Current progress in the muon $g-2$

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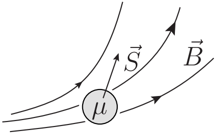
Outline

Section 1

Introduction

Brief introduction to $(g-2)_\mu$: what's that?

- How charged spin particle interacts in classical electromagnetic field $\vec{B}$



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Classical spinning particle $\to g = 1$

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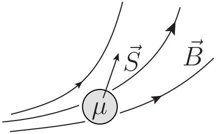

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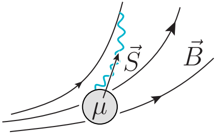

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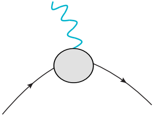

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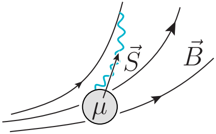

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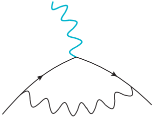

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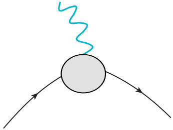

$$\mathcal{M} = -e\left(\overline{u}\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u\right)\tilde{A}_\mu(q)$$

$$g_{e(\mu)} = 2(F_1(0)+F_2(0)) \xrightarrow{Ward} g_{e(\mu)} = 2(1+F_2(0))$$

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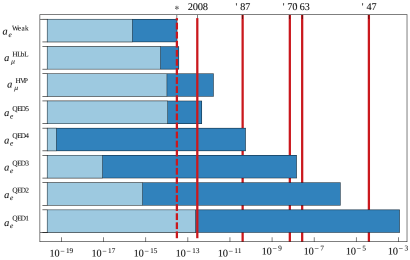
Culmination of QED renormalization (1948)

$$a_{e(\mu)} \equiv \frac{g_{e(\mu)}-2}{2} = F_2(0) = \frac{\alpha}{2\pi} = 0.00116$$

Brief introduction to $(g-2)_\mu$: what's that?

- But measurements just began! And higher loops to come...



- Most recent result

$$a_e^{\rm Ex} = 1159652180.73(28) \times 10^{-12}$$
$$a_e^{\rm Th} = 1159652181.61(23*) \times 10^{-12}$$
$$a_e^{\rm Th\text{-}Ex} = 0.88(36) \times 10^{-12} \; (2.4\sigma)$$

* Previously (72); now α(Cs) 2018

- Future ambitions $\Delta a_e^{\rm Exp} = 3 \times 10^{-14}$

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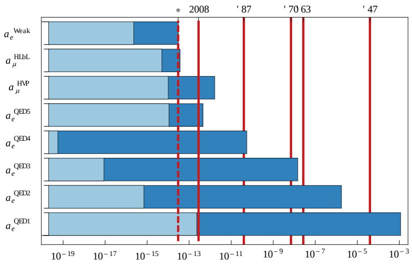

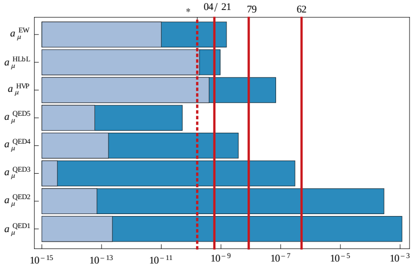

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- More sensitive to heavy physics

$$\delta a_\ell \sim m_\ell^2/M^2 \to m_\mu^2/m_e^2 \sim 10^4$$

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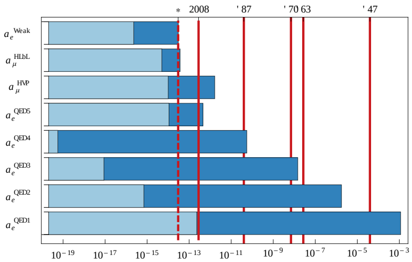

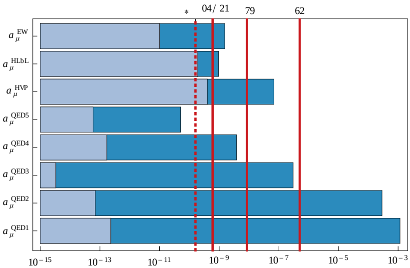

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- More sensitive to heavy physics

$$\delta a_\ell \sim m_\ell^2/M^2 \to m_\mu^2/m_e^2 \sim 10^4$$

- Makes μ more interesting!
- Indeed, there is a tension... but let's give experimentalists their credit first!

How do we measure that?

Brief introduction to $(g-2)_\mu$: how do we measure it?

- Crash course on (spin) precession!
- In classical EM

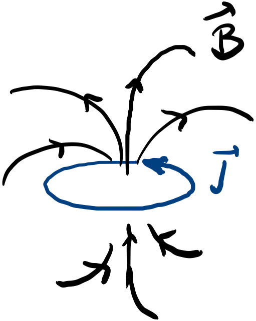
$$\vec{\mu} = \frac{1}{2}\int \vec{r}\times\vec{j} = \frac{Q}{2m}\vec{L} \Rightarrow \frac{eQ}{2m}g\vec{L}$$

- Torque on dipole from a $\vec{B}$ field

$$\vec{\tau} = \vec{\mu}\times\vec{B} = \frac{eQ}{2m}g\vec{L}\times\vec{B}$$

  - Implies spin precession; for $\vec{B}\perp\vec{L}$

$$\omega_p = \frac{\tau}{L} = \frac{-eQ}{2m}gB$$



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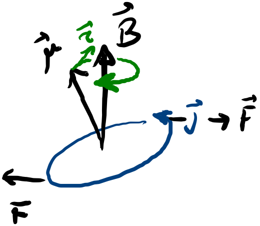

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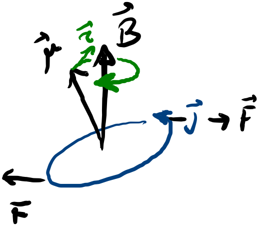


Brief introduction to $(g-2)_\mu$: how do we measure it?

- Crash course on (spin) precession!

- Also in QM: spin along $+\hat{x}$ and $\vec{B}$ along $\hat{z}$

$$\mathcal{H} = -\vec{\mu}\cdot\vec{B}, \quad \hat{\mathcal{H}}\,|\hat{z},\pm\rangle = E_\pm\,|\hat{z},\pm\rangle$$
$$\langle\vec{\mu}\rangle = g\,\tfrac{eQ}{2m}\langle\vec{S}\rangle \Rightarrow E_\pm = \mp\tfrac{e\hbar Q}{4m}gB$$

- Then, time-evolution implies

$$< S_x >= \frac{\hbar}{2}\cos(\frac{2Et}{\hbar}), \quad < S_y >= \frac{\hbar}{2}\sin(-\frac{2Et}{\hbar})$$

- This is, it precesses with $\omega_p = \frac{-eQ}{2m}g$

- Same as classical mechanics

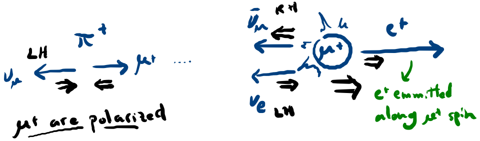
Time evolution:

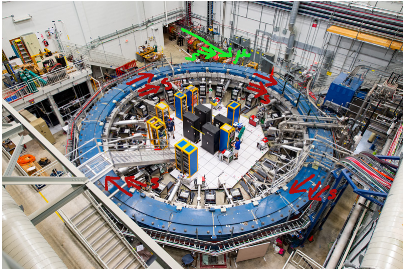
$$|+\hat{x},t\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-iE_+t} \\ e^{-iE_-t}\end{pmatrix}$$

Expectation value:
$\langle S_x\rangle = \frac{\hbar}{2}\langle +\hat{x},t|\,\sigma_x\,|+\hat{x},t\rangle$

Brief introduction to $(g-2)_\mu$: how do we measure it?

- a_μ measured via spin precession
- The short μ^+ lifetime in brief





- Let's boost the μ to the experiment

$$\omega_p = \frac{g_\mu e B}{2 m_\ell} = \frac{g_\mu}{2}\omega_c = \omega_c(1+a_\mu)$$

$$\Rightarrow a_\mu = \frac{\omega_p}{\omega_c} - 1 = \frac{(g-2)_\mu}{2} = F_2(0)$$

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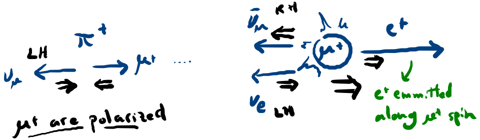

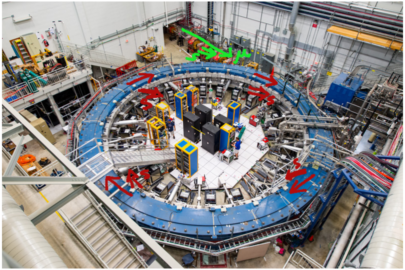

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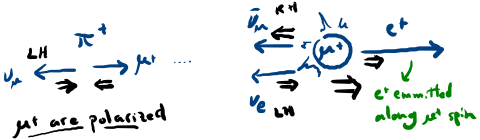

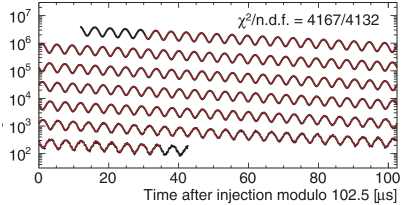

PRL 126, 141801 (2021)

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Brief introduction to $(g-2)_\mu$: Current status

- There is a tension suggesting possibility of New Physics since E821@BNL ('04)

$$
\begin{aligned}
a_\mu^{\mathrm{Ex'04}} &= 116592089(63) \times 10^{-11} \\
a_\mu^{\mathrm{Th'15}} &= 116591807(57) \times 10^{-11} \\
a_\mu^{\mathrm{Th'15\text{-}Ex}} &= -282(85) \times 10^{-11} \ (3.3\sigma)
\end{aligned}
$$

- Motivated experiments at FNL and JPARC with goal $\Delta a_\mu^{\mathrm{Ex}} = 16 \times 10^{-11}$

Brief introduction to $(g-2)_\mu$: Current status

- Motivated th. improvements [μ g-2 theory initiative, Phys.Rep.887 (2020)]

$$a_\mu^{\mathrm{Ex'04}} = 116592091(63) \times 10^{-11}$$
$$a_\mu^{\mathrm{Th}} = 116591810(43) \times 10^{-11}$$
$$a_\mu^{\mathrm{Th\text{-}Ex'04}} = -279(76) \times 10^{-11}\ (3.7\sigma)$$

- Motivated experiments at FNL and JPARC with goal $\Delta a_\mu^{\mathrm{Ex}} = 16 \times 10^{-11}$

Brief introduction to $(g-2)_\mu$: Current status

- Tension confirmed with 1st run at FNL

$$a_\mu^{\rm FNL'21} = 116592040(54) \times 10^{-11}$$
$$a_\mu^{\rm Th} = 116591810(43) \times 10^{-11}$$
$$a_\mu^{\rm Th\text{-}FNL'21} = -230(69) \times 10^{-11} \quad (3.3\sigma)$$

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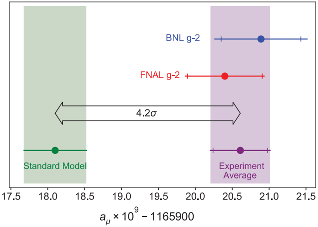

- Motivated experiments at FNL and JPARC with goal $\Delta a_\mu^{\rm Ex} = 16 \times 10^{-11}$

Brief introduction to $(g-2)_\mu$: Current status

- Current result: E821+FNL'21

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$$a_\mu^{\text{Th}} = 116591810(43) \times 10^{-11}$$
$$a_\mu^{\text{Th-Ex}} = -251(59) \times 10^{-11} \; (4.2\sigma)$$

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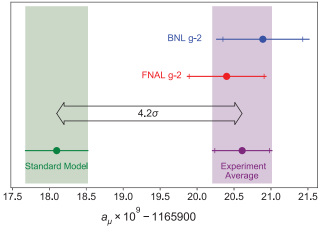

- Motivated experiments at FNL and JPARC with goal $\Delta a_\mu^{\text{Ex}} = 16 \times 10^{-11}$
- Rapidly changing and active field over past years

Brief introduction to $(g-2)_\mu$: Current status

- Imagine a theoretical paradise...

$$a_\mu^{\rm FNL} = 116592061(16) \times 10^{-11}$$
$$a_\mu^{\rm Th} = 116591810(0) \times 10^{-11}$$
$$a_\mu^{\rm Th\text{-}FNL} = -251(16) \times 10^{-11} \; (16\sigma)$$

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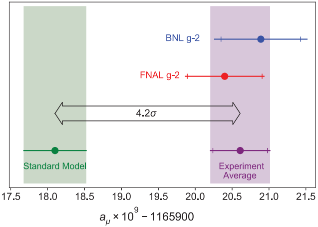

- FNL analysing run2+3 (factor of 2 improvement); run4 finished; 5 in future
- JPARC data taking in '25; 1st results in '27
- Soon will be a theorists (+exp) business again; Exciting times ahead!

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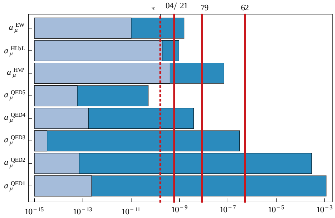

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- Next slides will be all about discussing (hadronic) uncertainties

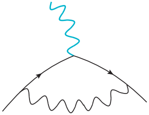
Section 2

Theoretical calculation I: QED part

QED Contributions

- 1-Loop: Schwinger term '48 (1 diagram) - universal (pure number)

$$a_\mu^{\mathrm{QED1}} = 116140973.321(23) \times 10^{-11}$$



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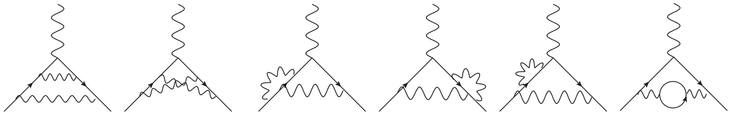
$$a_\mu^{\rm QED1} = 116140973.321(23) \times 10^{-11}$$

- 2-Loops: Petermann and Sommerfield '56; analytic (9 diagrams)

$$a_\mu^{\rm QED2} = 413217.626(7) \times 10^{-11}$$



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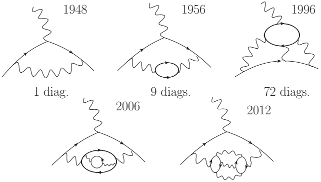
- 3-Loops: Laporta and Remidi '96; analytic (72 diagrams)
$$a_\mu^{\mathrm{QED}3} = 30141.9023(3) \times 10^{-11}$$



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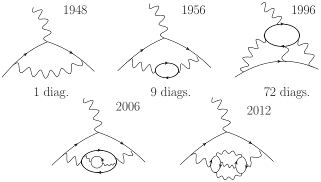
- 4-Loops: Kinoshita et al '06 numeric (891 diagrams) checks in '16,'17
$$a_\mu^{\mathrm{QED4}} = 381.00(2) \times 10^{-11}$$



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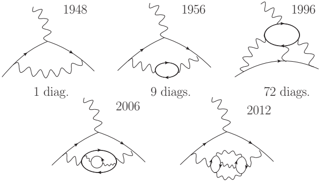
- 5-Loops: Kinoshita et al '12 (12672 diagrams) +checks
$$a_\mu^{\mathrm{QED5}} = 5.078(6) \times 10^{-11}$$



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$$a_\mu^{\mathrm{QED5}} = 5.078(6) \times 10^{-11}$$

- QED up to 5th order +6th order error estimate (Aoyama '19)

$$a_\mu^{\mathrm{QED}} = 116584718.931(30) \times 10^{-11}$$

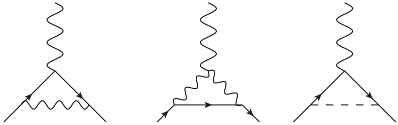
Section 3

Theoretical calculation II: EW part

EW Contributions

- 1-Loop: Jackiw-Weinberg & Bars-Yoshimura & Fujikawa-Lee-Sanda(R_ξ) '72

$$a_\mu^{\mathrm{EW;LO}} = 194.80(1) \times 10^{-11} \qquad \text{Stöckinger et al'13 with } m_H$$



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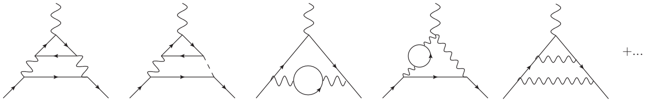
- 2-Loop: Czarnecki et al'95 & Knecht et al'02 & Stöckinger et al'13

$$a_\mu^{\mathrm{EW;NLO}} = -41.2(1.0) \times 10^{-11}$$



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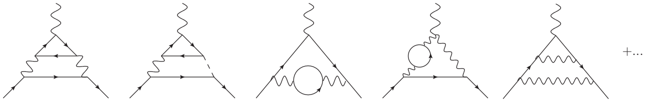
- 2-Loop: Czarnecki et al'95 & Knecht et al'02 & Stöckinger et al'13

$$a_\mu^{\mathrm{EW;NLO}} = -41.2(1.0) \times 10^{-11}$$

- Including NNLO error estimate (Stöckinger et al'13)

$$a_\mu^{\mathrm{EW}} = 153.6(1.0) \times 10^{-11}$$

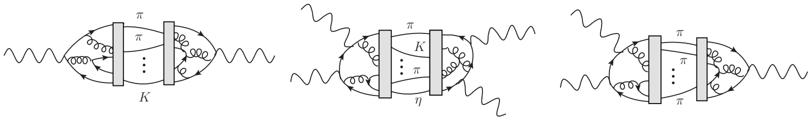


Section 4

Theoretical calculation III: Hadronic part

Hadronic Contributions

- QCD is a non-perturbative confining theory
- Perturbative calculations valid at short distances; otherwise hadrons!



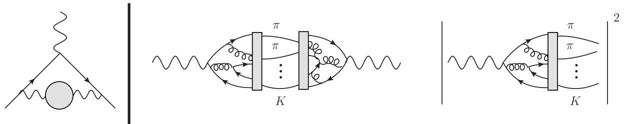
- Certainly the case at muon scales ($m_\mu \sim m_\pi$)
- Indeed (believe me) it is space-like (mostly) low-energies that matter for a_μ
- To convince you how bad pQCD: HVP in units of 10^{-11}

$$a_\mu^{\rm HVP}|_{m_q^{PDG}} = 223366, \quad a_\mu^{\rm HVP}|_{m=100 \text{ MeV}} = 5876, \quad a_\mu^{\rm HVP} = 6933,$$

- Hopefully convinces you not an expansion in α_s!
- Alternative techniques to deal with QCD non-pert. required!
- Dedicated workshops '13,'14,'16; (g-2) theory initiative '17,'18,'19,'21

Hadronic Contributions I: HVP

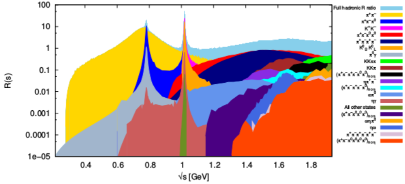
- Nature has solved QCD; use via the optical and Cauchy's th. to get $\hat{\Pi}(-Q^2)$



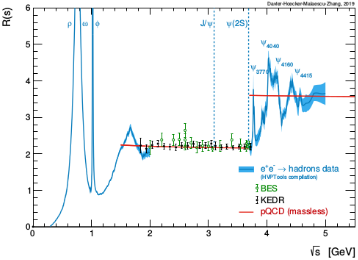
$$\mathrm{Im}\,\hat{\Pi}(q^2) \quad \sim \quad \sigma(e^+e^- \to \mathrm{hadrons})$$

$$\int d^4x e^{iq\cdot x} \langle 0|\, T\{j^\mu(x) j^\nu(0)\}\, |0\rangle = i(q^2 g^{\mu\nu} - q^\mu q^\nu)\hat{\Pi}(q^2) \leftarrow \hat{\Pi}(0)$$

- Oversimplifying: precise measurements for $e^+e^- \to \mathrm{hadrons}$ or the R-ratio



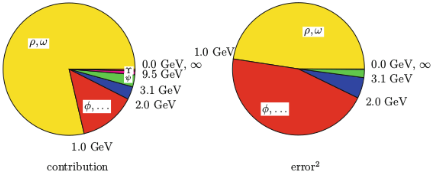
(a) The hadronic R-ratio.



Figs in KNT'18/DHMZ'19 (left/right)

Hadronic Contributions I: HVP

- Include over 27 channels with up to 6 mesons; dominated by ρ/ω (FJ Fig.)



- Data driven | Data+resonance profile | BHLS Model $(10^{-11}$ units)

DHMZ19 : 6940(40) KNT19 : 6928(24) | FJ17 : 6881(41) | BDJ19 : 6871(30)

- Analytic constraints on $a_\mu^{\mathrm{HVP}}[\pi\pi]|_{\leq 1\ \mathrm{GeV}}$, $a_\mu^{\mathrm{HVP}}[3\pi]|_{\leq 1.8\ \mathrm{GeV}}$ (CHHKS)
- Also $\tau^\pm \to \pi^\pm\pi^0$ data with isospin corrections [lattice might help]

Hadronic Contributions I: HVP

- Devil is in the details! Interpolation, errors,... non-trivial below 1% precision!

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Tab. 5 from Phys.Rep. 887 (2020)

- Data driven | Data+resonance profile | BHLS Model (10^{-11} units)

DHMZ19 : 6940(40) KNT19 : 6928(24) | FJ17 : 6881(41) | BDJ19 : 6871(30)

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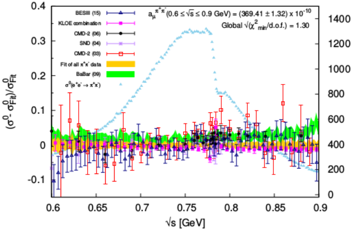
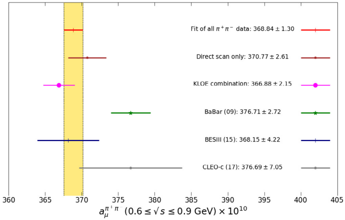
- Also $\tau^\pm \to \pi^\pm\pi^0$ data with isospin corrections [lattice might help]

Hadronic Contributions I: HVP

- BaBar vs. KLOE discrepancy $\Rightarrow$ exp. programme (CMD-3, BES-III, Belle II)





Figs. from KNT19 (left) and Phys.Rep. 887 (2020) (right).

- Data driven | Data+resonance profile | BHLS Model $(10^{-11}$ units)

$\mathrm{DHMZ19}: 6940(40)$ $\mathrm{KNT19}: 6928(24)$ | $\mathrm{FJ17}: 6881(41)$ | $\mathrm{BDJ19}: 6871(30)$

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- Also $\tau^\pm \to \pi^\pm\pi^0$ data with isospin corrections [lattice might help]

Hadronic Contributions I: HVP

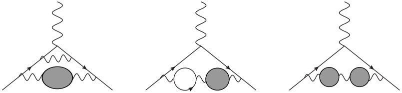
- Data driven | Data+resonance profile | BHLS Model   (10^{-11} units)

DHMZ19 : 6940(40)   KNT19 : 6928(24) | FJ17 : 6881(41) | BDJ19 : 6871(30)

- In WP merging, DHMZ19+KNT19 (avoid models) + analytic constraints

$$a_\mu^{\mathrm{HVP,LO}} = 6931(40) \times 10^{-11}$$

- Finally, higher orders (HO) corrections need be included



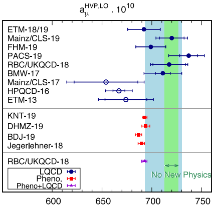
KNT19+Kurz14 (10^{-11} units)

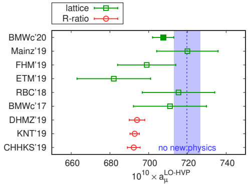
$$a_\mu^{\mathrm{HVP;HO}} = -98.3(0.7) + 12.4(0.1)$$

$$a_\mu^{\mathrm{HVP}} = 6845(40) \times 10^{-11}$$

Hadronic Contributions I: HVP

- Also lattice (euclidean) QCD getting close to precision needs [$\mathcal{O}(1\%)$ now]
- Note at this level requires SIB and QED $\to$ state of the art

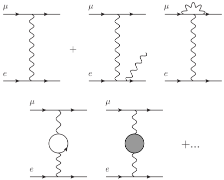


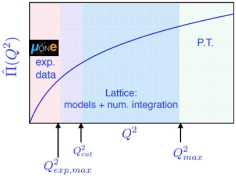


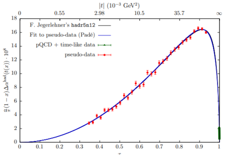
- Recent BMW20 result! Need more lattice Colls. there; Care with EWPO!

$$\text{BMW20:}7075(55) \quad \text{vs.} \quad \text{DR:}6931(28)_{exp}(7)_{QCD}(28)_{BaBar-KLOE}[40]$$

- MUonE Coll: measure $\hat{\Pi}(Q^2)$ at low Q^2 (Figs. from Marinkovic/Calame@Seattle'19)

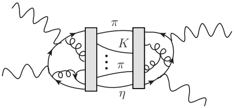






Hadronic Contributions II: HLbL

- For low-energies, QCD non-perturbative



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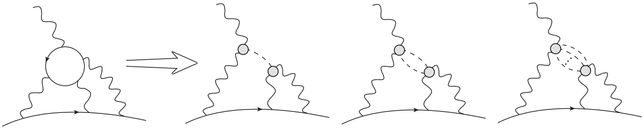

- Direct connection to exp not possible

From 1 (HVP) $\Pi^{\mu\nu}_{HVP} \longrightarrow$ 9 (HLbL) $\Pi^{\mu\nu\rho\sigma}_{HLbL}$ scalar functions
From 1 scale q^2 (HVP) $\longrightarrow$ 6 $\{q_i^2, s, t\}$ (HLbL) scales $\longrightarrow$ multiscale hard/soft

Hadronic Contributions II: HLbL

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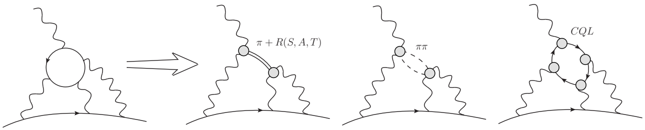

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- First EdR'94, guidance(organization scheme) from EFTs: ChPT+large-N_c

Reduces all to the relevant $\gamma^*\gamma^* \to R$ form factors $\to$ Syst. improvement?

Hadronic Contributions II: HLbL

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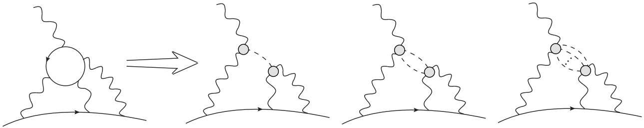

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Reduces all to the relevant $\gamma^*\gamma^* \to R$ form factors $\to$ Syst. improvement?

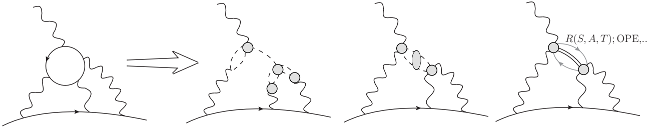
- Recently Disp. Rel for describing resonances

Dominant modes π, η, η' (already known); resonances in $\pi\pi$ rescattering
Higher multiplicity assumed irrelveant unless resonant
Multiscale need modelling $\Pi^{\mu\nu\rho\sigma}_{HLbL}(Q^2, Q^2, q^2, 0), \Pi^{\mu\nu\rho\sigma}_{HLbL}(Q^2, Q^2, Q^2, 0)$

- Let's see the current status!

Hadronic Contributions II: HLbL

- For low-energies, QCD non-perturbative



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- Let's see the current status!

Hadronic Contributions II: HLbL

- Pseudoscalar-pole contributions ✓

$a_\mu^\pi = 63.0(2.4)_{\mathrm{DR'18}},\ 63.6(2.7)_{\mathrm{CA'17}},\ 62.6(1.3)_{\mathrm{DSE\text{-}Gießen'19}},\ 61.4(2.1)_{\mathrm{DSE\text{-}Mex'19}}, 62.3(2.3)_{\mathrm{Latt}}$

Hadronic Contributions II: HLbL

- Pseudoscalar-pole contributions ✓

$$a_\mu^\eta = 16.3(1.4)_{\text{CA'17}},\ 15.8(1.1)_{\text{DSE-Gießen'19}},\ 14.7(1.9)_{\text{DSE-Mex'19}}$$

Hadronic Contributions II: HLbL

- Pseudoscalar-pole contributions ✓

$$a_\mu^{\eta'} = 14.5(1.9)_{\text{CA'17}},\ 13.3(0.8)_{\text{DSE-Gießen'19}},\ 13.6(0.8)_{\text{DSE-Mex'19}}$$

Hadronic Contributions II: HLbL

- Pseudoscalar-pole contributions ✓

$$a_\mu^{\text{PS-poles}} = 93.8(4.0)_{\text{WP}}$$

Hadronic Contributions II: HLbL

- Pseudoscalar-pole contributions ✓

$$a_\mu^{\text{PS-poles}} = 93.8(4.0)_{\text{WP}}$$

- $\pi\pi$ ✓and KK box contributions (no rescattering); $\pi\eta$ under study (DVdH)

$$a_\mu^{\pi\pi;\pi\text{-box}} = -15.9(2)_{\text{DR'17}}, -15.7(4)_{\text{DSE-Gießen'19}}$$
$$a_\mu^{KK\text{-box}} = -0.5_{\text{VMD}}, -0.7_{\text{DSE-Gießen'19}}$$

Hadronic Contributions II: HLbL

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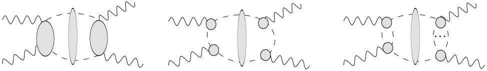
$$a_\mu^{KK\text{-box}} = -0.5_{\text{VMD}}, -0.7_{\text{DSE-Gießen'19}}$$

- Scalar $\pi\pi$ ✓ [KK ✗] [$\sim f_0(500)$; agreement with res. estimates]



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$$a_\mu^{\pi\pi;\pi\text{-LHC}} = -8(1)$$

Hadronic Contributions II: HLbL

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- Scalar $\pi\pi$ ✓ [KK ✗] [$\sim f_0(500)$; agreement with res. estimates]

$$a_\mu^{\pi\pi;\pi\text{-LHC}} = -8(1)$$

- Heavier S ($\notin \pi\pi$) DR ✗ $\rightarrow$ res. estimates

$$a_\mu^S = -\{3.1(1.8), 0.9(2)\}_{\text{PVdH}}, -\{2.2(^{+3.2}_{-0.7}), 1.0(^{+2.0}_{-0.4})\}_{\text{KNRR'18}}$$

Hadronic Contributions II: HLbL

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- Scalar $\pi\pi$ ✓ [KK ✗] [$\sim f_0(500)$; agreement with res. estimates]

$$a_\mu^{\pi\pi;\pi\text{-LHC}} = -8(1)$$

- Heavier S ($\not\subseteq \pi\pi$) DR ✗ $\rightarrow$ res. estimates

$$a_\mu^S = -\{3.1(1.8), 0.9(2)\}_{\text{PVdH}}, -\{2.2(^{+3.2}_{-0.7}), 1.0(^{+2.0}_{-0.4})\}_{\text{KNRR'18}}$$

- Tensor (D-wave) ($\pi\pi$ DR not ready but feasible) all are resonance estimates

$$a_\mu^T = 0.9(0.1)_{\text{DVdH'17}}$$

Hadronic Contributions II: HLbL

- Axials (would-be $3\pi, \eta 2\pi, 4\pi$) from DR ✗; res. estimates (work required!)

$$a_\mu^A = 6.4(2.0)_{\mathrm{PVdH'14}}, 7.6(2.7)_{\mathrm{FJ'17}}, 0.8(^{+3.5}_{-0.1})_{\mathrm{R\chi T'19}}, \\ (22.5 \div 40.6)_{\mathrm{Hol;LR}}, 28(2)_{\mathrm{Hol;CCAGI}}$$

Hadronic Contributions II: HLbL

- Axials (would-be $3\pi, \eta 2\pi, 4\pi$) from DR ✗; res. estimates (work required!)

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- Short distances: $(Q^2_{1,2} \gg Q^2_3)$ DR ✗; models. $(Q^2_{1,2,3} \gg \Lambda^2_{\rm QCD})$ ✓Q-loop (B'19)

$$a_\mu^{\rm SD} = 13(6)_{\rm L;Regge} + 4.6_{\rm T;QLoop}, (14 \div 23)_{\rm Hol};$$

Non-trivial matching (again, devil is in the details)!

Hadronic Contributions II: HLbL

- Axials (would-be $3\pi, \eta 2\pi, 4\pi$) from DR ✗; res. estimates (work required!)

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$$a_\mu^{\rm SD} = 13(6)_{\rm L;Regge} + 4.6_{\rm T;QLoop}, (14 \div 23)_{\rm Hol};$$

Non-trivial matching (again, devil is in the details)!

- Estimate (so far) in WP

$$a_\mu^{\text{PS-poles}} = 93.8(4.0), \quad a_\mu^{\pi\pi+KK\text{-box}} = -16.4(2), \quad a_\mu^{\pi\pi;S} = -8(1), \\ a_\mu^{S+T} = -1(3), \quad a_\mu^A = 6(6), \quad a_\mu^{{\rm SD}+c\bar{c}} = 15(10) + 3(1)$$

Hadronic Contributions II: HLbL

- Axials (would-be $3\pi, \eta 2\pi, 4\pi$) from DR ✗; res. estimates (work required!)

$$a_\mu^A = 6.4(2.0)_{\mathrm{PVdH'14}}, 7.6(2.7)_{\mathrm{FJ'17}}, 0.8(^{+3.5}_{-0.1})_{\mathrm{R\chi T'19}}, \\ (22.5 \div 40.6)_{\mathrm{Hol;LR}}, 28(2)_{\mathrm{Hol;CCAGI}}$$

- Short distances: $(Q^2_{1,2} \gg Q^2_3)$ DR ✗; models. $(Q^2_{1,2,3} \gg \Lambda^2_{\mathrm{QCD}})$ ✓ Q-loop (B'19)

$$a_\mu^{\mathrm{SD}} = 13(6)_{\mathrm{L;Regge}} + 4.6_{\mathrm{T;QLoop}}, (14 \div 23)_{\mathrm{Hol}};$$

Non-trivial matching (again, devil is in the details)!

- Estimate (so far) in WP

$$a_\mu^{\mathrm{HLbL}} = 92(19) \times 10^{-11}$$

Hadronic Contributions II: HLbL

- Axials (would-be $3\pi, \eta 2\pi, 4\pi$) from DR ✗; res. estimates (work required!)

$$a_\mu^A = 6.4(2.0)_{\rm PVdH'14}, 7.6(2.7)_{\rm FJ'17}, 0.8(^{+3.5}_{-0.1})_{\rm R\chi T'19}, (22.5 \div 40.6)_{\rm Hol;LR}, 28(2)_{\rm Hol;CCAGI}$$

- Short distances: $(Q^2_{1,2} \gg Q^2_3)$ DR ✗; models. $(Q^2_{1,2,3} \gg \Lambda^2_{\rm QCD})$ ✓Q-loop (B'19)

$$a_\mu^{\rm SD} = 13(6)_{\rm L;Regge} + 4.6_{\rm T;QLoop}, (14 \div 23)_{\rm Hol};$$

Non-trivial matching (again, devil is in the details)!

- Estimate (so far) in WP

$$a_\mu^{\rm HLbL} = 92(19) \times 10^{-11}$$

Updates since WP

- $\pi\pi$ with KK rescattering effects & new heavy S: $a_\mu^{\pi\pi;S+S} = -9(1)$ DHS'20
- SD (+axials) K'20(+MRS'20); SD and matching LP'20
- Improve pQCD Q-loop and extend to lowest possible Q^2
- Remarkable lattice improvements!

$a_\mu^{\rm HLbL} = 79(30)(18)_{\rm RBC\text{-}UKQCD}, 107(15)_{\rm Mainz}$

Section 5

Summary

Theoretical Summary

- Controlled QED and EW (error irrelevant)

$$a_\mu^{\rm QED} = 116584718.93(30) \times 10^{-11}, \qquad a_\mu^{\rm EW} = 153.6(1.0) \times 10^{-11}$$

- Hadronic are the bottleneck; devil in details; lattice progressing; (10^{-11} units)

$$a_\mu^{\rm HVP;LO} = 6931(40)_{\rm DR} \text{ vs } 7075(55)_{\rm BMW20} \qquad a_\mu^{\rm HLbL} = 92(19)_{ph} \text{ vs } 107(15)_{\rm Mainz}$$

- Discrepancy persists, and more data to come soon! (10^{-11} units)

$$a_\mu^{\rm Ex} = 116592061(41)\ a_\mu^{\rm Th} = 116591810(43)\ a_\mu^{\rm Th\text{-}Ex} = -251(59)\ (4\sigma)$$

- Let's see what nature has prepared for us

- Currently the theoretical a_μ estimate is all about hadronic physics. Exciting times ahead!