Physics of the tau lepton

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Leptons

$$L_\ell = \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L , \quad \ell^-_R$$

$$\ell = e, \mu, \tau$$

$$N_{(\nu_\ell, \ell^-)} = +1$$

$$N_{(\overline{\nu}_\ell, \ell^+)} = -1$$

$$\Delta N_\ell = 0$$

$$\mathrm{Br}(\mu^+ \to e^+ \gamma) < 4.2 \times 10^{-13}$$

$$90\%\mathrm{CL}, \ \mathrm{MEG}$$ [1]

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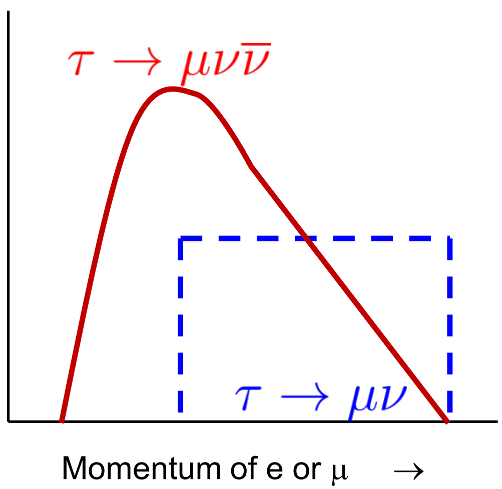
$\tau^- \to e^- \overline{\nu}_e \nu_\tau$



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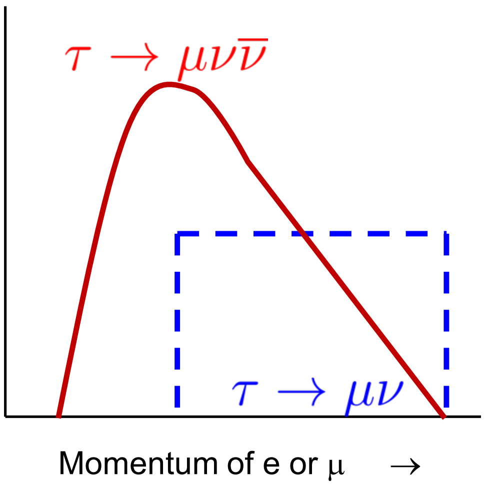

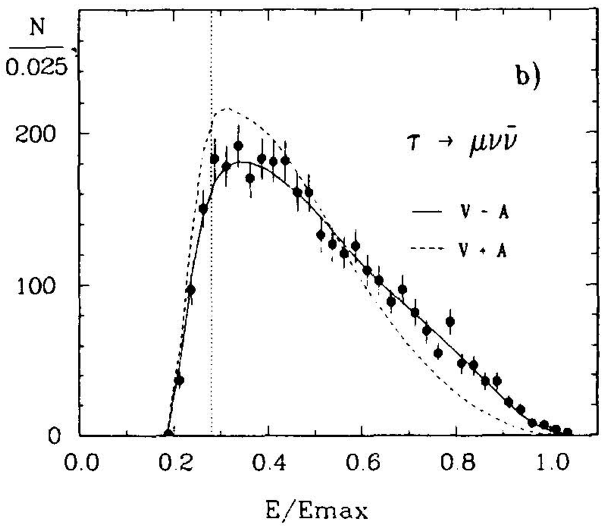

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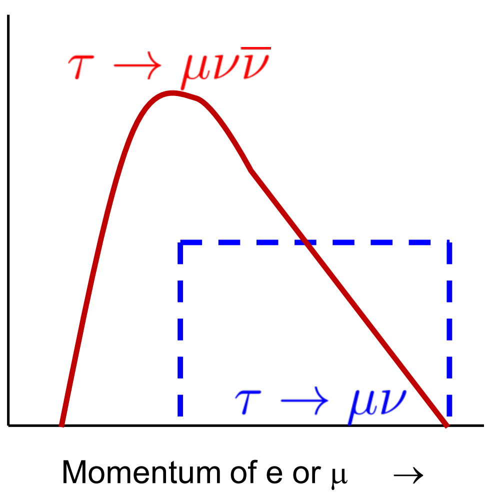

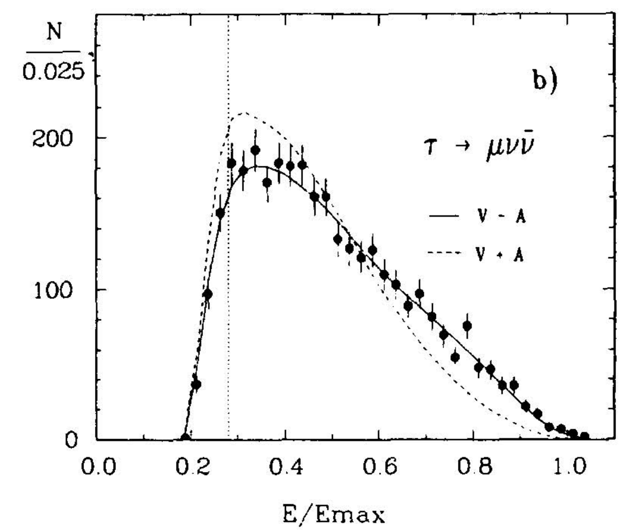

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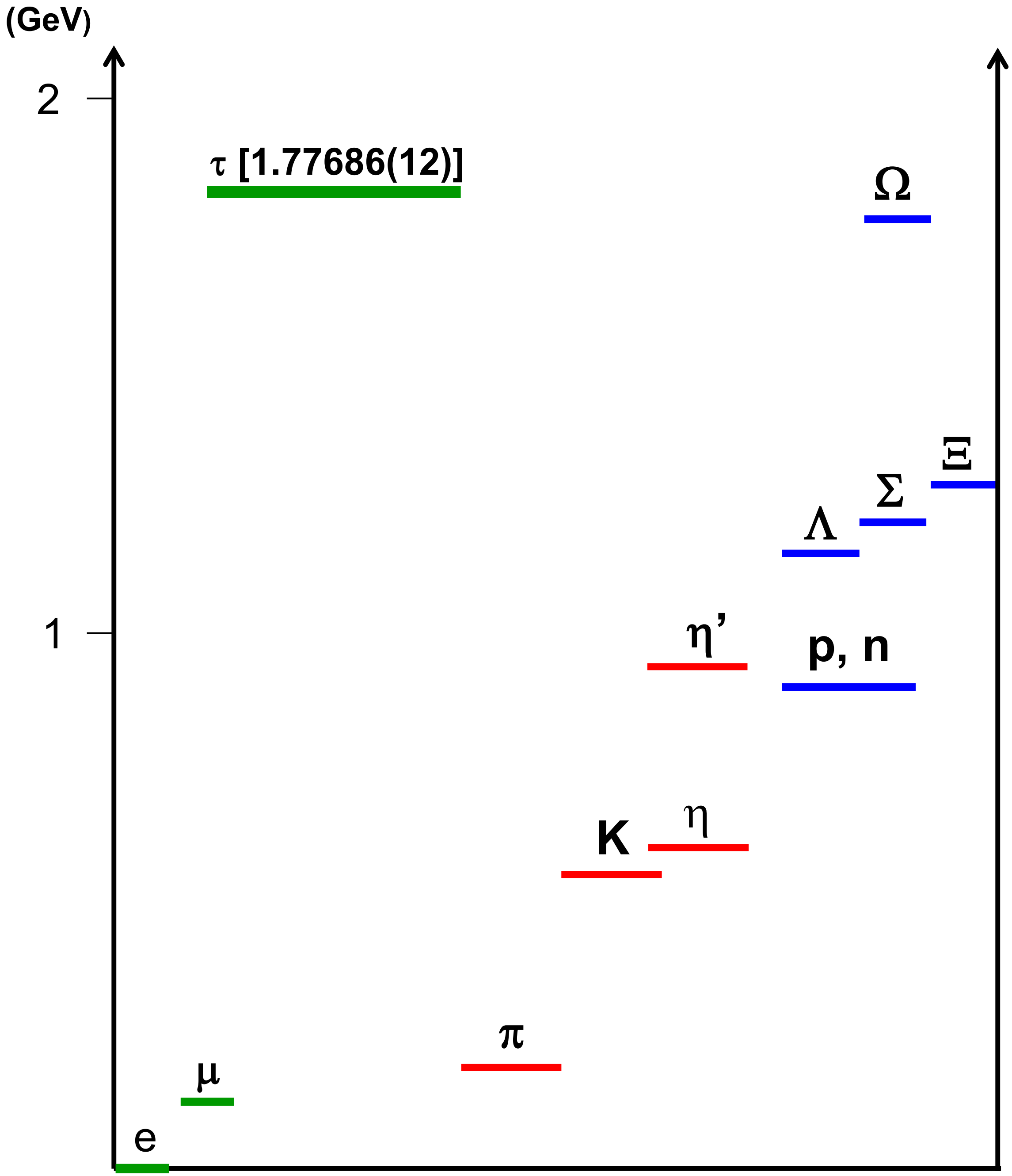
ν_τ → DONuT (Direct Observation of Nu Tau), 2000

Decay spectrum



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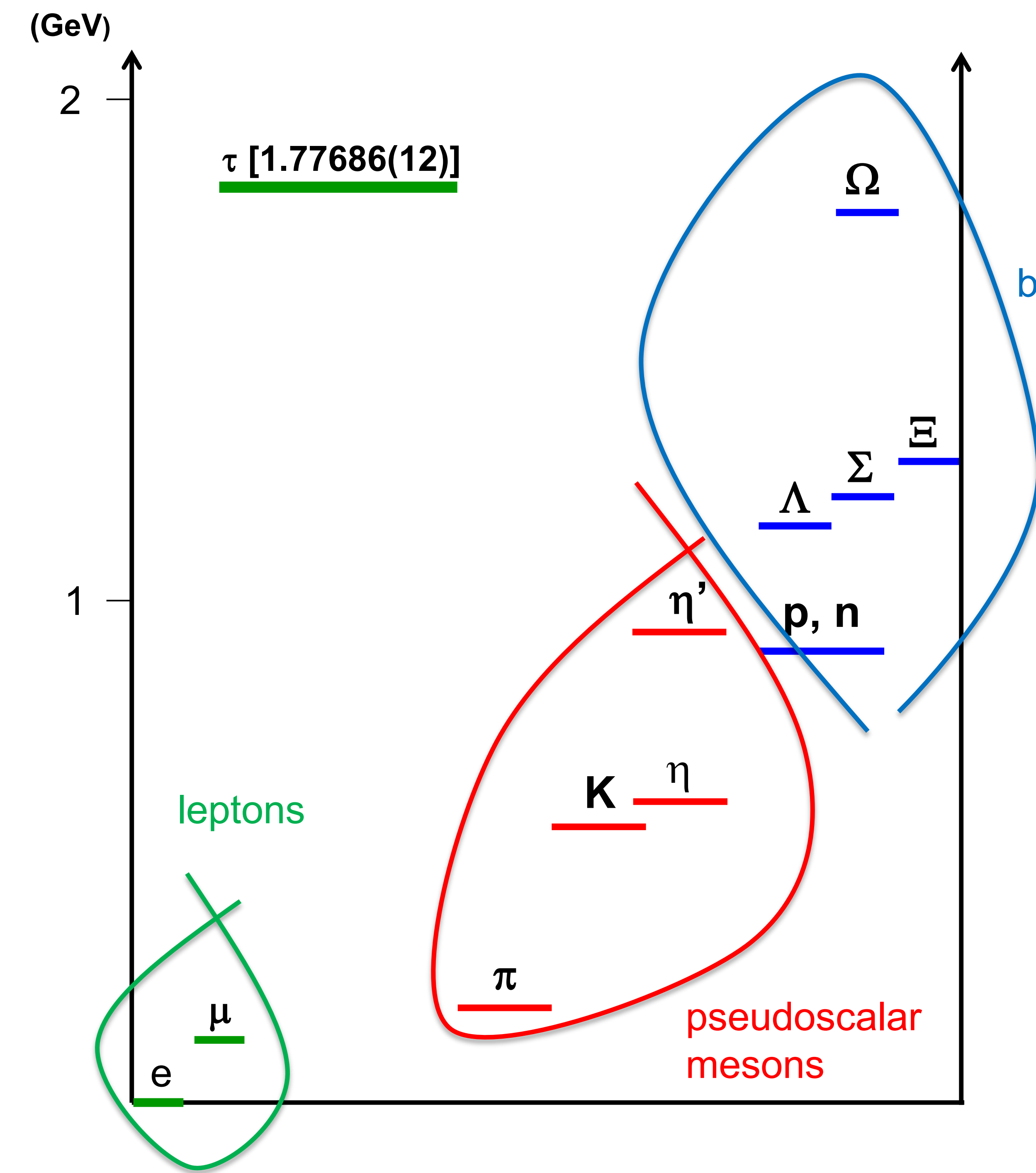

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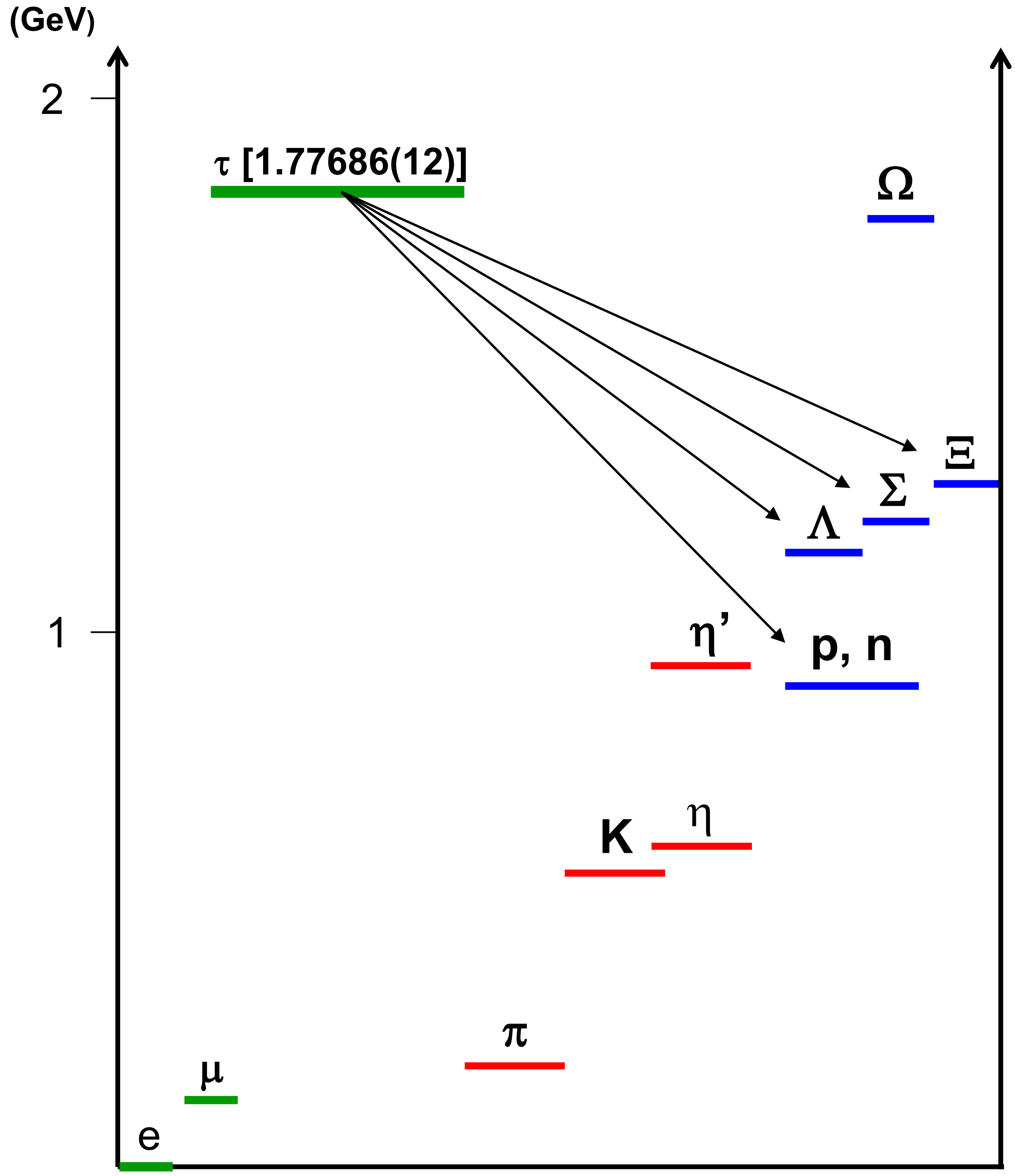

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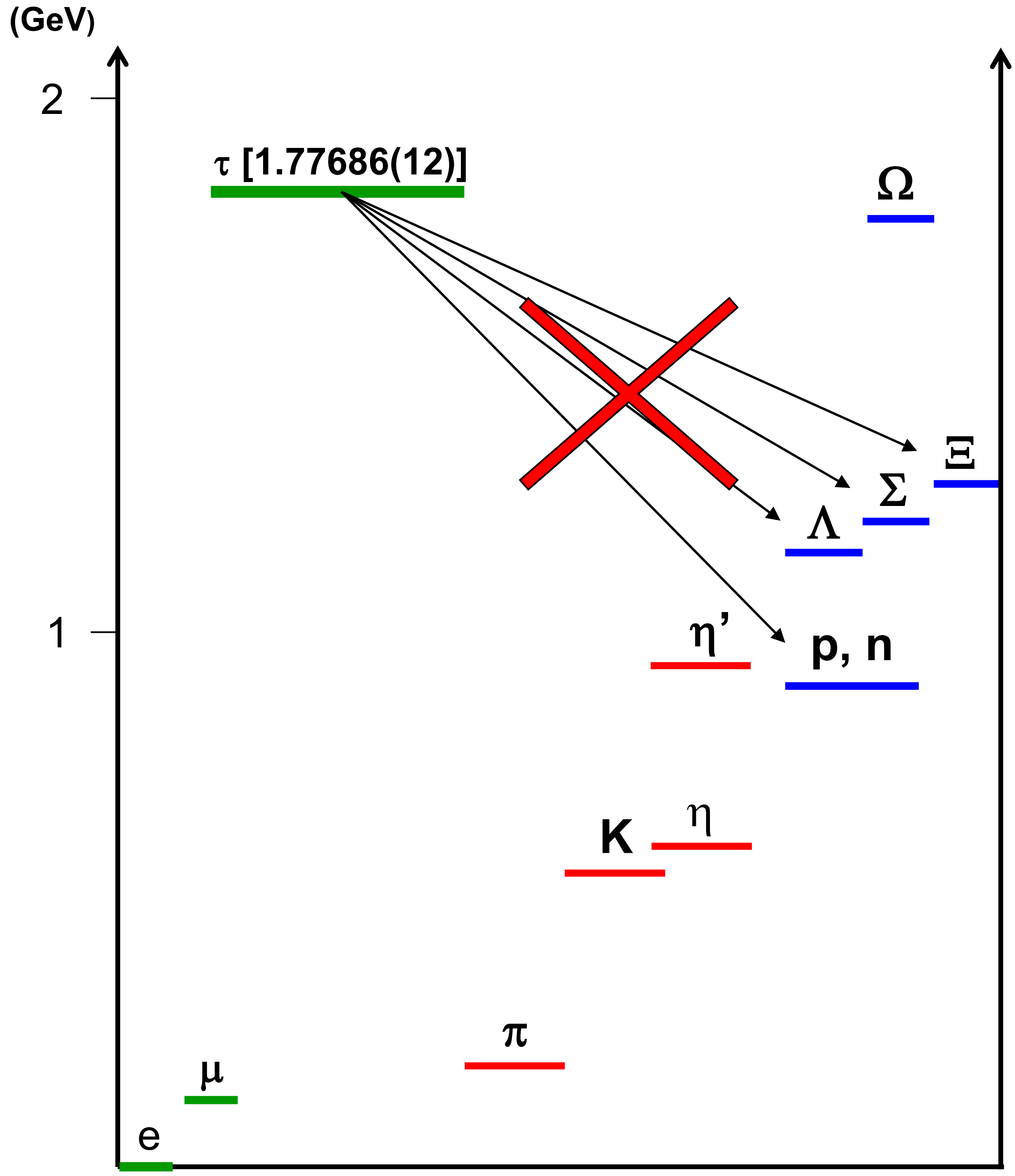
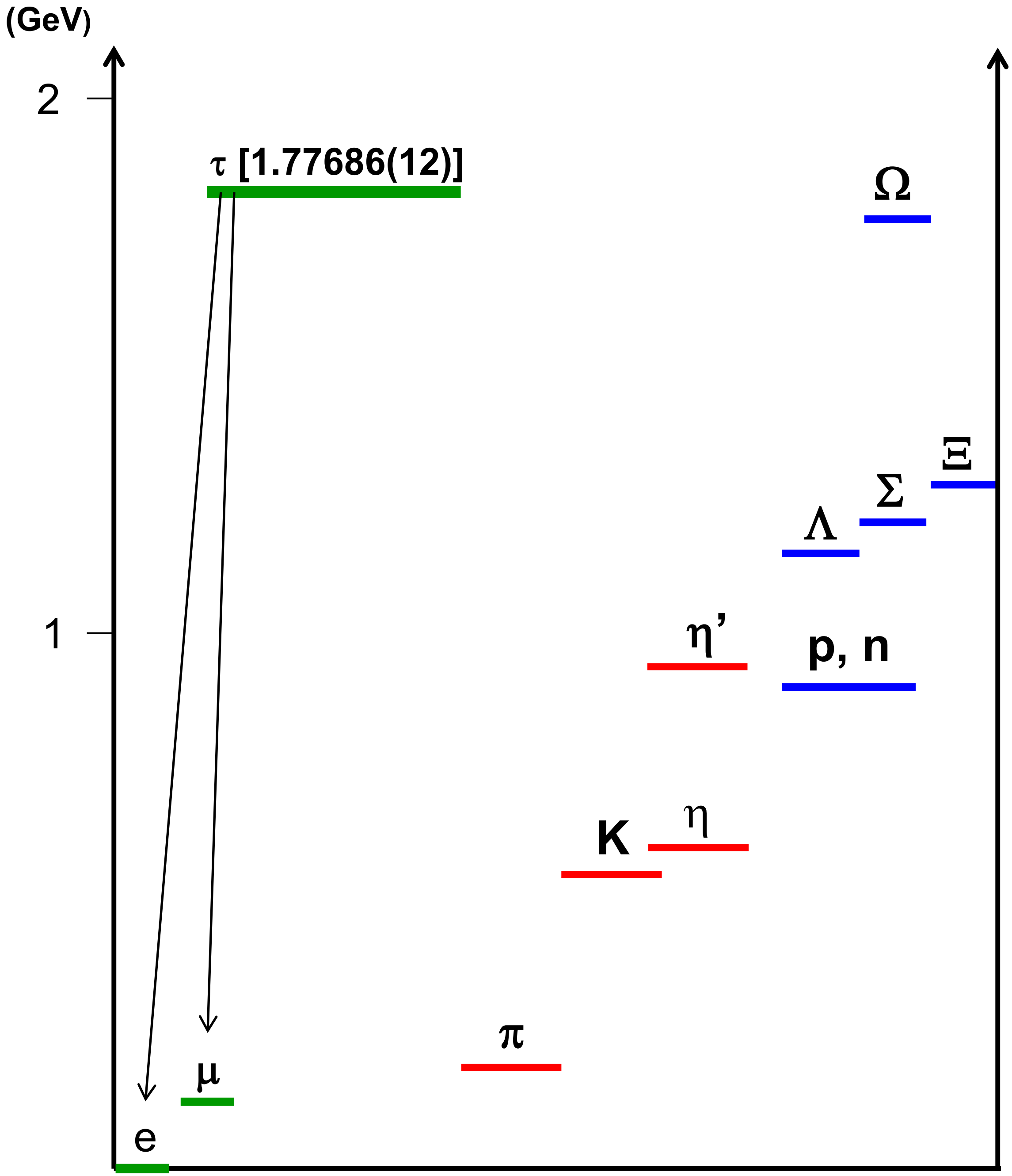
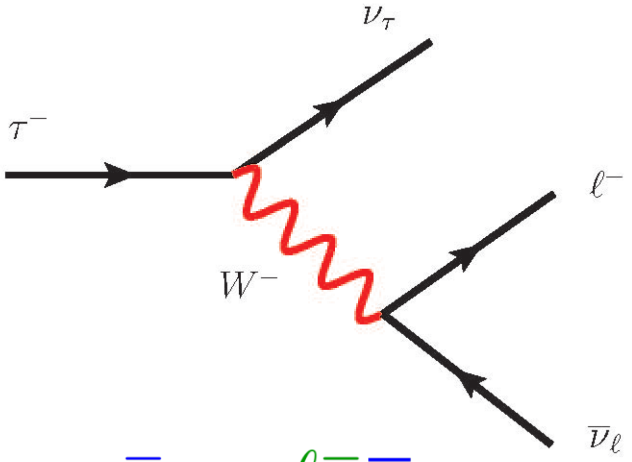


$$\Delta B = 0$$

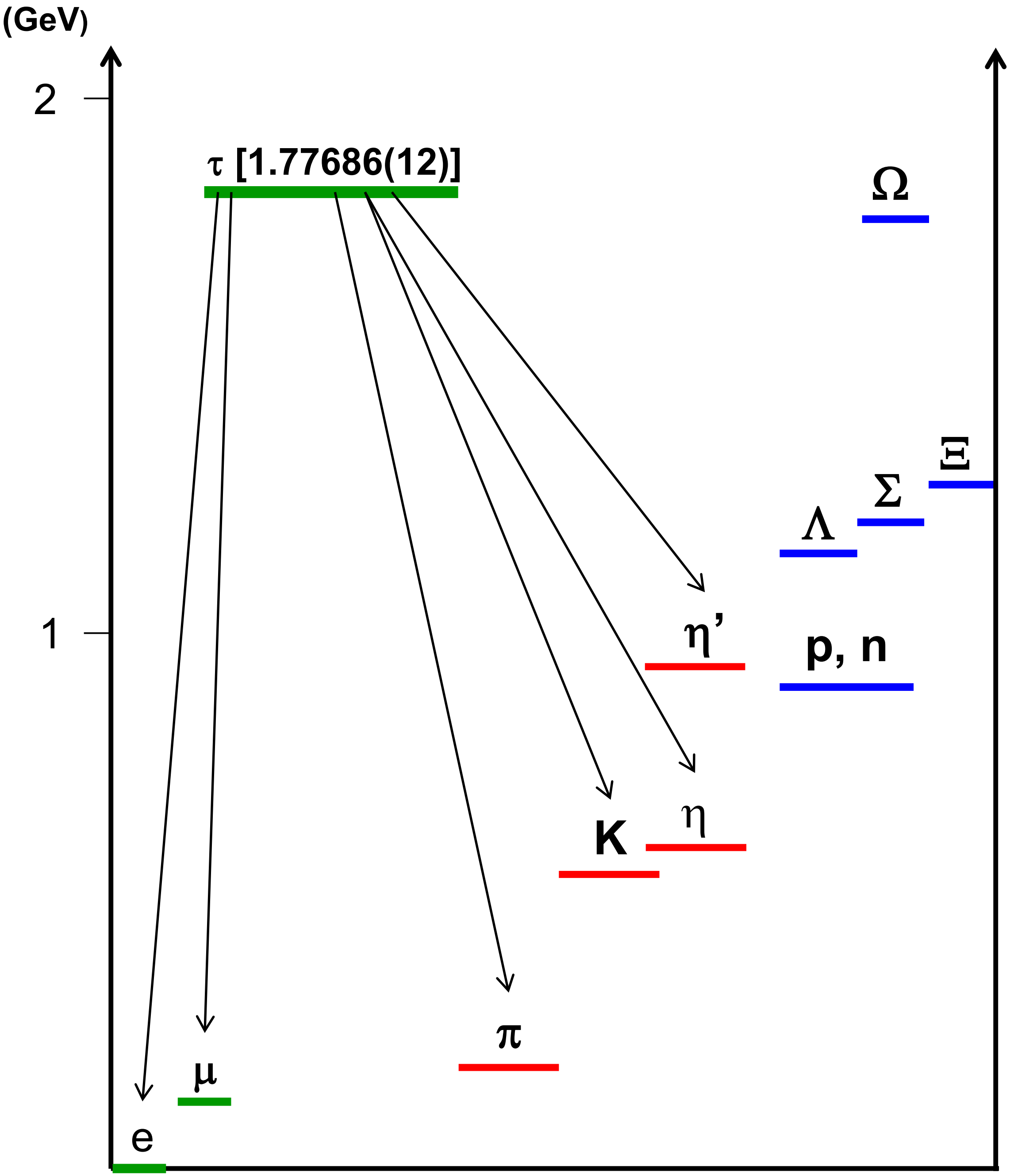
Decay spectrum

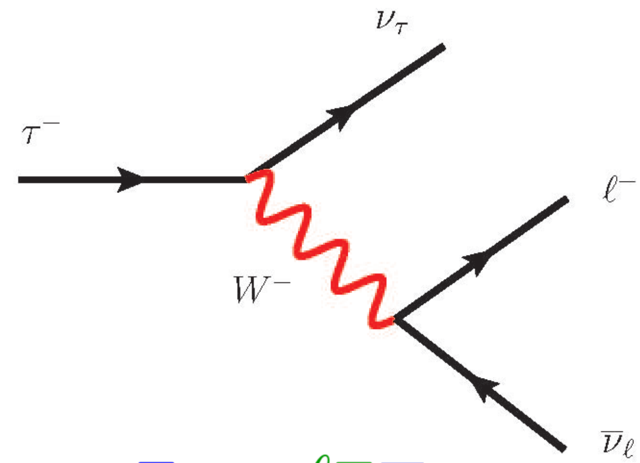




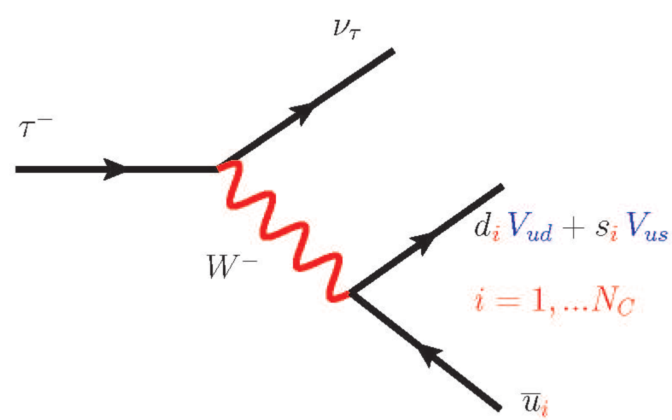
$$\tau^- \rightarrow \ell^- \overline{\nu}_\ell \nu_\tau$$

Decay spectrum



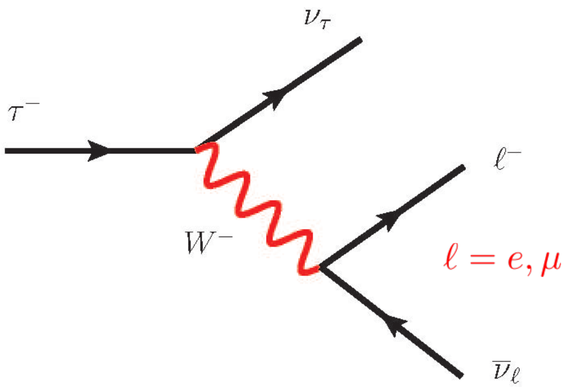


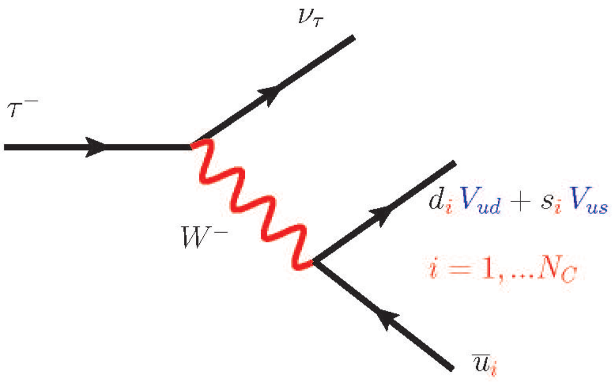
$$\tau^- \to \ell^- \overline{\nu}_\ell \nu_\tau$$



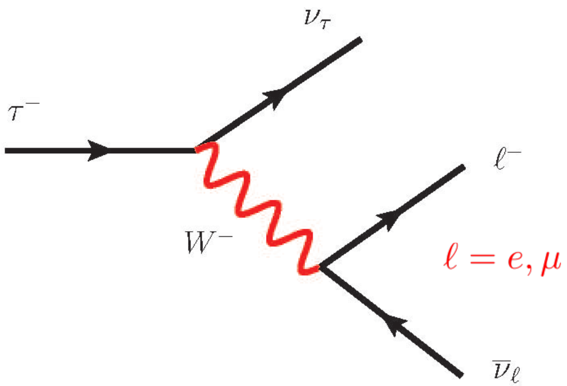
$$\tau^- \to (P, PP, \ldots)^- \nu_\tau$$

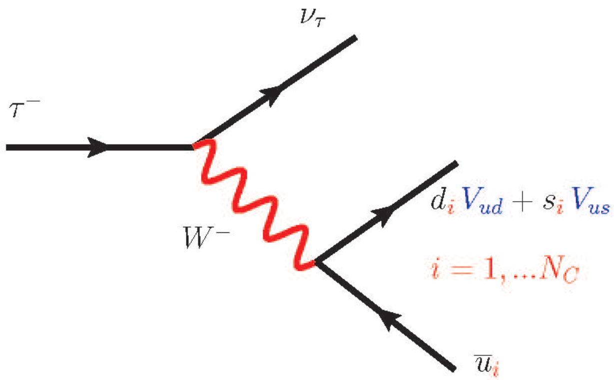
P ≡ Pseudoscalar meson





Process	Estimate	Experiment
$\mathrm{B}_e \equiv \mathrm{Br}(\tau \to e\overline{\nu}\nu)$ $\mathrm{B}_\mu \equiv \mathrm{Br}(\tau \to \mu\overline{\nu}\nu)$		
$\mathrm{Br}(\tau \to \text{non-strange hadrons})$		
$\mathrm{Br}(\tau \to \text{strange hadrons})$		





Process	Estimate	Experiment
$\mathrm{B}_e \equiv \mathrm{Br}(\tau \to e\overline{\nu}\nu)$	$\dfrac{1}{2 + N_C\left(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2\right)}$	$(17.82 \pm 0.04)\%$
$\mathrm{B}_\mu \equiv \mathrm{Br}(\tau \to \mu\overline{\nu}\nu)$	$\simeq 20\,\%$	$(17.39 \pm 0.04)\%$

$\mathrm{Br}(\tau \to \text{non-strange hadrons})$

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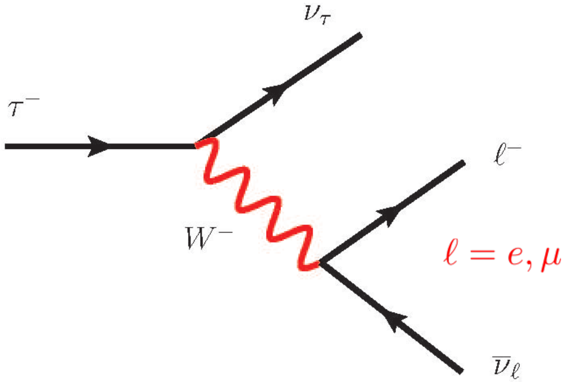

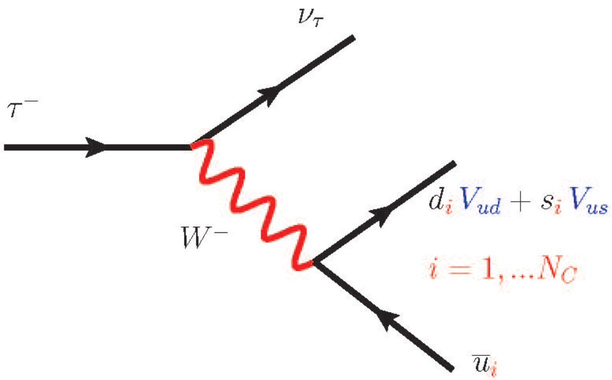

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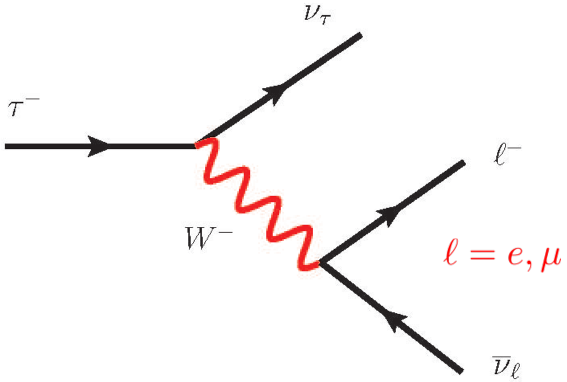
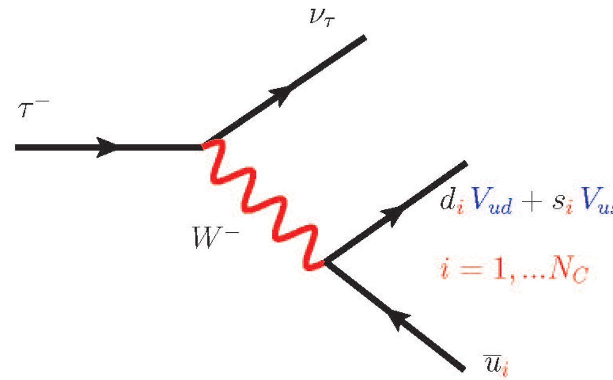
$B_e \equiv \mathrm{Br}(\tau \to e\overline{\nu}\nu)$ $B_\mu \equiv \mathrm{Br}(\tau \to \mu\overline{\nu}\nu)$	$\dfrac{1}{2 + N_C\left(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2\right)} \simeq 20\,\%$	$(17.82 \pm 0.04)\%$ $(17.39 \pm 0.04)\%$
$\mathrm{Br}(\tau \to \text{non-strange hadrons})$	$\dfrac{N_C\lvert V_{ud}\rvert^2}{2 + N_C\left(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2\right)} \simeq 58\,\%$	$(62 \pm 4)\%$
$\mathrm{Br}(\tau \to \text{strange hadrons})$		





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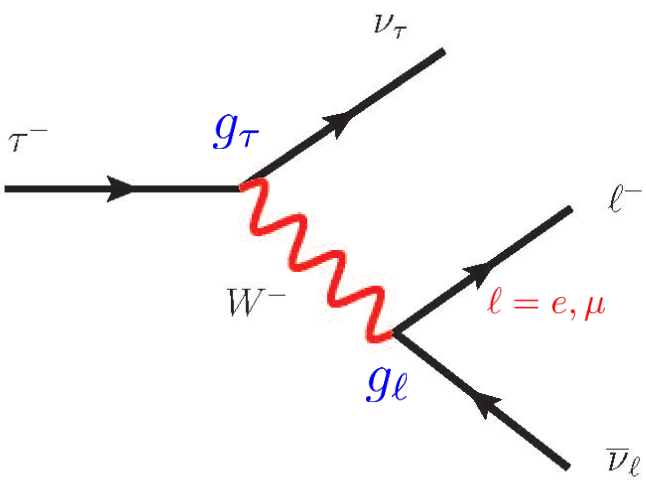
$Br(\tau \to \text{strange hadrons})$	$\frac{N_C \|V_{us}\|^2}{2 + N_C\left(\|V_{ud}\|^2 + \|V_{us}\|^2\right)}$ $\simeq 2\,\%$	$(2.6 \pm 0.7)\%$

Outline

- Leptonic decays
- Hadron decays
  I. Inclusive tau decays: $\alpha_S(M_\tau)$
  II. Exclusive tau decays: Hadronization of QCD currents
  E. g. $\tau \rightarrow \pi\pi\nu_\tau,\ \pi\pi\pi\nu_\tau$
- Breaking the SM rules
- Messages

1. Lepton decays



$$\Gamma\,(\tau \to \ell\,\overline{\nu}_\ell\,\nu_\tau) = \frac{G_F^2 M_\tau^5}{192\,\pi^3}\, f\left(M_\ell^2/M_\tau^2\right) r_{\rm EW}$$

$$f(x) = 1 - 8\,x + 8\,x^3 - x^4 - 12\,x^2 \ln x$$

$$f\left(\frac{M_e{}^2}{M_\tau^2}\right) = 0.999999, \qquad f\left(\frac{M_\mu{}^2}{M_\tau^2}\right) = 0.972559$$

$$r_{\rm EW} \underset{[3]}{=} \left(1 + \frac{3}{5}\frac{M_\tau^2}{M_W^2}\right)\left[1 + \frac{\alpha(M_\tau)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right] = 0.9960$$

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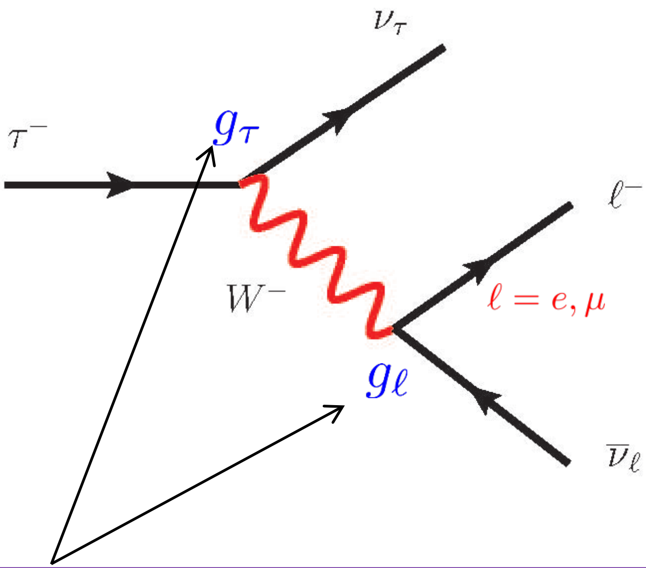

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Charged current universality $g_\tau = g_\mu = g_e$

$\mathrm{Br}\left(\mu^- \to e^- \overline{\nu}_e \nu_\mu\right) \simeq 100\,\%$

	$\lvert g_\tau / g_e \rvert$
$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$	$1.0028(15)$
$\Gamma_{W\to\tau}/\Gamma_{W\to e}$	$1.022(12)$

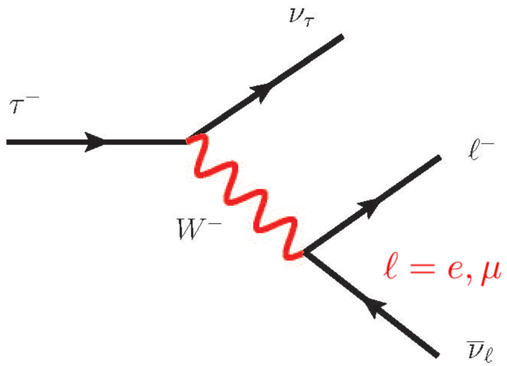
	$\lvert g_\tau / g_\mu \rvert$
$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$	$1.0011(14)$
$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$	$1.004(16)$

	$\lvert g_\mu / g_e \rvert$
$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$	$1.0017(16)$
$\Gamma_{W\to\mu}/\Gamma_{W\to e}$	$0.998(4)$

[4]

1. Lepton decays

Michel parameters [5]



$$\mathcal{M} = 4\frac{G_{\tau\ell}}{\sqrt{2}} \sum_{a=S,V,T}^{i,j=R,L} g_{ij}^{a} \langle \bar{\ell}_i | \Gamma^a | (\nu_\ell)_n \rangle \langle (\bar{\nu}_\tau)_m | \Gamma_a | \tau_j \rangle$$

$$\ell = \mu, e \qquad \Gamma^S = 1, \quad \Gamma^V = \gamma^\mu, \quad \Gamma^T = \sigma^{\mu\nu}/\sqrt{2}$$

$$\text{SM} \quad \longrightarrow \quad g_{LL}^V = 1, \quad g_{ij}^S = g_{ij}^T = g_{RR}^V = g_{RL}^V = g_{LR}^V = 0$$

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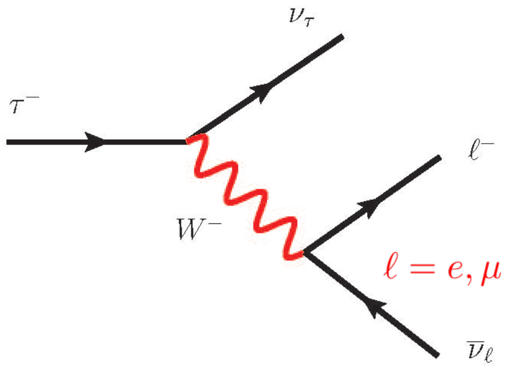

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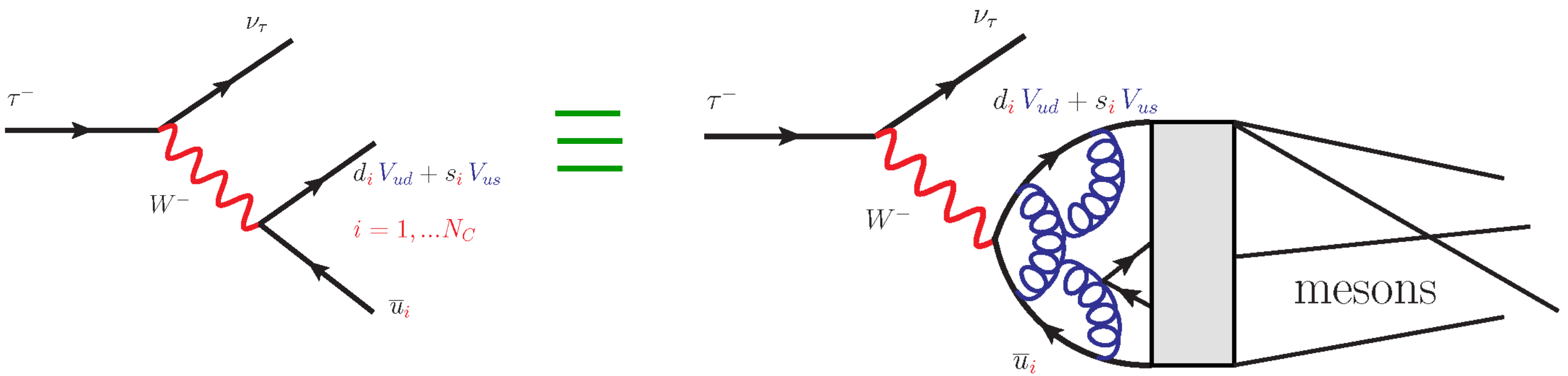
[6] $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ (95 % CL)

$\|g_{RR}^S\| < 0.72$	$\|g_{LR}^S\| < 0.95$	$\|g_{RL}^S\| \leq 2$	$\|g_{LL}^S\| \leq 2$
$\|g_{RR}^V\| < 0.18$	$\|g_{LR}^V\| < 0.12$	$\|g_{RL}^V\| < 0.52$	$\|g_{LL}^V\| \leq 1$
$\|g_{RR}^T\| \equiv 0$	$\|g_{LR}^T\| < 0.08$	$\|g_{RL}^T\| < 0.51$	$\|g_{LL}^T\| \equiv 0$

2. Hadron decays



$$\mathcal{M}\left(\tau \to \nu_\tau \mathrm{H}\right) = \frac{G_F}{\sqrt{2}}\, V_{\mathrm{CKM}}\, \overline{u}_{\nu_\tau} \gamma^\mu \left(1-\gamma_5\right) u_\tau \,\langle H|\, \left(V_\mu - A_\mu\right)\, e^{i\, \mathrm{L}_{\mathrm{QCD}}} |\Omega_{\mathrm{H}}\rangle$$

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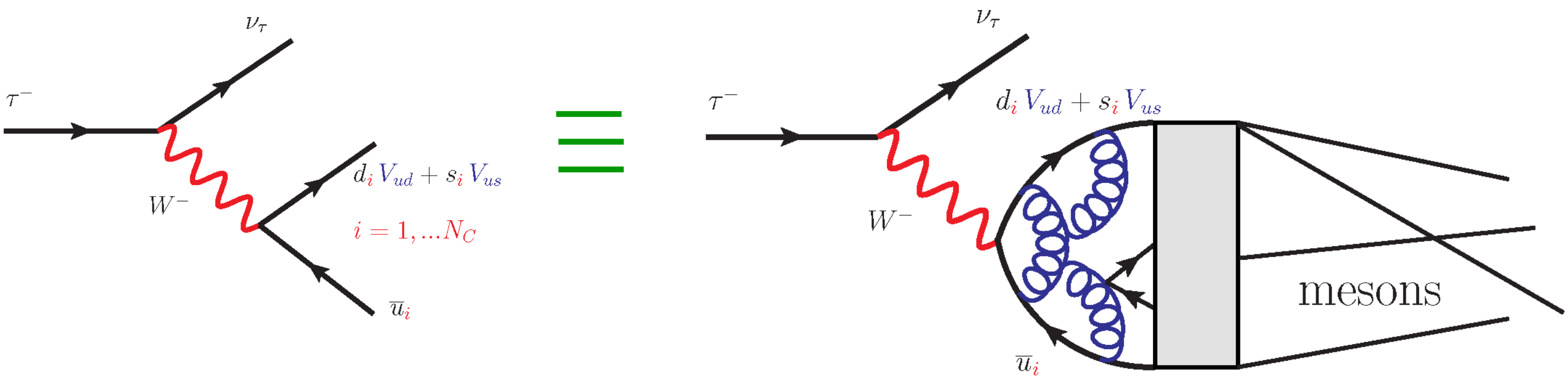

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form factors

$$\langle H |\, \left(V_\mu - A_\mu\right)\, e^{i\,\mathrm{L_{QCD}}} |\Omega_{\mathrm{H}}\rangle = \sum_i \left(\text{Lorentz structure}\right)^i{}_\mu\, \underbrace{F_i(Q^2, s, \ldots)}_{\text{form factors}}$$

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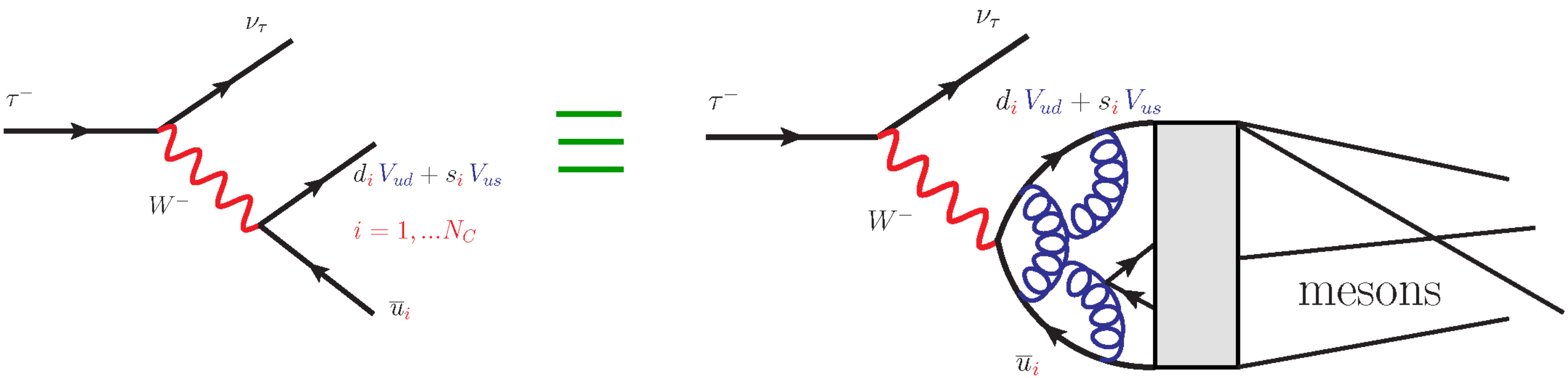

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$$d\Gamma\left(\tau \to \nu_\tau H\right) = \frac{G_F^2}{4\, M_\tau} \left|V_{\mathrm{CKM}}\right|^2 L_{\mu\nu}\, H^{\mu\nu}\, d\mathrm{PS}$$

$$\begin{cases} L_{\mu\nu} H^{\mu\nu} \overset{[7]}{=} \sum_X L_X W_X \\ W_X \equiv \textbf{structure functions} \end{cases}$$

What can we get?

1. Inclusive decays: full hadron spectra.

$$\tau^- \rightarrow \nu_\tau \, (\bar{u}d, \bar{u}s)$$

→ Study of Standard Model parameters : $\alpha_S(M_\tau)$, $|V_{us}|$, m_S

2. Exclusive decays: specific hadron spectrum.

$$\tau^- \rightarrow \nu_\tau \, (PP, PPP, ...)$$

P = pseudoscalar meson

→ Study of form factors, resonance parameters (M_R, Γ_R), hadronization of QCD currents.

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2. Exclusive decays: specific hadron spectrum. Non-precision physics

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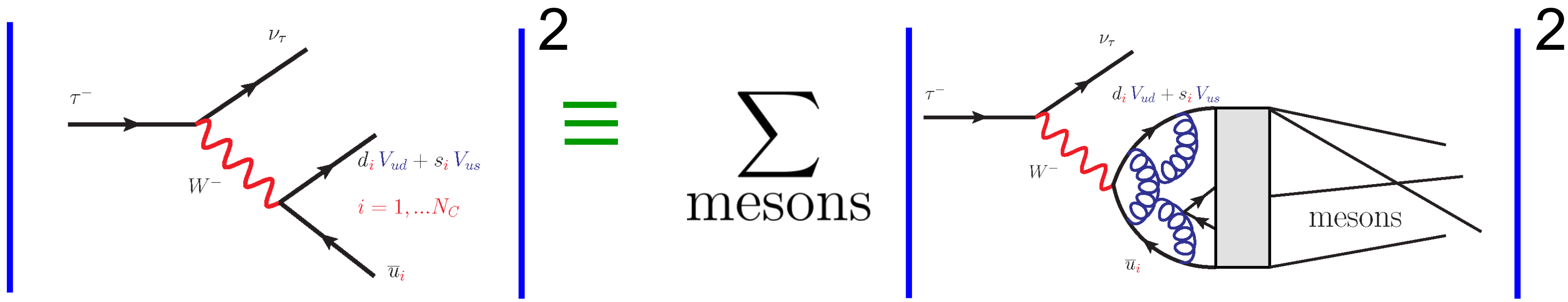
2.1 Inclusive hadron decays



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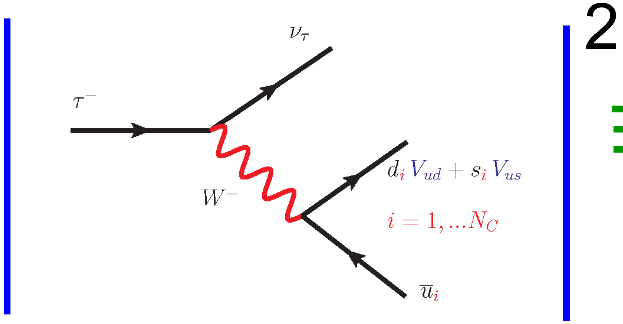
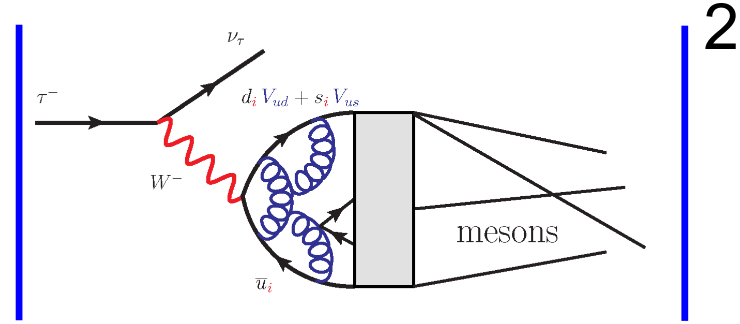


$\equiv \sum_{\text{mesons}}$



$$e^+e^- \longrightarrow \text{hadrons}$$

$$V_\mu^i = \overline{q}\,\gamma_\mu\,\frac{\lambda^i}{2}\,q, \qquad q = (u, d, s)^T$$

$$\sigma_{e^+e^-\to\text{had}}(q^2) = \frac{e^4}{2q^6}\,L^{\mu\nu}\sum_h (2\pi)^4\delta^4(p_h - q)\langle\Omega_h|\,J_\mu(0)\,|h\rangle\langle h|\,J_\nu(0)\,|\Omega_h\rangle$$

$$J_\mu = V_\mu^3 + \frac{1}{\sqrt{3}}V_\mu^8 = \frac{2}{3}\,\overline{u}\,\gamma_\mu\,u - \frac{1}{3}\,\overline{d}\,\gamma_\mu\,d - \frac{1}{3}\,\overline{s}\,\gamma_\mu\,s \qquad E \ll M_Z$$

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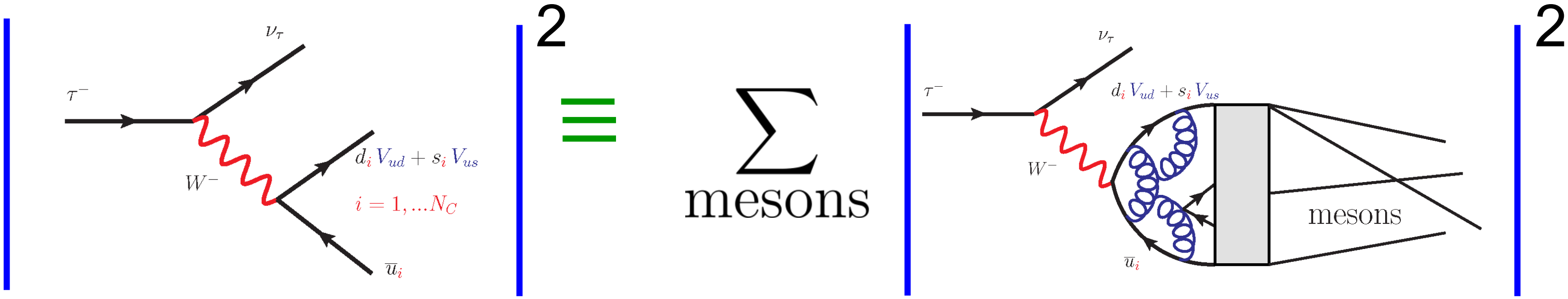

$e^+e^- \longrightarrow$ hadrons

$$V^i_\mu = \overline{q}\,\gamma_\mu\,\frac{\lambda^i}{2}\,q, \qquad q = (u,d,s)^T$$

$$\sigma_{e^+e^-\to\text{had}}(q^2) = \frac{e^4}{2q^6}\,L^{\mu\nu}\sum_h (2\pi)^4\delta^4(p_h - q)\langle\Omega_h|\,J_\mu(0)\,|h\rangle\langle h|\,J_\nu(0)\,|\Omega_h\rangle$$

$$J_\mu = V^3_\mu + \frac{1}{\sqrt{3}}V^8_\mu \;=\; \frac{2}{3}\,\overline{u}\,\gamma_\mu\,u - \frac{1}{3}\,\overline{d}\,\gamma_\mu\,d - \frac{1}{3}\,\overline{s}\,\gamma_\mu\,s \qquad E \ll M_Z$$

$$\sum_h (2\pi)^4\delta^4(p_h - q)\langle\Omega_h|J_\mu(0)|h\rangle\langle h|J_\nu(0)|\Omega_h\rangle = \int d^4x\, e^{iqx}\,\langle\Omega_h|J_\mu(x)J_\nu(0)|\Omega_h\rangle$$

$$= \int d^4x\, e^{iqx}\,\langle\Omega_h|\,[J_\mu(x), J_\nu(0)]\,|\Omega_h\rangle$$

$$\int d^4x\, e^{iqx} \langle \Omega_h | \left[J_\mu(x), J_\nu(0) \right] | \Omega_h \rangle \overset{[8]}{=} 2\,\mathrm{Im} \left[i \int d^4x\, e^{iqx} \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle \right]$$

$$i \int d^4x\, e^{iqx} \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle = \left(q_\mu q_\nu - q^2 g_{\mu\nu} \right) \Pi_V(q^2)$$

$$\sigma_{e^+e^- \to \mathrm{had}}(q^2) = \frac{16\pi^2\alpha^2}{q^2}\, \mathrm{Im}\, \Pi_V(q^2)$$

$$\int d^4x\, e^{iqx} \langle \Omega_h | \left[J_\mu(x), J_\nu(0) \right] | \Omega_h \rangle \stackrel{[8]}{=} 2\,\mathrm{Im}\left[i \int d^4x\, e^{iqx} \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle \right]$$

$$i \int d^4x\, e^{iqx} \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle = \left(q_\mu q_\nu - q^2 g_{\mu\nu} \right) \Pi_V(q^2)$$

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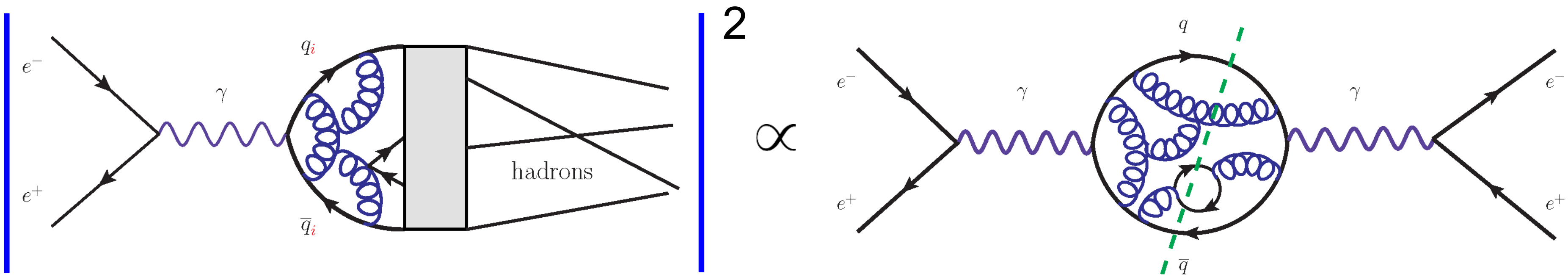

$$\int d^4x\, e^{iqx}\, \langle \Omega_h | \left[J_\mu(x), J_\nu(0) \right] | \Omega_h \rangle \overset{[8]}{=} 2\,\mathrm{Im}\left[i \int d^4x\, e^{iqx}\, \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle \right]$$

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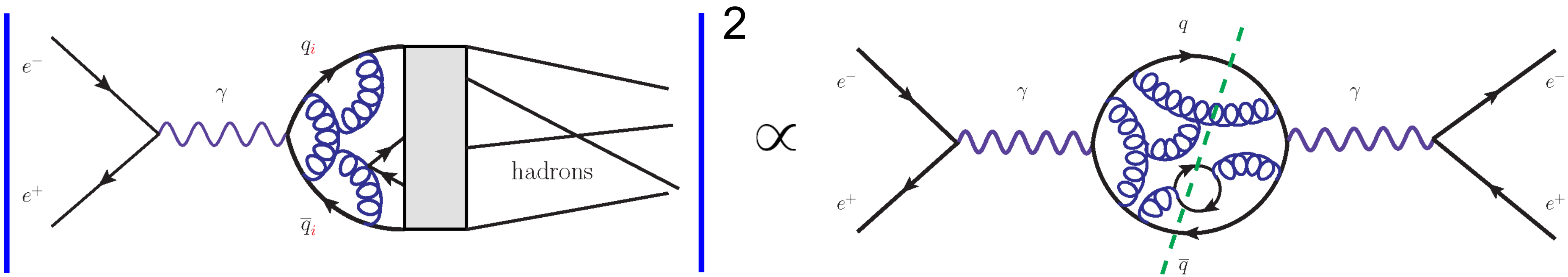
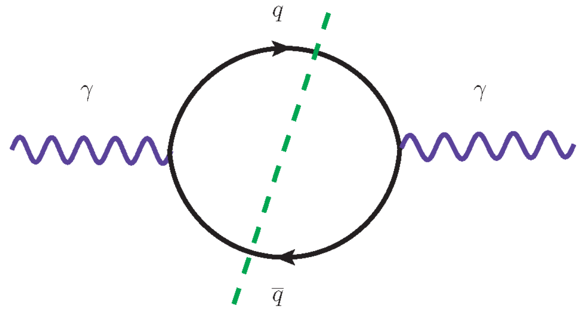
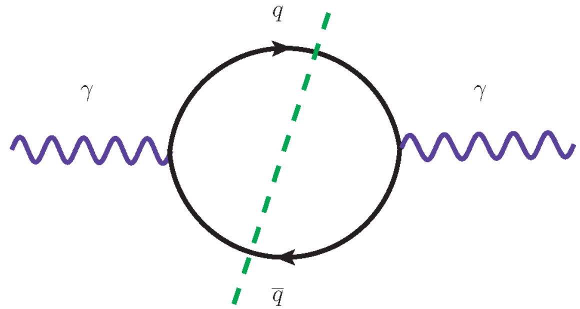


$$\sigma_{e^+e^-\to\mu^+\mu^-} = \frac{4\pi\alpha^2}{3\,q^2} \qquad R(q^2) = \frac{\sigma_{e^+e^-\to\mathrm{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = 12\,\pi\, \mathrm{Im}\, \Pi_V(q^2)$$

$$\mathrm{Im}\,\Pi_V(q^2) =$$



$$= \frac{N_C}{12\,\pi} \sum_i Q_i^2$$

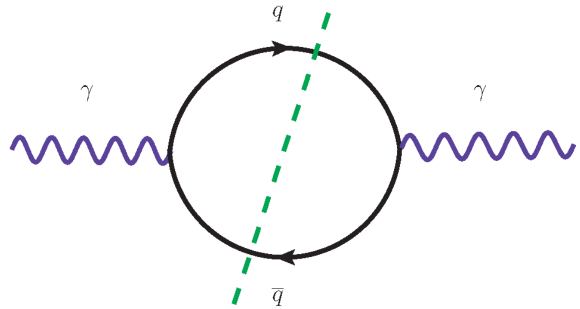
$$\mathrm{Im}\,\Pi_V(q^2) =$$ 

$$= \frac{N_C}{12\,\pi} \sum_i Q_i^2$$

$$R(q^2) = \frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} = 12\,\pi\,\mathrm{Im}\,\Pi_V(q^2) = N_C \sum_i Q_i^2 = 2,\ \frac{10}{3},\ \frac{11}{3}$$

$$N_F = 3,\quad 4,\quad 5$$

$$\mathrm{Im}\,\Pi_V(q^2) \;=$$



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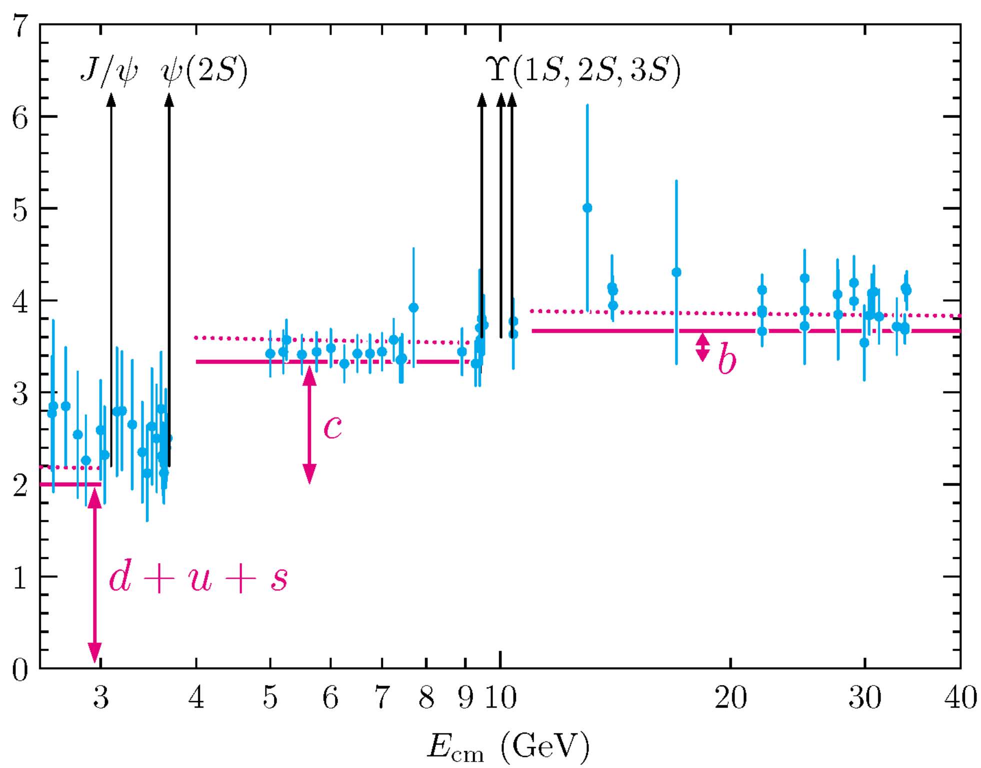
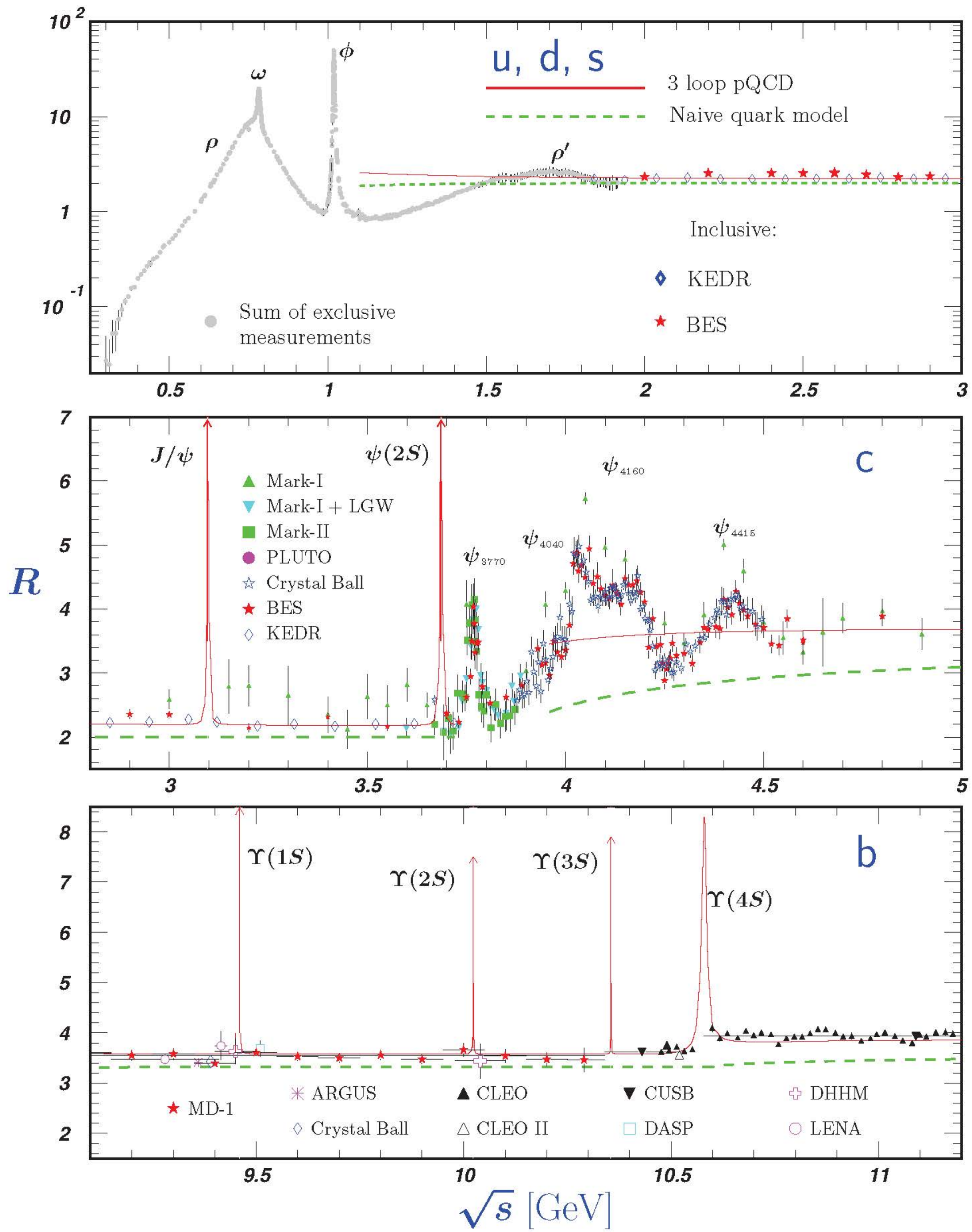
$$N_F = 3,\ \ 4,\ \ 5$$

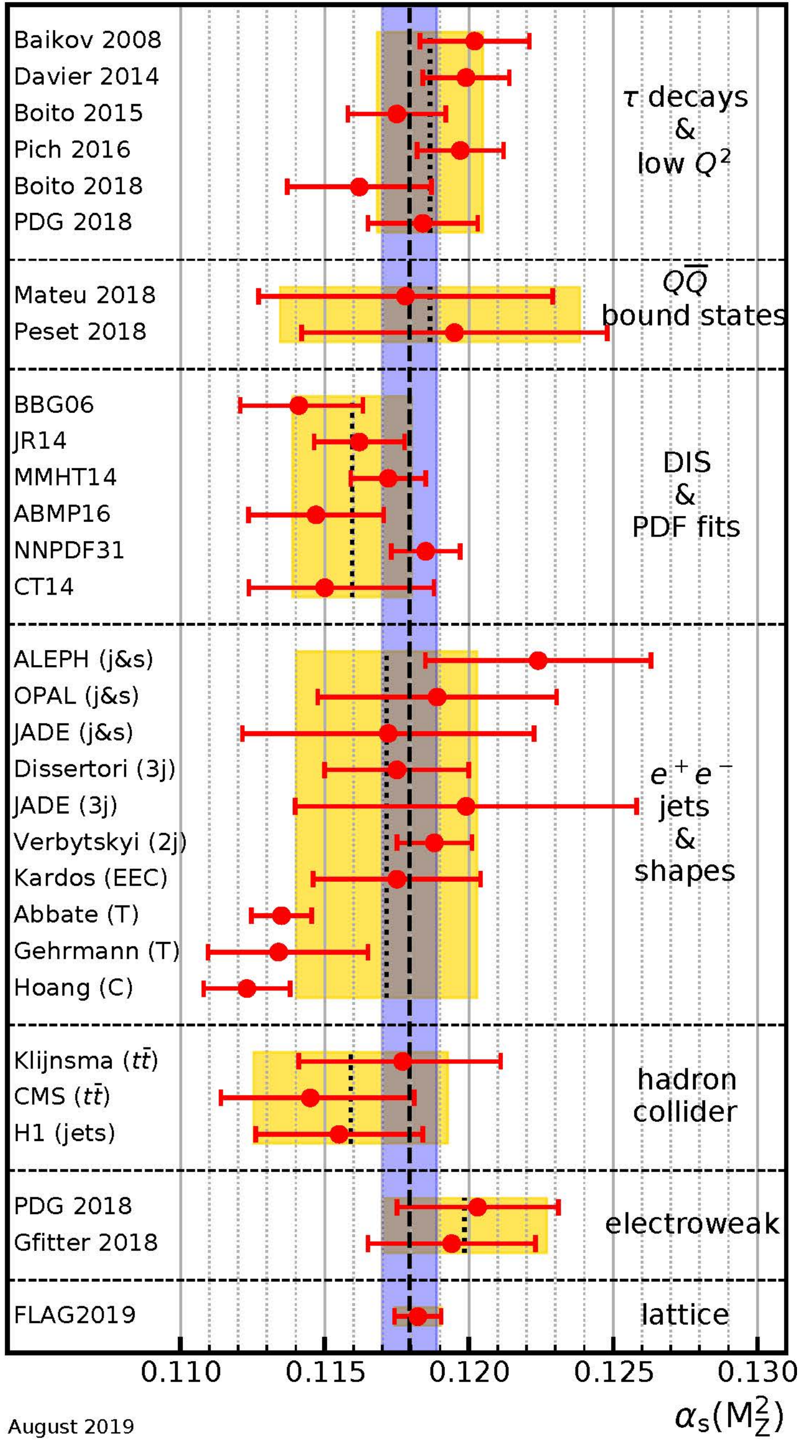
$R = \sigma(\mathrm{hadrons})/\sigma(\mu^+\mu^-)$



PDG

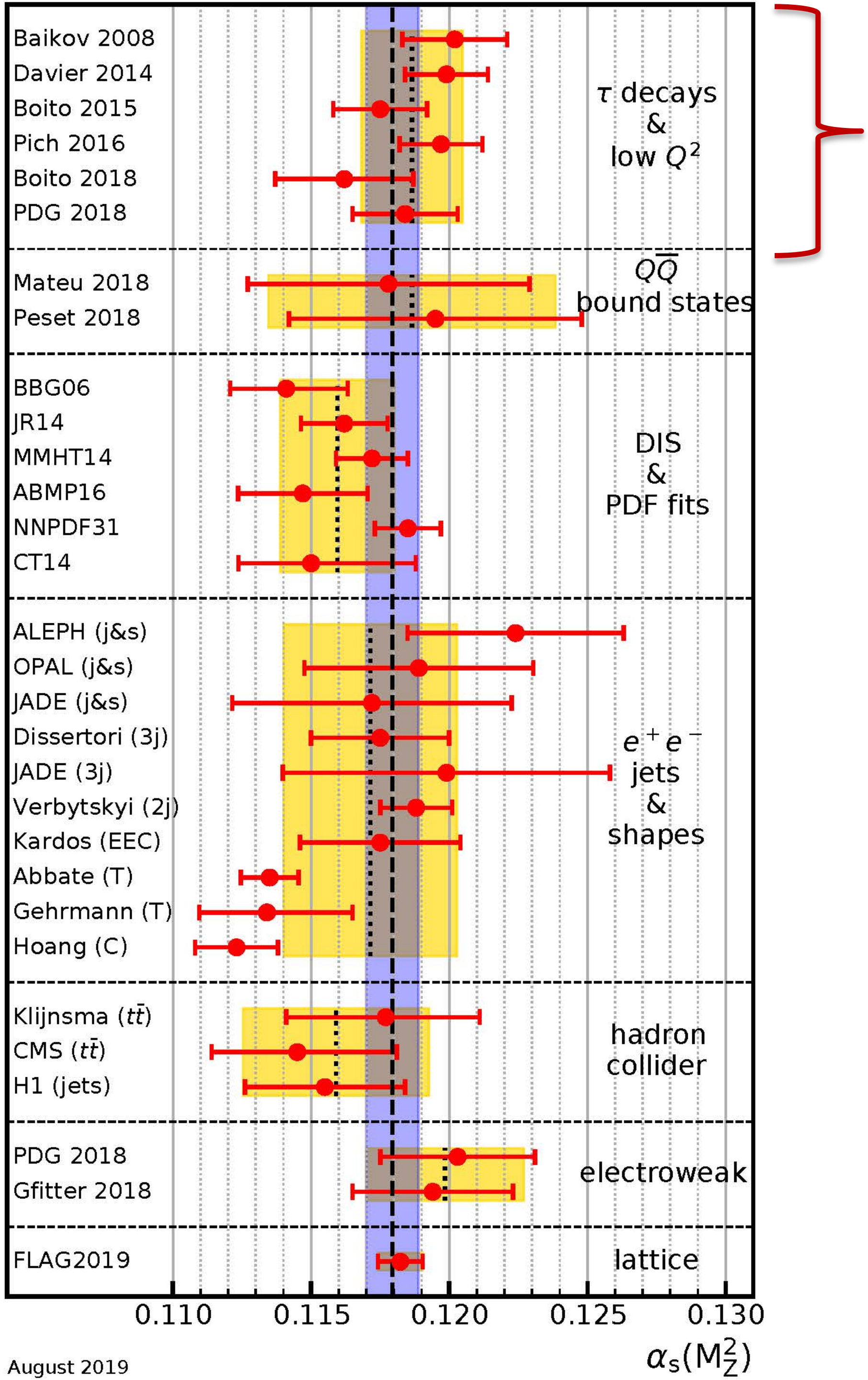

u, d, s
3 loop pQCD
Naive quark model
Inclusive:
KEDR
BES
Sum of exclusive measurements
ρ
ω
φ
ρ'
c
J/ψ
ψ(2S)
ψ4160
ψ4040
ψ3770
ψ4415
Mark-I
Mark-I + LGW
Mark-II
PLUTO
Crystal Ball
BES
KEDR
R
b
Υ(1S)
Υ(2S)
Υ(3S)
Υ(4S)
MD-1
ARGUS
Crystal Ball
CLEO
CLEO II
CUSB
DASP
DHHM
LENA
√s [GeV]

PDG



PDG

$$\alpha_S(M_Z^2)$$



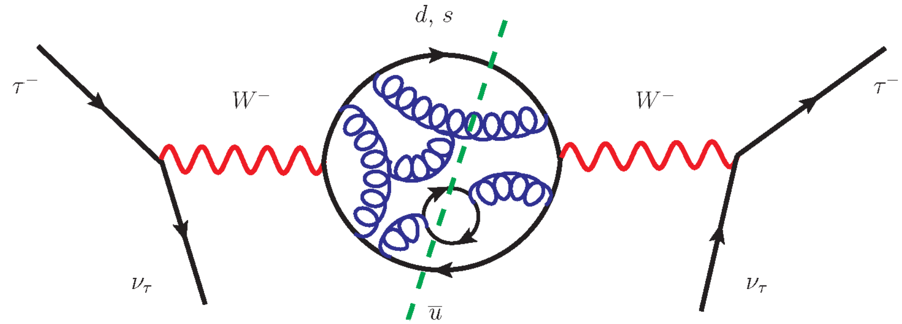
Inclusive hadron τ decays

$$\alpha_S(M_Z^2)$$

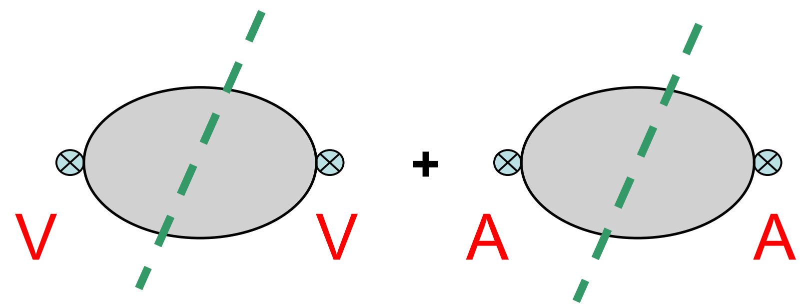
PDG

$$\tau \longrightarrow \nu_\tau \,\text{mesons}$$

$$\Gamma\left(\tau \longrightarrow \nu_\tau \,\text{mesons}\right) \quad \propto$$

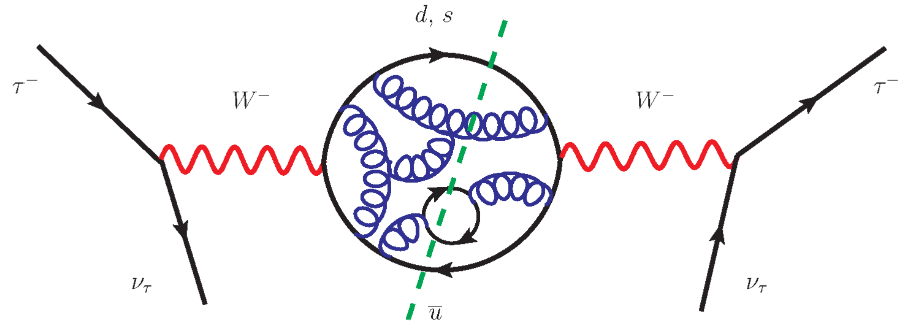


$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau \,\text{mesons}\right)}{\Gamma\left(\tau^- \to e^-\overline{\nu}_e \nu_\tau\right)} \quad \propto$$

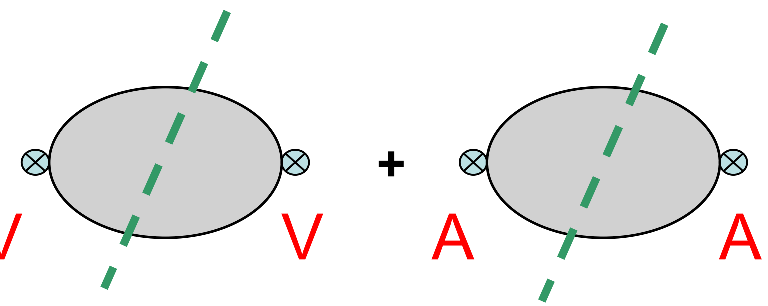


$\tau \longrightarrow \nu_\tau$ mesons

$$\Gamma\left(\tau \longrightarrow \nu_\tau \,\text{mesons}\right) \quad \propto$$



$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau \,\text{mesons}\right)}{\Gamma\left(\tau^- \to e^-\overline{\nu}_e\nu_\tau\right)} \quad \propto$$

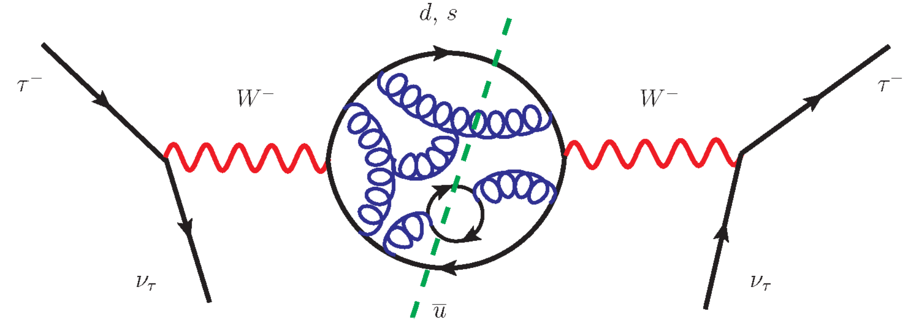


S=0 S=1

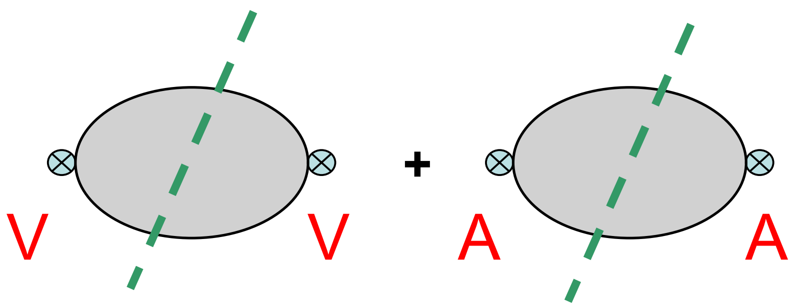
$$R_\tau = \underbrace{R_{\tau,V} + R_{\tau,A}}_{S=0} + \underbrace{R_{\tau,S}}_{S=1} \simeq \frac{N_C}{2}|V_{ud}|^2 + \frac{N_C}{2}|V_{ud}|^2 + N_C|V_{us}|^2$$

$$\tau \longrightarrow \nu_\tau \, \text{mesons}$$

$$\Gamma\left(\tau \longrightarrow \nu_\tau \, \text{mesons}\right) \quad \propto$$



$$R_\tau \equiv \frac{\Gamma\left(\tau^- \rightarrow \nu_\tau \, \text{mesons}\right)}{\Gamma\left(\tau^- \rightarrow e^- \overline{\nu}_e \nu_\tau\right)} \quad \propto$$



$$R_\tau = \underbrace{R_{\tau,V} + R_{\tau,A}}_{S=0} + \underbrace{R_{\tau,S}}_{S=1} \simeq \frac{N_C}{2}|V_{ud}|^2 + \frac{N_C}{2}|V_{ud}|^2 + N_C|V_{us}|^2$$

$$R_\tau \simeq N_C$$

$$R_\tau^{\rm exp} = \frac{\sum_i \Gamma(\tau \to \nu_\tau h_i)}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} = 3.6355 \pm 0.0081$$ [9]

$$R_\tau^{\rm exp} = \frac{1 - {\rm B}_e - {\rm B}_\mu}{{\rm B}_e} = 3.6370 \pm 0.0075$$

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$$R_\tau^{\exp} = \frac{1 - \mathrm{B}_e - \mathrm{B}_\mu}{\mathrm{B}_e} = 3.6370 \pm 0.0075$$

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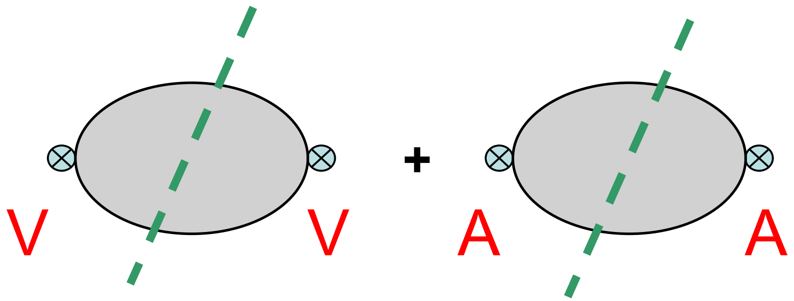

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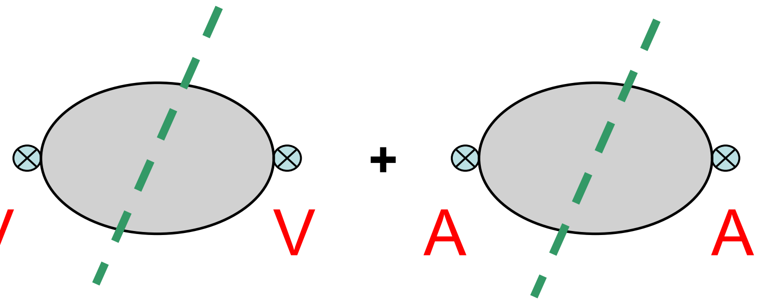

$$\Pi^{\mu\nu}_{ij,V}(q) \equiv i \int d^4x \, e^{iqx} \, \langle \Omega_h | \, T V^\mu_{ij}(x) V^\nu_{ij}(0)^\dagger \, | \Omega_h \rangle$$

$$\Pi^{\mu\nu}_{ij,A}(q) \equiv i \int d^4x \, e^{iqx} \, \langle \Omega_h | \, T A^\mu_{ij}(x) A^\nu_{ij}(0)^\dagger \, | \Omega_h \rangle$$

$$V^\mu_{ij} = \overline{\psi}_j \gamma^\mu \psi_i$$

$$A^\mu_{ij} = \overline{\psi}_j \gamma^\mu \gamma_5 \psi_i$$

i,j = flavour indices

J

$$\Pi^{\mu\nu}_{ij,V/A}(q) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) \Pi^{(1)}_{ij,V/A}(q^2) + q^\mu q^\nu \, \Pi^{(0)}_{ij,V/A}(q^2)$$

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2 \left(\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right)$$

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2 \left(\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right)$$

spectral functions

$$\mathrm{Im}\Pi^{(J)}{}_{ud(s),U} = \frac{1}{2\pi} u_J$$

$$U = V, A \longrightarrow u_J = v_J, \mathrm{a}_J$$

$$R_{\tau,V} = 1.783\,(11)_{exp}(2)_{V/A}$$

$$R_{\tau,A} = 1.695\,(11)_{exp}(2)_{V/A}$$

$$R_{\tau,S} = 0.1615\,(40)$$

[10,11]

[7]

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

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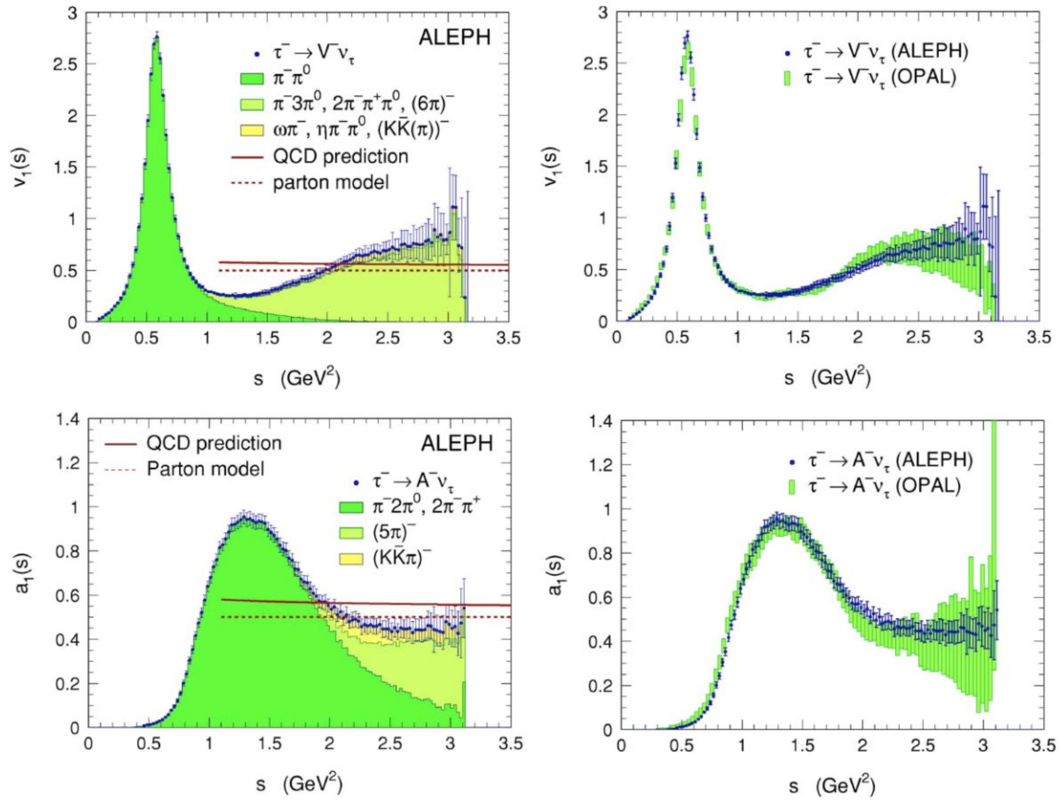
[10,11]



[10]

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

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spectral functions

$$\mathrm{Im}\Pi^{(J)}{}_{ud(s),U} = \frac{1}{2\pi}\, u_J$$

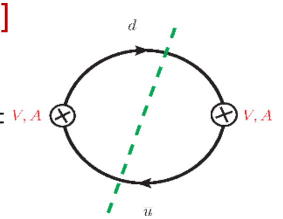
$$U = V, A \longrightarrow u_J = v_J, \mathrm{a}_J$$

$$R_{\tau,V} = 1.783\,(11)_{exp}(2)_{V/A}$$

$$R_{\tau,A} = 1.695\,(11)_{exp}(2)_{V/A}$$

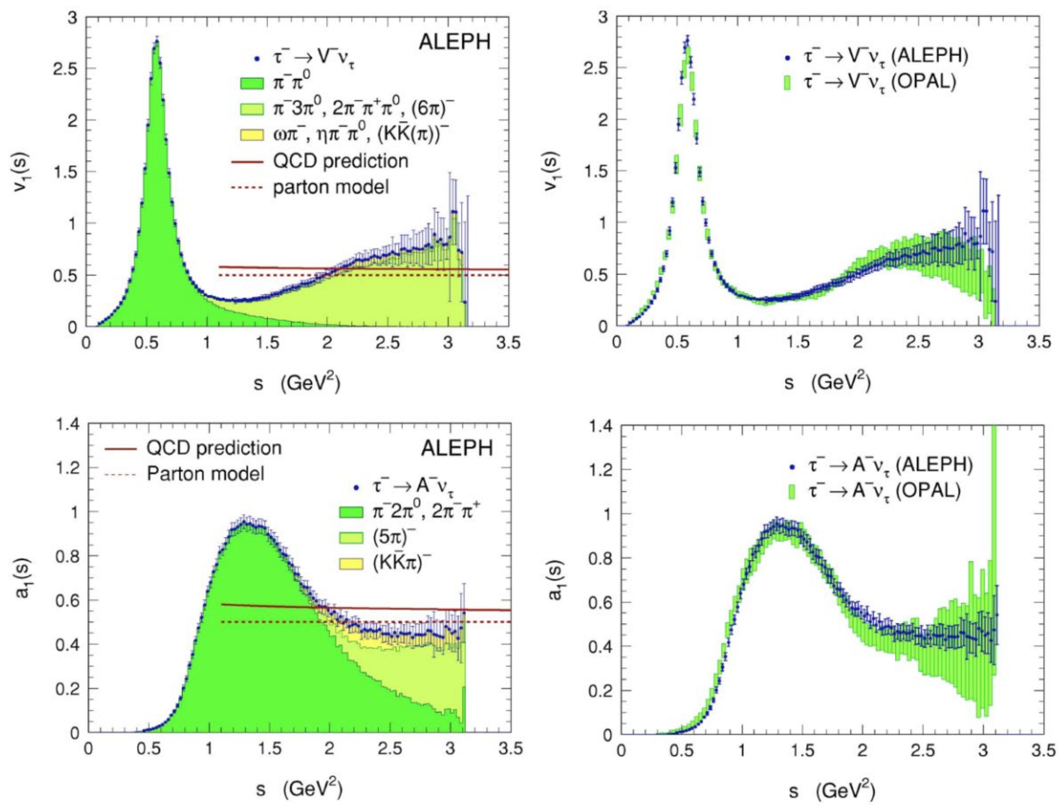
$$R_{\tau,S} = 0.1615\,(40)$$

[10,11]

$\mathrm{Im}\Pi^{(1)}_{V,A} =$ 

$= \frac{N_C}{12\pi}$

$$v_1 = \mathrm{a}_1 = \frac{N_C}{6} \qquad v_0 \simeq 0 \qquad \mathrm{a}_0 \propto \delta\left(s - M_\pi^2\right)$$



[10]

Working on the theoretical prediction of R_τ …. to get $\alpha_S(M_\tau)$,$|V_{us}|$ .....

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

Working on the theoretical prediction of R_τ .... to get $\alpha_S(M_\tau)$,$|V_{us}|$ .....

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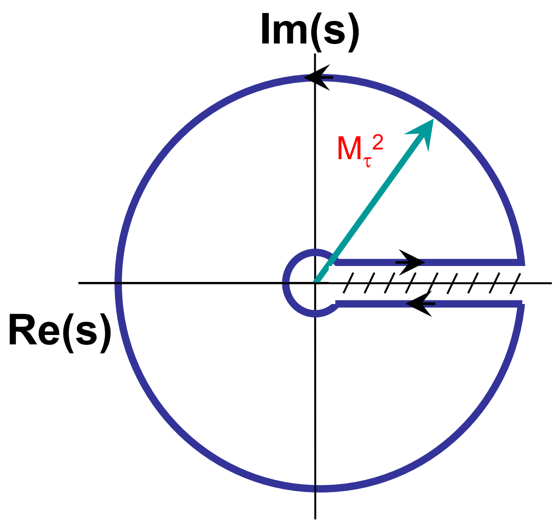
Cauchy's Theorem

$$\frac{1}{\pi}\int_0^{s_0} ds\, f(s)\, \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, f(s)\, \Pi(s)$$

- $\Pi(s)$ analytic everywhere except on the positive real axis
- $f(s)$ analytic

$$R_\tau \overset{[12]}{=} 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\Pi^{(0+1)}(s) - \frac{2s}{M_\tau^2}\Pi^{(0)}(s)\right]$$



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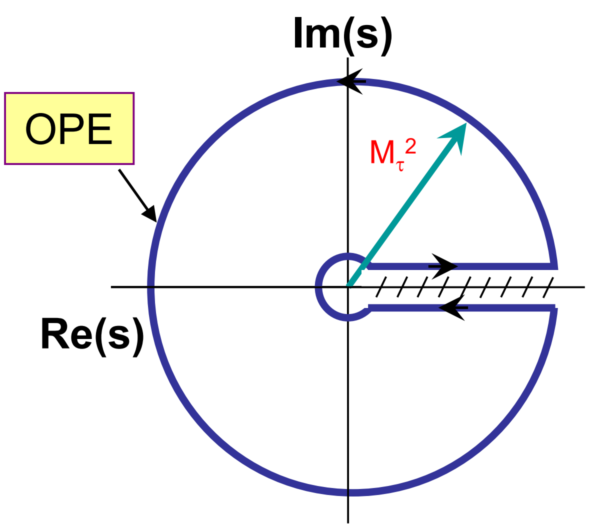

Operator Product Expansion (OPE)

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim \mathcal{O}=D} C_D^{(J)}(s,\mu)\,\langle \mathcal{O}_D(\mu)\rangle$$

$D=0 \rightarrow$ perturbative (expansion in $\alpha_S(\mu)$)

$D>0 \rightarrow$ non-perturbative (expansion in condensates)

Working on the theoretical prediction of R_τ .... to get $\alpha_S(M_\tau)$,$|V_{us}|$ .....

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

Cauchy's Theorem

$$\frac{1}{\pi}\int_0^{s_0} ds\, f(s)\,\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, f(s)\,\Pi(s)$$

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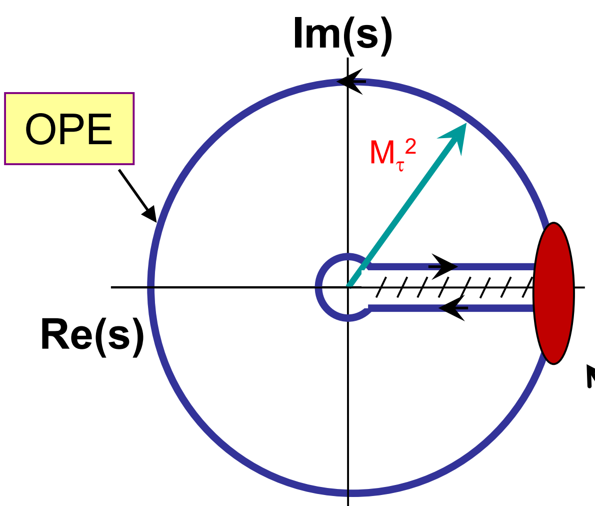

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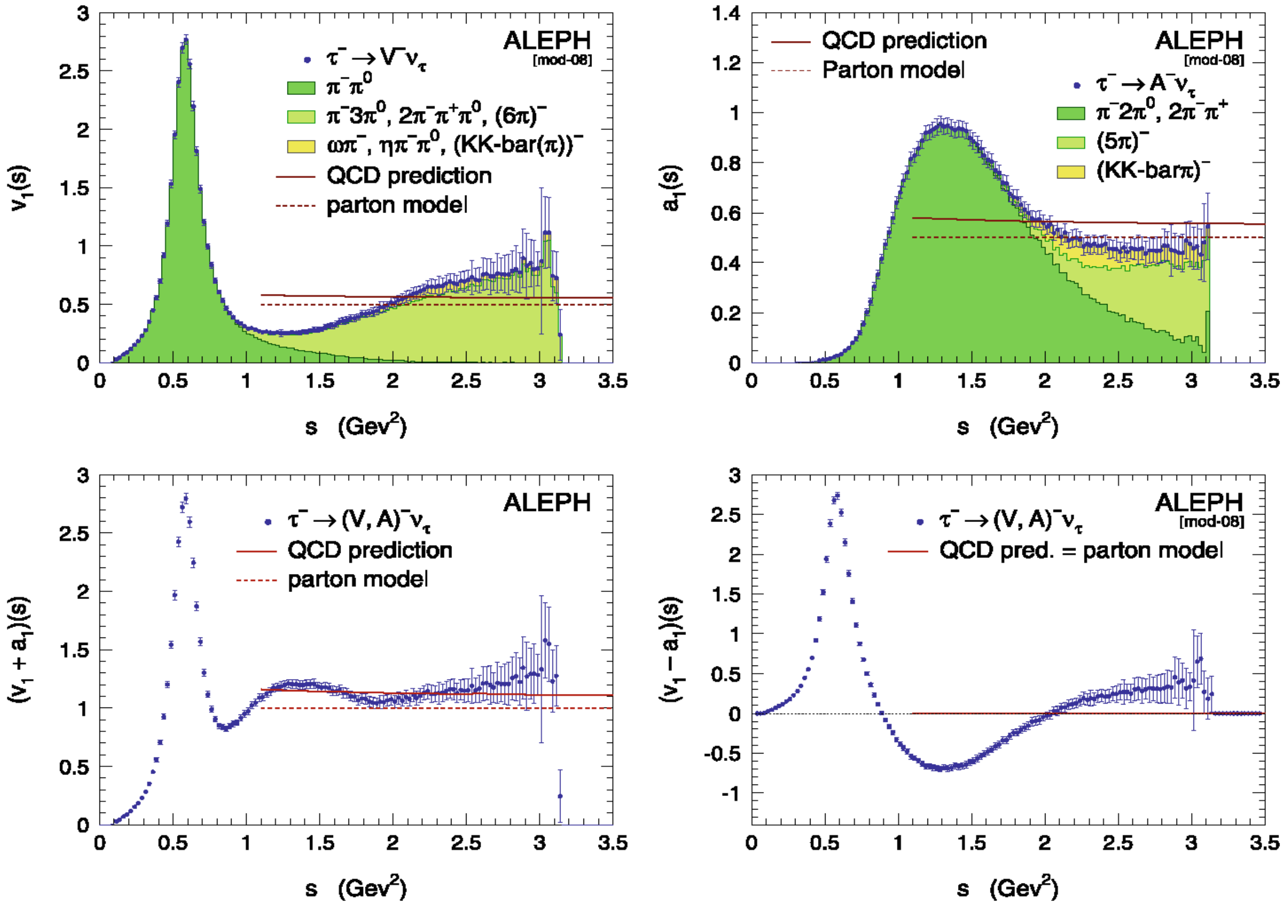

[10]

$$(V-A)\Big|_\chi \quad \propto \quad \text{non-perturbative} \qquad (m_u = m_d = m_s = 0)$$

$$(V+A) \quad \propto \quad \left(\text{perturbative} + \frac{1}{M_\tau^6}\,\text{non-perturbative}\right)$$

$\uparrow$

$\alpha_S(M_\tau)$

$$R_{\tau,V+A} = N_C \, |V_{ud}|^2 \, S_{\rm EW} \, \{1 + \delta_P + \delta_{NP}\}$$

$$S_{\rm EW} = 1.0201\,(3)$$

[3,13]

$$S_{\rm EW} \simeq 1 + \frac{3\,\alpha}{4\,\pi} \ln\left(\frac{M_Z^2}{M_\tau^2}\right) \left[\frac{4}{3} - \frac{\alpha_S}{3\,\pi}\right]$$

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$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots}\frac{1}{(-s)^{D/2}}\sum_{\dim\mathcal{O}=D} C_D^{(J)}(s,\mu)\,\langle\mathcal{O}_D(\mu)\rangle$$

Perturbative contribution

$m_q = 0$ [11,14,15]

$$\delta_P = \sum_{n=1} K_n\,A^{(n)}(\alpha_S)$$

$$A^{(n)}(\alpha_S) \equiv \frac{1}{2\pi i}\oint_{|s|=M_\tau^2}\frac{ds}{s}\left(\frac{\alpha_S(-s)}{\pi}\right)^n\left(1-2\frac{s}{M_\tau^2}+2\frac{s^3}{M_\tau^6}-\frac{s^4}{M_\tau^8}\right)$$

K_n known up to $\mathcal{O}\left(\alpha_S{}^4\right) \longrightarrow N_F = 3$

$$\begin{cases} K_0 = K_1 = 1 \\ K_2 = 1.63982 \\ K_3^{\overline{\rm MS}} = 6.37101 \\ K_4^{\overline{\rm MS}} = 49.07570 \end{cases}$$

$$a_\tau \equiv \alpha_S(M_\tau)/\pi$$

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$$a_\tau \equiv \alpha_S(M_\tau)/\pi$$

Fixed-order perturbation theory (FOPT)
Expansion of $A^{(n)}(\alpha_S)$ in powers of $\alpha_S(M_\tau^2)$

$$\delta_P = a_\tau + 5.2\,a_\tau^2 + 26.4\,a_\tau^3 + 127.1\,a_\tau^4$$

Contour-improved perturbation theory (CIPT)
Using the exact solution for $\alpha_S(s)$ given by the RG β-function equation

$$\delta_P = 1.4\,a_\tau + 2.5\,a_\tau^2 + 9.7\,a_\tau^3 + 64.3\,a_\tau^4$$

Non-perturbative contributions

$$\delta_{NP} = \frac{-1}{2\pi i}\oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)\sum_{D\geq 2}\frac{1}{(-s)^{D/2}}\, C_D(s,\mu)\,\langle\mathcal{O}_D(\mu)\rangle$$

$$\delta_{NP}\Big|_{C_D=\text{constant}} \simeq \frac{1}{M_\tau^2}C_2\langle\mathcal{O}_2\rangle - \frac{3}{M_\tau^6}\,C_6\,\langle\mathcal{O}_6\rangle - \frac{2}{M_\tau^8}\,C_8\,\langle\mathcal{O}_8\rangle + \ldots$$

Non-perturbative contributions

$$\delta_{NP} = \frac{-1}{2\pi i}\oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)\sum_{D\geq 2}\frac{1}{(-s)^{D/2}}C_D(s,\mu)\,\langle\mathcal{O}_D(\mu)\rangle$$

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[12,16]

$$C_2\,\langle\mathcal{O}_2\rangle \propto \left[1+\frac{16}{3}\frac{\alpha_S(M_\tau)}{\pi}\right](m_u^2(M_\tau)+m_d^2(M_\tau))$$

$$C_4\,\langle\mathcal{O}_4\rangle \propto \left(\frac{\alpha_S(M_\tau)}{\pi}\right)^2 \langle(\alpha_S/\pi)G_{\mu\nu}G^{\mu\nu}\rangle\,,$$

$$\langle m_u\overline{\psi}_u\psi_u + m_d\overline{\psi}_d\psi_d\rangle\,, \ldots.$$

$$C_6\,\langle\mathcal{O}_6\rangle \propto \frac{\alpha_S(M_\tau^2)}{\pi}\,\langle\overline{\psi}_u\Gamma\psi_d\overline{\psi}_d\Gamma\psi_u\rangle\,, \ldots.$$

$$C_8\,\langle\mathcal{O}_8\rangle \propto \langle(\alpha_S/\pi)G_{\mu\nu}G^{\mu\nu}\rangle^2\,, \ldots.$$

$$\langle\overline{\psi}_i\psi_i\rangle(2\,\text{GeV}) =$$

$-\,(283(2)\,\text{MeV})^3$ [Lattice] [17]

$-\,(267(16)\,\text{MeV})^3$ [Pheno] [18]

$$\langle\frac{\alpha_S}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle \simeq$$

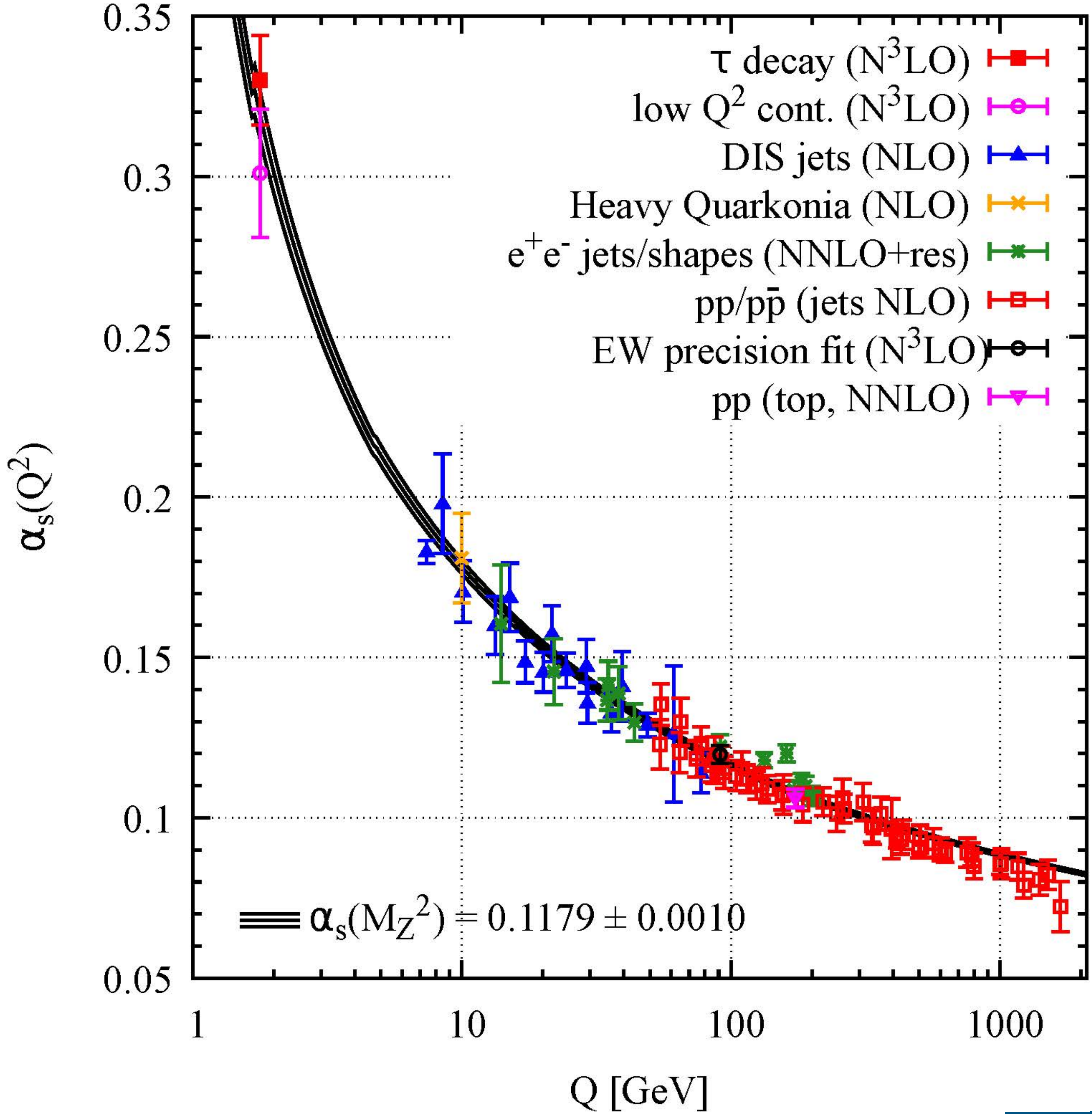
$0.012\,\text{GeV}^4$ [Sum Rules] [16]

$\alpha_S(M_\tau)$ Analyses

Reference	Method	δ_P	δ_{NP}	$\alpha_S(M_\tau)$	$\alpha_S(M_Z)$
Baikov et al. [15]	CIPT, FOPT	0.1998 (43)	-	0.332 (16)	0.1202 (19)
Davier et al. [11]	CIPT	0.2066 (70)	-0.0059 (14)	0.344 (09)	0.1212 (11)
Beneke-Jamin [19]	BSR + FOPT	0.2042 (50)	-0.007 (03)	0.316 (06)	0.1180 (08)
Maltman-Yavin [20]	PWM + CIPT	-	+0.012 (18)	0.321 (13)	0.1187 (16)
Narison [21]	CIPT, FOPT	-	-	0.324 (08)	0.1192 (10)
Caprini-Fischer [22]	BSR + CIPT	0.2037 (54)	-	0.322 (16)	-
Abbas et al. [23]	IFOPT	0.2037 (54)	-	0.338 (10)	-
Cvetic et al. [24]	β_{exp} + CIPT	0.2040 (40)	-	0.341 (08)	0.1211 (10)
Boito et al. [25]	FOPT, DV	-	-	0.308 (8)	0.1171 (10)
Pich-Rodríguez [26]	CIPT, FOPT	-	-	0.328 (13)	0.1197 (15)
C. Ayala et al. [27]	FOPT, PV	-	-	0.312 (7)	0.1176 (10)

CIPT : Contour-improved perturbation theory
FOPT : Fixed-order perturbation theory
BSR : Borel summation of renormalon series
IFOPT: Improved FOPT

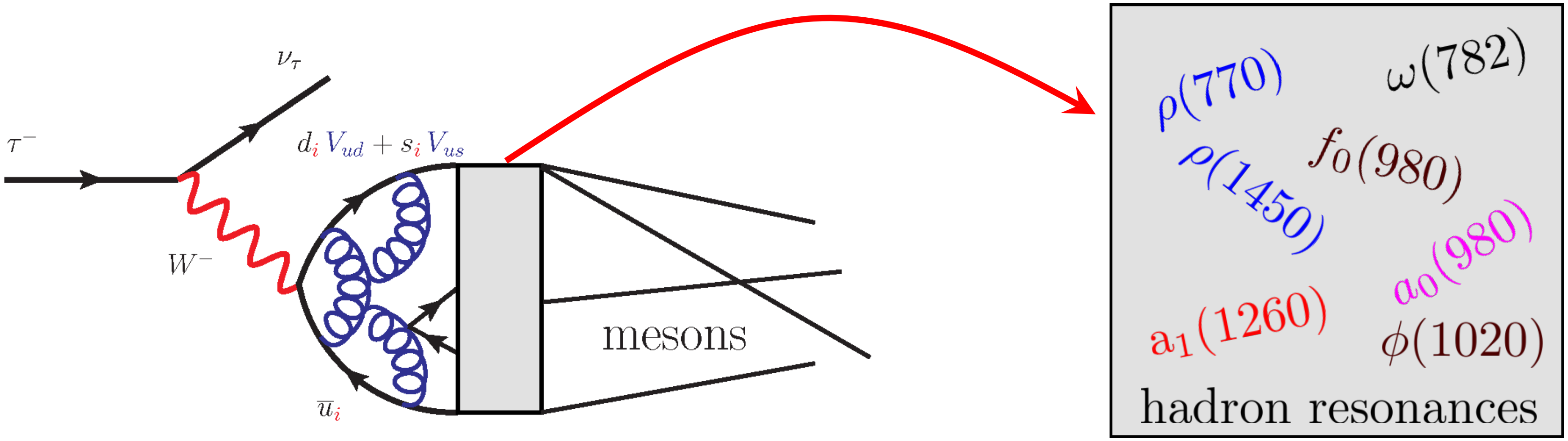
β_{exp} : Expansion in derivatives of α_s
PWM : Pinched-weight moments
DV : Duality violation
PV : Principal Value of Borel summation



τ decay (N3LO)
low Q2 cont. (N3LO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
e+e- jets/shapes (NNLO+res)
pp/pp̄ (jets NLO)
EW precision fit (N3LO)
pp (top, NNLO)
αs(MZ2) = 0.1179 ± 0.0010
αs(Q2)
Q [GeV]
0.35
0.3
0.25
0.2
0.15
0.1
0.05
1
10
100
1000

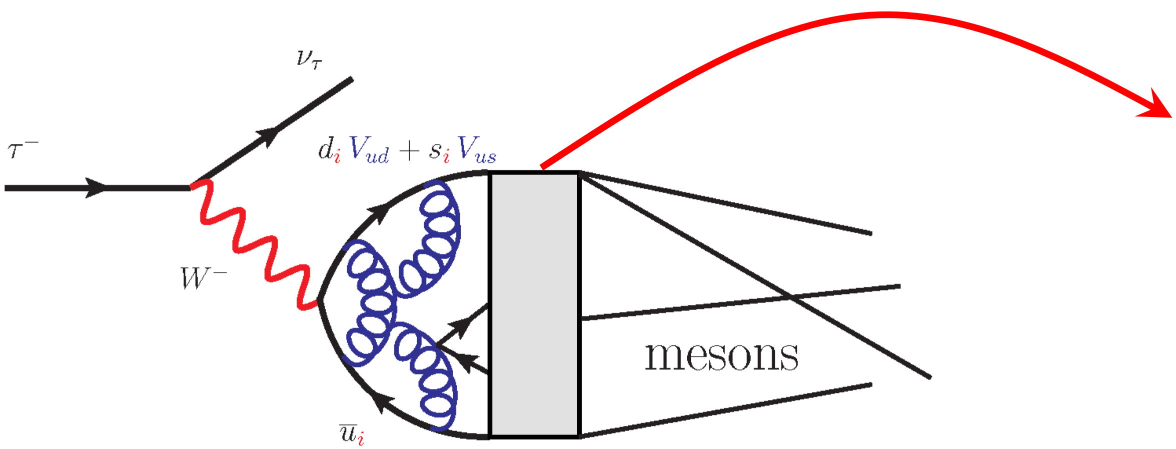
PDG

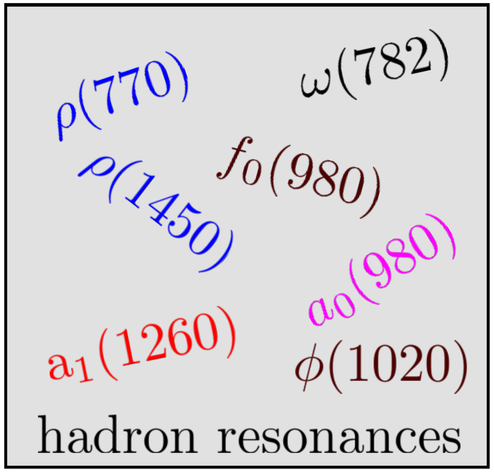
2.2 Exclusive hadron decays



$$\mathcal{M}\left(\tau \to \nu_\tau \mathrm{H}\right) = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}}\, \overline{u}_{\nu_\tau} \gamma^\mu \left(1-\gamma_5\right) u_\tau \left\langle H \right| \left(V_\mu - A_\mu\right) e^{i\,\mathrm{L}_{\mathrm{QCD}}} \left|\Omega_{\mathrm{H}}\right\rangle$$

2.2 Exclusive hadron decays





$$\mathcal{M}\left(\tau \to \nu_\tau \mathrm{H}\right) = \frac{G_F}{\sqrt{2}}\, V_{\mathrm{CKM}}\, \overline{u}_{\nu_\tau} \gamma^\mu \left(1-\gamma_5\right) u_\tau \,\langle H|\,\left(V_\mu - A_\mu\right)\, e^{i\,\mathrm{L}_{\mathrm{QCD}}}\,|\Omega_{\mathrm{H}}\rangle$$

$$\langle H|\,\left(V_\mu - A_\mu\right)\, e^{i\,\mathrm{L}_{\mathrm{QCD}}}\,|\Omega_{\mathrm{H}}\rangle = \sum_i \left(\text{Lorentz structure}\right)^i{}_\mu\, F_i(Q^2, s, \ldots)$$

form factors

$$d\Gamma\left(\tau \to \nu_\tau H\right) = \frac{G_F^2}{4\,M_\tau}\,|V_{\mathrm{CKM}}|^2\, L_{\mu\nu}\, H^{\mu\nu}\, d\mathrm{PS}$$

$$\begin{cases} L_{\mu\nu} H^{\mu\nu} = \sum_X L_X W_X \\ W_X \equiv \textbf{structure functions} \end{cases}$$

Examples

$$\mathrm{H} = PP \qquad\qquad P = \pi, K, \eta, \eta'$$

$$\langle P_1 P_2 | V_\mu \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle = F_V(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (p_1 - p_2)^\nu + F_S(q^2) \, q_\mu$$

$$q = p_1 + p_2 \qquad\qquad \partial^\mu V_\mu \propto (m_i - m_j)\, \overline{q}_i \, q_j$$

Examples

$$\mathrm{H} \;=\; PP \qquad\qquad P = \;\pi, K, \eta, \eta'$$

$$\langle P_1 P_2 | V_\mu \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \;=\; F_V(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (p_1 - p_2)^\nu + F_S(q^2) \; q_\mu$$

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$$\langle \pi^- \pi^0 | V_\mu \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \;=\; \sqrt{2}\, F_V(q^2) \; (p_- - p_0)_\mu$$

Vector form factor

Examples

$\mathrm{H} = PP \qquad P = \pi, K, \eta, \eta'$

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Vector form factor

$\mathrm{H} = PPP$

$$\langle P_1^- P_2^- P_3^+ | \, (V_\mu - A_\mu) \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle =$$

$$\left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) \left[F_1(Q^2, s, t) \, (p_1 - p_3)^\nu + F_2(Q^2, s, t) \, (p_2 - p_3)^\nu \right]$$

$$+ F_3(Q^2, s, t) \, Q_\mu + i F_4(Q^2, s, t) \, \varepsilon_{\mu\alpha\beta\gamma} \, p_3^\alpha p_2^\beta p_1^\gamma$$

$$Q = p_1 + p_2 + p_3$$
$$s = (p_2 + p_3)^2$$
$$t = (p_1 + p_3)^2$$

Examples

$\mathrm{H} = PP \qquad P = \pi, K, \eta, \eta'$

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$$Q = p_1 + p_2 + p_3$$
$$s = (p_2 + p_3)^2$$
$$t = (p_1 + p_3)^2$$

F_3 (crossed out): $\pi\pi\pi, KK\pi, m_\pi = 0$

F_4 (crossed out): $\pi\pi\pi, SU(2)_I$

Examples

$\mathrm{H} \;=\; PP \qquad\qquad P = \;\pi, K, \eta, \eta'$

$$\langle P_1 P_2 | V_\mu \, e^{i \mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \;=\; F_V(q^2) \; \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (p_1 - p_2)^\nu + F_S(q^2) \; q_\mu$$

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Vector form factor

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$$\left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) \left[\, F_1(Q^2, s, t) \, (p_1 - p_3)^\nu + F_2(Q^2, s, t) \, (p_2 - p_3)^\nu \, \right]$$

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$$Q = p_1 + p_2 + p_3 \qquad s = (p_2 + p_3)^2 \qquad t = (p_1 + p_3)^2$$

$\pi\pi\pi, KK\pi, m_\pi = 0$ (for F_3) $\qquad \pi\pi\pi, SU(2)_I$ (for F_4)

$$\tau \;\rightarrow\; \pi\pi\pi\nu_\tau$$

$$F_2(Q^2, s, t) = F_1(Q^2, t, s)$$

Bose symmetry, Axial-Vector only

Examples

$\mathrm{H} = PP \qquad P = \pi, K, \eta, \eta'$

$$\langle P_1 P_2 | V_\mu \, e^{i \mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle = F_V(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (p_1 - p_2)^\nu + F_S(q^2) \, q_\mu$$

$$q = p_1 + p_2 \qquad \partial^\mu V_\mu \propto (m_i - m_j) \, \overline{q}_i \, q_j$$

$$\langle \pi^- \pi^0 | V_\mu \, e^{i \mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle = \sqrt{2} \, F_V(q^2) \, (p_- - p_0)_\mu$$

Vector form factor

$\mathrm{H} = PPP$

$$\langle P_1^- P_2^- P_3^+ | \, (V_\mu - A_\mu) \, e^{i \mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle =$$

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$$Q = p_1 + p_2 + p_3$$
$$s = (p_2 + p_3)^2$$
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$\pi\pi\pi, KK\pi, m_\pi = 0$ (for F_3) $\qquad$ $\pi\pi\pi, SU(2)_I$ (for F_4)

$$\tau \to \pi\pi\pi\nu_\tau$$
$$F_2(Q^2, s, t) = F_1(Q^2, t, s)$$
Bose symmetry, Axial-Vector only

$$\tau \to KK\pi\nu_\tau$$
Vector and Axial-Vector

Phenomenological Lagrangians : Tree Level

$$\mathcal{L}_\chi^2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$\mathcal{L}_R = \sum_i \lambda_i \, \mathcal{O}_R^i(R,\phi)$$

$$\mathcal{L}_R^K \text{ (kinetic)}$$

Resonance Chiral Theory

RχT

[28,29]

Chiral Perturbation Theory

Resonance Fields

Large $- N_C$

Phenomenological Lagrangians : Tree Level

$$\mathcal{L}_\chi^2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$\mathcal{L}_R = \sum_i \lambda_i\, \mathcal{O}_R^i(R,\phi)$$

$\mathcal{L}_R^K$ (kinetic)

Resonance Chiral Theory

RχT

[28,29]

Chiral Perturbation Theory

Resonance Fields

Large $-\ N_C$

$$u_\mu = i\left[u^\dagger\left(\partial_\mu - i\, r_\mu\right)u - u\left(\partial_\mu - i\,\ell_\mu\right)u^\dagger\right]$$

$$\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u, \qquad \chi = 2B_0\,(s + i\,p)$$

$$u = \exp\left(\frac{i}{F\sqrt{2}}\Pi(\Phi)\right)$$

F = decay constant of the pion

$$B_0^2\, F = -\langle \overline{u}u\rangle$$

$$\mathcal{L}_R = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + \ldots$$

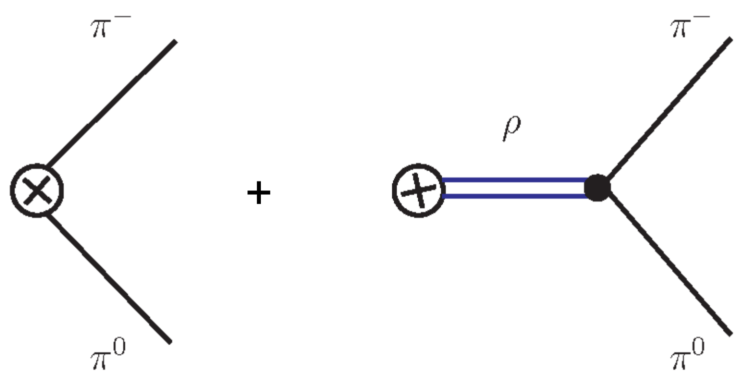
$$f_{\mu\nu}^\pm = uF_{\mu\nu}^L u^\dagger \pm u^\dagger F_{\mu\nu}^R u, \qquad F_{\mu\nu}^L = \partial_\mu\ell_\nu - \partial_\nu\ell_\mu - i\left[\ell_\mu,\ell_\nu\right]$$

$$\tau^- \to \pi^- \pi^0 \nu_\tau$$

[29,30,31]

$$\langle \pi^- \pi^0 | V_\mu \, e^{i\mathcal{L}_{\rm QCD}}) | \Omega_h \rangle \;=\; \sqrt{2}\, F_V(q^2)\; (p_- - p_0)_\mu$$

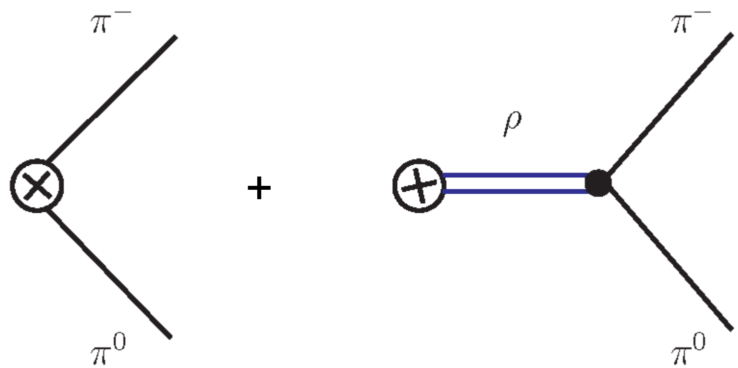
$F_V(q^2) =$



$\tau^- \to \pi^- \pi^0 \nu_\tau$ [30,31,32]

$$\langle \pi^- \pi^0 | V_\mu \, e^{i \mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \; = \; \sqrt{2}\, F_V(q^2) \; (p_- - p_0)_\mu$$

$F_V(q^2) \; =$



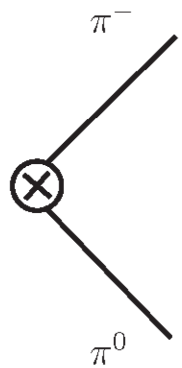
$$= \; 1 \; + \; \frac{F_V \, G_V}{F^2} \, \frac{q^2}{M_V^2 - q^2}$$

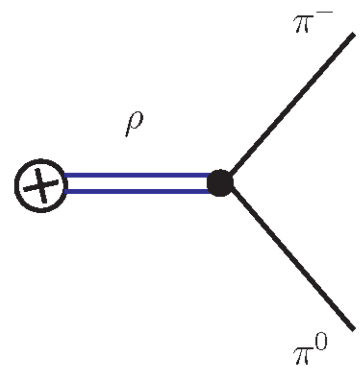
$1-$ Short-distance constraints

$$F_V(q^2) \underset{q^2 \to \infty}{\longrightarrow} \frac{1}{q^2} \qquad \longrightarrow \qquad F_V \, G_V = F^2$$

$$\tau^- \to \pi^- \pi^0 \nu_\tau$$ [29,30,31]

$$\langle \pi^- \pi^0 | V_\mu \, e^{i L_{\rm QCD}}) | \Omega_h \rangle \; = \; \sqrt{2}\, F_V(q^2) \; (p_- - p_0)_\mu$$

$F_V(q^2) \; =$ 

$+$ 

$= \; 1 + \dfrac{F_V \, G_V}{F^2} \dfrac{q^2}{M_V^2 - q^2}$

1 − Short-distance constraints

$$F_V(q^2) \underset{q^2 \to \infty}{\longrightarrow} \frac{1}{q^2} \qquad \longrightarrow \qquad F_V \, G_V = F^2$$

2 − Off-shell widths of resonances

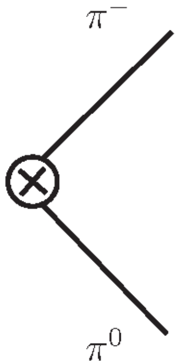
$$M_V^2 \;\longrightarrow\; M_V^2 - i M_V \Gamma_V(q^2)$$

$\tau^- \to \pi^- \pi^0 \nu_\tau$

[29,30,31]

$$\langle \pi^- \pi^0 | V_\mu \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \; = \; \sqrt{2}\, F_V(q^2)\; (p_- - p_0)_\mu$$

$F_V(q^2) \; = $



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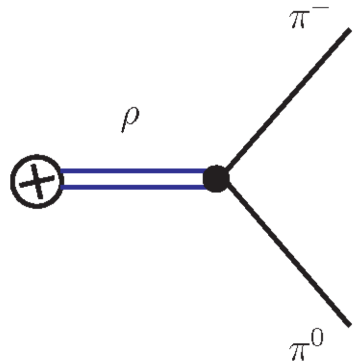


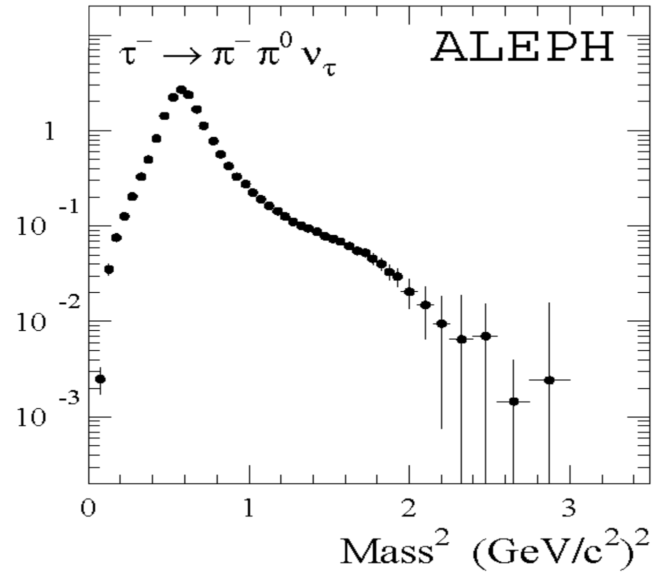
$$= \; 1 + \frac{F_V\, G_V}{F^2} \frac{q^2}{M_V^2 - q^2}$$

1 – Short-distance constraints

$$F_V(q^2) \underset{q^2 \to \infty}{\longrightarrow} \frac{1}{q^2} \qquad \longrightarrow \qquad F_V\, G_V = F^2$$

2 – Off-shell widths of resonances

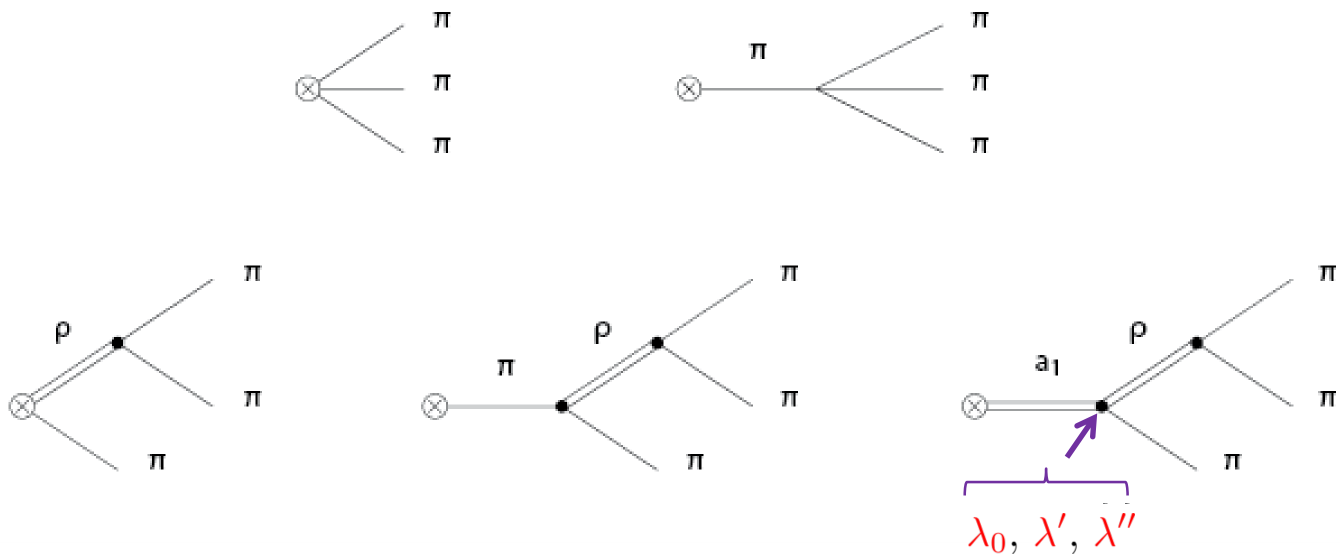
$$M_V^2 \;\longrightarrow\; M_V^2 - i M_V \Gamma_V(q^2)$$



$$\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau$$

[32]

$$F_1(Q^2, s, t) \; =$$



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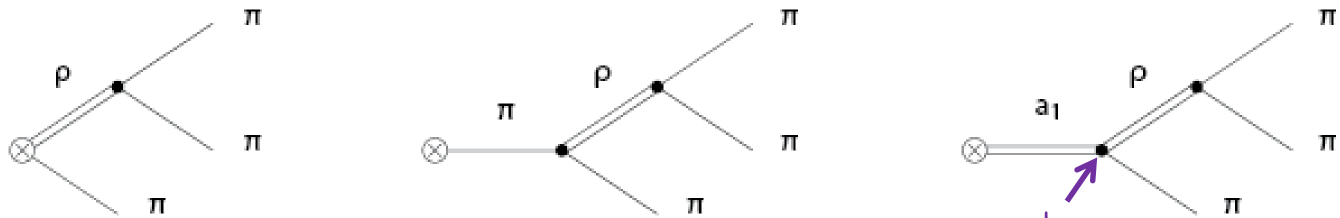

$\lambda_0,\ \lambda',\ \lambda''$

$$H(Q^2,x) = -\lambda_0\frac{M_\pi^2}{Q^2} + \lambda'\frac{x}{Q^2} + \lambda''$$

$$\begin{aligned} F_1(Q^2,s,t) \;&=\; -\frac{2\sqrt{2}}{3F} + \frac{\sqrt{2}F_V G_V}{3F^3}\left[\frac{3s}{s-M_V^2} - \left(\frac{2G_V}{F_V}-1\right)\left(\frac{2Q^2-2s-u}{s-M_V^2}+\frac{u-s}{t-M_V^2}\right)\right] \\ &+\frac{4F_A G_V}{3F^3}\frac{Q^2}{Q^2-M_A^2}\left[-(\lambda'+\lambda'')\frac{3s}{s-M_V^2} + H(Q^2,s)\frac{2Q^2+s-u}{s-M_V^2} + H(Q^2,t)\frac{u-s}{t-M_V^2}\right] \end{aligned}$$

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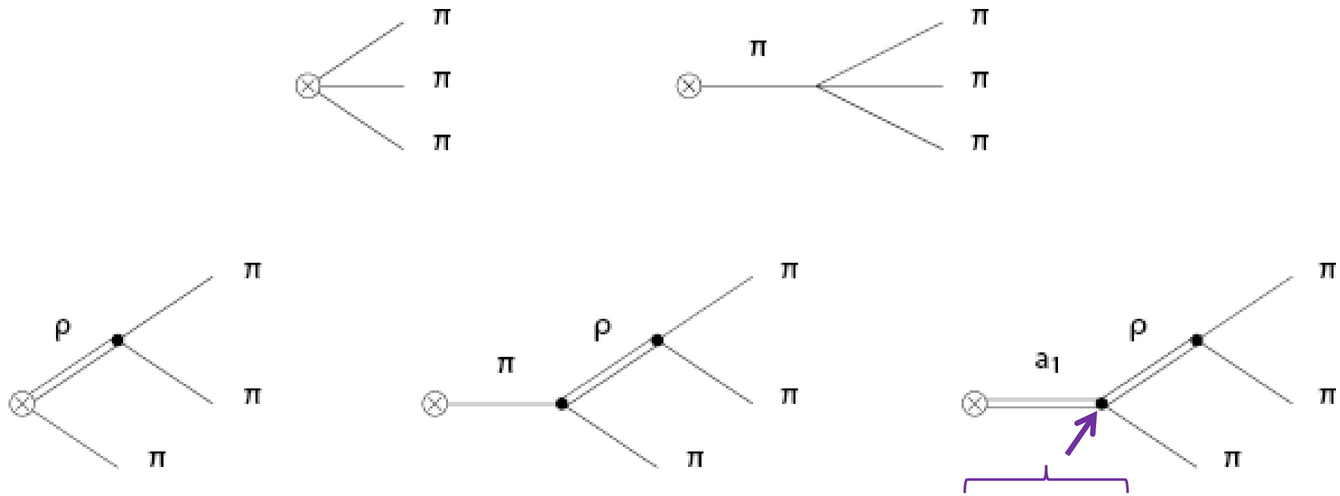

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Short-distance constraints

$$\mathrm{Im}\,\Pi_A(q^2) \underset{q^2\to\infty}{\longrightarrow} 0$$

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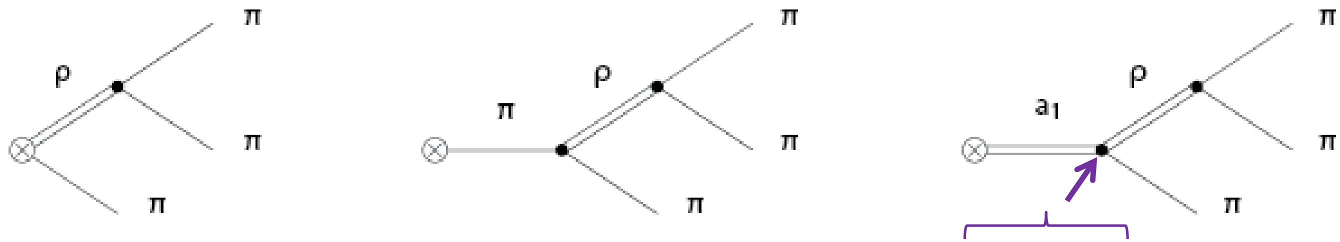

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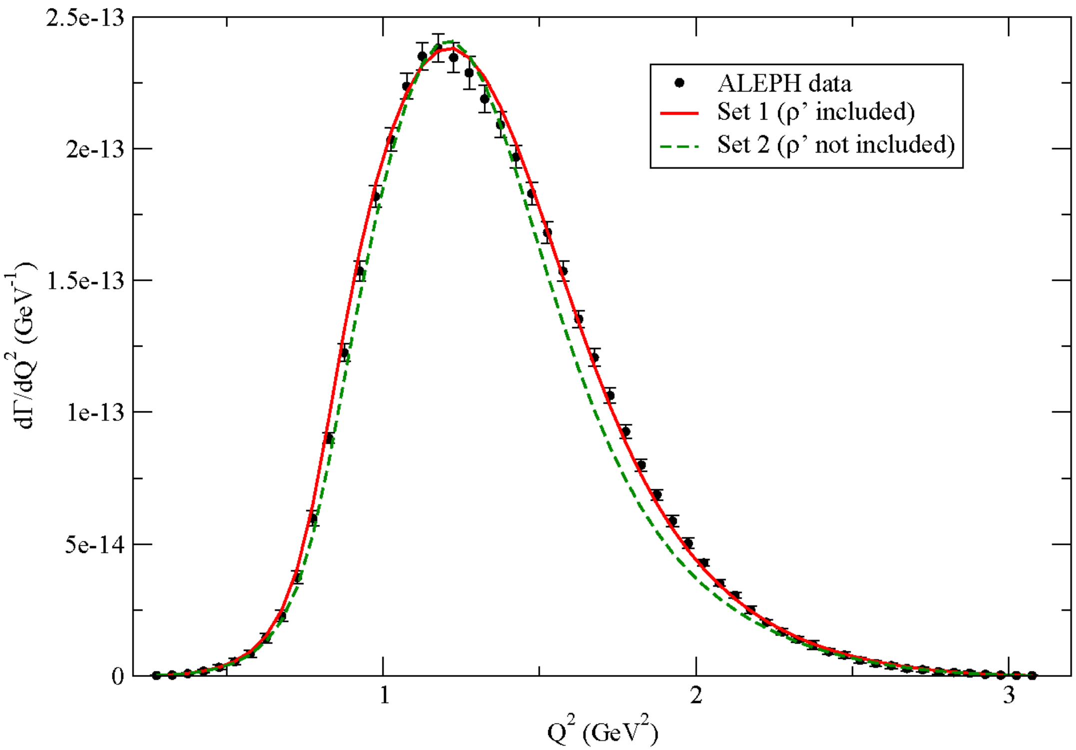
Short-distance constraints

$$\mathrm{Im}\, \Pi_A(q^2) \underset{q^2 \to \infty}{\longrightarrow} 0$$

$$\left\{ \begin{aligned} \lambda' &= \frac{M_A}{2\sqrt{2} M_V} \\ \lambda'' &= \frac{M_A^2 - 2M_V^2}{2\sqrt{2} M_V M_A} \\ \lambda_0 &= (\lambda' + \lambda'')/4 \end{aligned} \right.$$

[33]



Set 1

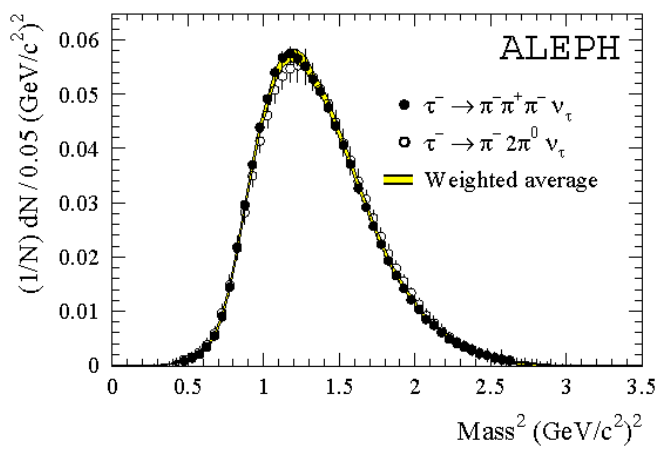
$$F_V = 0.180\,\text{GeV}, \qquad F_A = 0.149\,\text{GeV}$$
$$M_V = 0.775\,\text{GeV}, \qquad M_A = 1.120\,\text{GeV}$$

Set 2

$$F_V = 0.206\,\text{GeV}, \qquad F_A = 0.145\,\text{GeV}$$
$$M_V = 0.775\,\text{GeV}, \qquad M_A = 1.115\,\text{GeV}$$

$$\Gamma(\tau \to \pi\pi\pi\nu_\tau)\Big|_{\text{theo}} = 2.09 \times 10^{-13}\,\text{GeV}$$

$$\Gamma(\tau \to \pi\pi\pi\nu_\tau)\Big|_{\text{exp}} = 2.11(02) \times 10^{-13}\,\text{GeV}$$



Deviations from the Standard Model

$$\mathcal{L}_{CC} = -\frac{G_F}{\sqrt{2}} V_{uD} \Big[(1+\varepsilon_L^\tau)\, \bar\tau\, \gamma_\mu (1-\gamma_5)\, \nu_\tau\, \bar u\, \gamma^\mu (1-\gamma_5)\, D$$

[34]

$$+\, \varepsilon_R^\tau\, \bar\tau\, \gamma_\mu (1-\gamma_5) \nu_\tau\, \bar u \gamma^\mu (1+\gamma_5)\, D$$

$$+\, \bar\tau\, (1-\gamma_5)\, \nu_\tau\, \bar u\, (\varepsilon_S^\tau - \varepsilon_P^\tau \gamma_5)\, D$$

$D = d, s$

$$+\, \varepsilon_T^\tau\, \bar\tau\, \sigma_{\mu\nu} (1-\gamma_5)\, \nu_\tau\, \bar u\, \sigma^{\mu\nu} (1-\gamma_5)\, D \Big] + h.c.$$

SM $\longrightarrow$ $\varepsilon_L^\tau = \varepsilon_R^\tau = \varepsilon_S^\tau = \varepsilon_P^\tau = \varepsilon_T^\tau = 0$

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$\tau^- \to P^- \nu_\tau$

$P = \pi, K$

$\tau \to P_1 P_2\, \nu_\tau$

$(P_1, P_2) = (\pi,\pi), (K,\pi), (K,\eta)$

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$(P_1, P_2) = (\pi,\pi), (K,\pi), (K,\eta)$

$$\begin{pmatrix} \varepsilon_L^\tau - \varepsilon_L^e + \varepsilon_R^\tau - \varepsilon_R^e \\ \varepsilon_R^\tau \\ \varepsilon_P^\tau \\ \varepsilon_S^\tau \\ \varepsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.029 \pm 0.014 \\ 0.071 \pm 0.411 \\ -0.076 \pm 0.540 \\ 0.050 \pm 0.016 \\ -0.005 \pm 0.012 \end{pmatrix}$$

3. Breaking the SM rules

$$\left.\begin{array}{c}\text{SM}\\ m_\nu = 0\end{array}\right\} \longrightarrow \left[U(1)_{\mathrm{B}} \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau\right]_{\text{global}}$$

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LFV is large in the neutrino sector :

Oscillations

$$\theta_{12} \approx 30^0, \qquad \theta_{23} \approx 45^0, \qquad \theta_{13} \approx 0^0$$

Solar neutrinos

$$\nu_e \quad \Rightarrow \quad \frac{1}{3}\left(\nu_e + \nu_\mu + \nu_\tau\right)$$

$\times 10^8$

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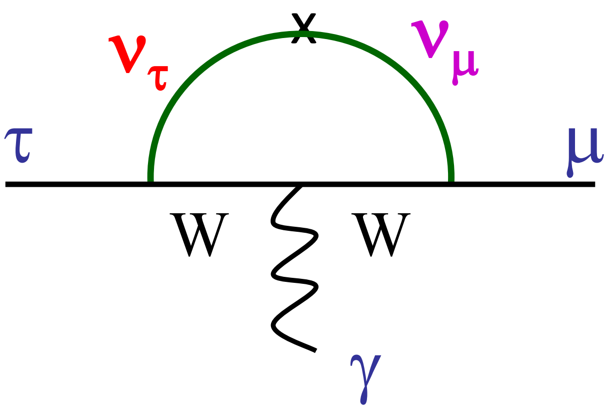
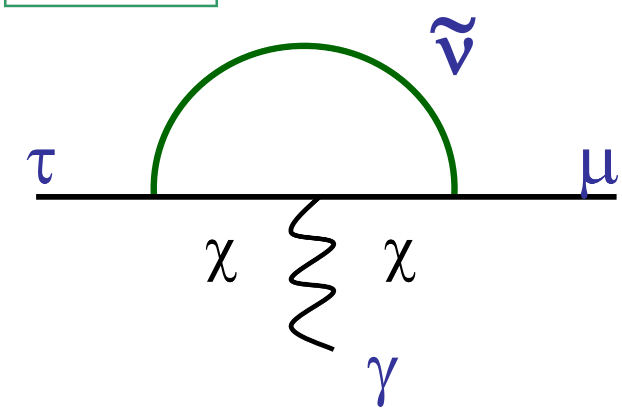
SM + ν_R



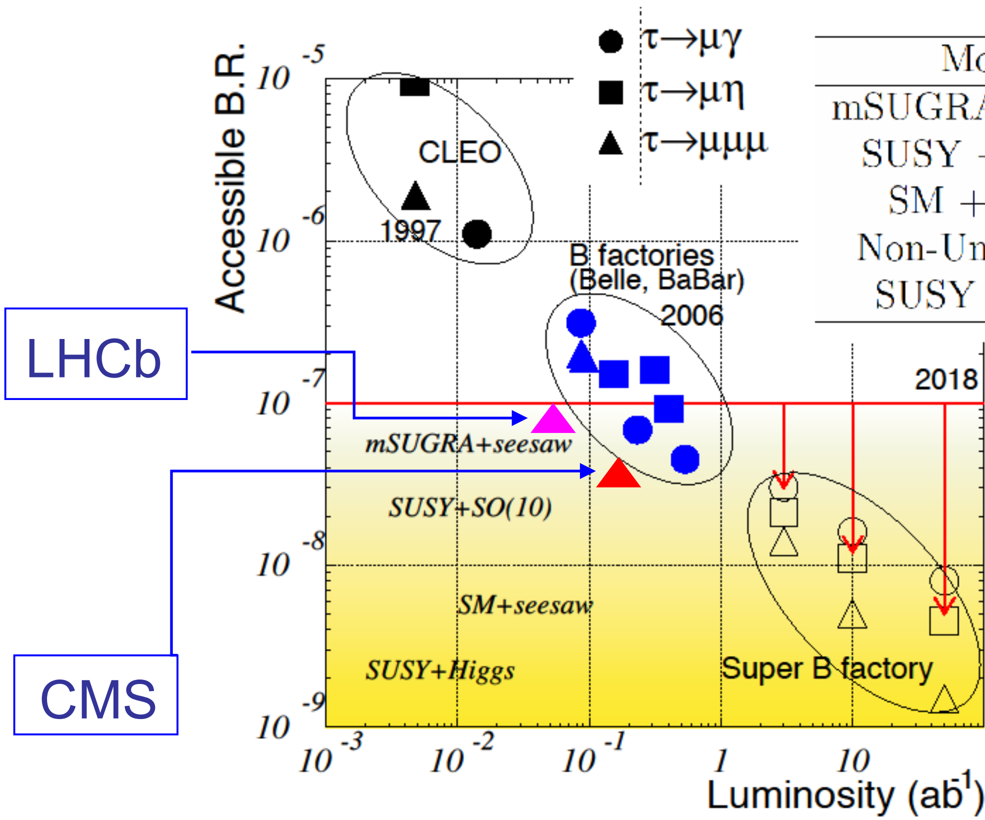
$$\frac{B(\tau \to \mu\,\gamma)}{B(\tau \to \mu \bar{\nu}_\mu \nu_\tau)} = \frac{3\alpha}{128\pi}\left(\frac{\Delta m_{23}^2}{m_W^2}\right)^2 \sin^2 2\theta_{mix} \approx O\left(10^{-53}\right)$$

BSM



some SUSY models can bring up to

$$\frac{B(\tau \to \mu\,\gamma)}{B(\tau \to \mu\bar{\nu}_\mu \nu_\tau)} \approx O\left(10^{-6} - 10^{-7}\right)$$



Models	$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
mSUGRA + seesaw	10^{-7}	10^{-9}
SUSY + SO(10)	10^{-8}	10^{-10}
SM + seesaw	10^{-9}	10^{-10}
Non-Universal Z'	10^{-9}	10^{-8}
SUSY + Higgs	10^{-10}	10^{-7}

Y. Kwon, SUSY08

An interesting feature…involving baryons

$U(1)_{\mathrm{B+L}}$ anomalous $\Big\} \longrightarrow$

$$\partial_\mu \mathcal{J}_{\mathrm{B}}{}^{\mu} = \partial_\mu \mathcal{J}_{\mathrm{L}}{}^{\mu} = \mathcal{O}(\hbar)$$

$$\Delta(B-L) = 0$$

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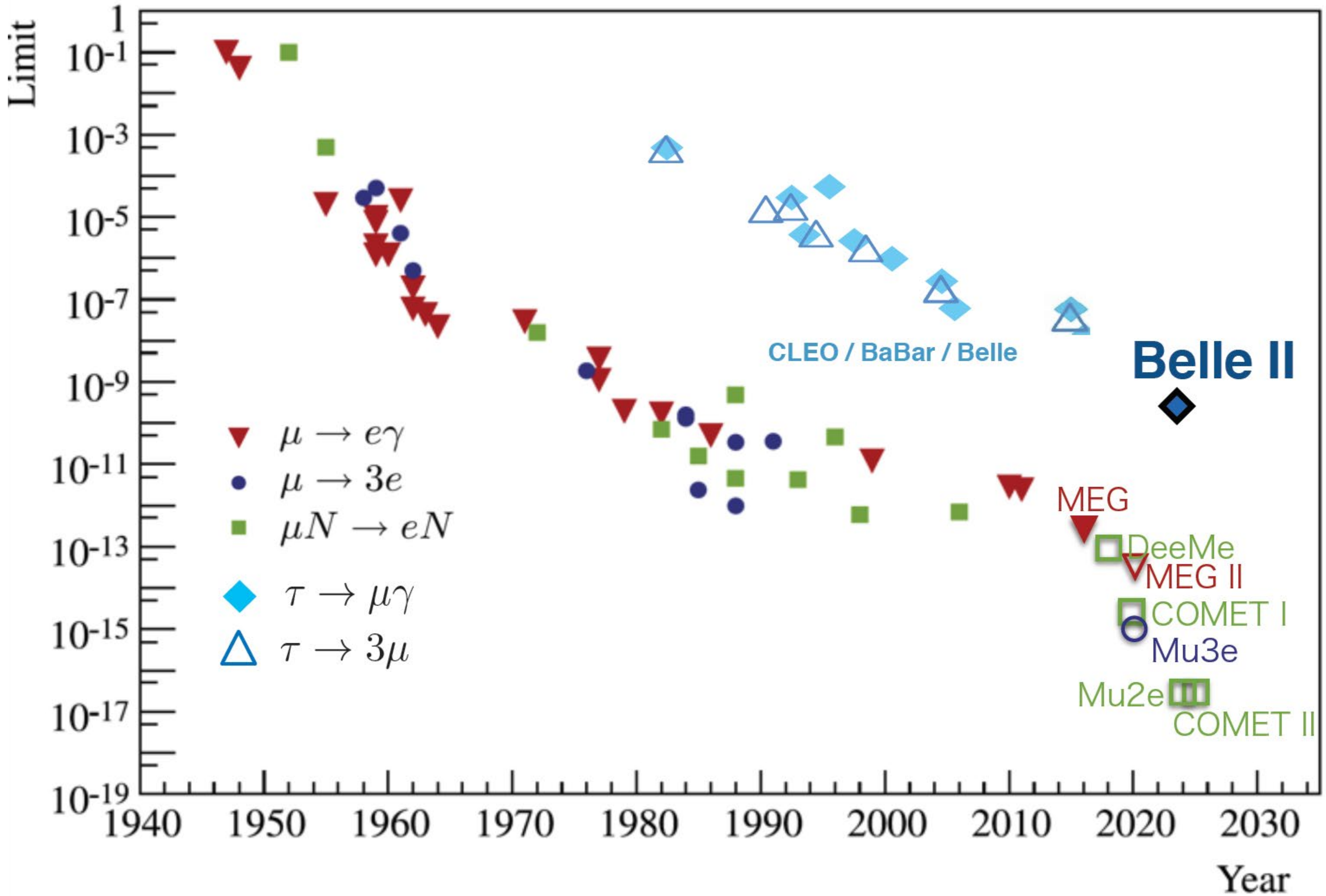
$$\partial_\mu \mathcal{J}_{\mathrm{B}}{}^{\mu} = \partial_\mu \mathcal{J}_{\mathrm{L}}{}^{\mu} = \mathcal{O}(\hbar)$$

$$\Delta(B-L)=0$$

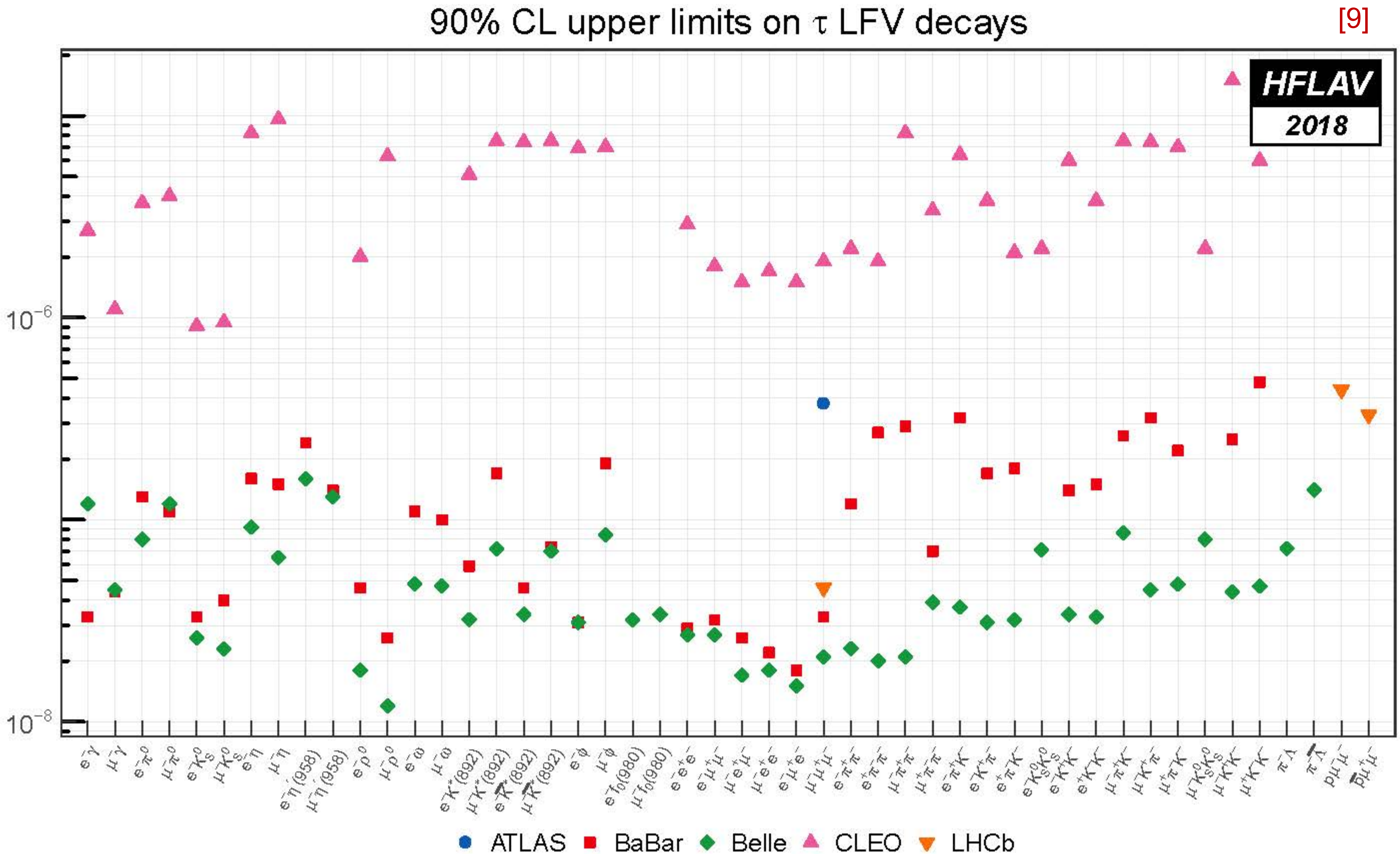
$$\Delta B = \Delta L = \pm 1$$

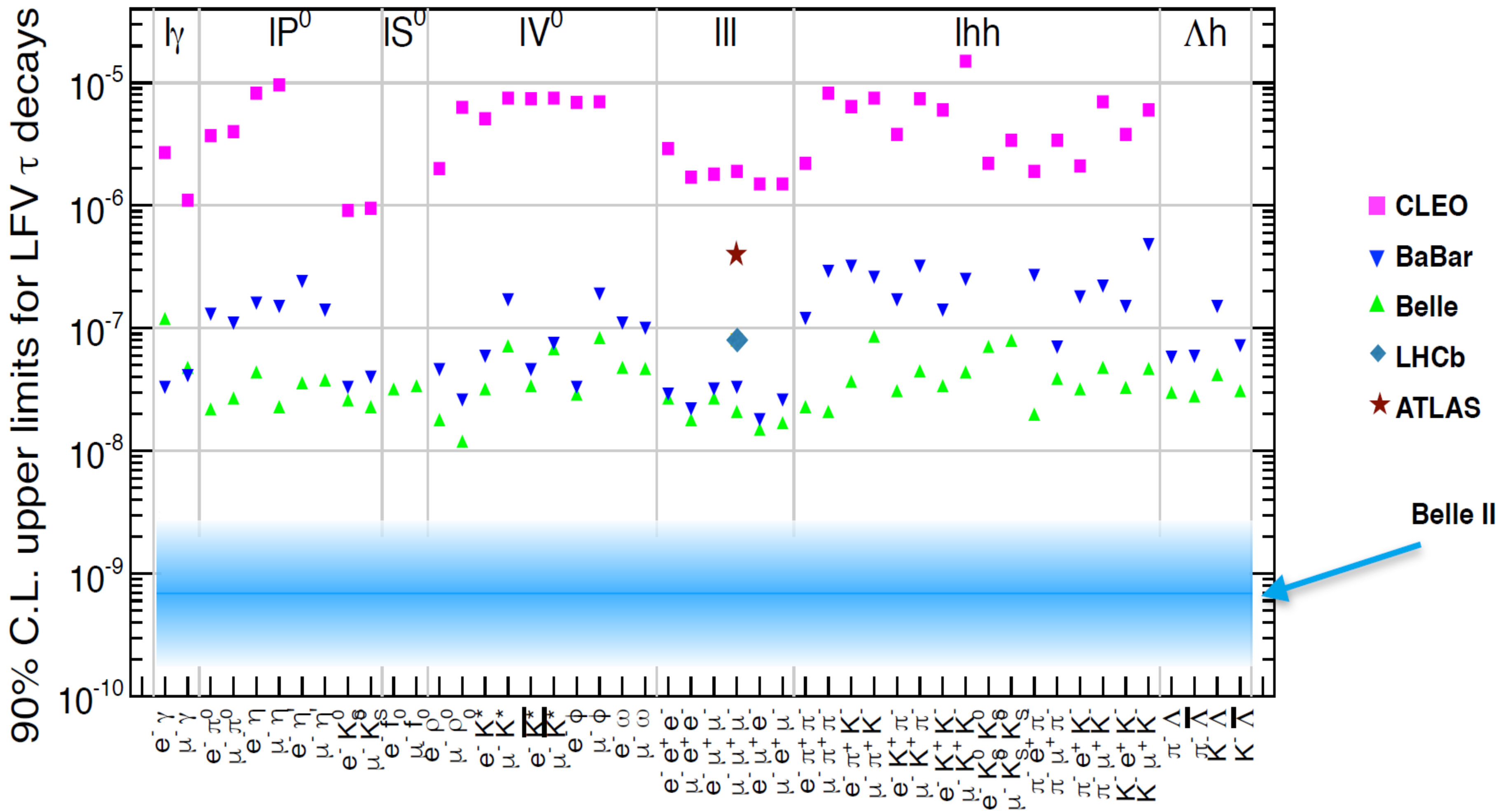
$\times 10^8$	Belle [35]	LHCb [36]
$Br(\tau^- \to \bar{p}\mu^+\mu^-)$	< 1.8	< 23
$Br(\tau^- \to p\mu^-\mu^-)$	< 4.0	< 44
$Br(\tau^- \to \bar{p}e^+e^-)$	< 3.0	
$Br(\tau^- \to pe^-e^-)$	< 3.0	
$Br(\tau^- \to \bar{p}e^+\mu^-)$	< 2.0	
$Br(\tau^- \to \bar{p}e^-\mu^+)$	< 1.8	



A. Rostomyan, TAU2018

[9]


90% CL upper limits on τ LFV decays
HFLAV
2018
10⁻⁶
10⁻⁸
ATLAS
BaBar
Belle
CLEO
LHCb



A. Rostomyan, TAU2018

Model-independent analysis in SMEFT [37]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{D>4} \left(\frac{1}{\Lambda^{D-4}} \sum_i C_i^{(D)} \mathcal{O}_i^{(D)} \right)$$ [38,39]

$$[\Lambda] = [E]$$

$$[\mathcal{O}_i^{(D)}] = [E^D]$$

$\mathcal{O}_i^{(D)}$ {
Field content of SM

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

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$$\tau \to \ell P : \quad P = \pi^0, K^0, \eta, \eta'$$
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$$\tau \to \ell V : \quad V = \rho^0(770), \omega(782), \phi(1020), K^{*0}(892), \bar{K}^{*0}(892)$$
$$\ell N(A,Z) \to \tau X : \quad N(A,Z) = \mathrm{Fe}(56,26), \mathrm{Pb}(208,82)$$

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Belle data

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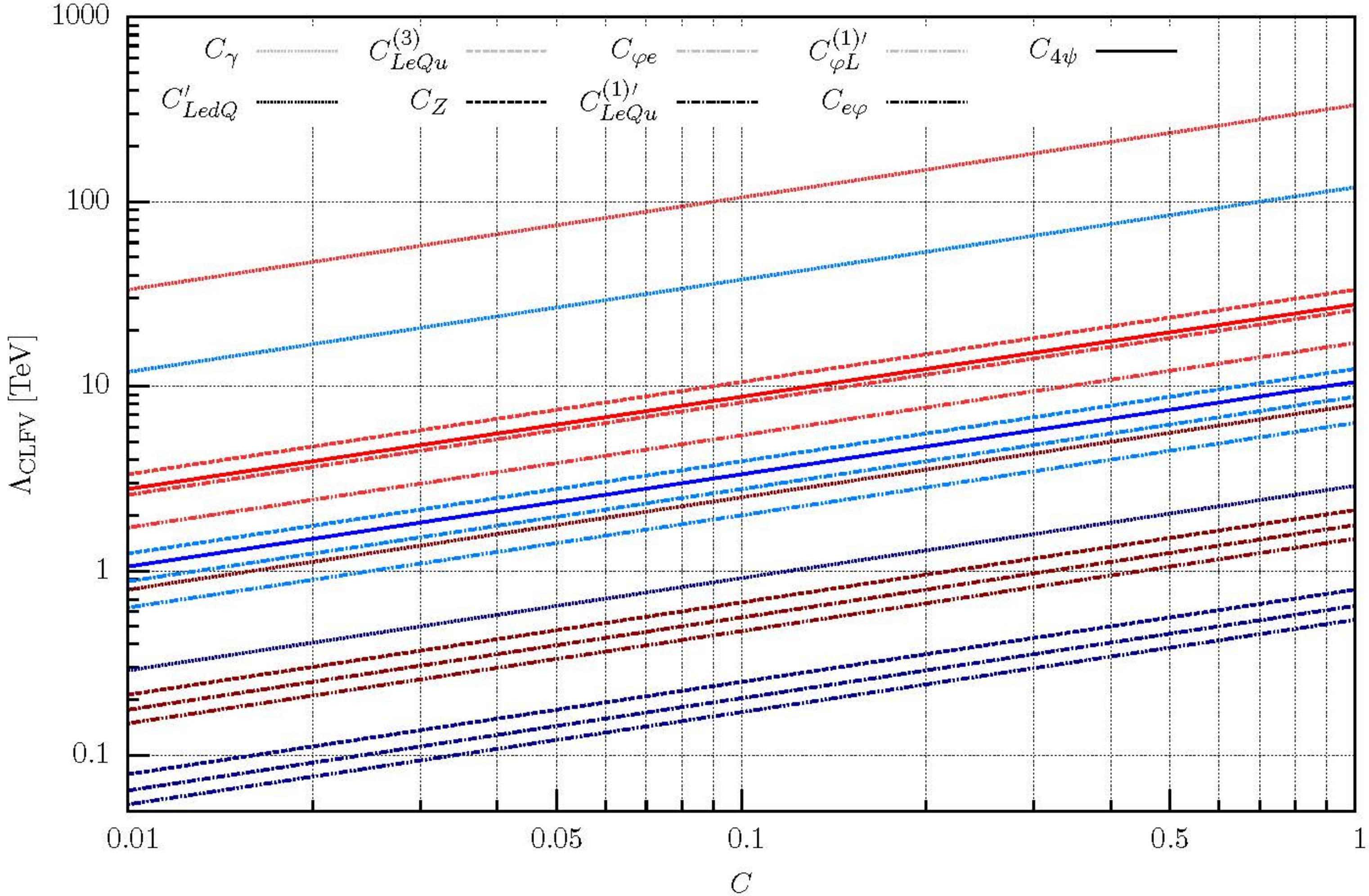
NA64 input

D = 6 ΔL = 1 operators

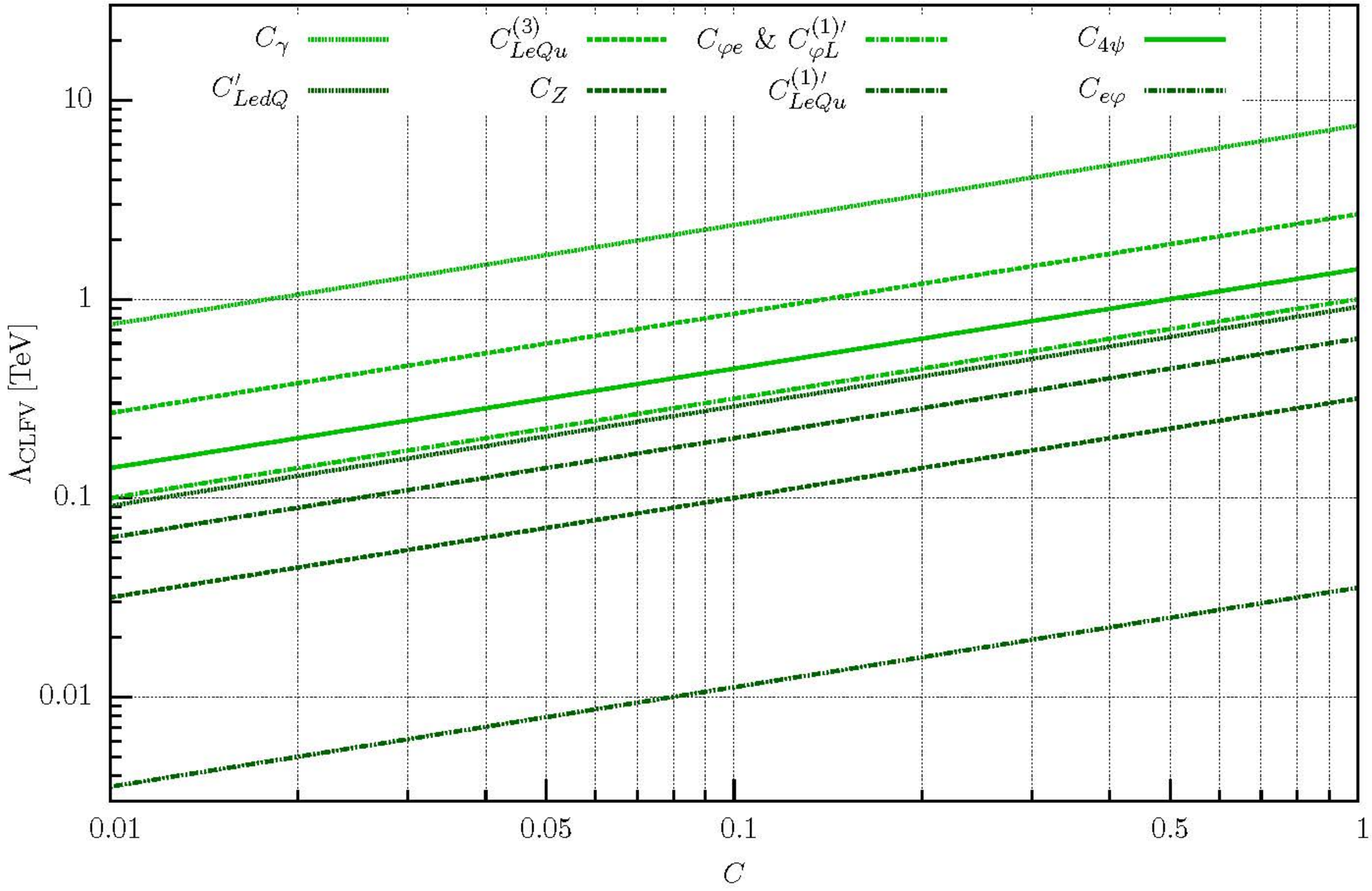
$\Lambda^2 \times$ Coupling	Operator	$\Lambda^2 \times$ Coupling	Operator
$C_{LQ}^{(1)}$	$\left(\bar{L}_p \gamma_\mu L_r\right)\left(\bar{Q}_s \gamma^\mu Q_t\right)$	$C_{e\varphi}$	$\left(\varphi^\dagger \varphi\right)\left(\bar{L}_p e_r \varphi\right)$
$C_{LQ}^{(3)}$	$\left(\bar{L}_p \gamma_\mu \sigma^I L_r\right)\left(\bar{Q}_s \gamma^\mu \sigma^I Q_t\right)$	$C_{\varphi e}$	$\left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)\left(e_p \gamma^\mu e_r\right)$
C_{eu}	$\left(\bar{e}_p \gamma_\mu e_r\right)\left(\bar{u}_s \gamma^\mu u_t\right)$	$C_{\varphi L}^{(1)}$	$\left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)\left(\bar{L}_p \gamma^\mu L_r\right)$
C_{ed}	$\left(\bar{e}_p \gamma_\mu e_r\right)\left(\bar{d}_s \gamma^\mu d_t\right)$	$C_{\varphi L}^{(3)}$	$\left(\varphi^\dagger i \overleftrightarrow{D}_{I\mu} \varphi\right)\left(\bar{L}_p \sigma_I \gamma^\mu L_r\right)$
C_{Lu}	$\left(\bar{L}_p \gamma_\mu L_r\right)\left(\bar{u}_s \gamma^\mu u_t\right)$	C_{eW}	$\left(\bar{L}_p \sigma^{\mu\nu} e_r\right) \sigma_I \varphi W_{\mu\nu}^I$
C_{Ld}	$\left(\bar{L}_p \gamma_\mu L_r\right)\left(\bar{d}_s \gamma^\mu d_t\right)$	C_{eB}	$\left(\bar{L}_p \sigma^{\mu\nu} e_r\right) \varphi B_{\mu\nu}$
C_{Qe}	$\left(\bar{Q}_p \gamma_\mu Q_r\right)\left(\bar{e}_s \gamma^\mu e_t\right)$		
C_{LedQ}	$\left(\bar{L}_p^j e_r\right)\left(\bar{d}_s Q_t^j\right)$		
$C_{LeQu}^{(1)}$	$\left(\bar{L}_p^j e_r\right) \varepsilon_{jk} \left(\bar{Q}_s^k u_t\right)$		
$C_{LeQu}^{(3)}$	$\left(\bar{L}_p^j \sigma_{\mu\nu} e_r\right) \varepsilon_{jk} \left(\bar{Q}_s^k \sigma^{\mu\nu} u_t\right)$		

Tau decays



HEPfit [40]

$\ell - \tau$ conversion in nuclei



HEPfit [40]

Tau decays

Bounds on $\Lambda_{\rm CLFV}$ [TeV]					
WC	Belle	Belle II	WC	Belle	Belle II
$C_{LQ}^{(1)}$	$\gtrsim 8.5$	$\gtrsim 26$	$C_{LeQu}^{(1)\,\prime}$	$\gtrsim 0.65$	$\gtrsim 1.8$
$C_{LQ}^{(3)}$	$\gtrsim 7.5$	$\gtrsim 21$	$C_{LeQu}^{(3)}$	$\gtrsim 12$	$\gtrsim 33$
C_{eu}	$\gtrsim 7.7$	$\gtrsim 22$	$C_{\varphi L}^{(1)\,\prime}$	$\gtrsim 6.3$	$\gtrsim 17$
C_{ed}, C_{Ld}	$\gtrsim 10$	$\gtrsim 26$	$C_{\varphi e}$	$\gtrsim 8.8$	$\gtrsim 26$
C_{Lu}	$\gtrsim 6.5$	$\gtrsim 20$	C_{γ}	$\gtrsim 120$	$\gtrsim 330$
C_{Qe}	$\gtrsim 11$	$\gtrsim 28$	C_{Z}	$\gtrsim 0.79$	$\gtrsim 2.1$
$C_{LedQ}^{\prime}$	$\gtrsim 2.9$	$\gtrsim 7.9$	$C_{e\varphi}$	$\gtrsim 0.54$	$\gtrsim 1.5$

4. Messages

Physics of the tau lepton has many interesting aspects:

- To explore QCD, both at perturbative level (inclusive processes) and in the non-perturbative energy región (study of hadronization).
- If we look for violation of Universality in the lepton families we will have to pay close attention to the processes that involve the tau lepton in comparison with those involving the lighter leptons.
- We already have seen neutral lepton flavour violation: the neutrinos mix. There seems to be no reason why charged lepton flavour violation should not happen in Nature: the quest on the experimental side is a major task.

EXERCISE

Within Resonance Chiral Theory, using the antisymmetric formulation to describe the vector resonances, determine the vector form factor of the pion $F_\pi(q^2)$ defined by

$$\langle \pi^-(p_-)\pi^0(p_0) \,|\, V_\mu^{1-i2} \,|\, 0\rangle \;=\; \sqrt{2}\, F_\pi(q^2)\,(p_- - p_0)_\mu \;,$$

where $V_\mu^i = \bar{q}\gamma_\mu \frac{\lambda^i}{2} q$ and $q = (u,\, d,\, s)^T$.

1) Obtain information on the couplings of the vector resonance to the pions, F_V and G_V by imposing the proper high-energy behaviour of the form factor.

2) In some extensions of the Standard Model it is suggested an extra contribution to the pion form factor, given by the following lagrangian

$$\mathcal{L} \;=\; i\, g_{\mathrm{BSM}}\, \langle f_{\mu\nu}^{+}\, u^\mu\, u^\nu \,\rangle \;.$$

What do you think? Is that posible for all values of g_{BSM} ?

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Stubbornly
Testing QCD

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Additional Topics

m_s and $|V_{us}|$ from inclusive tau data decays

$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} ds \left(1-\frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_\tau}{ds} = R_{\tau,V+A}^{kl} + R_{\tau,S}^{kl}$$

moments

(notice that $R_\tau^{00} = R_\tau$)

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moments

(notice that $R_\tau^{00} = R_\tau$)

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = N_C S_{\rm EW} \sum_{D\geq 2} \left(\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)}\right)$$

The most relevant contributions come from D=2,4 : $\delta^{(2)} \propto \frac{m^2}{M_\tau^2}, \quad \delta^{(4)} \propto \frac{m\langle \overline{q} q\rangle}{M_\tau^4}$

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$$\delta R_\tau^{kl}\Big|_{\rm theo} = f\left(|V_{ud}|, |V_{us}|, m_s\right)$$

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Joint fit

$$m_s(2\,\mathrm{GeV}) \simeq 76\,\mathrm{MeV}$$
$$|V_{us}| \simeq 0.2196$$

m_S and $|V_{us}|$ from inclusive tau data decays

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$$\delta R_\tau^{kl}\Big|_{\rm theo} = f\left(|V_{ud}|, |V_{us}|, m_s\right)$$

$$\begin{aligned} \delta R_\tau^{00}\big|_{\rm theo} &= 0.240(32) \\ R_{\tau,V+A}^{00} &= 3.4671(84) \\ R_{\tau,S}^{00} &= 0.162(28) \\ |V_{ud}| &= 0.97425(22) \end{aligned}$$

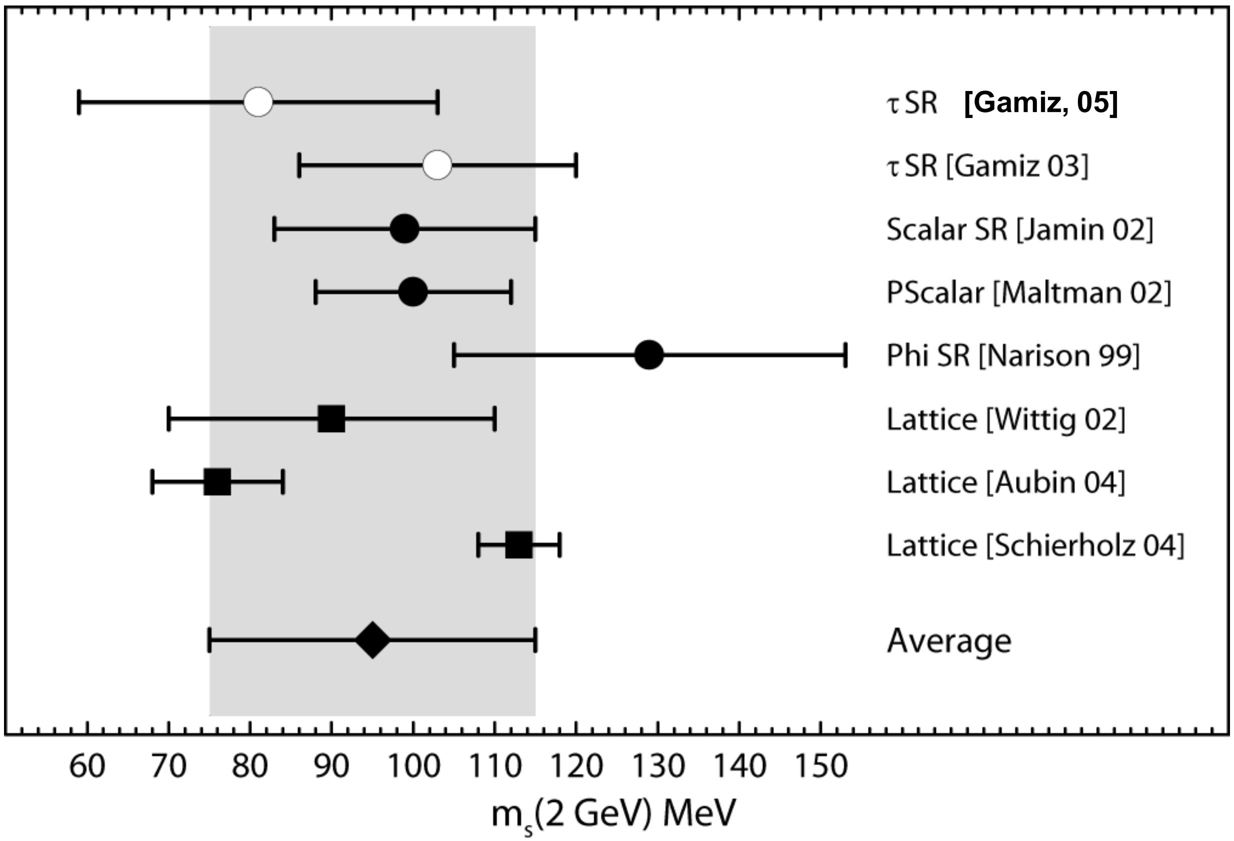
$$|V_{us}| = 0.2173(20)_{exp}(10)_{th}$$

[A.1, A.2]

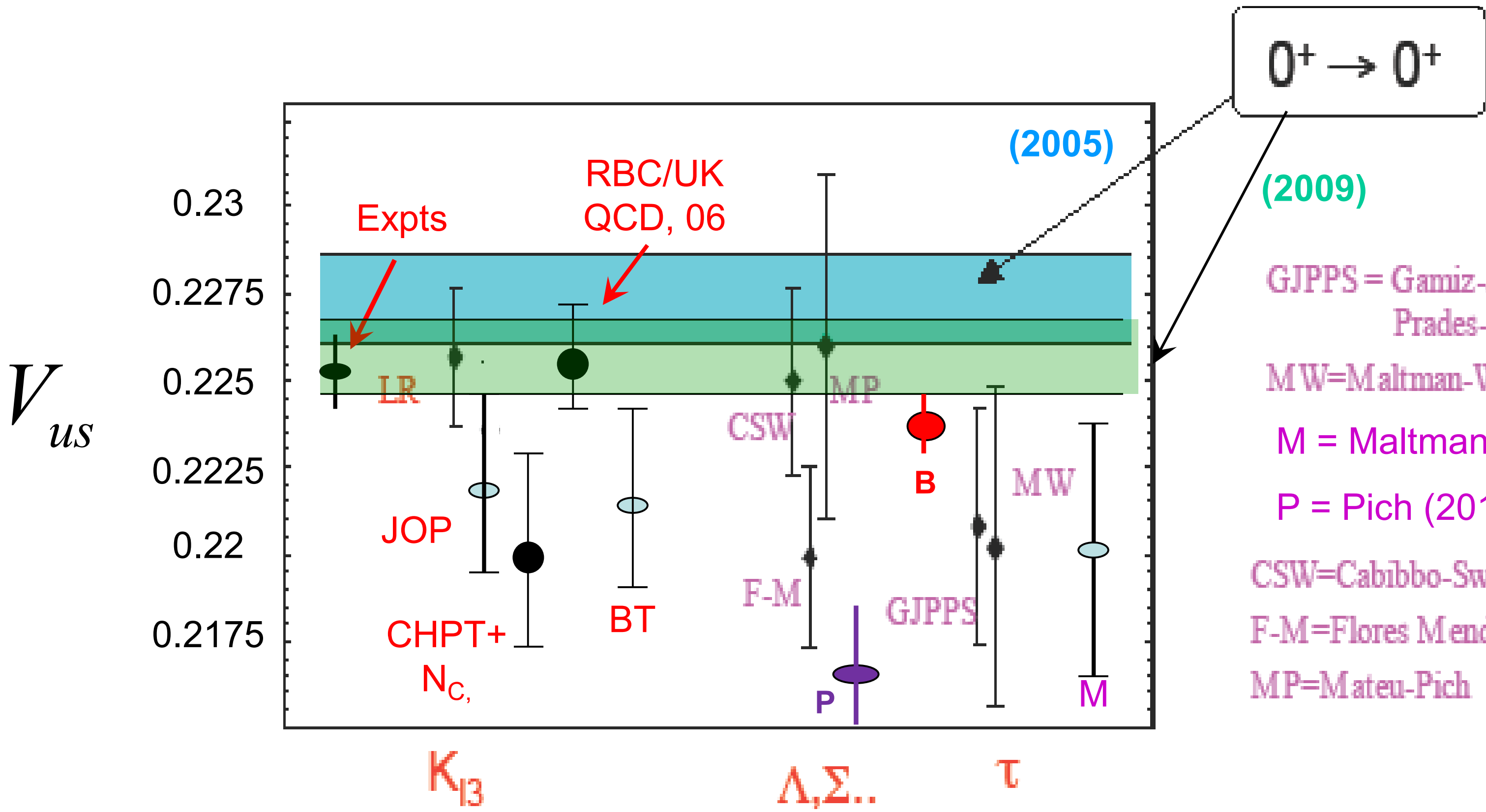
Joint fit

$$m_s(2\,\text{GeV}) \simeq 76\,\text{MeV}$$
$$|V_{us}| \simeq 0.2196$$

$$m_s(2\,\mathrm{GeV})|_{\mathrm{average}} = (95 \pm 20)\ \mathrm{MeV}$$



V_{us}



GJPPS = Gamiz-Jamin-Pich-Prades-Schwab (2005)

MW=Maltman-Wolfe (2007)

M = Maltman (2009)

P = Pich (2013)

CSW=Cabibbo-Swallow-Winston (2004)

F-M=Flores Mendieta (2004)

MP=Mateu-Pich (2005)

LR = Leutwyler-Roos (1984)

JOP = Jamin-Oller-Pich (2004)

BT = Bijnens-Talavera (2003)

CHPT+N_C = Cirigliano et al (2006)

Expts = FLAVIAnet WG (2010)

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