A new theoretical determination of $R_{\tau/P} = \frac{\Gamma(\tau \to P\nu_\tau(\gamma))}{\Gamma(P \to \mu\nu_\mu(\gamma))}$, $P = \pi, K$.

M. A. Arroyo-Ureña[1], G. Hernández-Tomé[1], G. López-Castro[1], P. Roig[1], I. Rosell[2].

[1]Centro de Investigación y de Estudios Avanzados del IPN.

[2]Universidad Cardenal Herrera-CEU.

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Aim of the work

1. Analyze lepton universality (LU) through the ratio

$$R_{\tau/P} = \frac{\Gamma(\tau \to P\nu_\tau(\gamma))}{\Gamma(P \to \mu\nu_\mu(\gamma))} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}(1+\delta R_{\tau/P}),\ P=\pi,\ K. \quad (1)$$

$\Gamma(P \to \mu\nu_\mu(\gamma))$ was calculated in Ref. [1].

$$g_\tau = g_\mu \text{ according to LU,} \quad (2)$$

$$R^{(0)}_{\tau/P} = \frac{M_\tau^3}{2m_P m_\mu^2}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}, \quad (3)$$

$$\delta R_{\tau/P} \to \text{Radiative corrections.} \quad (4)$$

$$\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%, \quad \delta R_{\tau/K} = (0.90 \pm 0.22)\%^1. \quad (5)$$

$$\Rightarrow \begin{cases} |g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026, \\ |g_\tau/g_\mu|_K = 0.9877 \pm 0.0063. \end{cases} \quad (6)$$

[1] Ref. [2]

Motivation

1. Phenomenology: Disagreement in LU tests.
   - Using $\frac{\Gamma(\tau\to P\nu_\tau(\gamma))}{\Gamma(P\to\mu\nu_\mu(\gamma))}$ with DF result, HFLAV reported:[2]
     - $|g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026$ (at 1.6σ of LU),
     - $|g_\tau/g_\mu|_K = 0.9877 \pm 0.0063$ (at 1.9σ of LU).
2. Theoretical issues within DF:
   - Hadronic form factors do not satisfy the correct QCD short distance behavior,
   - Violate unitarity, analicity, chiral limit,
   - DF used a cutoff in order to regalate loop integrals,
   - Underestimated uncertainties.

[2] Ref. [3]

$\tau \to P\nu_\tau$, $(P = \pi, K)$

- The decay width of the process $\tau \to P\nu_\tau$ was calculated in the context of an effective approach encoding the hadronization.
  - Large-N_c expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonaces such that the relevant Green functions are well-behaved at high energies[3]

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \underbrace{\frac{G_F^2 |V_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2}_{\text{LO result}} \underbrace{\left\{1 + \frac{2\alpha}{\pi} \log \frac{m_Z}{m_\rho}\right\}}_{\text{short-distance EW correction}} \underbrace{\left\{1 + \frac{\alpha}{\pi} G(m_P^2/M_\tau^2)\right\}}_{\text{structure independent (SI) contributions (point-like approximation) Ref [4]}} \times$$

$$\left\{1 - \frac{3\alpha}{2\pi} \log \frac{m_\rho}{M_\tau} + \delta_{\tau P}\big|_{\mathrm{rSD}} + \delta_{\tau P}\big|_{\mathrm{vSD}}\right\}$$

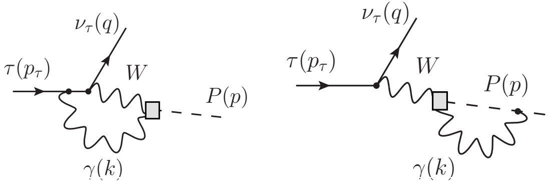
real-photon structure-dependent (rSD) contributions

virtual-photon structure-dependent (rSD) contributions

[3]Ref. [4]

$\tau \to P\nu_\tau,\ (P = \pi,\ K)$

- Virtual-photon structure-dependent contributions.



$$i\mathcal{M}[\tau \to P\nu_\tau]|_{\mathrm{SD}} = iG_F V_{uD} e^2 \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2 - M_\tau^2]} \left[i\epsilon_{\mu\nu\lambda\rho} k^\lambda p^\rho F_V^P(W^2,k^2) + F_A^P(W^2,k^2)\lambda_{1\mu\nu} + 2B(k^2)\lambda_{2\mu\nu} \right]$$

$$\begin{aligned}
F_V^P(W^2,k^2) &= \frac{-N_C M_V^4}{24\pi^2 F_P (k^2 - M_V^2)(W^2 - M_V^2)} \\
F_A^P(W^2,k^2) &= \frac{F_P}{2} \frac{M_A^2 - 2M_V^2 - k^2}{(M_V^2 - k^2)(M_A^2 - W^2)} \\
B(k^2) &= \frac{F_P}{M_V^2 - k^2}
\end{aligned}$$

Ref [5, 6]

$$\begin{aligned}
\ell^{\mu\nu} &= \bar{u}(q)\gamma^\mu(1-\gamma_5)[(\not{p}_\tau + \not{k}) + M_\tau]\gamma^\nu u(p_\tau) \\
\lambda_{1\mu\nu} &= \left[(p+k)^2 + k^2 - m_P^2\right] g_{\mu\nu} - 2k_\mu p_\nu \\
\lambda_{2\mu\nu} &= k^2 g_{\mu\nu} - \frac{k^2(p+k)_\mu p_\nu}{(p+k)^2 - m_P^2}
\end{aligned}$$

Results

- Virtual-photon structure-dependent contribution:

$$\delta R_{\tau/P} = \delta_{\tau P} - \delta_{P\mu}, \; (P = \pi, \; K). \tag{7}$$

$$\delta_{\tau\pi}|_{\mathrm{vSD}} = (-0.48 \pm 0.56)\%, \quad \delta_{\tau K}|_{\mathrm{vSD}} = (-0.45 \pm 0.57)\%^{4}, \tag{8}$$

$$\delta_{\pi\mu}|_{\mathrm{vSD}} = (0.54 \pm 0.12)\%, \quad \delta_{K\mu}|_{\mathrm{vSD}} = (0.43 \pm 0.12)\%^{5} \tag{9}$$

$$\boxed{\delta R_{\tau/\pi}|_{\mathrm{vSD}} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{\mathrm{vSD}} = (-0.88 \pm 0.58)\%}$$

- Uncertainties dominated by $\delta_{\tau P}|_{\mathrm{vSD}}$

[4] Ref. [3]
[5] Ref. [1]

Results

- Finally:

Contribution	$\delta R_{\tau/\pi}$	$\delta R_{\tau/K}$	Ref.
SI	$+1.05\%$	$+1.67\%$	5
rSD	$+0.15\%$	$+(0.18 \pm 0.05)\%$	6
vSD	$-(1.02 \pm 0.57)\%$	$-(0.88 \pm 0.58)\%$	new
Total	$+(0.18 \pm 0.57)\%$	$+(0.97 \pm 0.58)\%$	new

- Central values agree with **DF**

$$\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%, \quad \delta R_{\tau/K} = (0.90 \pm 0.22)\%. \qquad (10)$$

- But in that work:
  - Problematic hadronization: Form factors do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders,
  - A cutoff to regulate the loop integrals,
  - underestimated uncertainties.

Results

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\left(1+\delta R_{\tau/P}\right)$$

PDG

$\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$
$\delta R_{\tau/K} = (0.97 \pm 0.58)\%$

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

- ✓ π case: at 0.9σ of LU vs. 1.6σ of LU in HFLAV'21* using DF'95
- ✓ K case: at 1.8σ of LU vs. 1.9σ of LU in HFLAV'21* using DF'95

Conclusions

1. We calculate the radiative corrections $\delta_{\tau P}$ of the decay $\tau \to P\nu_\tau$ in the theoretical framework of a resonance extension of ChPT.
2. Using the observable

$$R_{\tau/P} = \frac{\Gamma(\tau \to P\nu_\tau(\gamma))}{\Gamma(P \to \mu\nu_\mu(\gamma))} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R_{\tau/P}^{(0)}(1+\delta R_{\tau/P}), \; P = \pi, \, K, \quad (11)$$

$$\delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \text{and} \delta R_{\tau/K} = (0.97 \pm 0.58)\%$$

$$\Rightarrow \begin{cases} |g_\tau/g_\mu|_\pi = 0.9964 \pm 0.0038 & (0.9\sigma \text{ of LU}), \\ |g_\tau/g_\mu|_K = 0.9857 \pm 0.0078 & (1.8\sigma \text{ of LU}). \end{cases} \quad (12)$$

- Reducing HFLAV disagreement with LU.

Thank you!

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