The structure of cosmic strings of a $U(1)$ gauge field for the conservation of $B - L$

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- Baryon number (B) conservation, Stückelberg (1938)
- Lepton number (L) conservation, Konopinsky and Mahmoud (1953)
- At very high energy, the SM allows B and L conservation violations (unobserved)
- However $B - L$ is still conserved
- Gauging leads to a $B - L$ anomaly, which can be cured by adding ν_R
- We can get a mass term for ν_R via the Higgs mechanism from an additional χ

The Higgs Mechanism in the SM

Lagrangian with $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_+ \\ \phi_0\end{pmatrix}$ and gauge fields $W_\mu(x)$ and $B_\mu(x)$ for $SU(2)_L$ and $U(1)_Y$

$$\mathcal{L}_H = D^\mu \Phi^\dagger D_\mu \Phi - V(\Phi) - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

$$V(\Phi) = m^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2, \quad D_\mu \Phi = \left[\partial_\mu + i\frac{g}{2}W_\mu^a(x)\sigma_a + i\frac{g'}{2}B_\mu(x)\right]\Phi.$$

Case of symmetry breaking with $m^2 < 0$, we can select the vacuum solution

$$\Phi = \frac{v}{\sqrt{2}}\begin{pmatrix}0 \\ 1\end{pmatrix}.$$

- Massive Higgs, W and Z bosons
- A massless photon
- Lepton and quarks acquire mass via Yukawa interactions

B and L violation

- B and L violation appears through the Adler-Bell-Jackiw anomaly

$$\partial^\mu J^B_\mu = -\frac{N_g}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}[W_{\mu\nu}(x)W_{\rho\sigma}(x)]$$

- Quantum corrections known as sphaleron processes allow the violation

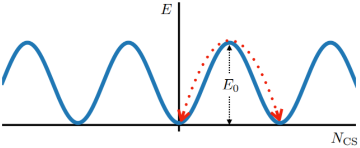


Figure: Sphaleron transition between sections of the EW vacua, image Papaefstathiou, A. et al. arXiv:1910.04761

- Each transition fulfills

$$\Delta B = \Delta L$$

- Hence the difference $B - L$ is still conserved
- $B - L$ is an exact symmetry and it is more natural if we gauge it

Anomaly in $B-L$ gauge

- The $B-L$ gauge field $\mathcal{A}_\mu$ for $U(1)_{B-L}$
- This leads to an anomaly

$$\partial^\mu J_\mu^{B-L} = -\frac{1}{16\pi^2}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}$$

- It comes from the fermion triangle diagram

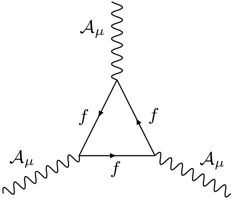


Figure: Fermion triangle diagram.

- Adding ν_R with $B-L=-1$ cancels the anomaly

Extension of the SM

Lagrangian with $SU(2)_L \times U(1)_{\alpha Y+\beta(B-L)}$ local symmetry

$$\mathcal{L}_b = D_\mu \Phi^\dagger D^\mu \Phi + d_\mu \chi^* d^\mu \chi - V(\Phi,\chi) - \frac{1}{4} W_a^{\mu\nu} W_{\mu\nu}^a - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu},$$

with the potential

$$V(\Phi,\chi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2 + \mu'^2 \chi^*\chi + \lambda'(\chi^*\chi)^2 - \kappa \Phi^\dagger \Phi \chi^* \chi$$

the covariant derivatives

$$D_\mu \Phi = \left(\partial_\mu + i\frac{g}{2} W_\mu^a(x)\sigma_a + i\frac{\alpha}{2}\mathcal{A}_\mu\right)\Phi, \quad d_\mu \chi = \left(\partial_\mu + 2i\beta\mathcal{A}_\mu\right)\chi,$$

and the field strength tensors

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g[W_\mu, W_\nu], \quad \mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu.$$

Extension of the SM

A first generation of matter

$$\mathcal{L}_{\mathsf{f}} = (\overline{\nu}_L, \overline{e}_L) i\gamma^\mu D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + (\overline{u}^c_L, \overline{d}^c_L) i\gamma^\mu D_\mu \begin{pmatrix} u^c_L \\ d^c_L \end{pmatrix} + \sum_{F=\nu,e,u^c,d^c} \overline{F}_R i\gamma^\mu d_\mu F_R$$

$$f_e(\overline{\nu}_L, \overline{e}_L)\Phi e_R + f_d(\overline{u}^c_L, \overline{d}^c_L)\Phi d^c_R + f_u(\overline{u}^c_L, \overline{d}^c_L)\widetilde{\Phi} u^c_R$$

$$+ f_\nu(\overline{\nu}_L, \overline{e}_L)\widetilde{\Phi}\nu_R + f_M \overline{\nu}_R \chi C \overline{\nu}^T_R + \text{h. c.}$$

With $\widetilde{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_0^* \\ -\phi_+^* \end{pmatrix}$.

- The neutrinos in the Majorana-like term give $B - L = 2$, thus χ must have $B - L = -2$.

Homotopy Groups

Consider two maps that preserve the base point $f, g : S^n \to \mathcal{M}$. These maps are homotopic if they can be deformed smoothly into one another. They form an equivalence class and each map is an element of the group called π_n.

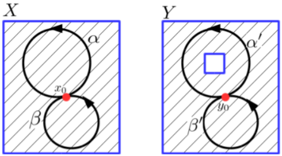


Figure: Two distinct topological spaces, image from Wikipedia.

The winding number of a curve is the number of times a curve turns over a point

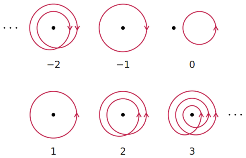
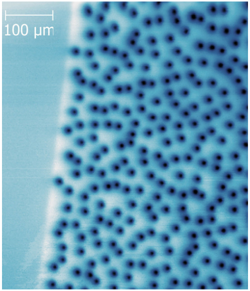


Figure: Winding number.

Topological Defects

Some examples

- Quantized magnetic flux lines in type-II superconductors
- Vortices in superfluids
- Dislocations in crystals
- Some proposals in and beyond the SM with cosmic strings



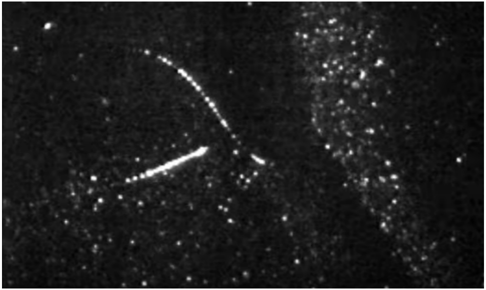


Figure: Vortices in the local magnetic field of a 200 nm thin film of YBCO superconductor, image from Wells, F. et al. Sci Rep 5, 8677 (2015). Visualization of filaments in ^{4}He superfluid, image from Fonda, E. et al. arXiv:1210.5194 (2012).

Vortices in type-II Superconductors

The free energy density F has a series expansion in ψ around T_c

$$F = F_0 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{H^2}{8\pi} + \frac{1}{2m}\left|i\hbar\nabla\psi + \frac{2e}{c}\vec{A}\psi\right|^2.$$

In order to obtain the eq. of motion, we take a variation with respect to ψ^*, requiring it to be minimal.

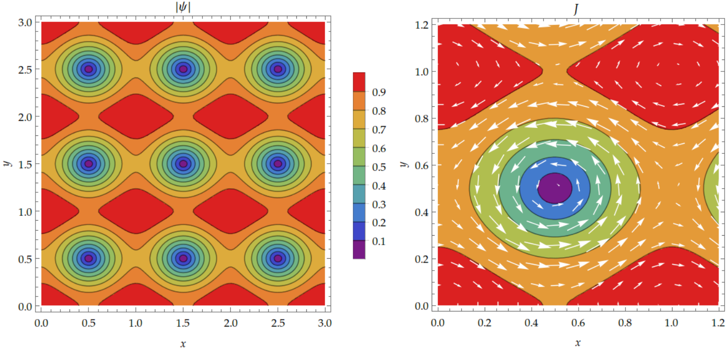


Figure: Contour lines of the solution $|\psi|$ with square lattice symmetry and current as a vector field in a the vortex.

Vortices and Topology

Generally for a scalar field ϕ which transforms under the representation of a compact Lie Group $\mathcal{G}$. Let $\mathcal{M}$ be the vacuum manifold and let $v \in \mathcal{M}$. Then for any $g \in \mathcal{G}$, $gv \in \mathcal{M}$.

- The points on $\mathcal{M}$ are in one-to-one correspondence with the cosets $\mathcal{M} = \mathcal{G}/\mathcal{H}$

A solution

$$\phi(\varphi) = g(\varphi)v,$$

might be regarded as defining a loop in $\mathcal{M}$, a map $S^1 \to \mathcal{M}$. Whether or not there is a vortex solution depends on the topological characterization of this loop.

- Here we use the symmetry group $U(1)$ and we have a non-trivial

$$\pi_1(U(1)) = \mathbb{Z}$$

Equations of motion for local $U(1)$ symmetry

We take the $U(1)_{\alpha Y+\beta(B-L)}$ as a local symmetry associated to a gauge field $\mathcal{A}_\mu$, the Lagrangian

$$\mathcal{L} = D_\mu\Phi^\dagger D^\mu\Phi + d_\mu\chi^* d^\mu\chi - V(\Phi,\chi) - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu},$$

with the potential

$$V(\Phi,\chi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 + \mu'^2\chi^*\chi + \lambda'(\chi^*\chi)^2 - \kappa\Phi^\dagger\Phi\chi^*\chi$$

the covariant derivatives (heavy W_μ decouples and fixed fermion background)

$$D_\mu\Phi = \partial_\mu\Phi + i\frac{\alpha}{2}\mathcal{A}_\mu\Phi, \quad d_\mu\chi = \partial_\mu\chi + 2i\beta\mathcal{A}_\mu\chi,$$

and the field strength tensor

$$\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu.$$

We take the ansatz in cylindrical coordinates (r, φ, z)

$$\Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \phi(r) e^{in\varphi}, \quad \chi = \xi(r) e^{in'\varphi}, \quad \mathcal{A}_\mu = \mathcal{A}_\varphi \hat{\varphi} = \frac{a(r)}{r} \hat{\varphi},$$

where ϕ, ξ, $a \in \mathbb{R}$. The equations of motion are

$$\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} - \frac{n^2}{r^2}\phi - \mu^2\phi - 2\lambda\phi^3 + \kappa\phi\xi^2 - h\frac{a}{r^2}\phi(2n + ha) = 0,$$

$$\frac{d^2\xi}{dr^2} + \frac{1}{r}\frac{d\xi}{dr} - \frac{n'^2}{r^2}\xi - \mu'^2\xi - 2\lambda'\xi^3 + \kappa\xi\phi^2 - h'\frac{a}{r^2}\xi(2n' + h'a) = 0,$$

$$\frac{d^2a}{dr^2} - \frac{1}{r}\frac{da}{dr} = 2h\phi^2(n + ha) + 2h'\xi^2(n' + h'a).$$

We use the boundary conditions

$$\phi(r = 0) = 0, \quad \xi(r = 0) = 0, \quad a(r = 0) = 0,$$

$$\lim_{r\to\infty} \phi(r) = v, \quad \lim_{r\to\infty} \xi(r) = v', \quad \lim_{r\to\infty} a(r) = -\frac{n}{h} = -\frac{n'}{h'}.$$

Cosmic Strings solutions

In all these plots we use $v = 0.5$, $v' = 1$, $\lambda = 1 = \lambda'$. The variation of these parameters does not modify the general behaviour of the solutions. Also $h = n$ and $h' = n'$. Winding numbers establish the behaviour of the solutions. $\kappa \in [-1, 1]$.

- Winding numbers: $n = 0$, $n' = 1$

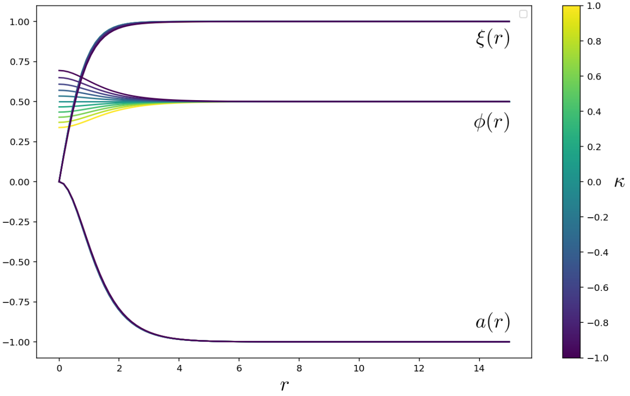


Figure: Cosmic strings solutions with Higgs and gauge fields.

Cosmic Strings solutions

- Winding numbers: $n = 1$, $n' = 1$

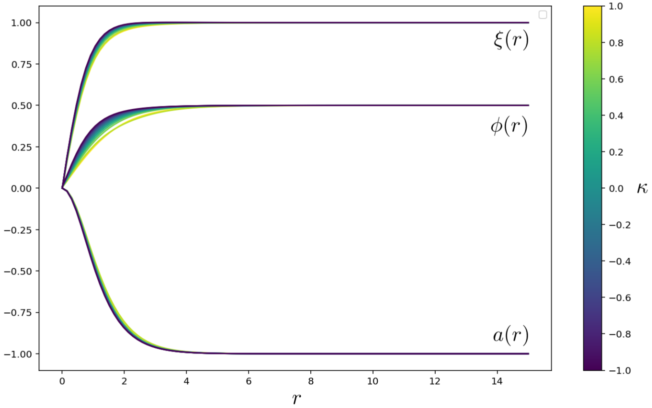


Figure: Cosmic strings solutions with Higgs and gauge fields.

Cosmic Strings solutions

- Winding numbers: $n = 1$, $n' = 2$

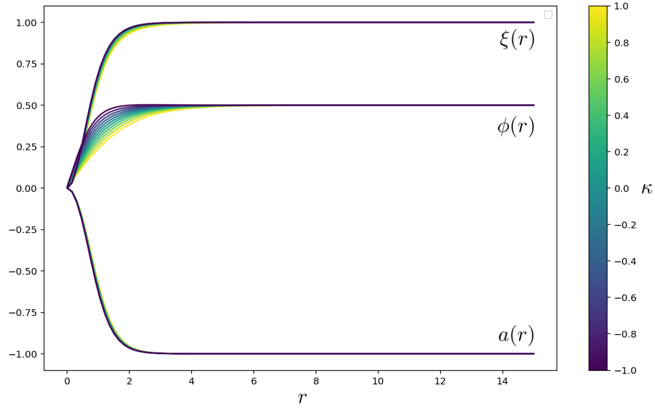


Figure: Cosmic strings solutions with Higgs and gauge fields.

Summary and Conclusions

- As $B - L$ is an exact symmetry of the SM, we gauge it with $\mathcal{A}_\mu$
- To cancel the anomaly we add ν_R
- We give mass to ν_R by adding χ
- Now neutrinos can have Dirac and Majorana mass terms
- We study cosmic string solutions and identify their profiles for $\mathcal{A}_\mu$, Φ and χ
- We find the behaviour of cosmic strings being dominated by the winding numbers
- Particles might change their masses while crossing the cosmic string

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