= Four-loop scattering amplitudes through the loop-tree duality =

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Outline

- Motivation
- Loop-Tree Duality
- N^4MLT
  - *Universal topology*
  - *Causal representation*
- A look at quantum

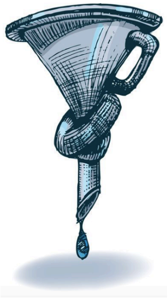
Motivation

Improve theoretical predictions

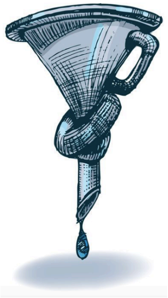
↳ Achieve higher perturbative orders

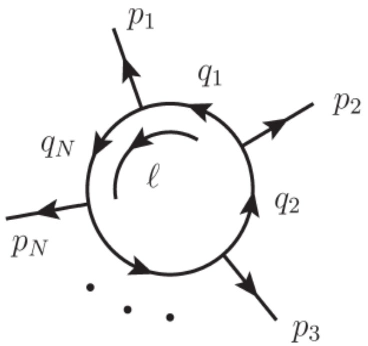
↳ Quantum fluctuations at high-energy scattering processes

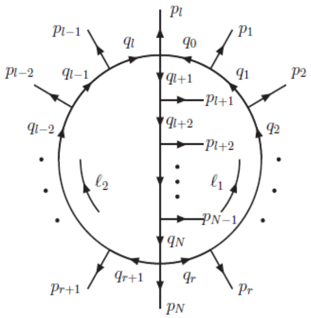
= Multiloop scattering amplitudes =

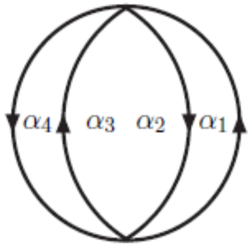


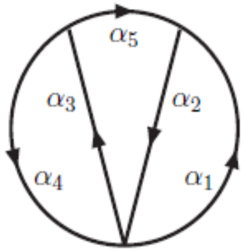
Loop diagrams











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○ S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J. C. Winter, "From loops to trees by-passing Feynman's theorem," JHEP 0809 (2008) 065.

- What does LTD do?

  Opens any loop diagram to a forest of non-disjoint trees.

- How does it do?

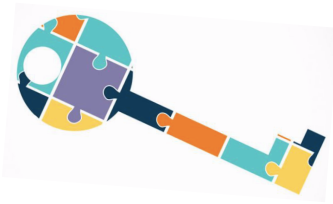
  Exploits the Cauchy residue theorem to reduce the dimension of the integration domain by one unit:

$$\int_{\boldsymbol{q}} \int dq_0 \prod_{j=1}^{N} G_F(q_j) = -2\pi i \int_{\vec{q}} \sum_{\boldsymbol{i}} Res_{\{Im\ q_{\{i,0\}}<0\}} \left[\prod_{j=1}^{N} G_F(q_j) \right]$$

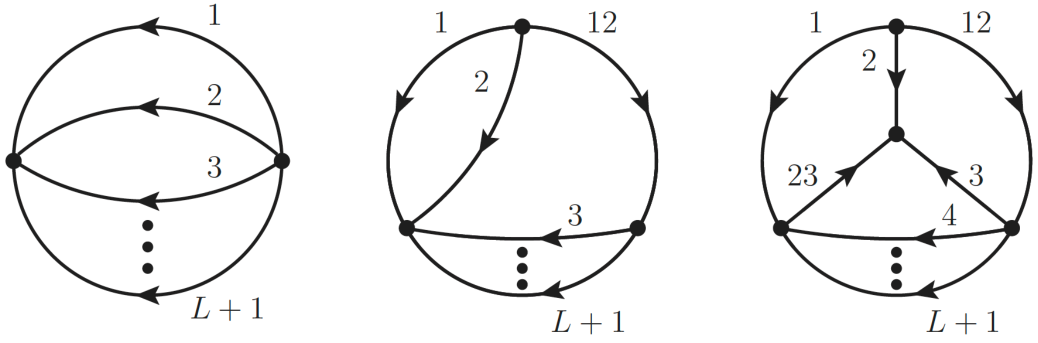
Minkowski space → Euclidean space

Reformulation of LTD to all orders

○ Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rodrigo, Sborlini, Torres, Tracz, "Open loop amplitudes and causality to all orders and powers from the loop-tree duality", Phys. Rev. Lett. 124 (2020) no.21, 211602



Causality



a) MLT b) NMLT c) N^2MLT

External particles not shown

- We found explicit and more compact analytic expressions with the LTD formalism to all orders

LTD

- A generic L-loop scattering amplitude with N external legs,

$$\mathcal{A}_N^{(L)}(1,\dots,n) = \int_{\ell_1,\dots,\ell_L} \mathcal{A}_F^{(L)}(1,\dots,n)$$

$$\mathcal{N}\left(\{\ell_i\}_L,\{p_j\}_N\right) G_F(1,\dots,n)$$

$$\prod_{i\in 1\cup\dots\cup n} \left(G_F(q_i)\right)^{a_i}$$

$$\frac{1}{q_{i,0}^2 - \left(q_{i,0}^{(+)}\right)^2}$$

$$\sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}$$

$$\int_{\ell_i} = -i\mu^{4-d}\int \frac{d^d\ell_i}{(2\pi)^d}$$

LTD

- The LTD representation is written in terms of nested residues,

$$\mathcal{A}_D^{(L)}(1,\ldots,r;r+1,\ldots,n) = -2\pi i \sum_{i_r \in r} Res\left(\mathcal{A}_D^{(L)}(1,\ldots,r-1;r,\ldots,n), Im\,\eta\cdot q_{i_r} < 0\right)$$

On-shell Off-shell

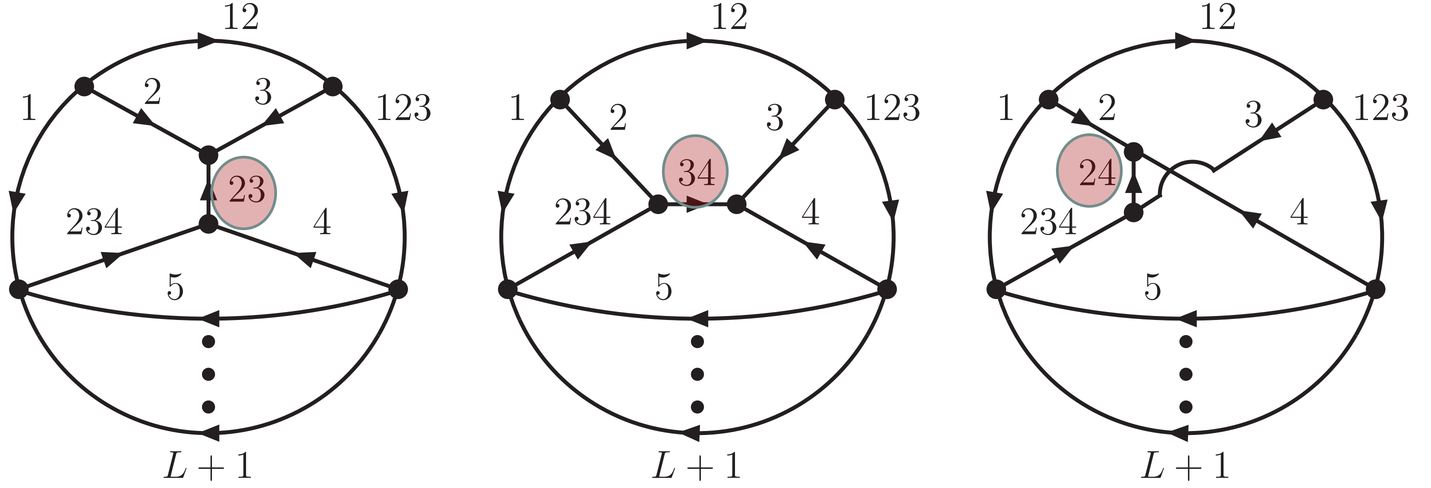
$$\eta^\mu = (1,\mathbf{0})$$

starting from

$$\mathcal{A}_D^{(L)}(1;2,\ldots,n) = -2\pi i \sum_{i_1 \in 1} Res\left(\mathcal{A}_F^{(L)}(1,\ldots,n), Im\,\eta\cdot q_{i_1} < 0\right)$$

N^4MLT universal topology

○ S. Ramírez-Uribe, R. J. Hernández-Pinto, G. Rodrigo, G. F. R. Sborlini, and W. J. Torres Bobadilla, "Universal opening of four-loop scattering amplitudes to trees," JHEP 04, 129 (2021).



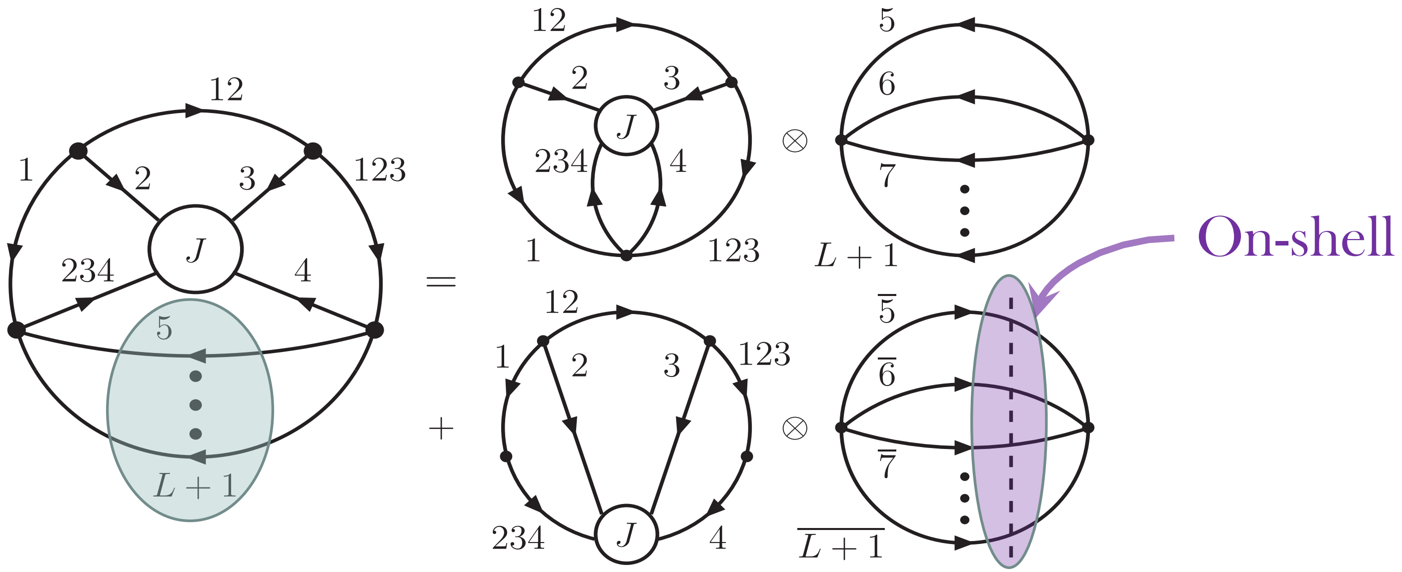
- $q_{i_s} = \ell_s + k_{i_s},\; s \in \{1, \ldots, L\}$
- $q_{i_{(L+1)}} = -\sum_{s=1}^{L} \ell_s + k_{i_{(L+1)}}$
- $q_{i_{12}} = -\ell_1 - \ell_2 + k_{i_{12}}$
- $q_{i_{123}} = -\ell_1 - \ell_2 - \ell_3 + k_{i_{123}}$
- $q_{i_{234}} = -\sum_{s=2}^{4} \ell_s + k_{i_{234}}$
- $q_{i_{rs}} = -\ell_r - \ell_s + k_{i_{rs}},\; r, s \in \{2, 3, 4\}$

Can we achieve a unified description?

t-, s- and u- kinematic channels

$J \equiv 23 \cup 34 \cup 24$

N^4MLT universal topology



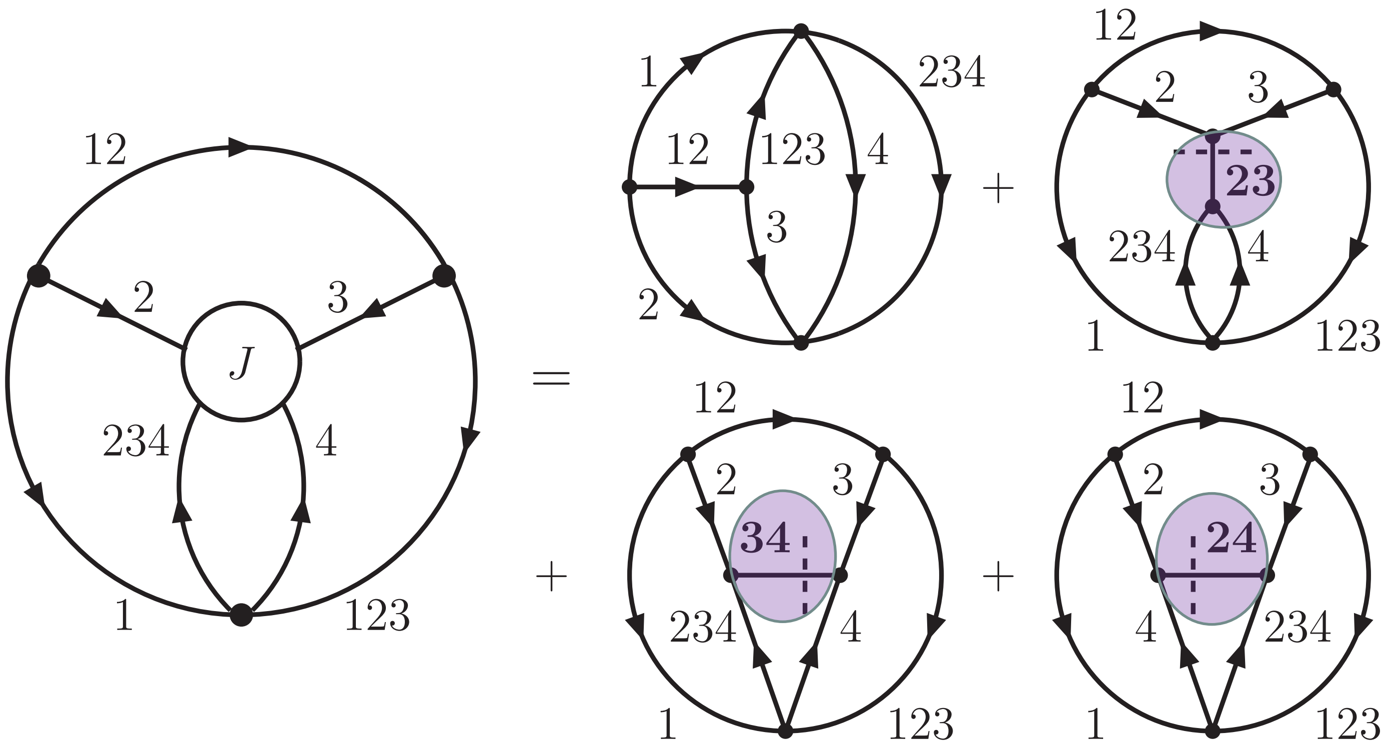
$$\mathcal{A}^{(L)}_{N^4MLT}(1,\dots,L+1,12,123,234,J)$$

$$= \mathcal{A}^{(4)}_{N^4MLT}(1,2,3,4,12,123,234,J) \otimes \mathcal{A}^{(L-4)}_{MLT}(5,\dots,L+1)$$

$$+\mathcal{A}^{(3)}_{N^2MLT}(1\cup 234,2,3,4\cup 123,12,J) \otimes \mathcal{A}^{(L-3)}_{MLT}(\bar{5},\dots,\overline{L+1})$$

Momentum flow reversed

N^4MLT universal topology

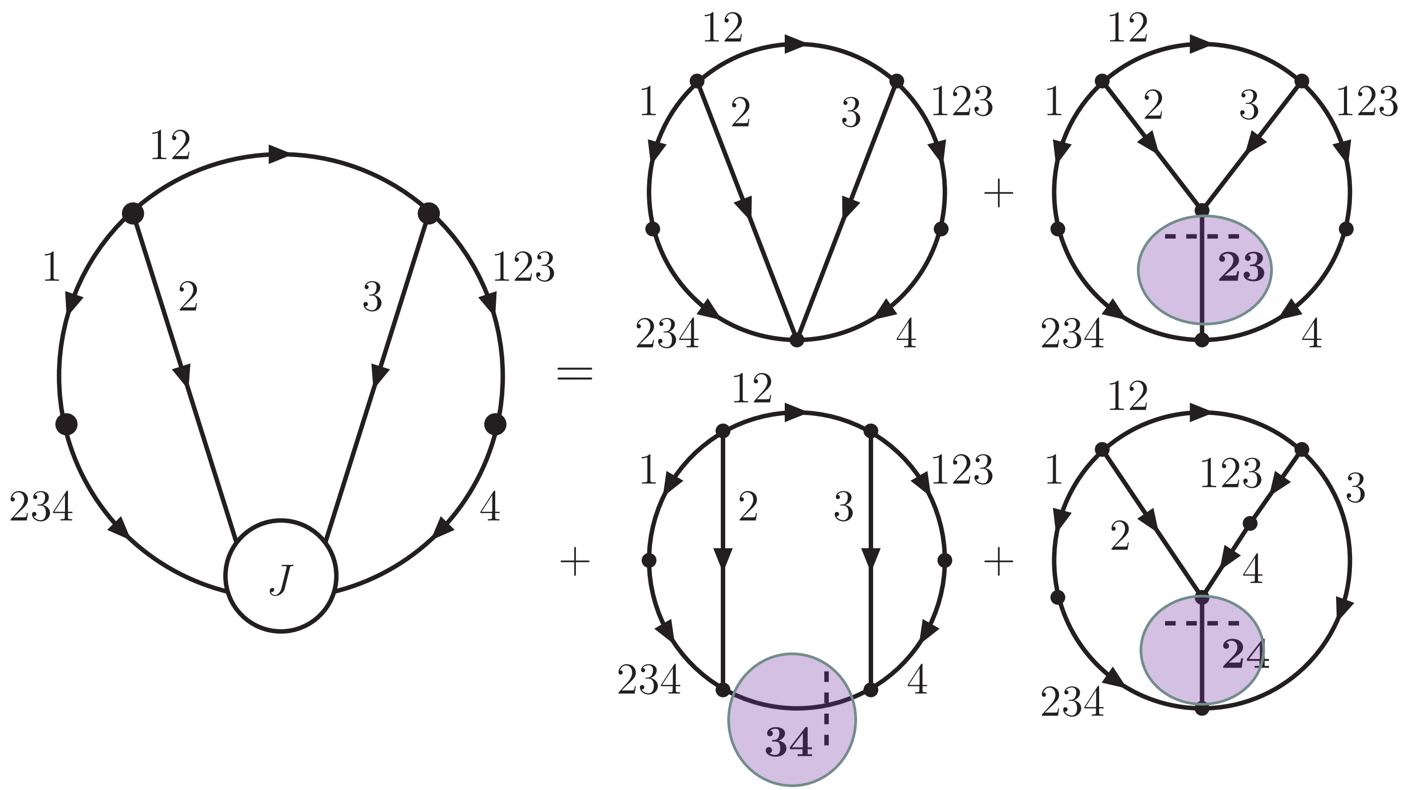


Off-shell

$$\mathcal{A}^{(4)}_{N^4MLT}(1,2,3,4,12,123,234,J) = \mathcal{A}^{(4)}_{N^2MLT}(1,2,3,4,12,123,234) \otimes \mathcal{A}^{(0)}(J) + \sum_{s\in J} \mathcal{A}^{(4)}_{D}(1,2,3,4,12,123,234,\boldsymbol{s})$$

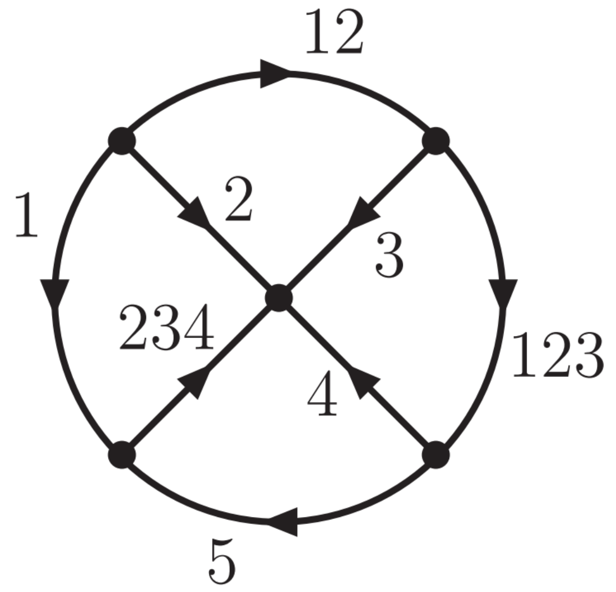
$\{\mathbf{23},\mathbf{34},\mathbf{24}\}$

N^4MLT universal topology



$$\mathcal{A}^{(3)}_{N^2MLT}(1\cup 234, 2, 3, 4\cup 123, 12, J) = \mathcal{A}^{(3)}_{NMLT}(1\cup 234, 2, 3, 4\cup 123, 12) \otimes \mathcal{A}^{(0)}(J) + \sum_{s\in J} \mathcal{A}^{(3)}_{D}(1, 2, 3, 4, 12, 123, 234, \boldsymbol{s})$$

N^3MLT causal representation



$$= \int_{\ell_1,\dots,\ell_L} \mathcal{A}^{(4)}_{N^3MLT}(1,2,3,4,5,12,123,234)$$

Universal opening

$$= \int_{\vec{\ell}_1,\dots,\vec{\ell}_L} \left[\mathcal{A}^{(4)}_{N^2MLT}(1,2,3,4,12,123,234) + \mathcal{A}^{(3)}_{NMLT}(1\cup 234, 2, 3, 4\cup 123, 12) \otimes \mathcal{A}^{(1)}_{D}(\bar{5})\right]$$

Adding them all together

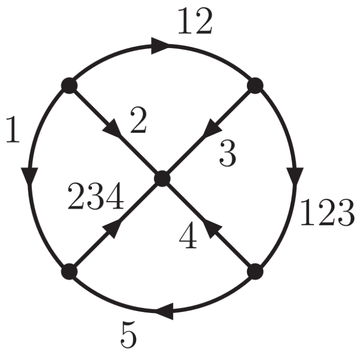
$$= \int_{\vec{\ell}_1,\dots,\vec{\ell}_L} \frac{\mathcal{N}_{N^3MLT}\left(\left\{q^{(+)}_{s,0}, k_{j,0}\right\}\right)}{\left(\prod_{s=1}^{L+4} 2q^{(+)}_{s,0}\right)\left(\prod_{i=1}^{13} \lambda_i^+ \lambda_i^-\right)}$$

$$\lambda_p^\pm = \sum_{i\in p} q^{(+)}_{i,0} \pm k_{p,0}$$

$k_{p,0}$ linear combination of external momenta

N^3MLT causal representation

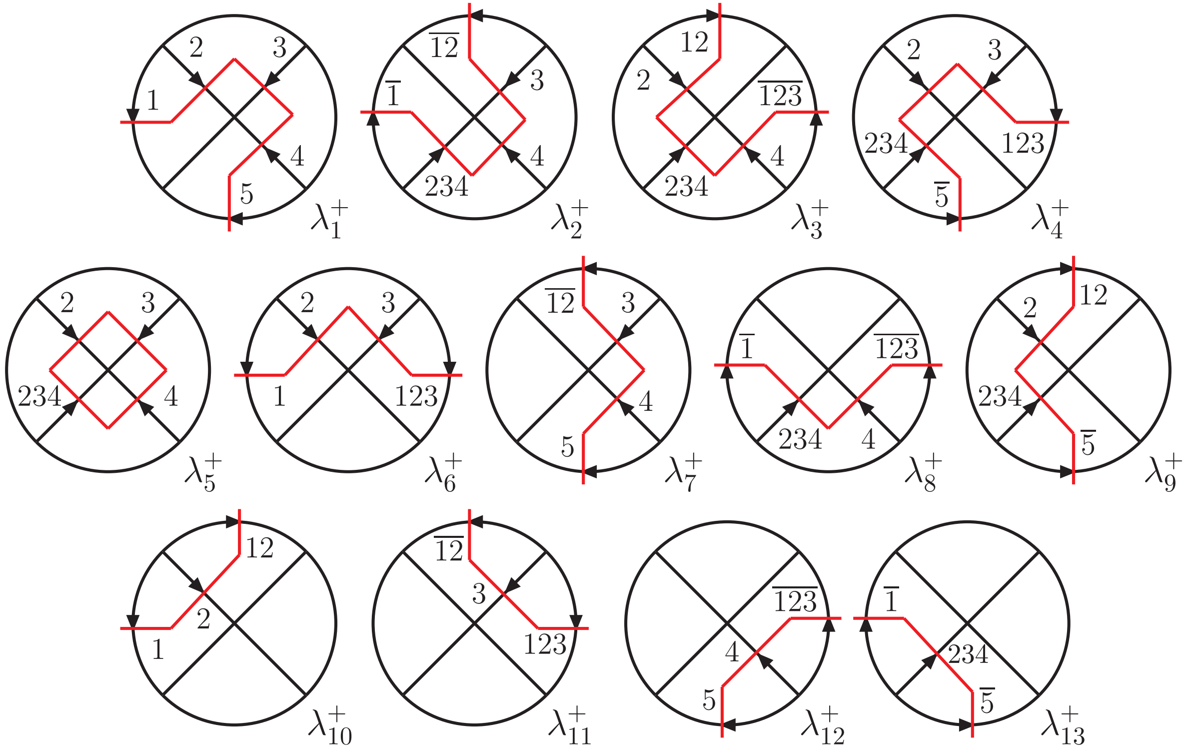
Reinterpreting in terms of four entangled thresholds



= LTD Causal representation =

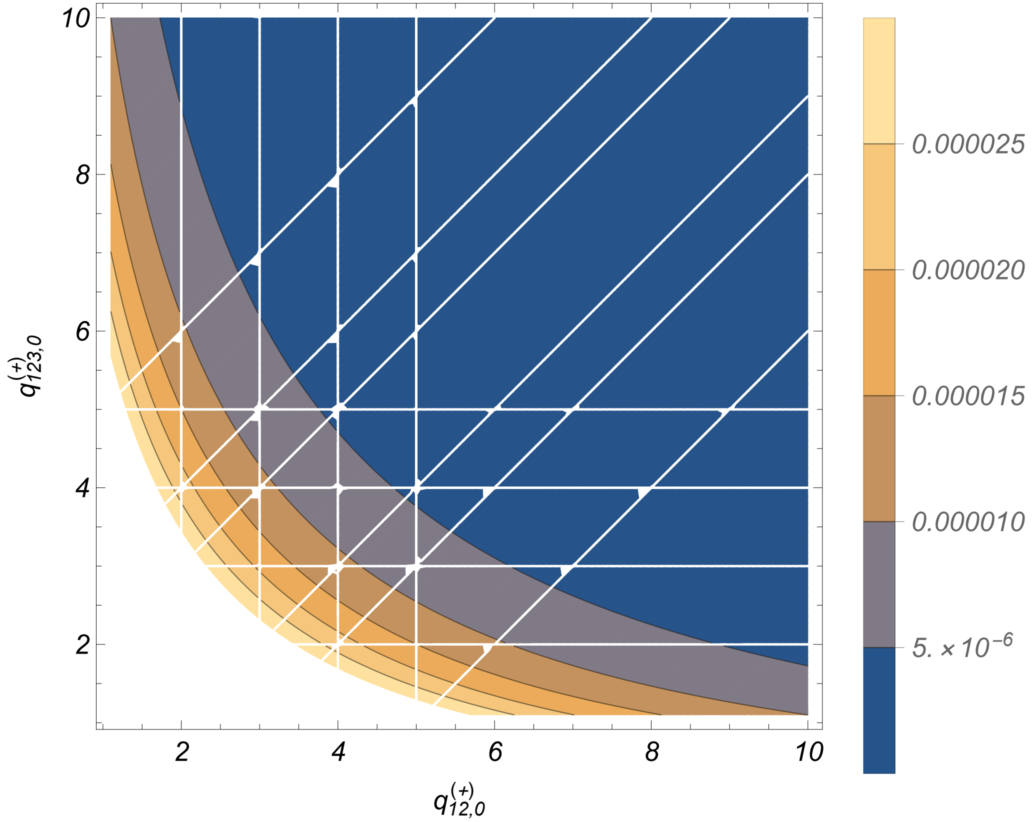
$$= \int_{\vec{\ell}_1,\dots,\vec{\ell}_L} \frac{1}{x_{L+4}} \sum_{\sigma} \frac{\mathcal{N}_{\sigma(i_1,\dots,i_4)}\left(\left\{q_{s,0}^{(+)}, k_{j,0}\right\}\right)}{\lambda_{\sigma(i_1)}\lambda_{\sigma(i_2)}\lambda_{\sigma(i_3)}\lambda_{\sigma(i_4)}}$$

$$x_{L+4} = \prod_{s=1}^{L+4} 2q_{s,0}^{(+)}$$



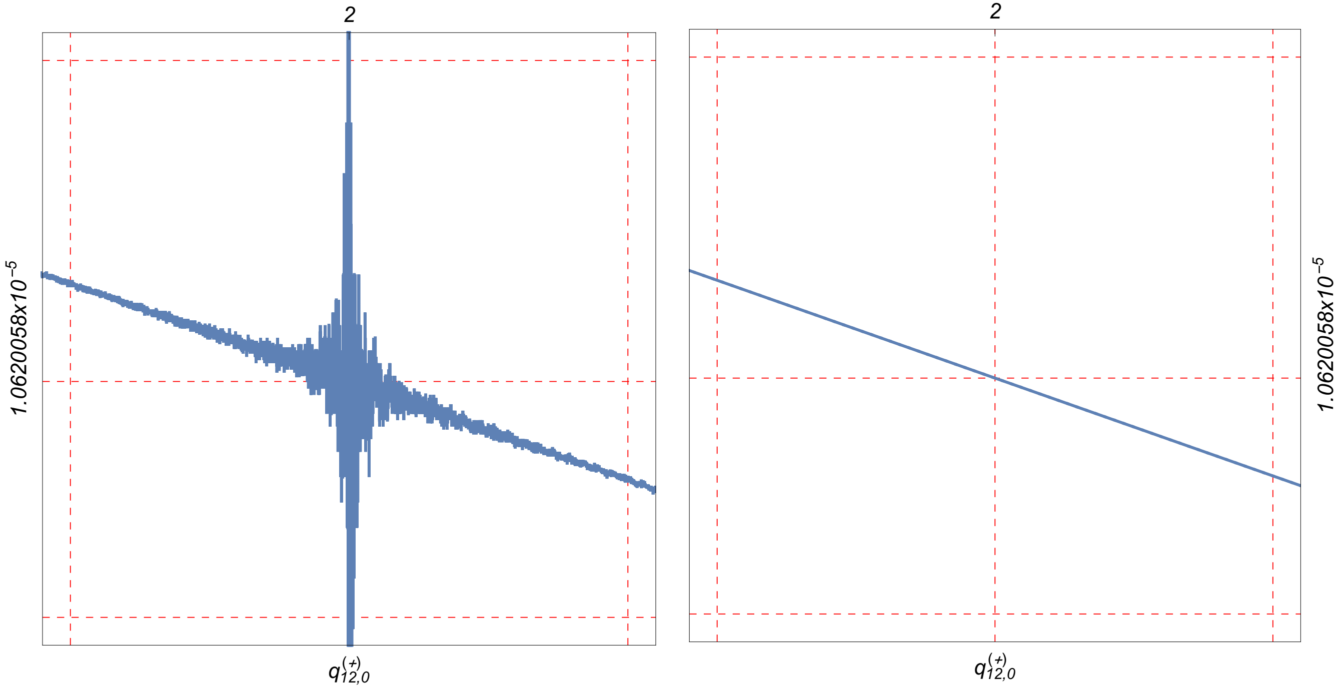
N^3MLT causal representation

Impact of noncausal singularities



Integrand-level behaviour of the noncausal LTD representation of a four-loop N^3MLT diagram. White lines are noncausal singularities.

Noncausal and causal evaluations of the N^3MLT configurations.



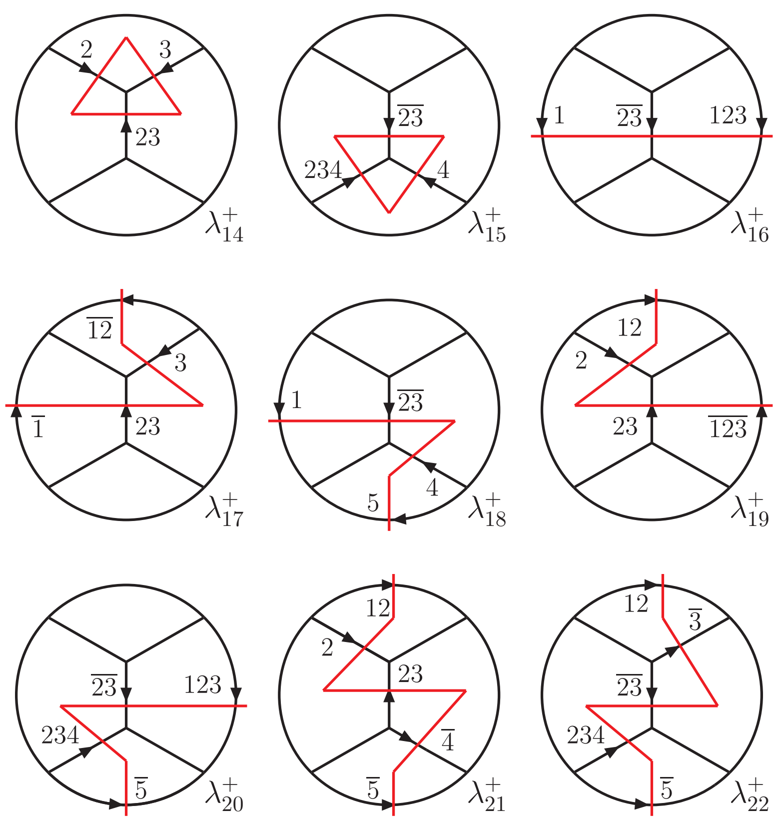
Numerical instabilities of the four-loop N^3MLT integrand arising due to noncausal singularities (left), which are absent in the manifestly causal representation (right).

N^4MLT causal representation

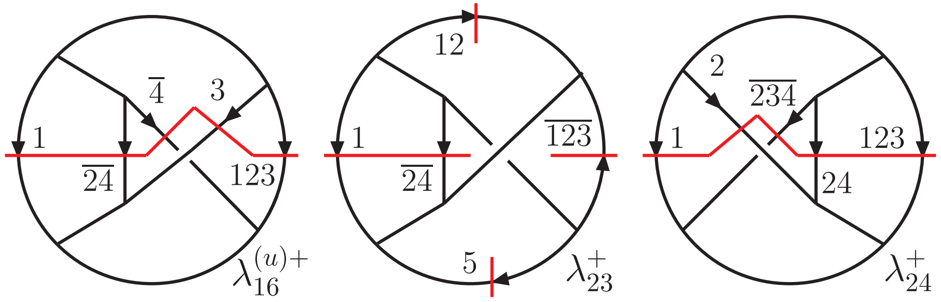
$$\mathcal{A}^{(L)}_{N^4MLT}(1,\dots,L+4,J) = \int_{\vec{\ell}_1,\dots,\vec{\ell}_L} \frac{1}{x_{\{t,s,u\},L+5}} \sum_{\sigma} \frac{\mathcal{N}_{\sigma(i_1,\dots,i_5)}\left(\left\{q^{(+)}_{s,0}, k_{j,0}\right\}\right)}{\lambda_{\sigma(i_1)}\lambda_{\sigma(i_2)}\lambda_{\sigma(i_3)}\lambda_{\sigma(i_4)}\lambda_{\sigma(i_5)}}$$

$$x_{\{t,s,u\},L+5} \rightarrow 2q^{(+)}_{\{23,34,24\},0}\, x_{L+4}$$

- Extra causal configurations of the t −channel



- For the s −channel a clockwise rotation is applied
  - Extra causal configurations of the u −channel

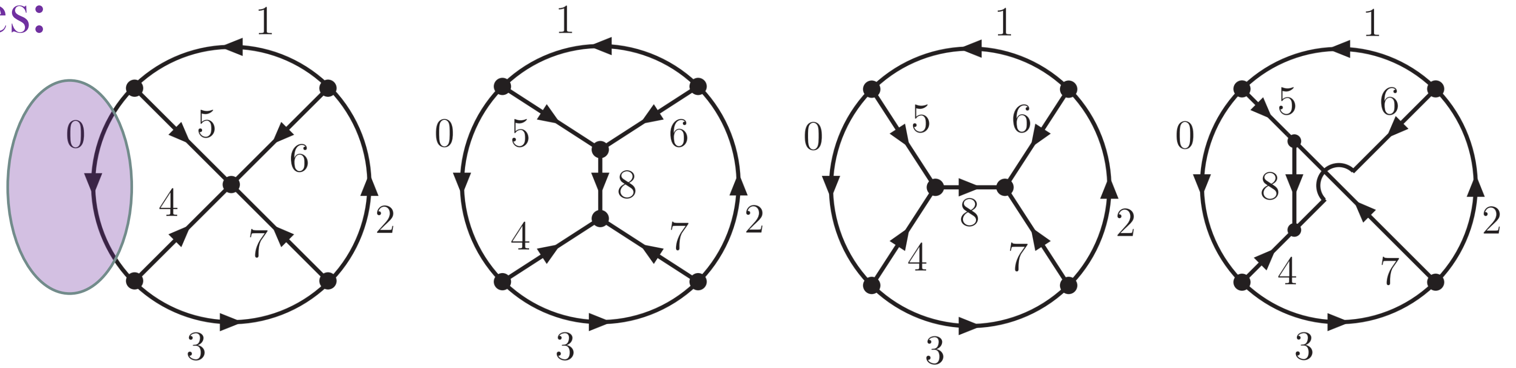


A look at quantum

Bootstrap the causal representation in the LTD of representative multiloop topologies.

- S. Ramírez-Uribe, A. E. Rentería-Olivo, G. Rodrigo, G. F. R. Sborlini, and L. Vale Silva, "Quantum algorithm for Feynman loop integrals", arXiv:2105.08703 [hep-ph]

Two possible states:
$|1\rangle$ *or* $|0\rangle$



Grover's quantum algorithm

A look at quantum

Grover's quantum algorithm

1. Superposition ($N = 2^n$)
2. Oracle
3. Diffusion

- Superposition

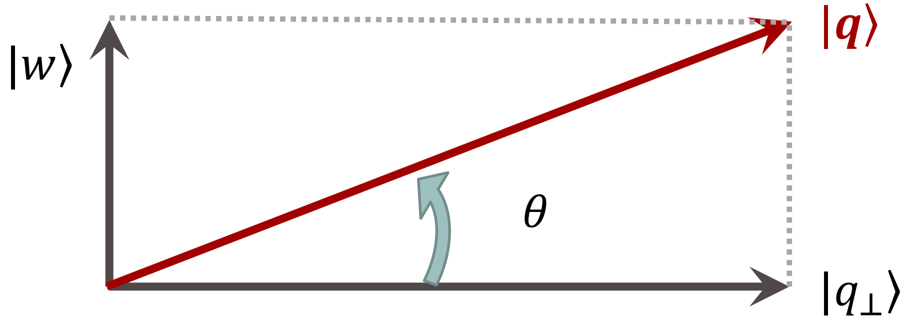
$$|q\rangle = \frac{1}{\sqrt{N}} \sum |x\rangle$$

$$|q\rangle = \cos\theta \, |q_\perp\rangle + \sin\theta \, |w\rangle$$

Orthogonal state

Winning state

Mixing angle

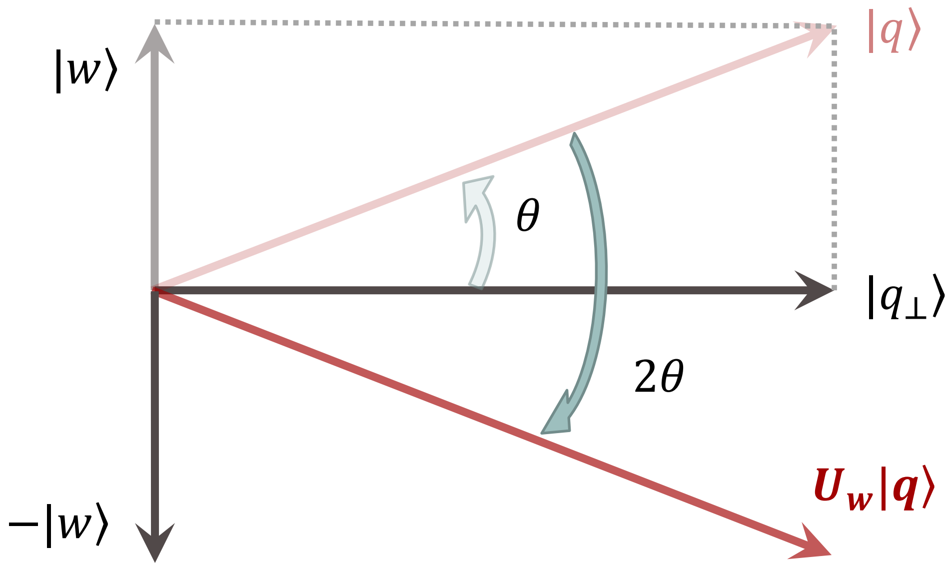


$$|w\rangle = \frac{1}{\sqrt{r}} \sum_{x \in w} |x\rangle \qquad |q_\perp\rangle = \frac{1}{\sqrt{N-r}} \sum_{x \in w} |x\rangle$$

$$\theta = \arcsin\sqrt{r/N}$$

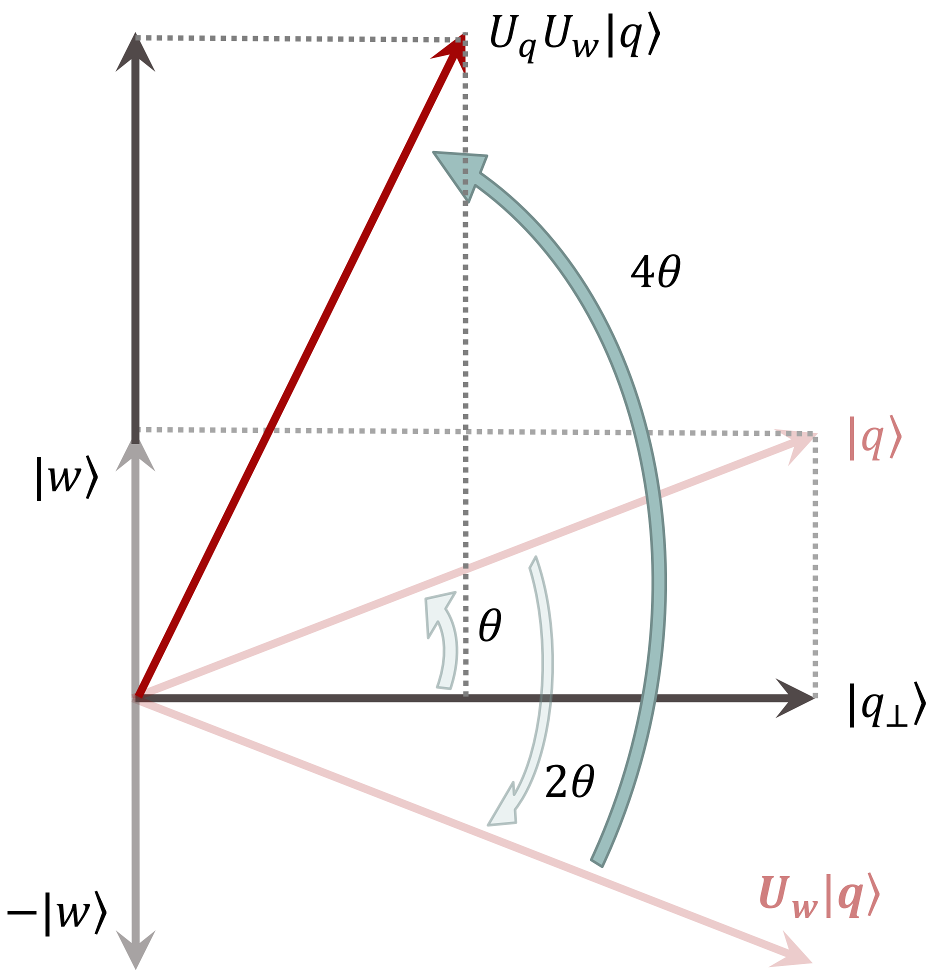
A look at quantum

- Oracle $U_w = \boldsymbol{I} - 2|w\rangle\langle w|$ $U_w|x\rangle = \begin{cases} -|x\rangle & if\ x \in w \\ |x\rangle & if\ x \notin w \end{cases}$



Flips the state $|x\rangle$ if $x \in |w\rangle$ and leaves it unchanged otherwise.

A look at quantum



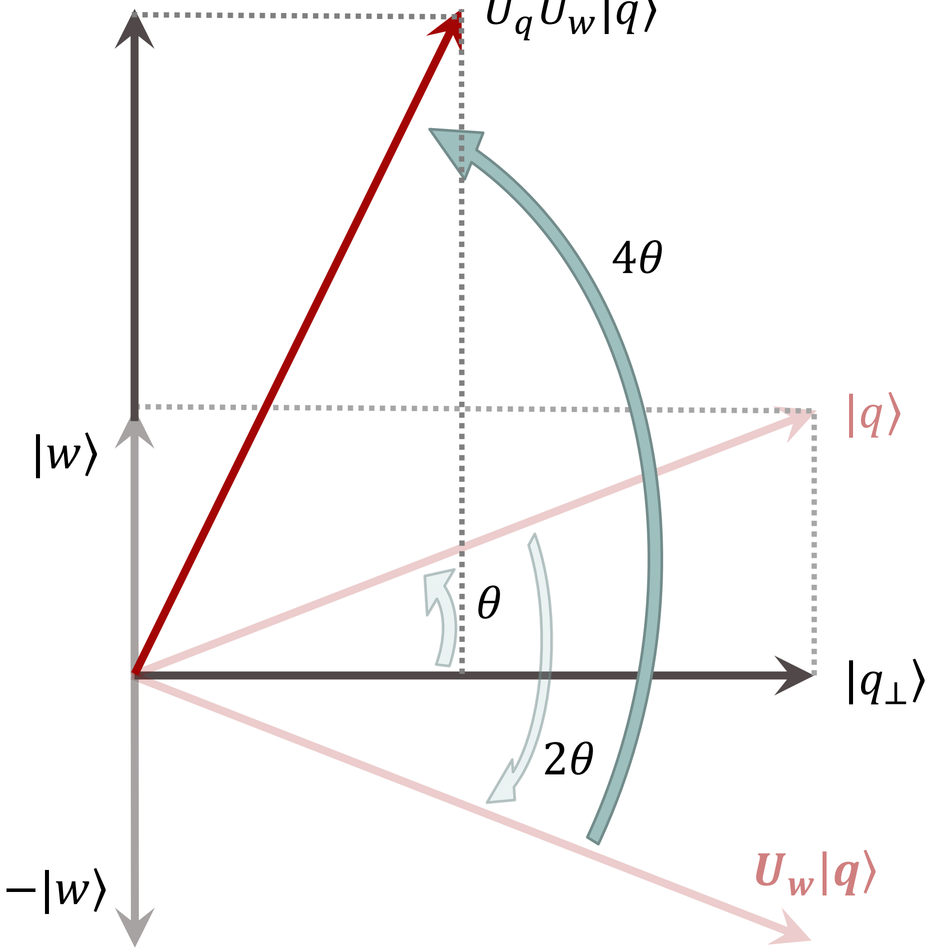
- **Diffusion** $U_q = 2|q\rangle\langle q| - \boldsymbol{I}$

  Performs a reflection around the initial state $|q\rangle$

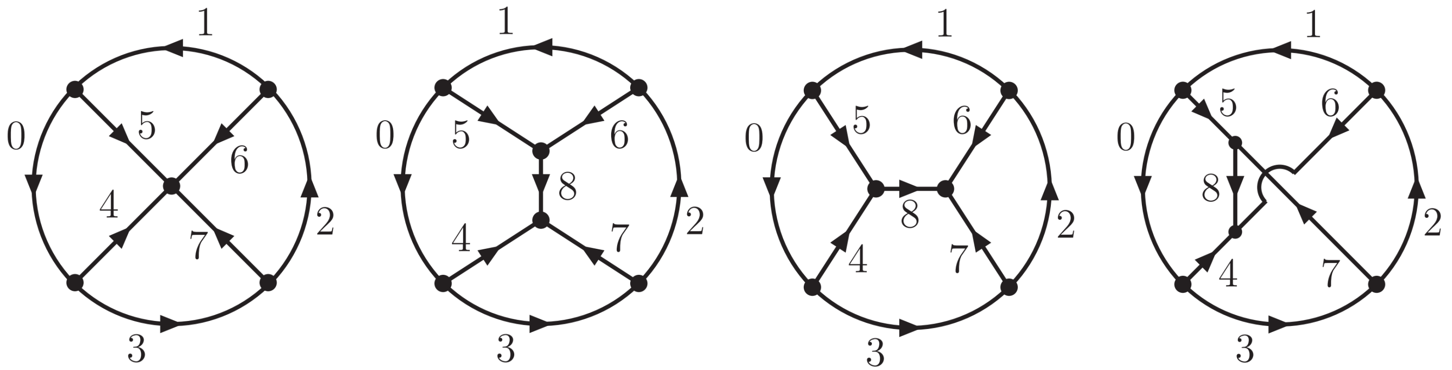
The iterative application of the oracle and diffusion operators t times leads to:

$$(U_q U_w)^t |q\rangle = \cos\theta_t\, |q_\perp\rangle + \sin\theta_t\, |w\rangle$$

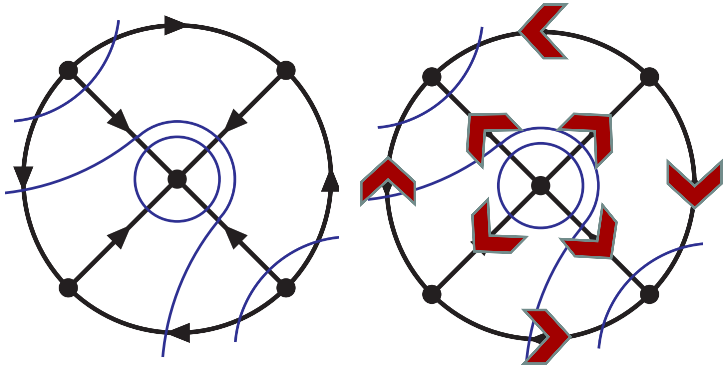
A look at quantum



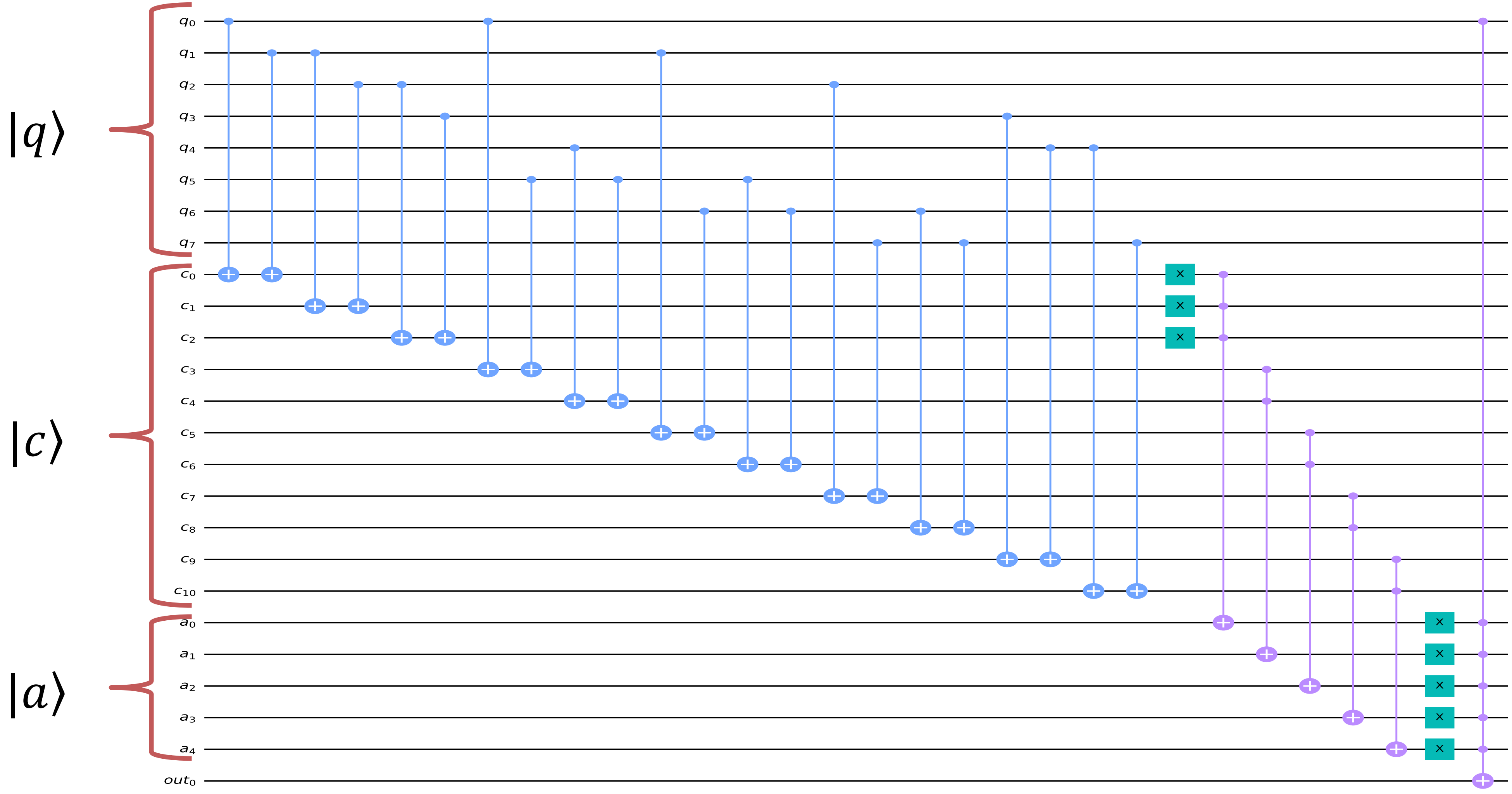
To consider → $\theta \leq \pi/6$ $(r \leq N/4)$?



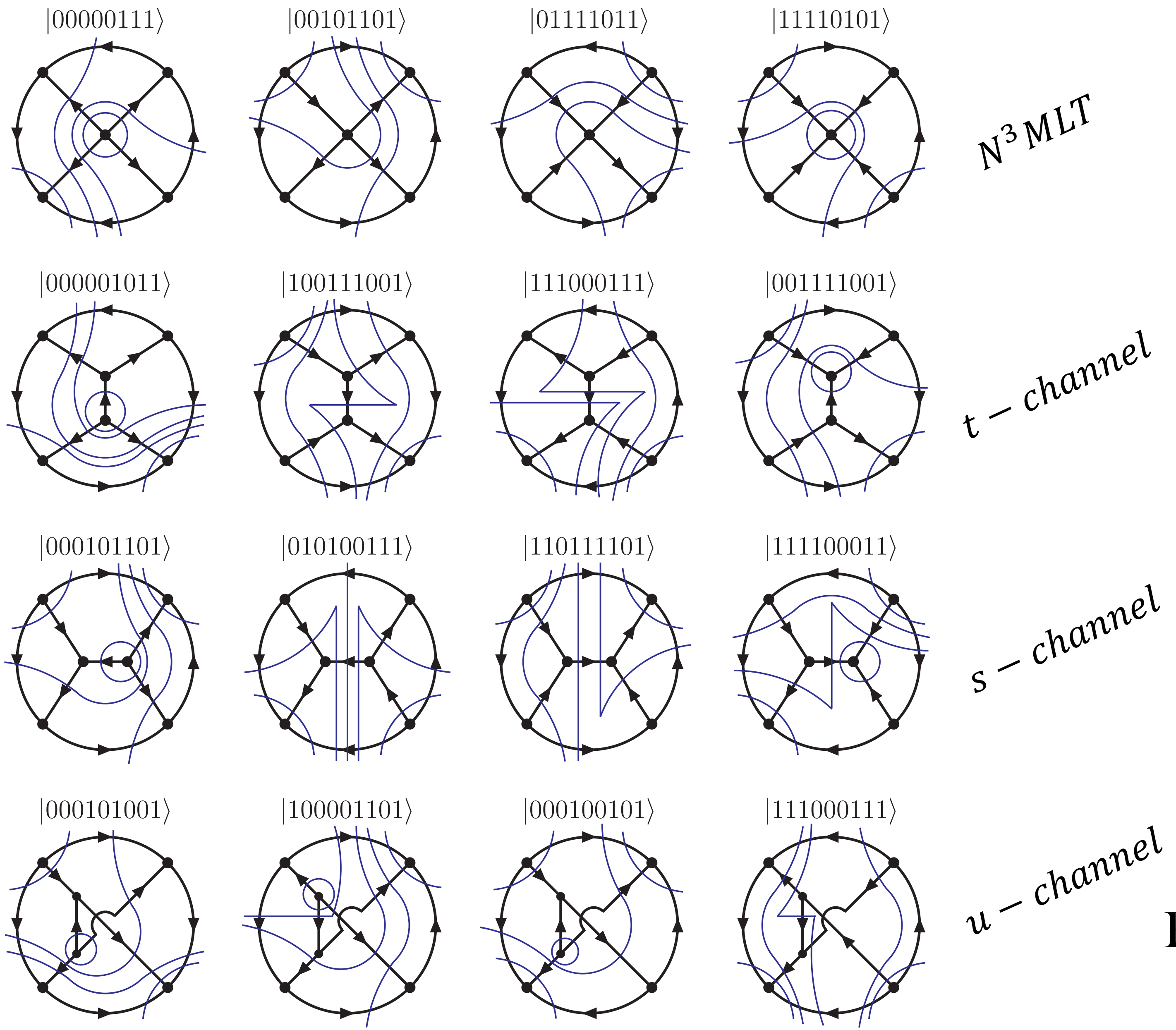
78/256 | 204/512 | 204/512 | 230/512

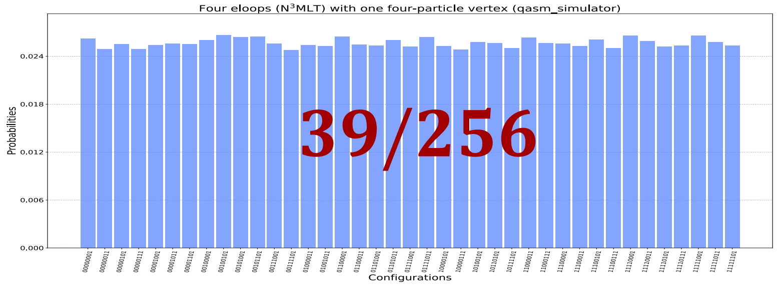


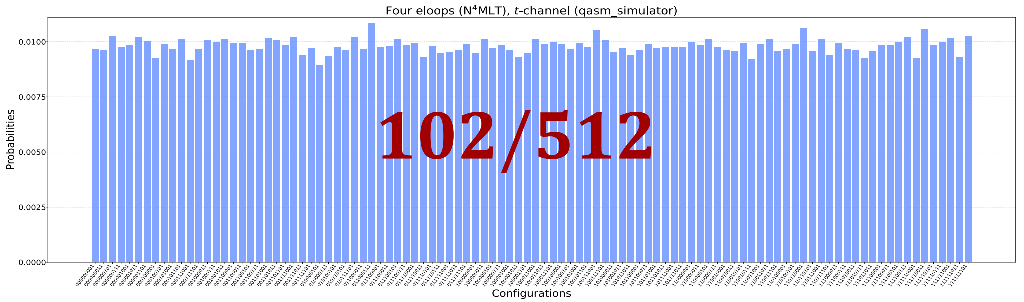
- Oracle

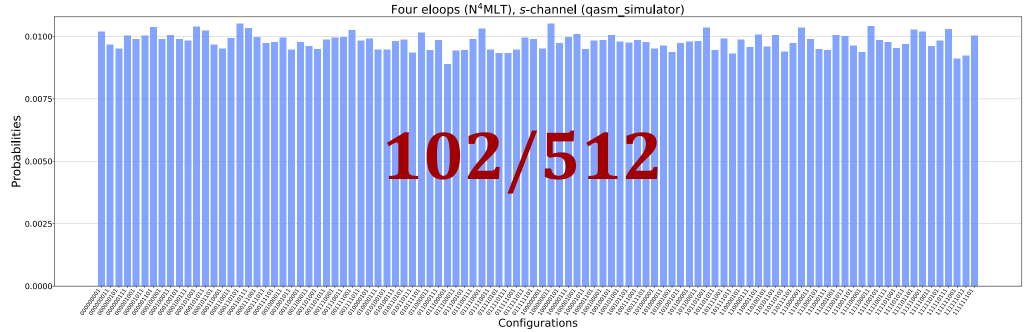

$|q\rangle$
$|c\rangle$
$|a\rangle$
q_0
q_1
q_2
q_3
q_4
q_5
q_6
q_7
c_0
c_1
c_2
c_3
c_4
c_5
c_6
c_7
c_8
c_9
c_{10}
a_0
a_1
a_2
a_3
a_4
out_0

A look at quantum









Requires 33 qubits > Qiskit capacity

Conclusions

- We have obtained a dual representation for selected loop topologies to all orders, which exhibits a nested form in terms of simpler topologies.
- The N^4MLT universal topology allow us to describe any scattering amplitude up to four loops.
- The causal LTD representation is interpreted in terms of entangled causal thresholds and allows a more efficient numerical evaluation of multiloop scattering amplitudes.
- Causal configurations of multiloop Feynman integrals have been efficiently identified with the application of Grover's quantum algorithm.


Gracias!