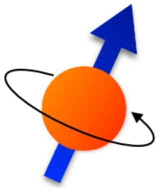
Muon g-2 Prediction and its Prospects

After a brief introduction, the talk will focus on describing the dispersive prediction of the leading-order (LO) hadronic vacuum polarization (HVP) to the muon g-2, which has the largest contribution to the uncertainty

Work in collaboration with M. Davier, A. Hoecker, B. Malaescu (DHMZ) and others

What's it and why it's relevant?

Muon's magnetic moment is connected to its spin by a g_μ factor:



$$\vec{\mu}_\mu = g_\mu \left(\frac{q_\mu}{2m_\mu} \right) \vec{s}$$

We introduce anomalous magnetic moment a_μ to quantify its deviation from 2:

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 0.001\cdots$$

One of the fewer observables that one can measure and predict with extraordinary precision

It receives contributions from all sectors of the SM

Any confirmed measurement/prediction discrepancy would be a sign of new physics (NP)

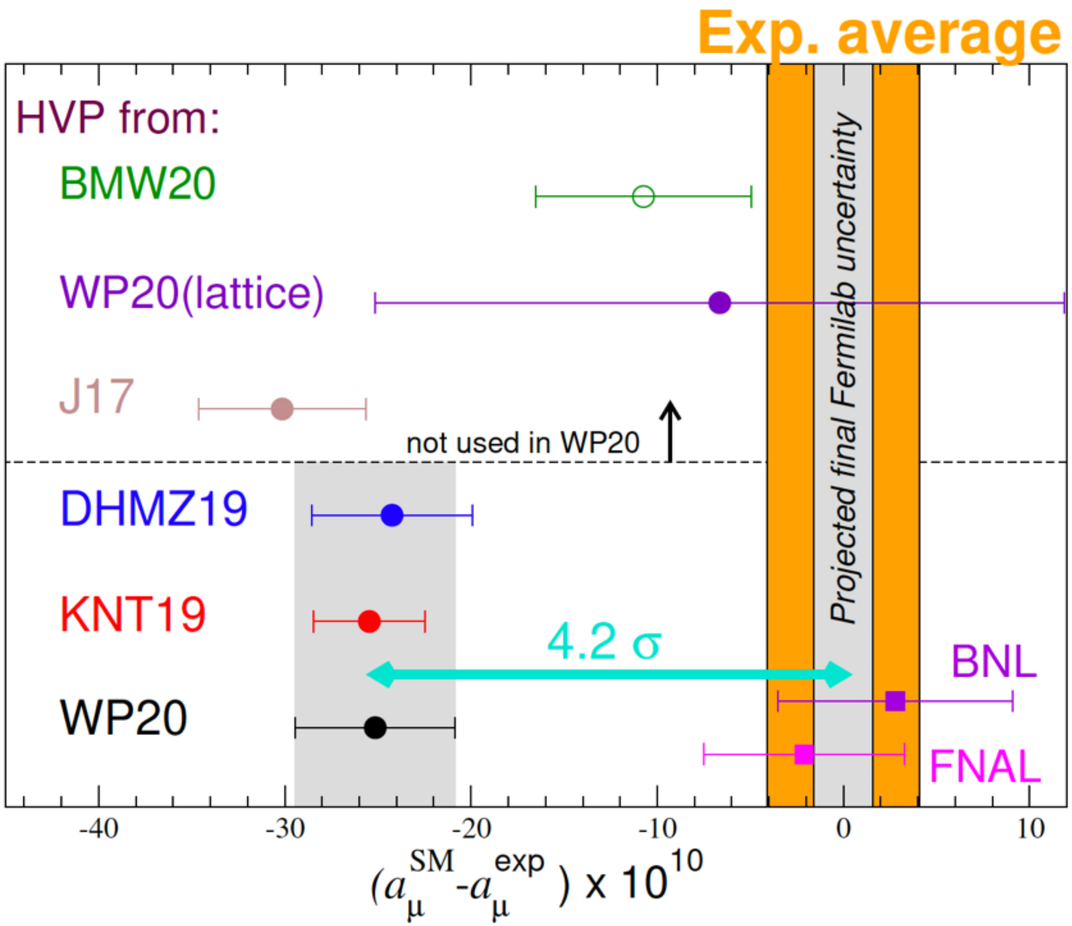
[Note: the sensitivity of a_e to NP is a factor $(m_\mu / m_e)^2$~43000 smaller]

More than 60 years of measurements

Experiment	Beam	Measurement	$\delta a_\mu / a_\mu$	Required th. terms
Columbia-Nevis (57)	μ^+	g=2.00±0.10		g=2
Columbia-Nevis (59)	μ^+	0.001 13(+16)(−12)	12.4%	α/π
CERN 1 (61)	μ^+	0.001 145(22)	1.9%	α/π
CERN 1 (62)	μ^+	0.001 162(5)	0.43%	$(\alpha/\pi)^2$
CERN 2 (68)	μ^+	0.001 166 16(31)	265 ppm	$(\alpha/\pi)^3$
CERN 3 (75)	$\mu^\pm$	0.001 165 895(27)	23 ppm	$(\alpha/\pi)^3$ + had
CERN 3 (79)	$\mu^\pm$	0.001 165 911(11)	7.3 ppm	$(\alpha/\pi)^3$ + had
BNL E821 (00)	μ^+	0.001 165 919 1(59)	5 ppm	$(\alpha/\pi)^3$ + had
BNL E821 (01)	μ^+	0.001 165 920 2(16)	1.3 ppm	$(\alpha/\pi)^4$ + had + weak
BNL E821 (02)	μ^+	0.001 165 920 3(8)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?
BNL E821 (04)	μ^-	0.001 165 921 4(8)(3)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?
New FNAL Run1 (21)	μ^+	0.001 165 920 40(54)	0.46 ppm	$(\alpha/\pi)^4$ + had + weak + ?

ppm = one part per million (10^{-6})

Current Situation



WP20: White Paper published in 2020 *Phys. Rept.* 887 (2020) 1 (link)

An outcome after several dedicated workshops since 2017

In the following, I shall review the prediction using our DHMZ19 as an example

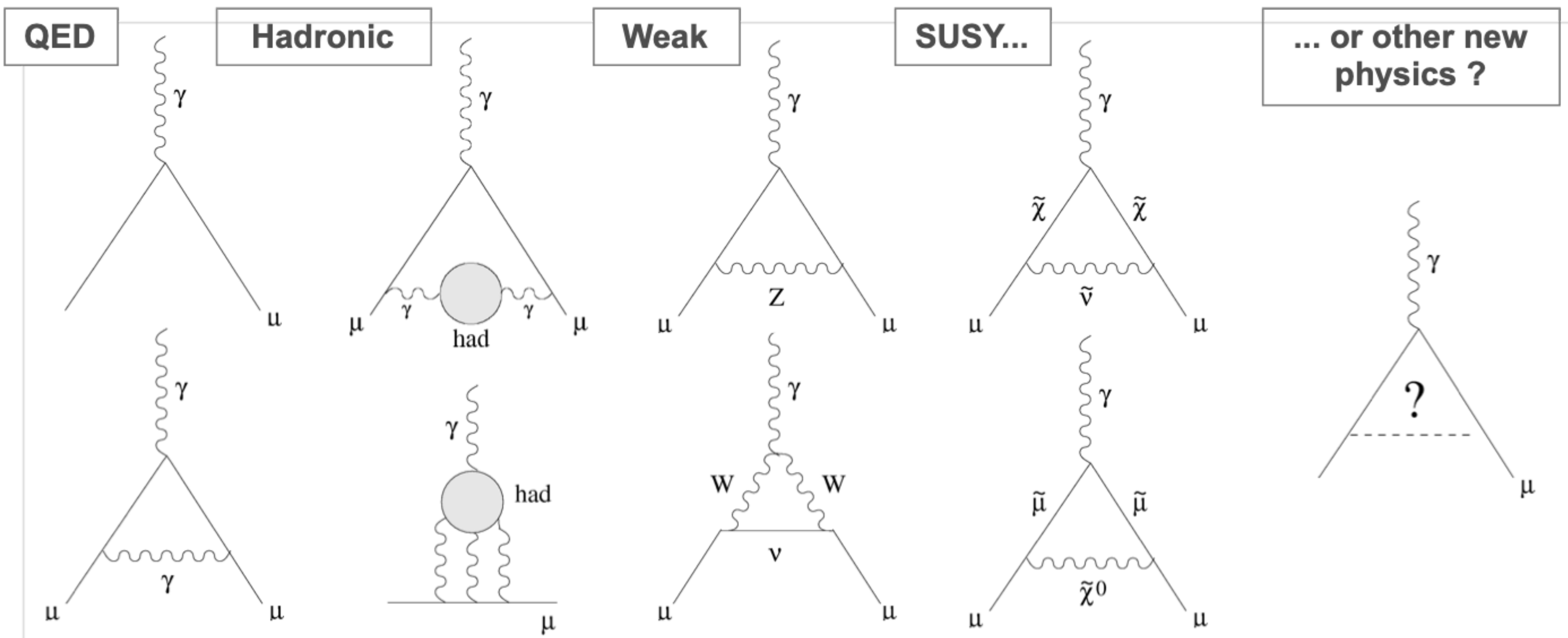
A discrepancy of 4.2σ
→ Strong evidence for new physics?

Theoretical Contributions

$$a_\mu^{\mathrm{th}} = a_\mu^{\mathrm{SM}} + a_\mu^{\mathrm{BSM}}$$

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{Weak}}$$

$$a_\mu^{\mathrm{BSM}}$$



Contributions of the SM components

QED:	$116\ 584\ 718.9\ (0.1) \times 10^{-11}$	[0.001 ppm]
Weak:	$153.6\ (1.0) \times 10^{-11}$	[0.01 ppm]
Hadronic …		
… Vacuum polarisation (HVP):	$6845.0\ (40.0) \times 10^{-11}$	[0.37 ppm]
… Light-by-Light (HLbL):	$92.0\ (18.0) \times 10^{-11}$	[0.15 ppm]

HVP has the largest uncertainty to the prediction

Numbers from WP20

Prediction QED

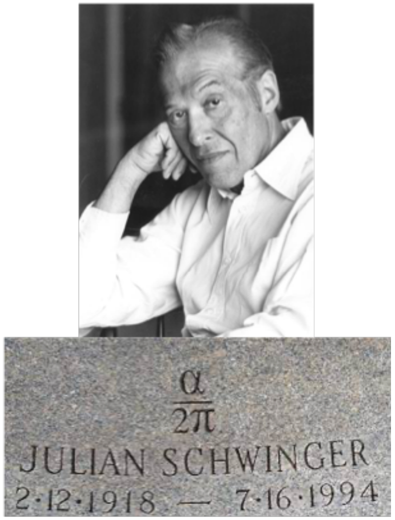
$$
\begin{array}{rlrll}
a_\mu^{\mathrm{QED}} & = \dfrac{\alpha}{2\pi} + A_2\left(\dfrac{\alpha}{\pi}\right)^2 + A_3\left(\dfrac{\alpha}{\pi}\right)^3 + A_4\left(\dfrac{\alpha}{\pi}\right)^4 + A_5\left(\dfrac{\alpha}{\pi}\right)^5 & & & \text{No of Feynman diagrams:} \\
 & = & 116\,140\,793.321\ (23) \times 10^{-11} & \mathcal{O}(\alpha) & 1 \\
 & + & 413\,217.626\ (7) \times 10^{-11} & \mathcal{O}(\alpha^2) & 7 \\
 & + & 30\,141.902\ (33) \times 10^{-11} & \mathcal{O}(\alpha^3) & 72 \\
 & + & 381.004\ (17) \times 10^{-11} & \mathcal{O}(\alpha^4) & 891 \\
 & + & 5.078\ (6) \times 10^{-11} & \mathcal{O}(\alpha^5) & 12\,672 \\
 & = & 116\,584\,718.931\ (104) \times 10^{-11} & &
\end{array}
$$

A_1=1/2 computed by Schwinger in 1947

A_2, A_3 known analytically,
A_2–A_4: cross-checked by different groups using different methods
A_5: calculated by one group with numerical means, only small portions double-checked

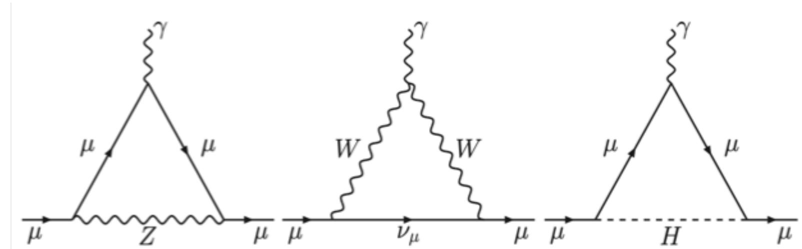
Aoyama, Hayakawa, Kinoshita, Nio(2012-2019)

Uncertainty dominated by estimate of higher order



Prediction Weak

The weak contributions are defined as all SM contributions that are not contained in the pure QED, the HVP, or the HLbL contributions



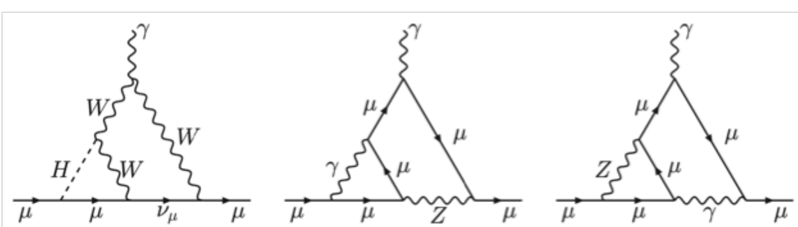
$$a_\mu^{\mathrm{Weak}(1)} = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{m_W^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right]$$

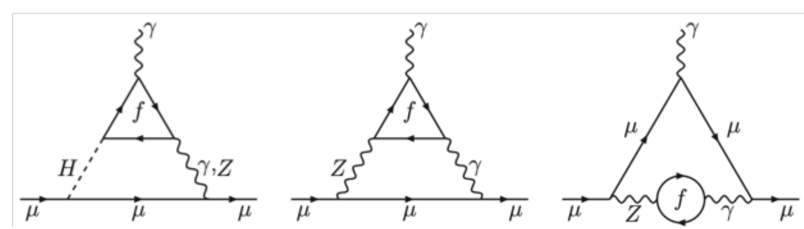
$$= \; 194.79\,(1)\times 10^{-11}$$

$$a_\mu^{\mathrm{Weak}(2)} = a_\mu^{\mathrm{bos}} + a_\mu^{e,\mu,u,c,d,s} + a_\mu^{\tau,t,b} + a_\mu^{H} + a_\mu^{\mathrm{no}\,H}$$

$$= [-19.96\,(1) - 6.91\,(20)(30) - 8.21\,(10) - 1.51\,(1) - 4.64\,(10)]\times 10^{-11}$$

$$a_\mu^{\mathrm{Weak}(\geq 3)} = 0\,(0.20)\times 10^{-11}$$



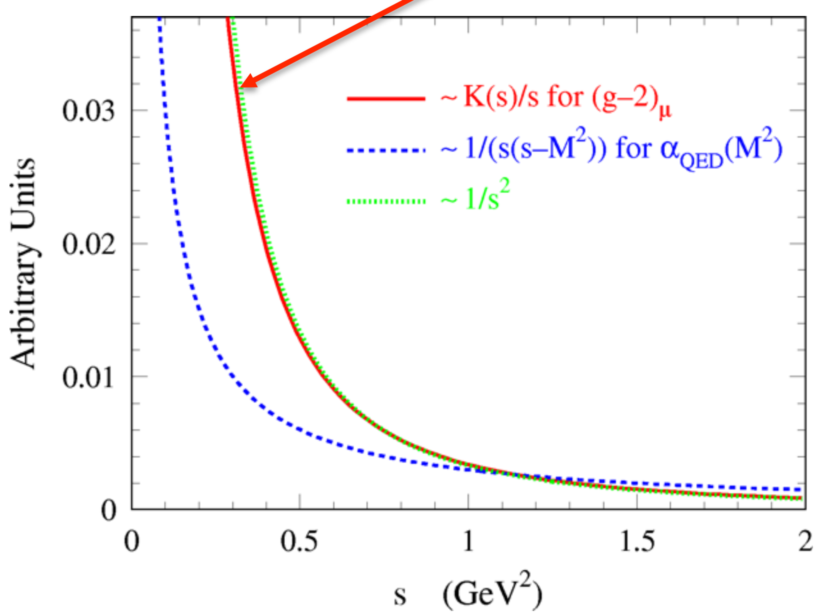


Prediction Hadronic

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}}$$

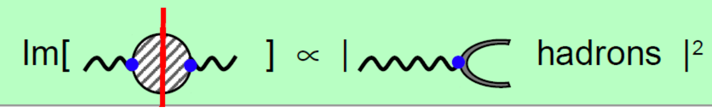
Based on analyticity and unitarity, the LO HVP contribution can be calculated using the dispersion relation [1] over $e^+e^- \rightarrow$ hadrons cross sections

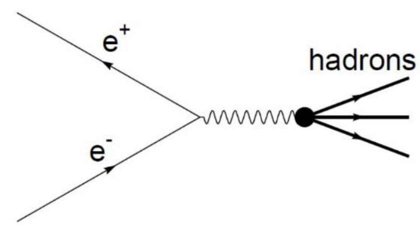
$$a_\mu^{\text{had,LO}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \boxed{\frac{K(s)}{s}} R(s)$$



Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

$$12\pi\ \text{Im}\Pi_\gamma(s) = \frac{\sigma^0\ [e^+e^- \rightarrow \text{hadrons}\ (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$





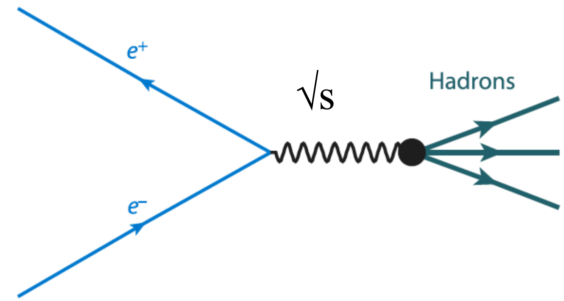
The QED kernel K(s) [2] has such an s dependence that low energy data contribute most:

$e^+e^- \rightarrow \pi^+\pi^-$ contributes ~73% (58% in uncertainty)

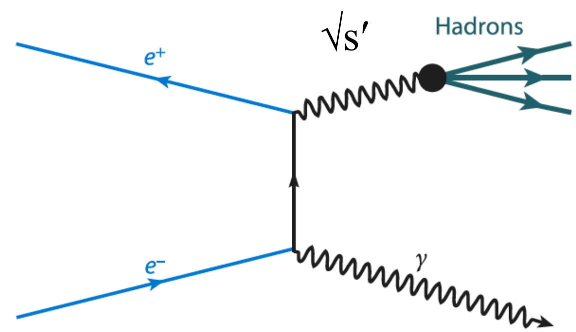
[1] Bouchiat and Michel, 1961
[2] Brodsky, de Rafael, 1968

Two Types of $\sigma(e^+e^- \to$ hadrons) Measurements

1. The scan method: e.g. CMD-2/3, SND at Novosibirsk
   - ➢ Advantages:
     - ➢ Well defined $\sqrt{s}$
     - ➢ Good energy resolution $\sim 10^{-3}\sqrt{s}$
   - ➢ Disadvantages:
     - ➢ Energy gap between two scans
     - ➢ Low luminosity at low energies
     - ➢ Limited $\sqrt{s}$ range of a given experiment



2. The ISR approach: e.g. BaBar, KLOE
   - ➢ Advantages:
     - ➢ $\sigma(e^+e^- \to$ hadrons) may be measured over $\sigma(e^+e^- \to \mu^+\mu^-)$ thus reducing some syst uncertainties
     - ➢ Continuous cross section measurement over a broad energy range down to threshold
   - ➢ Disadvantages:
     - ➢ Need to control background processes



$s'=(1-x)/s$

$x=2E_\gamma/\sqrt{s}$

A Large Number of Exclusive Processes below 1.8 GeV

Channel
$\pi^0\gamma$
$\eta\gamma$
$\pi^+\pi^-$
$\pi^+\pi^-\pi^0$
$2\pi^+2\pi^-$
$\pi^+\pi^-2\pi^0$
$2\pi^+2\pi^-\pi^0$ (η excl.)
$\pi^+\pi^-3\pi^0$ (η excl.)
$3\pi^+3\pi^-$
$2\pi^+2\pi^-2\pi^0$ (η excl.)
$\pi^+\pi^-4\pi^0$ (η excl., isospin)
$\eta\pi^+\pi^-$
$\eta\omega$
$\eta\pi^+\pi^-\pi^0(\text{non-}\omega,\phi)$
$\eta 2\pi^+2\pi^-$
$\omega\eta\pi^0$
$\omega\pi^0$ ($\omega\to\pi^0\gamma$)
$\omega 2\pi$ ($\omega\to\pi^0\gamma$)
ω (non-$3\pi,\pi\gamma,\eta\gamma$)
K^+K^-
K_SK_L
ϕ (non-$K\overline{K},3\pi,\pi\gamma,\eta\gamma$)
$K\overline{K}\pi$
$K\overline{K}2\pi$
$K\overline{K}\omega$
$\eta\phi$
$\eta K\overline{K}$ (non-ϕ)
$\omega 3\pi$ ($\omega\to\pi^0\gamma$)
7π ($3\pi^+3\pi^-\pi^0$ + estimate)

List of 30 channels evaluated in DHMZ19 (*Eur. Phys. J. C* 80 (2020) 3, 241, link)

DHMZ group involved in HVP evaluation since 1997

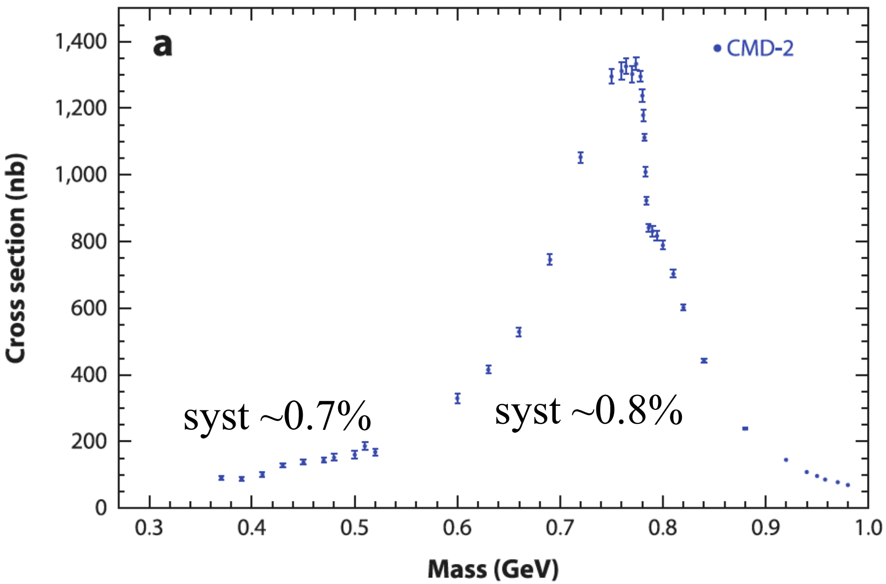
Result used as reference for the Brookhaven experiment: comparison revealed a deficit in the prediction at ~ 2-3σ level, hence our motivation to continue this effort for a more precise prediction

In the following, a few examples of measurements from different experiments in the dominant channels will be shown

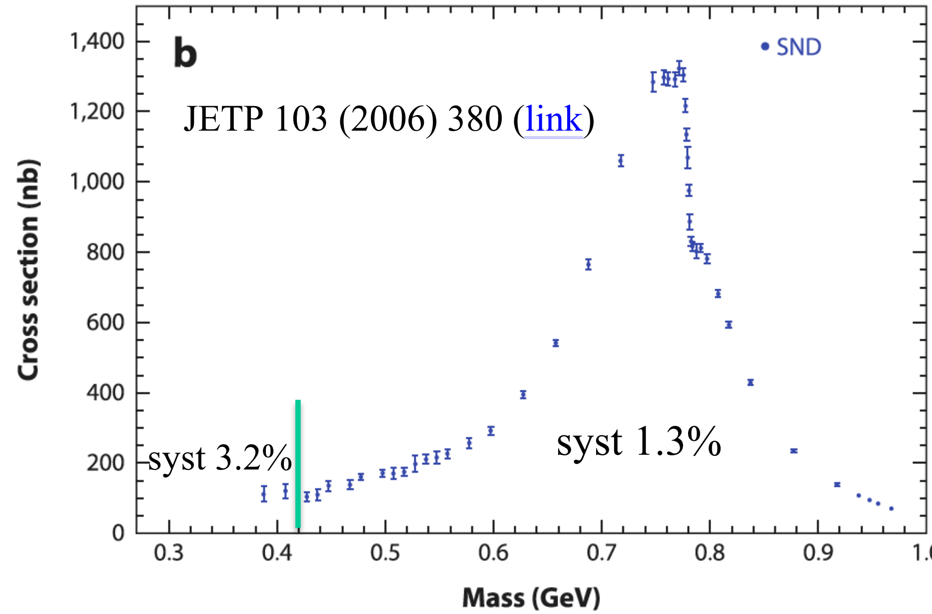
Then we discuss
- the combination of different measurements of a given channel
- comparison and tension between different measurements

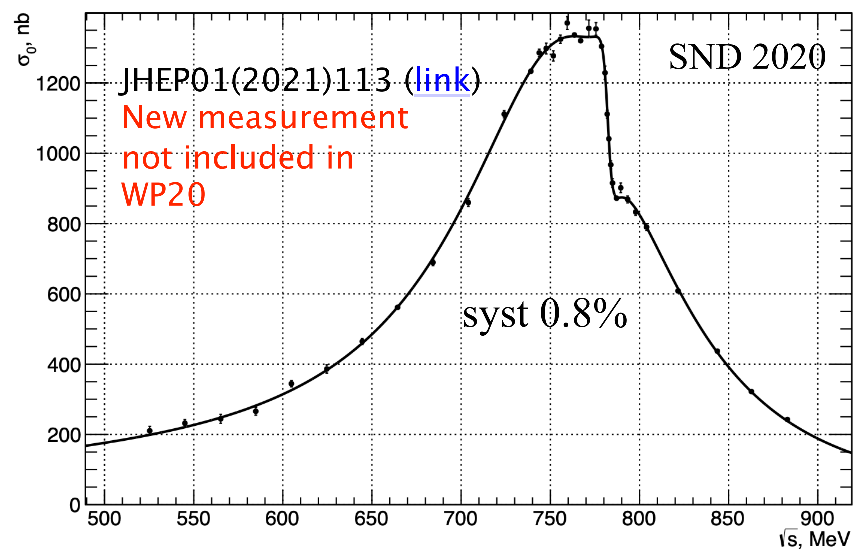
Measurements of 2π Channel: CMD-2/3, SND

CMD-2 (2006)
- Energy 0.35-0.52 GeV: JETP Lett. 84:413-417, 2006 (link)
- Energy 0.6-1.0 GeV: Phys. Lett. B 648: 28-38, 2007 (link)



Figures a, b from M. Davier, Ann. Rev. Nucl. Part. Sci. 63 (2013) 407 (link)

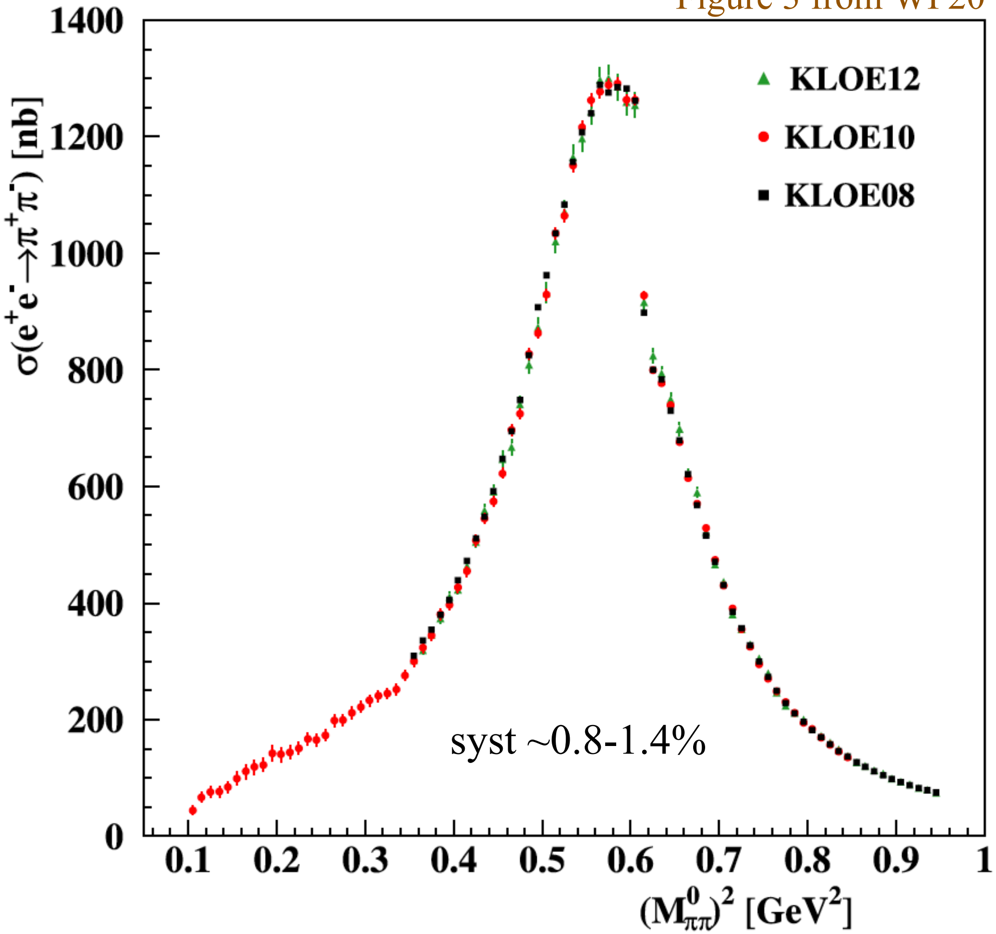




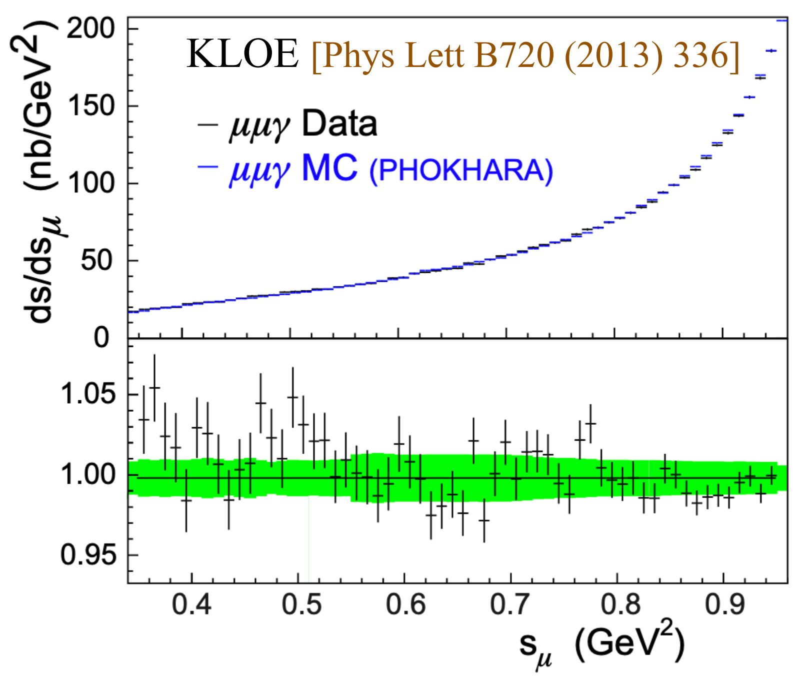
Measurements of 2π Channel: KLOE 08,10,12

$\sqrt{s}$=1.02 GeV ⇒ Soft ISR photons

Figure 3 from WP20

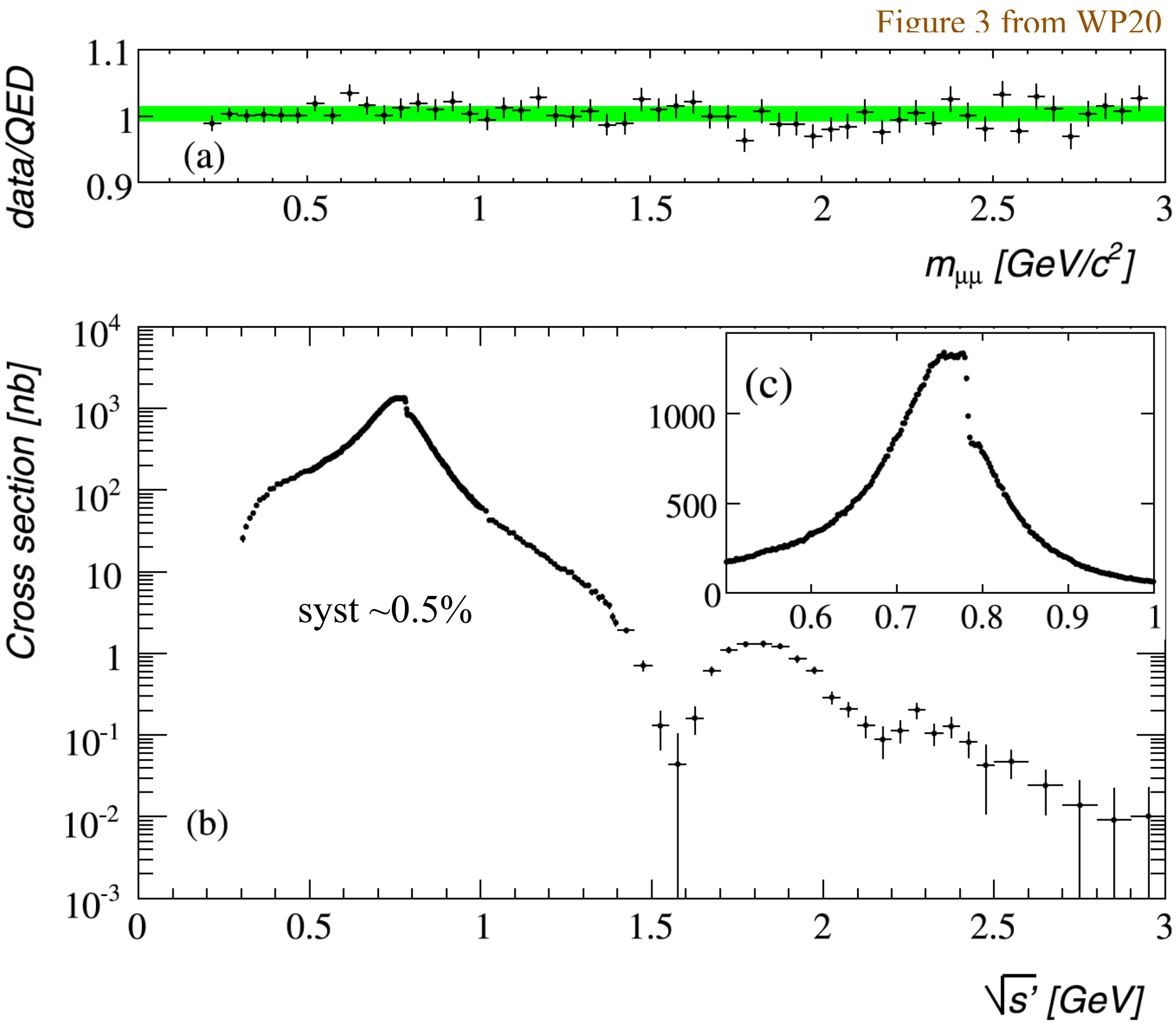


- KLOE12: photon at small angle and undetected, radiator function from measured $\mu^+\mu^-(\gamma)$ events
- KLOE10: photon at large angle and detected, radiator function from NLO QED
- KLOE08: photon at small angle and undetected, radiator function from NLO QED



Measurements of 2π Channel: BaBar 09

$\sqrt{s}$=10.58 GeV ⇒ Hard ISR photons



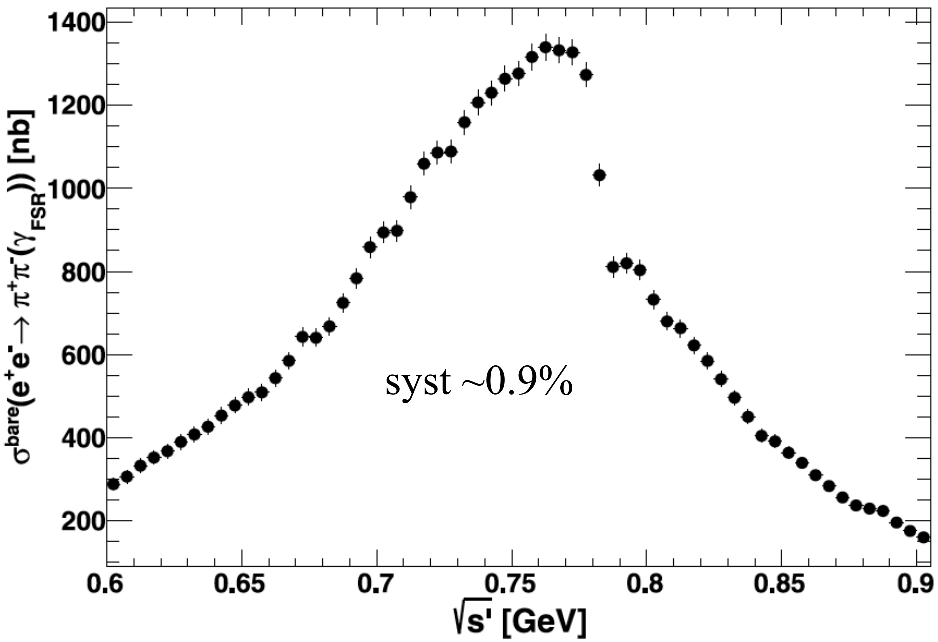
BABAR measurement covers a huge mass range from threshold to 3 GeV!

In BABAR, the ISR photon is detected at large angle

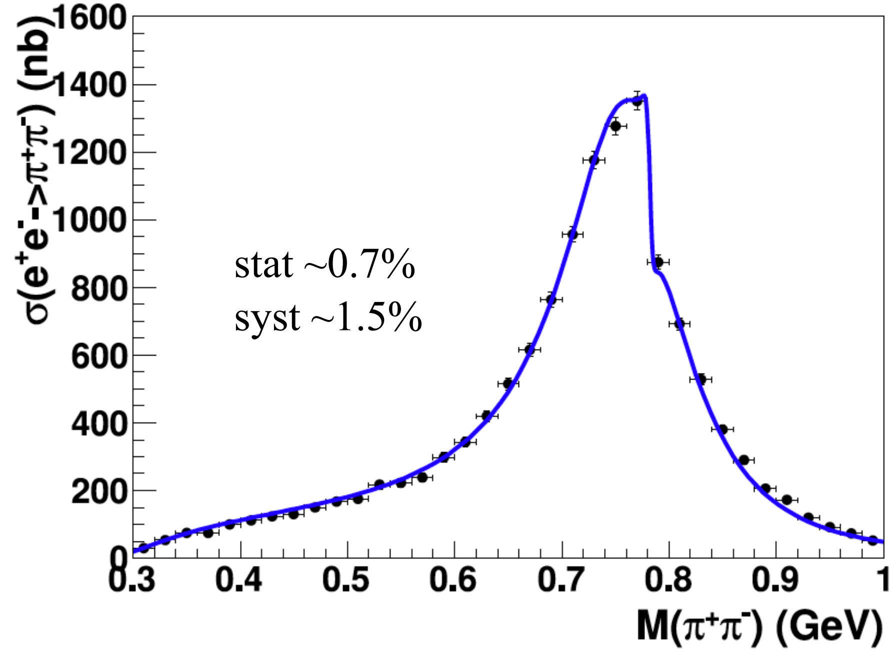
Both pion and muon pairs are measured and the ratio $\pi\pi(\gamma)/\mu\mu(\gamma)$ directly provides the $\pi\pi(\gamma)$ cross section

Measurements of 2π Channel: BESIII, CLEO-c

Original publication: Phys. Lett. B 753 (2016) 629
Erratum: Phys. Lett. B812 (2021) 135982 for updating the stat covariance matrix

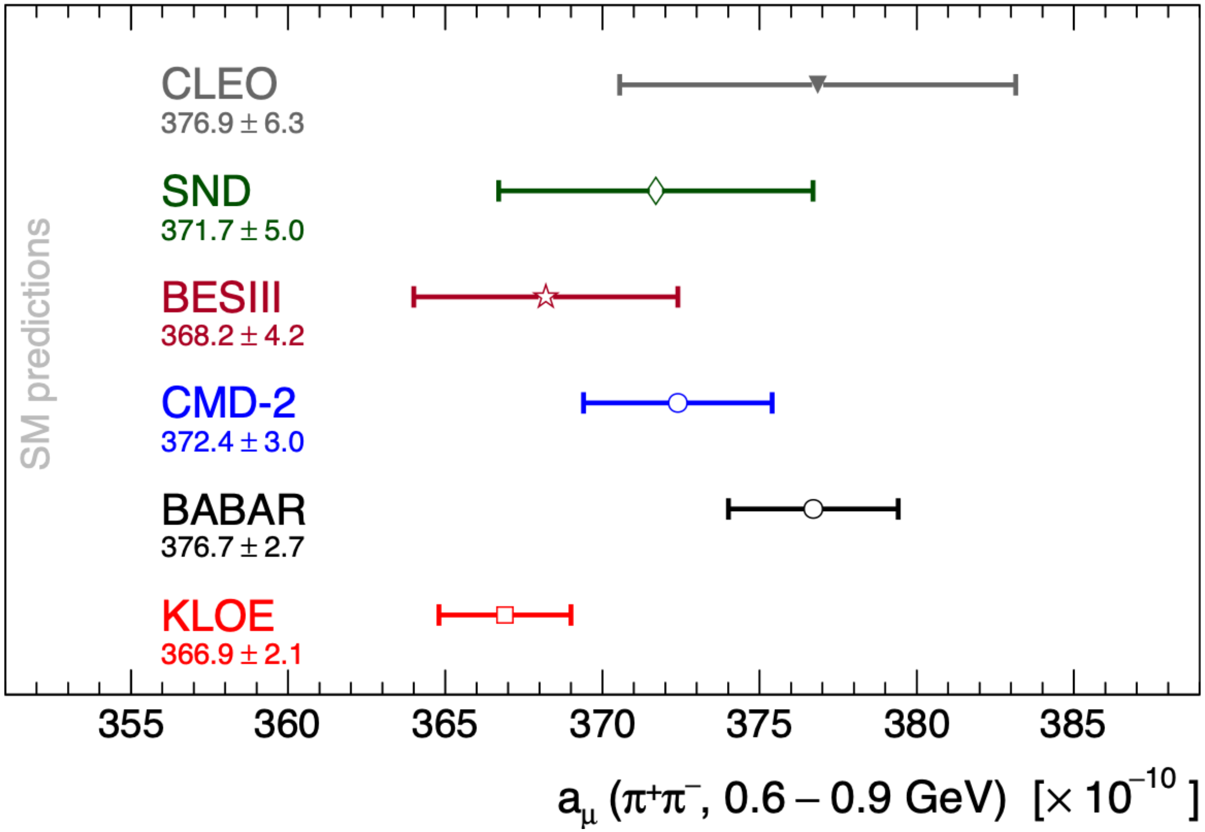


Phys. Rev. D 97, 032012 (2018)



Comparison

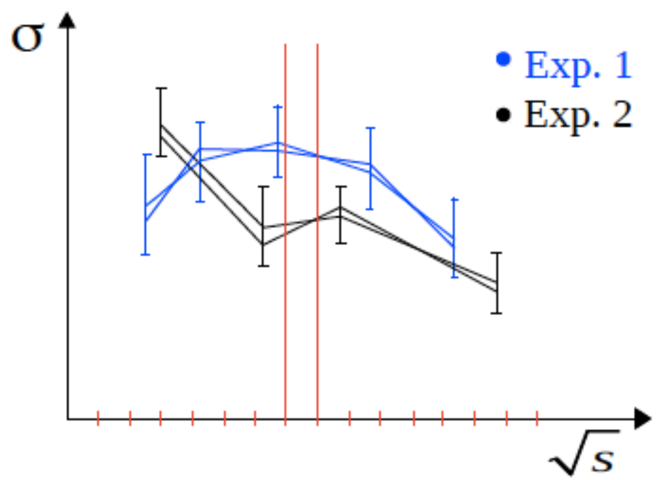
Figure from DHMZ, EPJC80 (2020) 241



BABAR and KLOE most precise but in clear discrepancy
Combination needs special treatment (see later)

Data Combination Using HVPTools*

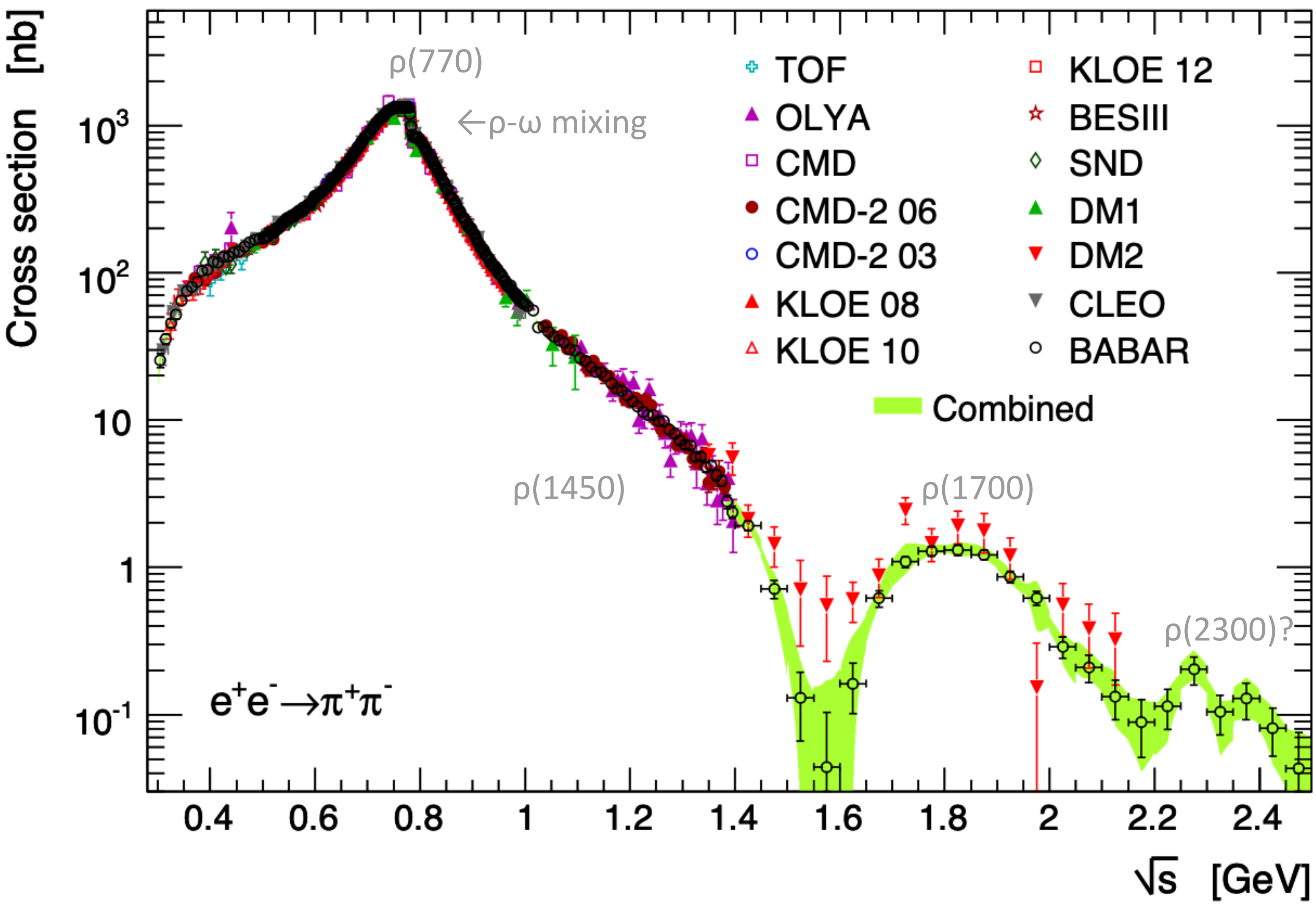
- ❐ Combine experimental spectra with arbitrary spacing/binning
- ❐ Linear/quadratic splines to interpolate between points/bins of each experiment
  - ➤ For binned measurements: preserve integral inside each bin
- ❐ Fluctuate data points taking into account correlations and redo the splines for each (pseudo)experiment
  - ➤ Each uncertainty fluctuated coherently for all points/bins that it impacts
  - ➤ Eigenvector decomposition for (stat & syst) covariance matrices
- ❐ Resulting combination shown in fine binning
  - ➤ Local error inflation following PDG prescription ($\sqrt{\chi^2/\mathrm{dof}}$) to take better into account data tension



* HVPTools, Davier et al., EPJC66 (2010) 127

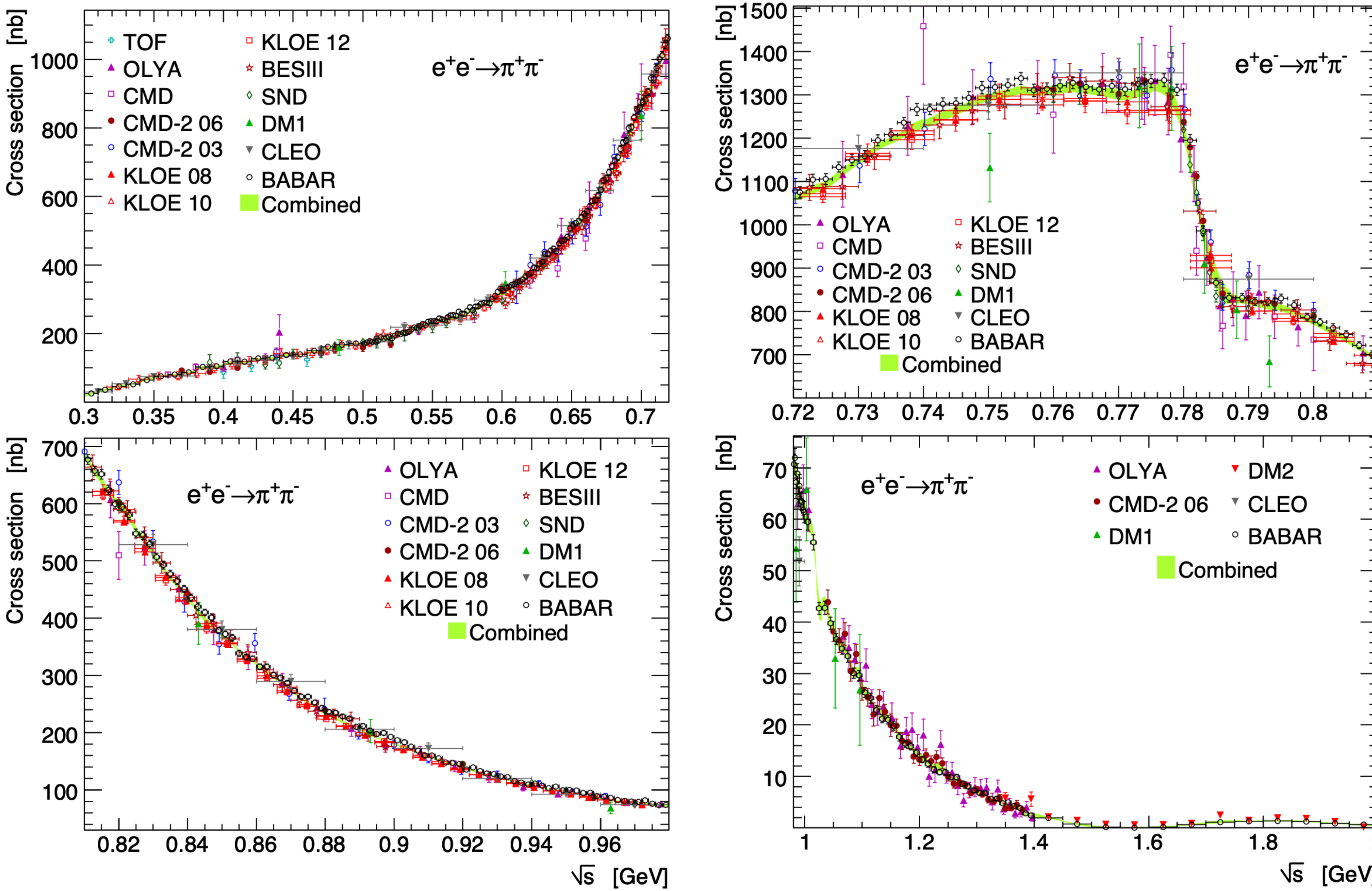
Combined 2π vs Individual Measurements

Figures from DHMZ, EPJC80 (2020) 241



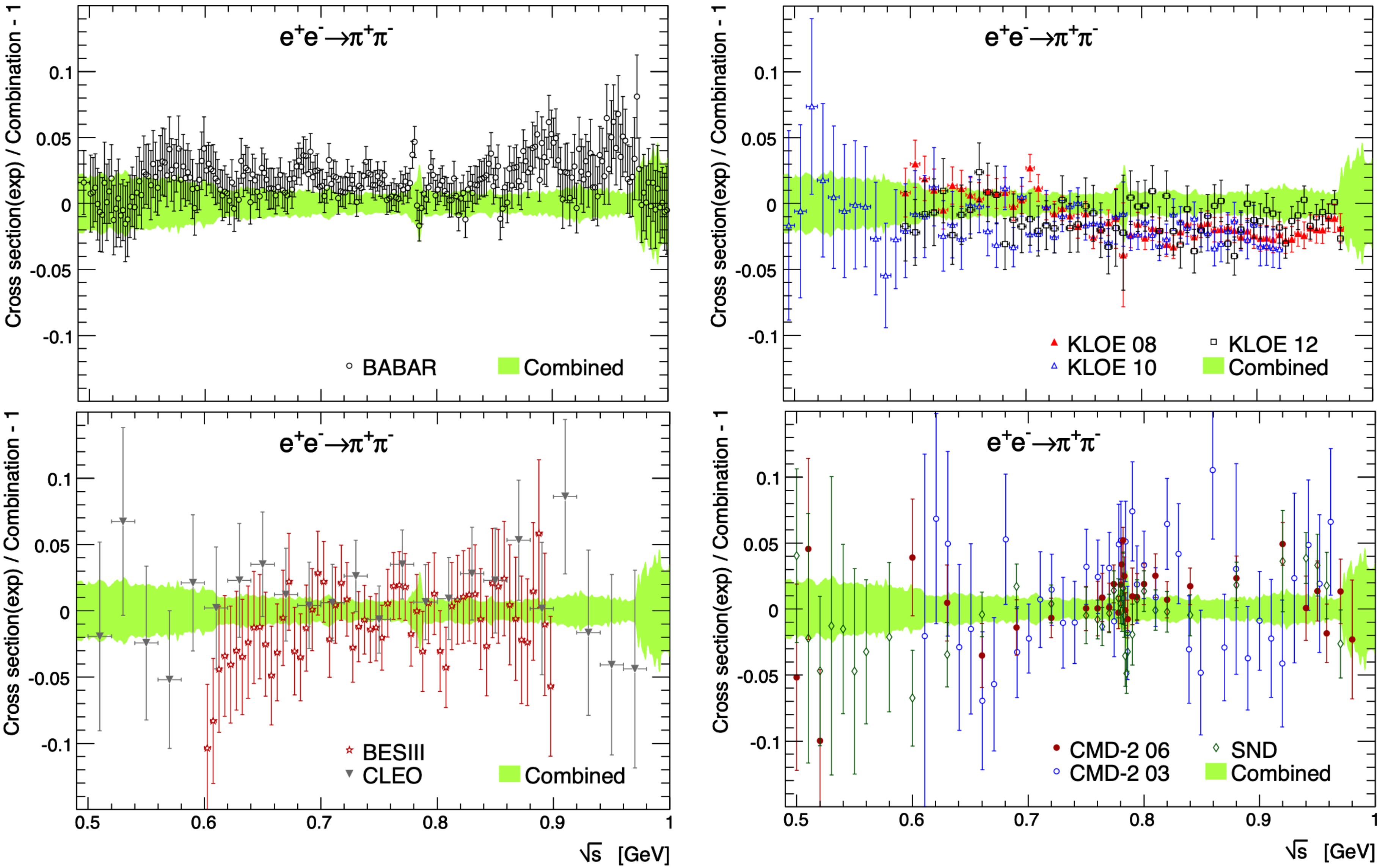
Combined 2π vs Individual Measurements

Figures from DHMZ, EPJC80 (2020) 241



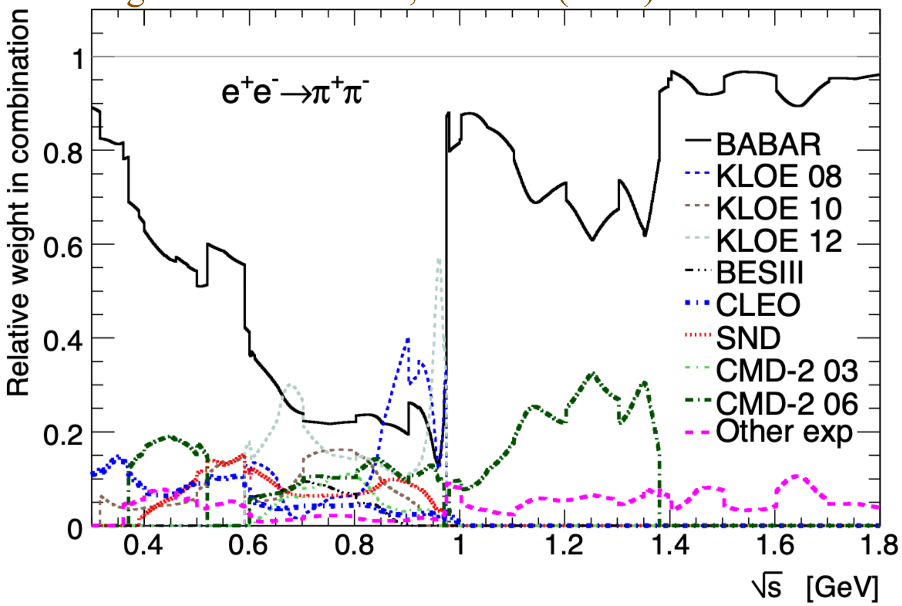
Combined 2π vs Individual Measurements

Figures from DHMZ, EPJC80 (2020) 241



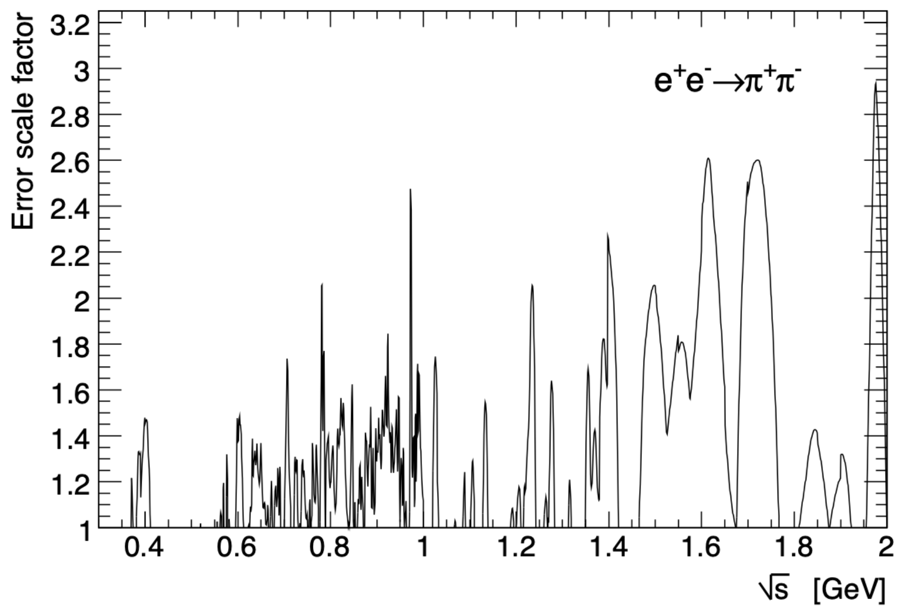
Relative Weights and Tension

Figures from DHMZ, EPJC80 (2020) 241



In the energy range of [0.6-0.9] GeV, BABAR and KLOE dominate, elsewhere BABAR has better precision

Inconsistency between measurements reflected by large local scale factor (SF) defined by $\sqrt{\chi^2/\mathrm{dof}}$ a la PDG
(~15% increase on δa_μ^{had})



Fit at Low Energy Based on Analyticity and Unitarity

Pion form factor F_π^0 extracted from $\pi^+\pi^-$ bare cross sections as in [1810.00007]

$$|F_\pi^0|^2 = |G(s) \times J(s)|^2$$

$$G(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}$$

$$J(s) = e^{1-\frac{\delta_1(s_0)}{\pi}} \left(1 - \frac{s}{s_0}\right)^{\left[1-\frac{\delta_1(s_0)}{\pi}\right]\frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi}\int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t(t-s)}}$$

$$\cot \delta_1(s) = \frac{\sqrt{s}}{2k^3}\left(m_\rho^2 - s\right)\left[\frac{2m_\pi^3}{m_\rho^2\sqrt{s}} + B_0 + B_1 \omega(s)\right]$$

$$k = \frac{\sqrt{s - 4m_\pi^2}}{2}$$

$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}$$

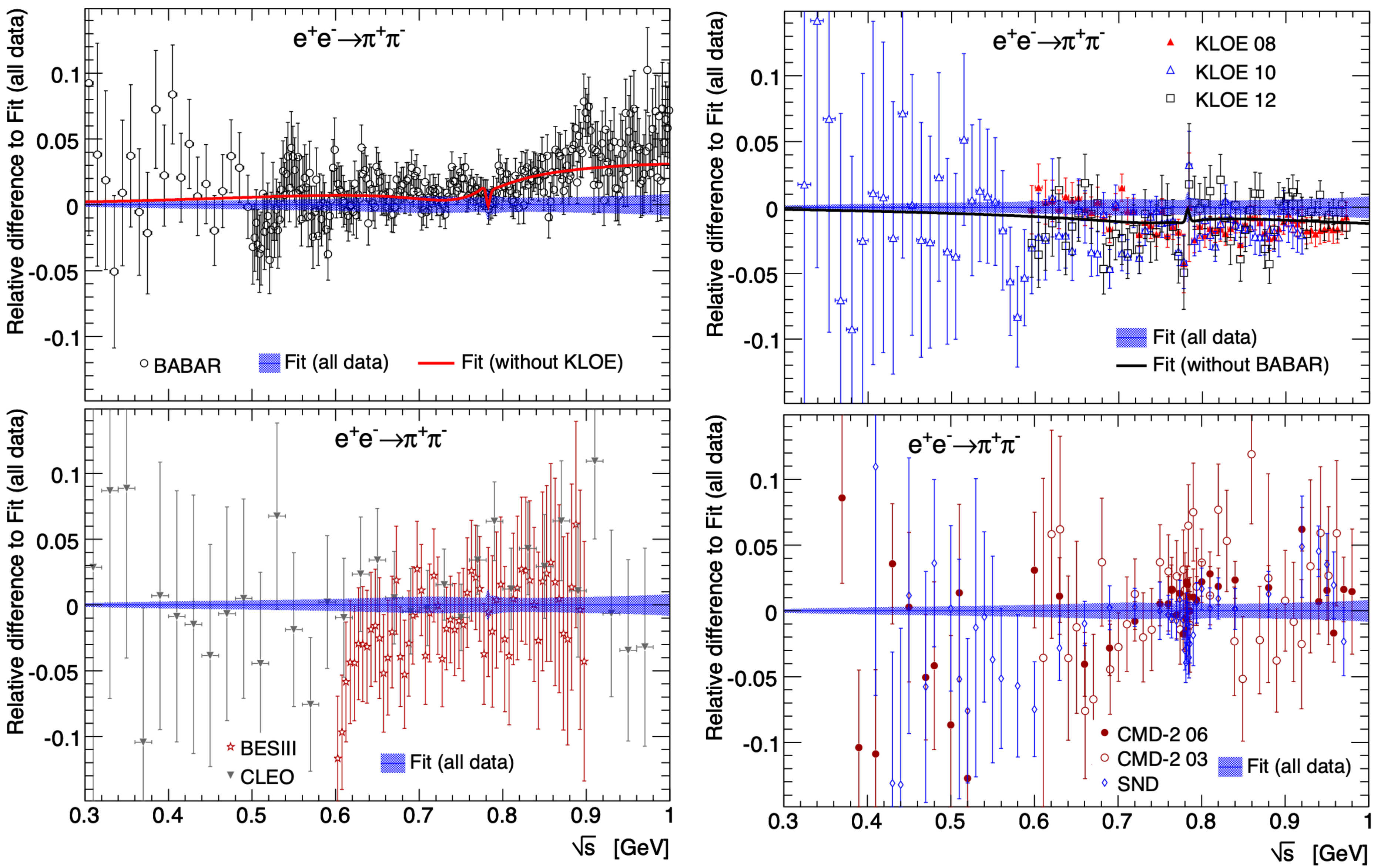
$$\sqrt{s_0} = 1.05 \text{ GeV}$$

$G(s)$ from [1611.09359]
$J(s)$ from [hep-ph/0106025, 0402285]
$\cot\delta_1(s)$ from [1102.2183]

Six free parameters to fit:
$\alpha_V, \kappa, m_\omega, m_\rho, B_0, B_1$
(Γ_ω fixed to PDG value)

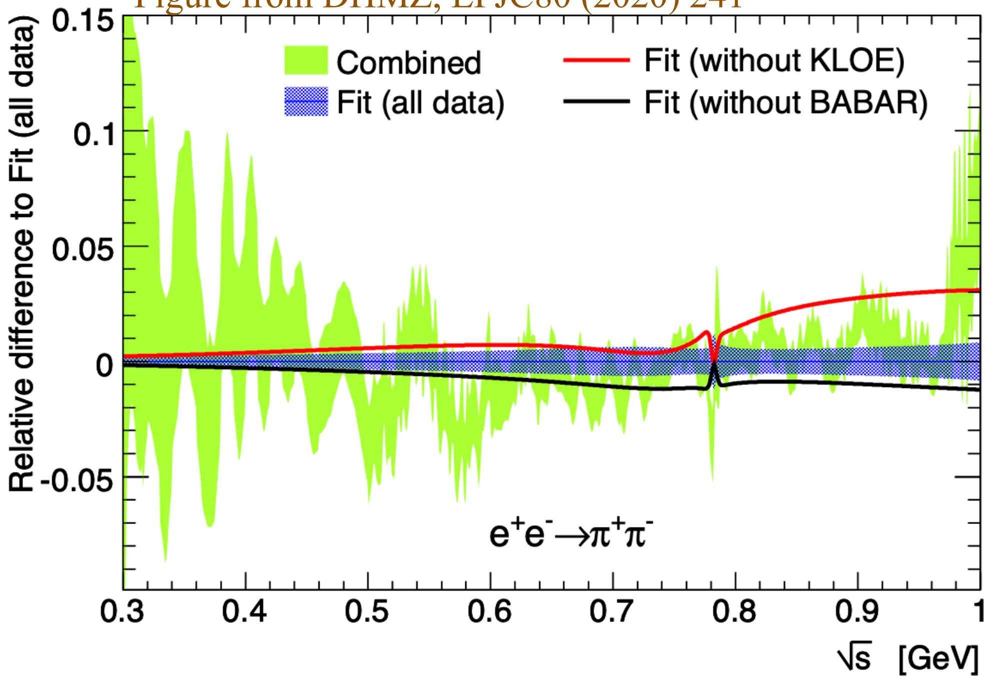
Fit Performed to 1 GeV, Results Used to 0.6 GeV

Figures from DHMZ, EPJC80 (2020) 241



Comparison Fit and Data Integration

Figure from DHMZ, EPJC80 (2020) 241



- ❒ Use fit only below 0.6 GeV for a_μ^{had} integral
  - ➤ Where the data are less precise and scarce
  - ➤ Less affected by potential uncertainties of inelastic contribution at high energy

$\sqrt{s}$ range [GeV]	a_μ^{had} $[10^{-10}]$ Fit	a_μ^{had} $[10^{-10}]$ Data Integration
0.3 - 0.6	$109.8 \pm 0.4_{exp} \pm 0.4_{para*}$	$109.6 \pm 1.0_{exp}$

⇒ The difference 0.2 ± 0.8 (correlation accounted for)

⇒ The fit improves the precision by ~2

* Parameter uncertainty corresponds to variations by removing the B_1 term in the phase shift formula and by varying $\sqrt{s_0}$ from 1.05 GeV to 1.3 GeV

Combined Results Fit [<0.6 GeV] + Data [0.6-1.8 GeV]

Take into account the correlation of 62% (based on pseudo-data samples) of the two regions

$\sqrt{s}$ range [GeV]	a_μ^{had} $[10^{-10}]$ All data	a_μ^{had} $[10^{-10}]$ All but BABAR	a_μ^{had} $[10^{-10}]$ All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{total}$	$505.0 \pm 2.1_{total}$	$510.6 \pm 2.2_{total}$

⇒ The difference "All but BABAR" and "All but KLOE" = 5.6
to be compared with 1.9 uncertainty with "All data"

- ➤ The local error inflation is not sufficient to amplify the uncertainty
- ➤ Global tension (normalisation/shape) not previously accounted for
- ➤ Potential underestimated uncertainty in at least one of the measurements?
- ➤ Other measurements not precise enough and are in agreement with BABAR or KLOE

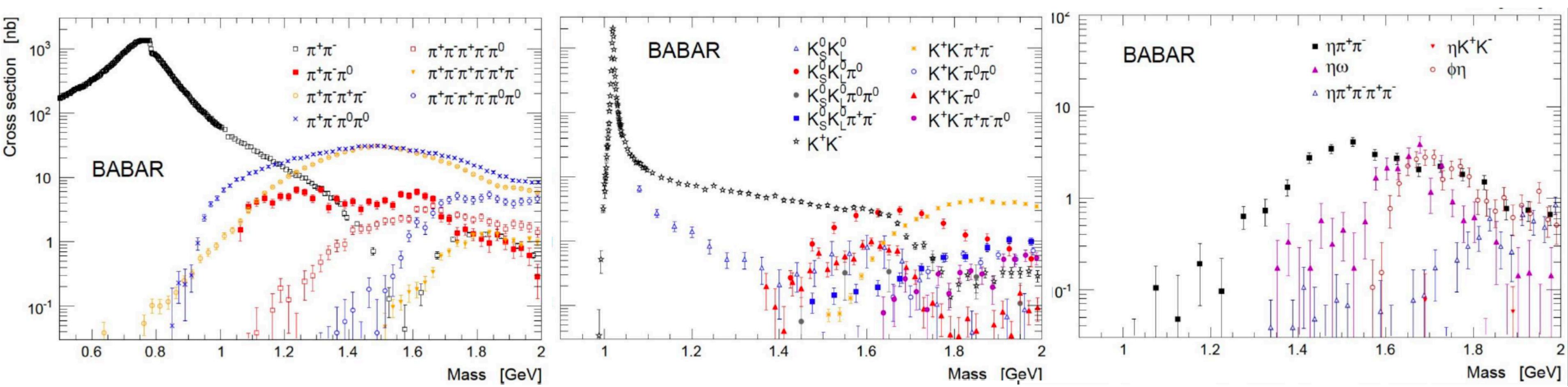
⇒ Given the fact we do not know which dataset is problematic, we decide to

- ➤ Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
- ➤ Take the mean value "All but BABAR" and "All but KLOE" as our central value

Other Channels e.g. Those Measured by BABAR

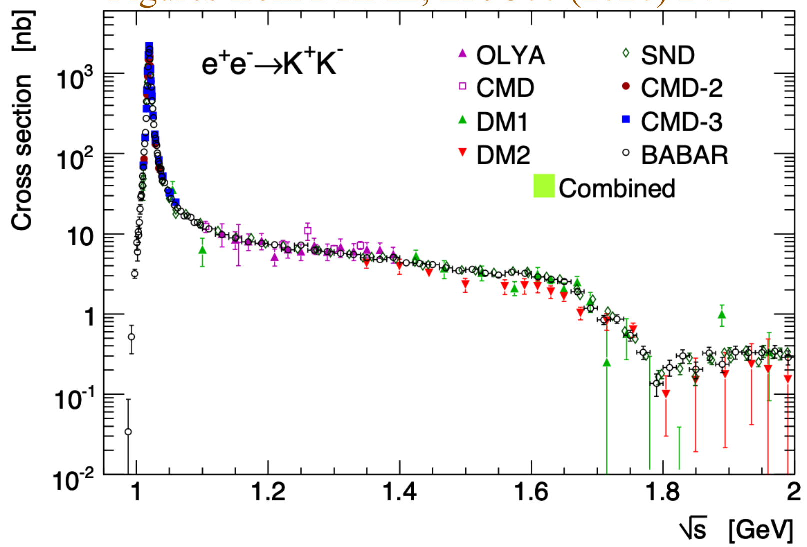
There are many exclusive channels (~ 40 processes) contributing to HVP

Here are some example measurements from BABAR (curtesy of M. Davier for the plots)

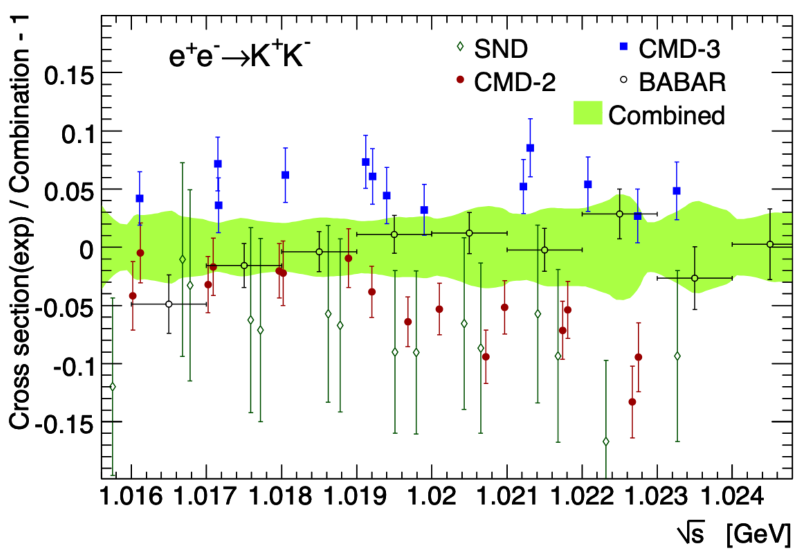


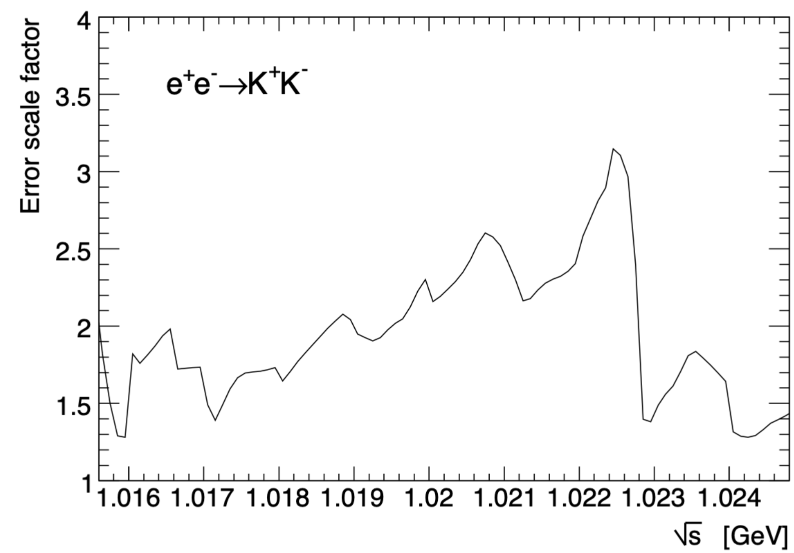
Tension in Other Channel (e.g. KK)

Figures from DHMZ, EPJC80 (2020) 241



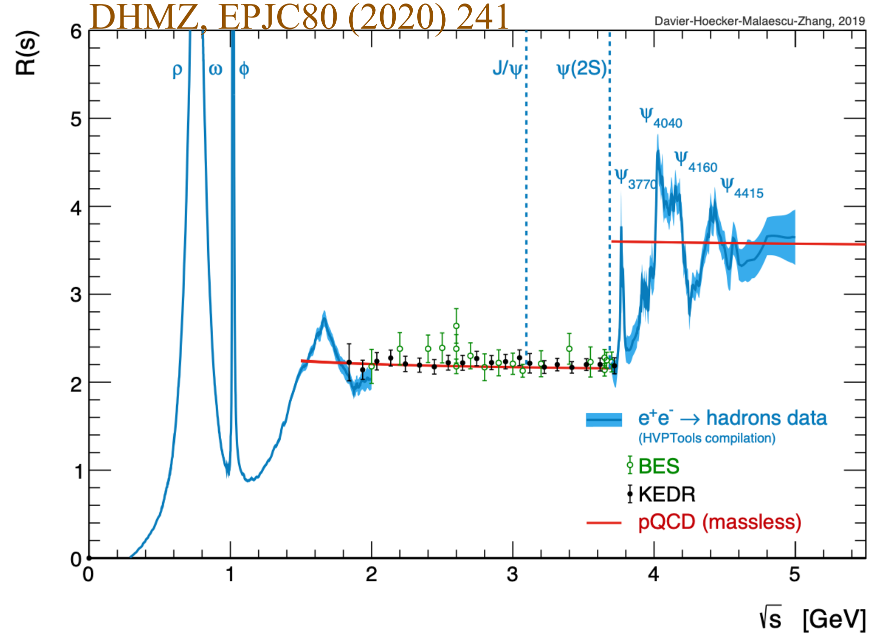
Several measurements with different precisions, CMD-2 and CMD-3 do not agree within the quoted uncertainties!
➜ Large error scaling factors
though this channel only contributes 3.3% to LO HVP and 1.2% to uncertainty-squared.



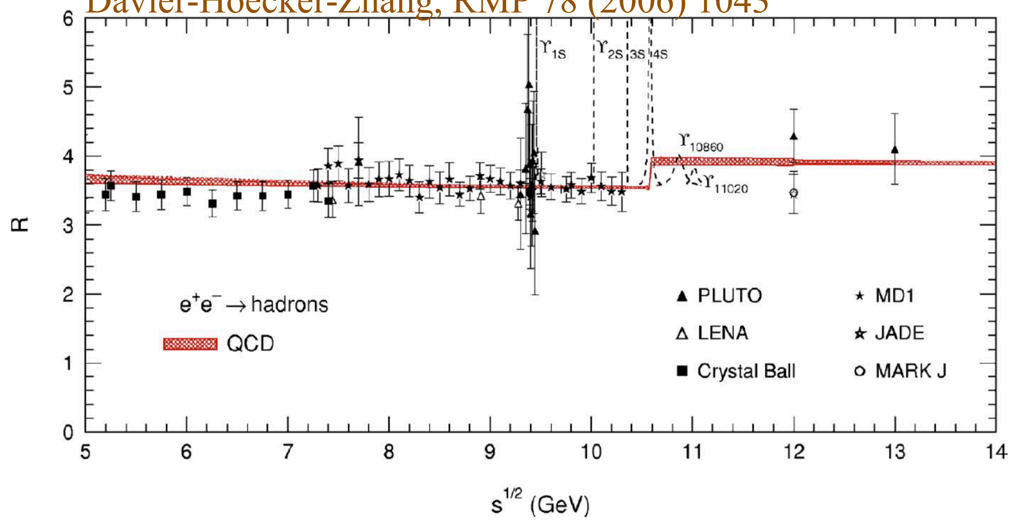


A Big Picture in Terms of R(s)

DHMZ, EPJC80 (2020) 241



Davier-Hoecker-Zhang, RMP 78 (2006) 1043



- [$\pi^0\gamma$-1.8GeV]
  - sum about 22→37 exclusive channels
  - estimate unmeasured channels using isospin relations (now < 0.1%)
- [1.8-3.7] GeV
  - good agreement between data and pQCD calculation → use 4-loop pQCD
  - J/ψ, ψ(2s): Breit-Wigner integral
- [3.7-5] GeV
  use data
- >5GeV
  use 4-loop pQCD calculation

For 1.8-2 GeV, the sum of exclusive channels gives 7.65 ± 0.31 to be compared with QCD 8.30 ± 0.09

The full difference of 0.65 is quoted as the "dual" uncertainty on next page

Overall Results

Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$	$\Delta\alpha(m_Z^2)[10^{-4}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07	7.09 ± 0.08
$\psi(2S)$ (BW integral)	1.57 ± 0.03	2.50 ± 0.04
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
R_{QCD} $[1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$	$24.27 \pm 0.18 \pm 0.28_{\text{dual}}$
R_{QCD} $[5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.17
R_{QCD} $[9.3 - 12.0\text{ GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
R_{QCD} $[12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.12
R_{QCD} $[> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
R_{QCD} $[> 40.0\text{ GeV}]_{t}$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$\boxed{693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}}$	$275.42 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_\psi \pm 0.55_{\text{QCD}}$

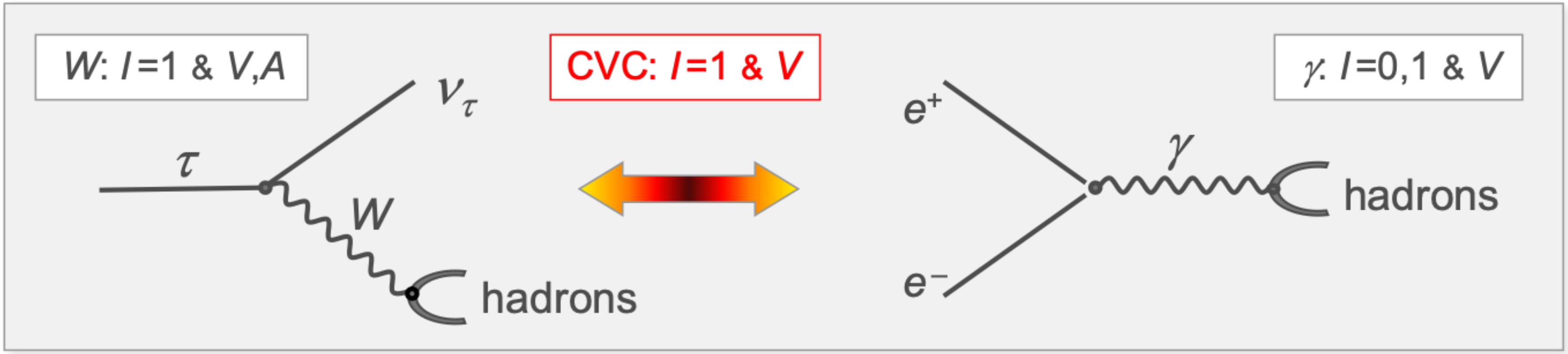
More than 30 exclusive channels (<1.8 GeV) evaluated

Estimation for missing modes based on isospin constraints becomes negligible (0.016%)

Table taken from DHMZ, EPJC80 (2020) 241

An Alternative Way Used to Evaluate HVP

Proposed by Alemany-Davier-Hoecker, EPJC 2 (1998) 123



Hadronic physics factorises in Spectral Functions:
Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions

Fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{BR\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{BR\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \; \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \; \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

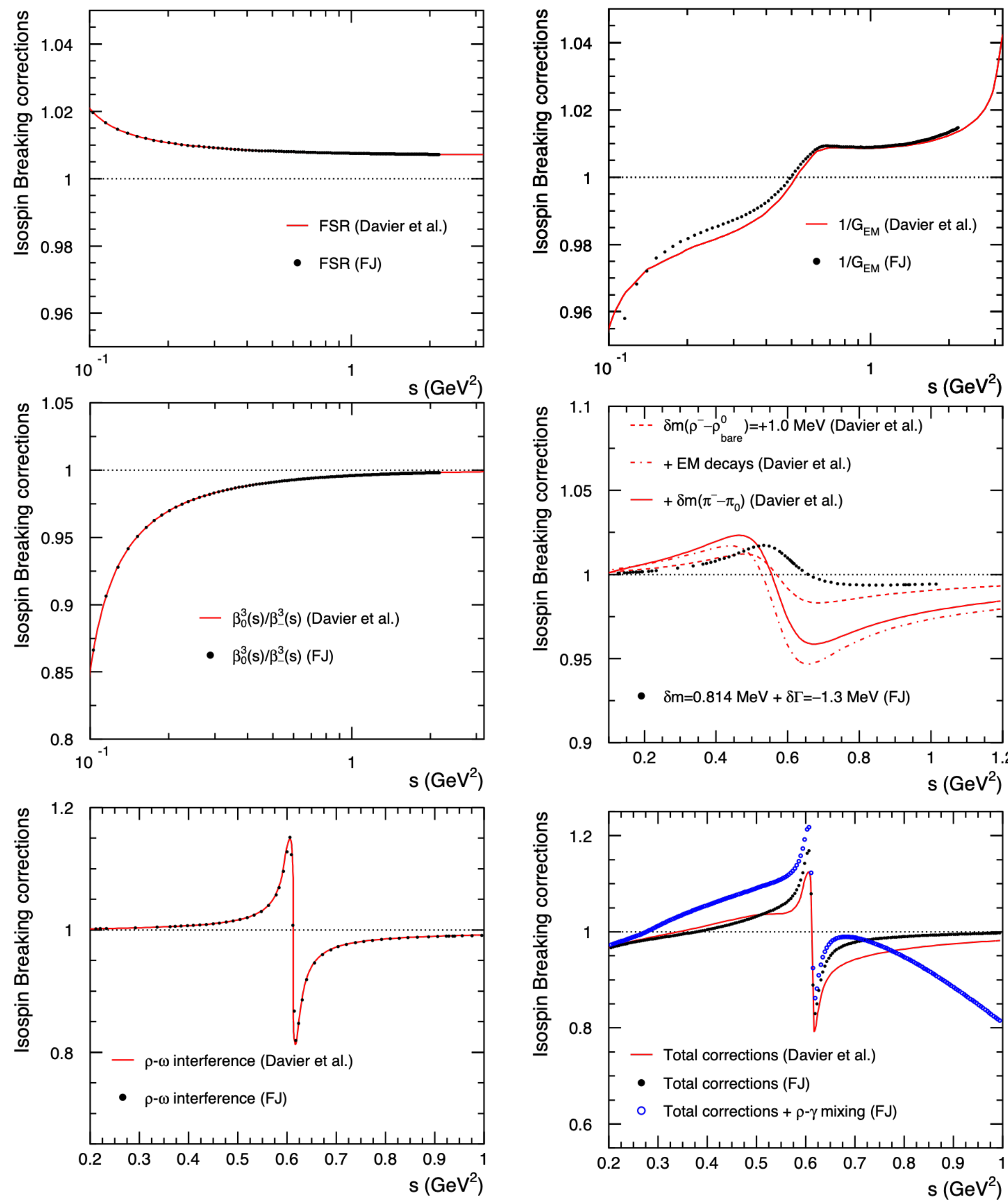
Branching fractions　Mass spectrum　Kinematic factors (PS)

Known Isospin Breaking Corrections

Davier et al., EPJC66 (2010) 127

$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{X^-}}{\mathcal{B}_e}\frac{1}{N_X}\frac{dN_X}{ds} \times \left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{\rm IB}(s)}{S_{\rm EW}},$$

$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2.$$



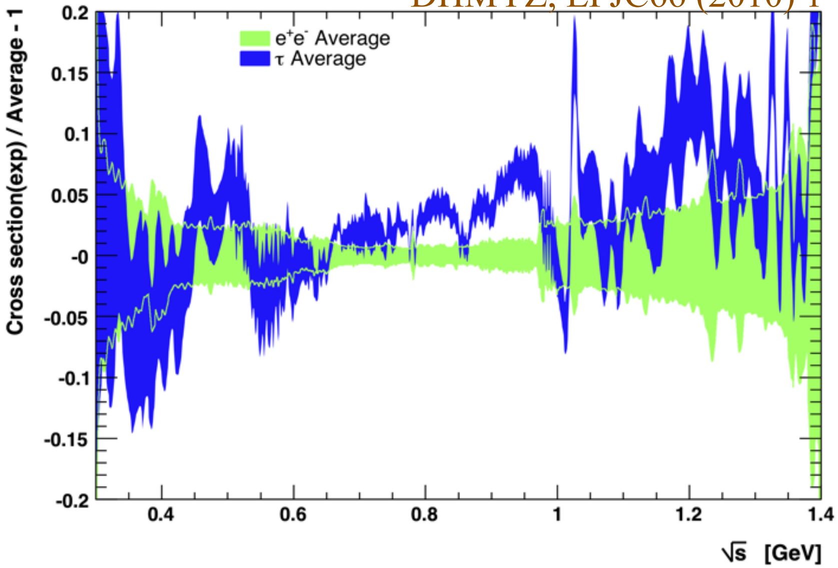
Good agreement between Davier et al. and FJ for most of the isospin breaking components

Figure 19 from WP20
Studies initiated in Davier et al., EPJC66 (2010) 127
Gabriel López Castro & Genaro Toledo Sánchez were our collaborators for this publication

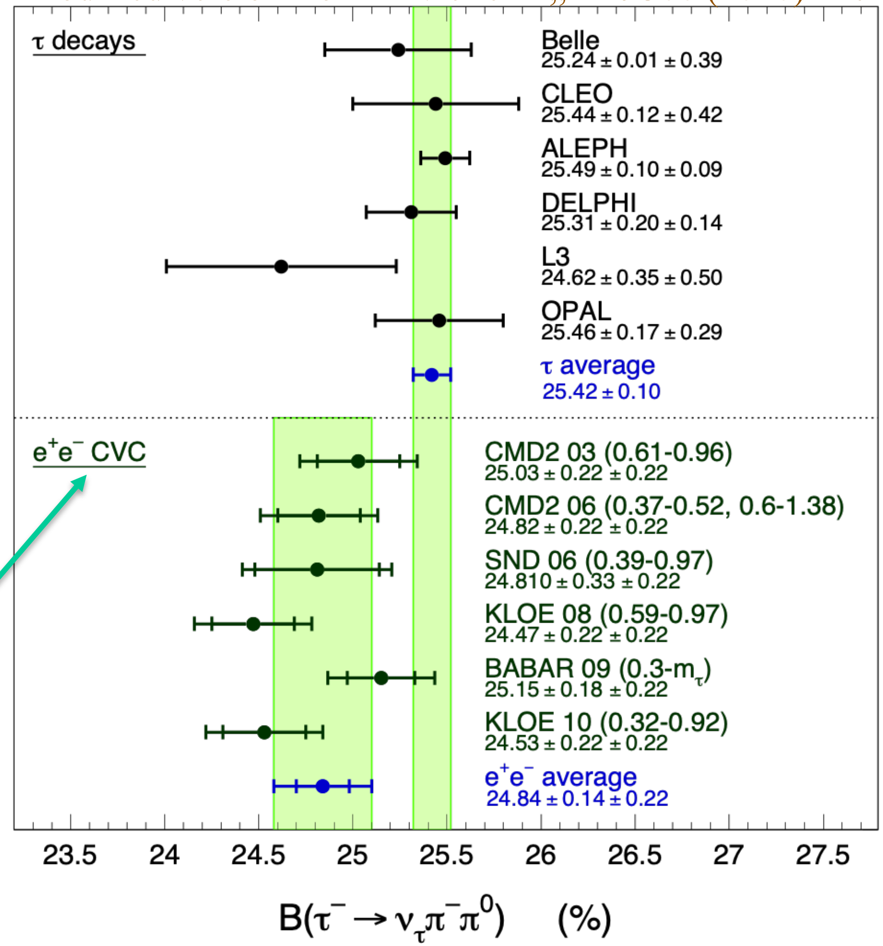
Open Issue in the 2π Channel

Take into account all known isospin breaking corrections except for the $\rho-\gamma$ mixing correction

DHMYZ, EPJC66 (2010) 1



Modified version from Davier et al,, EPJC66 (2010) 127



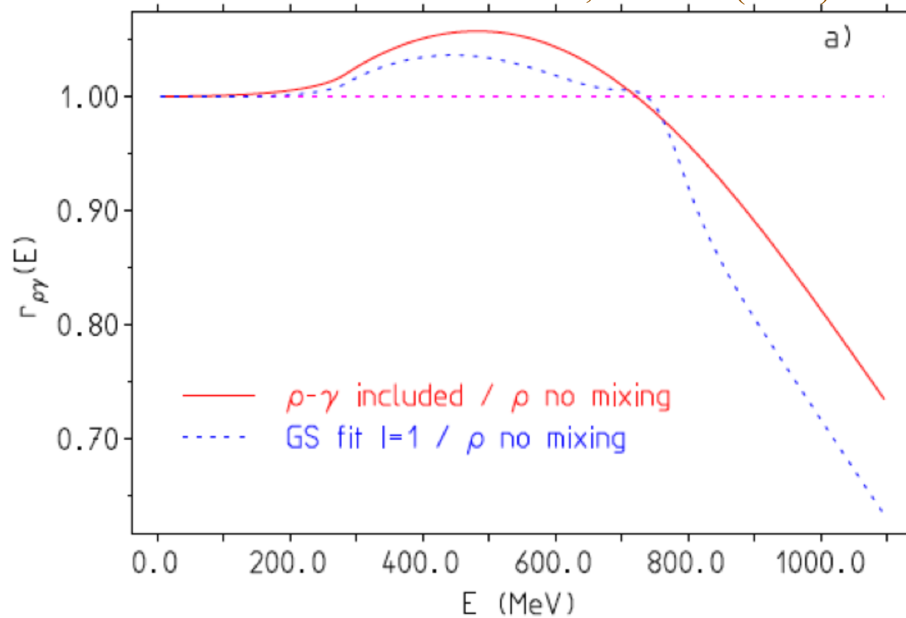
$$\mathcal{B}_X^{\rm CVC} = \frac{3}{2}\frac{\mathcal{B}_e |V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\rm min}}^{m_\tau^2} ds\, s\, \sigma_{X^0}^I \left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)$$

Clear difference in shape and in BR between e^+e^- and τ average

Gabriel López Castro & Genaro Toledo Sánchez were our collaborators for this publication

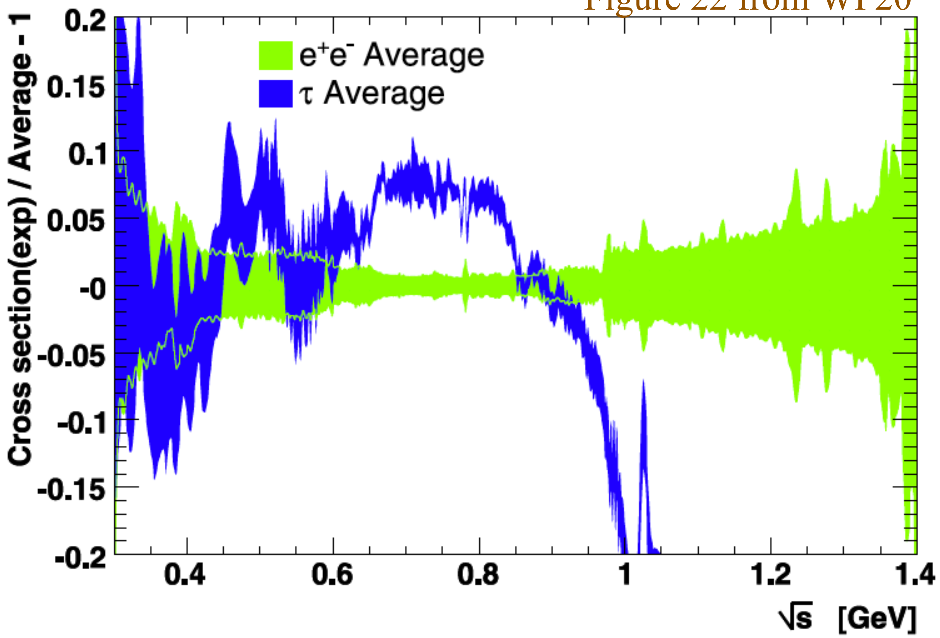
Additional EFT Based ρ–γ Mixing Correction

JS, EPJC71 (2011) 1632



Jegerlehner and Szafron proposed to use the missing ρ–γ mixing in τ data to explain the remaining e^+e^- and τ difference

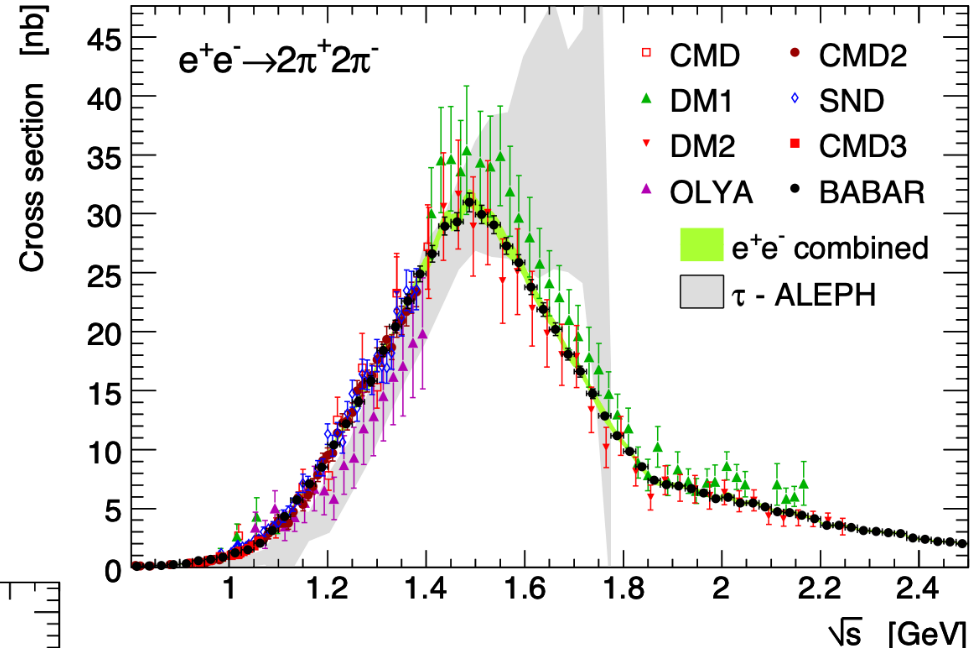
Figure 22 from WP20



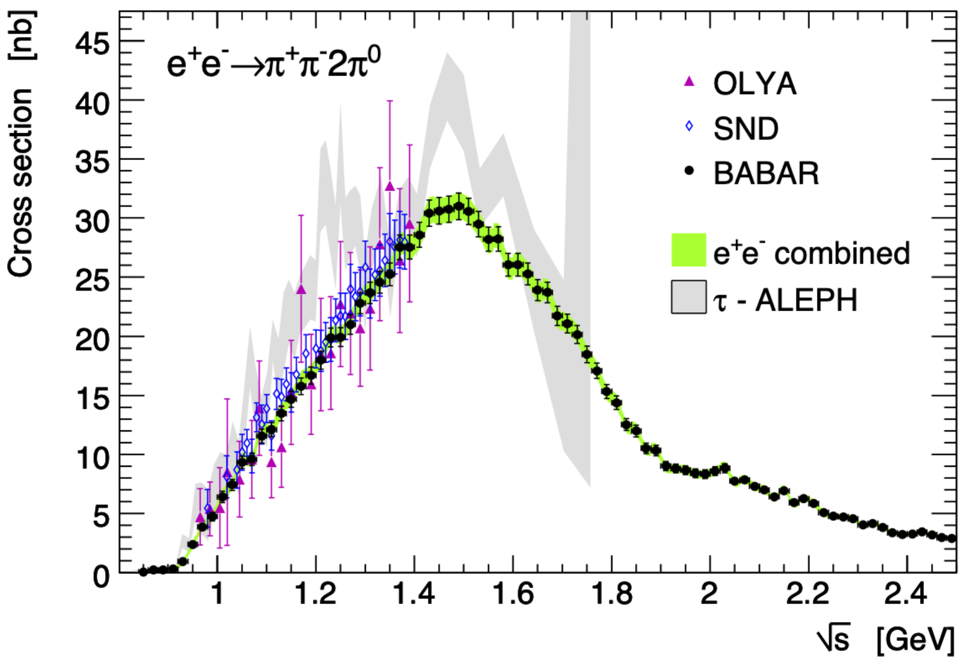
Applying the ρ–γ mixing correction makes the e^+e^- and τ difference worse in some of the mass range

Comparison of 4π Channels

The precision of e^+e^- data increased over time, a factor of 1.7−2.3 between 2011 and 2017



Figures from DHMZ, *EPJC* 77 (2017) 827



In comparison, τ data are now less precise

We no longer pursue the τ data-based evaluation due to the open issue in the 2π + less precise τ data

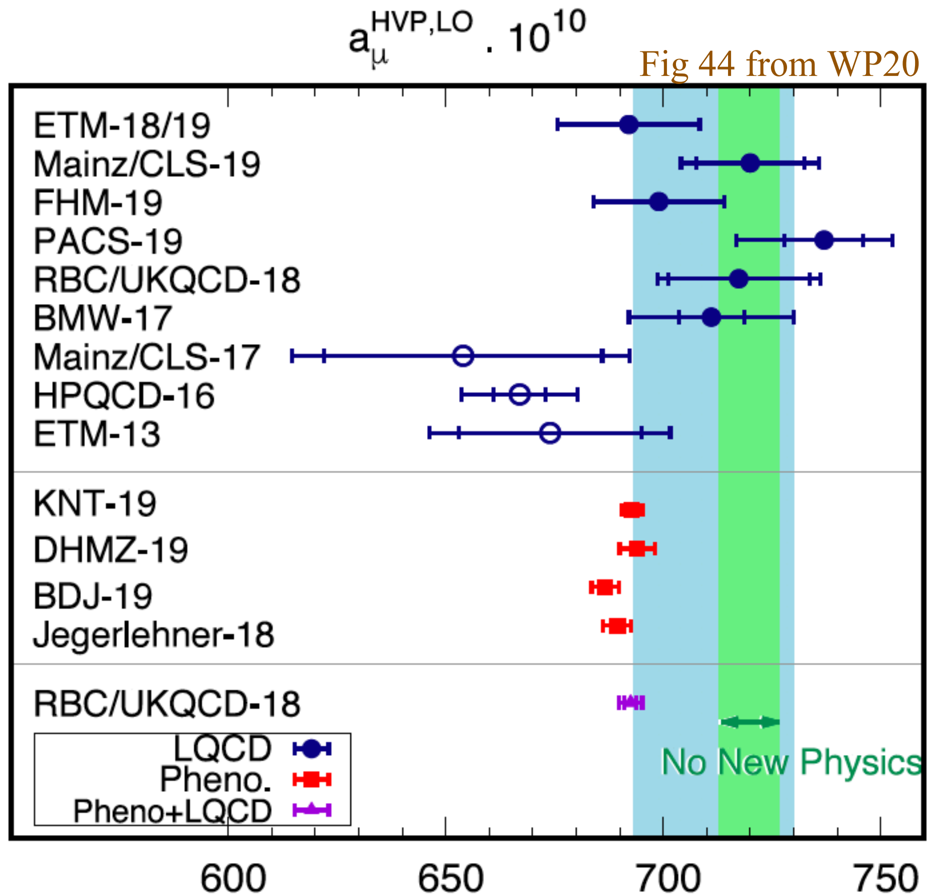
Alternative LO HVP Prediction from Lattice QCD

Lattice QCD allows to directly compute the real part of the two-point correlation function without invoking the resonances occurring on the imaginary axis

Several groups provided predictions, however their uncertainties are still large so that they were not used in providing the LO HVP prediction in the WP20

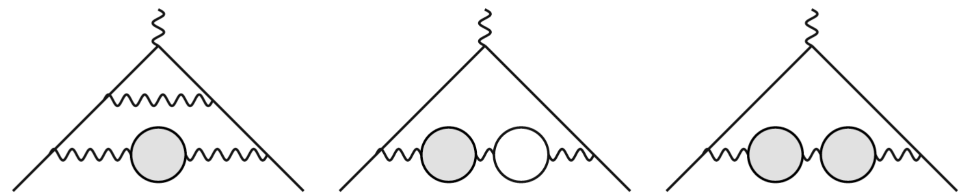
Recently, BMW has provided a syst dominating prediction varying
from v1: 712.4 (4.5) × 10^{-10}
to v3: 707.5 (5.5) × 10^{-10}
reaching 0.8% close to 0.6% (dispersive).

This prediction needs to be confirmed by other lattice groups with comparable precision

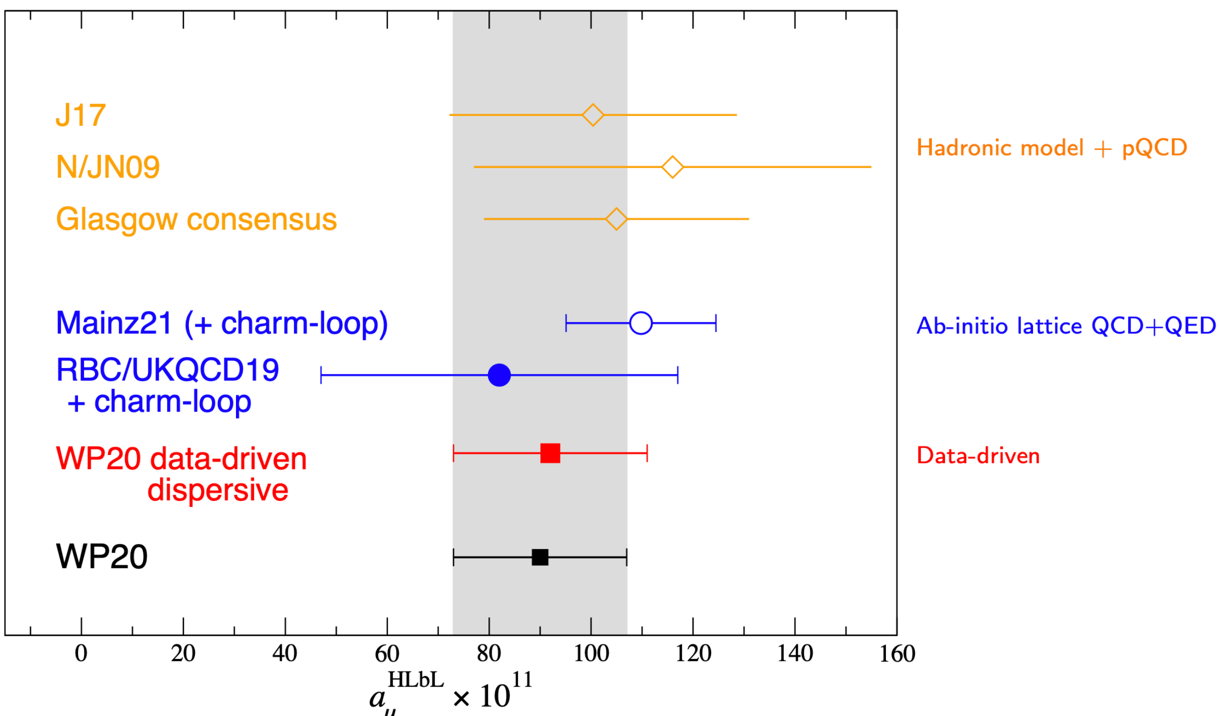


High Order HVP + HLbL Predictions

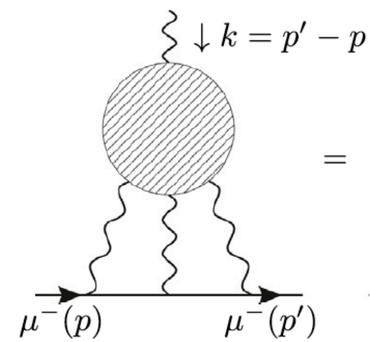
$$a_\mu^{\text{HVP, NLO}} = -9.83(7) \times 10^{-10}$$
$$a_\mu^{\text{HVP, NNLO}} = 1.24(1) \times 10^{-10}$$

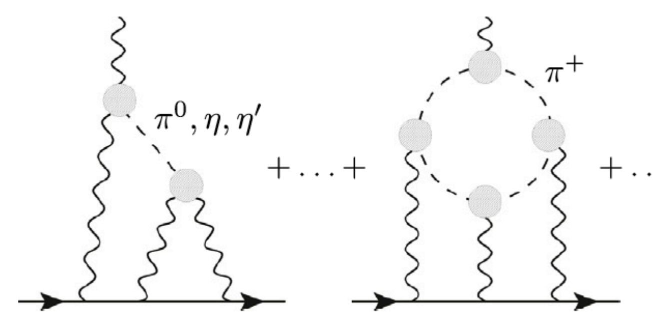


Status of hadronic light-by-light contribution



Values, graphs from WP20
Left figure from C. Lehner's CERN seminar talk





$$a_\mu^{\text{HLbL}}(\text{phenomenology + lattice QCD}) + a_\mu^{\text{HLbL, NLO}} = 92(18) \times 10^{-11}$$

Summary and Perspectives

- The current precision of the dispersive prediction ~ direct measurement
- The latter will be improved in the next years by a factor ~ 4
- On the theory side, the situation is less clear
  - We do expect more measurements (e.g. in 2π) from BABAR, Belle 2, BESIII, CMD-3, …
  - However the measurements have to be very precise in order to resolve the BABAR-KLOE discrepancy which prevented us from improving further the precision in the data combination
  - If the BMW prediction is confirmed, we also need to understand the difference between the dispersive and lattice predictions

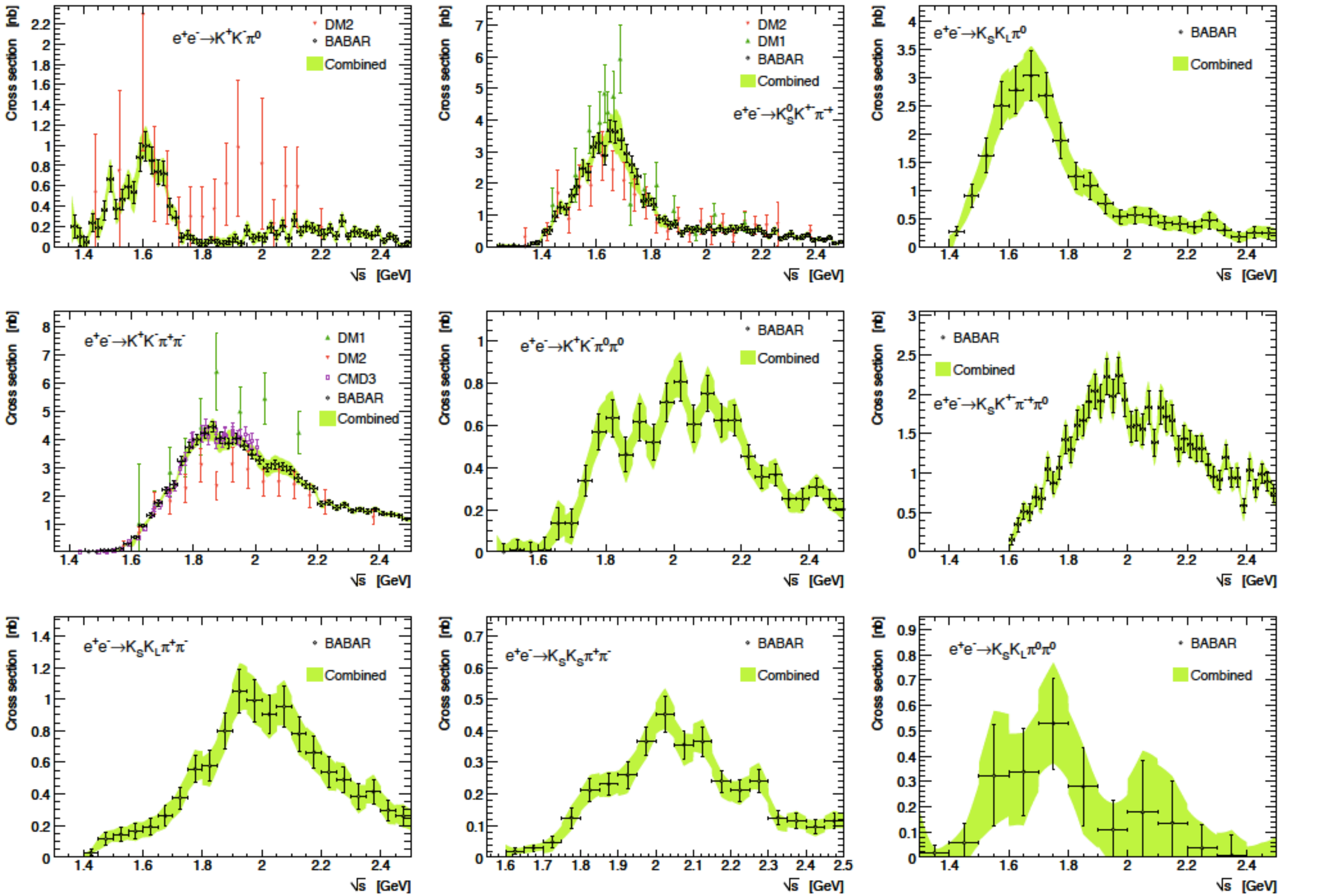
List of publications

1. ADH 1998, Eur.Phys.J.C 2 (1998) 123 [330 citations*]
2. DH 1998, Phys.Lett.B 419 (1998) 419 [219 citations]
3. DH 1998, Phys.Lett.B 435 (1998) 427 [292 citations]
4. DEHZ 2003, Eur.Phys.J.C 27 (2003) 497 [394 citations]
5. DEHZ 2003, Eur.Phys.J.C 31 (2003) 503 [430 citations]
6. DHMZ+ 2010, Eur.Phys.J.C 66 (2010) 127 [157 citations]
7. DHMYZ 2010, Eur.Phys.J.C 66 (2010) 1 [209 citations]
8. DHMZ 2011, Eur.Phys.J.C 71 (2011) 1515 [866 citations]
9. DHMZ 2017, Eur.Phys.J.C 77 (2017) 827 [259 citations]
10. DHMZ 2019, Eur.Phys.J.C 80 (2020) 241 [169 citations]
11. Theory initiative WP 2020, Phys.Rept. 887 (2020) 1 [171 citations]

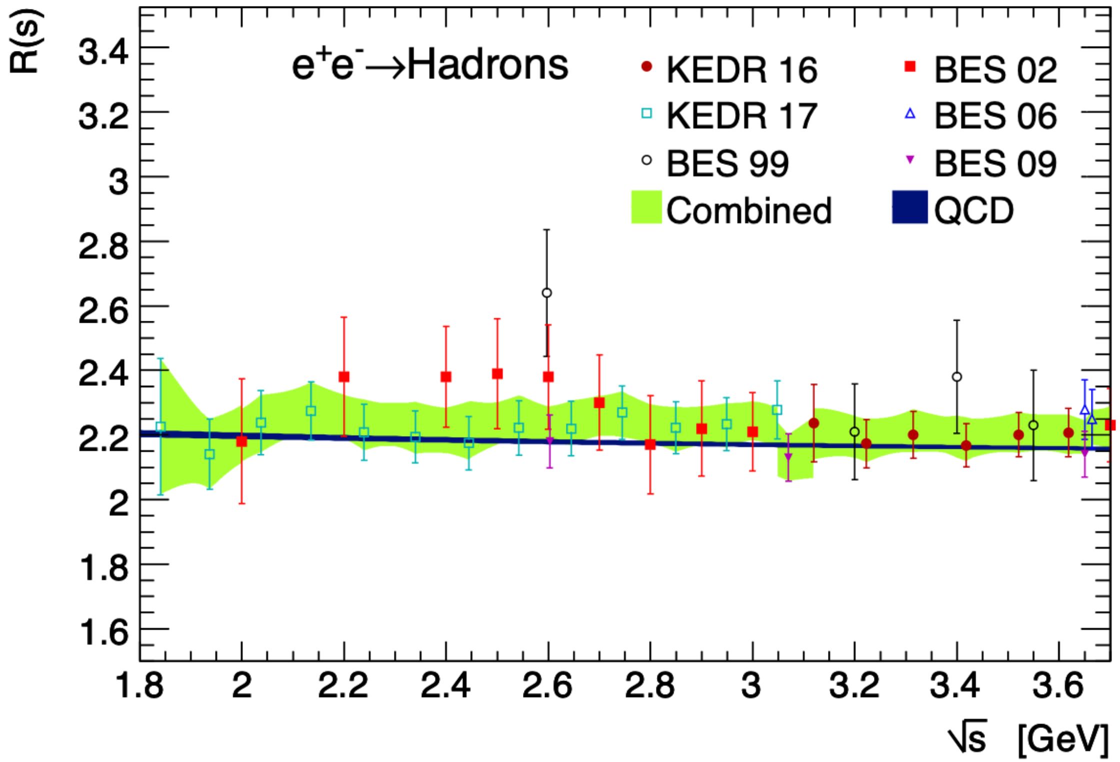
→Total number of citations: ~3500

* Status of April 9, 2021

KKbar+π's Channels



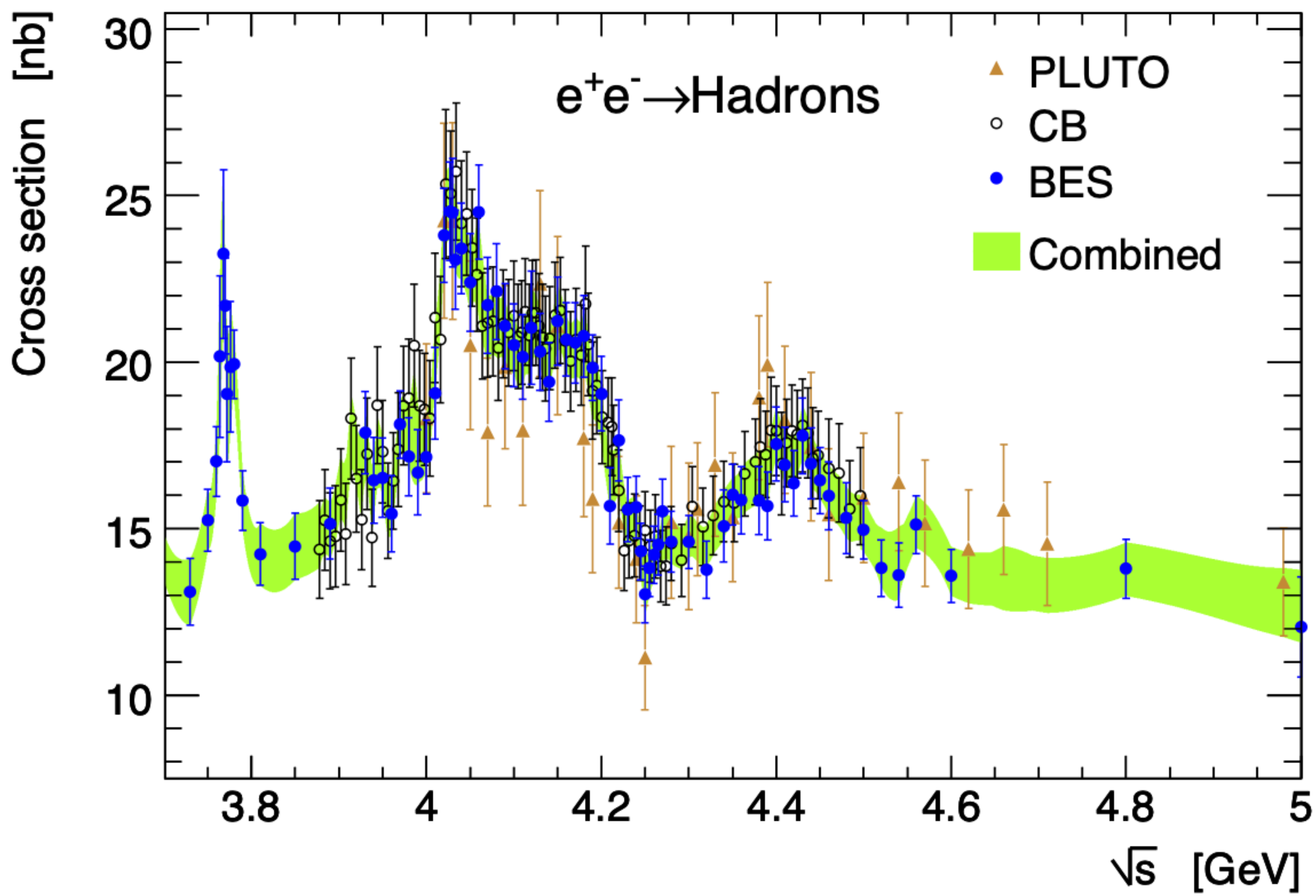
Contributions in the Region 1.8-3.7 GeV



Energy range [GeV]	1.8 - 2.0	2.0 - 3.7
Data	7.71 ± 0.32	25.82 ± 0.61
pQCD	8.30 ± 0.09	25.15 ± 0.19
Difference	0.59 → dual	agree < 1σ

pQCD evaluated from 4 loops + O(α_s^2) quark mass corrections
Uncertainties: α_s, truncation, FOPT/CIPT, m_q

Contributions from Charm Resonance Region



$7.29 \pm 0.05 \pm 0.30 \pm 0.00 \Rightarrow 1.05\%$ of $a_\mu^{\text{had, LO}}$

stat sys cor

Efforts on the prediction side

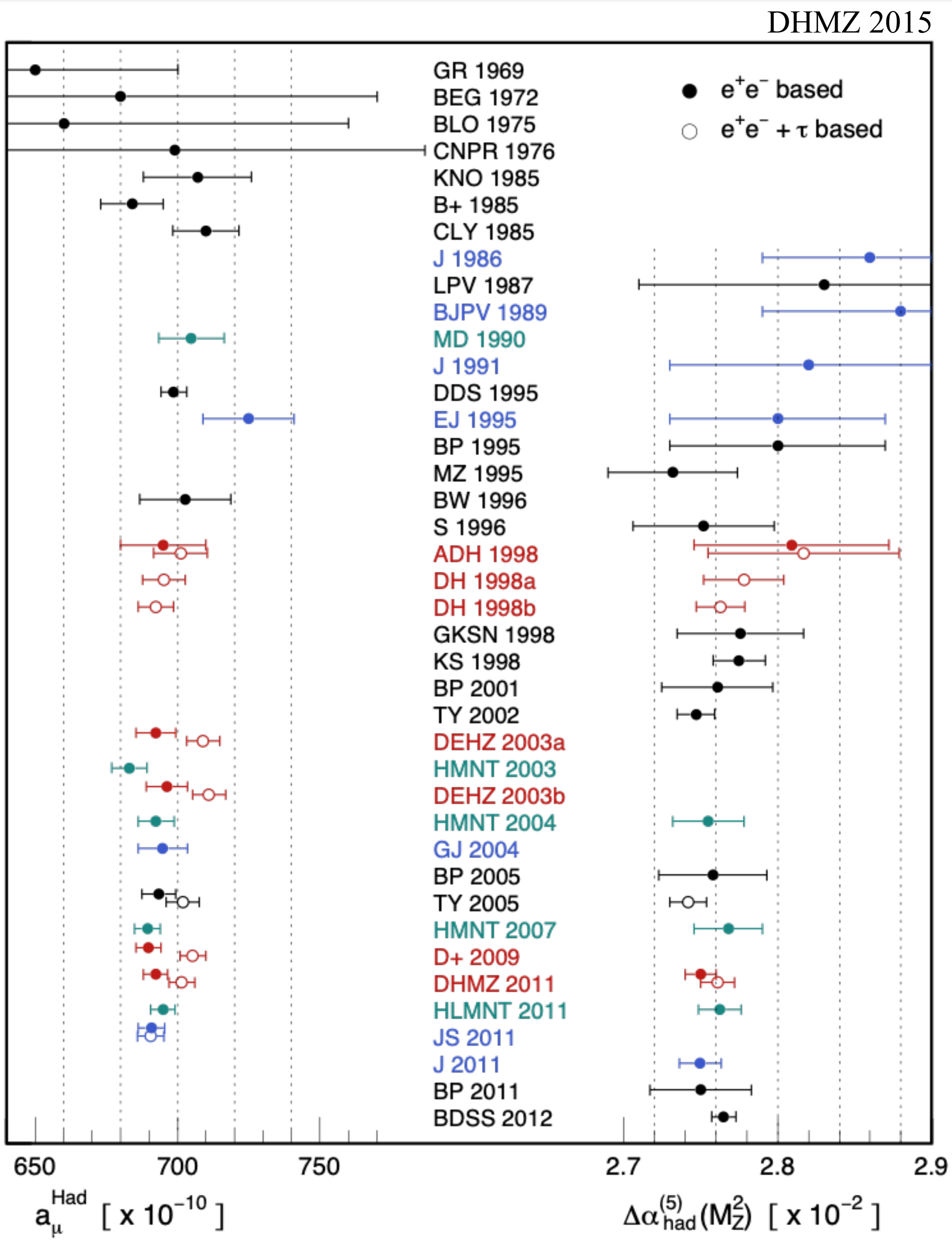


Fig. 3 prepared by Davier, Hoecker, Malaescu, Zhang for "Standard theory essays in the 60th anniversary of CERN"

Our efforts started by Michel with a first publication with Andreas and Richard in 1998 (ADH 1998)

Since then ZZ(02), Bogdan(09) and others joined the efforts

In total we have published 10 highly cited articles (link)

Our prediction has been one main reference used for comparing with the direct measurement

The precision of the HVP prediction is data-driven

It depends on

- the precision of e^+e^- annihilation (& tau) data
- state of the art techniques (HVPTools) for data interpolation, combination and error correlation treatment

Excellent Perspectives

FNAL run-1 ~6% of final data samples

The current precision is 0.35 ppm

Run 2/3 would improve by a factor ~2

Final expected precision ~0.14 ppm

J-PARC would provide a measurement with complete different systematic uncertainties

CMD-3, SND, BES-III, BABAR would provide more precision data

In particular BABAR's ongoing analysis has × 7 stat wrt the previous measurement and expects improved syst uncertainties

Ongoing activity between DHMZ and BMW to understand the difference in data-driven and Lattice predictions