Causal structures of Feynman diagrams through the Loop-Tree Duality formalism

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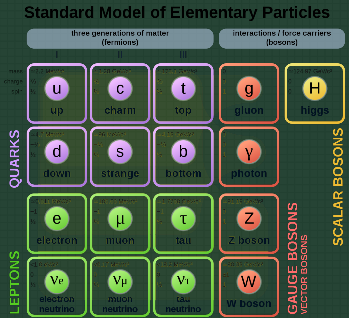
Standard Model of Elementary Particles
three generations of matter (fermions)
interactions / force carriers (bosons)
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e
electron
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tau
Z
Z boson
νe
electron neutrino
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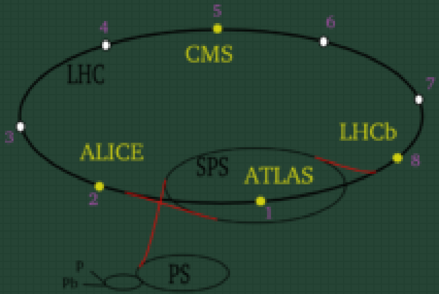
W boson
QUARKS
LEPTONS
GAUGE BOSONS
VECTOR BOSONS
SCALAR BOSONS


LHC
CMS
ALICE
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ATLAS
SPS
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Motivation

Theoretical predictions in QFTs can be improved both with non-perturbative analysis and reaching higher-order approximations within perturbative theory in the corresponding energy regime.

$$\text{Path integral} \Rightarrow \text{Perturbation theory}$$

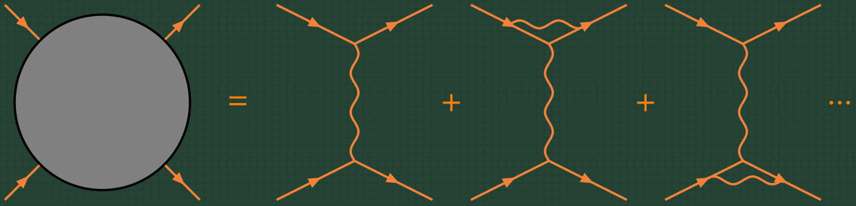
$$\frac{1}{Z}\int \mathcal{D}x\ e^{iS[x,\dot{x}]}\psi_0(x(t)) \Rightarrow \text{Feynman rules/diargams}$$

$$\text{(Minkowskian space)}$$

$$\text{Non analytical result} \Rightarrow \text{Regularizable sums}$$

Motivation

Perturbative computations:



Singularities!!!

Motivation

Singularities in QFT:

- High-energy regime (UV)
- Low-energy regime (IR)
  - Parallel particles seen as a single particle (Collinear)
  - Particles with zero energy (Soft)

Motivation

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Motivation

- How to tackle integrals?
  Among other methods, Loop-Tree Duality!! (LTD)
- What is LTD? A formalism developed to study loop-level Feynman diagrams through their spanning trees.
- How does the LTD work? Expoloiting Cauchy's residue theorem to reduce the dimensions of the integration space.

$$\begin{gathered}\text{Minkowskian space} \Rightarrow \text{Euclidean space} \\ q^2 = q_0^2 - \boldsymbol{q}^2 \Rightarrow q_0^{(+)} = \sqrt{\boldsymbol{q}^2 + m^2 - \imath 0}\end{gathered} \tag{1}$$

This implies:
$G_F(q) \propto (q^2 - m^2 + \imath 0)^{-1} = (q_0 + q_0^{(+)})^{-1}(q_0 - q_0^{(+)})^{-1}.$

Motivation

Loop-Tree Duality @1-loop

$$\mathcal{A}^{(1)}(\{p_n\}_N) = -\mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \mathcal{N}(\ell, \{p_n\}_N) \prod_{i=1}^{N} G_F(q_i)$$

Motivation

Loop-Tree Duality @1-loop

Applying LTD formalism to an arbitrary 1-loop Feynman diagram,

$$\mathcal{A}(p) = -\sum_{i=1}^{k} \int_{\ell} \tilde{\delta}(q_i) \mathcal{N}(\ell, \{p_n\}_N) \prod_{j \neq i}^{k} G_D(q_i; q_k) = -\sum_{i=1}^{k} \tilde{\mathcal{A}}_i. \quad (2)$$

where

- $G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - \imath 0 \eta k_{ij}}$, the *dual propagator*,
- $\tilde{\delta}(q_i) = 2\pi\imath\theta(q_{i,0})\delta(q_i^2 - m_i^2)$ sets internal particles on-shell with positive energy,
- η a future-like vector $(\eta^2 \geq 0\,,\, \eta_0 \geq 0)$. Convenient, $\eta^\mu = (1, \mathbf{0})$.

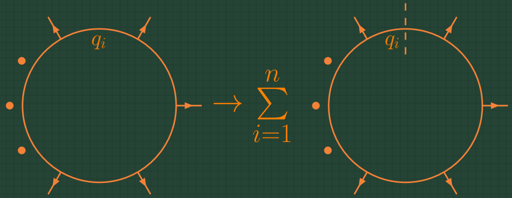
Motivation

Loop-Tree Duality @1-loop

Comparing Feynman propagator with dual propagator:

$$G_F(q_j) = \frac{1}{q_j^2 - m_j^2 + \imath 0} \quad , \quad G_D(q_i;q_j) = \frac{1}{q_j^2 - m_j^2 - \imath 0 \eta k_{i,j}}, \tag{3}$$

different prescription.



Motivation

Loop-Tree Duality @2-loop

$$\mathcal{A}^{(2)}(\{p_n\}_N) = -(\mu^{4-d})^2 \int\int \frac{d^d\ell_1}{(2\pi)^d}\frac{d^d\ell_2}{(2\pi)^d}\mathcal{N}(\ell_1,\ell_2,\{p_n\}_N) \times \prod_{i\in\alpha_1} G_F(q_i) \prod_{j\in\alpha_2} G_F(q_j) \prod_{k\in\alpha_3} G_F(q_k)$$

Motivation

Loop-Tree Duality @2-loop

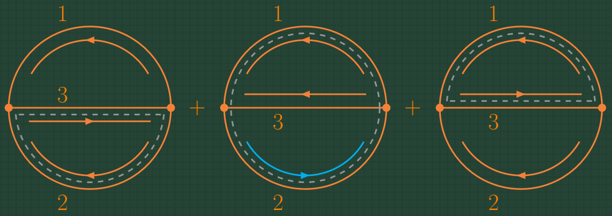
Studying LTD for 2-loop Feynman diagrams leads us to the relation

$$\mathcal{A}^{(2)}(\{p_n\}_N) = -(\mu^{4-d})^2 \int\int \frac{d^d\ell_1 d^d\ell_2}{(2\pi)^{2d}} [G_D(\alpha_1)G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_2 \cup \alpha_1)G_D(\alpha_3) - G_D(\alpha_1)G_F(\alpha_2)G_D(\alpha_3)], \quad (4)$$

where

$$G_D(\beta) = \sum_{i\in\beta} \tilde{\delta}(q_i) \prod_{\substack{j\in\beta \\ j\neq i}} G_D(q_i; q_j) \quad (5)$$

- Negative sign in Eq. (4)!



Loop-Tree Duality at work

$$\mathcal{A}^{(L)}(\{p_n\}_N) = \int_{\ell_1,\ldots,\ell_L} \mathcal{N}(\{\ell_i\}_L, \{p_n\}_N) \times G_F(1,\ldots,n)$$

Notation

Scattering amplitude:

$$\mathcal{A}^{(L)}(\{p_n\}_N) = \int_{\ell_1,\dots,\ell_L} \mathcal{N}(\{\ell_i\}_{i=1}^{n}, \{p_j\}_{j=1}^{N}) \times G_F(1,\dots,n), \tag{6}$$

where

$$\int_{\ell_i} = -\imath\mu^{4-d}\int \frac{d^d\ell_i}{(2\pi)^d} \quad , \quad \int_{\ell_1,\dots,\ell_{i+1}} = \int_{\ell_1,\dots,\ell_i}\int_{\ell_{i+1}}$$
$$G_F(1,\dots,n) = \prod_{i\in\alpha_1,\dots,\alpha_n} [G_F(q_i)]^{\gamma_i} \tag{7}$$

(α_i: sets of internal propagators with the same dependence on loop momenta.)

Iterated and nested residues

How to use Cauchy's residue theorem for two or more loops? Recall an L-loop scattering amplitude:

$$\mathcal{A}^{(L)} = \int_{\ell_1,\ldots,\ell_L} \mathcal{A}_F^{(L)}(1,\ldots,n). \tag{8}$$

Compute each residue assuming all other integration variables as real parameters.

$$\begin{gathered} \mathbb{C}^{\left(\mathbb{R}^L\right)} \rightarrow \mathbb{C}^{\left(\mathbb{R}^{L-1}\right)} \rightarrow \mathbb{C}^{\left(\mathbb{R}^{L-2}\right)} \rightarrow \ldots \mathbb{C} \\ (\mathrm{Res} \circ id)^L : \mathbb{C}^{\left(\mathbb{R}^L\right)} \rightarrow \mathbb{C} \end{gathered} \tag{9}$$

- Displaced poles.

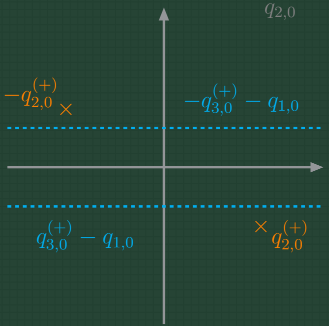
Iterated and nested residues

Fixing ideas!!

2-loop vacuum polarization diagram. Integrand: $G_F(1,2,12)$ (the argument 12 implies q_1+q_2).

$$\mathcal{I}^{(2)} = \frac{1}{\left(q_{1,0}^2 - q_{1,0}^{(+)2}\right)\left(q_{2,0}^2 - q_{2,0}^{(+)2}\right)\left((q_{1,0}+q_{2,0})^2 - q_{3,0}^{(+)2}\right)} \tag{10}$$

As a function of $q_{2,0}$, this integrand has a pole structure as

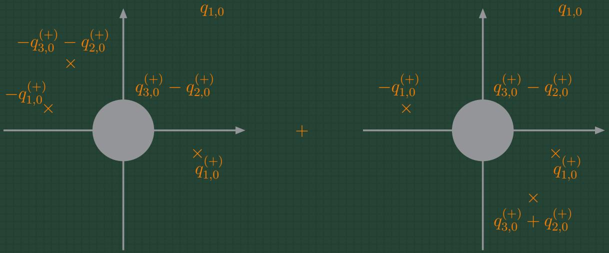


Iterated and nested residues

The first application of Cauchy's residue theorem leads us to two terms with pole structures,

$$\frac{1}{2q_{2,0}^{(+)}(q_{1,0}^2 - q_{1,0}^{(+)2})((q_{1,0} + q_{2,0}^{(+)})^2 - q_{3,0}^{(+)2})} \tag{11}$$

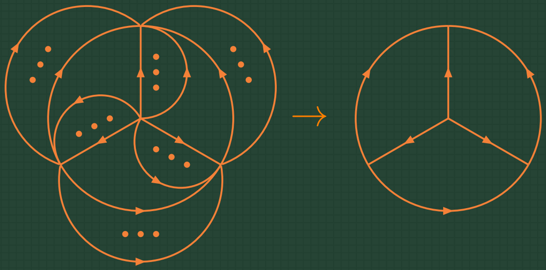
$$\frac{1}{2q_{3,0}^{(+)}(q_{1,0}^2 - q_{1,0}^{(+)2})((q_{3,0}^{(+)} - q_{1,0})^2 - q_{2,0}^{(+)2})} \tag{12}$$



Iterated residues vs nested residues

- Contributions of displaced poles: cancelled.
- We call *nested residues* to an analogue algorithm to iterated residues but blinded to displaced poles.
- We expect iterative relations.
- Nested residues speeds up the computations of iterated residues exploiting combinatorics.
- Non-causal divergences cancel.
- Higher powers of propagators represents no bottleneck.

Topological families and reduction of loop number

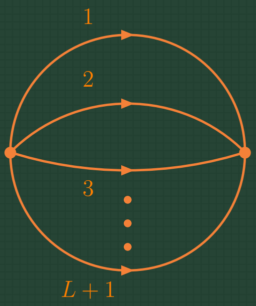


Maximal Loop Topology (MLT)

The topological family MLT with L loops is given by the integrand

$$\mathcal{I}^{(L)} = G_F(1, \ldots, L, 1 \ldots L). \tag{13}$$

Feynman diagram (with external particles omitted) associated to MLT is

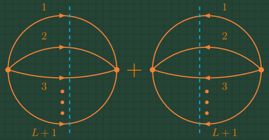


Topological families and reduction of loop number

Nested residues applied to MLT diagram with an external momentum p leads to

$$G_F(1,\ldots,L,1\ldots L) \to \sum \text{All tree} - \text{level subdiagrams} \quad (14)$$

- Partial fractions $\to$ causal structure!!



- Minimal diagrams!!

$$G_F(1,\ldots,L,L+1) \to \frac{1}{\prod_{k=1}^{L+1}\left(2q_{k,0}^{(+)}\right)}\left(\frac{1}{\sum_{i=1}^{L+2} q_{i,0}^{(+)} + p_0} + \frac{1}{\sum_{i=1}^{L+1} q_{i,0}^{(+)} - p_0}\right) \quad (15)$$

EQUIVALENCE BETWEEN MLT INSERTIONS AND A SINGLE PROPAGATOR!!!

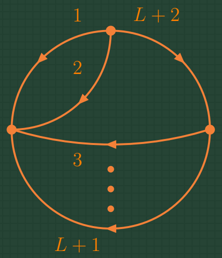
Topological families and reduction of loop number

Next-to Maximal Loop Topology (NMLT)

The topological family NMLT is defined by the integrand

$$\mathcal{I}^{(L)} = G_F(1,\ldots,L,1\ldots L,12), \tag{16}$$

and its associated Feynman diagram is



The minimal diagram is associated to a 1-loop triangle.

Topological families and reduction of loop number

Causal denominators:

$$\lambda_2^{\pm} = q_{1,0}^{(+)} + q_{1,0}^{(+)} + q_{L+2,0}^{(+)} \pm p_{2,0} \quad , \quad \lambda_3^{\pm} = \sum_{i=3}^{L+2} q_{i,0}^{(+)} \pm p_{3,0},$$

Nested residues applied to an NMLT diagram gives

$$\mathcal{I}^{(L)} = \frac{2}{\prod_{i=1}^{L+2} 2q_{i,0}^{(+)}} \left(\frac{1}{\lambda_1^+ \lambda_2^-} + \frac{1}{\lambda_2^+ \lambda_3^-} + \frac{1}{\lambda_3^+ \lambda_1^-} + (\lambda_i^+ \leftrightarrow \lambda_i^-) \right) \tag{17}$$

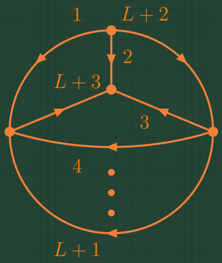
Topological families and reduction of loop number

Next-to-Next-to Maximal Loop Topology (N^2MLT)

The topological family N^2MLT is defined by the integrand

$$\mathcal{I}^{(L)} = G_F(1,\ldots,L,1\ldots L,12,23), \tag{18}$$

and its associated Feynman diagram is



The minimal diagram is associated to a 3-loop Mercedes-Benz-like diagram.

Topological families and reduction of loop number

Causal denominators:

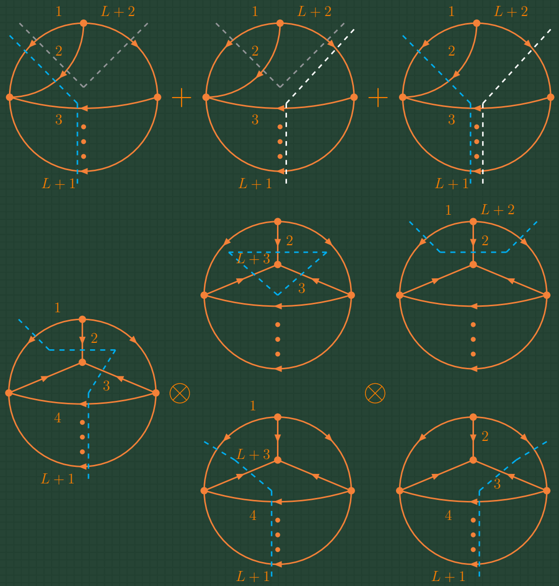
$$\lambda_4^{\pm} = q_{2,0}^{(+)} + q_{3,0}^{(+)} + q_{L+3,0}^{(+)} \pm p_{3,0} \quad , \quad \lambda_5^{\pm} = q_{1,0}^{(+)} + q_{L+3,0}^{(+)} + \sum_{i=4}^{L+1} q_{i,0}^{(+)} \pm p_{1,0},$$

$$\lambda_6^{\pm} = q_{1,0}^{(+)} + q_{3,0}^{(+)} + q_{L+2,0}^{(+)} + q_{L+3,0}^{(+)} \pm p_{23,0} \quad , \quad \lambda_7^{\pm} = q_{2,0}^{(+)} + \sum_{i=4}^{L+3} q_{i,0}^{(+)} \pm p_{12,0}.$$

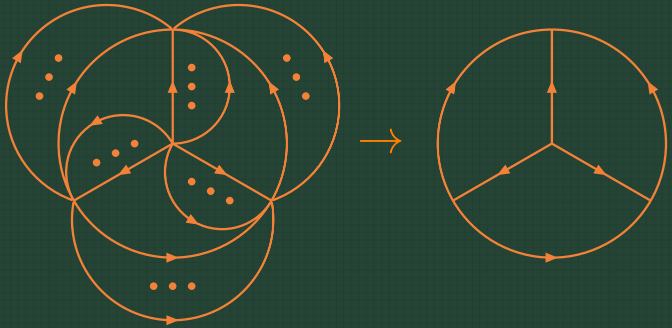
Causal structure of N^{2}MLT diagram:

$$\mathcal{I}^{(L)} \to \frac{2}{\prod_{i=1}^{L+2} 2q_{i,0}^{(+)}} \left(\frac{1}{\lambda_1^+} \left(\frac{1}{\lambda_2^-} + \frac{1}{{}_3^-} \right) \left(\frac{1}{\lambda_4^+} + \frac{1}{\lambda_5^+} \right) + \frac{1}{\lambda_6^+} \left(\frac{1}{\lambda_3^-} + \frac{1}{\lambda_5^-} \right) \left(\frac{1}{\lambda_2^+} + \frac{1}{\lambda_4^+} \right) \right.$$
$$\left. + \frac{1}{\lambda_7^+} \left(\frac{1}{\lambda_3^-} + \frac{1}{\lambda_4^-} \right) \left(\frac{1}{\lambda_2^+} + \frac{1}{\lambda_5^+} \right) + (\lambda_i^+ \leftrightarrow \lambda_i^-) \right) \tag{19}$$

Entanglement of causal thresholds



Diagramatic representation of general results



LTD formalism gives insight of Cutkosky cuts and their interplay.

Conclusiones

- Compact causal representations can be obtained through the LTD formalism.
- Scattering amplitudes can be obtained directly with this formalism, and no extra effort is needed.
- Nested residues takes an intricate interplay between causal and non-causal singularities, and naturally leads to a fully causal expressions.

THANK YOU!!