Transverse Momentum Dependent Parton Distribution Functions and QCD evolution equations

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XIX Mexican School of Particles and Fields, August 9-13, 2021 online event.

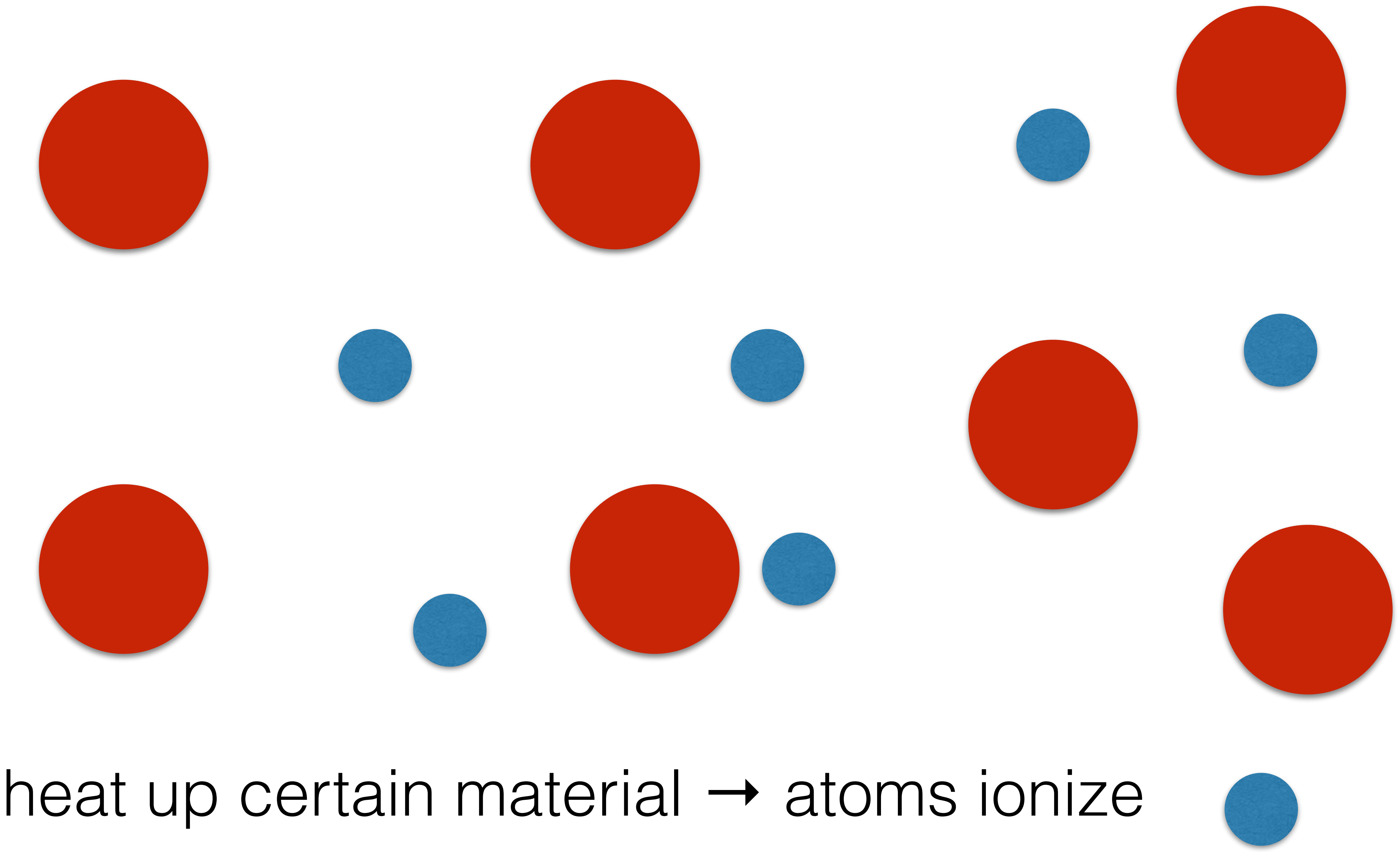
Let me start with something completely different: plasma and the Debye-Hückel model

a simple way to model the plasma:

Poisson equation for the electrostatic potential of an electron at $\vec{r} = 0$:

$$-\nabla^2\phi(\vec{r}) = \frac{-e}{\epsilon_0}\left[\delta^{(3)}(\vec{r}) + \rho g(\vec{r}) - \rho\right]$$

ions static, but other electrons move → $g(\vec{r})$



heat up certain material → atoms ionize

- gas of electrons with charge -q
- background of positively charged ions with charge density $+q\rho_{ion}$
- overall system is neutral
  → average charge density of electrons $\langle\rho\rangle_{electrons} = \rho_{ion}$

a good first model: $g(\vec{r}) = e^{\frac{-e\phi(\vec{r})}{k_B T}} \simeq 1 - \frac{e\phi(\vec{r})}{k_B T}$

where we take the high temperature limit

Debye screening length

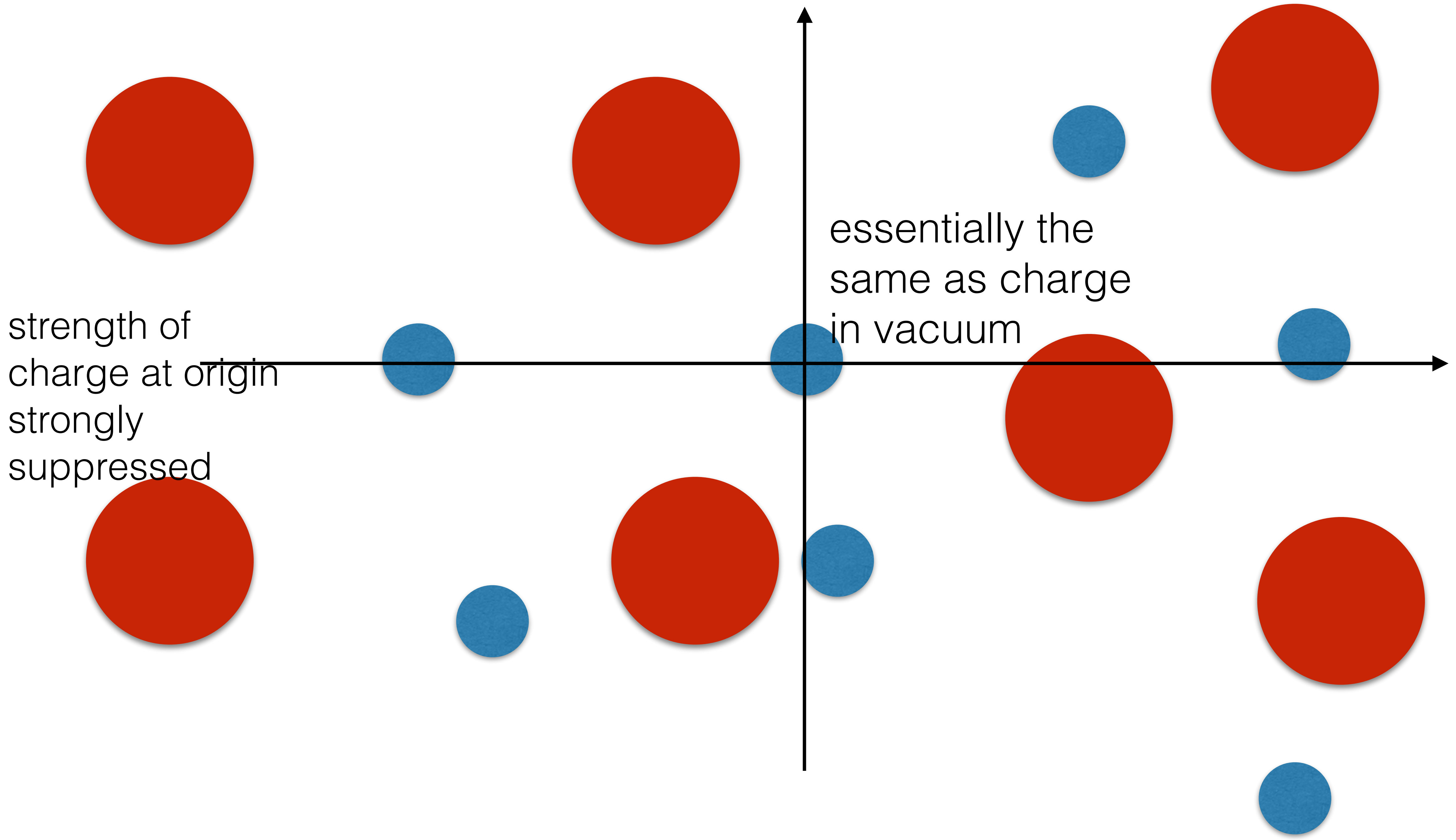
Poisson equation turns into

$$\left[\nabla^2 - \frac{1}{\lambda_D^2}\right]\phi(\vec{r}) = \frac{e}{\epsilon_0}\delta^{(3)}(\vec{r})$$

$$\lambda_D^2 = \frac{\epsilon_0 k_B T}{e^2 \rho}$$

Debye screening length

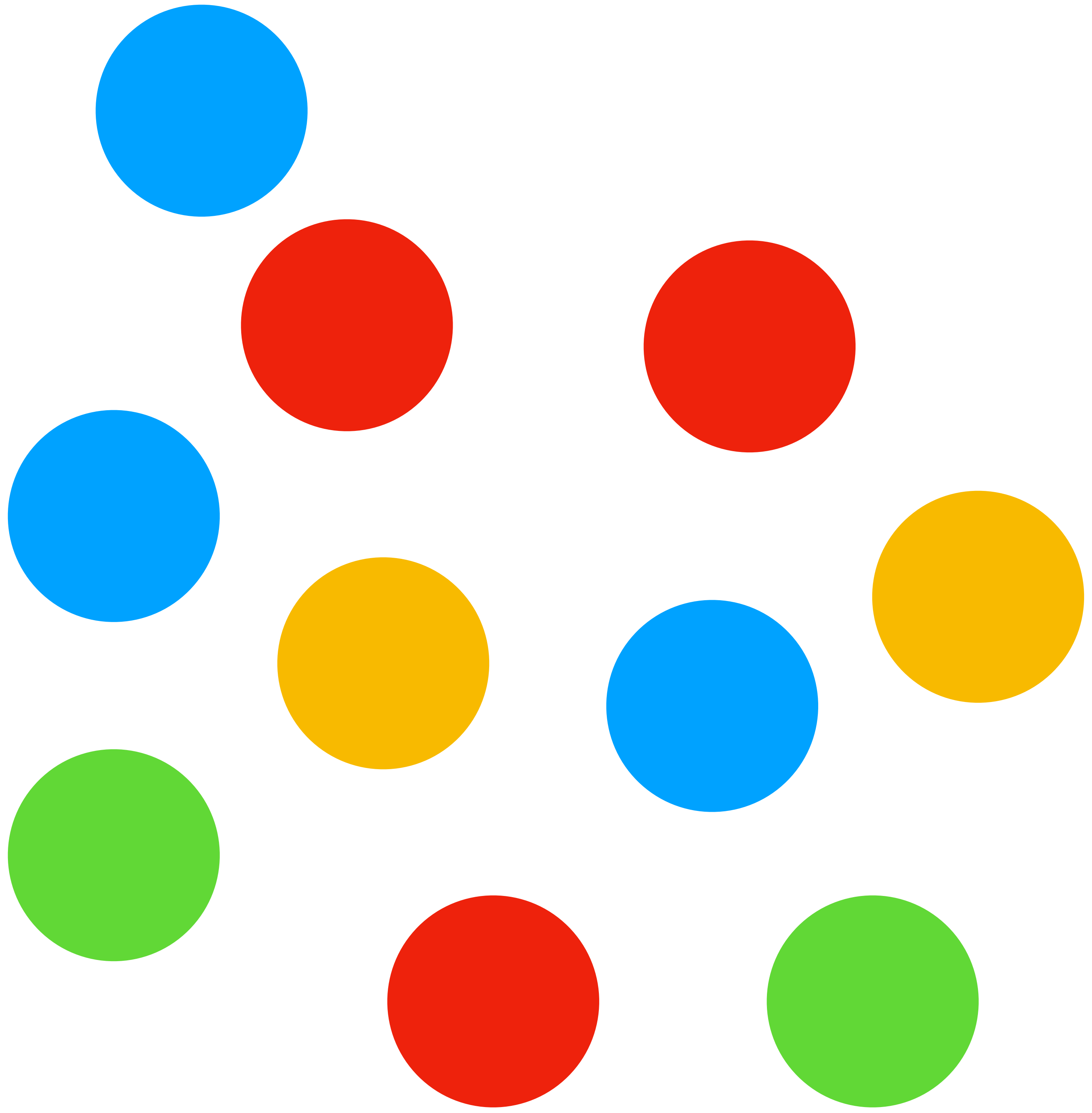
$$\phi(\vec{r}) = -\frac{e}{4\pi\epsilon_0|\vec{r}|}e^{-|\vec{r}|/\lambda_D}$$



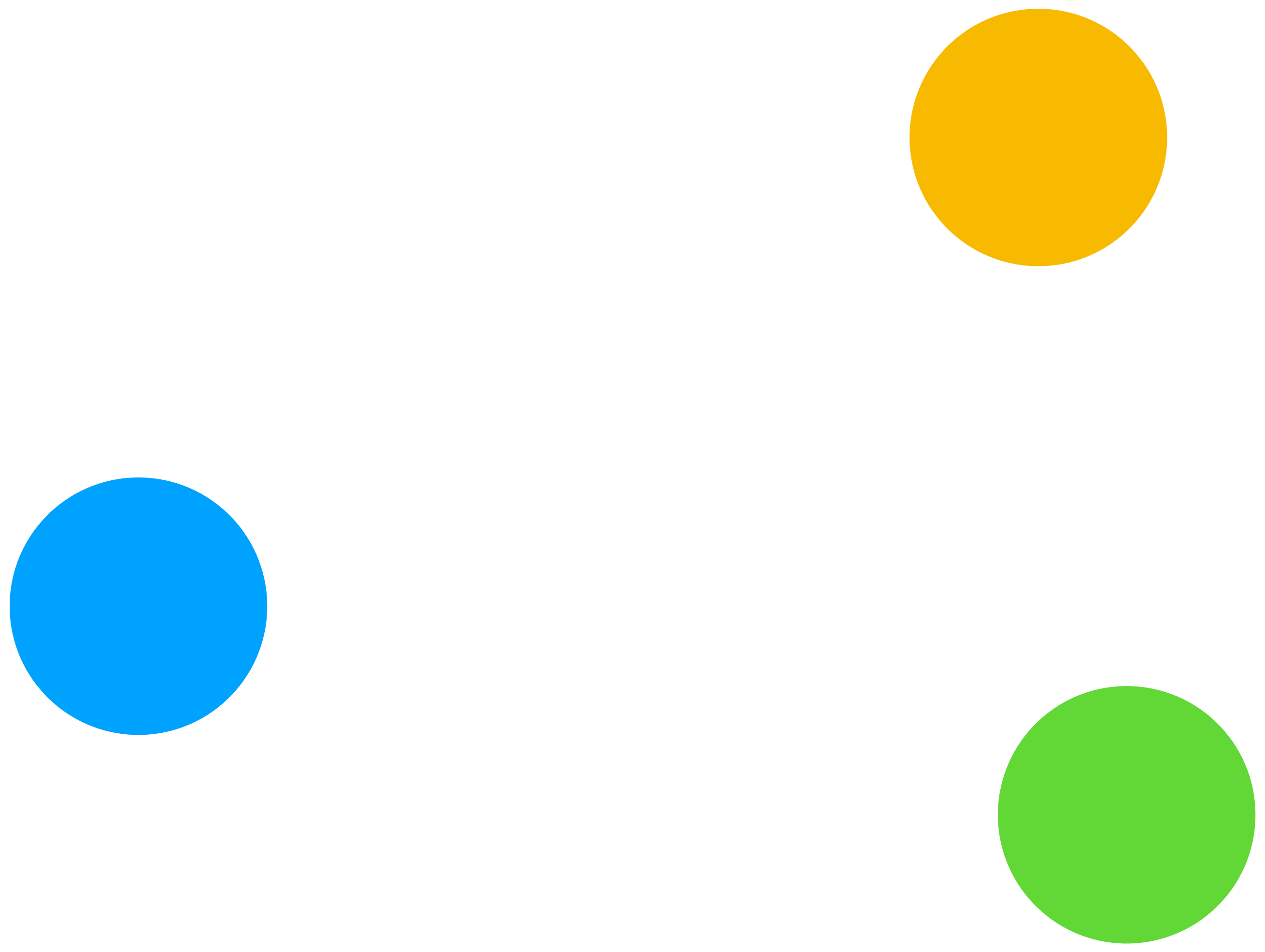
- electrostatic potential is exponentially suppressed for large distances
- charge placed at zero gets effectively reduced the further we get away

Let's turn to a QCD plasma instead → color charges

expect something very similar:
- close by color charges should behave similar to the vacuum
- far away charges should be screened
- in other words: we have a characteristic correlation length



plasma of quarks and gluons

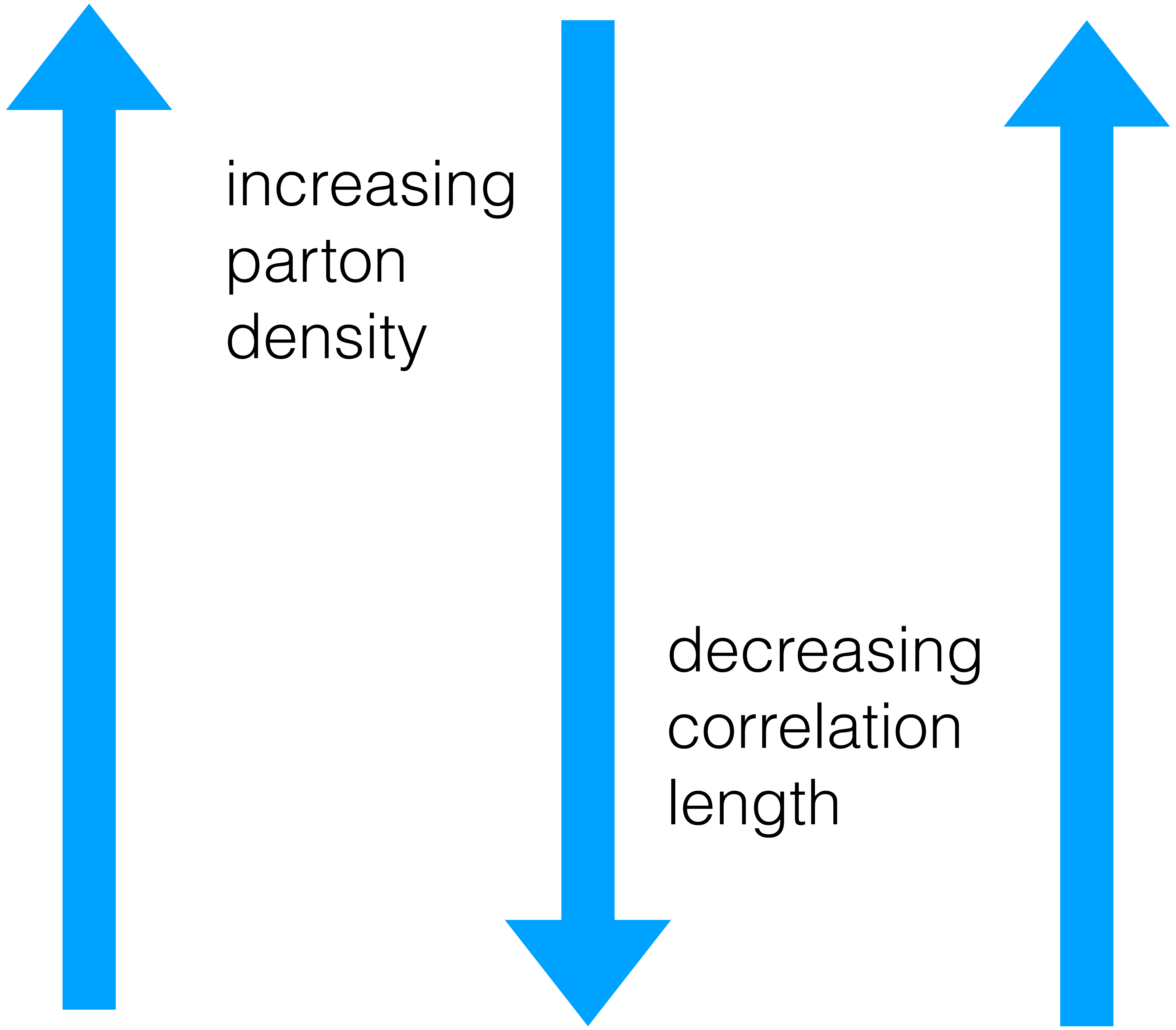


short correlation length in the plasma
= high parton (quarks, gluons) density

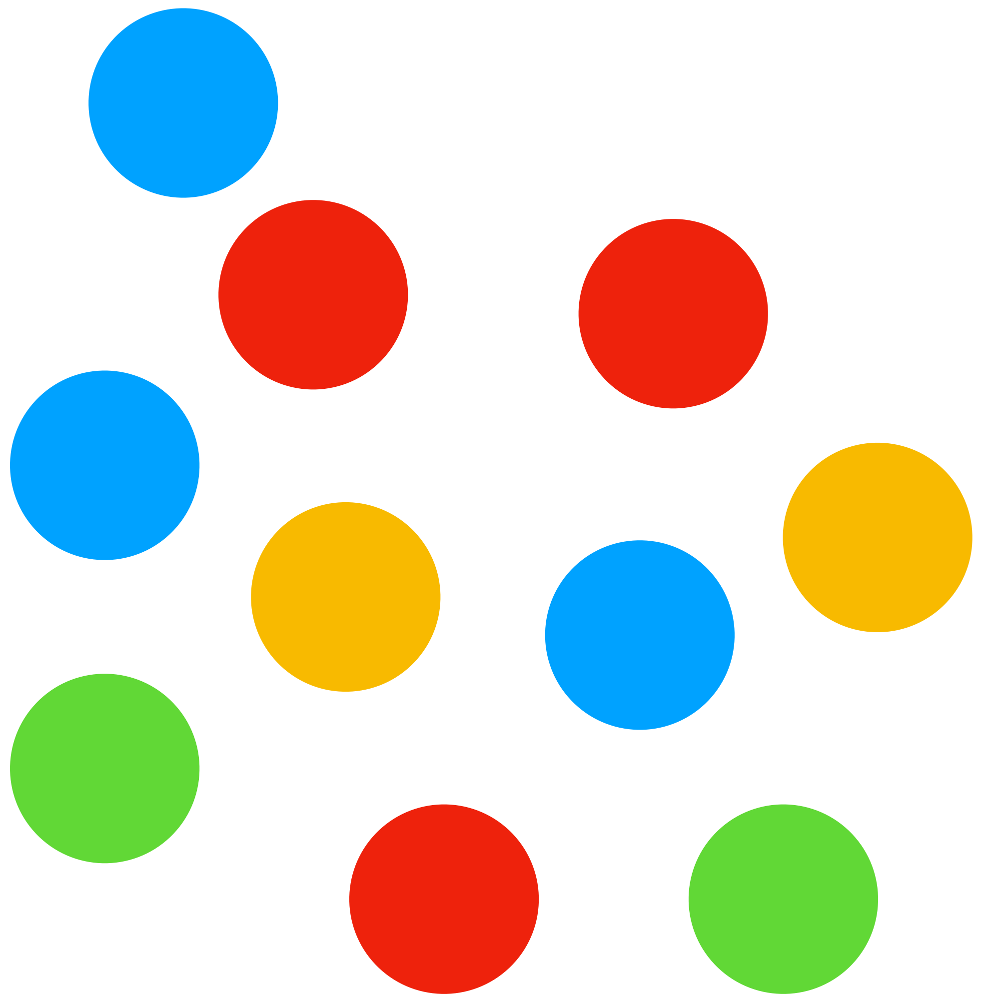
↔

long correlation length in the vacuum
= low parton densities

Short correlation length: that's interesting!



Heisenberg:
implies an energy scale which increases with density



$Q_s \sim \dfrac{1}{R_s}$ where

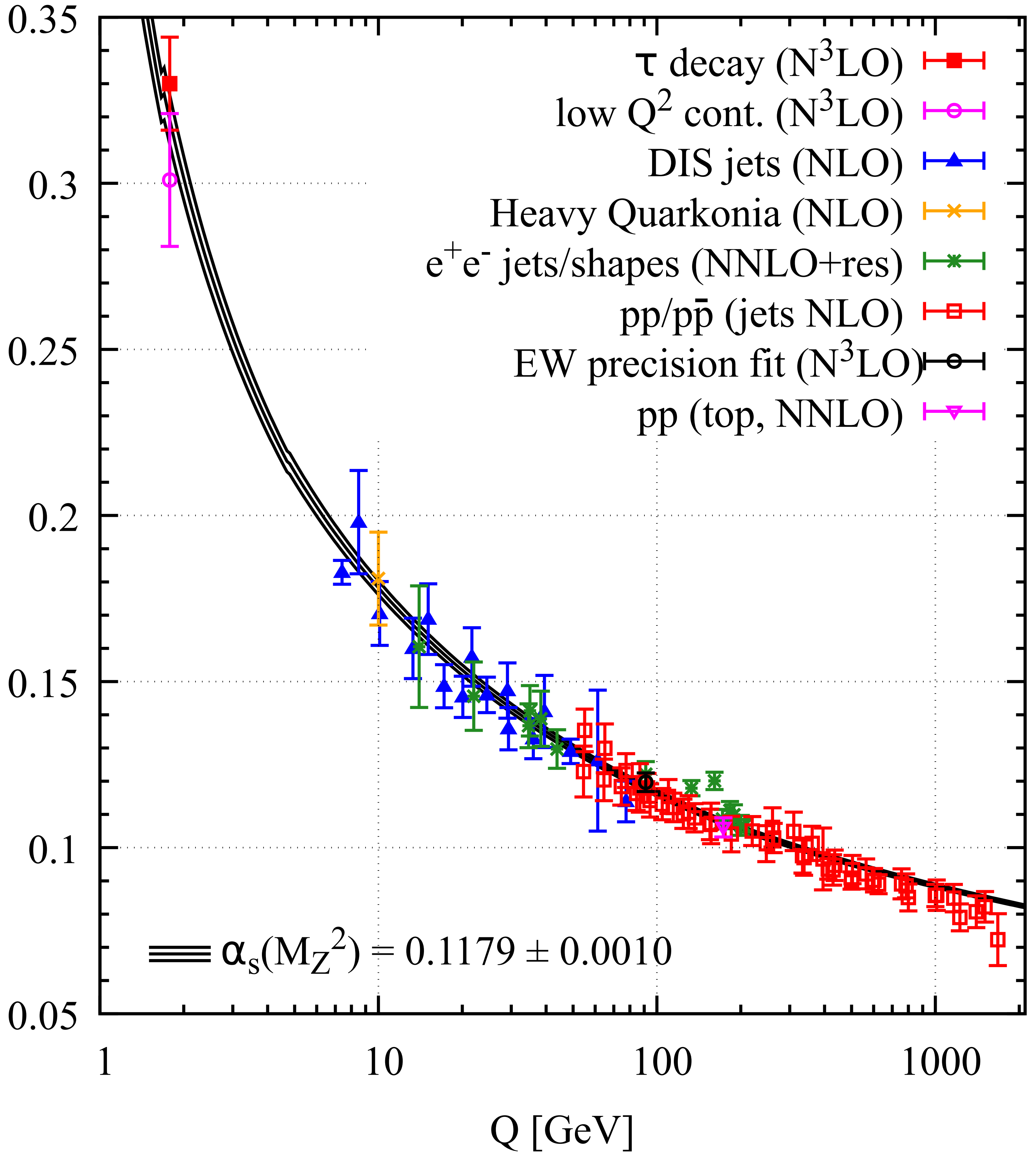
R_s: correlation length

high density creates an energy scale for the system which increases if the density increases

Why is that interesting?

QCD = strong interactions is characterized by asymptotic freedom

- at large distance and low energy scales, the coupling is large → quarks and gluons strongly bound into hadrons; non-perturbative physics (hard)
- at short distance and high energy (= hard) scales: QCD becomes a weakly coupled theory
- usually only realized for selected reactions (Deep Inelastic electron-proton scattering, high pT jet production, Higgs production, …)

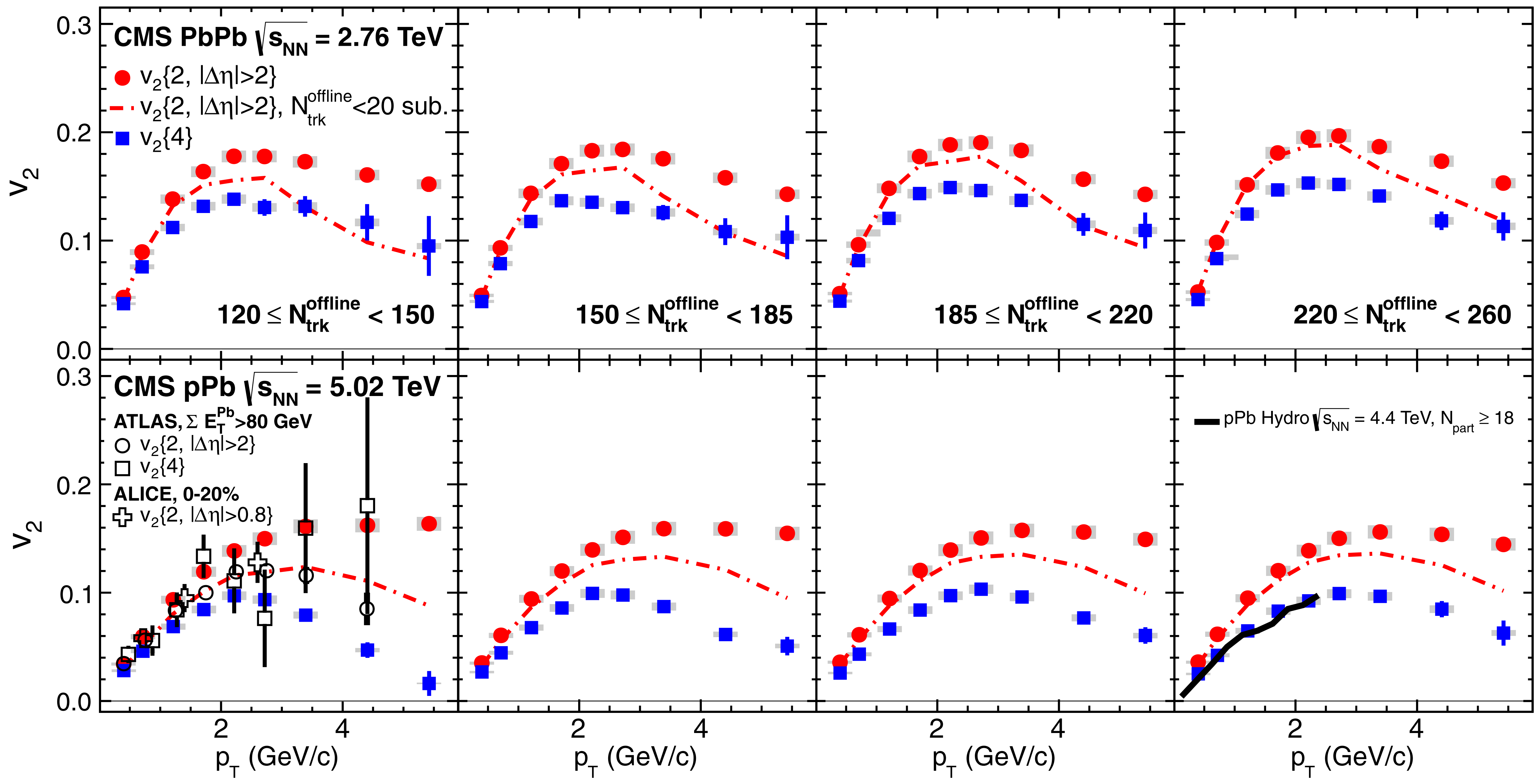


particle data group

here: an energy scale is generated dynamically & is increasing with density
at high enough densities: we should cross the magical boundary of $Q_s \sim 1$ GeV

prospect:

- theory description needs to deal with high parton densities (non-trivial, but sometimes possible)
- but can rely on weak coupling methods (strong coupling is perturbative)

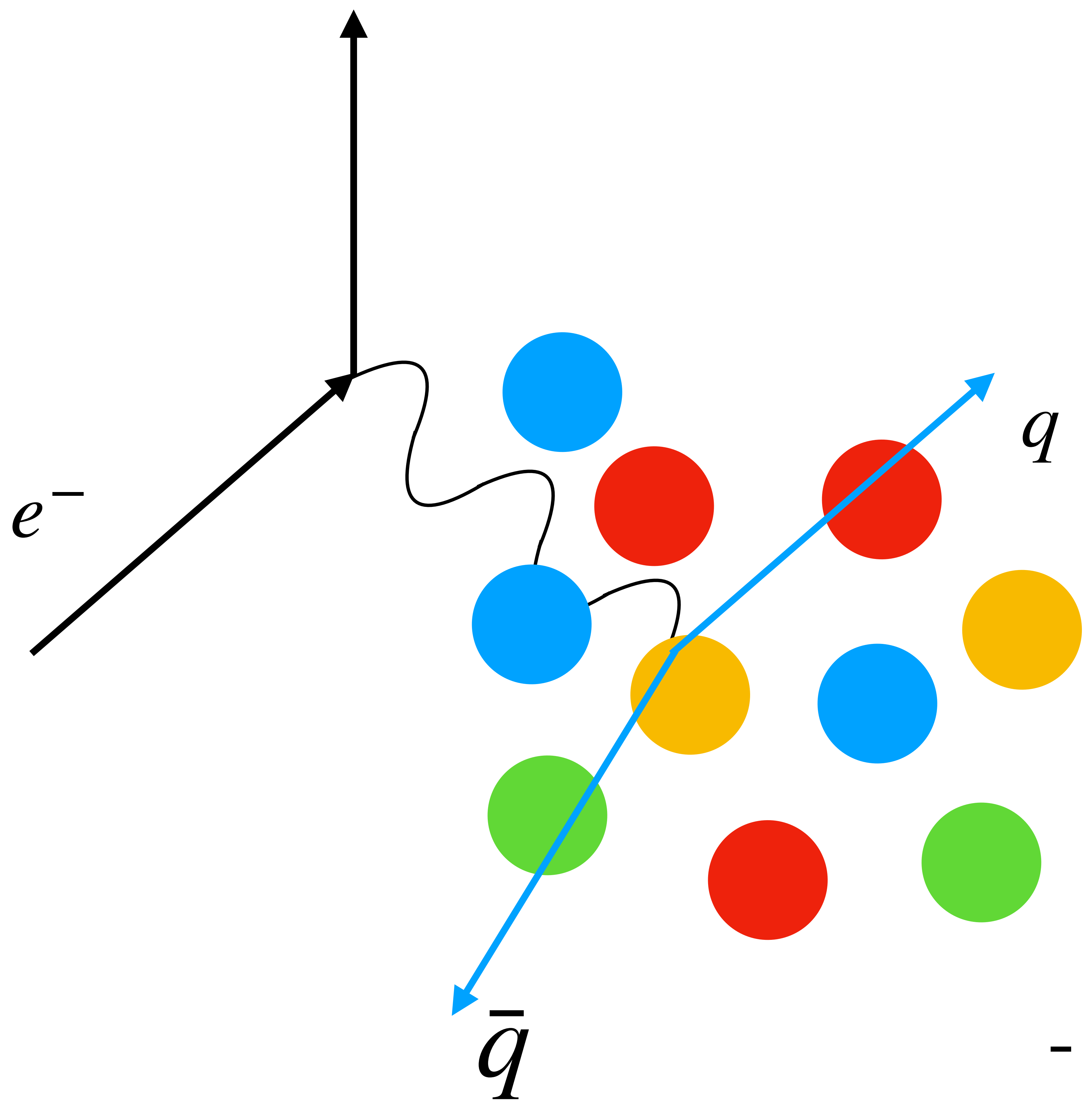


- see something related in data
- pT seems to grow with number of produced particles (didn't find the plot I wanted to show, but this one does the job)
- can we describe heavy ion collisions, high multiplicity events using weak coupling methods?
- = collision of 2 so-called Color Glass Condensates?

Answer: it's hard to tell ….. why? these are incredible complicated systems; many different effects are of relevance exploring such effects in heavy ion collisions/high multiplicity events is bit like understanding wave phenomena with one of those ….

we all know, that such a wave phenomena, are far more adequate to learn and understand waves ….
once we master those, we can start addressing the breaking wave

Our isolated system: electron ion collisions



Here:

- dihadron or dijet production in an electron ion collision
- if ion is replaced by dilute system (proton): expect that

$$\sum_{di-hadron} p_i \simeq p_\gamma + p_{prot.}$$

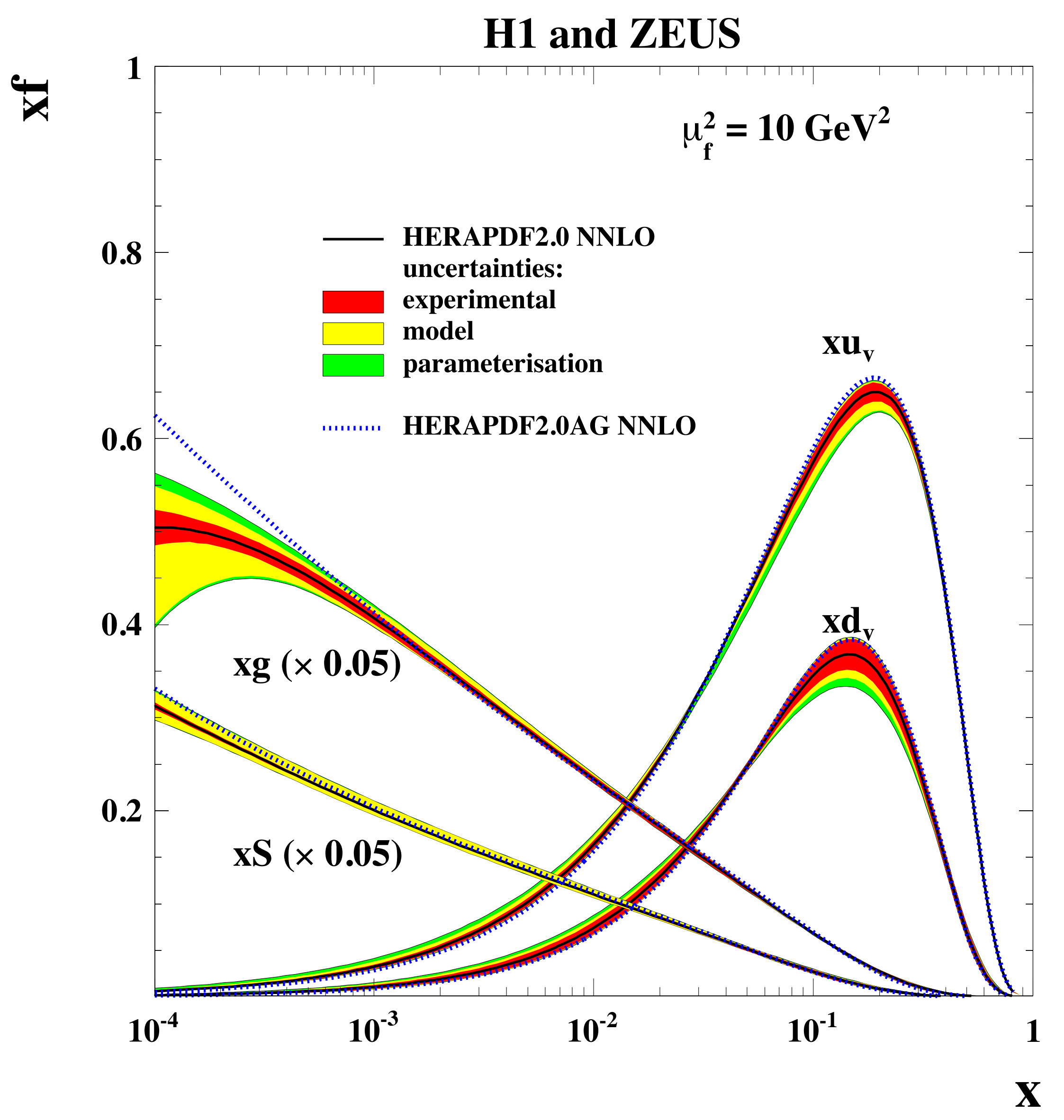
◉ only a small transverse momentum imbalance between momenta of colliding proton and photon

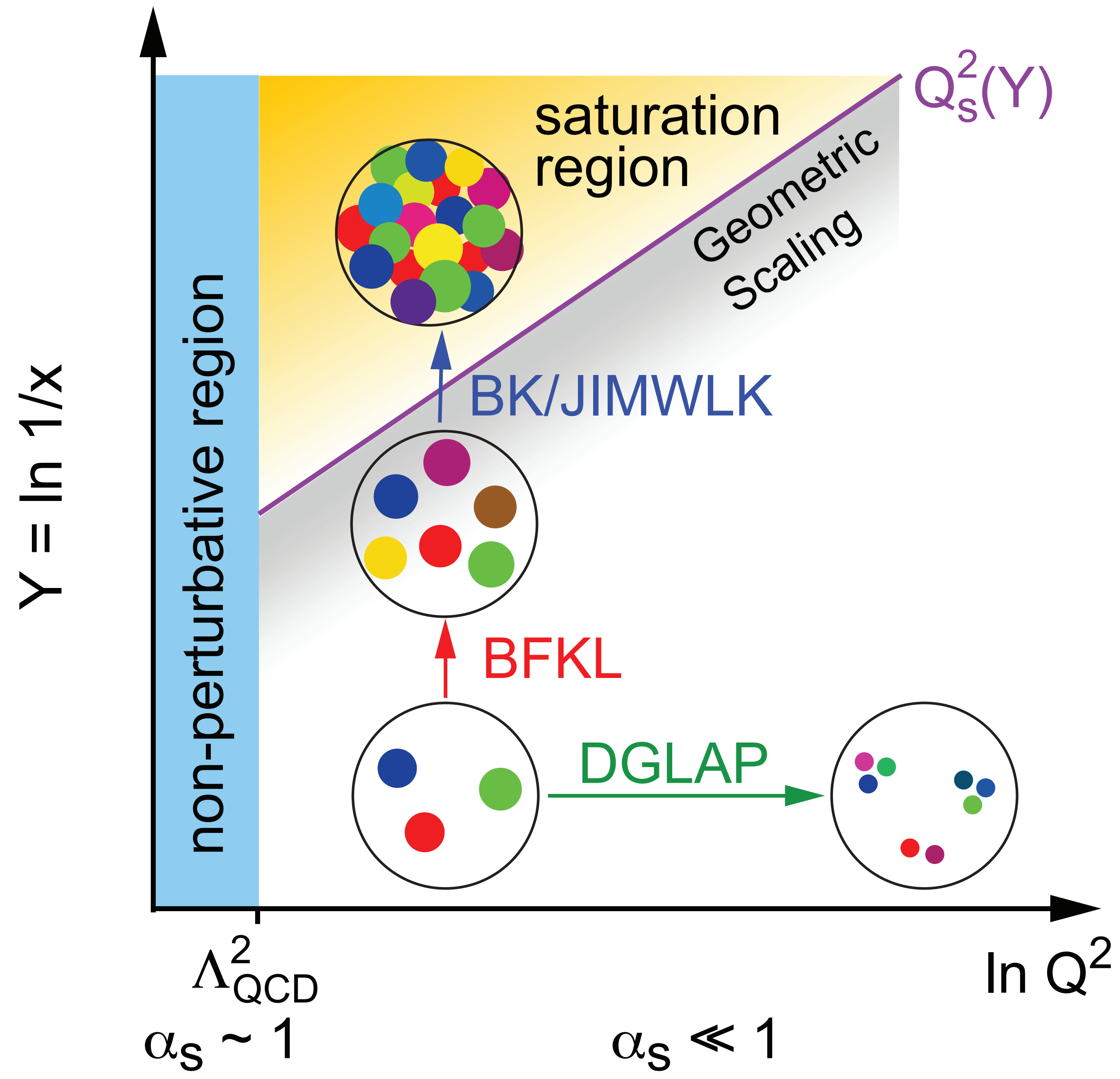
◉ transverse = transverse wrt. the collision axis

- dense system: expect transverse momentum imbalance of the order of the inverse correlation length $Q_s \sim \frac{1}{R_s}$
- can measure that in electron ion collisions

Gluon saturation

- dihadron and dijet production is a key process to search for effects of gluon saturation at the future Electron Ion Collider
- what is gluon saturation?

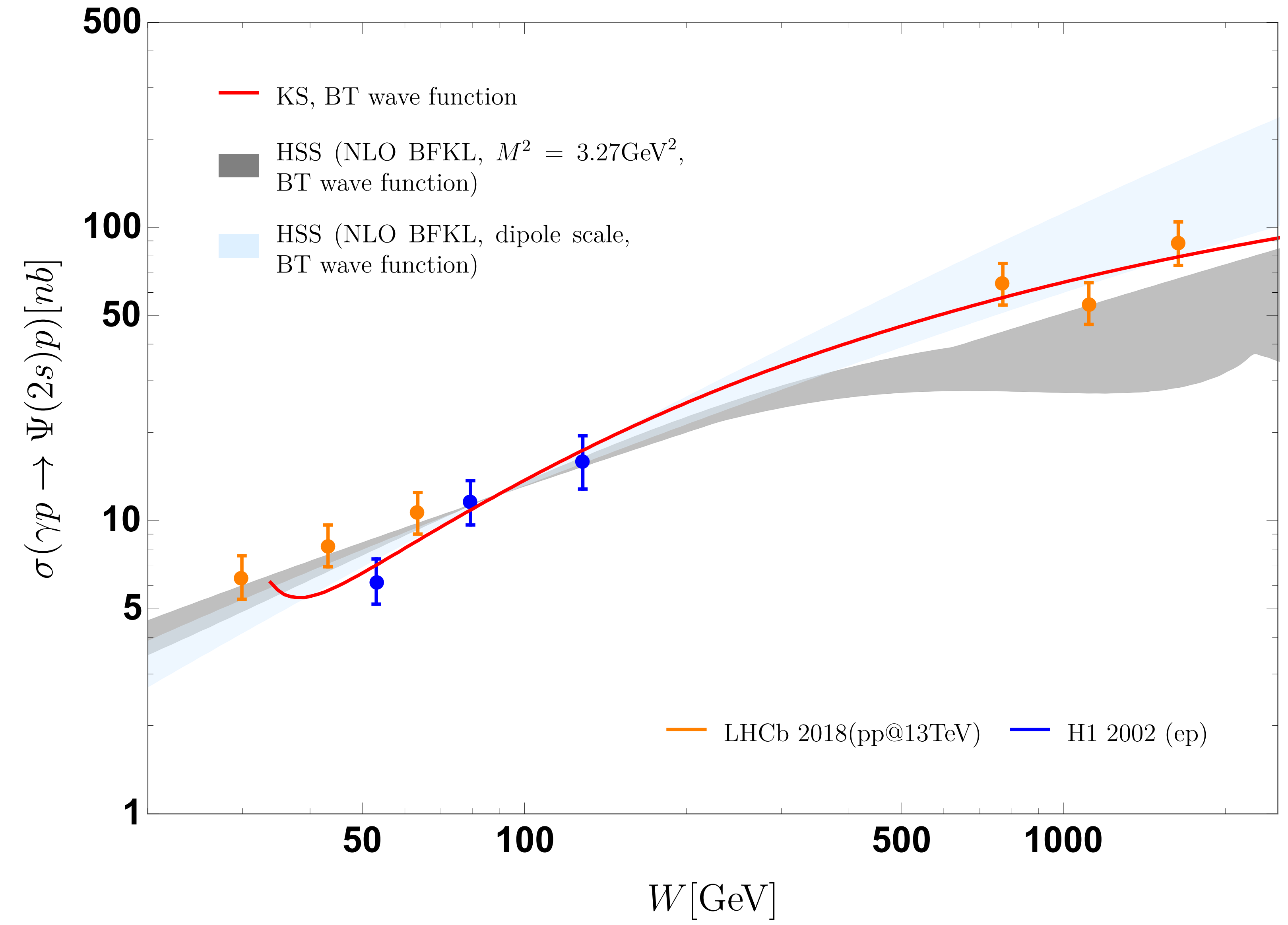


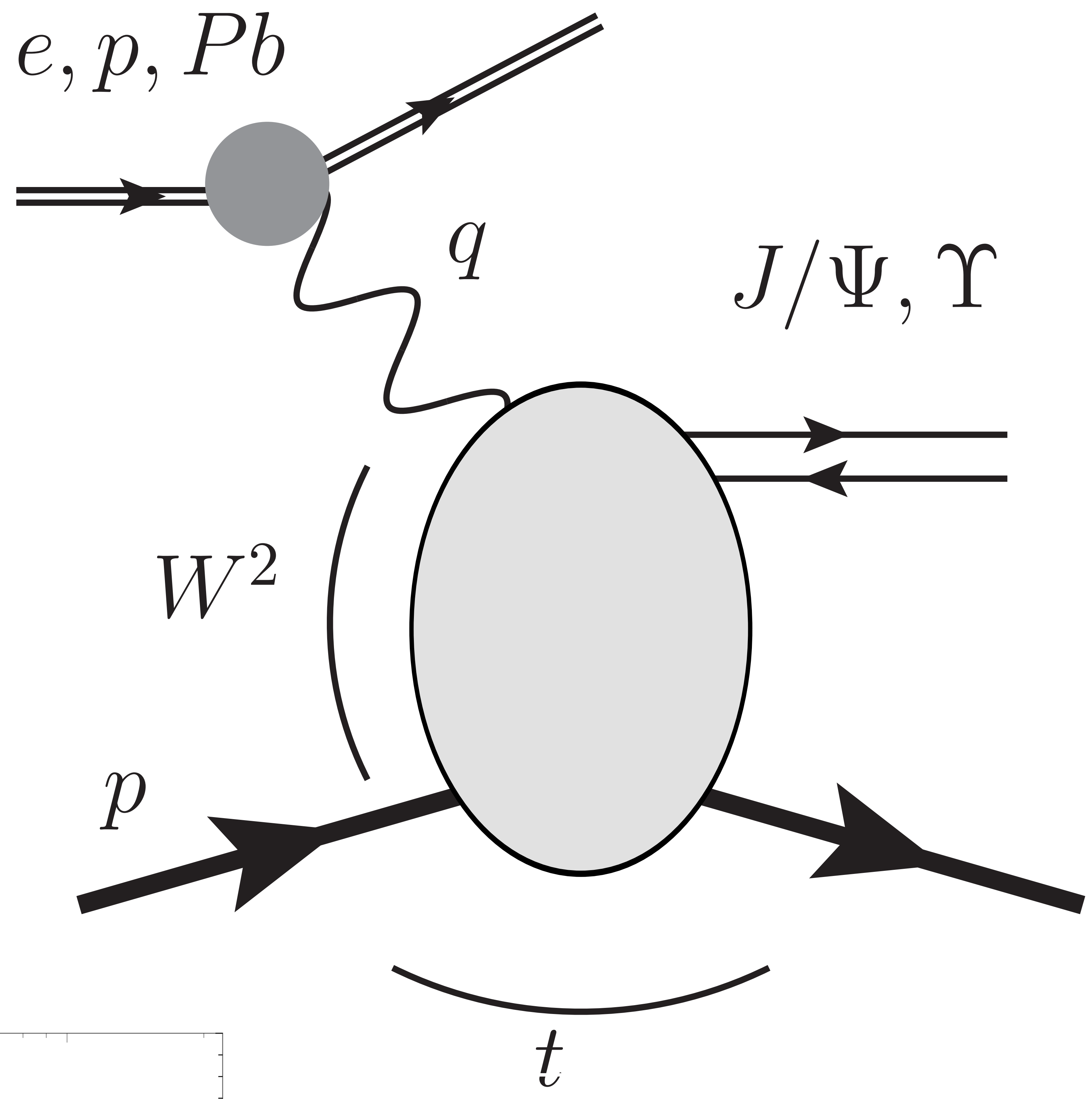


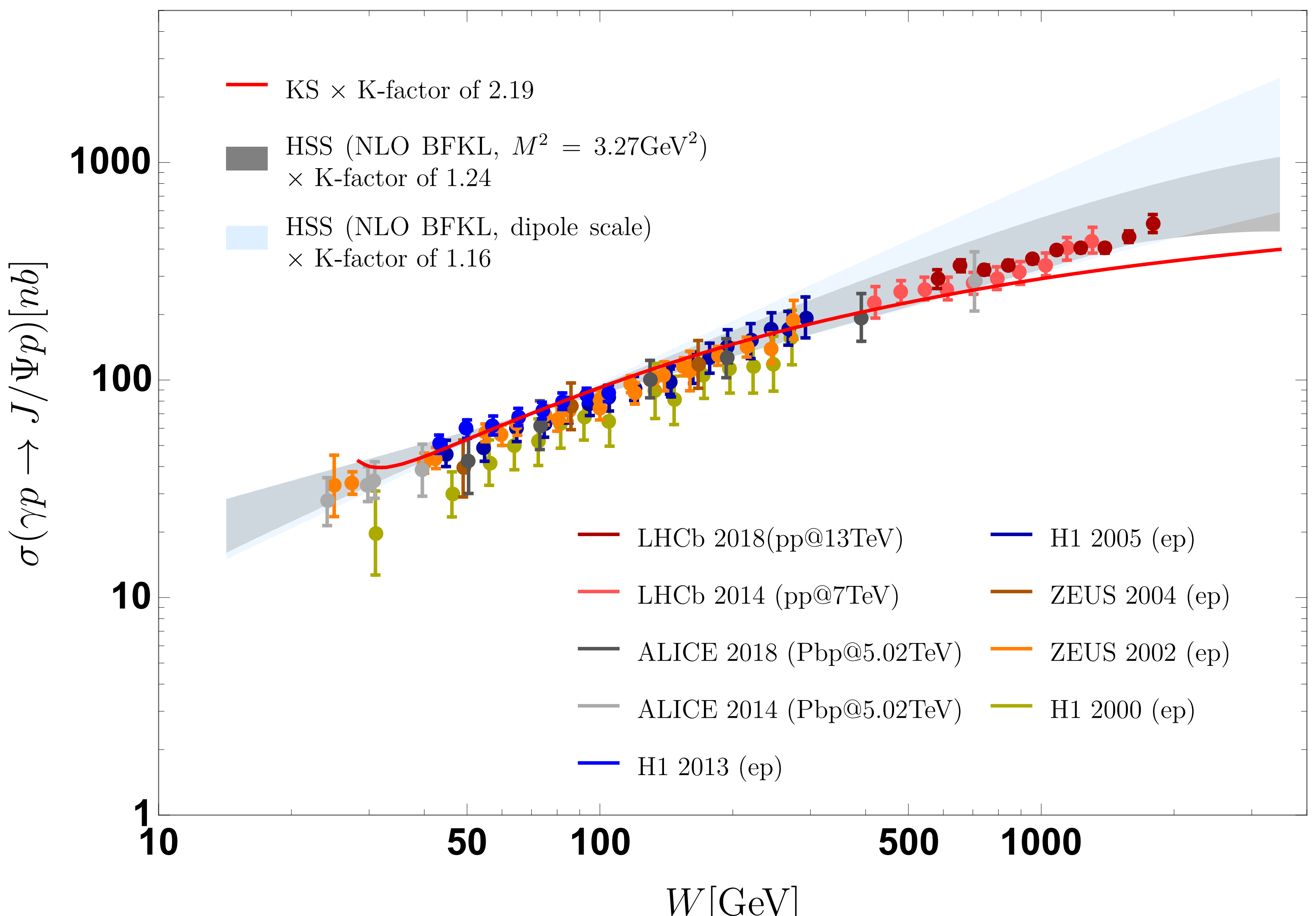
[Gribov, Levin & Ryskin Phys. Rept. 100 (1983)]
Color Glass Condensate effective theory:
[McLerran, Venugopalan PRD 49 (1994) 3352]

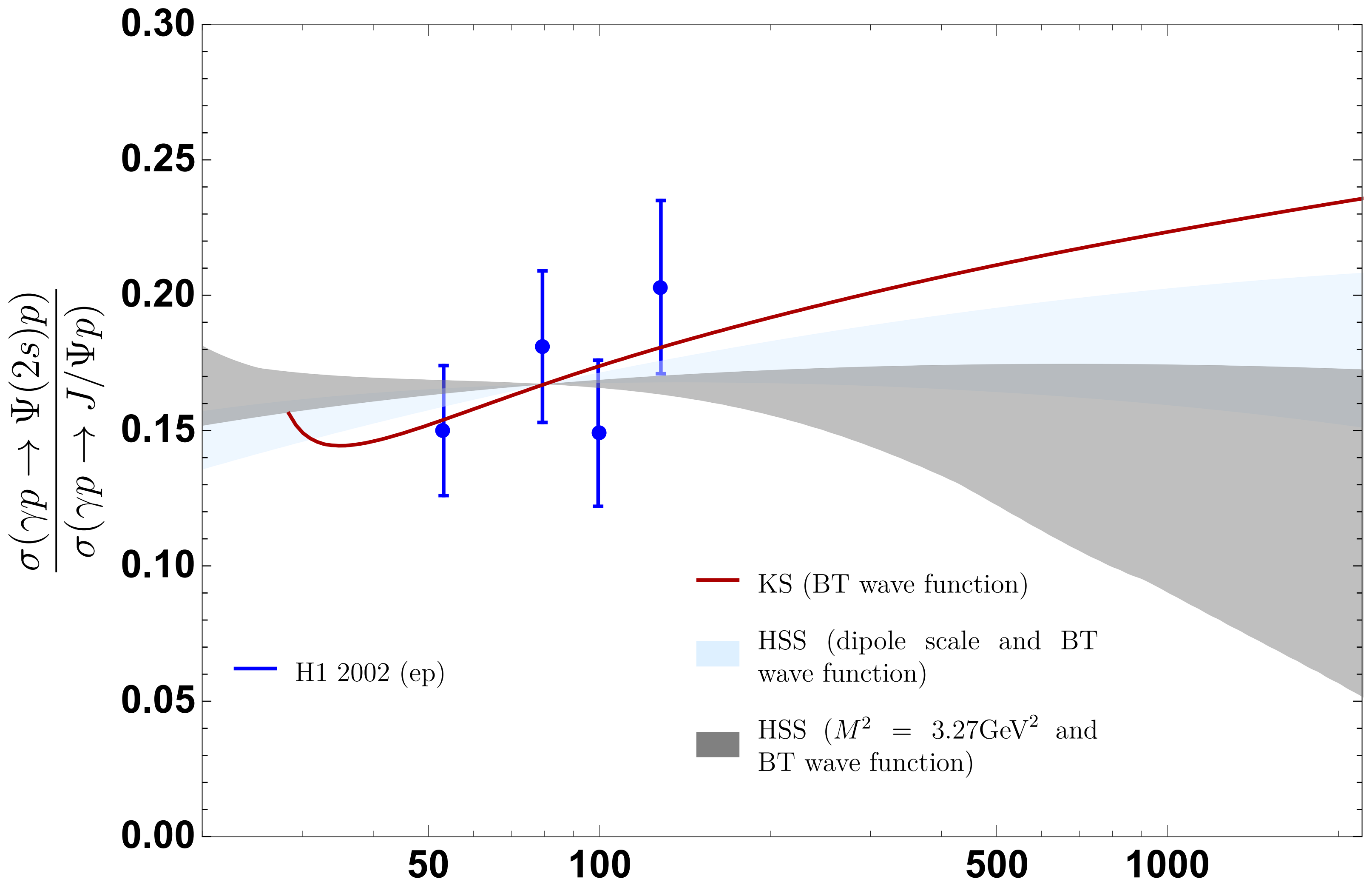
- observe power-like growth of gluon distribution towards low x = high center of mass energies
- if continued forever, violates unitarity bounds
- but: power-like growth drives us eventually into region of high parton densities
- can show: high densities slow down/stop growth of low x gluon: *saturation*

Can search for such effects through increasing the center of mass energy → for instance: exclusive charmonium production





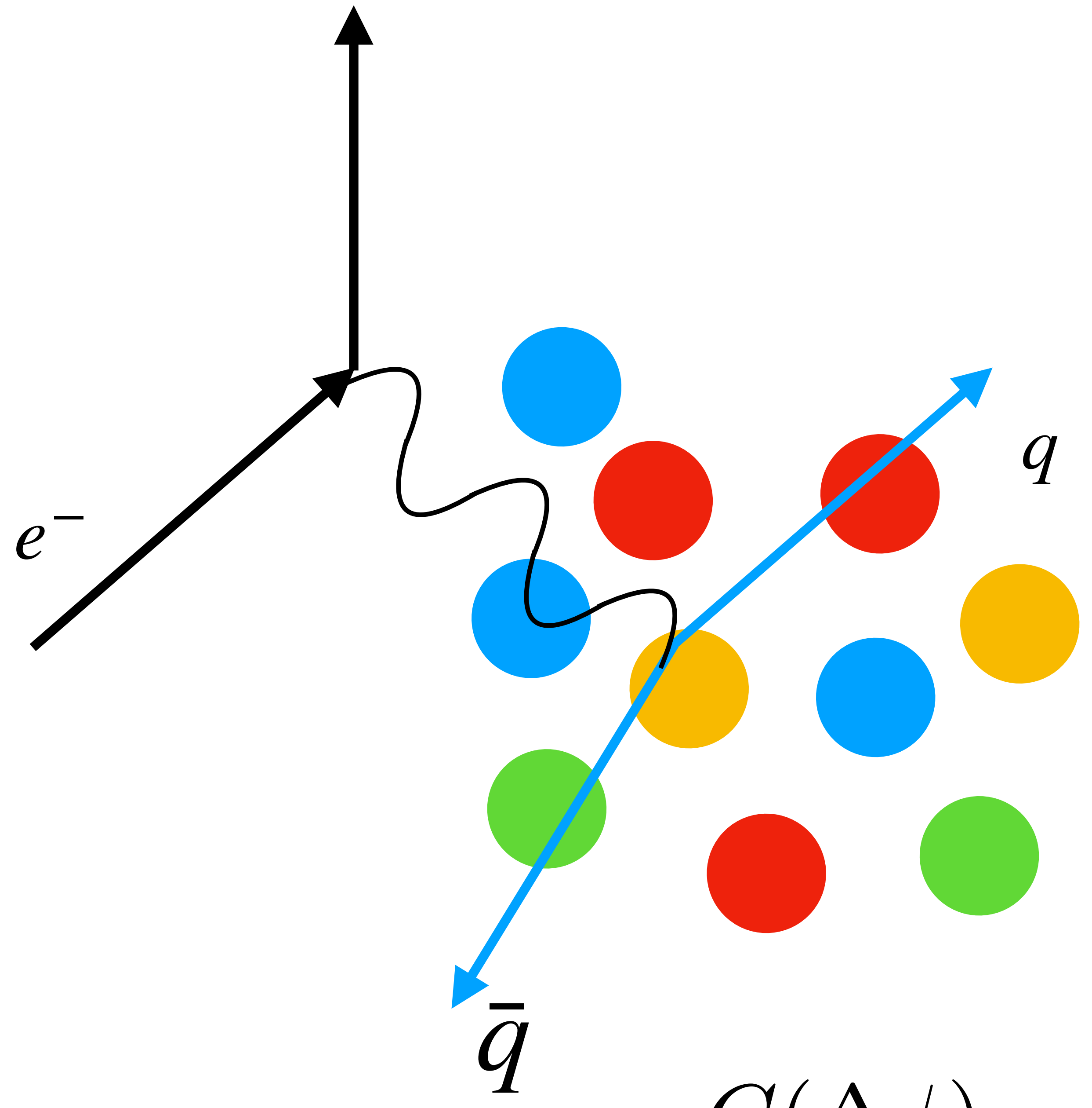


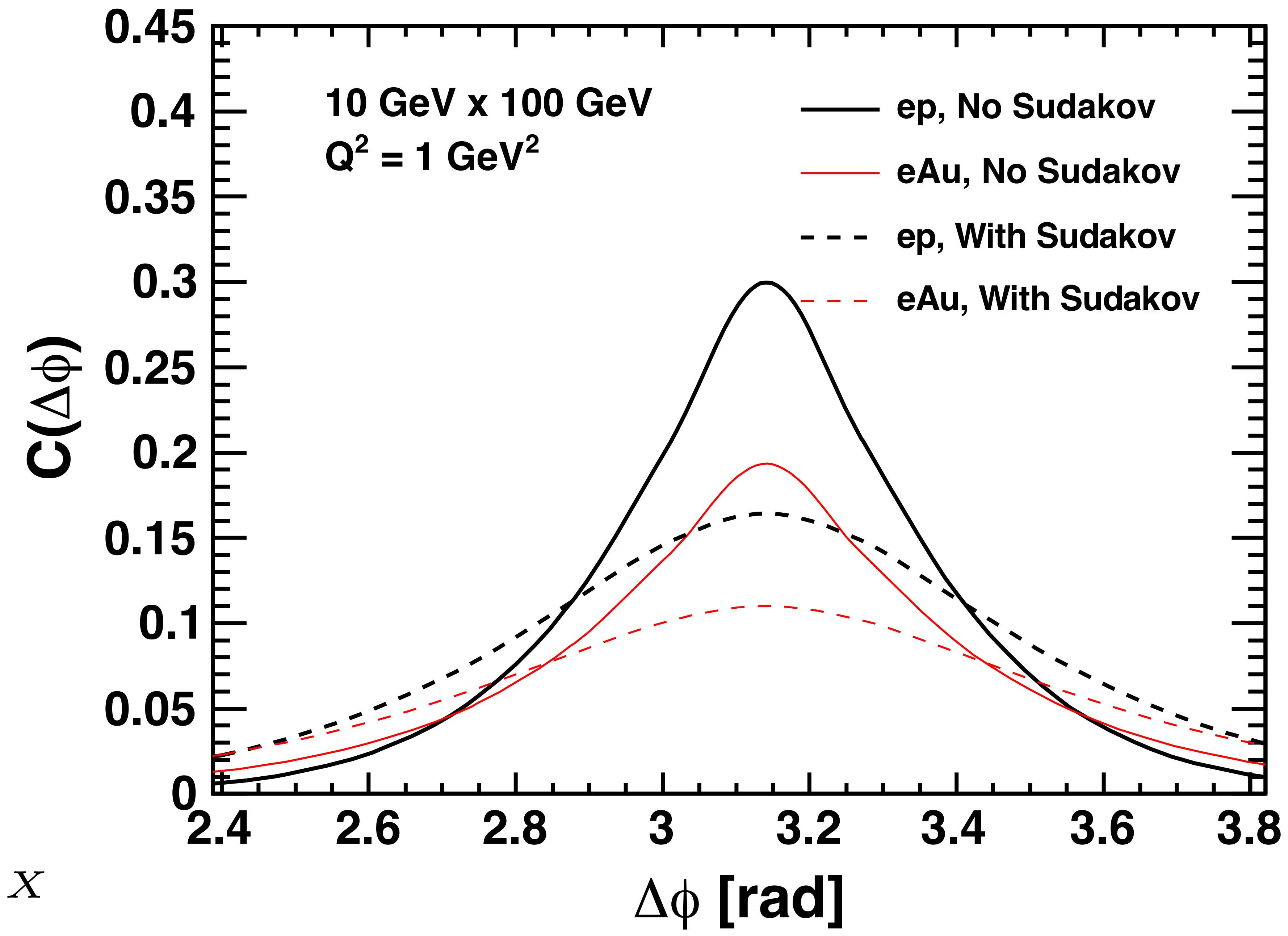


[MH, Padrón, 2011.02640]

Today: di-hadron decorrelations

[Zheng, Aschenauer, Lee, Xiao; 1403.2413]





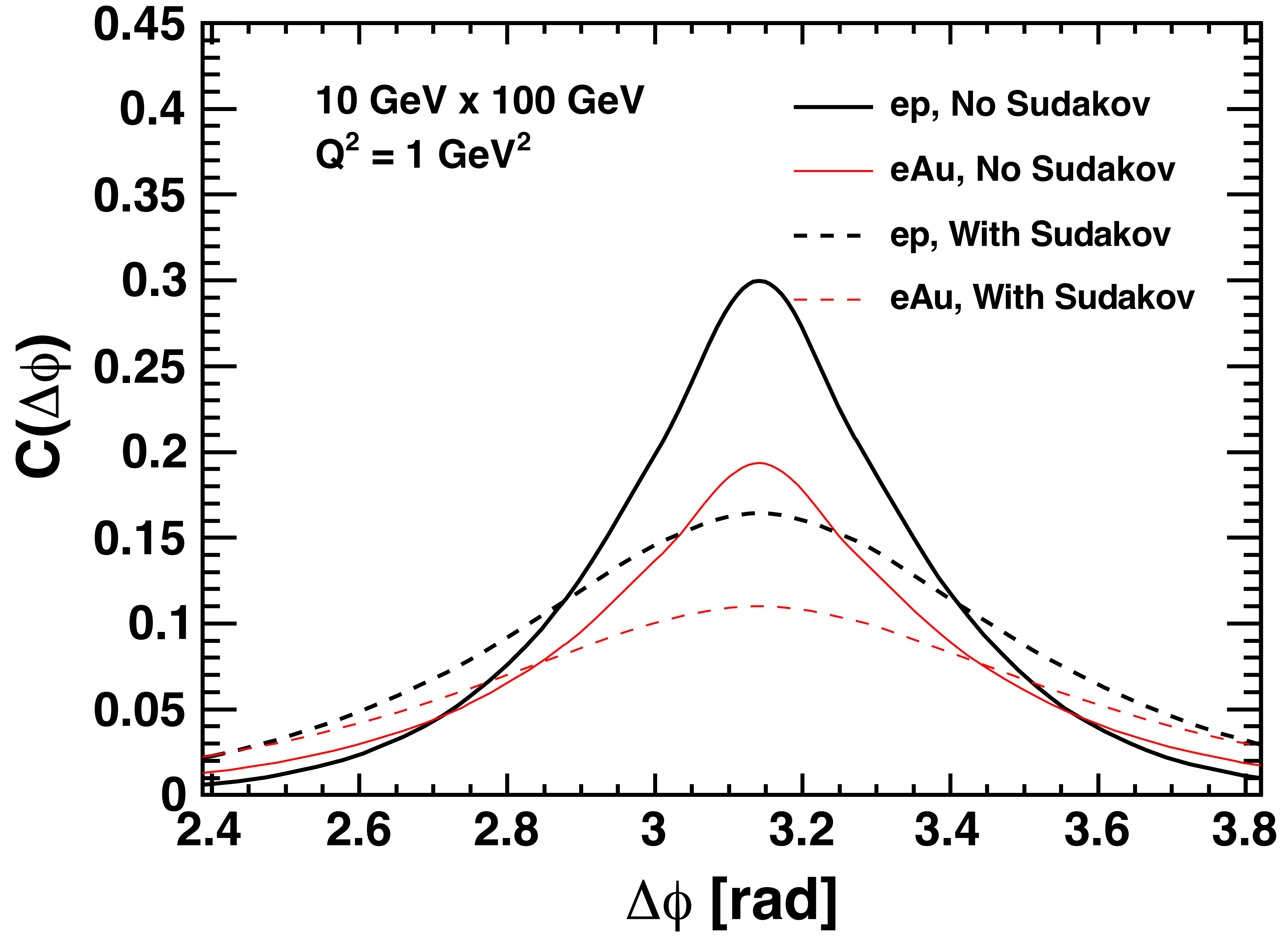
$$C(\Delta\phi) = \frac{1}{\frac{d\sigma_{\text{SIDIS}}^{\gamma^*+A\to h_1+X}}{dz_{h1}}} \frac{d\sigma_{\text{tot}}^{\gamma^*+A\to h_1+h_2+X}}{dz_{h1}dz_{h2}d\Delta\phi}.$$

$$\frac{d\sigma_{\text{tot}}^{\gamma^*+A\to h_1+h_2+X}}{dz_{h1}dz_{h2}d^2p_{h1\perp}d^2p_{h2\perp}} = C\int_{z_{h1}}^{1-z_{h2}} dz_q \frac{z_q(1-z_q)}{z_{h2}^2 z_{h1}^2} d^2p_{1\perp}d^2p_{2\perp}\mathcal{F}(x_g,q_\perp)\mathcal{H}_{\text{tot}}(z_q,k_{1\perp},k_{2\perp})$$
$$\times \sum_q e_q^2 D_q\left(\frac{z_{h1}}{z_q},p_{1\perp}\right)D_{\bar{q}}\left(\frac{z_{h2}}{1-z_q},p_{2\perp}\right),$$

extension to 3 particle correlation within the Color Glass Condensate: [Ayala, MH, Jalilian-Marian, Tejeda-Yeomans; 1604.08526, 1701.07143]

Sudakov form factor:

- the study includes already a first estimate of effects related to the so-called Sudakov form factor
- what is it?



$$\mathcal{F}(x_g, q_\perp) = \frac{1}{2\pi^2}\int d^2r_\perp e^{-iq_\perp r_\perp}\frac{1}{r_\perp^2}[1-\exp(-\frac{1}{4}r_\perp^2 Q_s^2)]\exp[-\frac{\alpha_s N_c}{4\pi}\ln^2\frac{K^2 r_\perp^2}{c_0^2}],$$

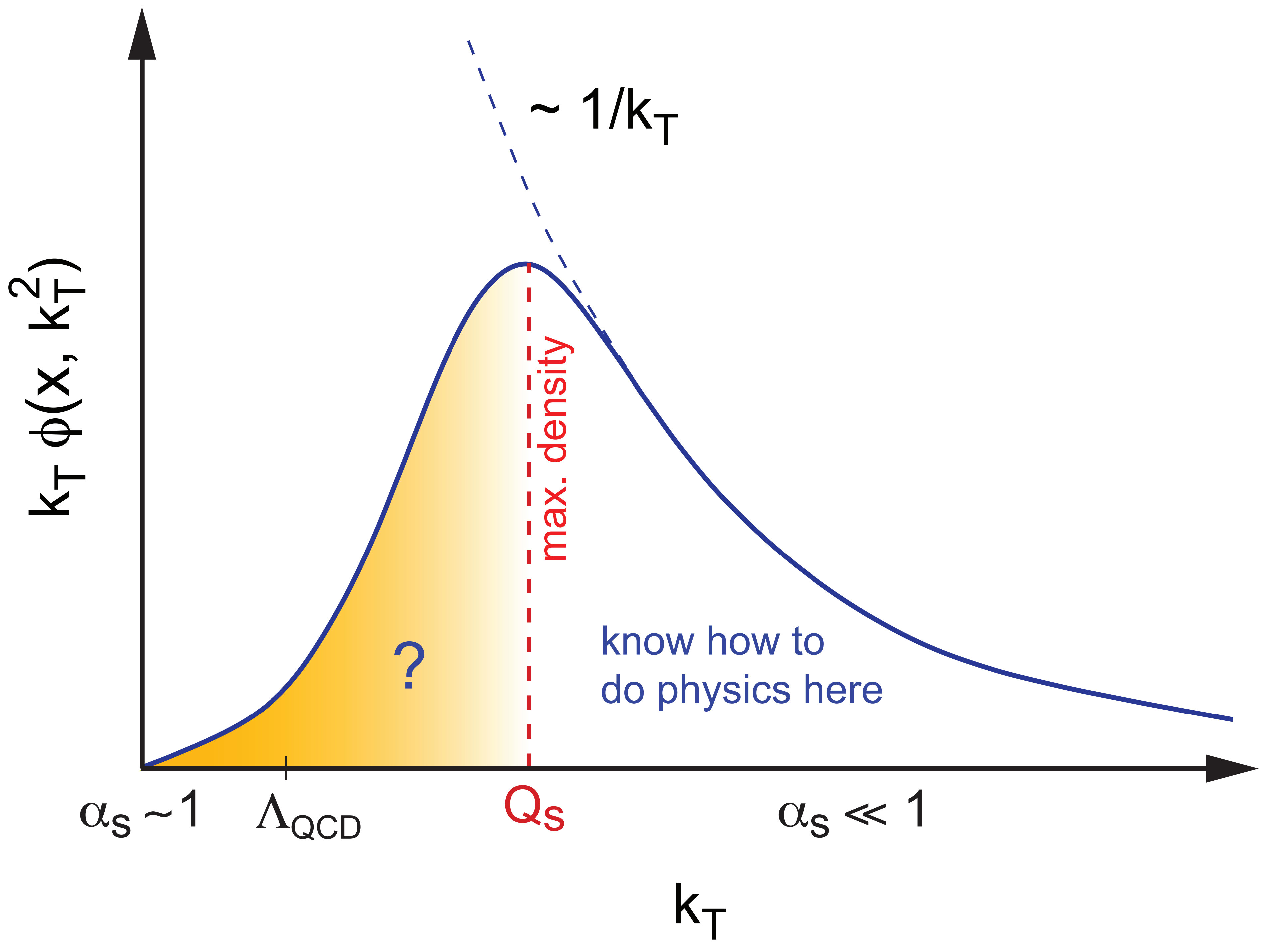
high density   Sudakov

simple model for the transverse momentum dependent gluon distribution used in [Zheng, Aschenauer, Lee, Xiao; 1403.2413]

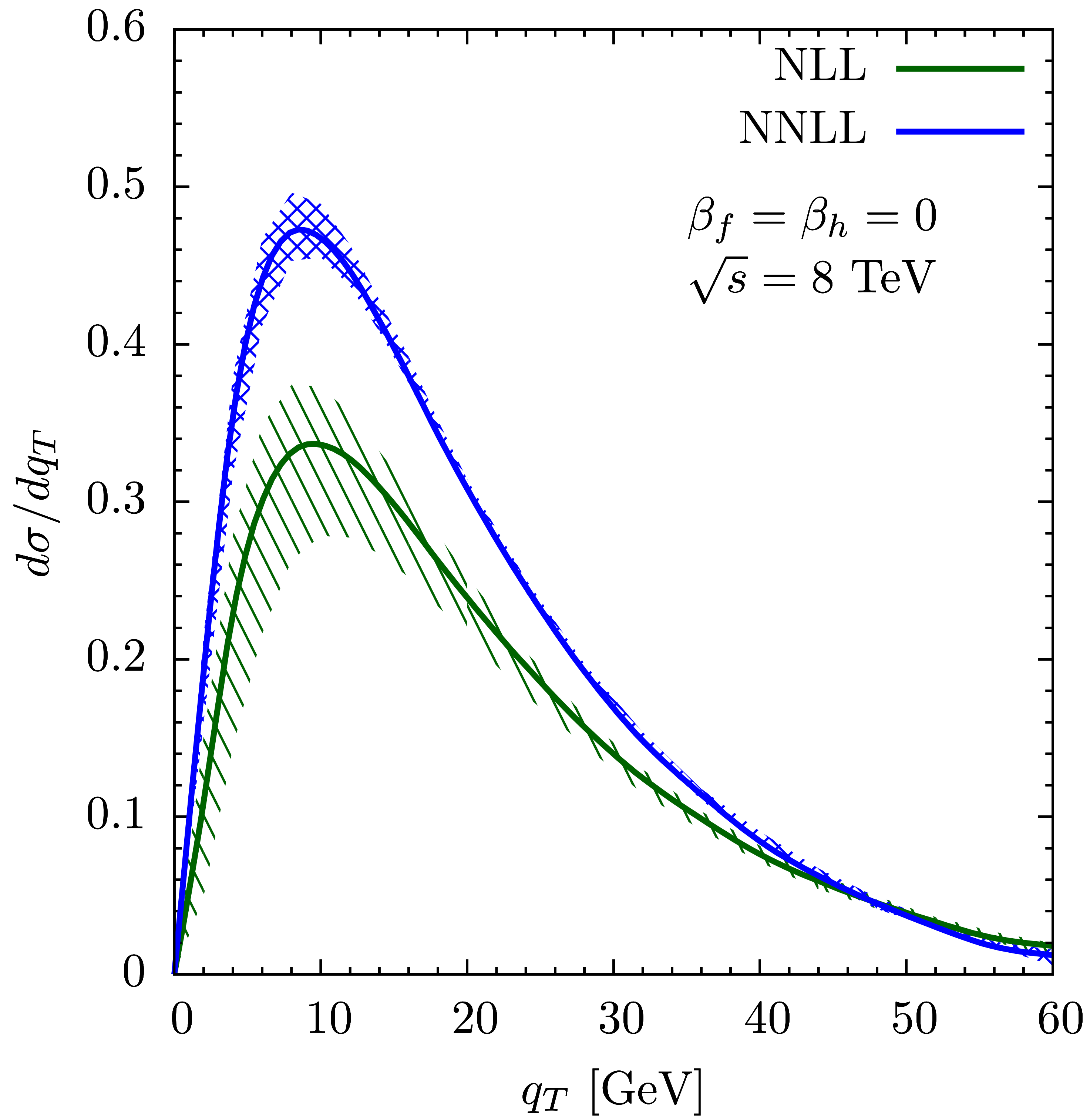
play a somehow similar role at first: crucial difference

- saturation factor depends through Q_s on density
- Sudakov form factor sums up emissions of soft gluons→ does not directly depend on density

To make good phenomenology at the future EIC: need to disentangle both→theory task



schematic picture of a TMD gluon distribution due to saturation/high density effects



TMD distribution of a Higgs boson due to Sudakov(=TMD) resummation;
no saturation

My own little contribution 🙂

- description of high density setup usually based on high energy factorization = factorization of QCD correlators in the limit of high center of mass energies
- care about relatively small (but perturbative pT), no high mass particles etc → in general a good approach, widely used
- previous studies [Xiao, Yuan, Zhou, NPB 921 (2017)] etc. recover renormalization group formulation through matching of Color Glass Condensate calculation scheme to collinear factorization (no saturation; conventional pQCD approach)

$$xG^{(1)}(x, k_\perp, \zeta_c = \mu_F = Q) = -\frac{2}{\alpha_S} \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_s(Q)) e^{-\mathcal{S}_{sud}(Q^2, r_\perp^2)} \times \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp) \ ,$$

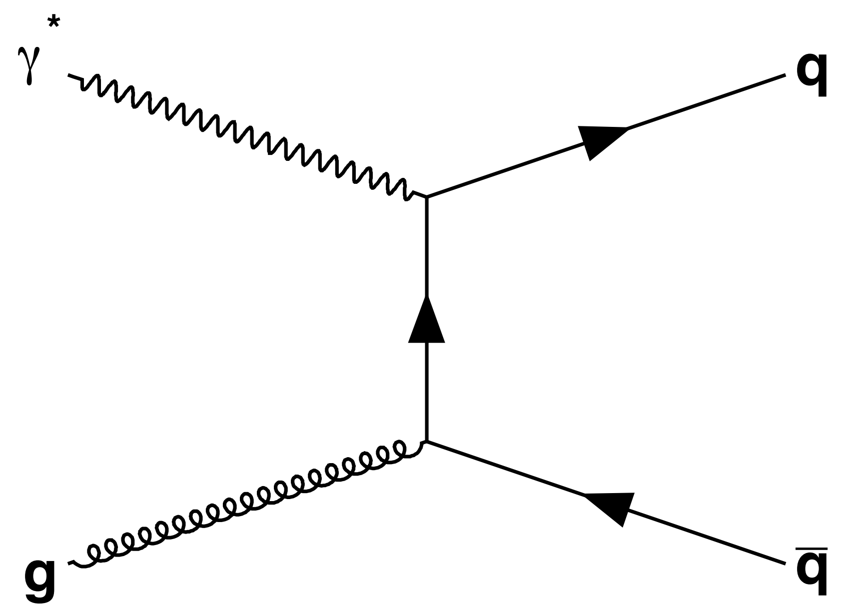
$$\mathcal{S}_{sud} = \int_{c_0^2/r_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B \right]$$

the problem:

- $B \neq 0$ in the collinear approach
- but their CGC calculation yields $B = 0$ at 1-loop

Why coincide?They're fixed by ultraviolet renormalization → universal

Soft-collinear factorization:



matrix element for hard coefficient

- consider event with hard scale M (here: pT of jet or hadron) $p_T \simeq |p_{1,T}| \simeq |p_{2,T}|$
- take formal limit $q_T/M \to 0$: factorization into hard coefficient (here: $q_T =$ the transverse momentum imbalance $|q_T| = |p_{1,T}| - |p_{2,T}|$) and TMD gluon distribution which carries the q_T dependence

$$xG^{(1)}(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{L}_\xi^\dagger \mathcal{L}_0 F^{+i}(0)|P\rangle$$

- possible operator definition of a TMD gluon distribution
- UV divergent→ requires renormalization
- physical reason: we took $M \to \infty$

big advantage: we can study this QCD operator using the renormalization group = determine its anomalous dimension

they are universal = independent of infrared physics what did we do? a different formalism for high energy factorized amplitudes

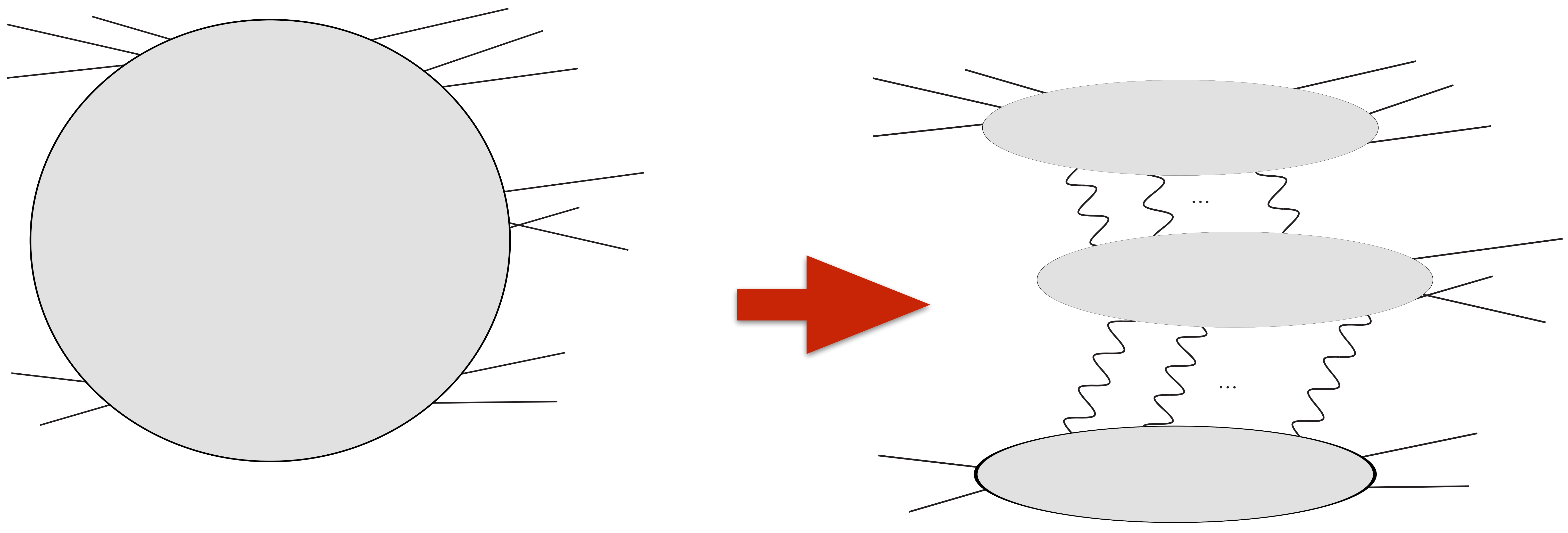
an action formalism for reggeized gluons: Lipatov's high energy effective action

- action for reggeized quarks: [Lipatov,Vyazovsky hep-ph/0009340]
- action for electroweak bosons: [Gomez Bock, MH, Sabio Vera, 2010.03621]

[Lipatov; hep-ph/9502308]

basic idea:

relevant kinematics: Multi-Regge-Kinematics (separated in rapidity & transverse momenta of same order of magnitude)



correlator with regions localized in rapidity, significantly separated from each other

factorize using auxiliary degree of freedom = the reggeized gluon

- idea: factorize QCD amplitudes in the high energy limit through introducing a new kind of field: *the reggeized gluon $A_\pm$* (conventional QCD gluon: v_μ)

  kinematics (strong ordering in light-cone momenta between different sectors):
  $$\partial_+ A_-(x) = 0 = \partial_- A_+(x).$$

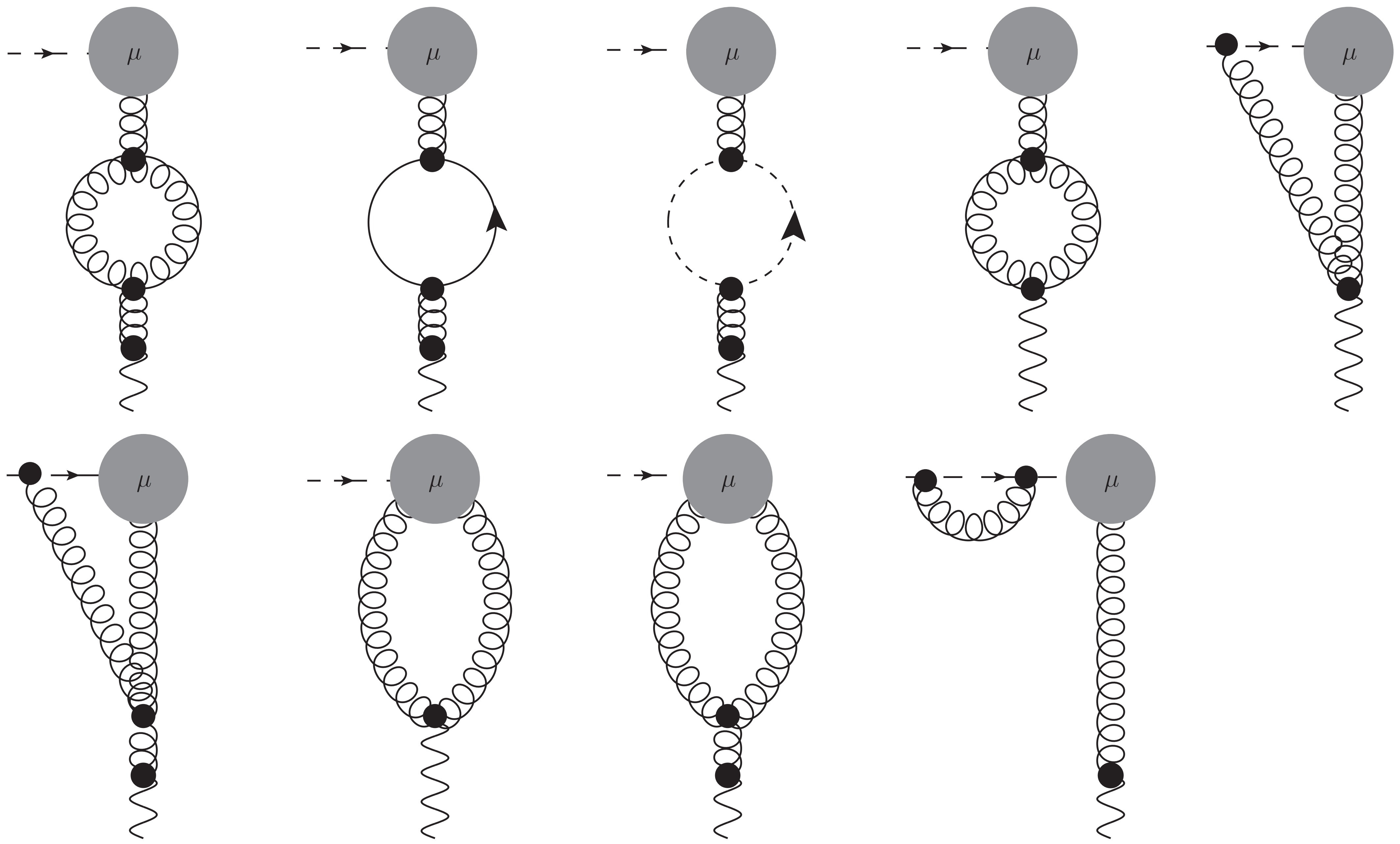
  underlying concept:

  - reggeized gluon globally charged under SU(N$_C$)
  $$A_\pm(x) = -it^a A^a_\pm(x)$$

  - but invariant under local gauge transformation
  $$\delta_{\rm L} v\mu = \frac{1}{g}[D_\mu, \chi_L] \quad \text{vs.} \quad \delta_{\rm L} A_\pm = \frac{1}{g}[A_\pm, \chi_L] = 0$$

  → gauge invariant factorization of QCD correlators calculation is more cumbersome → work in dilute approximation (but within high energy factorization) 😩

- not what we finally want
- but the first step to get eventually the right result also for the high density case



key observation:

- the set of virtual corrections (which carry the UV divergence) is greatly enhanced within Lipatov's effective action
- CGC approach: essentially only the last diagram (modulo details)

add real corrections + soft factor
→ obtain complete 1-loop coefficient and

$$\gamma_G = \frac{d \ln \mathcal{Z}_G}{d \ln \mu} = \frac{\alpha_s}{2\pi}\left[\beta_0 + 2C_A \ln \frac{\mu^2}{(q^-)^2 e^{2y_c}}\right]$$

complete 1-loop anomalous dimension (including the "B" term) [MH, 2107.06203]

Many more interesting details

- obtain matching coefficients of TMD gluon distribution to high energy factorization unintegrated gluon distribution
- unpolarized and linearly polarized TMD gluon distribution differ also due to 1-loop correction, not only due to high density effects (as found at Born level)
- clarification of the relation between Collins-Sopers-Sternman rapidity evolution of soft gluons and Balitsky-Fadin-Kuraev-Lipatov rapidity evolution from high energy factorization etc.
- next step: do all this for high densities (on it, but it's technically tricky ….)

Summing up: -Why is it interesting?

- relates to core questions of phenomenology of a future collider project which will be realized for sure in our lifetime (if you're not too old and your health is good)
- it's Quantum Field Theory at work; combines various non-trivial features
- not covered: TMD distributions are a very rich field by themselves (apart from their relation to gluon saturation) and needed to increase our understanding of the motion of quarks and gluons in a hadron and how spin and other quantities arise due to multi-particle dynamics (maybe Aurore will tell you about that; I hope …)

want know more about it? work on it? get in contact: martin.hentschinski@udlap.mx