MESON AND BARYON MASS: A UNIFIED TREATMENT

XXXV Annual Meeting of the Division of Particles and Fields

DPC-SMF

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May 11-13, 2021
Online event

MOTIVATION

- Hadron studies have provided strong indications that quark-quark correlations forming a diquark play an important role in hadron physics. These structures are a consequence of the chiral dynamic symmetry breaking (DCSB) which is responsible for 98% of the mass of visible matter in the universe. Phenomenology suggests that these structures may be crucial for the formation of exotic hadrons.
- The diquarks come in pairs with their corresponding mesons, in this way when knowing the characteristics of the mesons, the diquarks are known and then that of the baryons.

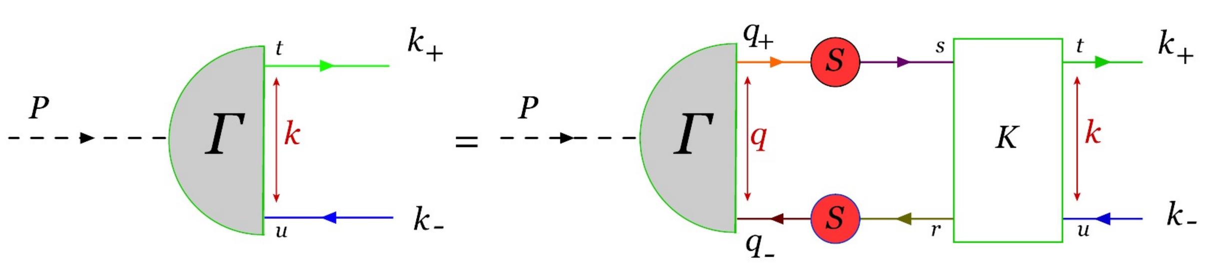


Figure 1. Bethe-Salpeter (BS) equation diagrammatically, in QFT, is a completely relativistic description of bound states of two particles used to describe Mesons.

$$\left[\Gamma_H^{f_1\bar{f}_2}(k;P)\right]_{tu} = \int \frac{d^4q}{(2\pi)^4} \left[\chi_H^{f_1\bar{f}_2}(q;P)\right]_{sr} K_{tu}^{rs}(q,k;P),$$

$$\chi_H^{f_1\bar{f}_2}(q;P) = S_{f_1}(q_+)\Gamma_H^{f_1\bar{f}_2}(q;P)S_{\bar{f}_2}(q_-),$$

- For diquarks the BS equation is modified by a factor 1/2.

- Once the properties of the mesons are known with some changes in the BS equation, the properties of the diquarks in the different channels are obtained according to their quantum numbers.
- Using a quark-diquark model, the Faddeev equation is solved to find the mass of the baryons.
- The Faddeev equation considers all possible combinations of diquarks within the Baryon.

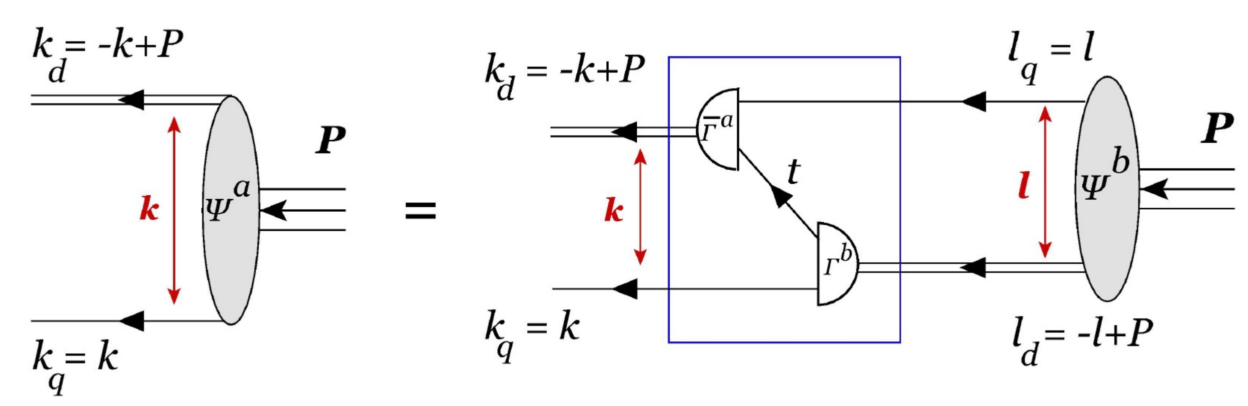


Figure 2 Faddeev's diagrammatic equation used to calculate baryon masses.

$$\begin{pmatrix} \mathcal{S}^{\Psi}(l;P)\,u^{\Psi}(P) \\ \mathcal{A}^{\Psi}_{\mu}(l;P)\,u^{\Psi}(P) \end{pmatrix} = -4 \int \frac{d^4 l}{(2\pi)^4} \mathcal{M}(k,l,P) \begin{pmatrix} \mathcal{S}^{\Psi}(l;P)\,u^{\Psi}(P) \\ \mathcal{A}^{\Psi}_{\nu}(l;P)\,u^{\Psi}(P) \end{pmatrix}$$

METHODOLOGY

GAP EQUATION

- Quark and gluons acquire mass effectively within hadrons due to dynamic mass generation (DCSB).
- The Quark propagator appears in the BS equation, which is obtained from the solution of the Schwinger-Dyson SDE equation of the quark, also known as the gap equation:

$$S^{-1}(p) = i\gamma\cdot p + m + \int \frac{d^4q}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma_\nu(p,q).$$

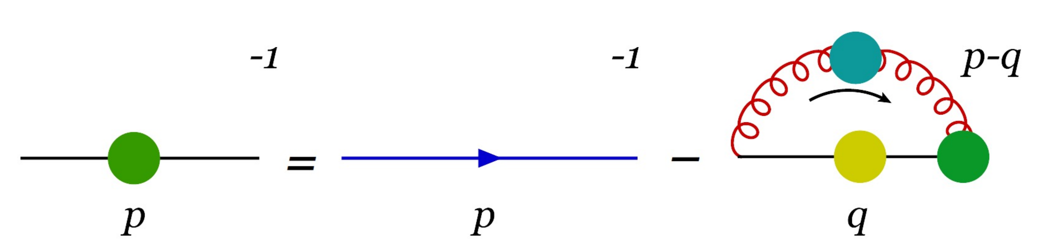


Figure 3 Diagrammatic Schwinger-Dyson equation used to calculate the dressed mass of quarks.

- Once the gap equation is solved, we use the results to solve the BS equation and thus obtain the masses and amplitudes of the mesons and diquarks.
- We use the static approximation in the Faddeev equation for the quark that is emitted from one diquark and binds to another:

$$S(p) \to S^T = \frac{g_B^2}{M_f},$$

- With the CI model, we have for the Faddeev amplitudes:

$$\mathcal{S}^{\Psi}(l,P) \to \mathcal{S}(P) = s(P)\mathbb{1}_D,$$
$$\mathcal{A}^{\Psi}_{\mu}(l,P) \to \mathcal{A}^{i}_{\mu}(P) = a_1^i(P)\, i\gamma_5\gamma_\mu + a_2^i(P)\,\gamma_5\hat{P}_\mu.$$

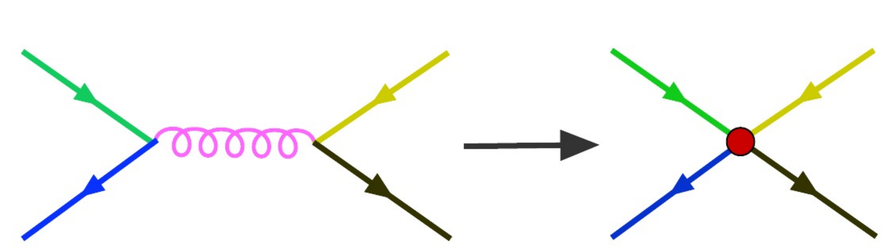
Where s and a are scalars and i represents the flavor of the diquarks.

CI CONTACT INTERACTION MODEL

- The CI model is a truncation that regularizes the UV divergences to preserve the QCD symmetries.
- The gluon propagator is considered a constant in the Landau gauge.

$$g^2 D_{\mu\nu}(k) = g^2\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]\frac{1}{k^2} \to \frac{4\pi\alpha_{IR}}{m_g^2}\delta_{\mu\nu} \equiv \delta_{\mu\nu}\frac{1}{m_G^2},$$

- Where mg is the dynamically generated mass scale, α_{IR} is a parameter that determines the force in the infrared.
- The Feynman diagram of a scatter qq:



- The quark-gluon vertex is taken at the lowest level, that is, the bare vertex is taken:

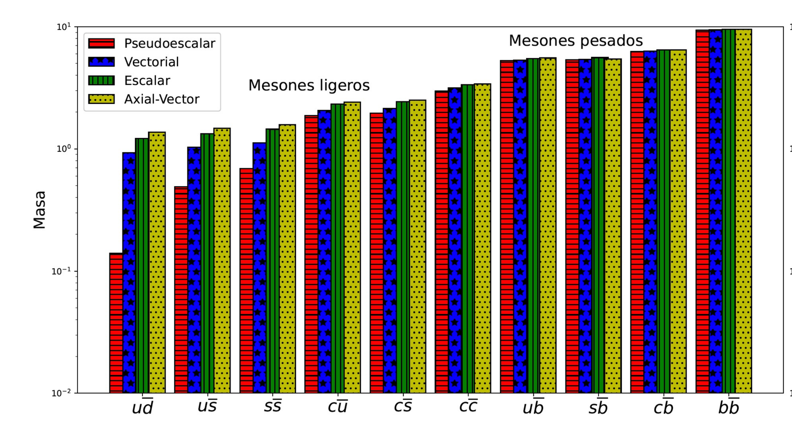
$$\Gamma_\nu^a(p,q;\mu) \to \frac{\lambda^a}{2}\gamma_\nu.$$

- It is the gauge by which the sensitivity of the differences between the bare and the full vertex is less noticeable, these two approximations are also known as the Rainbow-Lader (RL) approximation.
- With this model, the masses of the bound states are calculated in this work, but it is also possible to calculate decay constants, shape factors, among other characteristics of hadrons.

RESULTS

MESONS AND DIQUARKS

- With the CI model, the results for the masses and amplitudes of BS for the mesons have an error of approximately 5%:
- The way to check the results of the quarks is to obtain the masses of the baryons.



BARIONS

- The Faddeev amplitudes are formed with all the possible combinations of quarks within the baryon respecting the quantum numbers of parity, spin, mass, etc:

$$u = \begin{bmatrix} s \\ a_1^+ \\ a_1^0 \\ a_2^+ \\ a_2^0 \end{bmatrix}, \quad u_{P(udu)} = \begin{bmatrix} [ud]u \\ \{uu\}d \\ \{ud\}u \\ \{uu\}d \\ \{ud\}u \end{bmatrix}, \quad u_{\Sigma^+(usu)} = \begin{bmatrix} [us]u \\ \{uu\}s \\ \{us\}u \\ \{uu\}s \\ \{us\}u \end{bmatrix}, \quad u_{\Xi_{cc}^{++}(ucc)} = \begin{bmatrix} [uc]c \\ \{cc\}u \\ \{uc\}c \\ \{cc\}u \\ \{uc\}c \end{bmatrix}$$

- The interaction matrix is described in terms of the flavor matrix and the interaction kernel:

$$\mathcal{M}_{[q_1q_3][q_1q_2]} = \underbrace{t^{q_1 T}\, t^{[q_1q_2]}\, t^{[q_1q_3]^T}\, t^{q_3}}_{t^f} \underbrace{\left[\Gamma_{[q_1q_2]}(l_{q_1q_2})\, S^T_{q_1}\, \bar{\Gamma}_{[q_1q_3]}(-k_{q_1q_3})\, S_{q_3}(l_{q_3})\, \Delta_{[q_1q_2]}(l_{q_1q_2})\right]}_{\mathcal{K}_{[q_1q_3][q_1q_2]}},$$

- Baryons with spin 1/2.

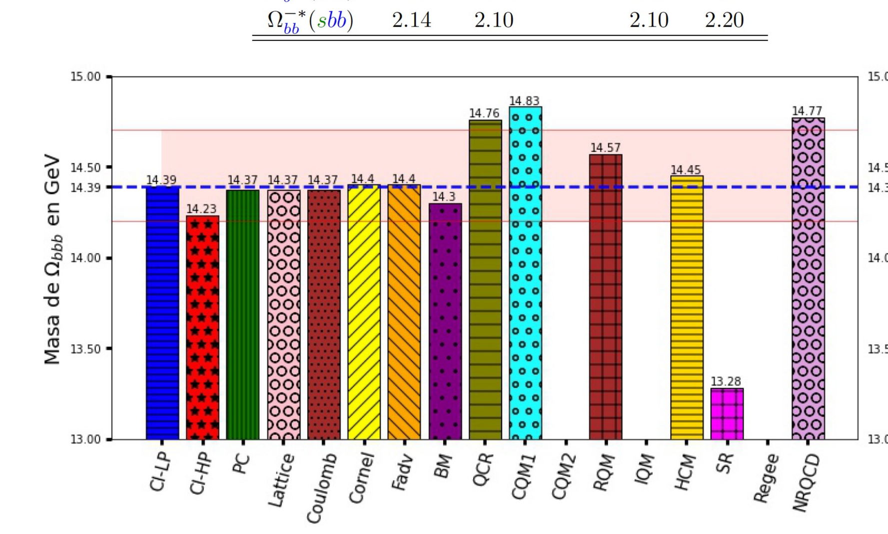
Baryon	(Exp.,+)	(CI,+)	(Exp.,-)	(CI,-)	(QRS,-)
$N(uud)$	0.94	1.14	1.54	1.82	1.542
$\Sigma(uus)$	1.19	1.36	1.75	1.96	1.581
$\Xi(sus)$	1.31	1.43	–	2.04	1.620
$\Xi_{cc}^{++}(ucc)$	3.62	3.64	–	3.80	3.790
$\Omega_{cc}^{+}(scc)$	–	3.76		3.95	3.829
$\Omega_c^0(ssc)$	2.69	2.82		2.99	2.744
$\Sigma_c^{++}(uuc)$	2.45	2.58		2.64	2.666
$\Xi_{bb}^0(ubb)$	–	10.06		10.17	10.289
$\Omega_{bb}^-(sbb)$	–	10.14		10.32	10.328
$\Omega_b^-(ssb)$	6.04	6.01		6.47	5.994
$\Sigma_b^+(uub)$	5.81	5.78		6.36	5.916
$\Omega(cbb)$	–	11.09		11.22	11.413
$\Omega(ccb)$	–	8.01		8.17	8.164

- Baryons with spin 3/2.

Baryon	(Lat.,+)	(CI,+)	(Exp.,-)	(CI,-)	(QRS,-)
$\Delta(uuu)$	1.23*	1.39	1.65	2.07	1.726
$\Sigma^*(uus)$	1.39*	1.51	1.67	2.16	1.785
$\Xi^*(sus)$	1.53*	1.63	1.82	2.26	1.843
$\Omega(sss)$	1.67*	1.76	–	2.36	1.902
Ω_{ccc}^{++*}	4.80	4.93		5.28	5.027
Ω_{bbb}^{-*}	14.37	14.23		14.39	14.771
Ω_{ccb}^{+*}	8.01	8.03		8.28	8.275
Ω_{cbb}^{0*}	11.20	11.12		11.35	11.523
$\Sigma_c^{++*}(uuc)$	0.53*	0.57		0.67	0.59
$\Xi_{cc}^{++*}(ucc)$	0.75	0.79		0.89	0.83
$\Omega_c^{0*}(ssc)$	0.58*	0.61		0.72	0.63
$\Omega_{cc}^{+*}(scc)$	0.78	0.82		0.92	0.84
$\Sigma_b^{+*}(uub)$	1.21*	1.23		1.32	1.28
$\Xi_{bb}^{0*}(ubb)$	2.11	2.12		2.10	2.19
$\Omega_b^{-*}(ssb)$	1.26	1.28		1.52	1.30
$\Omega_{bb}^{-*}(sbb)$	2.14	2.10		2.10	2.20

	Ω_{ccc}^{+++*}	Ω_{bbb}^{-*}	Ω_{ccb}^{++*}	Ω_{cbb}^{0*}
CI-LP	4.78	14.39	8.03	11.10
CI-HP	4.93	14.23	8.03	11.12
PC	4.76	14.37	7.96	11.17
Lattice	4.80	14.37	8.01	11.20
Coulomb	4.76	14.37	7.98	11.19
Cornell	4.80	14.40	8.04	11.24
Fadv	4.80	14.40	8.02	11.22
BM	4.79	14.30	8.03	11.20
QCR	4.92	14.76	8.20	11.48
CQM1	4.97	14.83	8.26	11.55
CQM2	4.63			
RQM	4.80	14.57	8.02	11.29
IQM	4.77			
HCM	4.74	14.45	8.10	11.38
SR	4.67	13.28	7.44	10.46
Regee	4.82			
NRQCD	4.90	14.77	8.24	11.53

- Comparison of the CI model with others.



CONCLUSION

- The CI model is simple to implement, it provides good results (of the order of 5%) for small moments. If the results are carefully interpreted with this model, valuable information can be extracted, such as the relationships between hadrons and their structure.
- It works as a useful substitute, in domains where full QCD is difficult to implement.

1. L. X. Gutiérrez-Guerrero, Adnan Bashir, Marco A. Bedolla, E. Santopinto. Masses of Light and Heavy Mesons and Baryons: A Unified Picture. Phys. Rev. D 100, 114032 (2019) DOI: 10.1103/PhysRevD.100.114032.

THANKS

I thank Dr. Laura Xiomara Gutiérrez Guerrero and Dr. Adnan Bashir, for allowing me to be part of this work. This work is part of the master's thesis by Gustavo Paredes Torres. In the same way, I thank the organizing committee of the XXXV Annual Meeting of the Division of Particles and Fields of the SMF for giving me the opportunity to participate in said event.