The 3d O(4) model and the QCD phase diagram in the chiral limit

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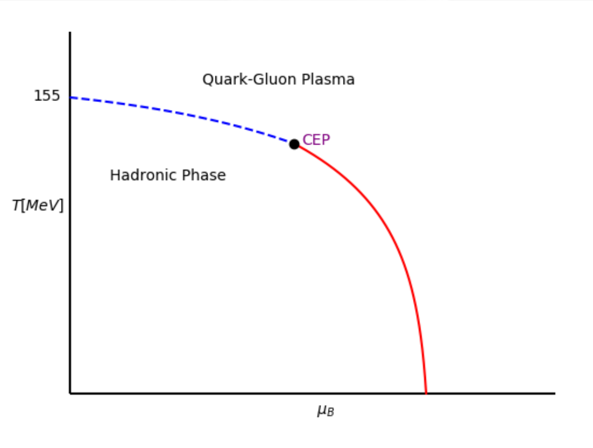
QCD phase diagram

- Low baryon density: crossover
- $T_c \approx 155$ MeV at $\mu_B = 0$
- Hypothetical Critical End Point (CEP)
- High μ_B, conjecture first order phase transition



Two flavor QCD in the chiral limit

- $m_{\mathrm{u}}, m_{\mathrm{d}} \ll \Lambda_{\mathrm{QCD}} \approx 300$ MeV
- Global symmetry at $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$:

$$\mathrm{U}(2)_{\mathrm{L}} \otimes \mathrm{U}(2)_{\mathrm{R}} = \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(2)_{\mathrm{R}} \otimes \mathrm{U}(1)_{\mathrm{V}} \otimes \mathrm{U}(1)_{\mathrm{A}}$$

- Lagrangian:

$$\mathcal{L} = \sum_{f=\mathrm{u,d}} \left(\overline{q}_{\mathrm{L},f} i \not{D} q_{\mathrm{L},f} + \overline{q}_{\mathrm{R},f} i \not{D} q_{\mathrm{R},f} \right) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

- Chiral Symmetry Breaking (CSB):

$$\mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(2)_{\mathrm{R}} \longrightarrow \mathrm{SU}(2)_{\mathrm{L=R}}$$

- Nambu-Goldstone bosons: π^0, π^+ and π^-
- Low baryon density: second order phase transition
- Nonzero quark masses: massive bosons and crossover

- Effective theory of QCD,

$$S[\vec{\sigma}] = \frac{1}{2}F_\pi^2 \int d^4x\, \partial_\mu \vec{\sigma}(x) \cdot \partial_\mu \vec{\sigma}(x), \quad \vec{\sigma}(x) \in \mathrm{O}(4)/\mathrm{O}(3) \simeq \mathrm{S}^3,$$

  where $F_\pi \approx 92$ MeV.
- $\pi/2$ Wick rotation of time axis,

$$t_\mathrm{E} = it = 1/T = \beta$$

- High temperature dimensional reduction.
- Euclidean action,

$$S_\mathrm{E}[\vec{\sigma}] = \frac{1}{2}\beta F_\pi^2 \int d^3x\, \partial_i \vec{\sigma}(x) \cdot \partial_i \vec{\sigma}(x)$$

- Spontaneous symmetry breaking locally isomorphic to the CSB,

$$\mathrm{O}(4) \simeq \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{SU}(2)_\mathrm{R} \longrightarrow \mathrm{O}(3) \simeq \mathrm{SU}(2)_{\mathrm{L=R}}$$

- Same universality class as the CSB (Skyrme 1961, 1962; Wilczek 1992).

- Topological sectors in 3d.
- Topological charge identify the 3rd homotopy group classes,

$$\pi_3(\mathrm{O}(4)/\mathrm{O}(3) \simeq \mathrm{S}^3) = \mathbb{Z}$$

- Winding number around S^3.
- Conserved under infinitesimal transformations.
- Represents the baryon number (Witten 1983; Adkins et. al 1983). The $\vec{\sigma}(x)$ fields are mesonic but the topological excitations are baryonic.
- Chemical potential μ_B without sign problem, in contrast with direct QCD simulations.

Lattice regularization

- Lattice regularization,

$$S_{\mathrm{E}}[\vec{\sigma}] = -\beta_{\mathrm{lat}} \left(\sum_{\langle xy \rangle} \vec{\sigma}_x \cdot \vec{\sigma}_y + \mu_{B,\mathrm{lat}} Q[\vec{\sigma}] \right) ,$$

  sum over nearest neighbors. Lattice units ($a = 1$) and $\beta_{\mathrm{lat}} = \beta F_\pi^2$, $\mu_{B,\mathrm{lat}} = \mu_B / F_\pi^2$.
- Physical units are assigned using the critical temperature at $\mu_B = 0$, $\beta_{c,\mathrm{lat}} = 0.93590$ (Oevers 1996) and $T_c \approx 155$ MeV (Borsanyi et al. 2010),

$$\beta = \frac{\beta_c}{\beta_{c,\mathrm{lat}}} \beta_{\mathrm{lat}} \approx 0.00689 \ \mathrm{MeV}^{-1} \beta_{\mathrm{lat}} \qquad \mu_B = \frac{\beta_{c,\mathrm{lat}}}{\beta_c} \mu_{B,\mathrm{lat}} \approx 145 \ \mathrm{MeV} \mu_{B,\mathrm{lat}}$$

- L^3 lattice volume.
- $\vec{\sigma}(x)$ with periodic boundary conditions. $\vec{\sigma}(x)$ covers all S^3, thus $Q[\vec{\sigma}(x)]$ is integer.

Magnetization density:

$$m(\beta_{\mathrm{lat}}) = \frac{1}{L^3}\langle|\vec{M}[\vec{\sigma}]|\rangle$$

Energy density:

$$\epsilon(\beta_{\mathrm{lat}}) = \frac{1}{L^3}\langle H[\vec{\sigma}]\rangle$$

Topological charge density:

$$q(\beta_{\mathrm{lat}}) = \frac{1}{L^3}\langle Q[\vec{\sigma}]\rangle$$

Magnetic susceptibility:

$$\chi_M(\beta_{\mathrm{lat}}) = \frac{\beta_{\mathrm{lat}}}{L^3}\left(\langle|\vec{M}[\vec{\sigma}]|^2\rangle - \langle|\vec{M}[\vec{\sigma}]|\rangle^2\right)$$

Specific heat:

$$c_V(\beta_{\mathrm{lat}}) = \frac{\beta_{\mathrm{lat}}^2}{L^3}\left(\langle(H[\vec{\sigma}])^2\rangle - \langle H[\vec{\sigma}]\rangle^2\right)$$

Topological susceptibility:

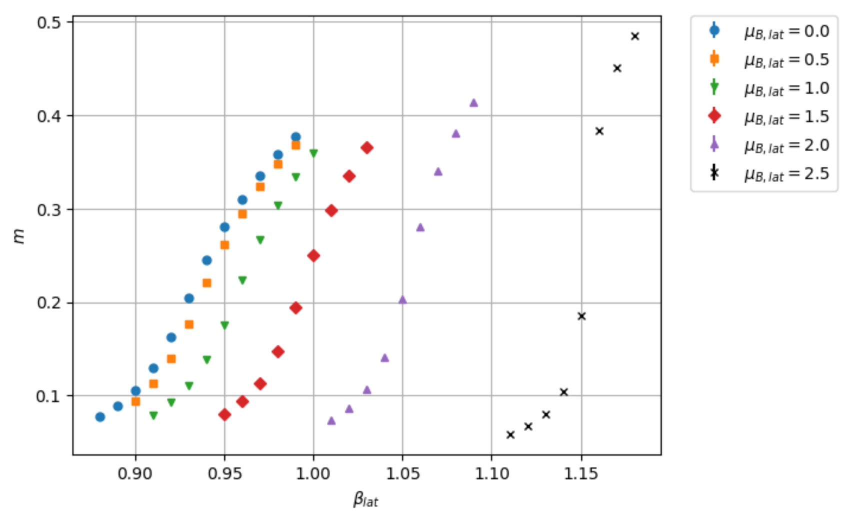
$$\chi_Q(\beta_{\mathrm{lat}}) = \frac{1}{L^3}\left(\langle(Q[\vec{\sigma}])^2\rangle - \langle Q[\vec{\sigma}]\rangle^2\right)$$

- Multicluster algorithm. Collective spin updates.
- Diminish the auto-correlation time τ of the observables, especially of the topological charge.
- T near T_c.
- Baryon chemical potentials,

$\mu_{B,\mathrm{lat}}$	0 0.9	0.1 1	0.2 1.1	0.3 1.2	0.4 1.3	0.5 1.4	0.6 1.5	0.7 2.0	0.8 2.5
$\mu_B[\mathrm{MeV}]$	0 130.6	14.5 145.1	29 159.6	43.5 174.1	58 188.6	72.5 203.1	87 217.6	101.5 290.1	116.1 362.7

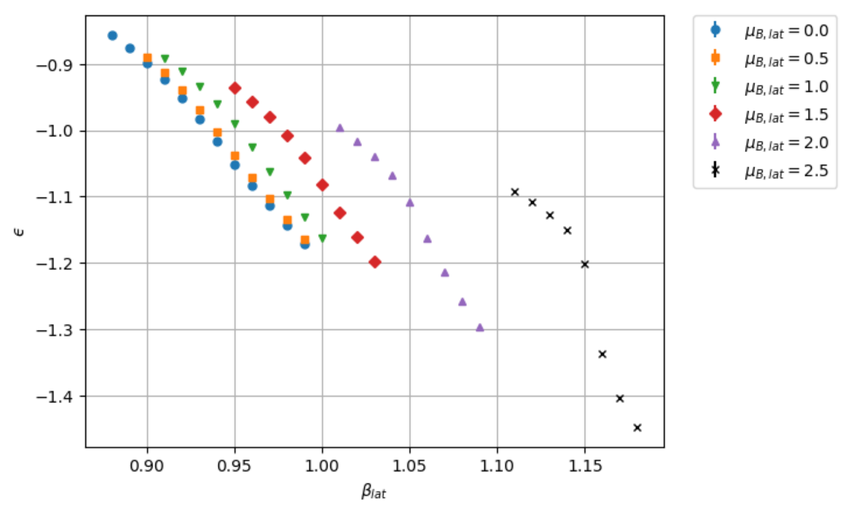
- L^3 volume lattices.
- Statistics 10^4 field configurations.
- Measurements such that $\tau \approx 1/2$, perfectly decorrelated measurements.
- A previous attempt by M. A. Nava Blanco.

Magnetization density



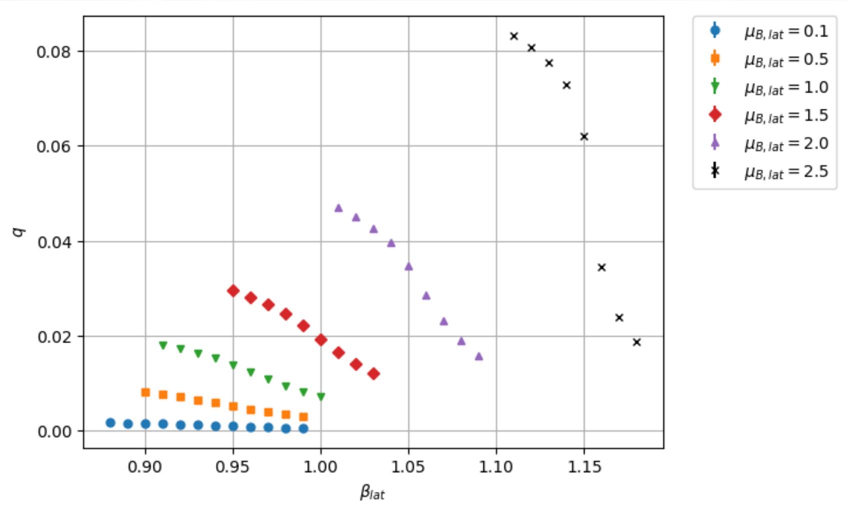
Magnetization density in a lattice of volume 20^3.

Energy density



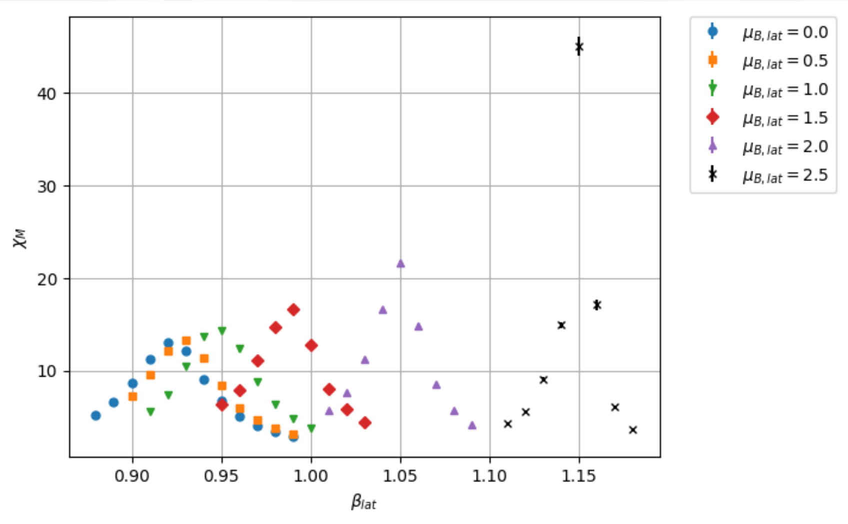
Energy density in a lattice of volume 20^3.

Topological charge density



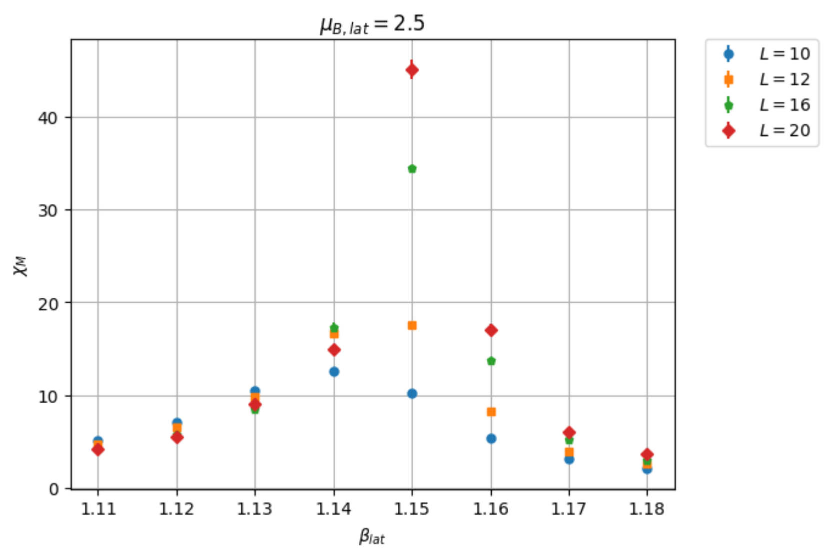
Topological charge density in a lattice of volume 20^3.

Magnetic suscpetibility



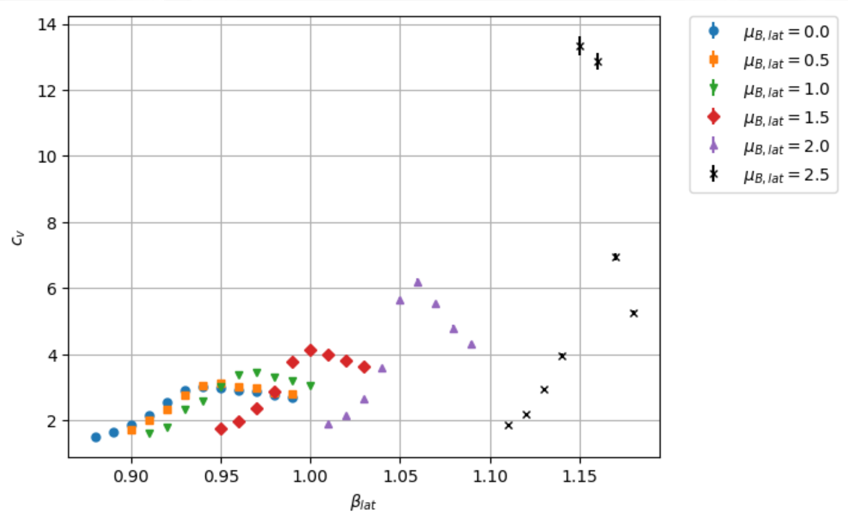
Magnetic susceptibility in a lattice of volume 20^3.

Magnetic susceptibility

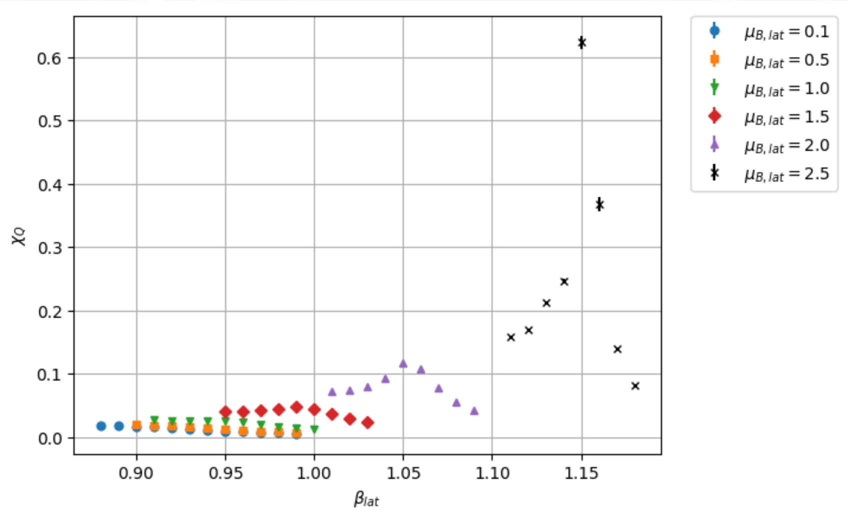


Magnetic susceptibility at $\mu_{B,\mathrm{lat}} = 2.5$ in lattices of volume L^3.

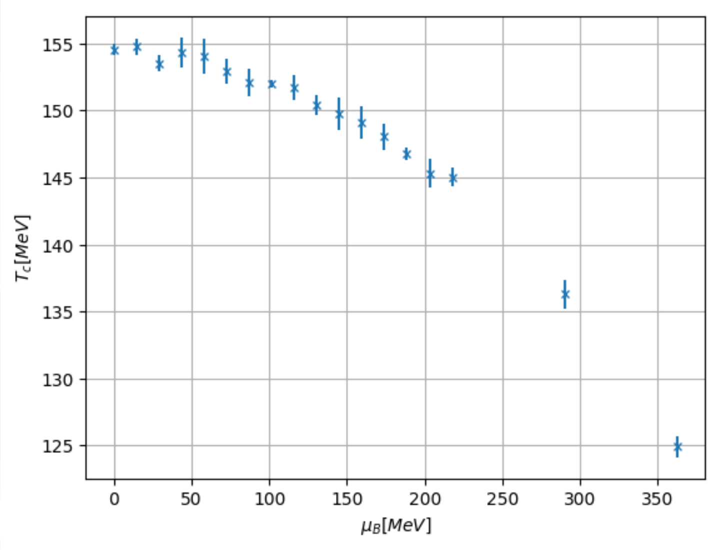
Specific heat



Specific heat in a lattice of volume 20^3.



Topological susceptibility in a lattice of volume 20^3.



Phase diagram in physical units.

Reference	T_{CEP}[MeV]	$\mu_{B,\mathrm{CEP}}$[MeV]
Contrera et al. (2016)	69.9-128.6	223.3-319.1
Cui et al. (2017)	38	245
Kovácks and Wolf (2017)	53	885
Rougemont et al. (2017)	< 130	> 400
Sharma (2017)	< 145	$2T_{\mathrm{CEP}}$
Antoniou et. al (2018)	119-162	252-258
Ayala et al. (2018)	18-45	315-349
Goswami et al. (2018)	195.23-200.6	$\pi/3T_{\mathrm{CEP}}$
Knaute et al. (2018)	111.5	611.5
Li et al. (2019)	100	240
Martínez and Raya (2019)	49	310
Motta et al. (2019)	122	862
Zhao et al. (2019)	237	101
Ayala et al. (2020)	40-51	271-291
Wu et al. (2020)	69-72	813-971
Shi et al. (2020)	116-127	135-160
Zhao et al. (2020)	328-330	72-76
Our work	< 125	> 363

CEP estimations. For a previous table see Ayala et. al (2018).

Conclusion

- We have studied the 3d O(4) non-linear σ model, an effective theory of QCD with two flavors in the chiral limit.
- The 3d O(4) model allow us to study the phase diagram of QCD with two flavors in the chiral limit at non-zero μ_B without sign problem.
- We could follow the critical line up to $\mu_B = 363$ MeV, where we find $T_c = 125$ MeV.
- The critical line is monotonically decreasing in μ_B.
- Only second order phase transitions up to $\mu_B = 363$ MeV.
- Indications of a possible first order phase transition at somewhat larger μ_B.
- Study with quark masses included: poster by José Antonio García Hernández.

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