The First Measurement of the Muon Anomalous Magnetic Moment from the Fermilab Muon $g-2$ Collaboration

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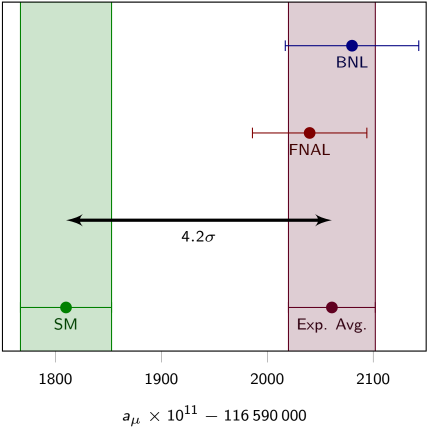
Abstract

This colloquium will cover the physics and methods behind Fermilab's Muon g-2 experiment, along with the long-awaited results from Run-1. The experiment was undertaken to resolve the tension between the Standard Model and the previous measurement taken at Brookhaven National Labs. The measured value of the muon anomalous magnetic moment is $a_\mu(FNAL) = 116592040(54) \times 10^{-11}$. This result is in good agreement with Brookhaven's previous measurement. The new world average, $a_\mu(Exp) = 116592061(41) \times 10^{-11}$, shows a difference from the theoretical value, $a_\mu(SM) = 116591810(43) \times 10^{-11}$, of 4.2 standard deviations, strongly hinting at physics beyond the Standard Model. The experiment requires the simultaneous measurement of the muon precession frequency, the magnetic field, and the muons' distribution in the field. All three of these measurements will be discussed in context, along with the main systematic corrections and uncertainties.

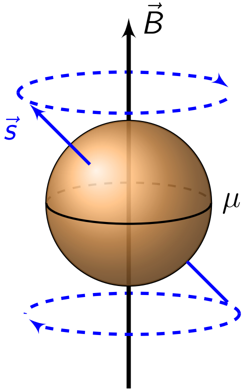
Spoilers (460 ppb Uncertainty)

- Fermilab measurement agrees with Brookhaven
- Reasonable to combine measurements because they are statistics dominated
- New world average is 4.2σ from the Standard Model prediction (2020 white paper)



Outline

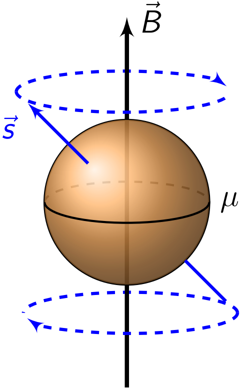
What is a magnetic moment? Where does it come from?



- Elementary particles can have inherent angular momentum called spin, in charged particles spin leads to magnetic moment
- Magnetic moment of "dressed" particle different from "bare" particle due to interactions with vacuum, shifts the magnetic dipole moment

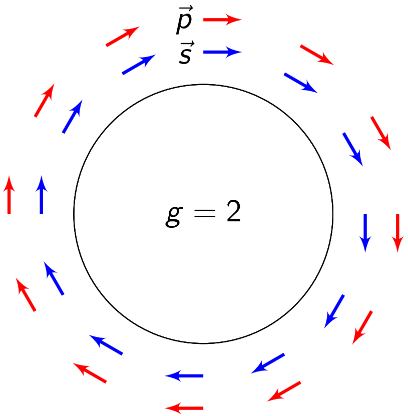
How does a magnetic moment behave?

- A magnetic dipole not parallel to external magnetic field precesses about the field



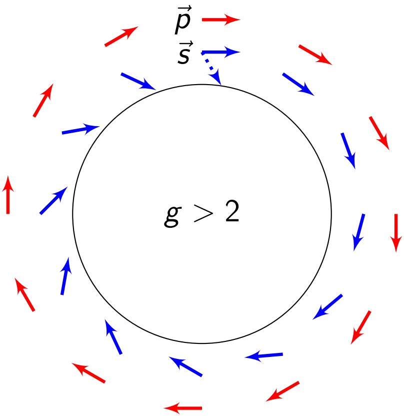
How does a magnetic moment behave?

- A magnetic dipole not parallel to external magnetic field precesses about the field
- If g-factor equals 2, spin precesses at same rate as momentum in circular motion.

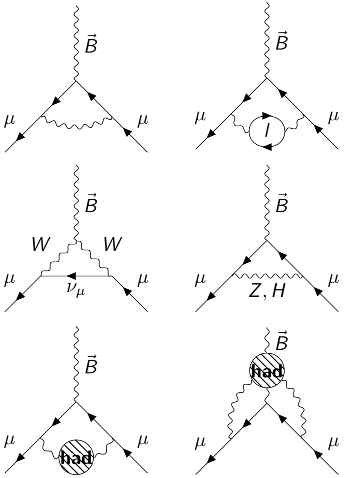


How does a magnetic moment behave?

- A magnetic dipole not parallel to external magnetic field precesses about the field
- Muon g-factor slightly larger than 2, its spin precesses faster than momentum

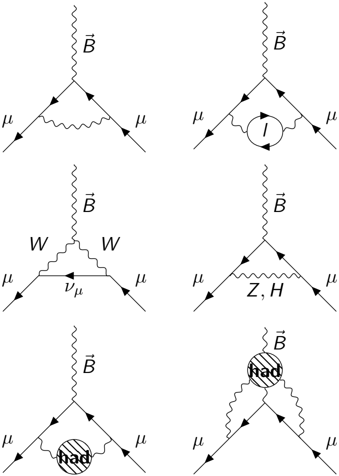


The state of the muon $g-2$ theory



- Theory takes QED, electroweak, and QCD into account
- QCD terms are leading source of uncertainty
- Recent white paper used data-driven approach to calculate hadronic sector contributions

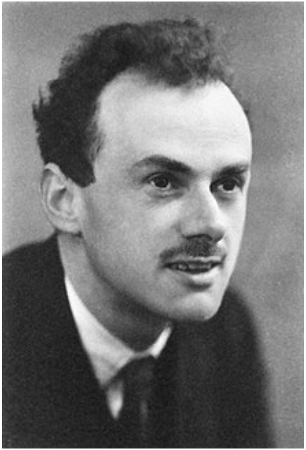
Why do we care about magnetic moments?



- Magnetic moments are shifted by interactions with virtual particles, precision measurements can investigate particle interactions
- Experimental value of muon g-factor differs from Standard Model predictions, hinting at new physics.

The history of the muon g-factor

- 1928, Dirac equation, $g = 2$



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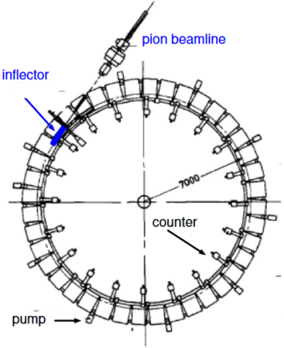
- 1962, CERN muon g-factor experiments



The history of the muon g-factor

- 1928, Dirac equation, $g = 2$
- 1947, Schwinger term, $a_l = \frac{\alpha}{2\pi}$
- 1962, CERN muon g-factor experiments
- 2006, BNL experiment finds hints of discrepancy with Standard Model

Spin precession equations

The "anomalous precession frequency" is the rate at which the muon's spin and momentum dephase.

$$\vec{\omega_a} = \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m} a_\mu \vec{B}$$

where $a_\mu = \frac{1}{2}(g-2)$. With the right experiment, this rate is *(nearly)* proportional to the anomalous magnetic moment.

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$$\vec{\omega_a} = -\frac{q}{m}\left(a_\mu \vec{B} - a_\mu \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left[a_\mu - \frac{1}{\gamma^2-1}\right]\frac{\vec{\beta}\times\vec{E}}{c}\right)$$

Extracting a_μ from spin precession measurements

With simultaneous measurements of anomalous precession frequency and magnetic field, we determine the anomalous magnetic moment, a_μ

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$$a_\mu = \frac{\omega_a}{B} \frac{m}{q}$$

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{g_e}{2} \frac{m_\mu}{m_e}$$

This experiment measured the ratio of the anomalous precession frequency, ω_a, to the precession frequency of a spherical water sample in the same volume, $\tilde{\omega}'_p$. Other terms from the literature are used to convert this ratio to a_μ. ± 25 ppb

A schematical view of the ratio $\omega_a/\tilde{\omega}'_p(r, y, \theta, t)$

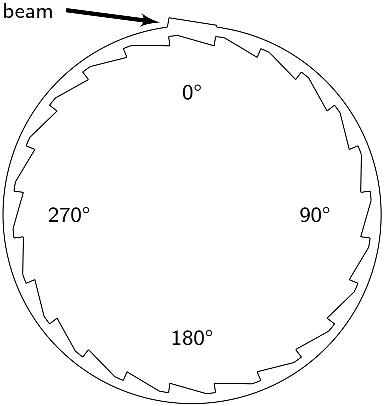
$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{g_e}{2} \frac{m_\mu}{m_e}$$

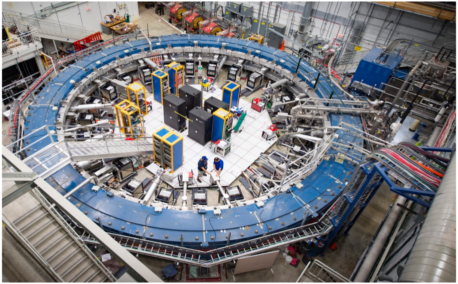
$$\vec{\omega_a} = -\frac{q}{m}\left(a_\mu \vec{B} - a_\mu \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left[a_\mu - \frac{1}{\gamma^2-1}\right]\frac{\vec{\beta}\times\vec{E}}{c}\right)$$

Insert equation for ω_a into the measured ratio in equation for a_μ, collect correction terms to write a schematical equation:

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \langle \omega_p \otimes \rho_\mu \rangle\ \ (1 + B_{qt} + B_{kick})}$$

The Muon $g-2$ storage ring





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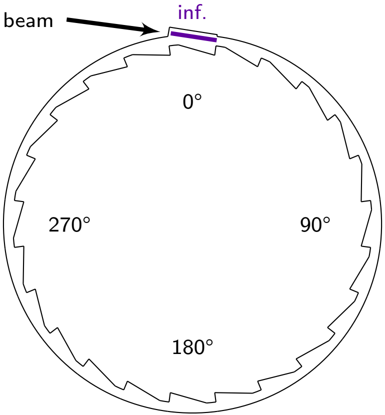

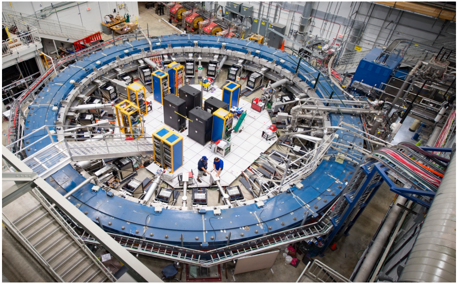

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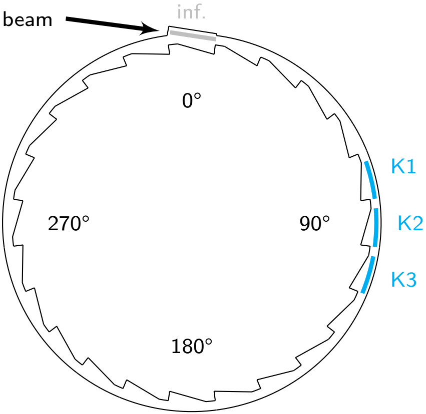

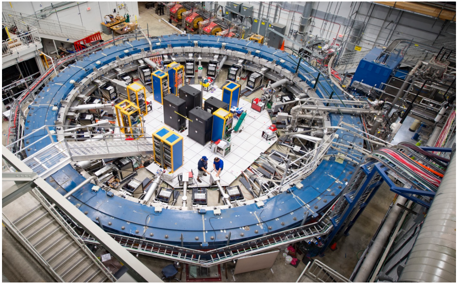

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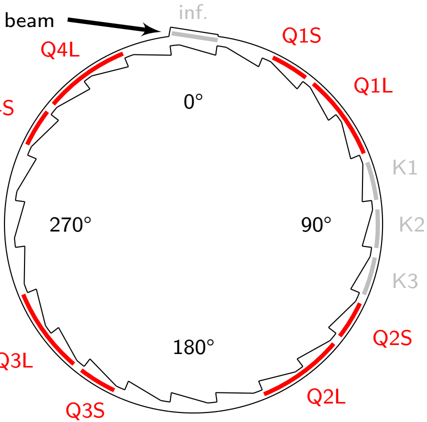

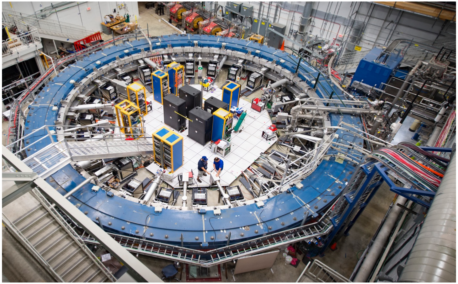

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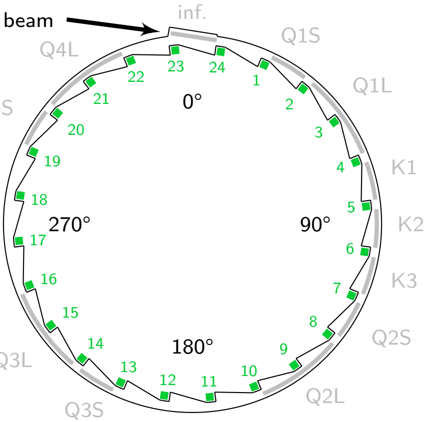

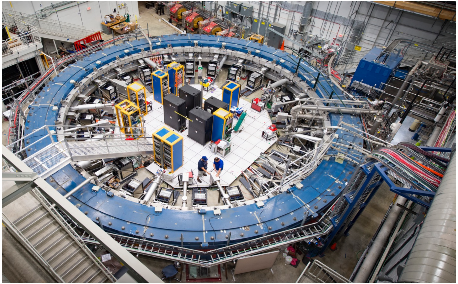

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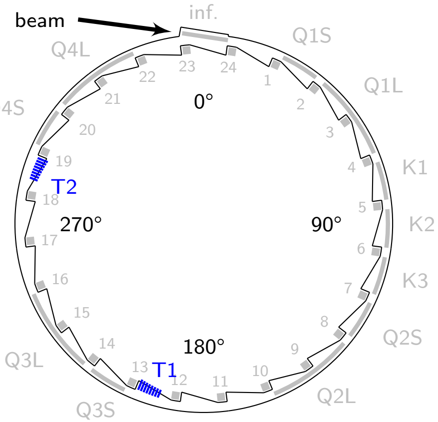
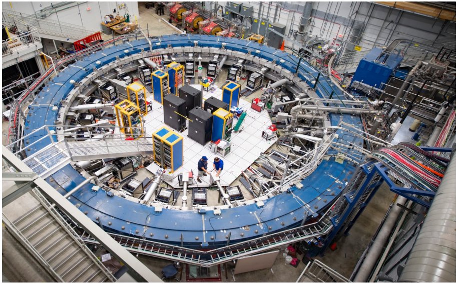
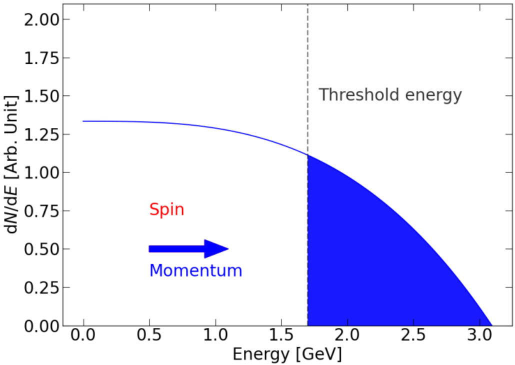




Measuring the precession frequency with positrons

- High-momentum positrons preferentially emitted along the muon's spin
- In lab frame, more high-energy positrons emitted when spin parallel to momentum



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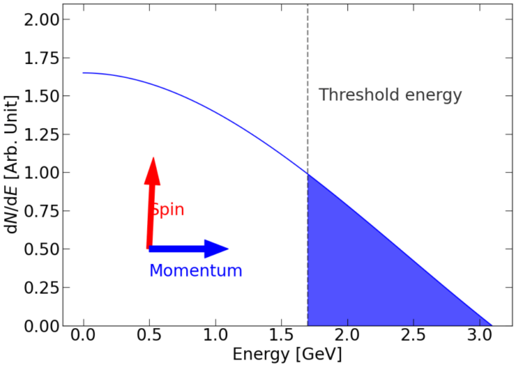

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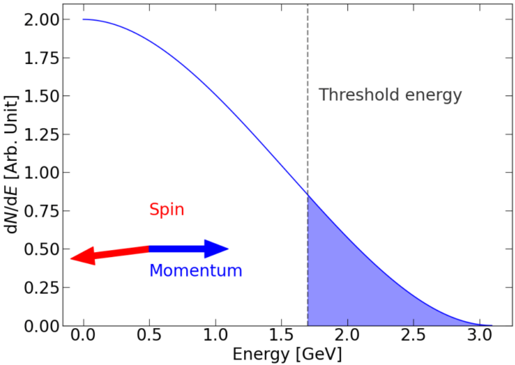

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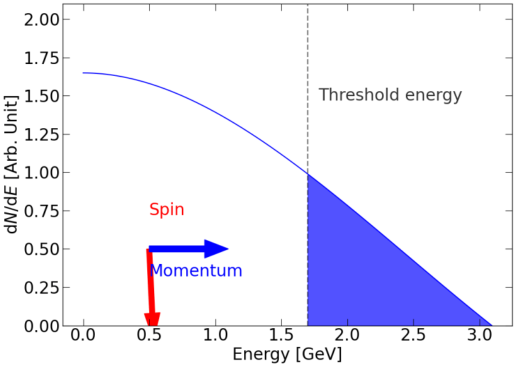

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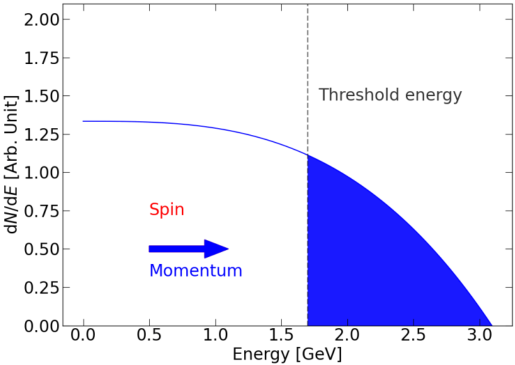
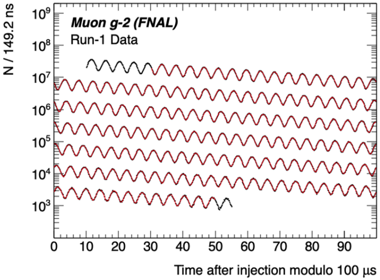


Measuring the precession frequency with positrons

- High-momentum positrons preferentially emitted along the muon's spin
- In lab frame, more high-energy positrons emitted when spin parallel to momentum
- Modulation of positron energy spectrum encodes anomalous precession frequency

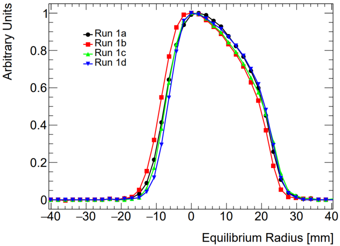
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\, \omega_{a,\text{meas}}\, (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}} \left\langle \omega_p \bigotimes \rho_\mu \right\rangle (1 + B_{qt} + B_{kick})}$$



Stat.: ±434 ppb, Syst.: ±56 ppb

The corrections to ω_a

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$



$$c_e = 2n(1-n)\beta_0^2 \frac{\left\langle x_e^2 \right\rangle}{R_0^2}$$

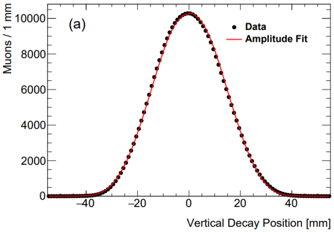
489 ± 53 ppb

c_e E-field correction minimized using magic momentum, remainder corrected by studying mean radius which is correlated to momentum dispersion

$$\frac{q}{m}\left[a_\mu - \frac{1}{\gamma^2 - 1}\right]\frac{\vec{\beta} \times \vec{E}}{c}$$

The corrections to ω_a

$$\frac{\omega_a}{\tilde{\omega}_p'} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$



$$c_p = \frac{n}{2} \frac{\left\langle y^2 \right\rangle}{R_0^2}$$

180 ± 13 ppb

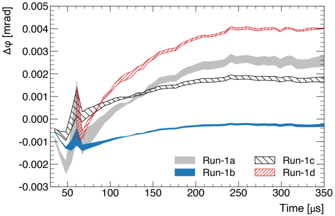
c_e The E-field

c_p Pitch correction from the small vertical component of muons' momentum, calculated by studying vertical position distribution from trackers

$$\frac{q}{m} a_\mu \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta}$$

The corrections to ω_a

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$



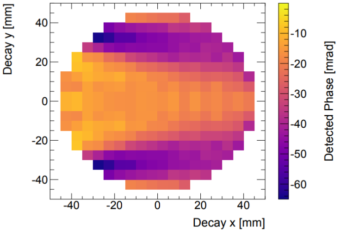
$$\frac{\mathrm{d}\phi_0}{\mathrm{d}t} = \frac{\mathrm{d}\phi_0}{\mathrm{d}\langle p\rangle}\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}$$

-11 ± 5 ppb

- c_e The E-field
- c_p Pitch correction
- c_{ml} Muon loss correction from initial phase-momentum correlation in muons, ss muons are lost in time, there is time dependent change in phase

The corrections to ω_a

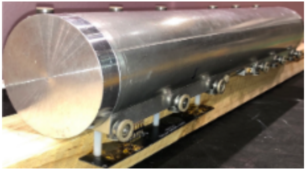
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

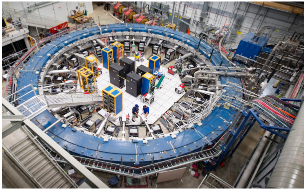


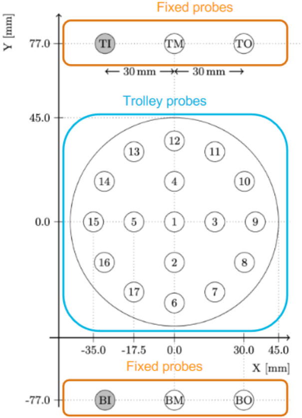
-158 ± 75 ppb

- c_e The E-field
- c_p Pitch correction
- c_{ml} Muon loss correction
- c_{pa} Phase acceptance correction caused by decay-position dependence of positron phase, early-to-late beam motion modulation leads to time-dependent phase

The magnetic field measurement systems



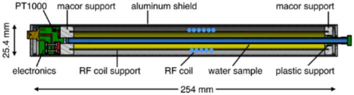


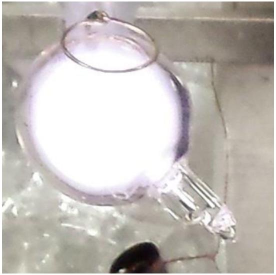


The field calibration chain

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Absolute calibration done at ANL, cross-checked with ^{3}He





$\pm$32 ppb

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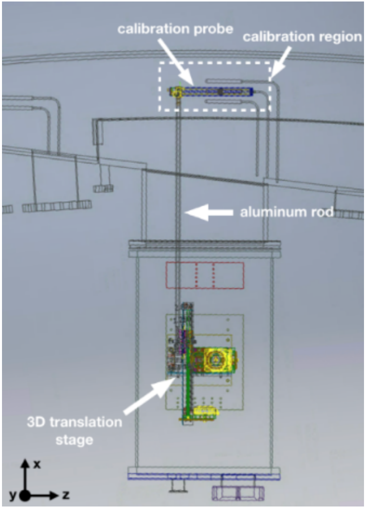
- Absolute calibration done at ANL, cross-checked with ^{3}He
- Plunging probe calibrates trolley

$\pm$32 ppb



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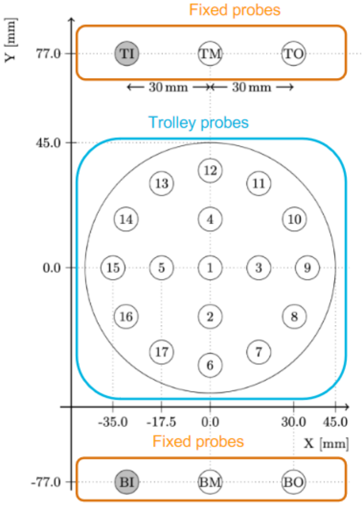
- Absolute calibration done at ANL, cross-checked with ^{3}He
- Plunging probe calibrates trolley
- Trolley synchronizes with fixed probes

±32 ppb

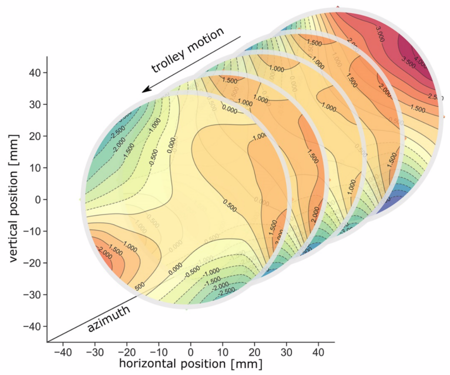


Getting field maps from the trolley

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Trolley has an array of 17 NMR probes, travels around ring, takes measurements at $\sim$ 8000 azimuthal locations
- Measurements in same volume muons are stored in

-13 ± 25 ppb



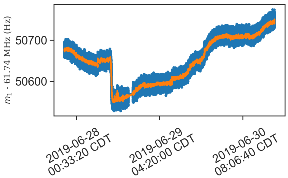
Interpolating between trolley runs with the fixed probes

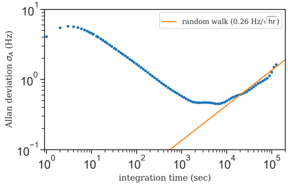
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Fixed probes synchronized to the trolley during the trolley runs
- Fixed probes track field between trolley runs, additional data points to interpolate field

±(24 to 44) ppb

Super-long Trolley Runs

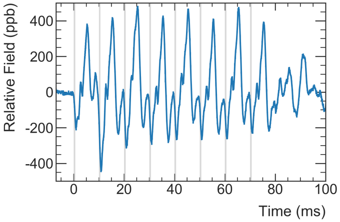




The fast transient systematics

- Electric quadrupoles vibrate generating eddy currents that perturb the field
- Perturbation too fast for fixed probes to pick up normally, shielded by the vacuum chambers
- Measurements made with special probe to determine effect of perturbation

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

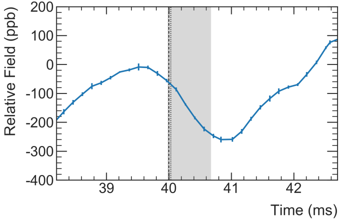


-17 ± 92 ppb

The fast transient systematics

- Electric quadrupoles vibrate generating eddy currents that perturb the field
- Perturbation too fast for fixed probes to pick up normally, shielded by the vacuum chambers
- Measurements made with special probe to determine effect of perturbation

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

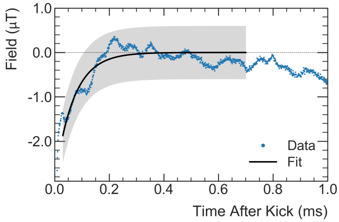


-17 ± 92 ppb

The fast transient systematics

- Pulsed magnetic kick generates larges eddy currents, perturb the magnetic field
- Like quad transient, too fast to be picked up by regular analysis
- Used optical Faraday magnetometer to study the transient

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{a,\text{meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

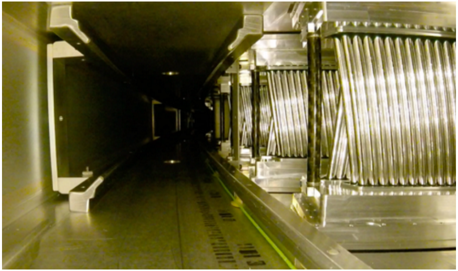


-27 ± 37 ppb

Straw trackers

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Straw trackers track decay positron path
- Tracks can be extrapolated back to muon decay position
- Decay vertices used to determine muon distribution in storage region.



$(-3 \text{ to } 1) \pm (11 \text{ to } 20)$ ppb

Straw trackers

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Straw trackers track decay positron path
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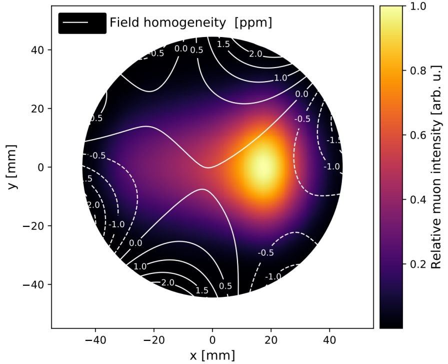
$(-3 \text{ to } 1) \pm (11 \text{ to } 20)$ ppb



Straw trackers

$$\frac{\omega_a}{\tilde{\omega}_p'} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Straw trackers track decay positron path
- Tracks can be extrapolated back to muon decay position
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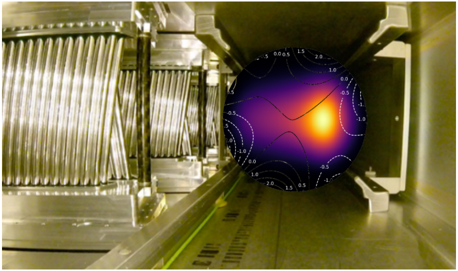
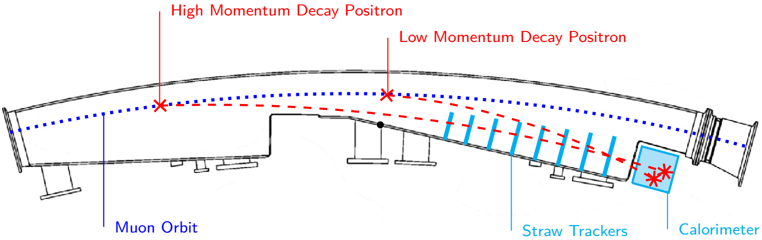


$(-3 \text{ to } 1) \pm (11 \text{ to } 20)$ ppb

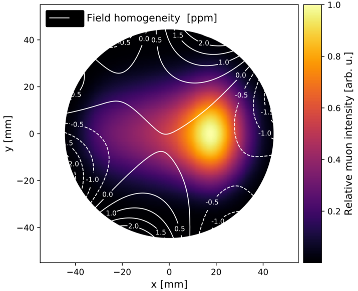
Straw trackers



Combining the field maps and muon distribution

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

- Magnetic field maps weighted by muon distribution determined by trackers
- Trackers measure at two locations storage ring, use beam dynamics simulations to extrapolate distribution around ring



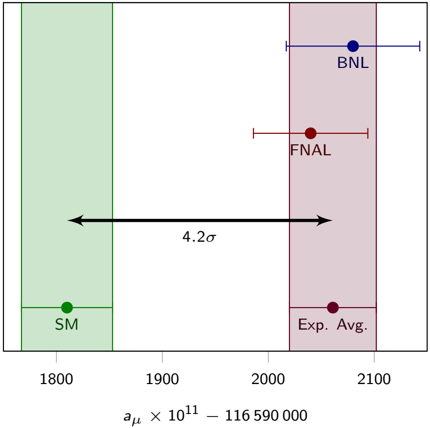
Collecting Uncertainties

Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	–	434
ω_a^m (systematic)	–	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}}\langle\omega_p'(x,y,\phi) \times M(x,y,\phi)\rangle$	–	56
B_k	-27	37
B_q	-17	92
$\mu_p'(34.7°)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

- Dominated by statistics
- Largest systematics uncertainties are phase acceptance and quadrupole transient
- Both systematics are expected to be drastically reduced in following runs

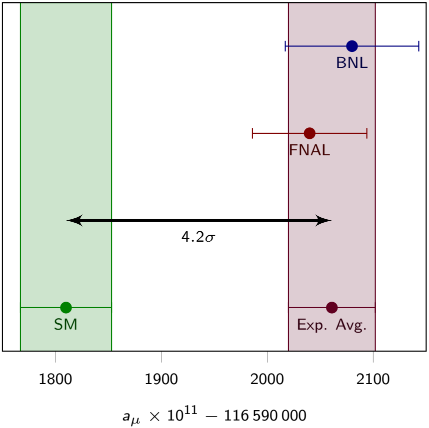
Results from Fermilab Muon $g-2$ Run-1

- Fermilab measurement agrees with Brookhaven
- Reasonable to combine measurements because they are statistics dominated

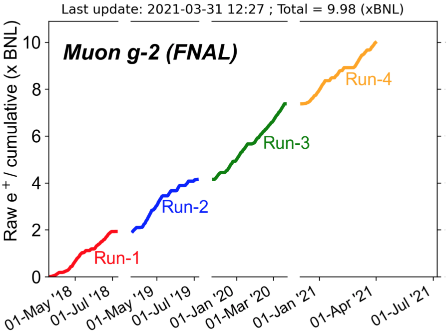


Results from Fermilab Muon $g - 2$ Run-1

- Fermilab measurement agrees with Brookhaven
- Reasonable to combine measurements because they are statistics dominated
- New world average is 4.2σ from the Standard Model prediction (2020 white paper)



What's next from Muon $g-2$



- Already collected four times the data used in results presented here
- Systematic uncertainties brought down by combination of upgrades and more systematic studies

Acknowledgments

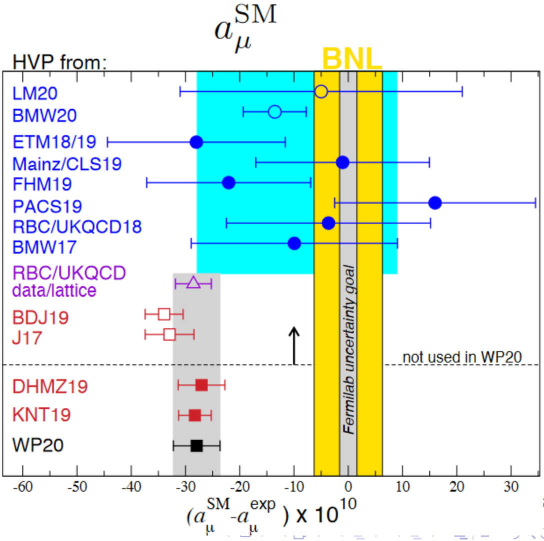
Thanks to:

- University of Michigan, Ann Arbor
- The Chupp Group, both $g - 2$ affiliated and not
- Muon $g - 2$ field team
- Muon $g - 2$ team in general
- Fermilab
- My wife, Lily

Backup Slides

Lattice QCD Results

- Large range of lattice values that agree with both 2020 white paper and experiment
- The BMW20 calculation still needs to be vetted by independent lattice groups
- More lattice results are upcoming, worth keeping an eye on

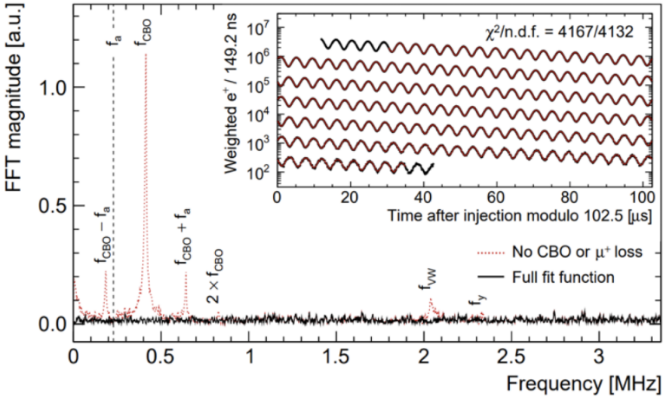


Blinding

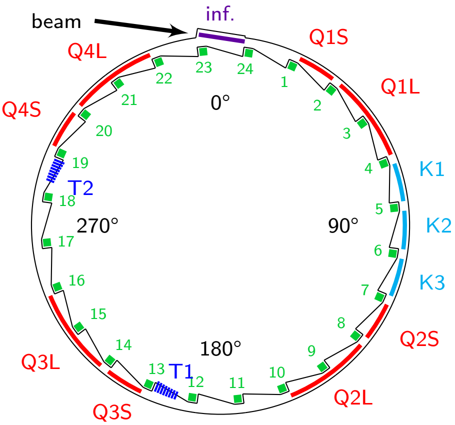
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}}\ \omega_{\text{a,meas}}\ (1 + c_e + c_p + c_{ml} + c_{pa})}{f_{\text{field}}\ \left\langle \omega_p \bigotimes \rho_\mu \right\rangle\ (1 + B_{qt} + B_{kick})}$$

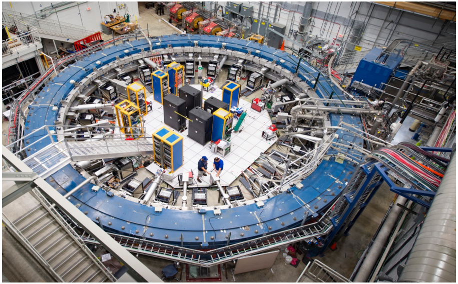
- Experiment blinded at ω_a clock
- Blinders not part of Muon $g-2$, Fermilab personnel
- Clock frequencies kept in sealed envelopes until unblinding

The frequencies in the measurement

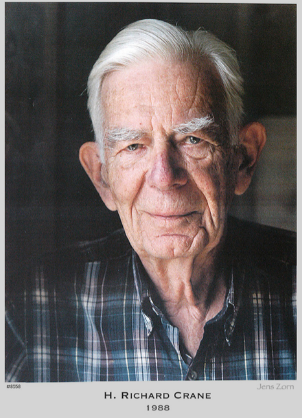


The Muon $g-2$ storage ring





Michigan's Contributions 1



- 1947, first measurement of electron $g-2$ by H. Foley (UMich Ph. D.), bound state
- 1953, first free electron g-factor measured by H. R. Crane at UMich

Michigan's Contributions 1

g-minus 2
To celebrate the pioneering studies of the free electron's magnetism
by H.R. Crane and his students at the University of Michigan 1950-1960
Bronze & stainless steel 84"x42"x42"
Jens Zorn 2004

- 1947, first measurement of electron $g-2$ by H. Foley (UMich Ph. D.), bound state
- 1953, first free electron g-factor measured by H. R. Crane at UMich
- 1961, free electron measurement of $g-2$, uncertainty of 0.2%

Michigan's Contributions 2

Tim Chupp, Midhat Farooq*, Joe Grange*, Eva Krageloh, David Aguillard, Tianyu (Justin) Yang*, Jonathan Sanchez-Lopez, ATB

- Magnetic field measurements
- Helium-3 Magnetometry
- Tracking and EDM measurements
- Opened the right door...

A 2019 picture...