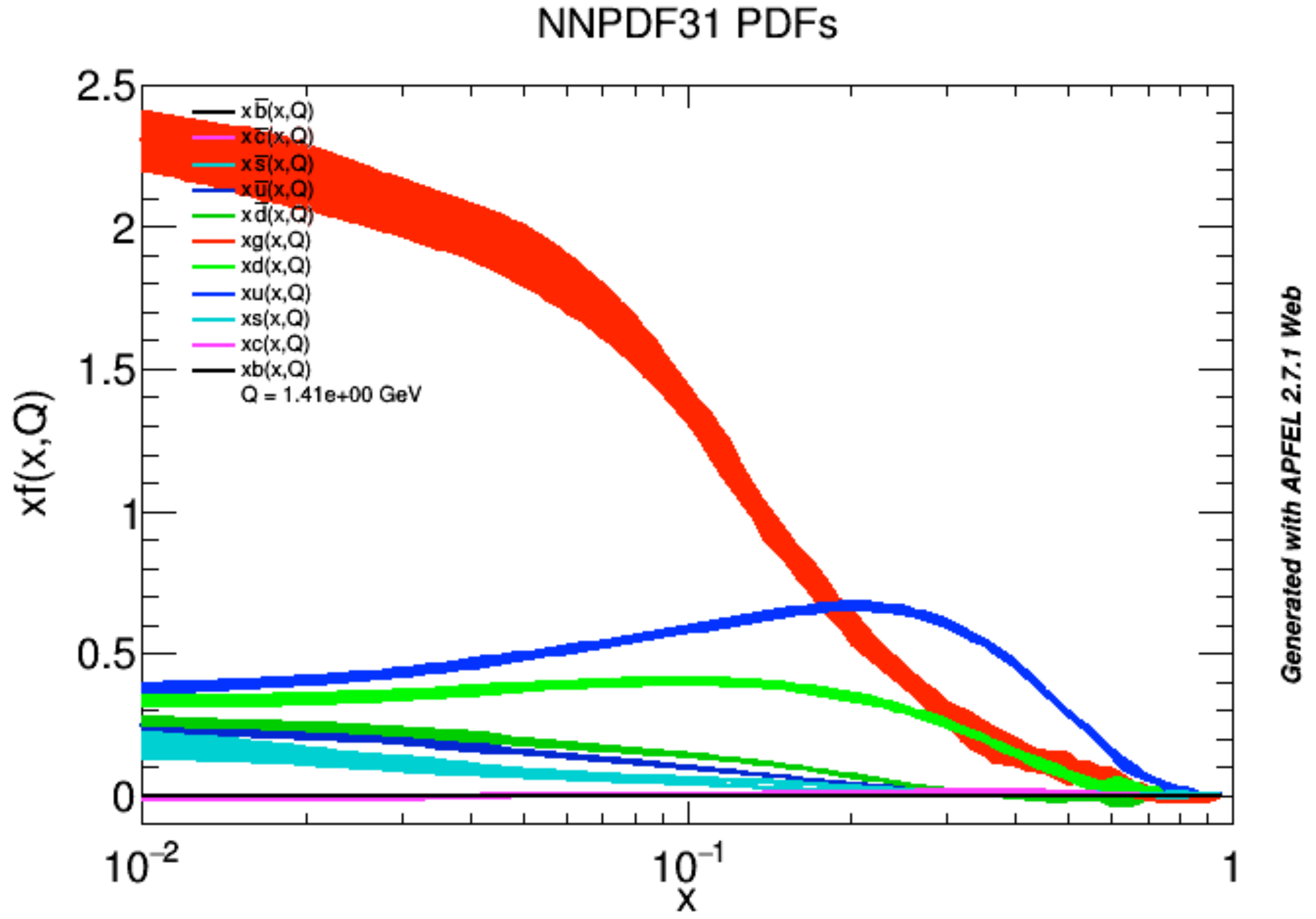
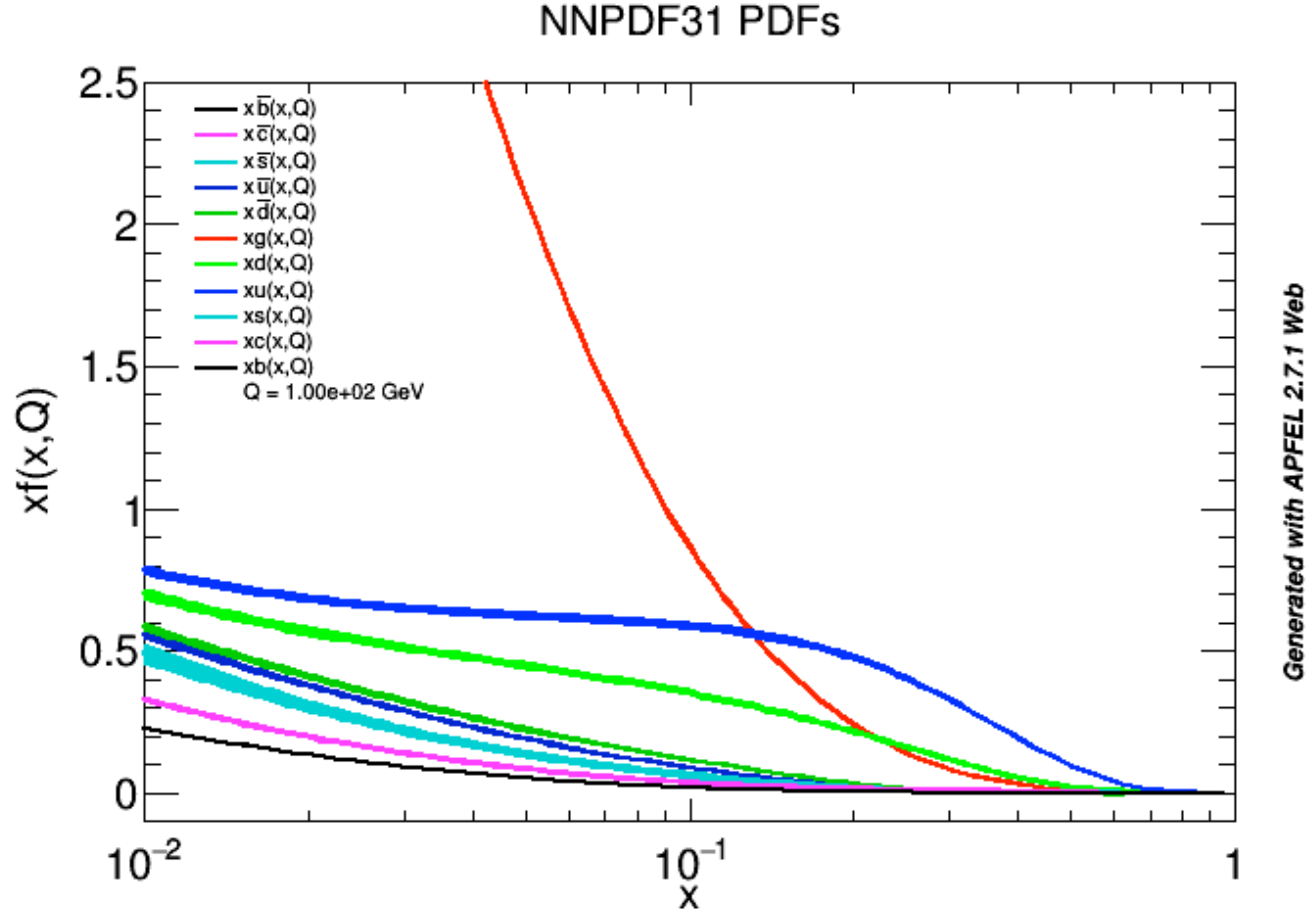
- gluon density evolves faster: colour charge $C_A = 3$ versus quark colour charge $C_F = 4/3$







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- up quark = 2 x down quark
- gluon density evolves faster: colour charge $C_A = 3$ versus quark colour charge $C_F = 4/3$
- more antiquarks at high energies from gluon splitting

PDFs strategy in a nutshell

- Make an **ansatz** for the functional form of the PDFs at some fixed low scale value ($Q_0 \sim 1\ \mathrm{GeV}$): e.g. in MRST/MSTW

$$x\, u_V = A_u\, x^{\eta_1} (1-x)^{\eta_2} (1 + \epsilon_u \sqrt{x} + \gamma_u x) \qquad u_V = u - \bar{u}$$

$$x\, d_V = A_d\, x^{\eta_3} (1-x)^{\eta_4} (1 + \epsilon_d \sqrt{x} + \gamma_d x) \qquad d_V = d - \bar{d}$$

$$x\, g = A_g\, x^{-\lambda_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x)$$

- **NNPDF** use **neural networks** and does not need such explicit functional form
- Collect data at various (x, Q^2) from different experiments (e.g. DIS), use DGLAP equations to evolve down to Q_0 and fit parameters, including α_S
- Ensure sum rules: (Gottfried, momentum, …).

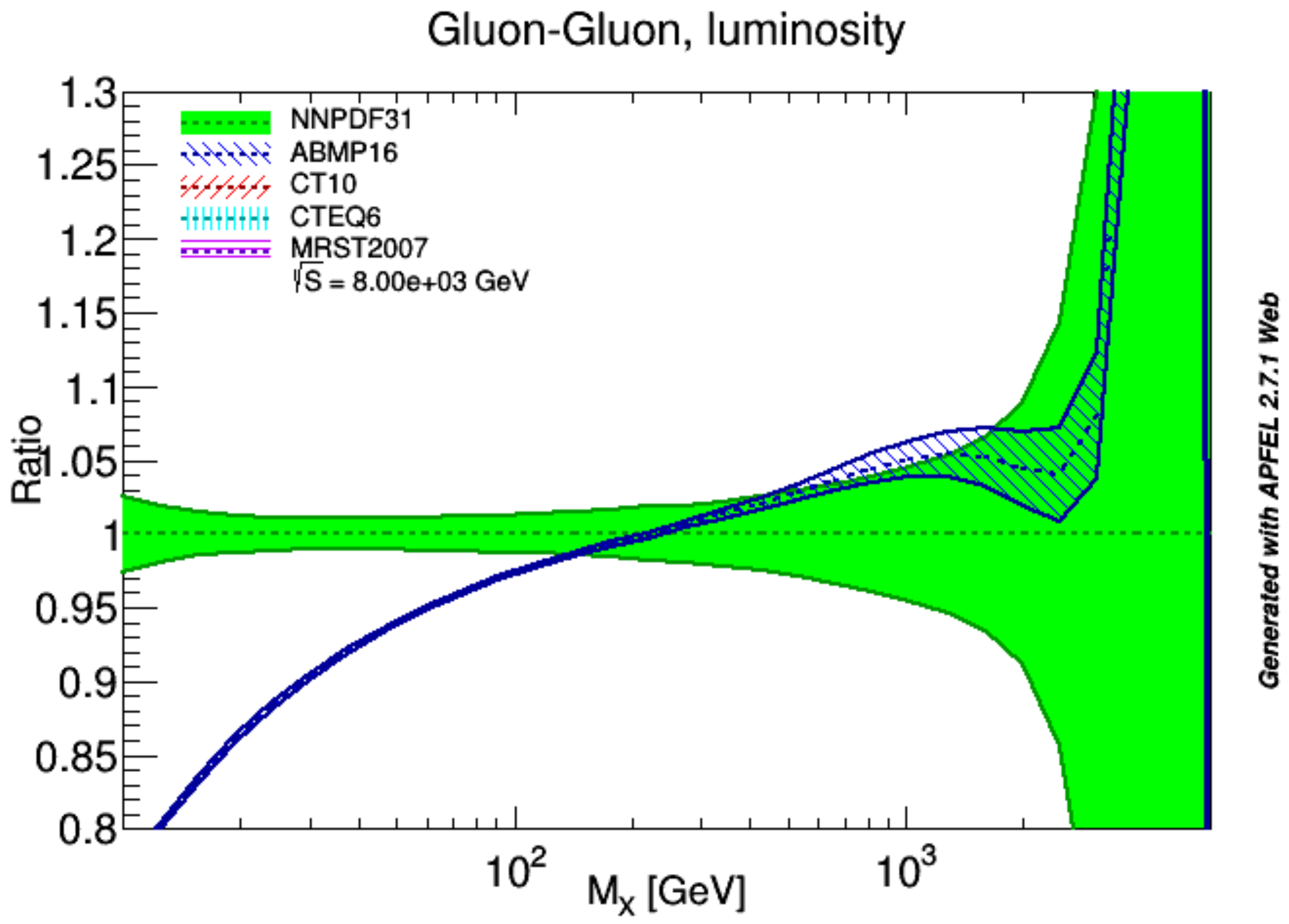
$$\int dx x \sum_i f_i(x, Q^2) = 1$$

Parton densities

- **Differences are due to different:**

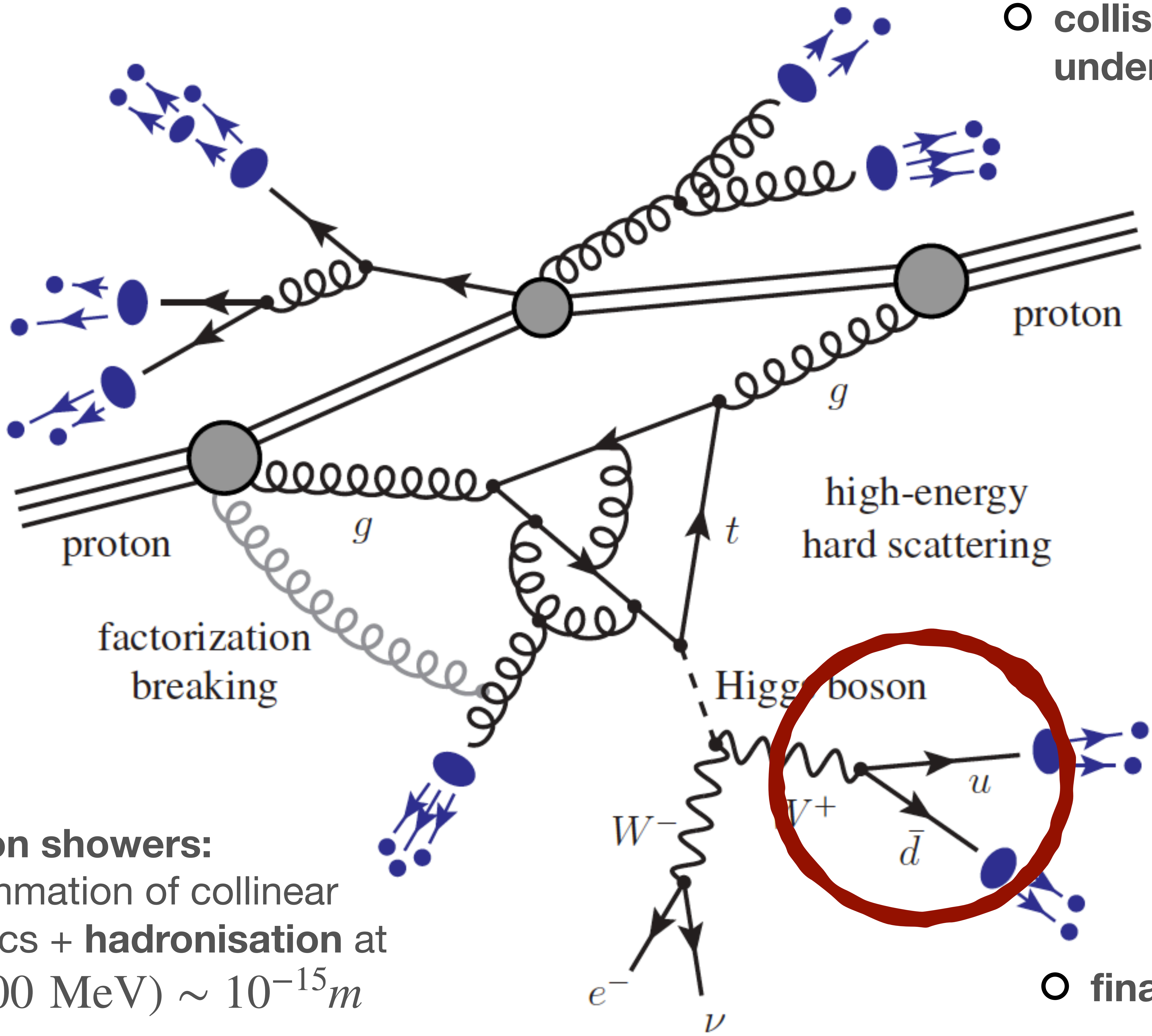
Data sets in fits, parameterization of starting distributions, order of pQCD evolution, power law contributions, nuclear target corrections, resummation corrections (ln 1/x, …), treatment of heavy quarks, strong coupling, choice of factorization and renormalization scales.

- at least 5-10% uncertainty in theoretical predictions



JETS

factorisation into short distance (hard scattering = high energy) and long distance (initial and final state = low energy)



- **collision remnants / underlying event**
- **initial-state** parton densities $1/\mathrm{GeV} = 10^{-16}m$
- High-energy **collision** at $1/\mathrm{TeV} \sim 10^{-19}m$
- **Parton showers:** resummation of collinear physics + **hadronisation** at $1/(200\ \mathrm{MeV}) \sim 10^{-15}m$
- **final state:** e.g. jets
- **final state:** e.g. leptons

What´s a jet

What´s a jet

- a bunch of energetic and collimated particles

What´s a jet

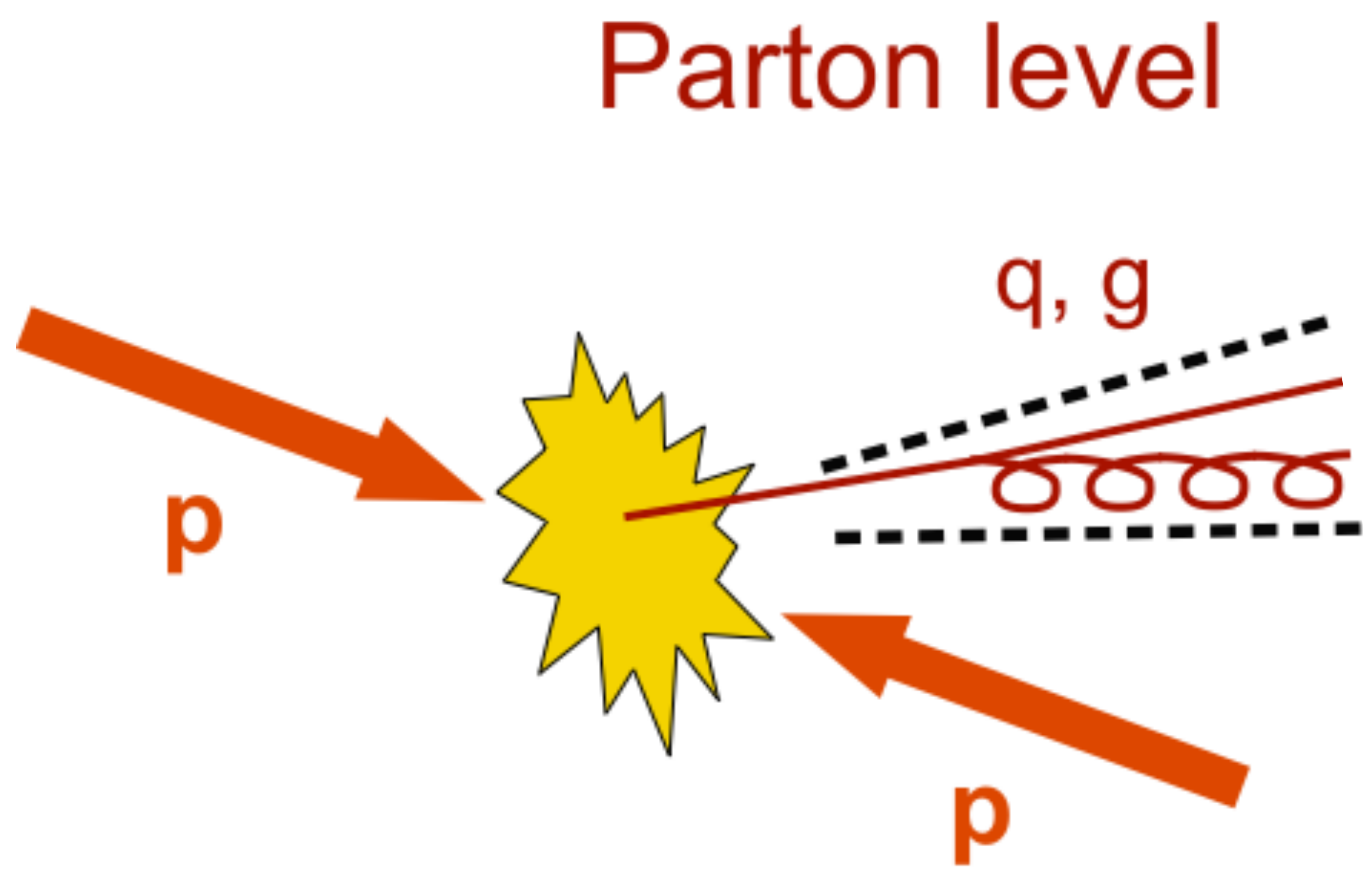
- a bunch of energetic and collimated particles
- 60% of LHC papers use jets [Salam, Soyez]

Why and how do we see jets?

Gluon emission

$$\alpha_S \int \frac{dE}{E} \frac{d\theta}{\theta}$$

higher probability at small angle (collinear) and small energy (soft)



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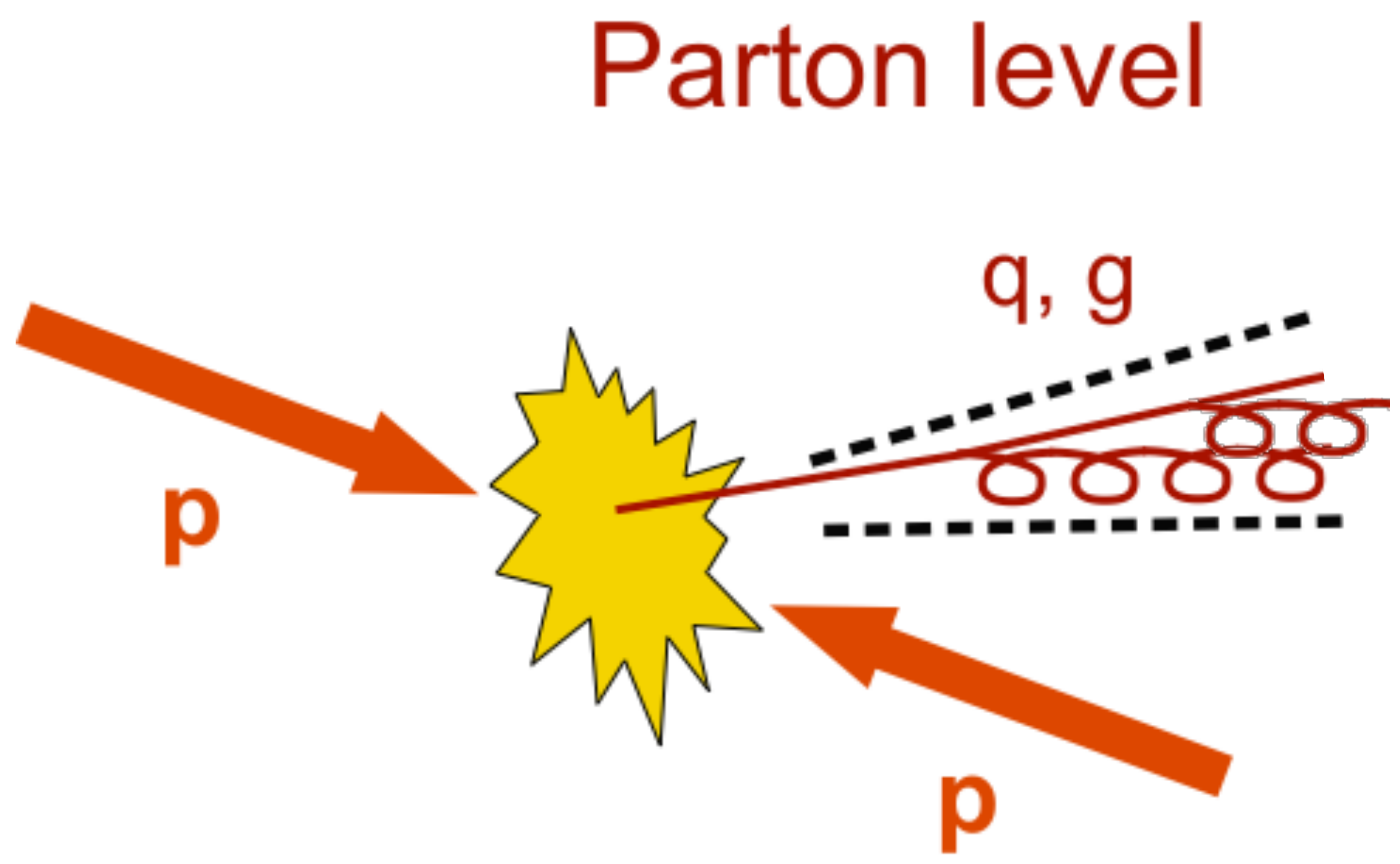

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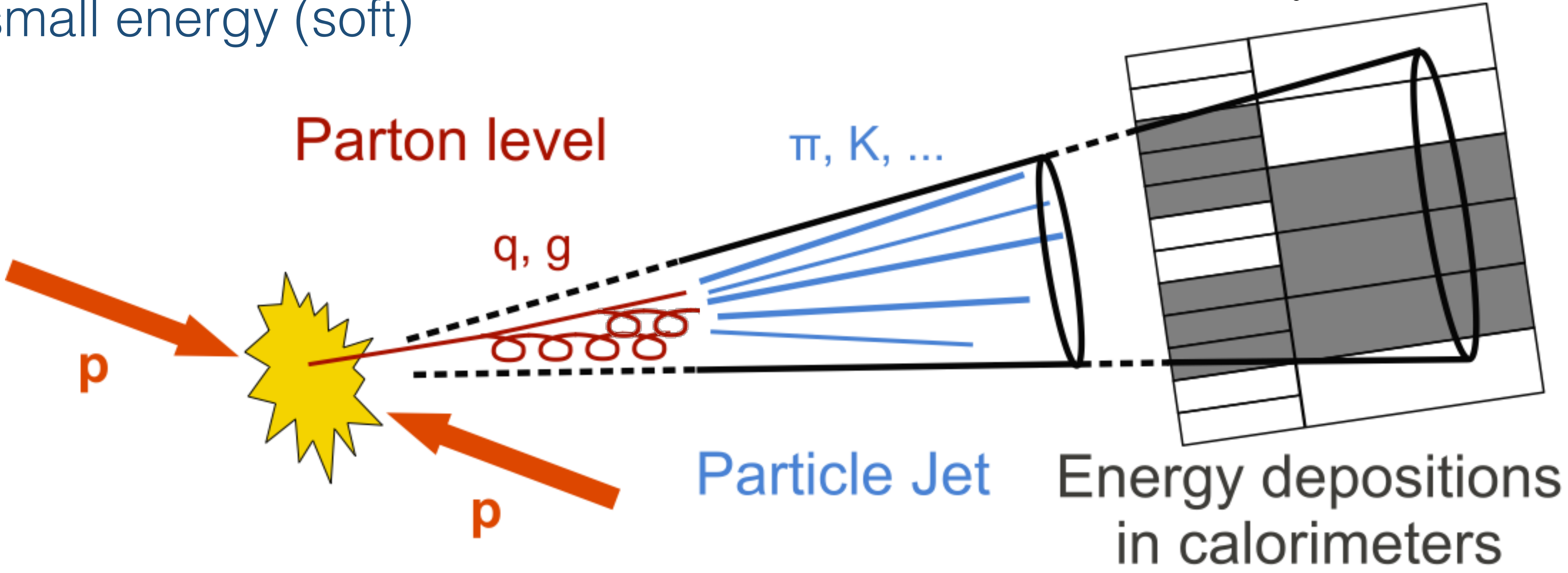
Non-perturbative transition to hadrons

$$\alpha_S \sim 1 \quad \Lambda_{QCD} \sim 200\text{MeV}$$



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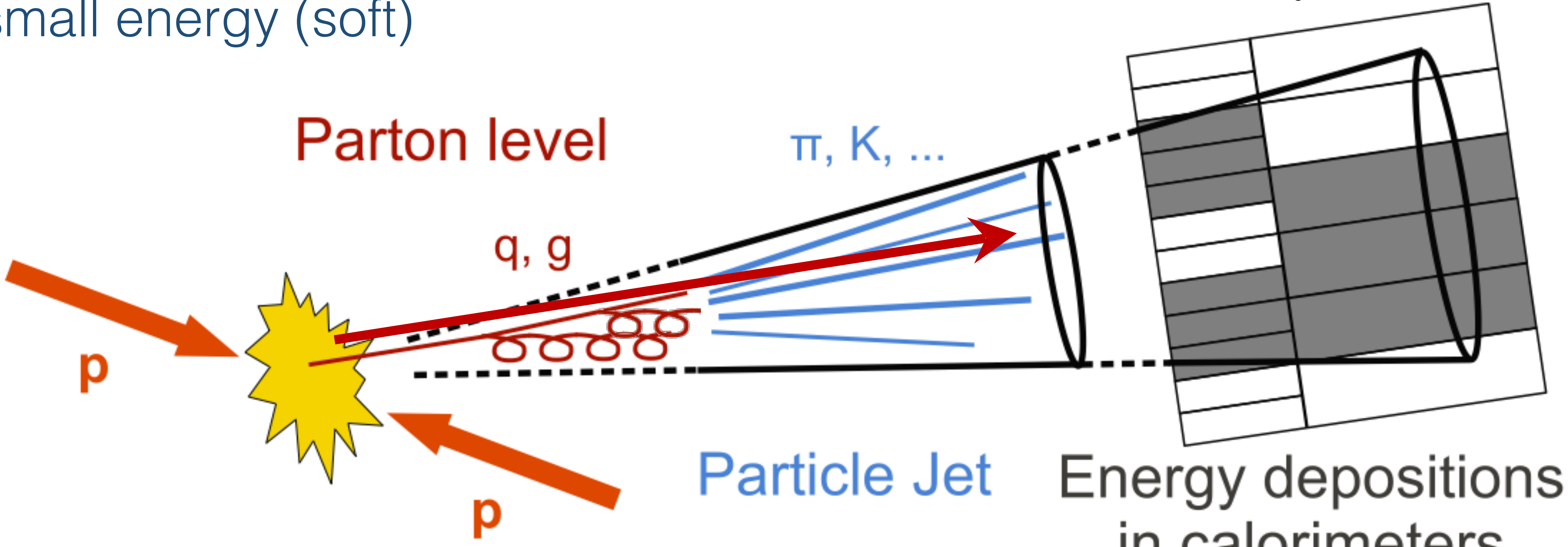

$\{j_k\}$ jet definition $\longleftarrow$ $\{p_i\}$

jets

final-state 4-momenta

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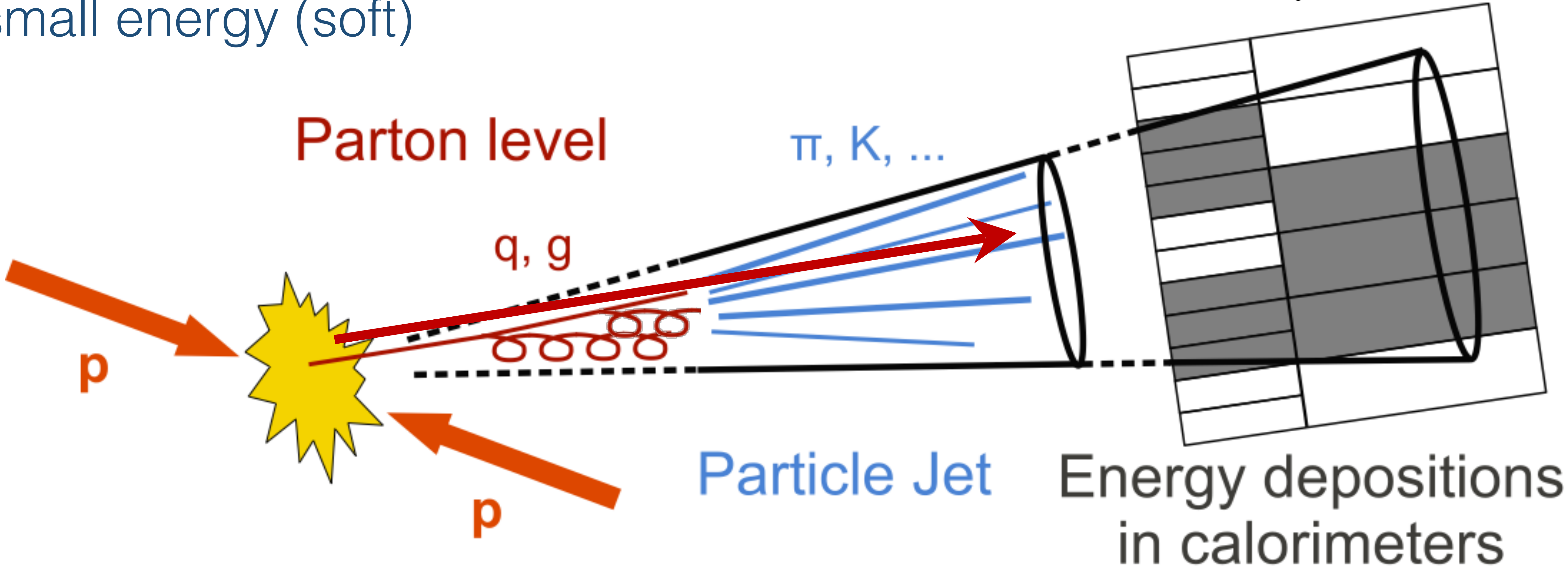


hard partons $\xleftarrow{\text{interpretation}}$ $\{j_k\}$ jets $\xleftarrow{\text{jet definition}}$ $\{p_i\}$ final-state 4-momenta

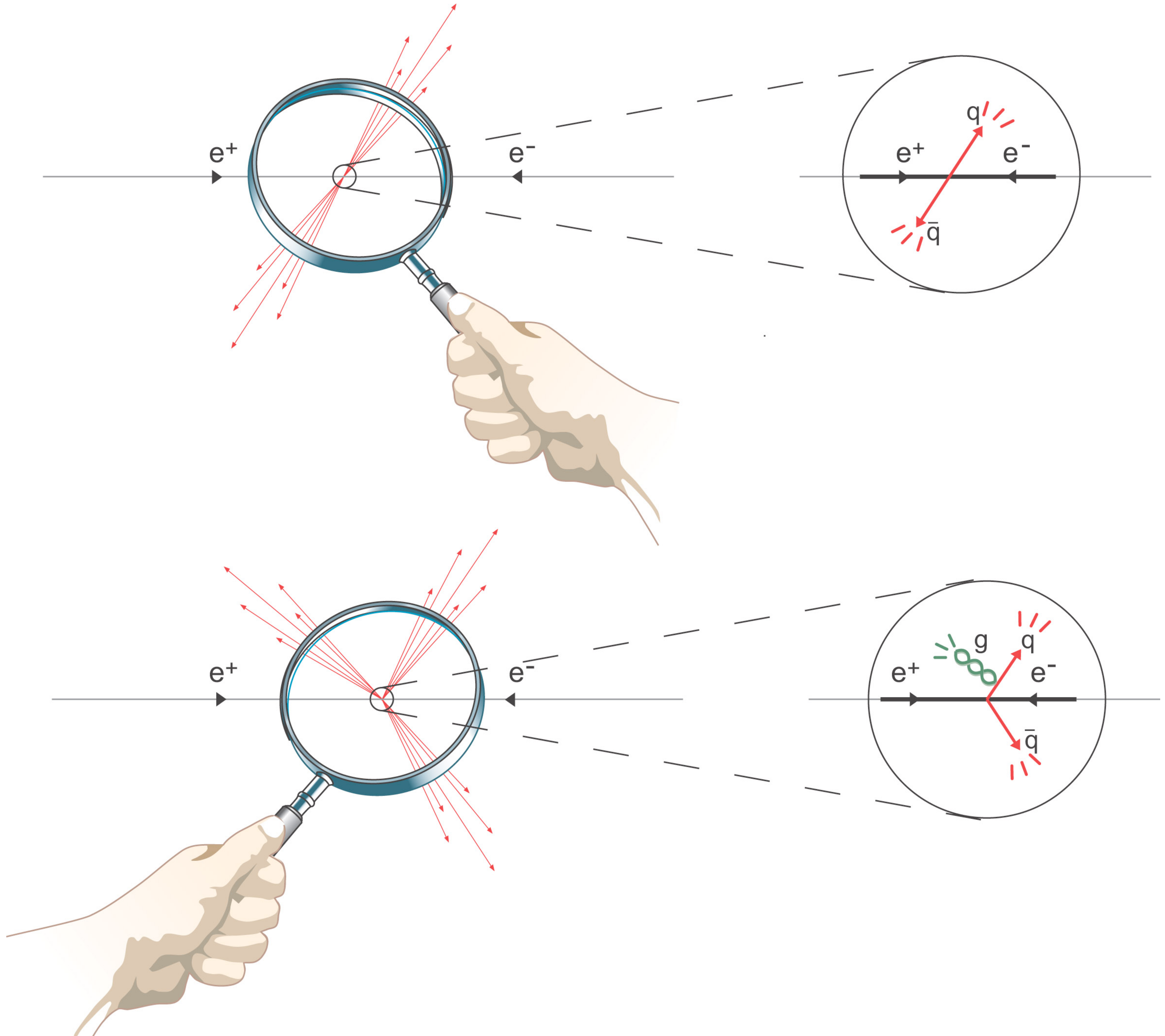
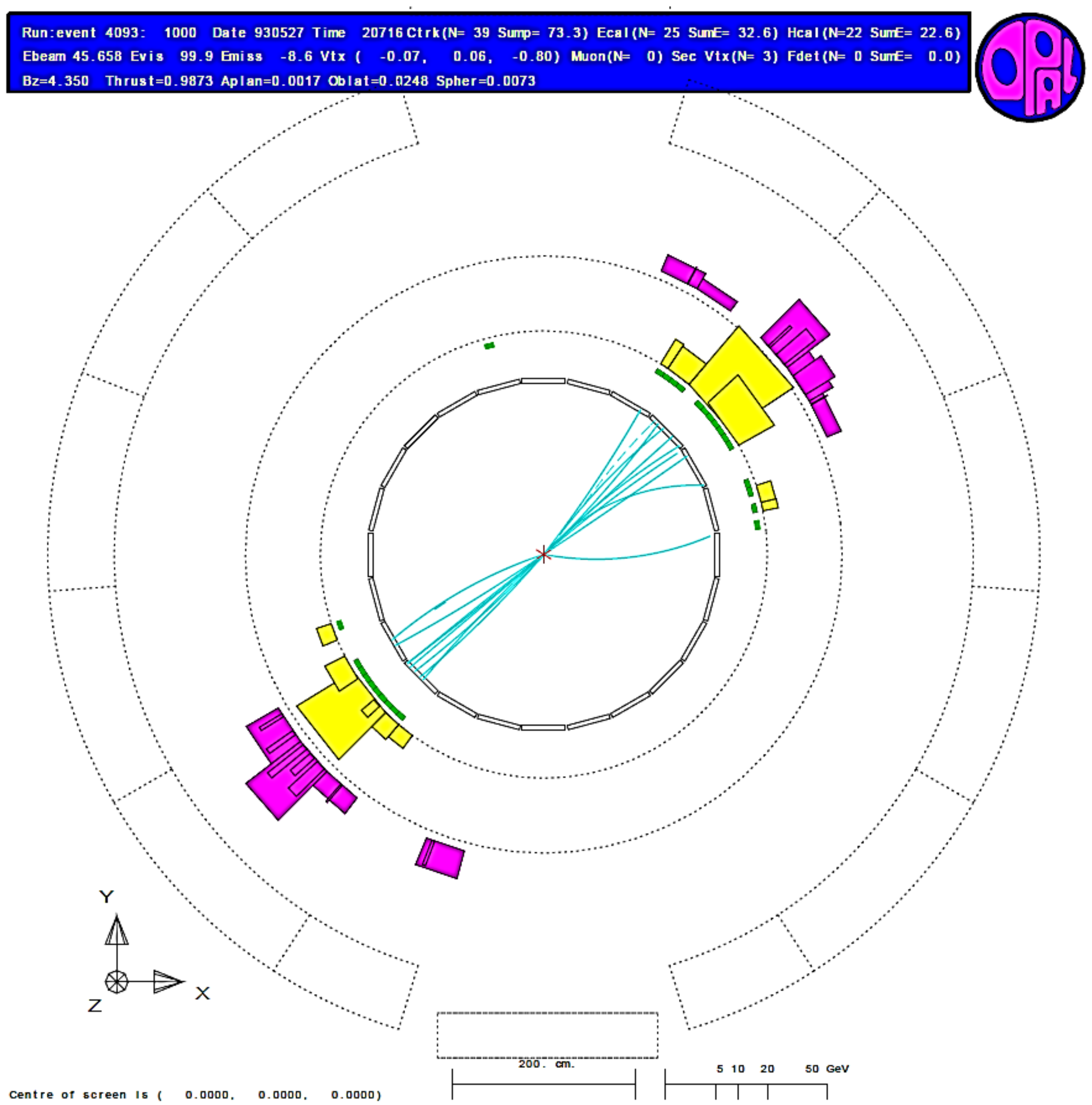


Illustration: Typoform



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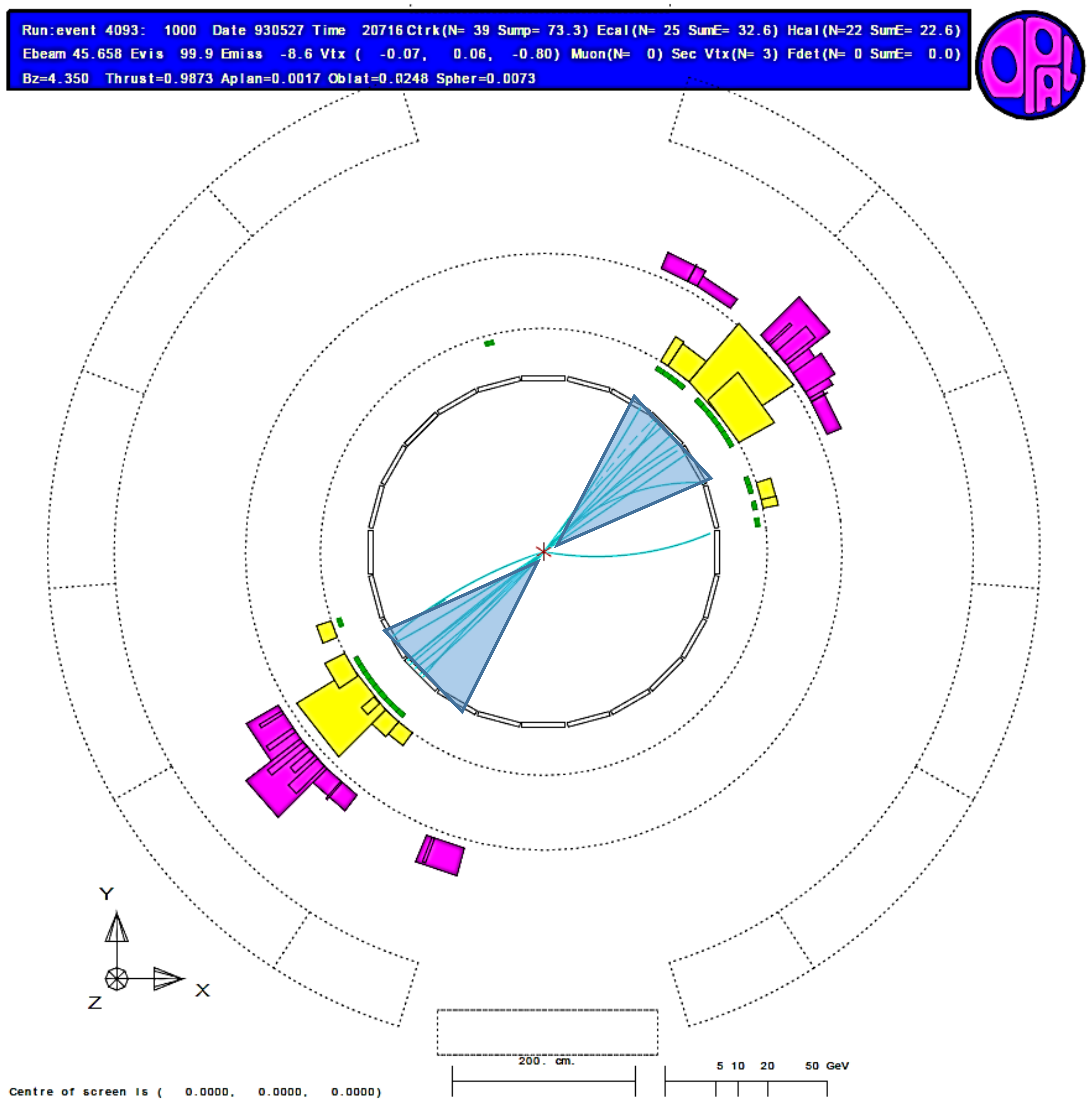
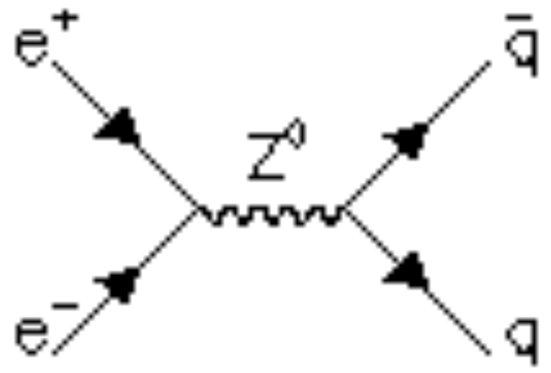
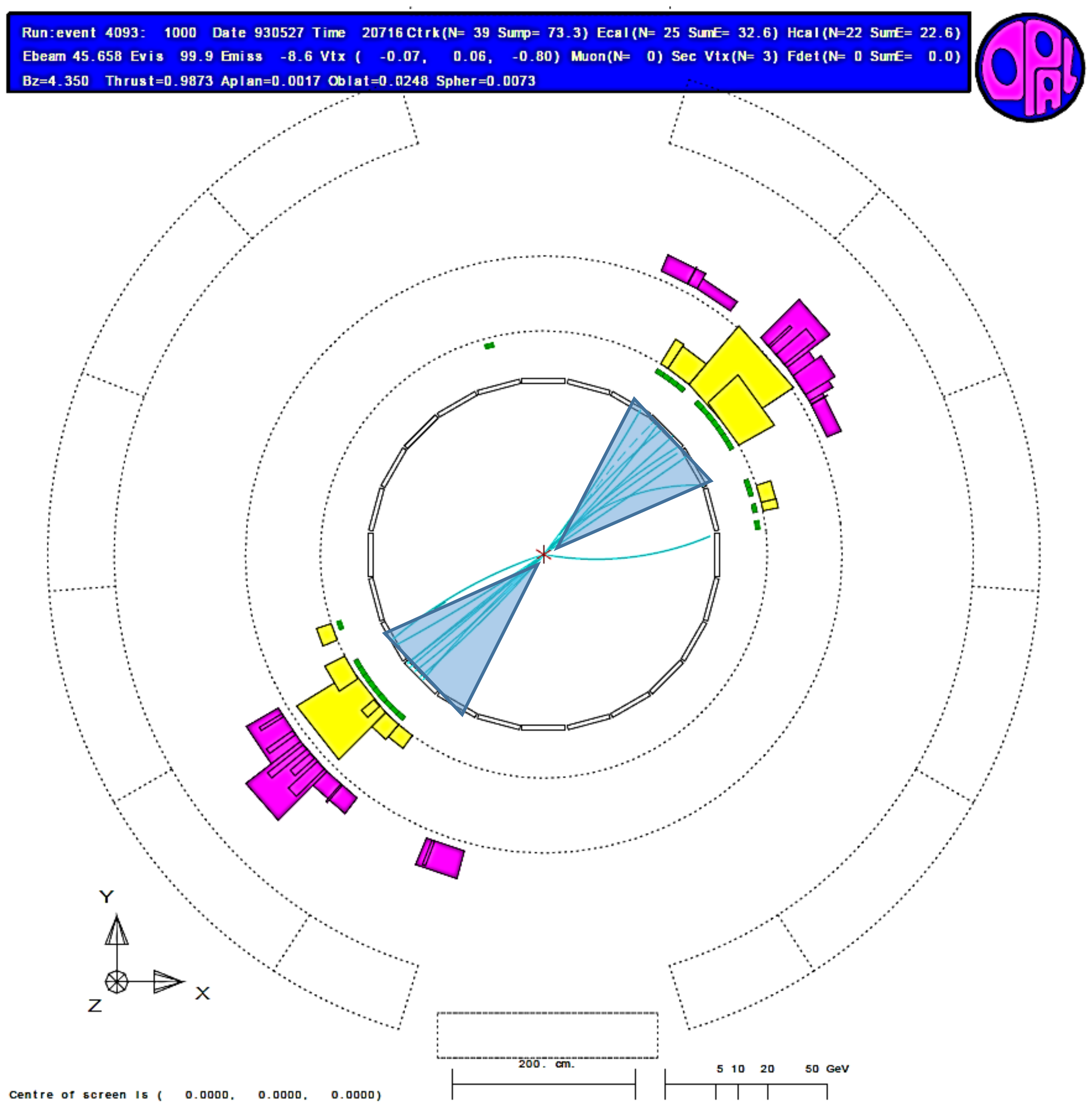
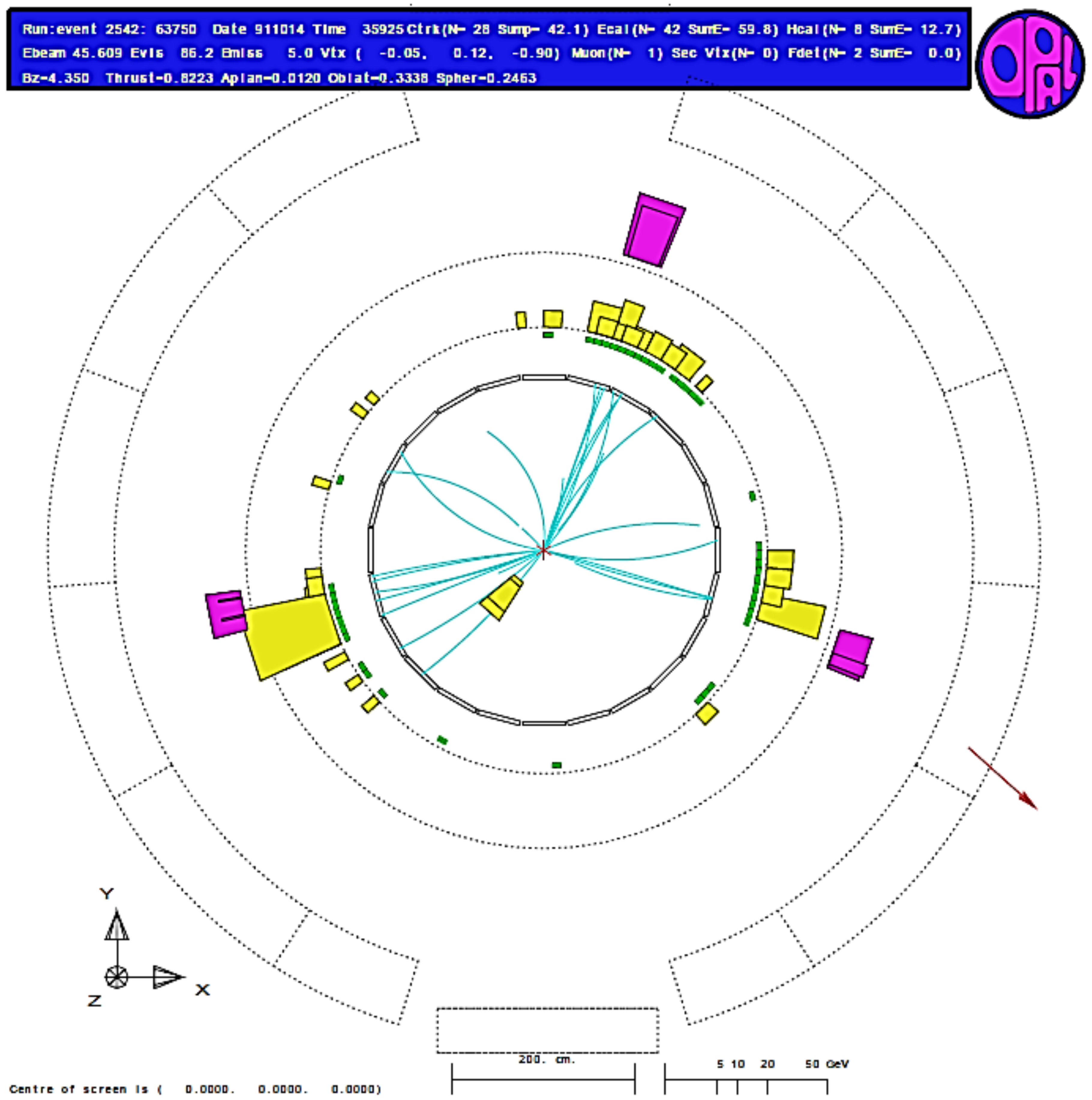
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
X
Z
200. cm.
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

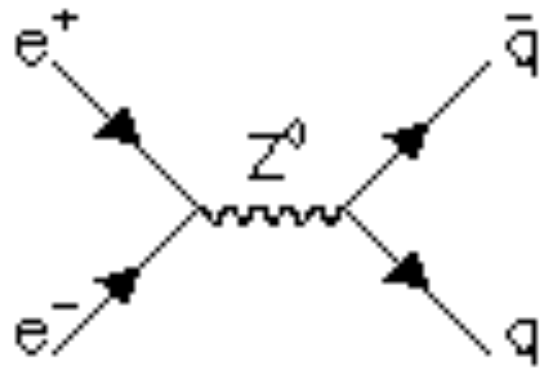




- Clearly
a two-jet event







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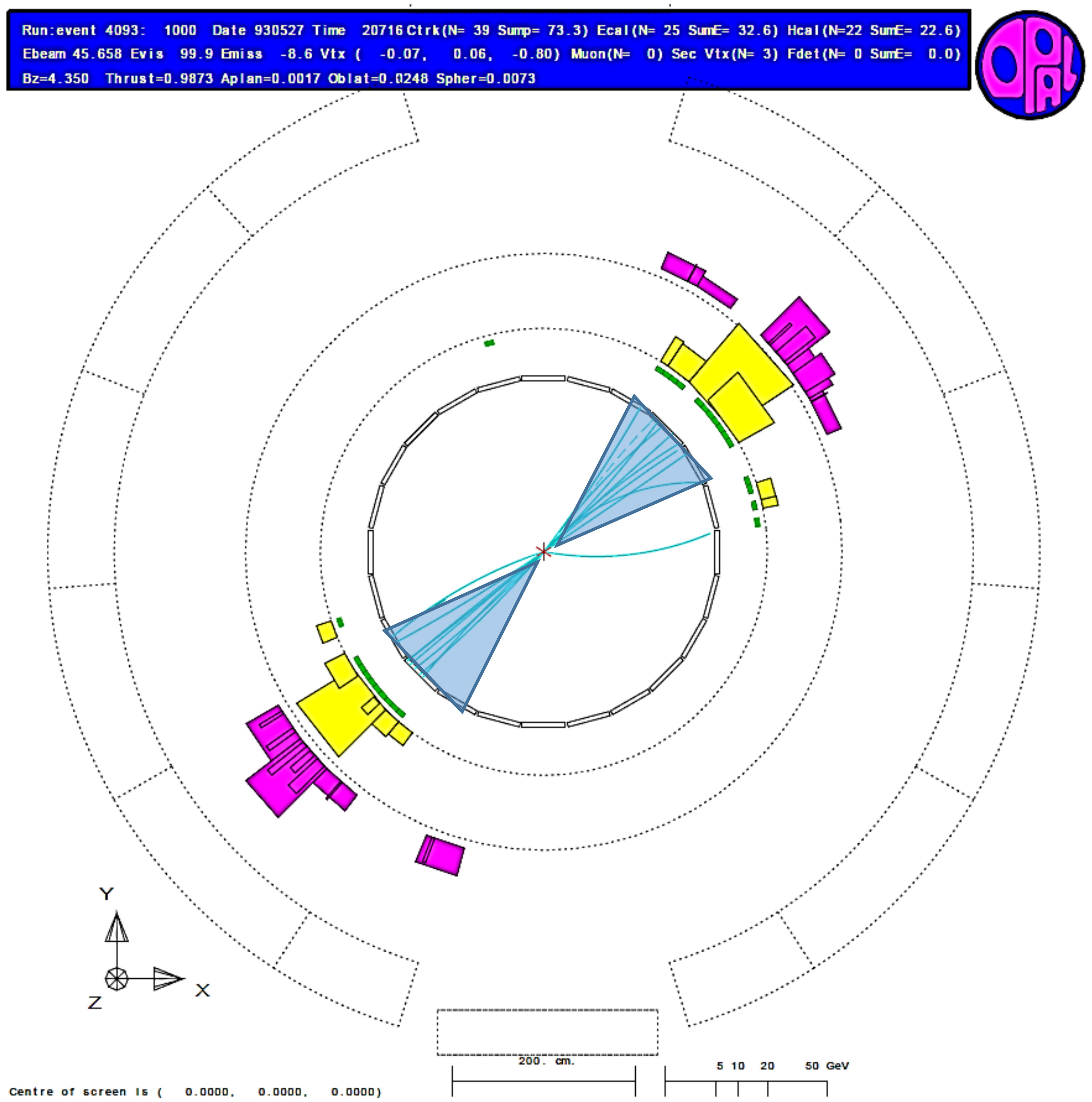

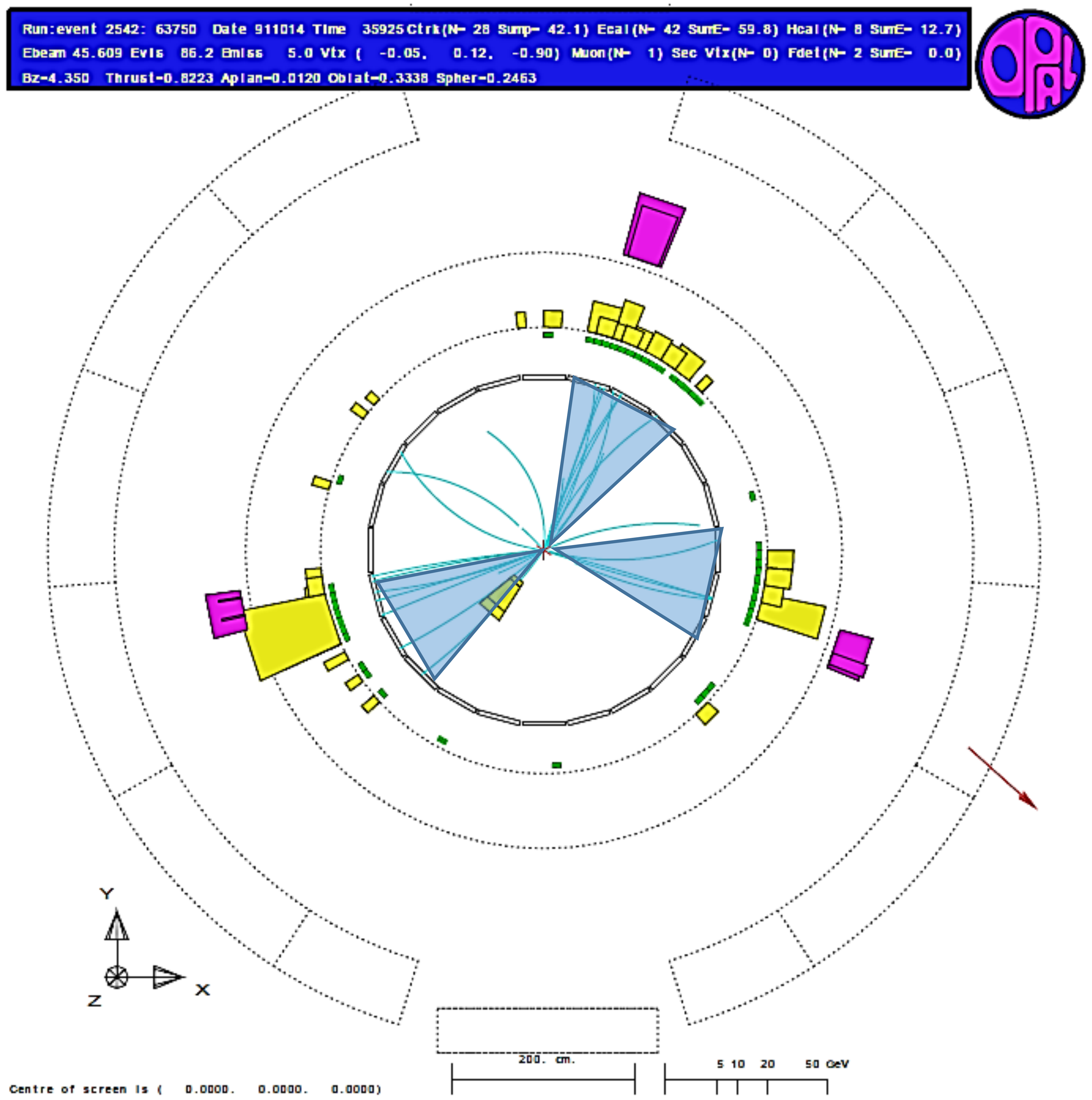

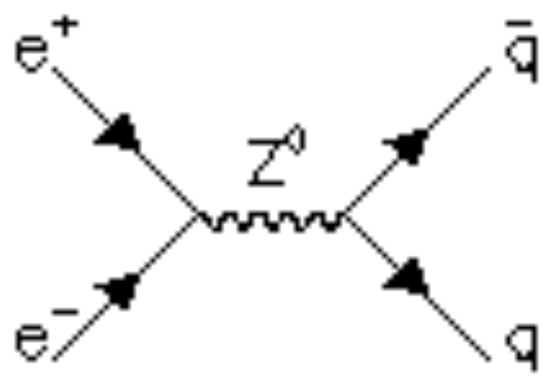

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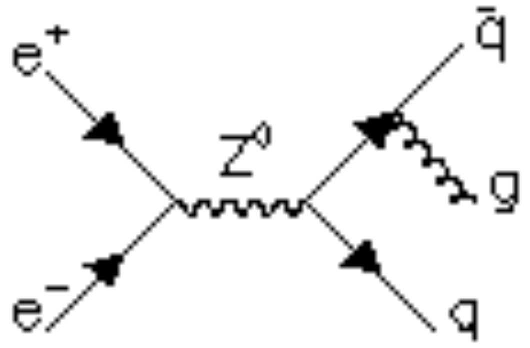

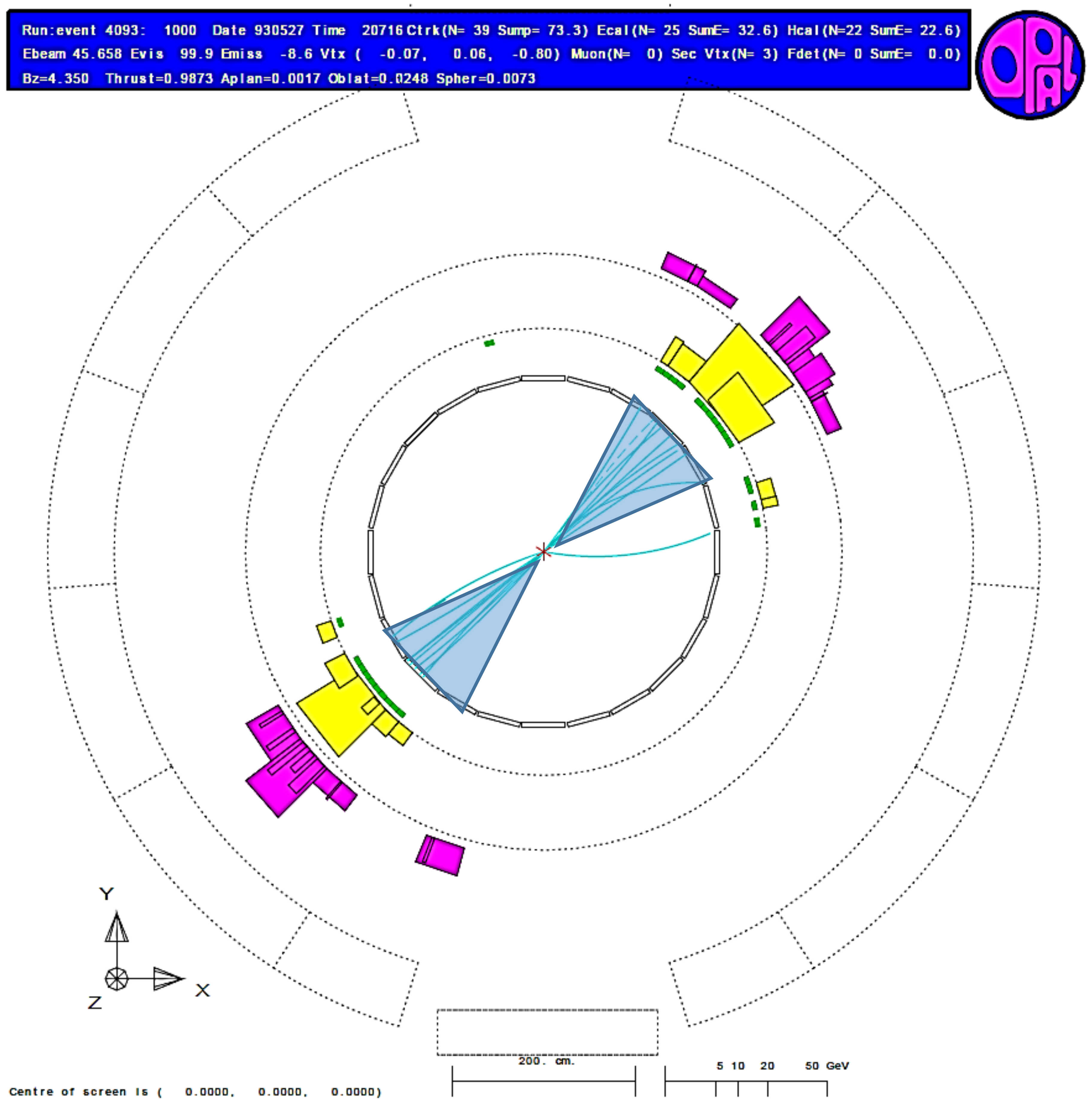

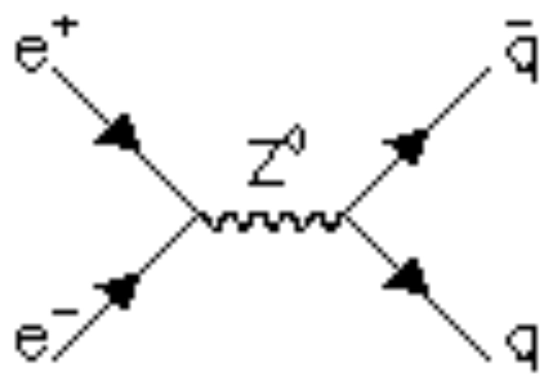

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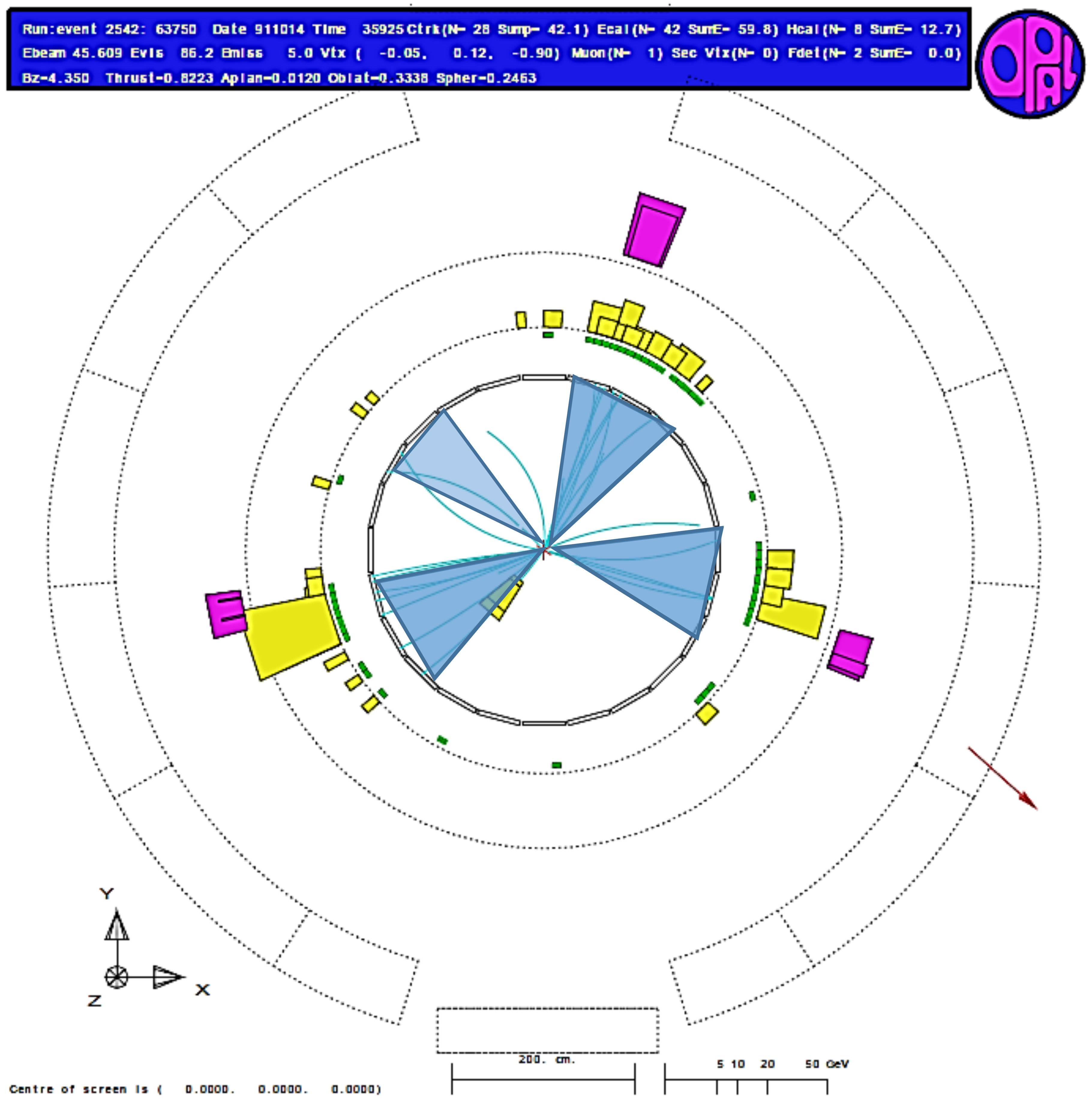

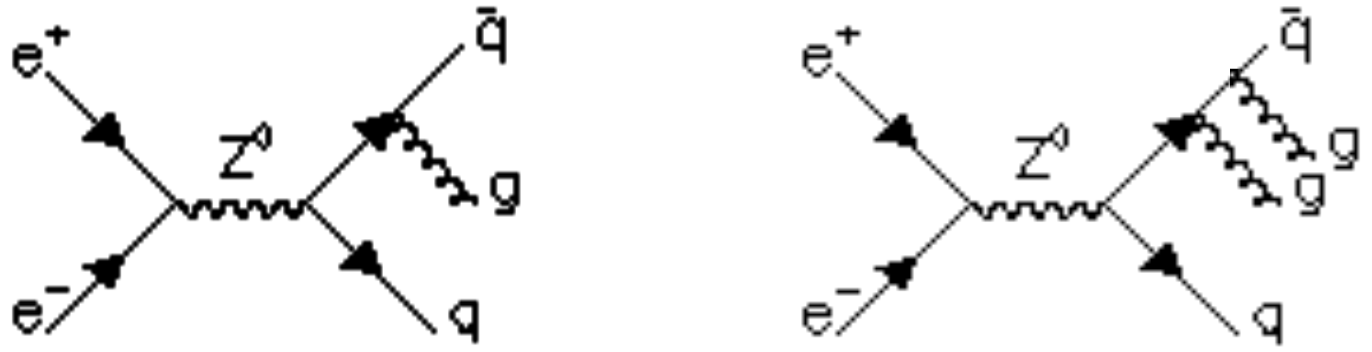

- Three- or four-jet event ?
- Depends on the jet resolution parameter

The k_T algorithm at hadron colliders

[Catani, Dokshitzer, Seymour, Webber, 93]
[Ellis, Soper, 93]

- Define distance among particles: e.g. $d_{ij} = (p_i + p_j)^2$

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Inclusive k_T

$$d_{ij} = \min(p_{Ti}^2, p_{T_j}^2)\frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = p_{Ti}^2 \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- Compute the smallest distance d_{ij} or d_{iB}
- If d_{ij}, cluster i and j together
- If d_{iB}, call i a jet and remove from the list of particles
- Repeat until no particle is left
- Two parameters R and minimal transverse momentum $p_{Ti} > p_{T,\min}$

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$$y = \frac{1}{2}\log\frac{E+p_z}{E-p_z} \neq \eta = -\log(\tan(\theta/2)) \quad \text{for massive particles}$$

The anti-k_T algorithm

[Cacciari, Salam, Soyez 08]

- k_T **has a physical meaning:** the stronger divergence between a pair of particles, the more likely it is they will be associated with each other

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- Cone-shaped cones but it is IRC safe, contrary to **cone algorithms** widely used at Tevatron
- Easier to energy jet energy scale right

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- Cambridge/Aachen: $d_{ij} = \dfrac{\Delta R_{ij}^2}{R^2}$

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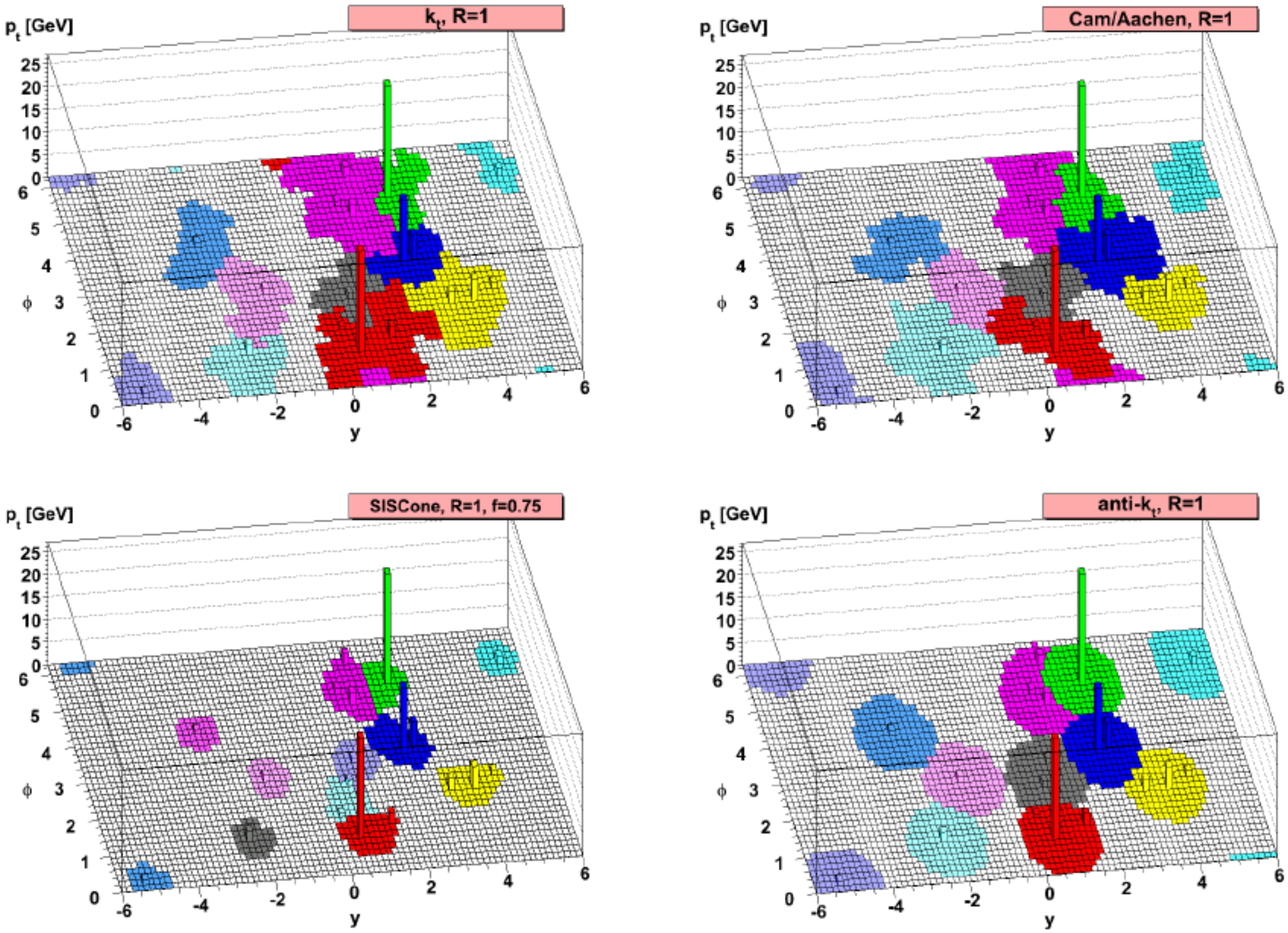

CONCLUSIONS

- Exciting times ahead with huge amount of collider data
- Challenging our theoretical understanding to meet experimental demands
- Parallel developments in maths and quantum computing
- QCD is the most non-Abelian theory that can be probed at high-energy colliders