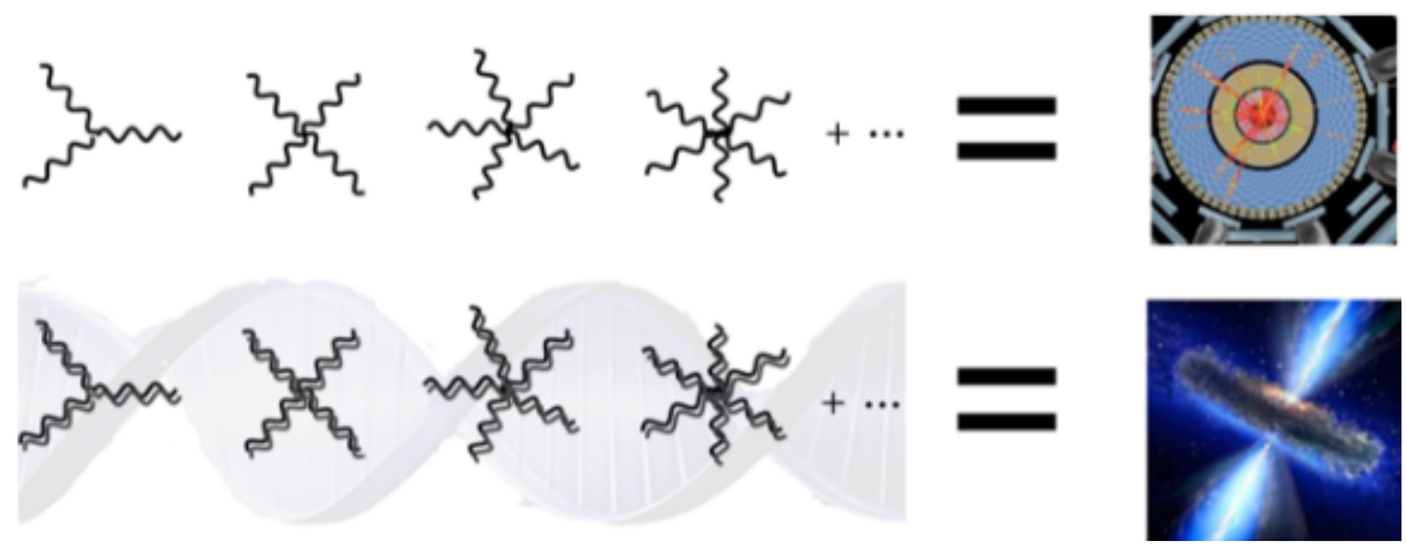
Gravity as the square of gauge theory



Credit: Carrasco

Andres Luna

XIX Mexican School of Particles and Fields

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High Energy Physics – Theory

[Submitted on 4 Apr 2010 (v1), last revised 30 Jul 2010 (this version, v2)]

Perturbative Quantum Gravity as a Double Copy of Gauge Theory

Zvi Bern, John Joseph M. Carrasco, Henrik Johansson

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Published in: *Phys.Rev.Lett.* 105 (2010) 061602 • e-Print: 1004.0476 [hep-th]

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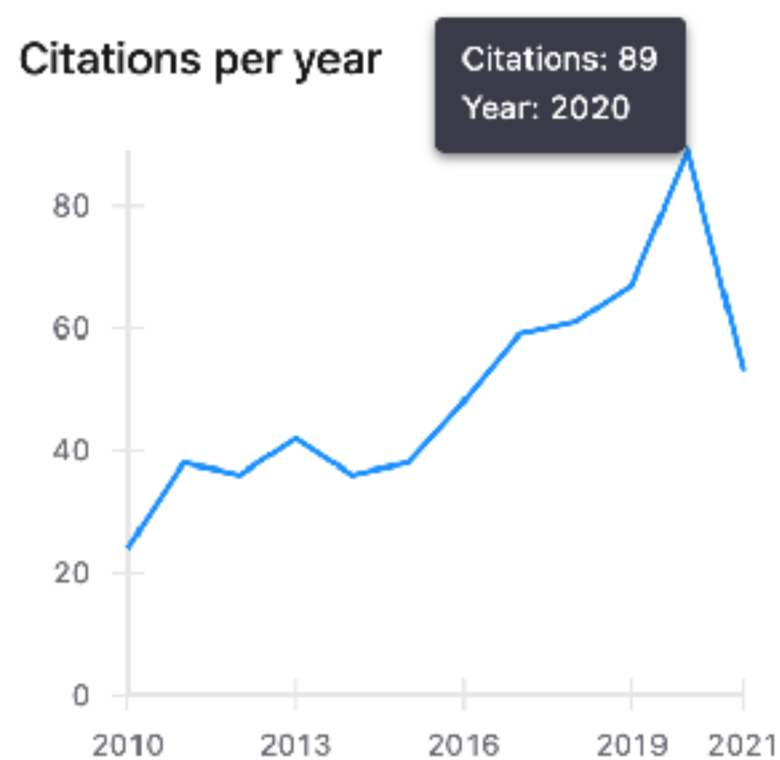
591 citations



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Perturbative Quantum Gravity as a Double Copy of Gauge Theory

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arXiv.org > hep-th > arXiv:1004.0693

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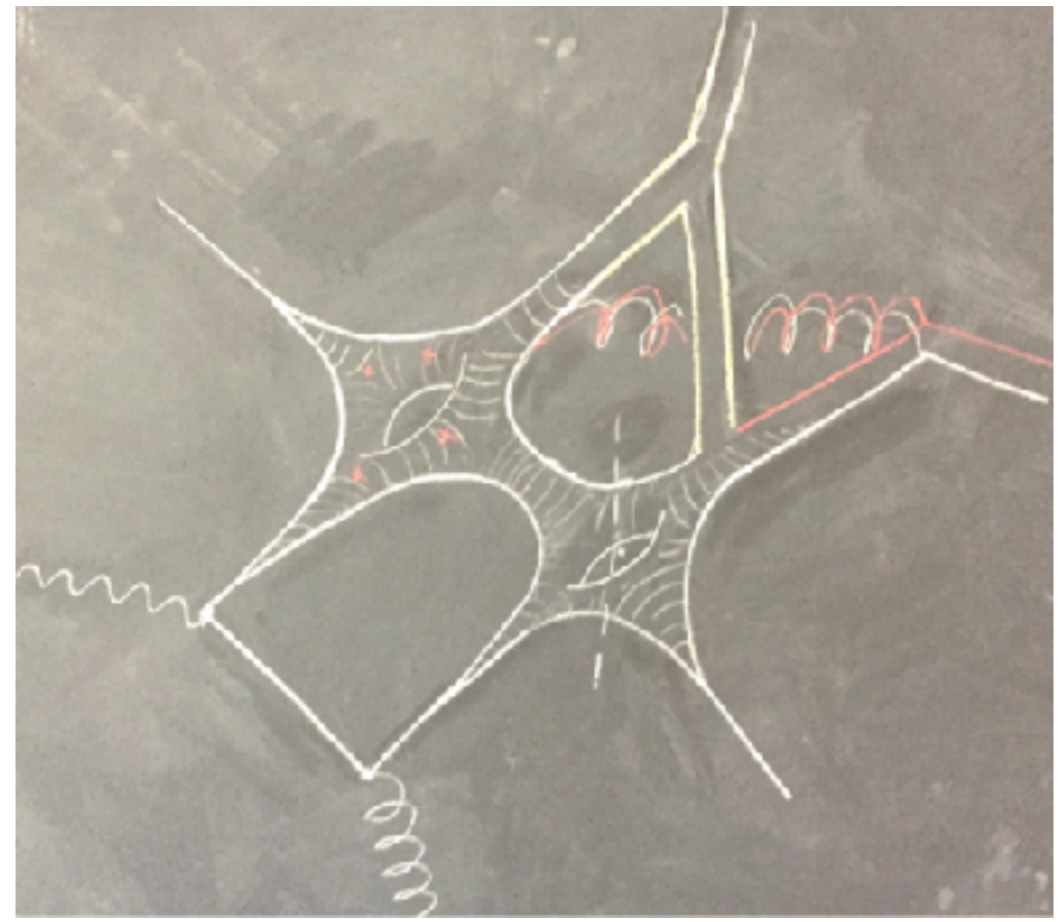
High Energy Physics – Theory

[Submitted on 5 Apr 2010]

Gravity as the Square of Gauge Theory

Zvi Bern, Tristan Dennen, Yu-tin Huang, Michael Kiermaier

Amplitudes



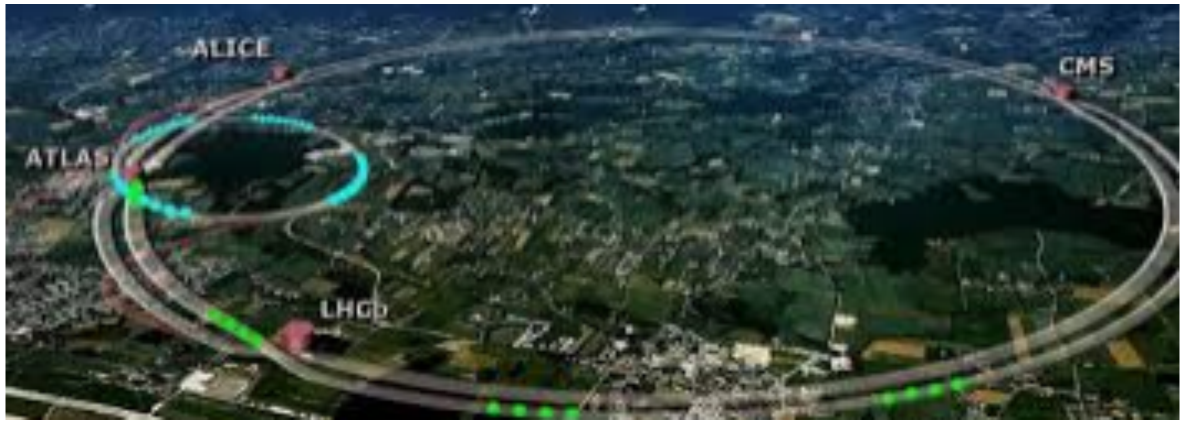
17:00

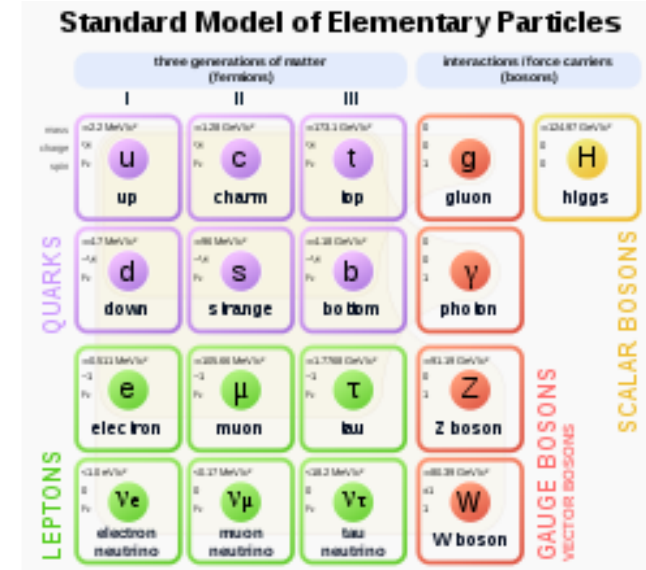
Scattering amplitudes in gravity and QFT

Mr. Bryan Larios

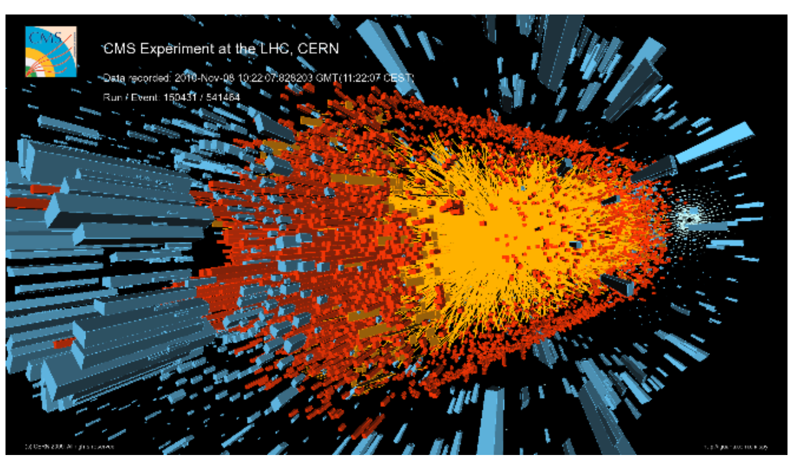
17:00 - 18:00

18:00



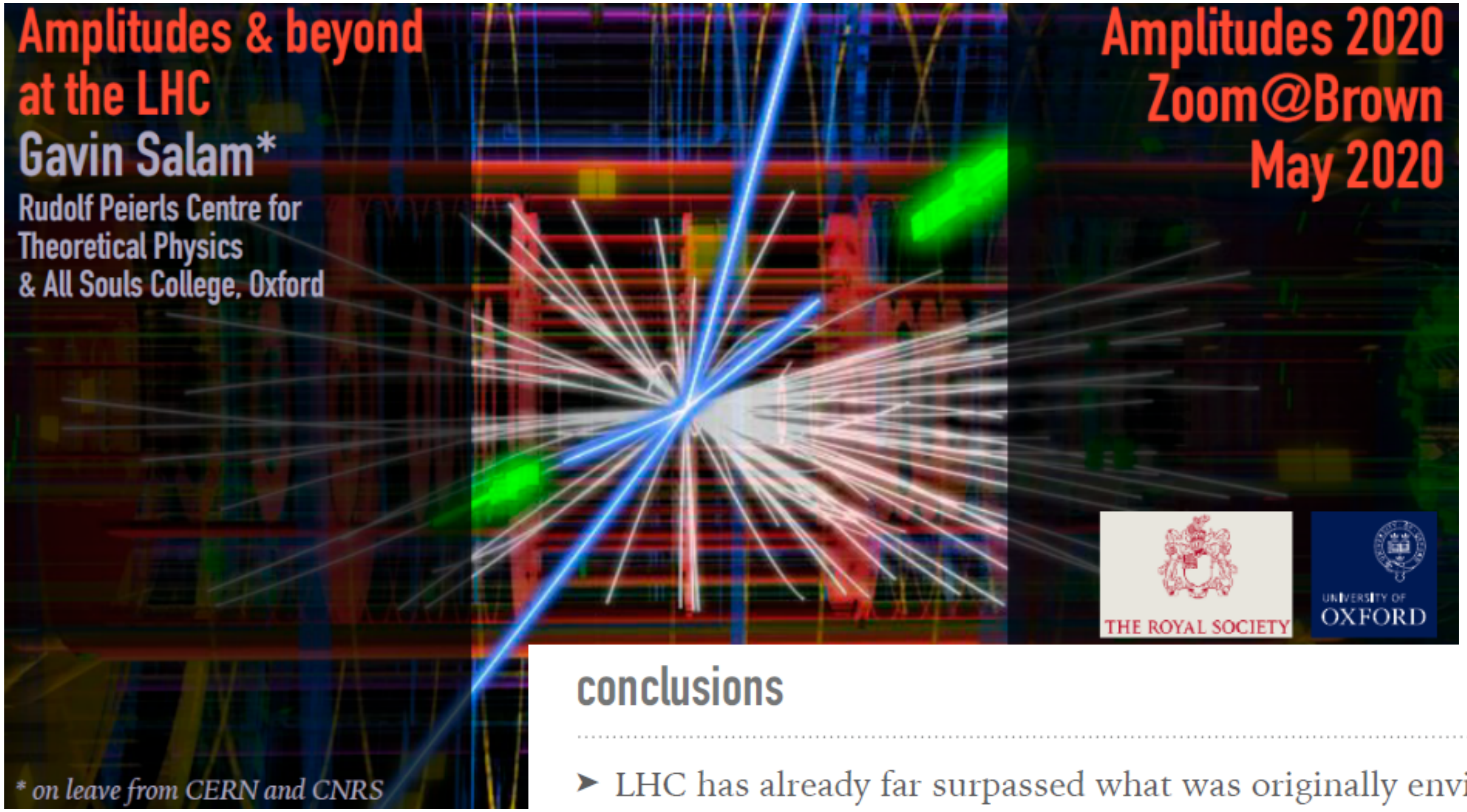


In the LHC, they perform experiments which test the standard model, the gauge theory that describes the fundamental building blocks of our universe, and their interactions (excluding gravity)



To describe the interaction between particles in an accelerator, it is necessary to know the differential cross section, whose principal ingredient is the Scattering Amplitude

$$d\sigma = \frac{|\mathcal{A}|^2}{64\pi^2 E_{\rm cm}} d\Omega$$

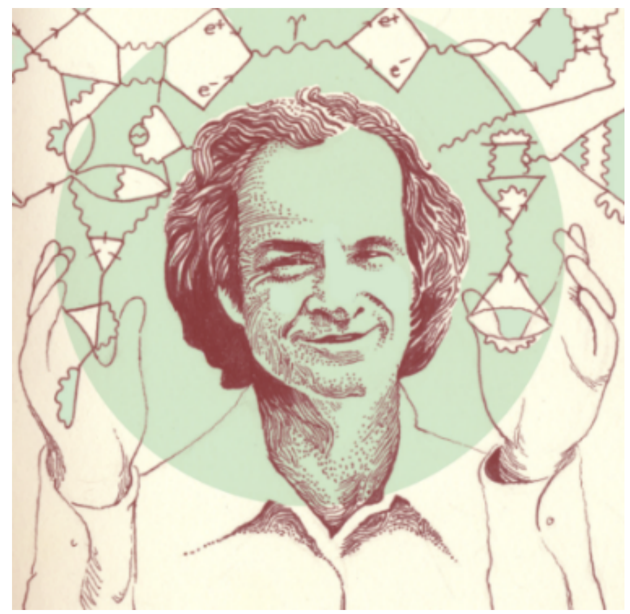


conclusions

- LHC has already far surpassed what was originally envisaged in terms of its potential for accurate measurements (e.g. Z production with < 1% accuracy)
- relative to current results, $20-80\times$ more stats on its way, i.e. potential for $4-9\times$ higher accuracy
- with perturbation theory as our only rigorous tool, progress in calculating amplitudes is essential to successful physics exploitation of this wealth of data
- amplitudes (and associated perturbative IRC safe cross sections) are not the only issue — parton showering, matching/merging, hadronisation all become increasingly important as one pushes the boundaries of accuracy and information-extraction in LHC events.

- with perturbation theory as our only rigorous tool, progress in calculating amplitudes is essential to successful physics exploitation of this wealth of data

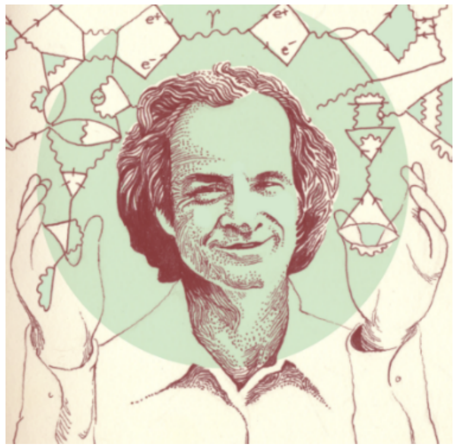
How can we compute scattering amplitudes?



1. Deduce Feynman rules from Lagrangian.

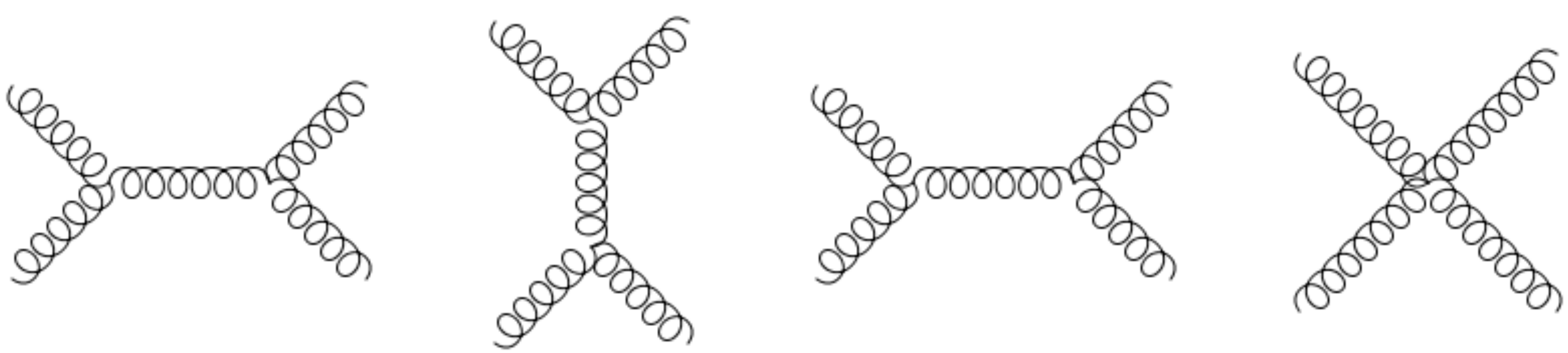
2. Draw all relevant diagrams.

3. Assign value following rules and sum.



Example: 4-gluon Amplitude

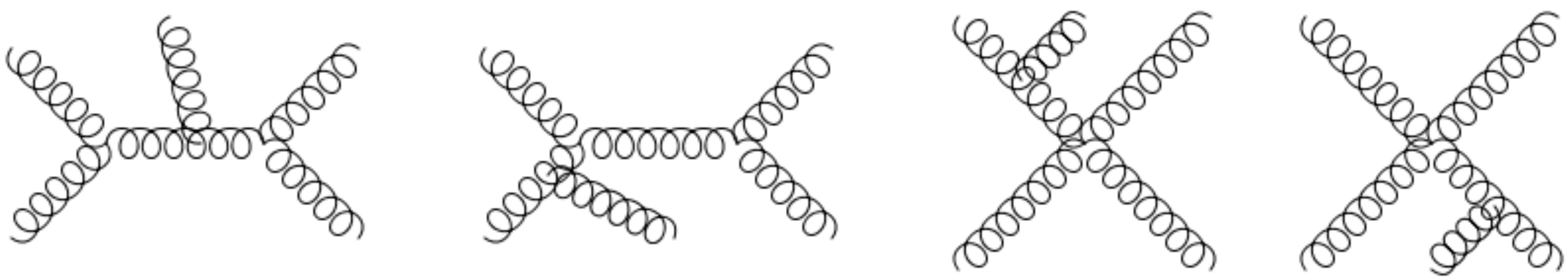
$g + g \;\rightarrow\; g + g$ 4 diagrams



Example: 5-gluon Amplitude

$$g + g \rightarrow g + g + g$$

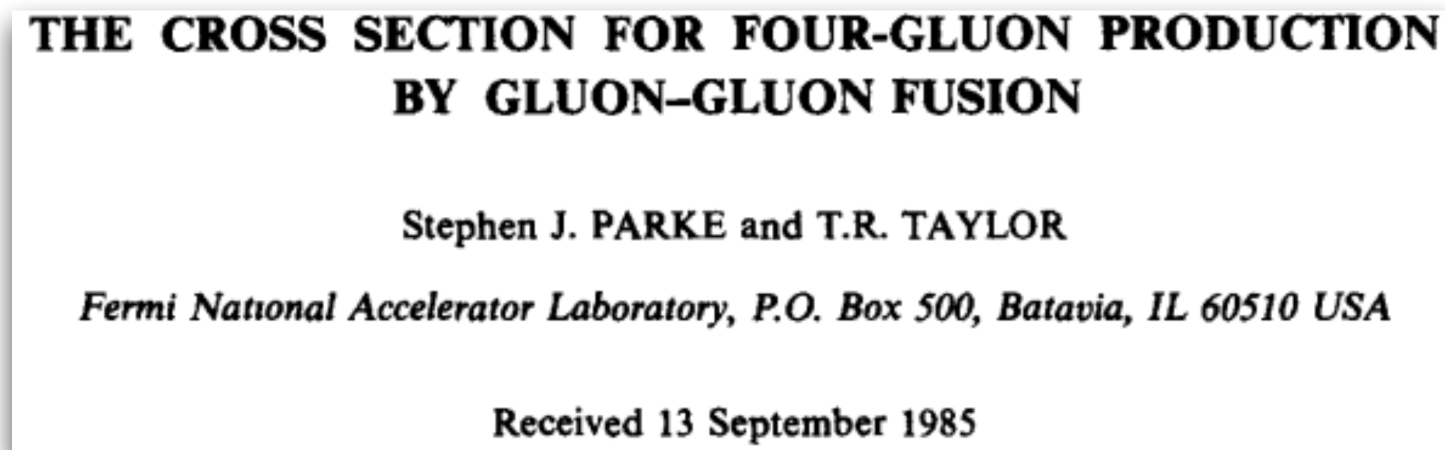
25 diagrams



and 21 other diagrams...

$$g + g \;\rightarrow\; g + g + g + g$$ 220 diagrams

...around 100 pages of computations...

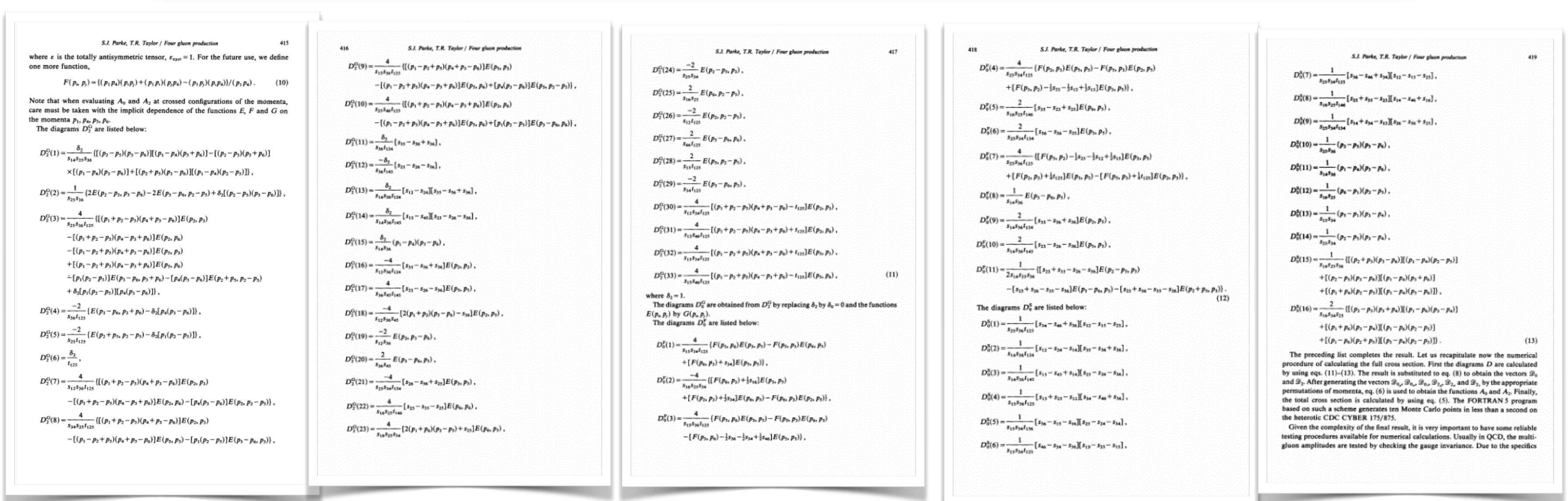


THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985



The result fits in 5 pages

Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

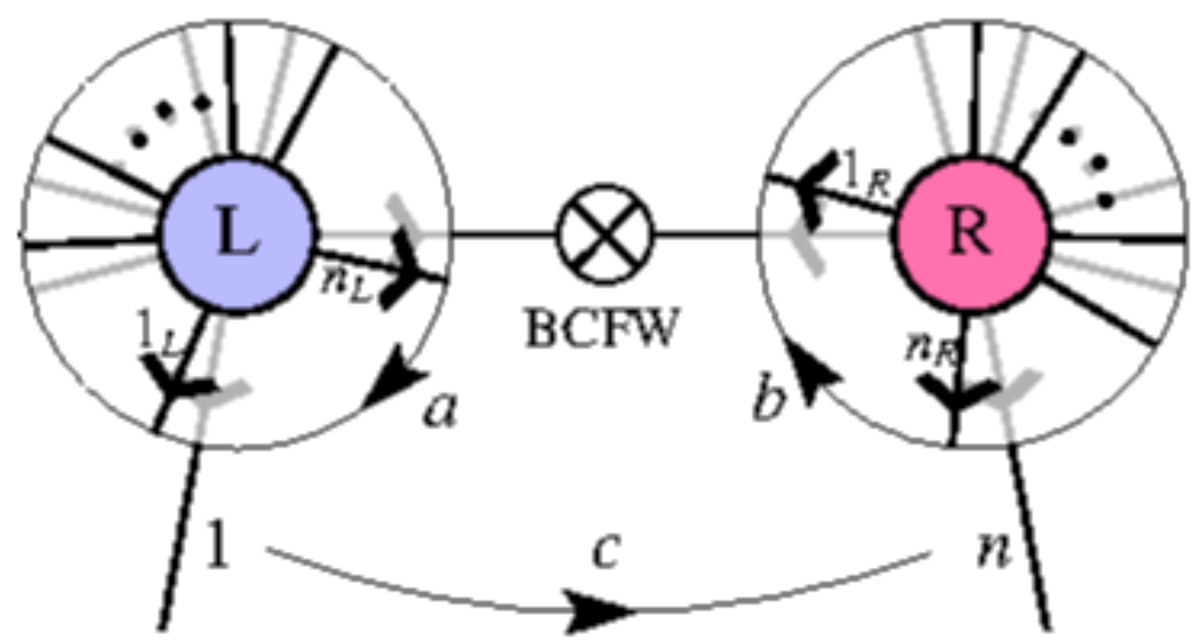
$$A_m^{\mathrm{tree}}(1^+, 2^+, \ldots, i^-, \ldots, j^-, \ldots, m^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle m-1, m \rangle \langle m, 1 \rangle}$$

$$\langle p\, q \rangle \, [p\, q] \;=\; 2\, p \cdot q \qquad p_{a\dot{b}} \;\equiv\; p_\mu \, (\sigma^\mu)_{a\dot{b}} \qquad p_{a\dot{b}} \;=\; -\, |p]_a \, \langle p|_{\dot{b}}$$

Takeaway:
Amplitudes are often simpler than expected
(when expectations are based on Feynman diagrams)

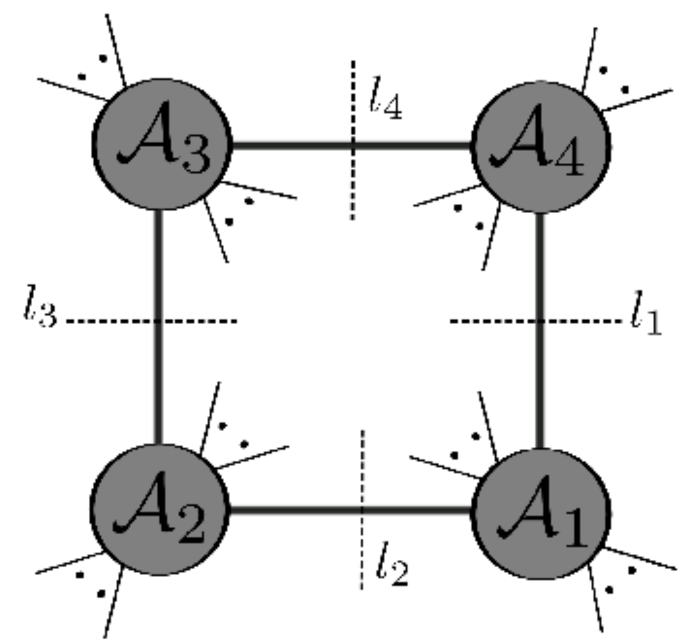
Modern Amplitudes Program: Avoid Lagrangians (Feynman diagrams). Instead use (recycle) amplitudes as building blocks

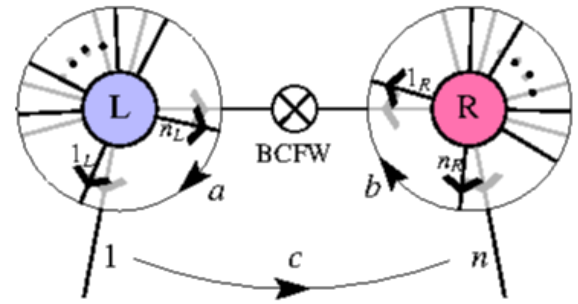
Recursion relations:
Build trees out of trees.
Britto, Cachazo, Feng, Witten (BCFW) 2005

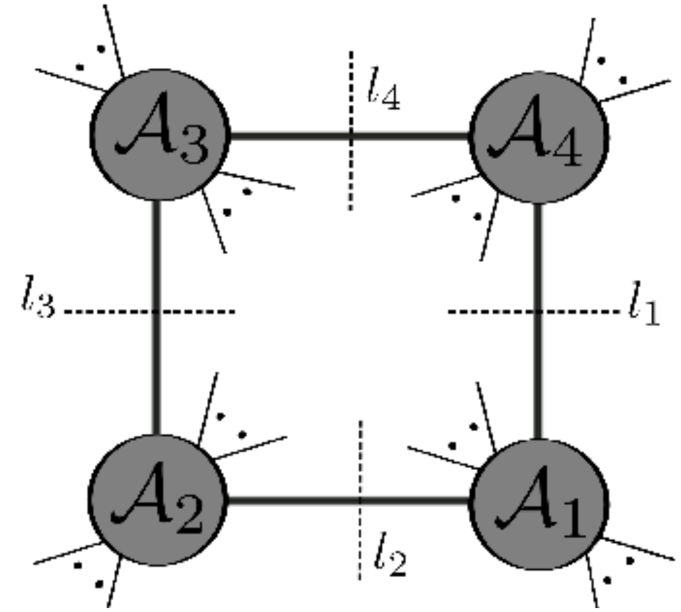


Modern Amplitudes Program: Avoid Lagrangians (Feynman diagrams). Instead use (recycle) amplitudes as building blocks

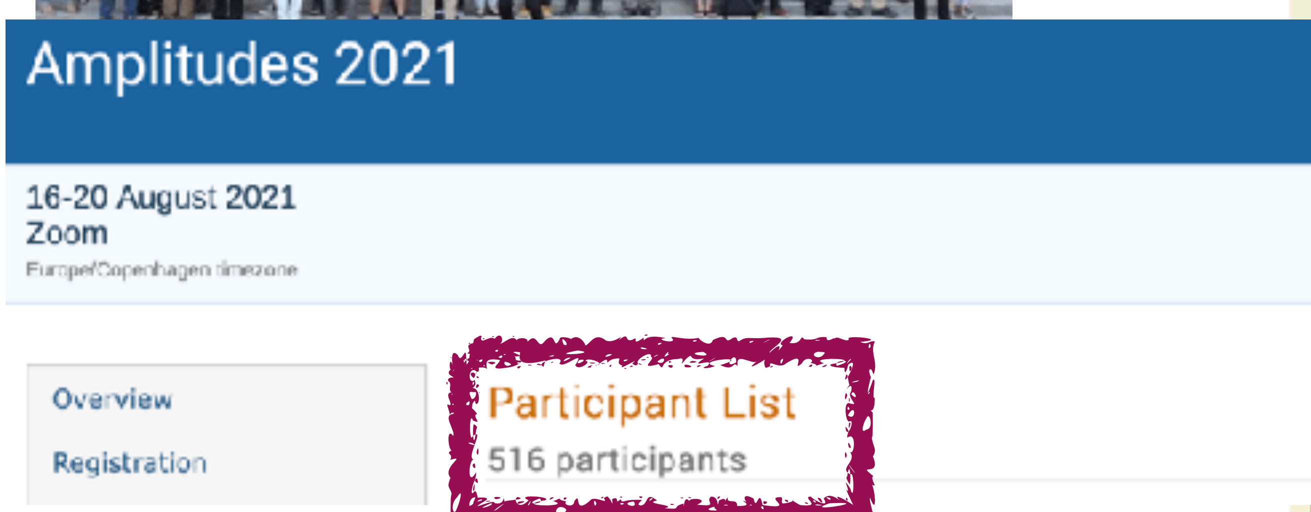
Unitarity method:
Build loops out of trees.
Bern, Dixon, Kosower.

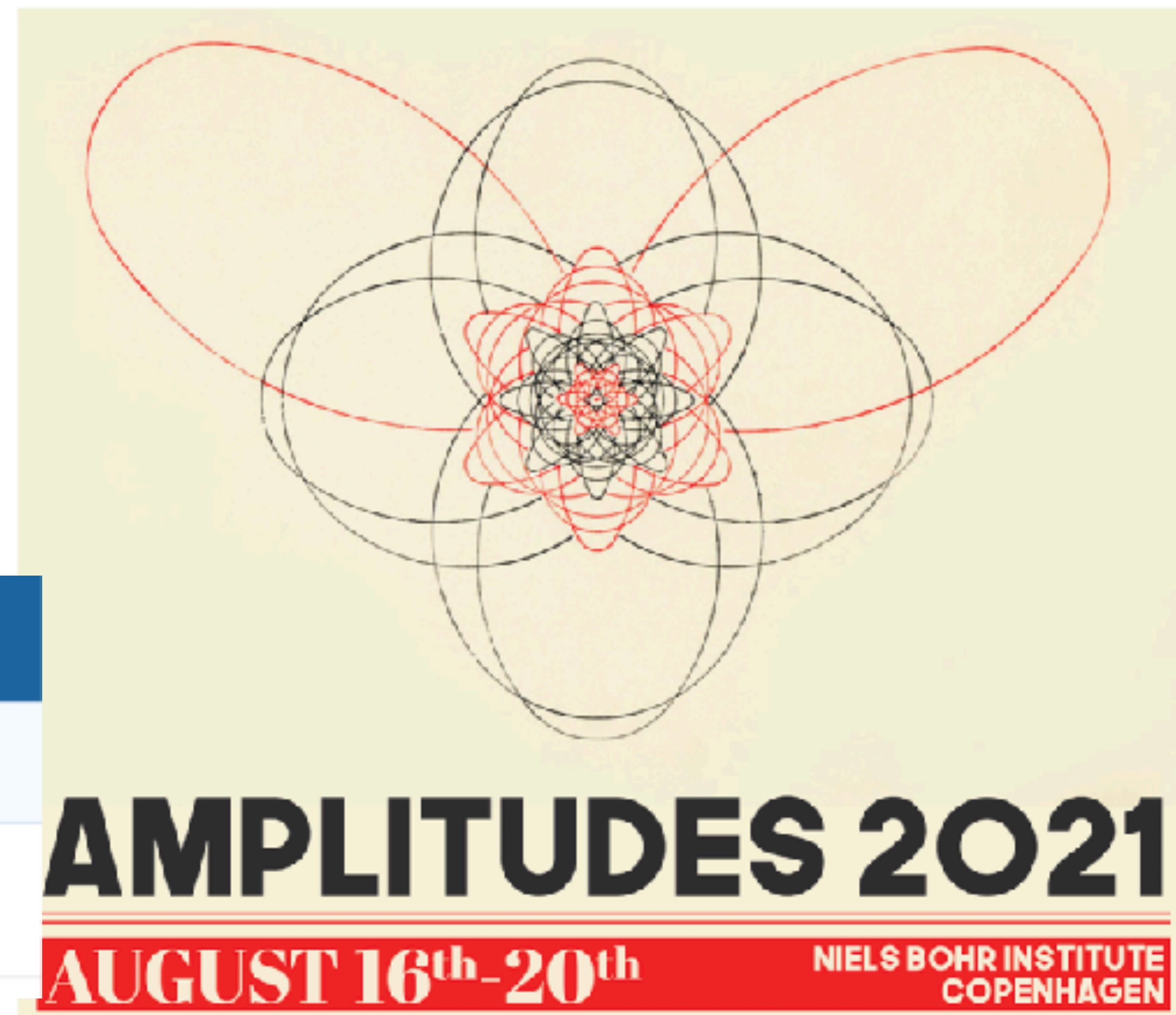


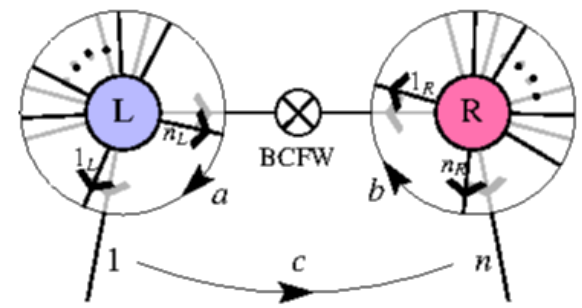

L
R
BCFW
a
b
c
1
n

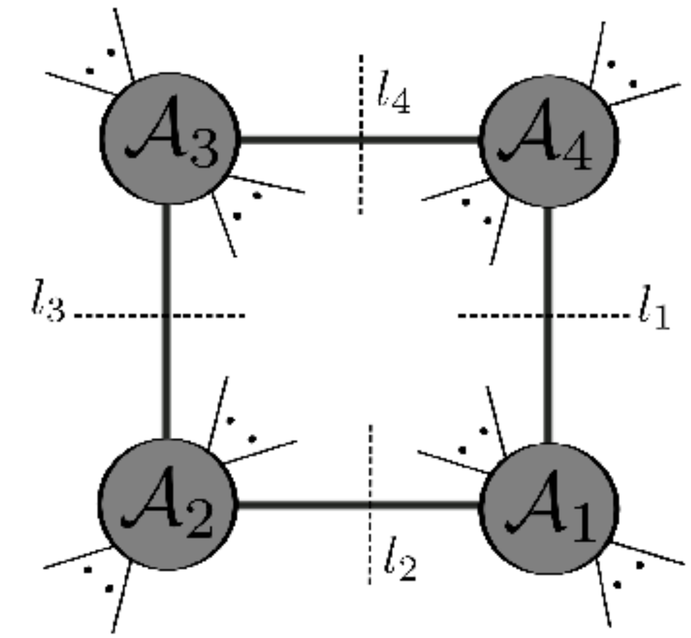

A_3
A_4
A_2
A_1
l_4
l_3
l_1
l_2


Amplitudes 2019


Amplitudes 2021
16-20 August 2021
Zoom
Europe/Copenhagen timezone
Overview
Registration
Participant List
516 participants


AMPLITUDES 2021
AUGUST 16th-20th
NIELS BOHR INSTITUTE
COPENHAGEN





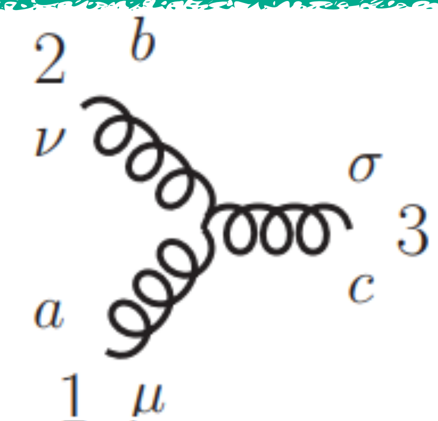
Cool! But nothing we couldn't achieve with infinite time (and infinite RAM, and infinite students, etc.)...

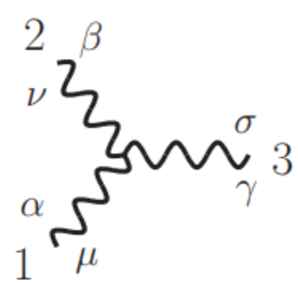
Structures

Let's compare Yang-Mills and Einstein-Hilbert

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$\mathcal{L}_{\text{EH}} = \frac{2}{\kappa^2}\sqrt{-g}R$$





…

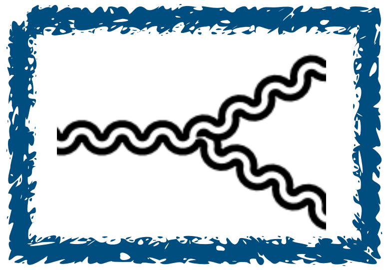
$$V^{abc}_{3\,\mu\nu\sigma}(p_1,p_2,p_3) = g f^{abc}\Big[(p_1-p_2)_\sigma \eta_{\mu\nu} + \text{cyclic}\Big]$$

$$\begin{aligned}
&G_{3\,\mu\rho,\nu\lambda,\sigma\tau}(p_1,p_2,p_3)\\
&\quad = i\text{Sym}\Big[-\frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\rho}\eta_{\nu\lambda}\eta_{\sigma\tau}) - \frac{1}{2}P_6(p_{1\nu}p_{1\lambda}\eta_{\mu\rho}\eta_{\sigma\tau}) + \frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\nu}\eta_{\rho\lambda}\eta_{\sigma\tau})\\
&\qquad + P_6(p_1\cdot p_2\eta_{\mu\rho}\eta_{\nu\sigma}\eta_{\lambda\tau}) + 2P_3(p_{1\nu}p_{1\tau}\eta_{\mu\rho}\eta_{\lambda\sigma}) - P_3(p_{1\lambda}p_{2\mu}\eta_{\rho\nu}\eta_{\sigma\tau})\\
&\qquad + P_3(p_{1\sigma}p_{2\tau}\eta_{\mu\nu}\eta_{\rho\lambda}) + P_6(p_{1\sigma}p_{1\tau}\eta_{\mu\nu}\eta_{\rho\lambda}) + 2P_6(p_{1\nu}p_{2\tau}\eta_{\lambda\mu}\eta_{\rho\sigma})\\
&\qquad + 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1\cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\Big],
\end{aligned}$$

They look nothing like each other!

Take a closer look at Einstein-Hilbert's 3-graviton vertex

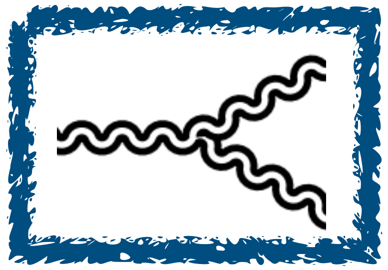
$$\mathcal{L}_{\mathrm{EH}} = \frac{2}{\kappa^2}\sqrt{-g}R$$



$$\begin{aligned} &G_{3\,\mu\rho,\nu\lambda,\sigma\tau}(p_1,p_2,p_3) \\ &\quad= i\mathrm{Sym}\Big[-\frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\rho}\eta_{\nu\lambda}\eta_{\sigma\tau}) - \frac{1}{2}P_6(p_{1\nu}p_{1\lambda}\eta_{\mu\rho}\eta_{\sigma\tau}) + \frac{1}{2}P_3(p_1\cdot p_2\eta_{\mu\nu}\eta_{\rho\lambda}\eta_{\sigma\tau}) \\ &\qquad + P_6(p_1\cdot p_2\eta_{\mu\rho}\eta_{\nu\sigma}\eta_{\lambda\tau}) + 2P_3(p_{1\nu}p_{1\tau}\eta_{\mu\rho}\eta_{\lambda\sigma}) - P_3(p_{1\lambda}p_{2\mu}\eta_{\rho\nu}\eta_{\sigma\tau}) \\ &\qquad + P_3(p_{1\sigma}p_{2\tau}\eta_{\mu\nu}\eta_{\rho\lambda}) + P_6(p_{1\sigma}p_{1\tau}\eta_{\mu\nu}\eta_{\rho\lambda}) + 2P_6(p_{1\nu}p_{2\tau}\eta_{\lambda\mu}\eta_{\rho\sigma}) \\ &\qquad + 2P_3(p_{1\nu}p_{2\mu}\eta_{\lambda\sigma}\eta_{\tau\rho}) - 2P_3(p_1\cdot p_2\eta_{\rho\nu}\eta_{\lambda\sigma}\eta_{\tau\mu})\Big]\,, \end{aligned}$$

Take a closer look at Einstein-Hilbert's 3-graviton vertex

$$\mathcal{L}_{\text{EH}} = \frac{2}{\kappa^2}\sqrt{-g}R$$



$$
\begin{aligned}
&\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \to 2\eta^{\mu\tau}\eta^{\nu\sigma}{k_1}^{\lambda}{k_1}^{\rho} + 2\eta^{\mu\sigma}\eta^{\nu\tau}{k_1}^{\lambda}{k_1}^{\rho} - \boxed{2\eta^{\mu\nu}\eta^{\sigma\tau}{k_1}^{\lambda}{k_1}^{\rho}} + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}{k_1}^{\sigma}{k_1}^{\rho} + 2\eta^{\lambda\sigma}\eta^{\mu\nu}{k_1}^{\tau}{k_1}^{\rho} + \eta^{\mu\tau}\eta^{\nu\sigma}{k_2}^{\lambda}{k_1}^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}{k_2}^{\lambda}{k_1}^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}{k_2}^{\mu}{k_1}^{\rho} + \\
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&\eta^{\lambda\nu}\eta^{\sigma\tau}{k_3}^{\mu}{k_1}^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}{k_3}^{\nu}{k_1}^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}{k_3}^{\nu}{k_1}^{\rho} - \eta^{\lambda\mu}\eta^{\sigma\tau}{k_3}^{\nu}{k_1}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\tau}{k_3}^{\sigma}{k_1}^{\rho} + \\
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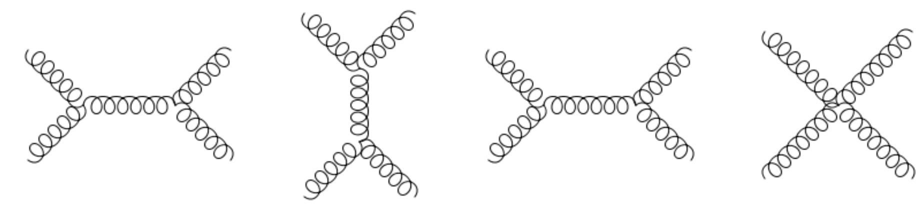
&\eta^{\lambda\tau}\eta^{\nu\rho}{k_2}^{\mu}{k_3}^{\sigma} + \eta^{\lambda\nu}\eta^{\rho\tau}{k_2}^{\mu}{k_3}^{\sigma} + \eta^{\lambda\tau}\eta^{\mu\rho}{k_2}^{\nu}{k_3}^{\sigma} + \eta^{\lambda\mu}\eta^{\rho\tau}{k_2}^{\nu}{k_3}^{\sigma} - \eta^{\lambda\tau}\eta^{\mu\nu}{k_2}^{\rho}{k_3}^{\sigma} + \\
&\eta^{\lambda\nu}\eta^{\mu\tau}{k_2}^{\rho}{k_3}^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\tau}{k_2}^{\rho}{k_3}^{\sigma} + 2\eta^{\lambda\rho}\eta^{\nu\tau}{k_3}^{\mu}{k_3}^{\sigma} + 2\eta^{\lambda\rho}\eta^{\mu\tau}{k_3}^{\nu}{k_3}^{\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}{k_1}^{\lambda}{k_3}^{\tau} + \\
&\eta^{\mu\rho}\eta^{\nu\sigma}{k_1}^{\lambda}{k_3}^{\tau} + \eta^{\lambda\nu}\eta^{\mu\rho}{k_1}^{\sigma}{k_3}^{\tau} + \eta^{\lambda\mu}\eta^{\nu\rho}{k_1}^{\sigma}{k_3}^{\tau} + \eta^{\mu\sigma}\eta^{\nu\rho}{k_2}^{\lambda}{k_3}^{\tau} + \eta^{\mu\rho}\eta^{\nu\sigma}{k_2}^{\lambda}{k_3}^{\tau} - \\
&\eta^{\mu\nu}\eta^{\rho\sigma}{k_2}^{\lambda}{k_3}^{\tau} + \eta^{\lambda\sigma}\eta^{\nu\rho}{k_2}^{\mu}{k_3}^{\tau} + \eta^{\lambda\nu}\eta^{\rho\sigma}{k_2}^{\mu}{k_3}^{\tau} + \eta^{\lambda\sigma}\eta^{\mu\rho}{k_2}^{\nu}{k_3}^{\tau} + \eta^{\lambda\mu}\eta^{\rho\sigma}{k_2}^{\nu}{k_3}^{\tau} - \\
&\eta^{\lambda\sigma}\eta^{\mu\nu}{k_2}^{\rho}{k_3}^{\tau} + \eta^{\lambda\nu}\eta^{\mu\sigma}{k_2}^{\rho}{k_3}^{\tau} + \eta^{\lambda\mu}\eta^{\nu\sigma}{k_2}^{\rho}{k_3}^{\tau} + 2\eta^{\lambda\rho}\eta^{\nu\sigma}{k_3}^{\mu}{k_3}^{\tau} + 2\eta^{\lambda\rho}\eta^{\mu\sigma}{k_3}^{\nu}{k_3}^{\tau} - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}{k_3}^{\sigma}{k_3}^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}{k_3}^{\sigma}{k_3}^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}{k_3}^{\sigma}{k_3}^{\tau} - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot \\
&k_2 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_2 + \\
&2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_2 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_2 - \\
&\eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_2 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_2 + \\
&2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_2 - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_3 + \\
&2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_3 - \\
&\eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_3 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - \\
&\eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_3 - \\
&\eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2\cdot k_3 - \\
&\eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_2\cdot k_3 + \eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_2\cdot k_3 - \\
&\eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2\cdot k_3 + \eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \\
&2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2\cdot k_3
\end{aligned}
$$

There are 171 total terms

$$2\eta^{\mu\nu}\eta^{\sigma\tau}{k_1}^{\lambda}{k_1}^{\rho}$$

For a 4-point amplitude…



The 4-vertex has 2850 terms

$$
\begin{aligned}
\frac{\delta S^3}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma\tau}\delta\varphi_{\rho\lambda}} \rightarrow\;& 2\eta^{\mu\tau}\eta^{\nu\sigma}k_1{}^{\lambda}k_1{}^{\rho} + 2\eta^{\mu\sigma}\eta^{\nu\tau}k_1{}^{\lambda}k_1{}^{\rho} - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_1{}^{\lambda}k_1{}^{\rho} + \\
& 2\eta^{\lambda\tau}\eta^{\mu\nu}k_1{}^{\sigma}k_1{}^{\rho} + 2\eta^{\lambda\sigma}\eta^{\mu\nu}k_1{}^{\tau}k_1{}^{\rho} + \eta^{\mu\tau}\eta^{\nu\sigma}k_2{}^{\lambda}k_1{}^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}k_2{}^{\lambda}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_2{}^{\mu}k_1{}^{\rho} + \\
& \eta^{\lambda\sigma}\eta^{\nu\tau}k_2{}^{\mu}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\nu\sigma}k_3{}^{\mu}k_1{}^{\rho} + \eta^{\lambda\sigma}\eta^{\nu\tau}k_3{}^{\mu}k_1{}^{\rho} - \\
& \eta^{\lambda\nu}\eta^{\sigma\tau}k_3{}^{\mu}k_1{}^{\rho} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_3{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_3{}^{\nu}k_1{}^{\rho} - \eta^{\lambda\mu}\eta^{\sigma\tau}k_3{}^{\nu}k_1{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\tau}k_3{}^{\sigma}k_1{}^{\rho} + \\
& \eta^{\lambda\mu}\eta^{\nu\tau}k_3{}^{\sigma}k_1{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_3{}^{\tau}k_1{}^{\rho} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_3{}^{\tau}k_1{}^{\rho} + 2\eta^{\mu\nu}\eta^{\rho\tau}k_1{}^{\lambda}k_1{}^{\sigma} + 2\eta^{\mu\nu}\eta^{\rho\sigma}k_1{}^{\lambda}k_1{}^{\tau} - \\
& 2\eta^{\lambda\rho}\eta^{\mu\nu}k_1{}^{\sigma}k_1{}^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^{\sigma}k_1{}^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^{\sigma}k_1{}^{\tau} + \eta^{\mu\tau}\eta^{\nu\rho}k_1{}^{\sigma}k_2{}^{\lambda} + \eta^{\mu\rho}\eta^{\nu\tau}k_1{}^{\sigma}k_2{}^{\lambda} + \\
& \eta^{\mu\sigma}\eta^{\nu\rho}k_1{}^{\tau}k_2{}^{\lambda} + \eta^{\mu\rho}\eta^{\nu\sigma}k_1{}^{\tau}k_2{}^{\lambda} + \eta^{\nu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_2{}^{\mu} + \eta^{\nu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_2{}^{\mu} + \eta^{\lambda\tau}\eta^{\nu\rho}k_1{}^{\sigma}k_2{}^{\mu} - \\
& \eta^{\lambda\rho}\eta^{\nu\tau}k_1{}^{\sigma}k_2{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_1{}^{\sigma}k_2{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1{}^{\tau}k_2{}^{\mu} - \eta^{\lambda\rho}\eta^{\nu\sigma}k_1{}^{\tau}k_2{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_1{}^{\tau}k_2{}^{\mu} + \\
& 2\eta^{\nu\rho}\eta^{\sigma\tau}k_2{}^{\lambda}k_2{}^{\mu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_2{}^{\nu} + \eta^{\mu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_2{}^{\nu} + \eta^{\lambda\tau}\eta^{\mu\rho}k_1{}^{\sigma}k_2{}^{\nu} - \eta^{\lambda\rho}\eta^{\mu\tau}k_1{}^{\sigma}k_2{}^{\nu} + \\
& \eta^{\lambda\mu}\eta^{\rho\tau}k_1{}^{\sigma}k_2{}^{\nu} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1{}^{\tau}k_2{}^{\nu} - \eta^{\lambda\rho}\eta^{\mu\sigma}k_1{}^{\tau}k_2{}^{\nu} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1{}^{\tau}k_2{}^{\nu} + 2\eta^{\mu\rho}\eta^{\sigma\tau}k_2{}^{\lambda}k_2{}^{\nu} + \\
& 2\eta^{\lambda\tau}\eta^{\rho\sigma}k_2{}^{\mu}k_2{}^{\nu} + 2\eta^{\lambda\sigma}\eta^{\rho\tau}k_2{}^{\mu}k_2{}^{\nu} - 2\eta^{\lambda\rho}\eta^{\sigma\tau}k_2{}^{\mu}k_2{}^{\nu} + \eta^{\mu\tau}\eta^{\nu\sigma}k_1{}^{\lambda}k_2{}^{\rho} + \eta^{\mu\sigma}\eta^{\nu\tau}k_1{}^{\lambda}k_2{}^{\rho} + \\
& \eta^{\lambda\nu}\eta^{\mu\tau}k_1{}^{\sigma}k_2{}^{\rho} + \eta^{\lambda\mu}\eta^{\nu\tau}k_1{}^{\sigma}k_2{}^{\rho} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_1{}^{\tau}k_2{}^{\rho} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_1{}^{\tau}k_2{}^{\rho} + 2\eta^{\mu\tau}\eta^{\nu\sigma}k_2{}^{\lambda}k_2{}^{\rho} + \\
& 2\eta^{\mu\sigma}\eta^{\nu\tau}k_2{}^{\lambda}k_2{}^{\rho} - 2\eta^{\mu\nu}\eta^{\sigma\tau}k_2{}^{\lambda}k_2{}^{\rho} + 2\eta^{\lambda\nu}\eta^{\sigma\tau}k_2{}^{\mu}k_2{}^{\rho} + 2\eta^{\lambda\mu}\eta^{\sigma\tau}k_2{}^{\nu}k_2{}^{\rho} + \eta^{\nu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_3{}^{\mu} + \\
& \eta^{\nu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_3{}^{\mu} - \eta^{\nu\rho}\eta^{\sigma\tau}k_1{}^{\lambda}k_3{}^{\mu} + \eta^{\lambda\tau}\eta^{\nu\rho}k_1{}^{\sigma}k_3{}^{\mu} + \eta^{\lambda\nu}\eta^{\rho\tau}k_1{}^{\sigma}k_3{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_1{}^{\tau}k_3{}^{\mu} + \\
& \eta^{\lambda\nu}\eta^{\rho\sigma}k_1{}^{\tau}k_3{}^{\mu} + \eta^{\nu\tau}\eta^{\rho\sigma}k_2{}^{\lambda}k_3{}^{\mu} + \eta^{\nu\sigma}\eta^{\rho\tau}k_2{}^{\lambda}k_3{}^{\mu} + \eta^{\lambda\tau}\eta^{\rho\sigma}k_2{}^{\nu}k_3{}^{\mu} + \eta^{\lambda\sigma}\eta^{\rho\tau}k_2{}^{\nu}k_3{}^{\mu} + \\
& \eta^{\lambda\tau}\eta^{\nu\sigma}k_2{}^{\rho}k_3{}^{\mu} + \eta^{\lambda\sigma}\eta^{\nu\tau}k_2{}^{\rho}k_3{}^{\mu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_1{}^{\lambda}k_3{}^{\nu} + \eta^{\mu\sigma}\eta^{\rho\tau}k_1{}^{\lambda}k_3{}^{\nu} - \eta^{\mu\rho}\eta^{\sigma\tau}k_1{}^{\lambda}k_3{}^{\nu} + \\
& \eta^{\lambda\tau}\eta^{\mu\rho}k_1{}^{\sigma}k_3{}^{\nu} + \eta^{\lambda\mu}\eta^{\rho\tau}k_1{}^{\sigma}k_3{}^{\nu} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_1{}^{\tau}k_3{}^{\nu} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_1{}^{\tau}k_3{}^{\nu} + \eta^{\mu\tau}\eta^{\rho\sigma}k_2{}^{\lambda}k_3{}^{\nu} + \\
& \eta^{\mu\sigma}\eta^{\rho\tau}k_2{}^{\lambda}k_3{}^{\nu} + \eta^{\lambda\tau}\eta^{\rho\sigma}k_2{}^{\mu}k_3{}^{\nu} + \eta^{\lambda\sigma}\eta^{\rho\tau}k_2{}^{\mu}k_3{}^{\nu} + \eta^{\lambda\tau}\eta^{\mu\sigma}k_2{}^{\rho}k_3{}^{\nu} + \eta^{\lambda\sigma}\eta^{\mu\tau}k_2{}^{\rho}k_3{}^{\nu} + \\
& 2\eta^{\lambda\tau}\eta^{\rho\sigma}k_3{}^{\mu}k_3{}^{\nu} + 2\eta^{\lambda\sigma}\eta^{\rho\tau}k_3{}^{\mu}k_3{}^{\nu} - 2\eta^{\lambda\rho}\eta^{\sigma\tau}k_3{}^{\mu}k_3{}^{\nu} + \eta^{\mu\tau}\eta^{\nu\rho}k_1{}^{\lambda}k_3{}^{\sigma} + \eta^{\mu\rho}\eta^{\nu\tau}k_1{}^{\lambda}k_3{}^{\sigma} + \\
& \eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^{\tau}k_3{}^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^{\tau}k_3{}^{\sigma} + \eta^{\mu\tau}\eta^{\nu\rho}k_2{}^{\lambda}k_3{}^{\sigma} + \eta^{\mu\rho}\eta^{\nu\tau}k_2{}^{\lambda}k_3{}^{\sigma} - \eta^{\mu\nu}\eta^{\rho\tau}k_2{}^{\lambda}k_3{}^{\sigma} + \\
& \eta^{\lambda\tau}\eta^{\nu\rho}k_2{}^{\mu}k_3{}^{\sigma} + \eta^{\lambda\nu}\eta^{\rho\tau}k_2{}^{\mu}k_3{}^{\sigma} + \eta^{\lambda\tau}\eta^{\mu\rho}k_2{}^{\nu}k_3{}^{\sigma} + \eta^{\lambda\mu}\eta^{\rho\tau}k_2{}^{\nu}k_3{}^{\sigma} - \eta^{\lambda\tau}\eta^{\mu\nu}k_2{}^{\rho}k_3{}^{\sigma} + \\
& \eta^{\lambda\nu}\eta^{\mu\tau}k_2{}^{\rho}k_3{}^{\sigma} + \eta^{\lambda\mu}\eta^{\nu\tau}k_2{}^{\rho}k_3{}^{\sigma} + 2\eta^{\lambda\rho}\eta^{\nu\tau}k_3{}^{\mu}k_3{}^{\sigma} + 2\eta^{\lambda\rho}\eta^{\mu\tau}k_3{}^{\nu}k_3{}^{\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}k_1{}^{\lambda}k_3{}^{\tau} + \\
& \eta^{\mu\rho}\eta^{\nu\sigma}k_1{}^{\lambda}k_3{}^{\tau} + \eta^{\lambda\nu}\eta^{\mu\rho}k_1{}^{\sigma}k_3{}^{\tau} + \eta^{\lambda\mu}\eta^{\nu\rho}k_1{}^{\sigma}k_3{}^{\tau} + \eta^{\mu\sigma}\eta^{\nu\rho}k_2{}^{\lambda}k_3{}^{\tau} + \eta^{\mu\rho}\eta^{\nu\sigma}k_2{}^{\lambda}k_3{}^{\tau} - \\
& \eta^{\mu\nu}\eta^{\rho\sigma}k_2{}^{\lambda}k_3{}^{\tau} + \eta^{\lambda\sigma}\eta^{\nu\rho}k_2{}^{\mu}k_3{}^{\tau} + \eta^{\lambda\nu}\eta^{\rho\sigma}k_2{}^{\mu}k_3{}^{\tau} + \eta^{\lambda\sigma}\eta^{\mu\rho}k_2{}^{\nu}k_3{}^{\tau} + \eta^{\lambda\mu}\eta^{\rho\sigma}k_2{}^{\nu}k_3{}^{\tau} - \\
& \eta^{\lambda\sigma}\eta^{\mu\nu}k_2{}^{\rho}k_3{}^{\tau} + \eta^{\lambda\nu}\eta^{\mu\sigma}k_2{}^{\rho}k_3{}^{\tau} + \eta^{\lambda\mu}\eta^{\nu\sigma}k_2{}^{\rho}k_3{}^{\tau} + 2\eta^{\lambda\rho}\eta^{\nu\sigma}k_3{}^{\mu}k_3{}^{\tau} + 2\eta^{\lambda\rho}\eta^{\mu\sigma}k_3{}^{\nu}k_3{}^{\tau} - \\
& 2\eta^{\lambda\rho}\eta^{\mu\nu}k_3{}^{\sigma}k_3{}^{\tau} + 2\eta^{\lambda\nu}\eta^{\mu\rho}k_3{}^{\sigma}k_3{}^{\tau} + 2\eta^{\lambda\mu}\eta^{\nu\rho}k_3{}^{\sigma}k_3{}^{\tau} - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot \\
& k_2 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_2 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_2 + \eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_2 + \\
& 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_2 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_2 - \\
& \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_2 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_2 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_2 + \\
& 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_2 - \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_1\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_1\cdot k_3 + \\
& 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_1\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_1\cdot k_3 + 2\eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_1\cdot k_3 - \\
& \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_1\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_1\cdot k_3 + 2\eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_1\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - \\
& \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_1\cdot k_3 - 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_1\cdot k_3 + \eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_1\cdot k_3 - \\
& \eta^{\lambda\tau}\eta^{\mu\sigma}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\sigma}\eta^{\mu\tau}\eta^{\nu\rho}k_2\cdot k_3 - \eta^{\lambda\tau}\eta^{\mu\rho}\eta^{\nu\sigma}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\tau}\eta^{\nu\sigma}k_2\cdot k_3 - \\
& \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau}k_2\cdot k_3 + 2\eta^{\lambda\rho}\eta^{\mu\sigma}\eta^{\nu\tau}k_2\cdot k_3 + \eta^{\lambda\tau}\eta^{\mu\nu}\eta^{\rho\sigma}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\tau}\eta^{\rho\sigma}k_2\cdot k_3 - \\
& \eta^{\lambda\mu}\eta^{\nu\tau}\eta^{\rho\sigma}k_2\cdot k_3 + \eta^{\lambda\sigma}\eta^{\mu\nu}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\nu}\eta^{\mu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \eta^{\lambda\mu}\eta^{\nu\sigma}\eta^{\rho\tau}k_2\cdot k_3 - \\
& 2\eta^{\lambda\rho}\eta^{\mu\nu}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\nu}\eta^{\mu\rho}\eta^{\sigma\tau}k_2\cdot k_3 + 2\eta^{\lambda\mu}\eta^{\nu\rho}\eta^{\sigma\tau}k_2\cdot k_3
\end{aligned}
$$

$$
\left\langle \frac{\delta S^3}{\delta\varphi^-_{\mu\nu}\delta\varphi^-_{\sigma\tau}\delta\varphi^+_{\rho\lambda}} \right\rangle_{\text{on-shell}} \rightarrow 4\left(k_1{}^{\sigma}\eta^{\mu\rho} - k_2{}^{\mu}\eta^{\rho\sigma}\right)\left(k_1{}^{\tau}\eta^{\nu\lambda} - k_2{}^{\nu}\eta^{\lambda\tau}\right)
$$

Takeaway: On-shell good (Off-shell bad)

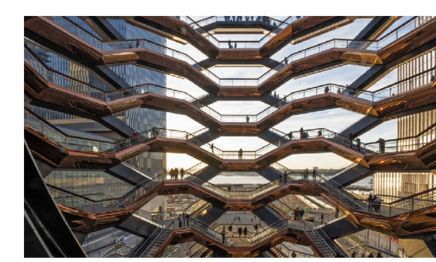
$$\left\langle \frac{\delta S^3}{\delta\varphi^-_{\mu\nu}\,\delta\varphi^-_{\sigma\tau}\,\delta\varphi^+_{\rho\lambda}} \right\rangle_{\text{on-shell}} \to 4\left(k_1{}^{\sigma}\eta^{\mu\rho} - k_2{}^{\mu}\eta^{\rho\sigma}\right)\left(k_1{}^{\tau}\eta^{\nu\lambda} - k_2{}^{\nu}\eta^{\lambda\tau}\right)$$

$$\left\langle \frac{\delta S^3}{\delta A^{-a}_{\mu}\,\delta A^{-b}_{\sigma}\,\delta A^{+c}_{\rho}} \right\rangle_{\text{on-shell}} \to -2i f^{abc}\left(k_1{}^{\sigma}\eta^{\mu\rho} - k_2{}^{\mu}\eta^{\rho\sigma}\right)$$

On-shell good (Off-shell bad)

Color = Kinematics

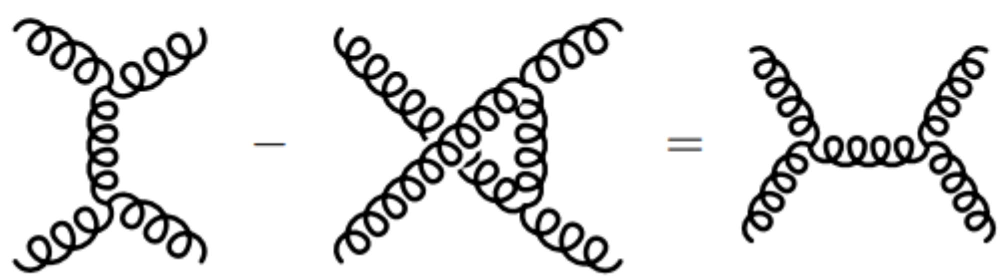
Gravity = Gauge ^ 2



Consider a gauge theory amplitude...

$$\mathcal{A}_m = g^{m-2} \sum_{i \in \Gamma} \frac{n_i \, c_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

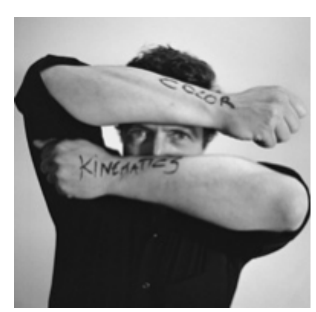
It satisfies an algebraic relation



$$f^{dae} f^{ebc} - f^{dbe} f^{eac} = f^{abe} f^{ecd}$$

$$c_i - c_j = c_k$$

$$n_i - n_j = n_k$$

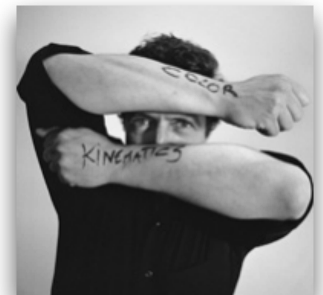


Color = Kinematics

The BCJ double copy

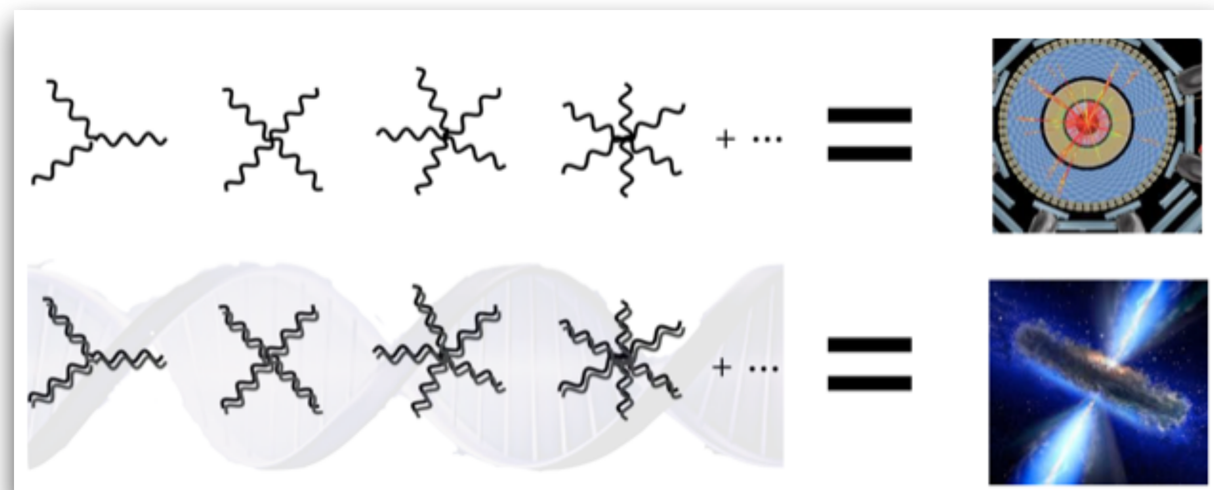
$$\frac{1}{g^{n-2}}\mathcal{A}_n = \sum_{\text{diags. } i} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

$$\frac{-i}{(\kappa/2)^{n-2}}\mathcal{M}_n = \sum_{\text{diags. } i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

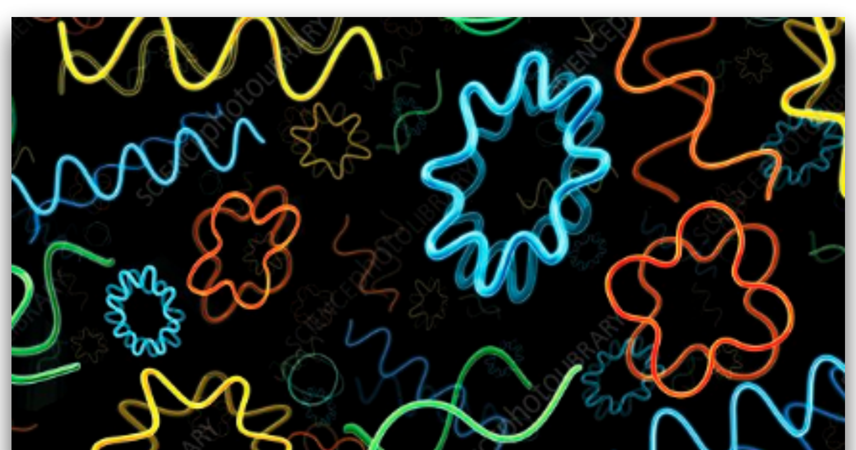


Bern, Carrasco and Johansson ‘08

Gravity = Gauge ^ 2



Credit: Carrasco



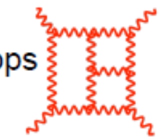
$$M_3^{\text{tree}}(1,2,3) = i A_3^{\text{tree}}(1,2,3)\, A_3^{\text{tree}}(1,2,3)\,,$$

$$\underbrace{M_4^{\text{tree}}(1,2,3,4)}_{\textbf{Gravity}} = -i s_{12} \underbrace{A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)}_{\textbf{Yang-Mills}}$$

LOOPS

Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

3 loops  $\sim 10^{20}$ TERMS — No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS — More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

Credit: Bern

The double copy works at loop level…

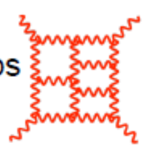
Feynman Diagrams for Gravity

SUPPOSE WE WANT TO CHECK IF CONSENSUS OPINION IS TRUE

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

More terms than atoms in your brain!

- Calculations to settle this seemed utterly hopeless!
- Seemed destined for dustbin of undecidable questions.

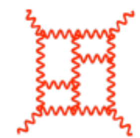
Credit: Bern

Unitarity Method + Color/Kinematics Duality

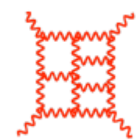
Z B, Carrasco, Dixon, Johansson, Roiban

For N = 8 supergravity.

3 loops  ~ 10 TERMS

4 loops  $\sim 10^{2}$ TERMS

Much more manageable!

5 loops  $\sim 10^{3}$ TERMS (Not yet done)

We now have the ability to settle the 35 year debate and determine the true UV behavior gravity theories.

Credit: Bern

arXiv.org > hep-th > arXiv:1804.09311

Search

He

High Energy Physics - Theory

[Submitted on 25 Apr 2018]

Ultraviolet Properties of N = 8 Supergravity at Five Loops

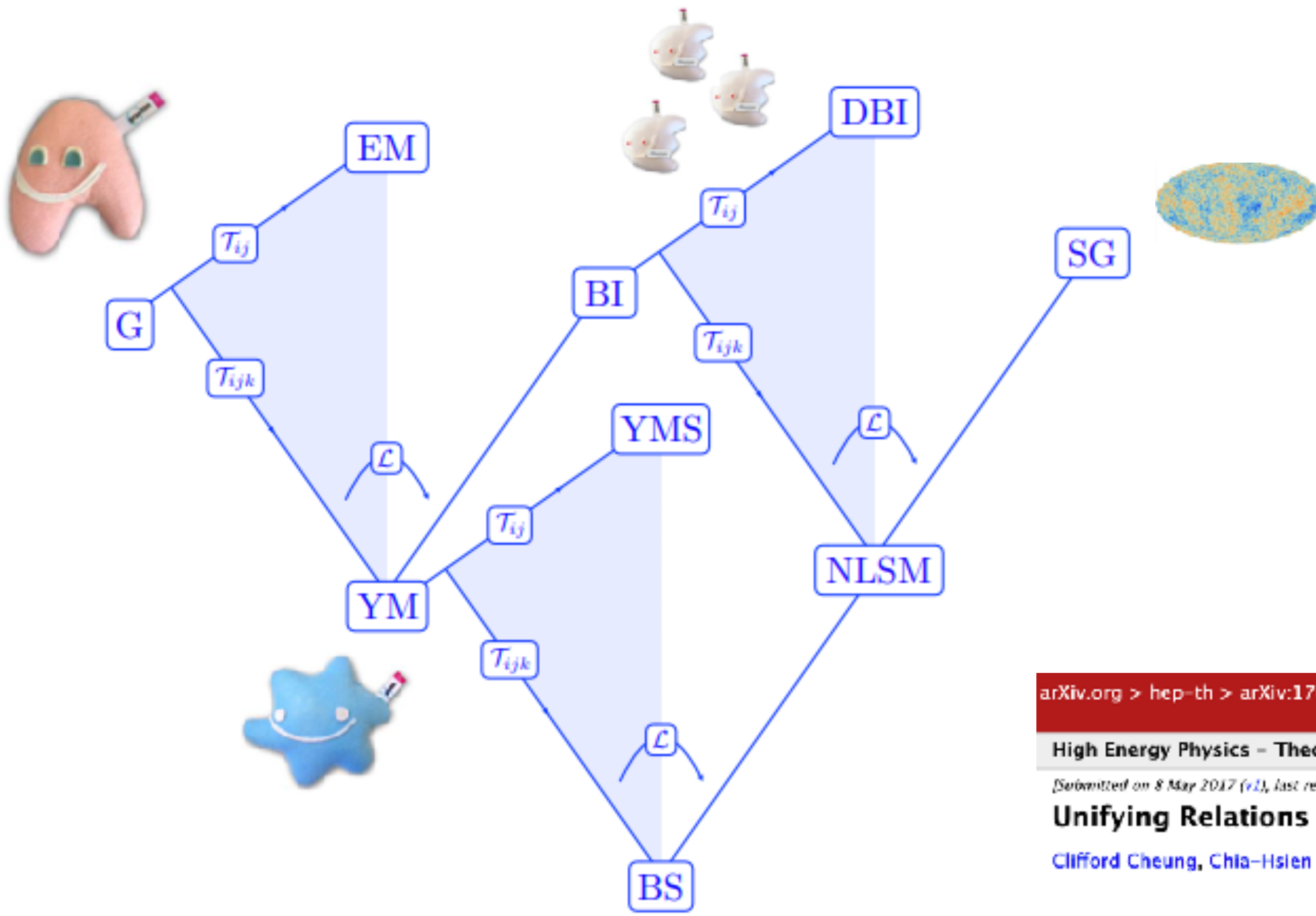
Zvi Bern, John Joseph Carrasco, Wei-Ming Chen, Alex Edison, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, Mao Zeng

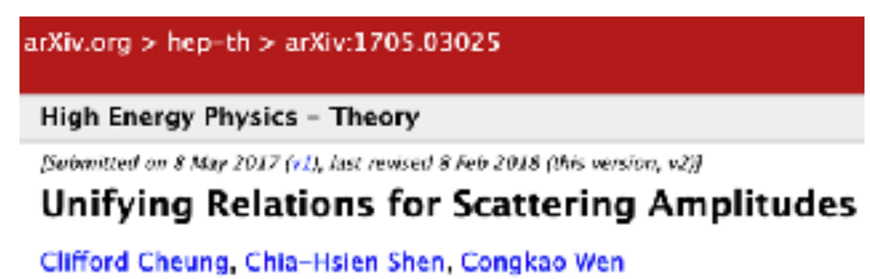
We use the recently developed generalized double-copy construction to obtain an improved representation of the five-loop four-point integrand of $N = 8$ supergravity whose leading ultraviolet behavior we analyze using state of the art loop-integral expansion and reduction methods. We find that the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5$, corresponding to a $D^8 R^4$ counterterm. This ultraviolet behavior stands in contrast to the cases of four-dimensional

…rendering possible otherwise impossible calculations

Webs

The double copy extends throughout a web of theories…




arXiv.org > hep-th > arXiv:1705.03025

High Energy Physics – Theory

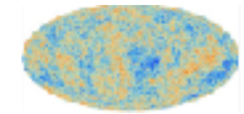
[Submitted on 8 May 2017 (v1), last revised 8 Feb 2018 (this version, v2)]

Unifying Relations for Scattering Amplitudes

Clifford Cheung, Chia-Hsien Shen, Congkao Wen

Figure 1: Diagram depicting the unified web of theories. The corners represent extended gravity (G), Einstein-Maxwell (EM) theory, Yang-Mills (YM) theory, Born-Infeld (BI) theory, Dirac-Born-Infeld (DBI) scalar theory, nonlinear sigma model (NLSM), special Galileon (SG), Yang-Mills scalar (YMS) theory, and biadjoint (BS) theory. The arrows correspond to the transmutation operators, $\mathcal{T}_{ij}$, $\mathcal{T}_{ijk}$, and $\mathcal{L}$. The shaded regions and edges correspond to hybrid theories.

… an intricate web of theories…



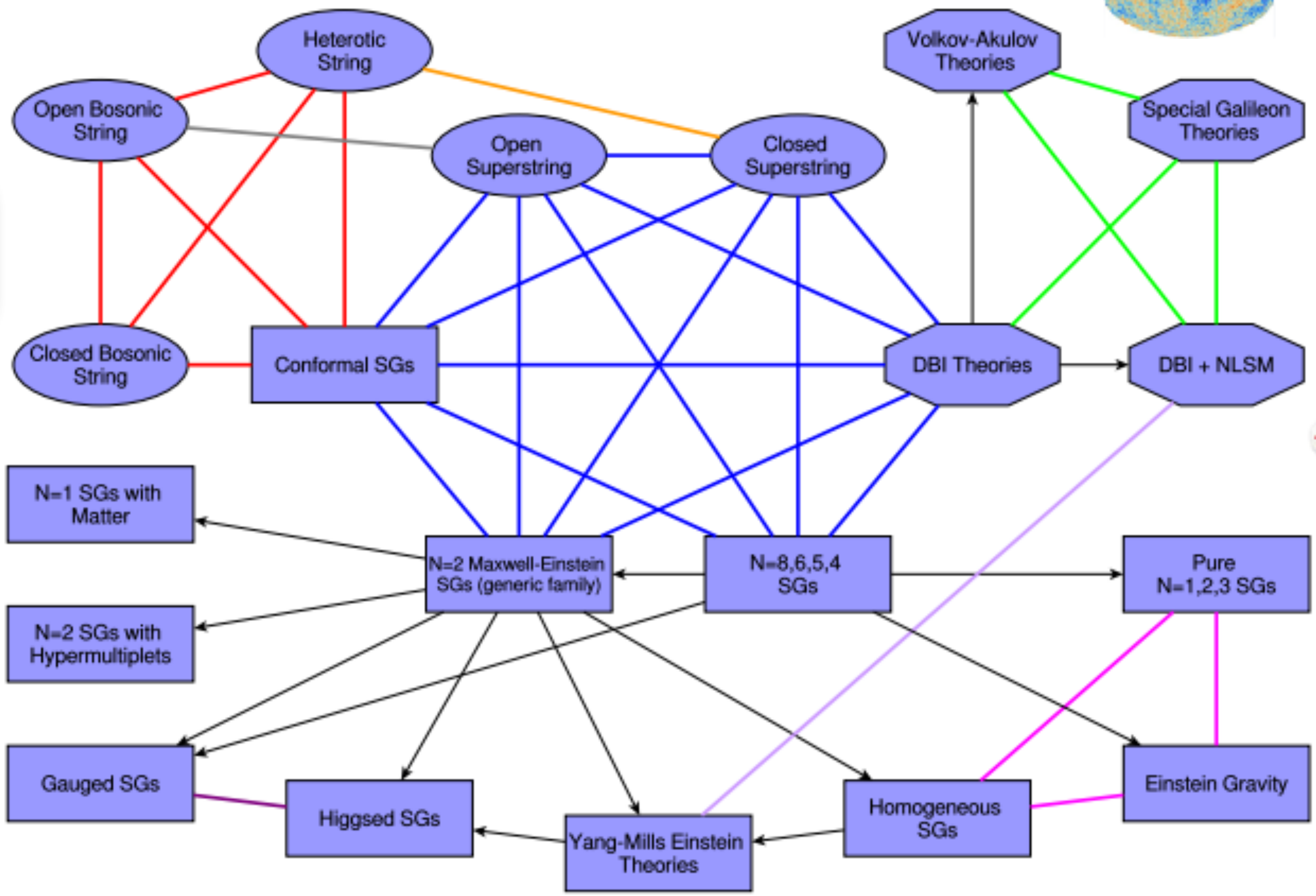


Figure 17: Schematic rendition of the web of theories. Nodes represent the main double-copy-

arXiv.org > hep-th > arXiv:1909.01358

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High Energy Physics – Theory

[Submitted on 3 Sep 2019]

The Duality Between Color and Kinematics and its Applications

Zvi Bern, John Joseph Carrasco, Marco Chiodaroli, Henrik Johansson, Radu Roiban

...including classical solutions...

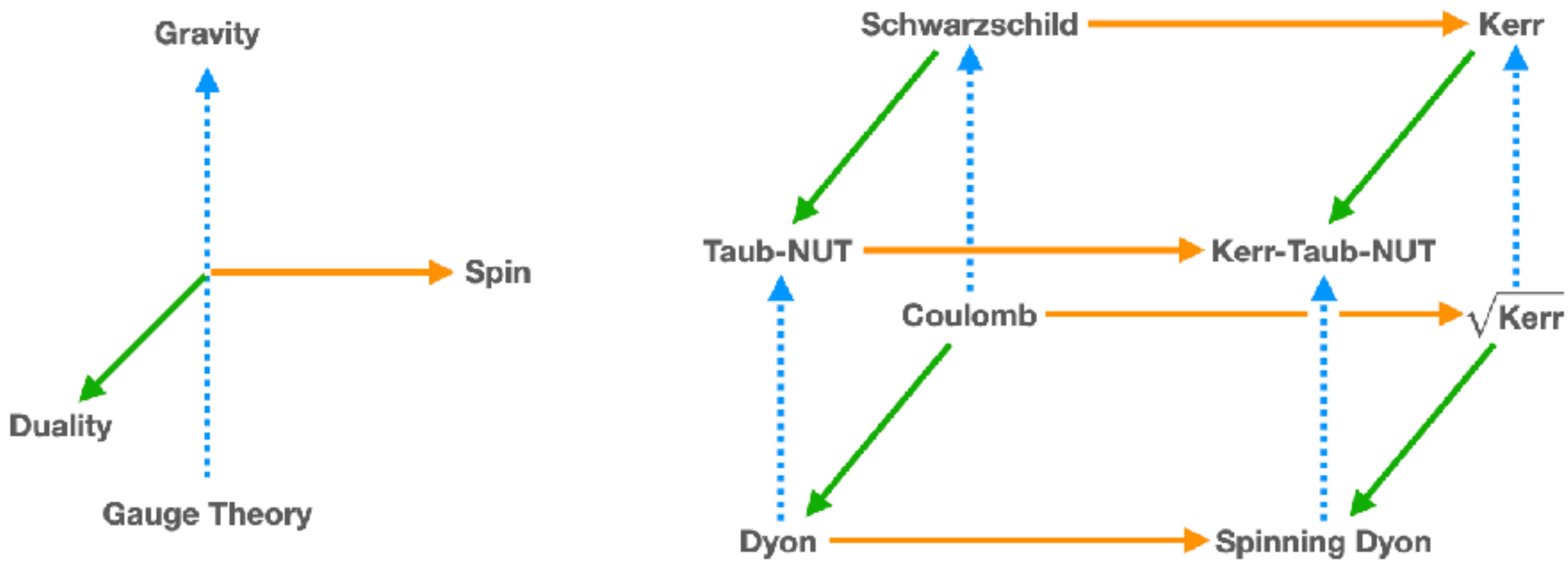


Figure 1. Classical solutions related by the double copy (dashed blue arrow), duality (green arrow) and the Newman-Janis shift (orange arrow). The double copy provides a map between solutions of

arXiv.org > hep-th > arXiv:2010.07861

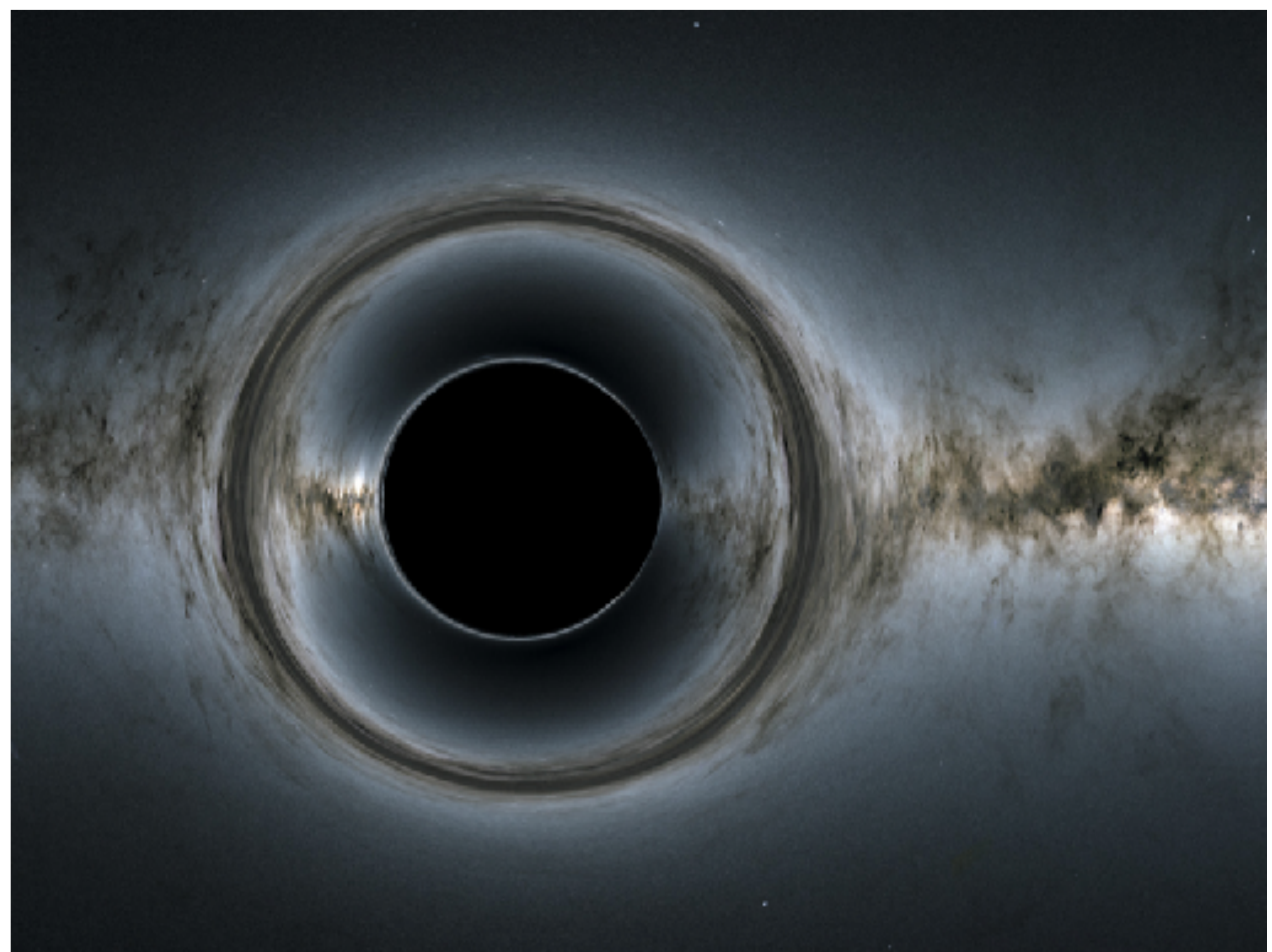
High Energy Physics – Theory

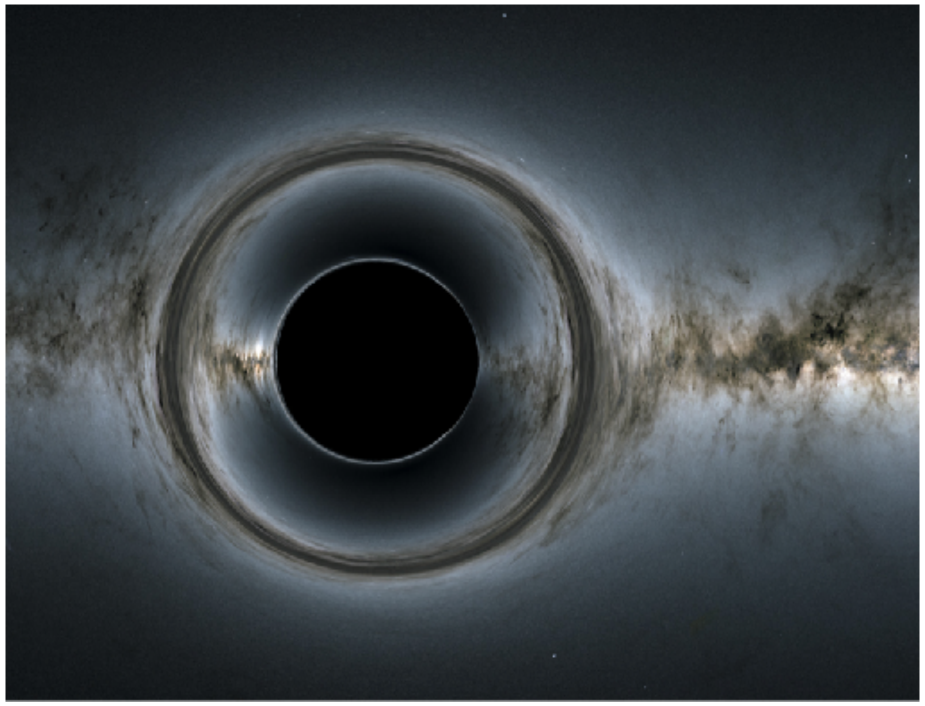
[Submitted on 15 Oct 2020 (v1), last revised 18 Oct 2020 (this version, v2)]

Amplitudes from Coulomb to Kerr–Taub–NUT

William T. Emond, Yu-tin Huang, Uri Kol, Nathan Moynihan, Donal O'Connell

What about black holes?





arXiv.org > hep-th > arXiv:1410.0239

High Energy Physics – Theory

[Submitted on 1 Oct 2014 (v1), last revised 8 Jan 2015 (this version, v2)]

Black holes and the double copy

Ricardo Monteiro, Donal O'Connell, Chris D. White

$$\frac{1}{g^{n-2}}\mathcal{A}_n = \sum_{\text{diags.}\, i} \frac{n_i c_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

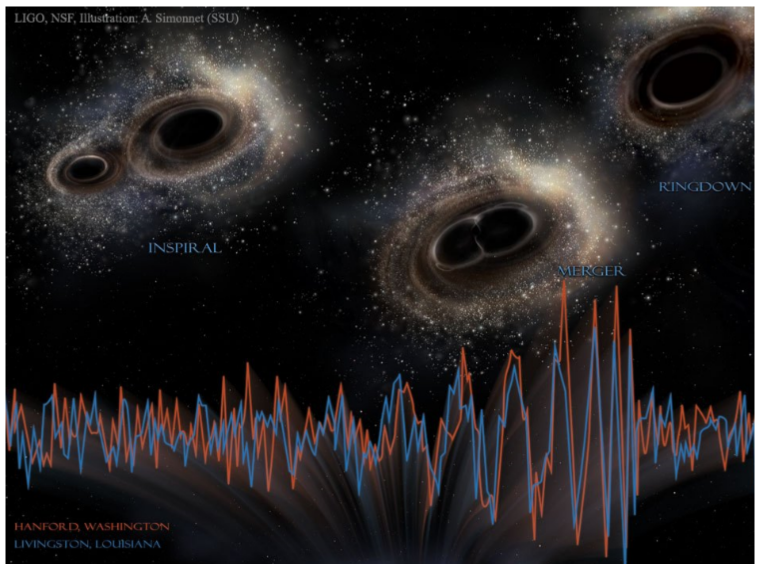
$$\frac{-i}{(\kappa/2)^{n-2}}\mathcal{M}_n = \sum_{\text{diags.}\, i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

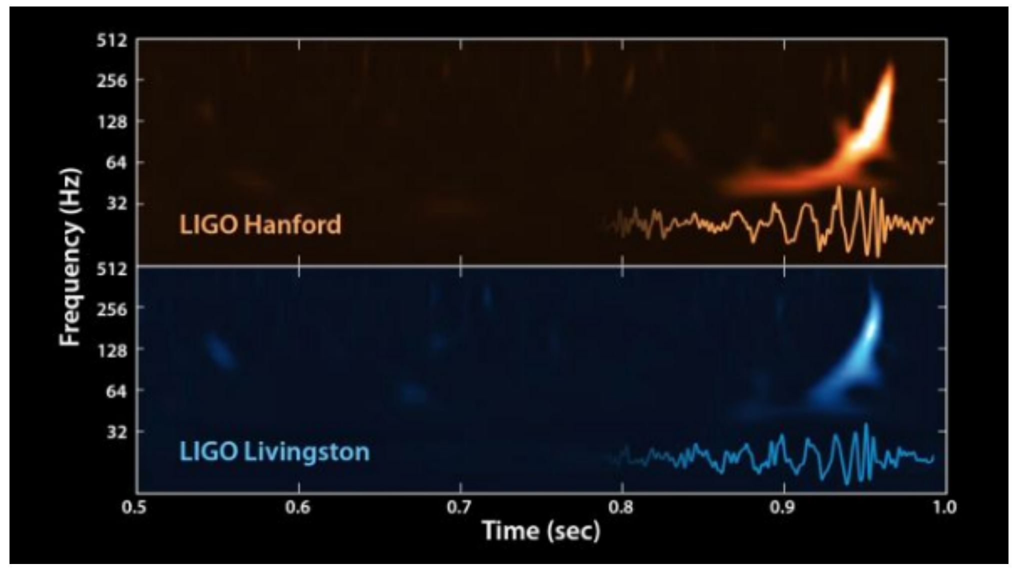
$$A^\mu_a = c_a \phi k^\mu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$\equiv \eta_{\mu\nu} + k_\mu k_\nu \phi$$

Exact solutions. Nice! But mostly static...

Can we apply Amplitudes/Double copy methods to the field of gravitational waves?





arXiv.org > hep-th > arXiv:1908.01493

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High Energy Physics – Theory

[Submitted on 5 Aug 2019 (v1), last revised 14 Feb 2020 (this version, v2)]

Black Hole Binary Dynamics from the Double Copy and Effective Theory

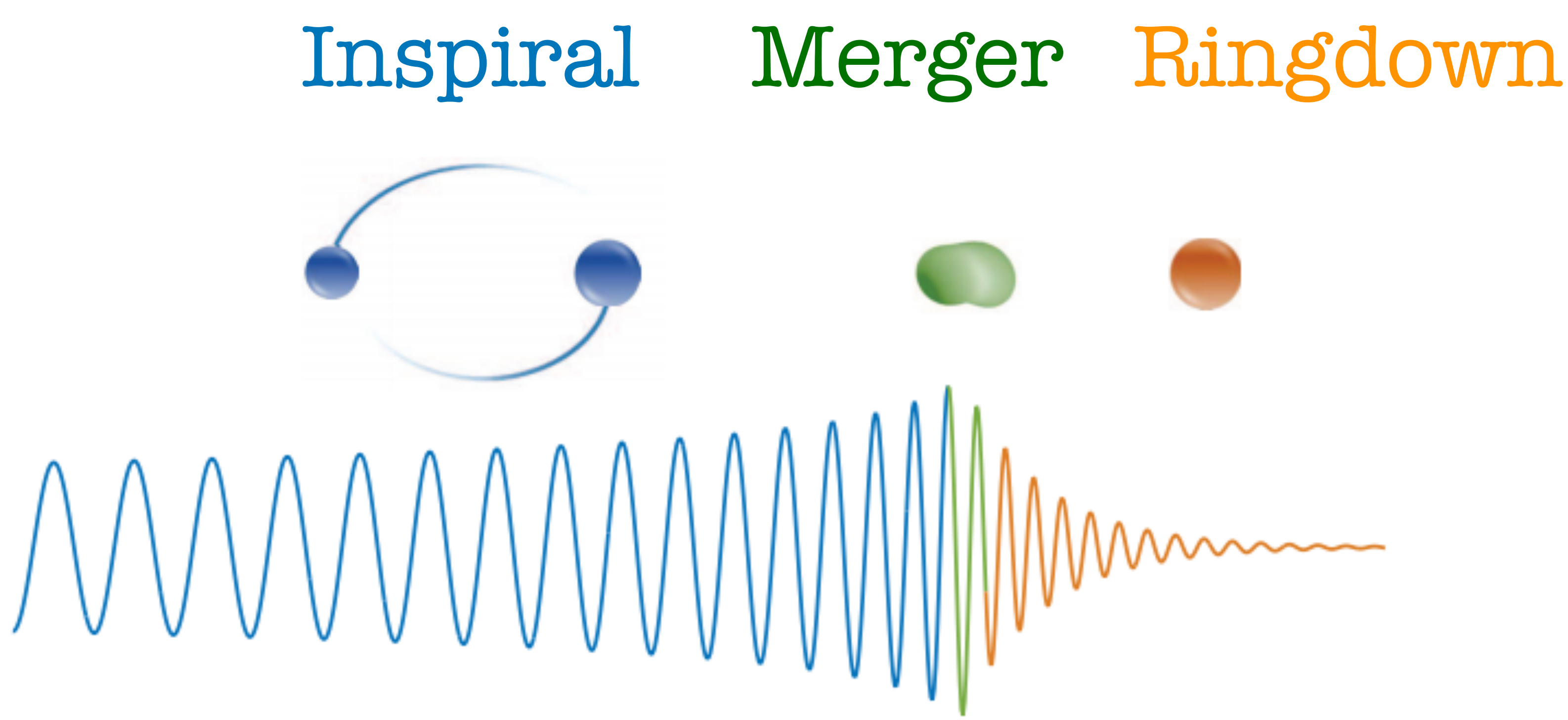
Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, Mao Zeng

Published in: *Phys.Rev.Lett.* 122 (2019) 20, 201603 • e-Print: 1901.04424 [hep-th]

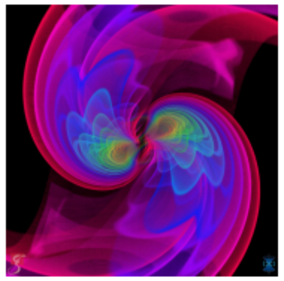
pdf DOI cite

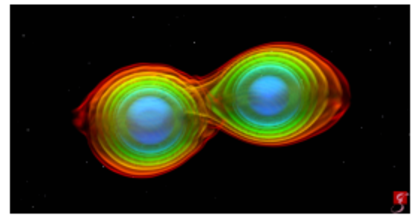
187 citations

Can we apply Amplitudes/Double copy methods to the field of gravitational waves?



The natural candidate is the inspiral phase, where perturbation theory is applied



The Need for High-Precision Gravitational Waveforms

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

(Albert Einstein Institute)

Department of Physics, University of Maryland

"QCD Meets Gravity IV", NORDITA, Stockholm — Dec 10-14, 2018

Need more efficient ways to solve two-body problem, analytically

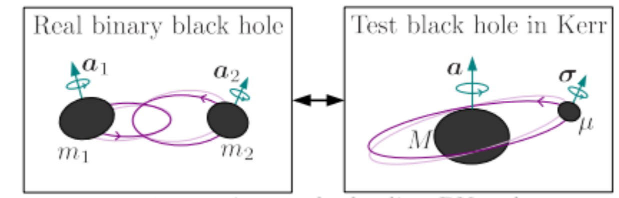
- In test-body limit, spinning EOB Hamiltonian includes linear terms in spin of test body at all PN orders.

(Barausse et al. 10, Barausse & AB 11, 12; Vines et al. 15)

$$(\sigma + \sigma^2)\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

- Is EOB mapping unique at all orders?

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$



exact mapping at the leading PN orders

Using unbound orbits, using scattering angle as adiabatic invariant, at 1PM: mapping unique & 2-body relativistic motion equivalent to 1-body motion in Kerr. (Damour 16, Bini et al. 17-18, Vines 17)

see Steinhoff's & Damour's talks

$$\frac{Gm}{rc^2}\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

$GM/rc^2 \ll v^2/c^2 \sim 1$

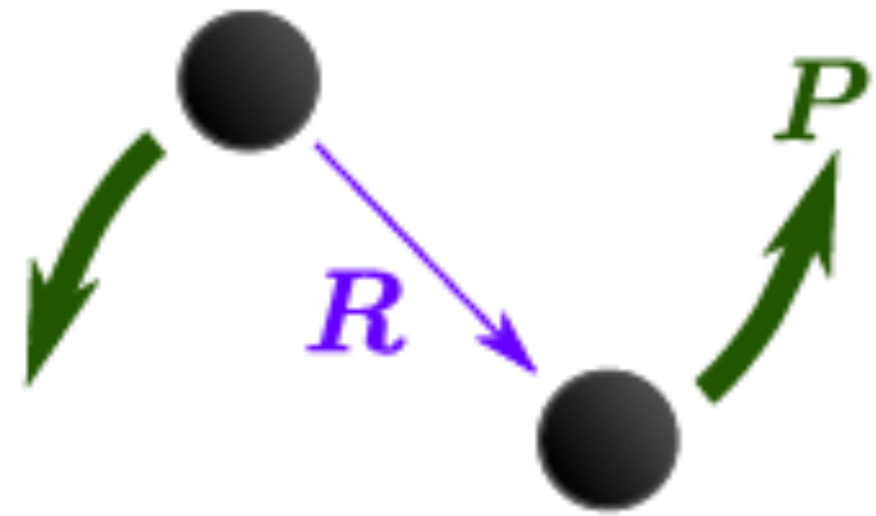
- Results at leading PN order but all orders in spin.

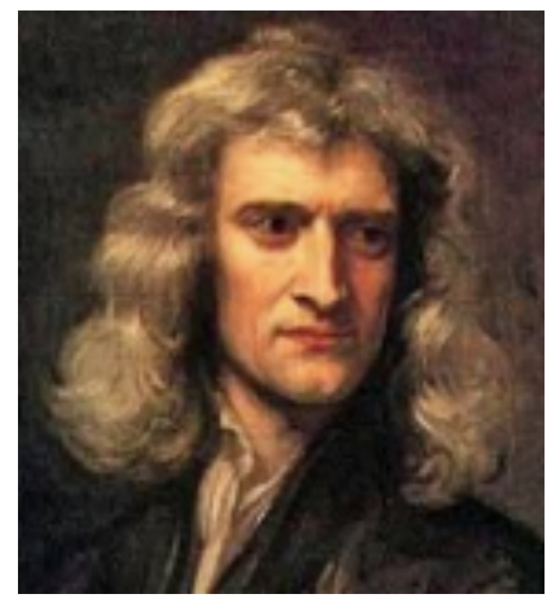
(Vines & Steinhoff 16; Vines & Harte 16, Siemonsen, Steinhoff & Vines 17)

$$\frac{v^2}{c^2}\left(S_i + S_i^2 + \cdots\right)$$

Need more efficient ways to solve two-body problem, analytically

The two-body problem





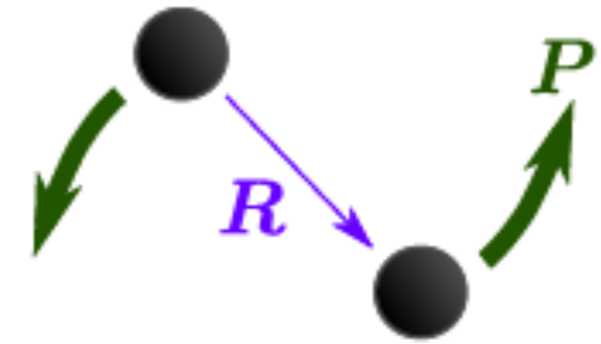
$$\frac{H}{\mu} = \boxed{\frac{P^2}{2} - \frac{Gm}{R}}$$

Newton $\sim \mathcal{O}(G)$

$m = m_A + m_B, \quad \nu = \mu/M \quad \mu = m_A m_B/m$

$$\frac{H}{\mu} = \boxed{\frac{P^2}{2} - \frac{Gm}{R}}$$

Newton $\sim \mathcal{O}(G)$



THE GRAVITATIONAL EQUATIONS AND THE PROBLEM OF MOTION

By A. Einstein, L. Infeld, and B. Hoffmann

(Received June 16, 1937)

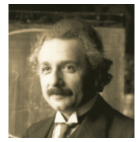
Introduction. In this paper we investigate the fundamentally simple question of the extent to which the relativistic equations of gravitation determine the motion of ponderable bodies.

degree of accuracy. In the present part we deal with the actual application of this method, carrying the calculation to such a stage that the main deviation from the Newtonian laws of motion is determined.

$$+\boxed{\frac{1}{c^2}\left\{-\frac{P^4}{8} + \frac{3\nu P^4}{8} + \frac{Gm}{R}\left(-\frac{P_R^2\nu}{2} - \frac{3P^2}{2} - \frac{\nu P^2}{2}\right) + \frac{G^2m^2}{2R^2}\right\}}$$

1PN: Einstein, Infeld, Hoffmann '38

$m = m_A + m_B, \quad \nu = \mu/M \quad \mu = m_A m_B/m$

virial theorem

$$v^2 \sim \frac{GM}{r} \ll 1$$

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

1PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots\right) G$

2PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots\right) G^2$

3PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots\right) G^3$

4PM $\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots\right) G^4$

5PM $\left(1 + v^2 + v^4 + v^6 + \cdots\right) G^5$

$\vdots$

Post-Newtonian approximation

Using ADM Hamiltonian, EFT (NRGR) and Self-force

0PN: Newton 1666

1PN: Einstein, Infeld, Hoffmann '38

2PN: Ohta et. al. '73

3PN: Damour, Jaranowski, Schaefer, Blanchet, Faye ca. '97

4PN: Bini, Damour, Jaranowski, Schaefer, Blanchet, Faye, Marchand, Foffa, Sturani, Mastrolia, Sturm, Porto, Rothstein... ca. '13

5PN*: Bini, Damour, Geralico, Foffa, Mastrolia, Sturani, Bobadilla ca. '19

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN

1PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots \right) G$

2PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots \right) G^2$

3PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots \right) G^3$

4PM $\left(1 + v^2 + v^4 + v^6 + v^8 + \cdots \right) G^4$

5PM $\left(1 + v^2 + v^4 + v^6 + \cdots \right) G^5$

$\vdots$

Post-Minkowskian approximation

Bertotti, Kerr, Plebanski, Portilla, Westpfahl, Goller, Bel, Damour, Deruelle, Ibanez, Martin, Ledvinka, Schaefer, Bicak...

An invitation by Damour…

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bodies into a corresponding (effective one-body) Hamiltonian description has been recently introduced [Phys. Rev. D **94**, 104015 (2016)]. Using this technique, we derive, for the first time, to second-order in Newton's constant (i.e. one classical loop) the Hamiltonian of two point masses having an arbitrary (possibly relativistic) relative velocity. The resulting (second post-Minkowskian) Hamiltonian is found to have a tame high-energy structure which we relate both to gravitational self-force studies of large mass-ratio binary systems, and to the ultra high-energy quantum scattering results of Amati, Ciafaloni and Veneziano. We derive several consequences of our second post-Minkowskian Hamiltonian: (i) the need to use special phase-space gauges to get a tame high-energy limit; and (ii) predictions about a (rest-mass independent) linear Regge trajectory behavior of high-angular-momenta, high-energy circular orbits. Ways of testing these predictions by dedicated numerical simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

simulations are indicated. We finally indicate a way to connect our classical results to the quantum gravitational scattering amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

Classical physics from Quantum field theory?

Progress of Theoretical Physics, Vol. 46, No. 5, November 1971

Quantum Theory of Gravitation vs. Classical Theory[*)]

——*Fourth-Order Potential*——

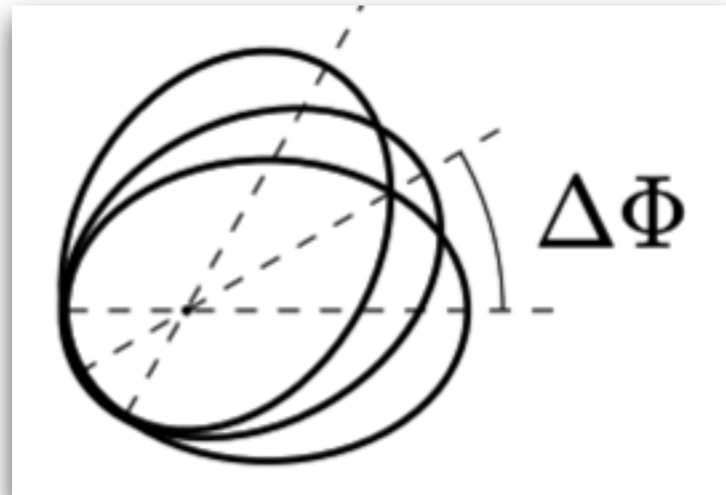
Yoichi IWASAKI

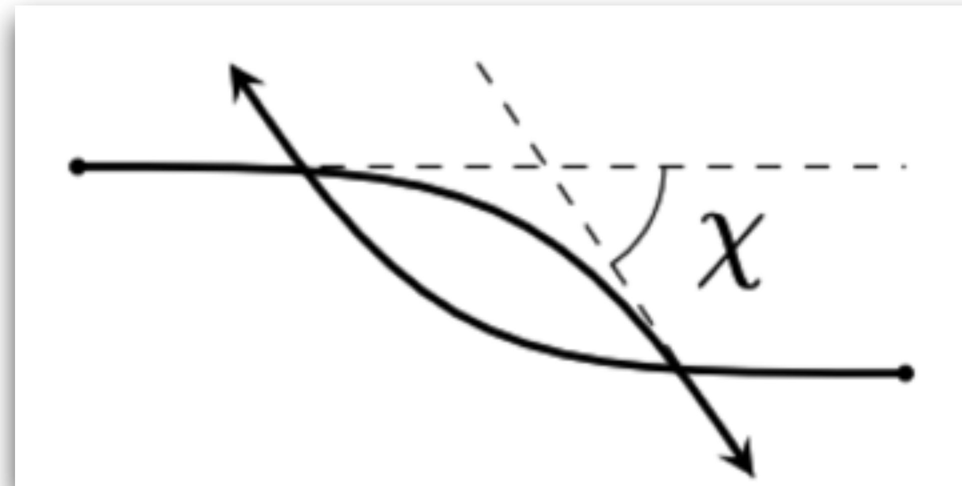
Trees AND loops?

Here we want to point out that there seems to exist an erroneous belief[*),**)] that only tree diagrams contribute to the classical process. Contrary to this belief, the quadratic term in k corresponds to fourth-order diagrams each of which contains a closed loop; it is a "radiative correction" term. Since the quantum

Iwasaki knew it 50 years ago...

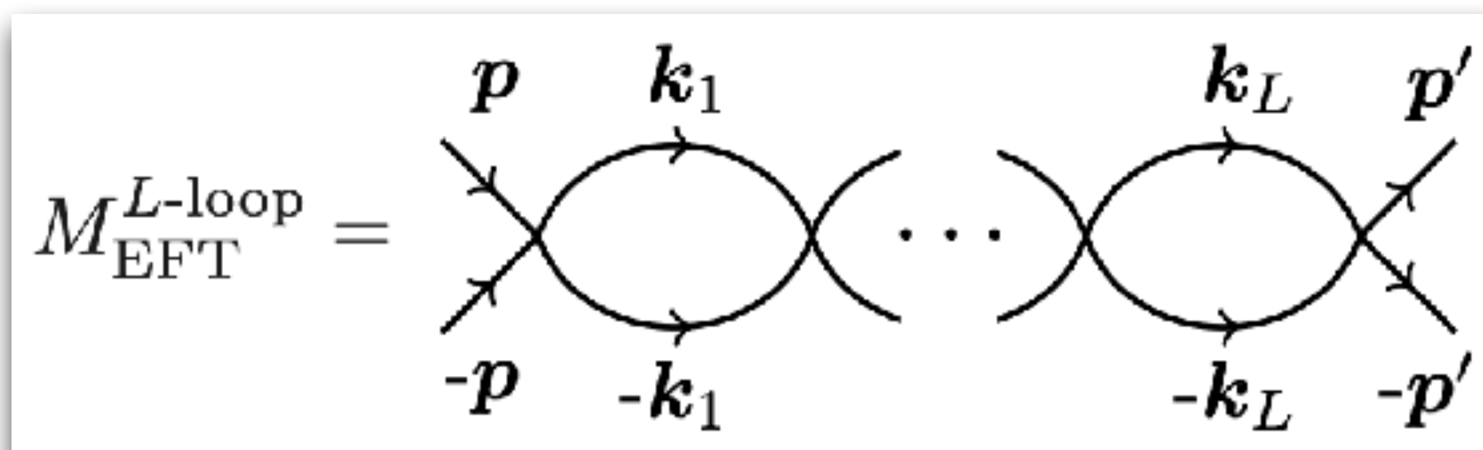
Scattering is not inspiral…

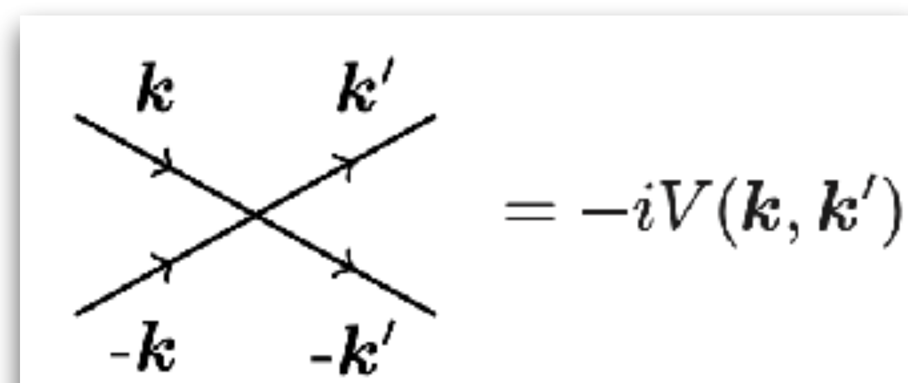




…but one may use the potential to describe both.

Amplitude is not potential…





… but one may use an Effective Field Theory to extract it

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}+\cdots)\,G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}+\cdots)\,G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}+\cdots)\,G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8+\cdots)\,G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6+\cdots)\,G^5$

$$\vdots$$

Damour asked for 3PM

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

$$\begin{array}{ll}
\text{1PM} & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots)\, G \\
\text{2PM} & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots)\, G^2 \\
\text{3PM} & (1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots)\, G^3 \\
\text{4PM} & (1 + v^2 + v^4 + v^6 + v^8 + \cdots)\, G^4 \\
\text{5PM} & (1 + v^2 + v^4 + v^6 + \cdots)\, G^5 \\
 & \vdots
\end{array}$$

arXiv.org > hep-th > arXiv:1901.04424

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High Energy Physics – Theory

[Submitted on 14 Jan 2019]

Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, Mao Zeng

Use unitarity, spinor helicity and double copy to get Amplitude

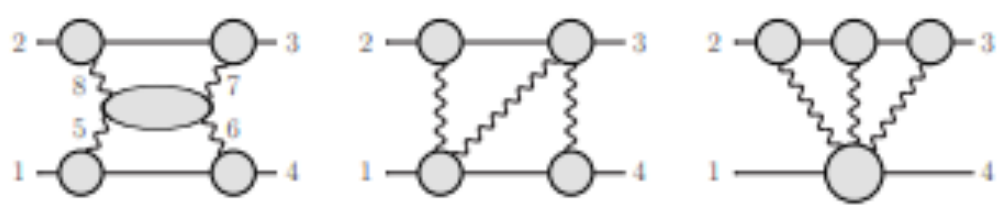


FIG. 1. Unitarity cuts needed for the classical scattering amplitude. The shaded ovals represent tree amplitudes while the exposed lines depict on-shell states. The wiggly and straight lines denote gravitons and massive scalars, respectively.

$$A_4(1^s, 2^+, 3^+, 4^s) = i\,\frac{m_1^2\,[2\,3]}{\langle 2\,3\rangle\, t_{12}},$$

$$A_4(1^s, 2^+, 3^-, 4^s) = i\,\frac{\langle 3|\,1\,|2]^2}{t_{23}\, t_{12}},$$

$$A_4(1^-, 2^-, 3^+, 4^+) = i\,\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle},$$

$$A_4(1^-, 2^+, 3^-, 4^+) = i\,\frac{\langle 1\,3\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle},$$

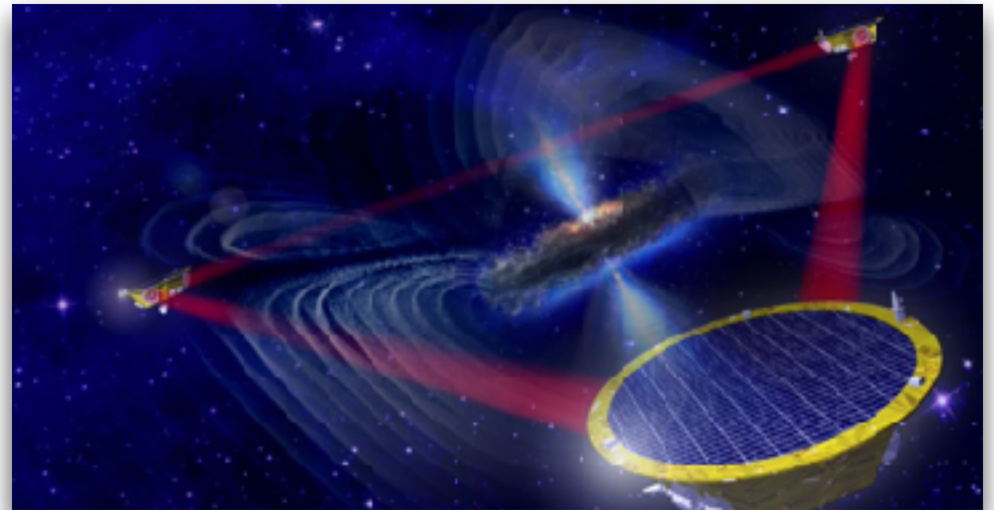
Use EFT to extract potential

$$H(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{r}),$$

$$V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2 (1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3 \xi^2}\right],$$

$$\begin{aligned}
c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\Bigg[& \frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \\
& - \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2} \\
& + \frac{2\nu^3 (3 - 4\xi)\sigma\left(1 - 2\sigma^2\right)^2}{\gamma^4 \xi^3} + \frac{\nu^4 (1 - 2\xi)\left(1 - 2\sigma^2\right)^3}{2\gamma^6 \xi^4}\Bigg],
\end{aligned}$$



The next generation of gravitational wave experiments (LISA) will require high precision in analytic results.

We aim to contribute, and are working on refining our techniques and apply them to different relevant problems

arXiv.org > hep-th > arXiv:2010.08559

High Energy Physics – Theory

[Submitted on 16 Oct 2020]

Leading Nonlinear Tidal Effects and Scattering Amplitudes

Zvi Bern, Julio Parra-Martinez, Radu Roiban, Eric Sawyer, Chia-Hsien Shen

arXiv.org > hep-th > arXiv:2101.07254

High Energy Physics – Theory

[Submitted on 18 Jan 2021]

Scattering Amplitudes and Conservative Binary Dynamics at $\mathcal{O}(G^4)$

Zvi Bern, Julio Parra-Martinez, Radu Roiban, Michael S. Ruf, Chia-Hsien Shen, Mikhail P. Solon, Mao Zeng

arXiv.org > hep-th > arXiv:2101.07255

High Energy Physics – Theory

[Submitted on 18 Jan 2021]

Gravitational Bremsstrahlung from Reverse Unitarity

Enrico Herrmann, Julio Parra-Martinez, Michael S. Ruf, Mao Zeng

arXiv.org > hep-th > arXiv:2102.10137

High Energy Physics – Theory

[Submitted on 19 Feb 2021]

Quadratic-in-Spin Hamiltonian at $\mathcal{O}(G^2)$ from Scattering Amplitudes

Dimitrios Kosmopoulos, Andres Luna

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Summary

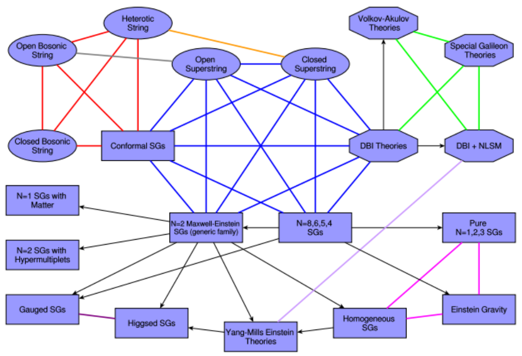
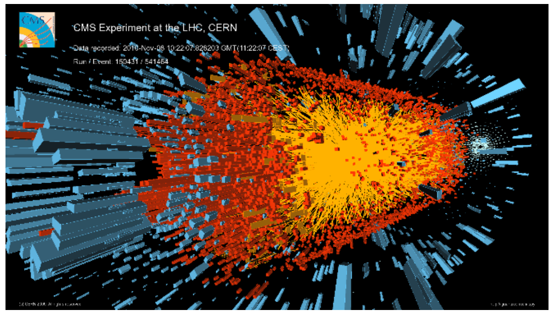
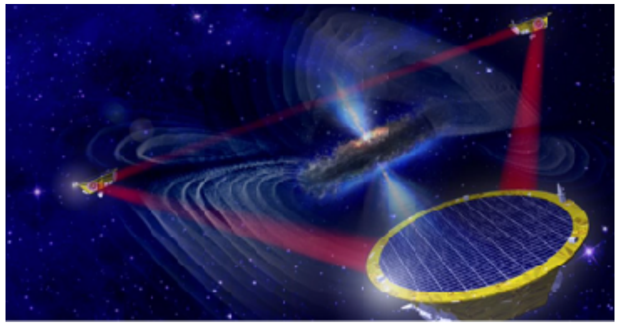


Figure 17: Schematic rendition of the web of theories. Nodes represent the main double-copy-





Thank you!

Questions?