

Contributions to  $ZZV^*$  ( $V = \gamma, Z, Z'$ ) couplings from  $CP$  violating  
flavor changing couplings  
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## Theoretical framework

$$V_\mu(q) \text{ (wavy line)} \rightarrow \text{Vertex} \rightarrow \begin{cases} Z_\alpha(p_1) \text{ (wavy line)} \\ Z_\beta(p_2) \text{ (wavy line)} \end{cases} = ei\Gamma_{ZZV}^{\alpha\beta\mu}(p_1, p_2, q)$$

Figura: Nomenclature for the TNGBCs  $ZZV^*$  ( $V = \gamma, Z, Z'$ ).

- The TNGBC  $ZZV^*$  ( $V = \gamma, Z$ ) coupling can be parametrized by two form factors<sup>1</sup>:

$$\Gamma_{ZZV^*}^{\alpha\beta\mu}(p_1, p_2, q) = \frac{i(q^2 - m_V^2)}{m_Z^2} \left[ f_4^V (q^\alpha g^{\mu\beta} + q^\beta g^{\mu\alpha}) - f_5^V \epsilon^{\mu\alpha\beta\rho} (p_1 - p_2)_\rho \right], \quad (1)$$

- The form factor  $f_5^V$  is  $CP$ -conserving, whereas  $f_4^V$  is  $CP$ -violating.
- From Eq. (1) it is evident that when the  $V^*$  gauge boson becomes on-shell ( $q^2 = m_V^2$ ),  $\Gamma_{ZZV^*}^{\alpha\beta\mu}(p_1, p_2, q)$  vanishes.

<sup>1</sup>G.J. Gounaris, J. Layssac, F.M. Renard; Phys. Rev. D62, 073013 (2000); DOI 10.1103/PhysRevD.62.073013

# Theoretical framework

- As far as the TNGBCs with a new neutral gauge boson  $Z'$ , they remain almost unexplored. We will parametrize the  $ZZZ'^*$  coupling as follows

$$\Gamma_{ZZZ'^*}^{\alpha\beta\mu}(p_1, p_2, q) = \frac{iq^2}{m_{Z'}^2} \left[ f_4^{Z'} \left( q^\alpha g^{\mu\beta} + q^\beta g^{\mu\alpha} \right) - f_5^{Z'} \epsilon^{\mu\alpha\beta\rho} (p_1 - p_2)_\rho \right], \quad (2)$$

where we only consider contributions of dimension-six operators.

- We also note in Eq. (2) that this TNGBC does not vanish for an on-shell  $Z'$  gauge boson.

# Theoretical framework

- FCNC couplings mediated by the  $Z$  gauge boson can be expressed by the following Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{e}{2s_W c_W} Z^\mu \bar{F}_i \gamma_\mu \left( g_{VZ}^i - \gamma^5 g_{AZ}^i \right) F_i \\ & -\frac{e}{2s_W c_W} Z^\mu \bar{F}_i \gamma_\mu \left( g_{VZ}^{ij} - \gamma^5 g_{AZ}^{ij} \right) F_j, \end{aligned} \quad (3)$$

- Here  $g_{VZ,AZ}^i$  are the diagonal SM couplings, whereas the non-diagonal couplings  $g_{VZ,AZ}^{ij}$  ( $i \neq j$ ) will be taken as complex since we are interested in the  $CP$ -violating contribution.
- Because of the hermiticity of Lagrangian (3):  $g_{VZ,AZ}^{ij*} = g_{VZ,AZ}^{ji}$ .
- The Lagrangian of Eq. (3) can be written in terms of the chiral couplings denoted by  $\epsilon_{L_{ij},R_{ij}}^Z$ , which are given in terms of the vector and vector-axial couplings  $g_{VZ,AZ}^{ij}$  as follows

$$g_{VZ,AZ}^{ij} = \frac{\epsilon_{L_{ij}}^Z \pm \epsilon_{R_{ij}}^Z}{2}. \quad (4)$$

# Theoretical framework

- FCNC  $Z'$  coupling in terms of the chiral couplings  $\epsilon_{Lij,Rij}^{Z'}$  can be written as

$$\mathcal{L}_{Z'}^{FCNC} = -g_{Z'} \sum_{i=f_{SM}} Z'_\mu \bar{\mathbf{F}}_i \gamma^\mu \left( \epsilon_{L_i}^{Z'} P_L + \epsilon_{R_i}^{Z'} P_R \right) \mathbf{F}_i, \quad (5)$$

where  $\mathbf{F}_i$  is a massive fermion triplet in the flavor basis,  $\mathbf{F}_\ell^T = (e, \mu, \tau)$ ,  $\mathbf{F}_d^T = (d, s, b)$ , and  $\mathbf{F}_u^T = (u, c, t)$ , with  $\epsilon_{L_i}^{Z'}$  and  $\epsilon_{R_i}^{Z'}$  being  $3 \times 3$  matrices containing the corresponding  $Z'$  couplings.

**Cuadro:** Models in which there is a new neutral gauge boson with FCNC couplings.

New heavy neutral gauge boson	Model	Gauge group
$Z_h$	Sequential $Z$	$SU_L(2) \times U_Y(1) \times U'(1)$
$Z_{LR}$	Left-right symmetric	$SU_L(2) \times SU_R(2) \times U_Y(1)$
$Z_\chi$	Gran Unification	$SO(10) \rightarrow SU(5) \times U(1)$
$Z_\psi$	Superstring-inspired	$E_6 \rightarrow SO(10) \times U(1)$
$Z_\eta \equiv \sqrt{3/8}Z_\chi - \sqrt{5/8}Z_\psi$	Superstring-inspired	$E_6 \rightarrow \text{Rank-5 group}$

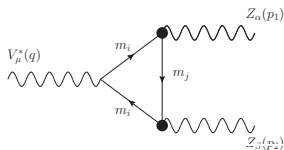
Analytical results: ZZ $\gamma^*$  coupling

Figura: Generic Feynman diagram required for the contribution of FCNC couplings to TNGBC ZZ $\gamma^*$ .

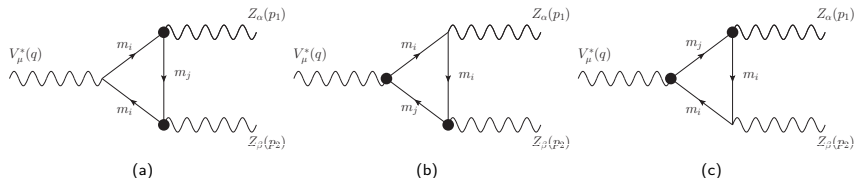
- In this case there are 4 contributing Feynman diagrams
- It is not possible to induce  $CP$  violation in the ZZ $\gamma^*$  coupling, which indeed was verified in our calculation.
- Therefore there are only contributions to the form factor  $f_5^\gamma$ , which can be written as

$$f_5^\gamma = - \sum_i \sum_{j \neq i} \frac{N_i Q_i e^2 m_Z^2 \operatorname{Re} \left( g_{AZ}^{ij*} g_{VZ}^{ij} \right)}{8\pi^2 s_W^2 c_W^2 \left( q^2 - 4m_i^2 \right)^2 q^2} R_{ij}, \quad (6)$$

where  $m_i$ ,  $N_i$  and  $Q_i$  are the mass, color number and electric charge of the fermion  $f_i$ .

- The analytical expression for  $R_{ij}$  is somewhat cumbersome and it is given in terms of Passarino-Veltman scalar functions.

## Analytical results: ZZZ\* coupling



**Figure:** Generic Feynman diagram required for the contribution of FCNC couplings to TNGBC ZZZ\* .

- The calculation of this coupling is more intricate than the previous one since there are 36 contributing
- The  $CP$ -conserving form factor  $f_5^Z$ , it can be written as

$$f_5^Z = - \sum_i \sum_{j \neq i} \frac{e^2 N_i m_Z^2}{16\pi^2 c_W^3 s_W^3 (q^2 - 4m_Z^2)} \times \left\{ g_{AZ}^i \left( |g_{AZ}^{ij}|^2 + |g_{VZ}^{ij}|^2 \right) (R_{1ij} + R_{2ij}) \right. \\ \left. + 2g_{VZ}^i \text{Re} \left( g_{AZ}^{ij*} g_{VZ}^{ij} \right) (R_{1ij} - R_{2ij}) + g_{AZ}^i \left[ |g_{AZ}^{ij}|^2 - |g_{VZ}^{ij}|^2 \right] R_{3ij} + (i \leftrightarrow j) \right\}, \quad (7)$$

where the  $R_{kij}$  ( $k = 1, 2, 3$ ) functions are in terms of Passarino-Veltman scalar functions.

## Analytical results: ZZZ\* coupling

- As for the  $CP$ -violating form factor  $f_4^Z$ , it reads

$$f_4^Z = - \sum_i \sum_{j \neq i} \frac{N_i e^2 m_i m_j m_Z^2}{24\pi^2 c_W^3 s_W^3 (q^2 - m_Z^2) (q^2 - 4m_Z^2) q^2} \text{Im} \left( g_{AZ}^{ij*} g_{VZ}^{ij} \right) g_{AZ}^i S_{ij}, \quad (8)$$

where  $S_{ij}$  is again in terms of Passarino-Veltman scalar functions.

- It is easy to see that  $f_4^Z$  vanishes for real couplings. Thus, a non-vanishing  $f_4^Z$  requires complex FCNC couplings.
- Furthermore, we can also see that we need different complex phases for  $g_{VZ}^{ij}$  and  $g_{AZ}^{ij}$  to obtain a non-vanishing  $CP$ -violating form factor.
- Since  $f_4^Z$  is proportional to  $m_i m_j$  we expect that the main contribution comes from FCNC couplings associated with the top quark. We would like to stress that the result of Eq. (8) has never been reported in the literature.



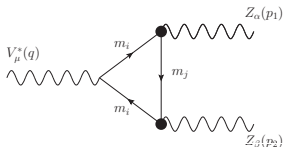
Analytical results: ZZZ'<sup>\*</sup> coupling

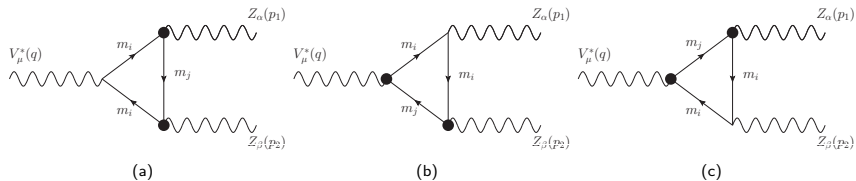
Figura: Generic Feynman diagram required for the contribution of FCNC couplings to TNGBC ZZZ'<sup>\*</sup>.

- We first present the diagonal case, where there is no flavor violation ( $m_i = m_j$ ).
- We only need to add one extra diagram obtained after the exchange  $p_{1\mu} \leftrightarrow p_{2\nu}$ .
- The  $CP$ -conserving form factor  $f_5^{Z'}$  reads

$$f_5^{Z'} = - \sum_i \frac{eN_i m_{Z'}^2}{16\pi^2 c_W^2 s_W^2 q^2 (q^2 - 4m_{Z'}^2)^2} \times \left\{ g_{AZ'}^i \left[ (g_{VZ}^i)^2 L_{1i} + (g_{AZ}^i)^2 L_{2i} \right] + g_{VZ'}^i g_{VZ}^i g_{AZ}^i L_{3i} \right\}, \quad (9)$$

where the  $L_{ji}$  ( $j = 1, 2, 3$ ) functions are in terms of Passarino-Veltman scalar functions.

- The  $CP$ -violating form factor  $f_4^{Z'}$  is not induced at the one-loop level in this scenario.

Analytical results: ZZZ'<sup>\*</sup> coupling

**Figura:** Generic Feynman diagram required for the contribution of FCNC couplings to TNGBC ZZZ'<sup>\*</sup>.

- There are 12 contributing Feynman diagrams in total.
- The *CP*-conserving form factor can be written as

$$\begin{aligned}
 f_5^{Z'} = & - \sum_i \sum_{j \neq i} \frac{e N_i m_{Z'}^2}{16 \pi^2 c_W^2 s_W^2 q^2 (q^2 - 4 m_Z^2)^2} \left\{ 2 g_{AZ}^i \left[ \text{Re} \left( g_{VZ}^{ij} g_{VZ'}^{ij*} \right) U_{1ij} \right. \right. \\
 & + \left. \text{Re} \left( g_{AZ'}^{ij} g_{AZ}^{ij*} \right) U_{2ij} \right] + 2 g_{VZ'}^i \text{Re} \left( g_{VZ}^{ij} g_{AZ}^{ij*} \right) U_{3ij} \\
 & + \left. 2 g_{VZ}^i \left[ \text{Re} \left( g_{VZ}^{ij} g_{AZ'}^{ij*} \right) U_{4ij} + \text{Re} \left( g_{VZ'}^{ij} g_{AZ}^{ij*} \right) U_{5ij} \right] + g_{AZ'}^i \left[ \left| g_{AZ}^{ij} \right|^2 U_{6ij} + \left| g_{VZ}^{ij} \right|^2 U_{7ij} \right] \right\}, \quad (10)
 \end{aligned}$$

## Analytical results: ZZZ'<sup>\*</sup> coupling

- The CP-violating one reads

$$f_4^{Z'} = \sum_i \sum_{j \neq i} \frac{eN_i m_{Z'}^2}{12\pi^2 c_W^2 s_W^2 q^6 (q^2 - 4m_Z^2)} \left\{ g_{VZ}^i \left[ \text{Im} \left( g_{VZ'}^{ij} g_{VZ}^{ij*} \right) T_{1ij} + \text{Im} \left( g_{AZ'}^{ij} g_{AZ}^{ij*} \right) T_{2ij} \right] \right. \\ \left. + g_{AZ'}^i \text{Im} \left( g_{VZ}^{ij} g_{AZ}^{ij*} \right) T_{3ij} + g_{AZ}^i \left[ \text{Im} \left( g_{AZ'}^{ij} g_{VZ}^{ij*} \right) T_{4ij} + \text{Im} \left( g_{VZ'}^{ij} g_{AZ}^{ij*} \right) T_{5ij} \right] \right\}, \quad (11)$$

where the  $U_{kij}$  ( $k = 1 \dots 7$ ) and  $T_{kij}$  ( $k = 1 \dots 5$ ) are in terms of Passarino-Veltman scalar functions.

- We note that  $f_5^{Z'}$  ( $f_4^{Z'}$ ) depends only on the real (imaginary) part of the combinations of products of the vector and axial couplings.
- We also note that it is not necessary that both  $Z$  and  $Z'$  gauge bosons have simultaneously complex FCNC couplings to induce the CP-violating form factor.

# Numerical Analysis

**Cuadro:** Bounds on the FCNC couplings of the  $Z$  gauge boson, with 95 % C.L., from the current experimental limits on FCNC  $Z$  decays. The second row stands for the limit when either  $\epsilon_{Rij}^Z$  or  $\epsilon_{Lij}^Z$  is taken as vanishing and the other one non-vanishing.

	$\bar{t}c$	$\bar{t}u$	$\bar{c}u$	$\bar{d}_i d_j$	$\bar{\ell}_i \ell_j$	$\bar{\nu}_i \nu_j$
$ \epsilon_{Lij}^Z  \simeq  \epsilon_{Rij}^Z $	0.016	0.013	$10^{-2}$	$10^{-2}$	$10^{-3}$	$10^{-3}$
$ \epsilon_{Lij,Rij}^Z $	0.032	0.026	$10^{-2}$	$10^{-2}$	$10^{-3}$	$10^{-3}$

- The form factors are complex.
- Real and imaginary parts of  $f_5^\gamma$  are of the order of  $10^{-3}$ .
- For the  $ZZZ^*$  the magnitude of the real and imaginary parts are  $|f_5^Z| \sim 10^{-6}$  and  $|f_4^Z| \sim |\text{Im}(\mathcal{g}_{AZ}^{tc*} \mathcal{g}_{VZ}^{tc})| \times 10^{-5}$ .
- SM predictions for the diagonal case<sup>2</sup> are of order  $10^{-2}$ .

<sup>2</sup>G.J. Gounaris, J. Layssac, F.M. Renard; Phys. Rev. D62, 073013 (2000); DOI 10.1103/PhysRevD.62.073013

# Numerical Analysis

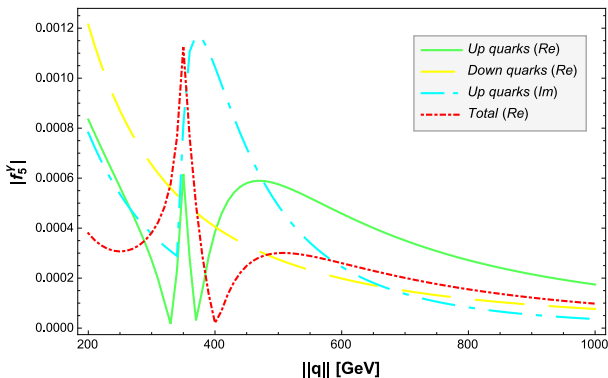
- For the TNGBC  $ZZZ'^*$ , in the diagonal case we find  $|\text{Re}f_5^{Z'}| \sim 10^{-1} - 10^{-2}$  and  $|\text{Im}f_5^{Z'}| \sim 10^{-2} - 10^{-3}$ .
- In the non-diagonal the real and imaginary parts of  $f_5^{Z'}$  could be of the order of  $10^{-4}$
- As far as the  $CP$ -violating form factor  $f_4^{Z'}$  is concerned, we obtain similar estimates for its real and imaginary parts, of the order of  $10^{-7} - 10^{-8}$ .
- For more details see:

Hernández-Juárez, A.I., Moyotl, A., Tavares-Velasco, G.; Eur. Phys. J. C 81, 304 (2021);  
<https://doi.org/10.1140/epjc/s10052-021-09093-w>

# Conclusions

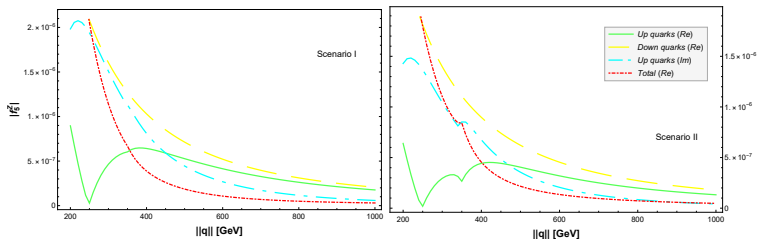
- New contributions to the TNGBC  $ZZV^*$  ( $V = \gamma, Z$ ) are possible through FCNC of the  $Z$  boson.
- Results for the  $ZZZ'^*$  vertex have been studied.
- New sources of  $CP$  violation are possible in the TNGBC  $ZZV^*$  ( $V = Z, Z'$ )

## Backup



**Figura:** Behavior of the FCNC contributions to the  $f_5^\gamma$  form factor as a function of the momentum of the photon for  $\phi_{L_{ij}} = 0.1$ ,  $\phi_{R_{ij}} = 0$ ,  $|\epsilon_{R_{ij}}^Z| = 0.9|\epsilon_{L_{ij}}^Z|$  and the  $|\epsilon_{L_{ij}}^Z|$  values shown in Table 2. Only the non-negligible contributions are shown: up quarks ( $tc$ ,  $tu$  and  $cu$ ) and down quarks ( $bs$ ,  $bd$  and  $sd$ ). The total imaginary contribution coincides with the respective up quark contribution since the down quark contribution (not shown in the plot) is negligible.

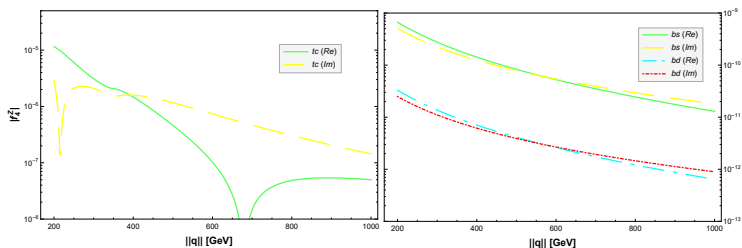
## Backup



**Figura:** Behavior of the FCNC contributions to the  $f_5^Z$  form factor as a function of the momentum of the virtual Z gauge boson in the two scenarios discussed in the text. Only the non-negligible contributions are shown: up quarks ( $tc$ ,  $tu$  and  $cu$ ) and down quarks ( $bs$ ,  $bd$  and  $sd$ ). The total imaginary contributions coincide with the respective up quark contributions since the down quark contribution (not shown in the plot) is negligible.



## Backup



**Figura:** Behavior of the  $f_4^Z$  form factor as a function of the momentum  $|q|$  of the virtual  $Z$  gauge boson. We have extracted a factor of  $\text{Im} \left( g_{AZ}^{tc*} g_{VZ}^{qq'} \right)$  from the respective contribution. All other contributions not shown in the plots are well below the  $10^{-10}$  level.

## Backup

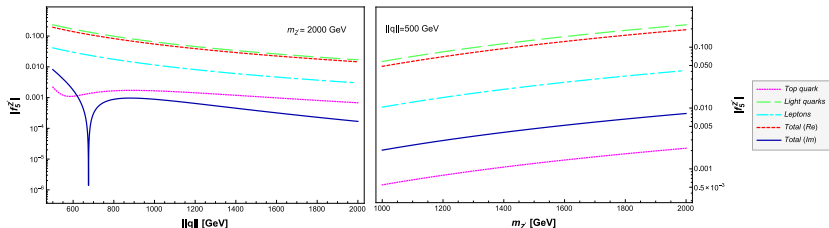


Figura: Behavior of the  $f_5^{Z'}$  form factor as a function of the transfer momentum  $|q|$  (left plot) and the  $m_{Z'}$  mass (right plot) in the diagonal case .

## Backup

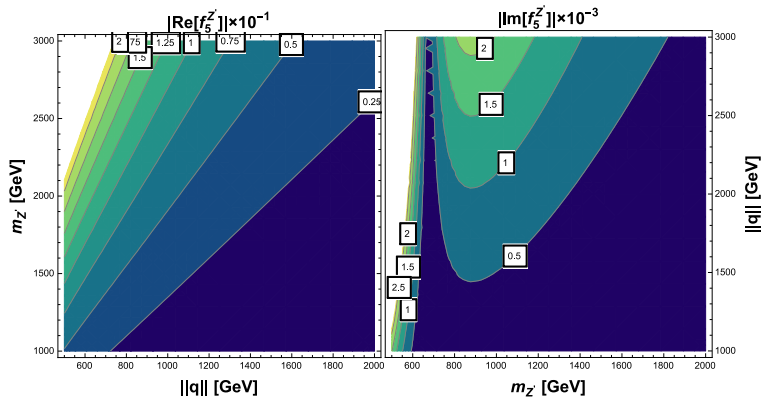


Figura: Contour lines of the  $f_5^{Z'}$  form factor in the  $|q|$  vs  $m_{Z'}$  plane in the diagonal case.

## Backup

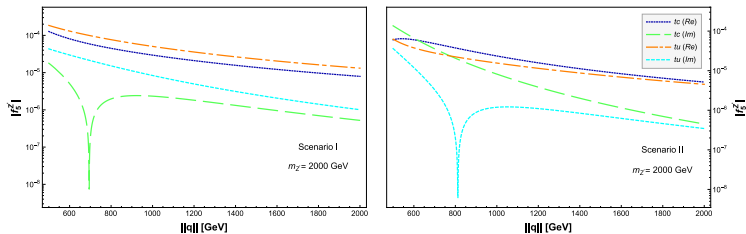


Figura: Behavior of the  $f_5^{Z'}$  form factor in the non-diagonal case as a function of the transfer momentum  $|q|$  of the  $Z'$  gauge boson.

## Backup

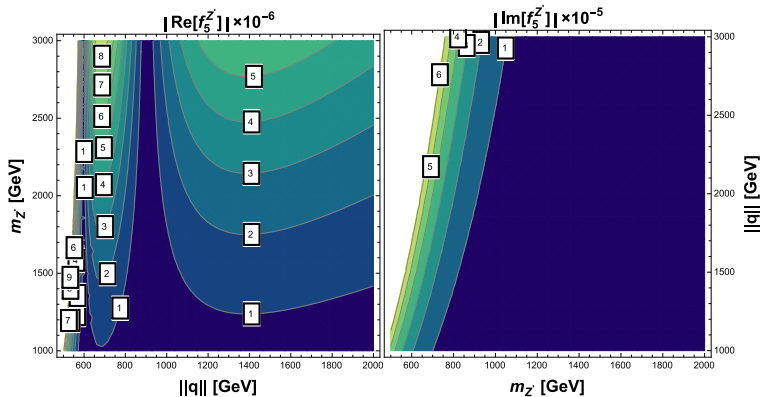


Figura: Contour lines in the  $|q|$  vs  $m_{Z'}$  plane of the real and imaginary parts of the  $f_5^{Z'}$  form factor in the non-diagonal case and scenario II.

## Backup

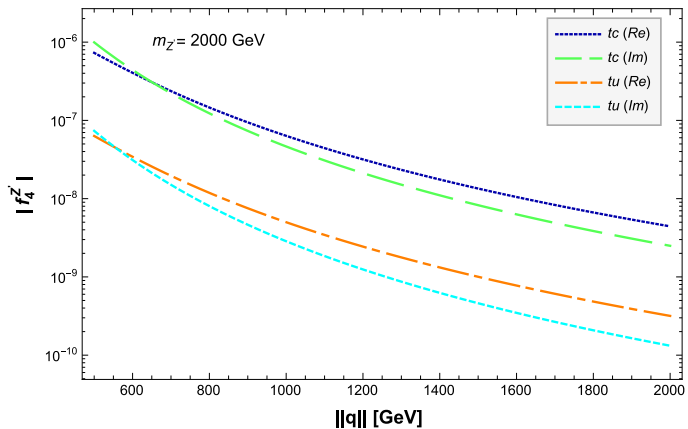


Figura: Behavior of the  $f_4^{Z'}$  form factor as a function of  $|q|$  in the non-diagonal case and scenario I.

## Backup

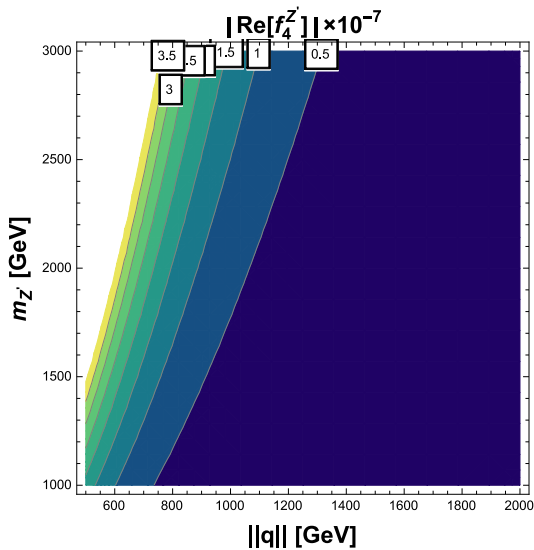


Figura: Contour lines of the real part of  $f_4^{Z'}$  in the  $q$  vs  $m_{Z'}$  plane in the non-diagonal case and scenario I.