CAUSAL REPRESENTATION OF PHYSICAL OBSERVABLES THROUGH THE LOOP-TREE DUALITY

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OUTLINE

- Introduction and Motivation
- Loop-Tree Duality theorem and the Four-Dimensional Unsubtraction method
- Integral representations of multi-loop integrals
- Conclusions and Outlook

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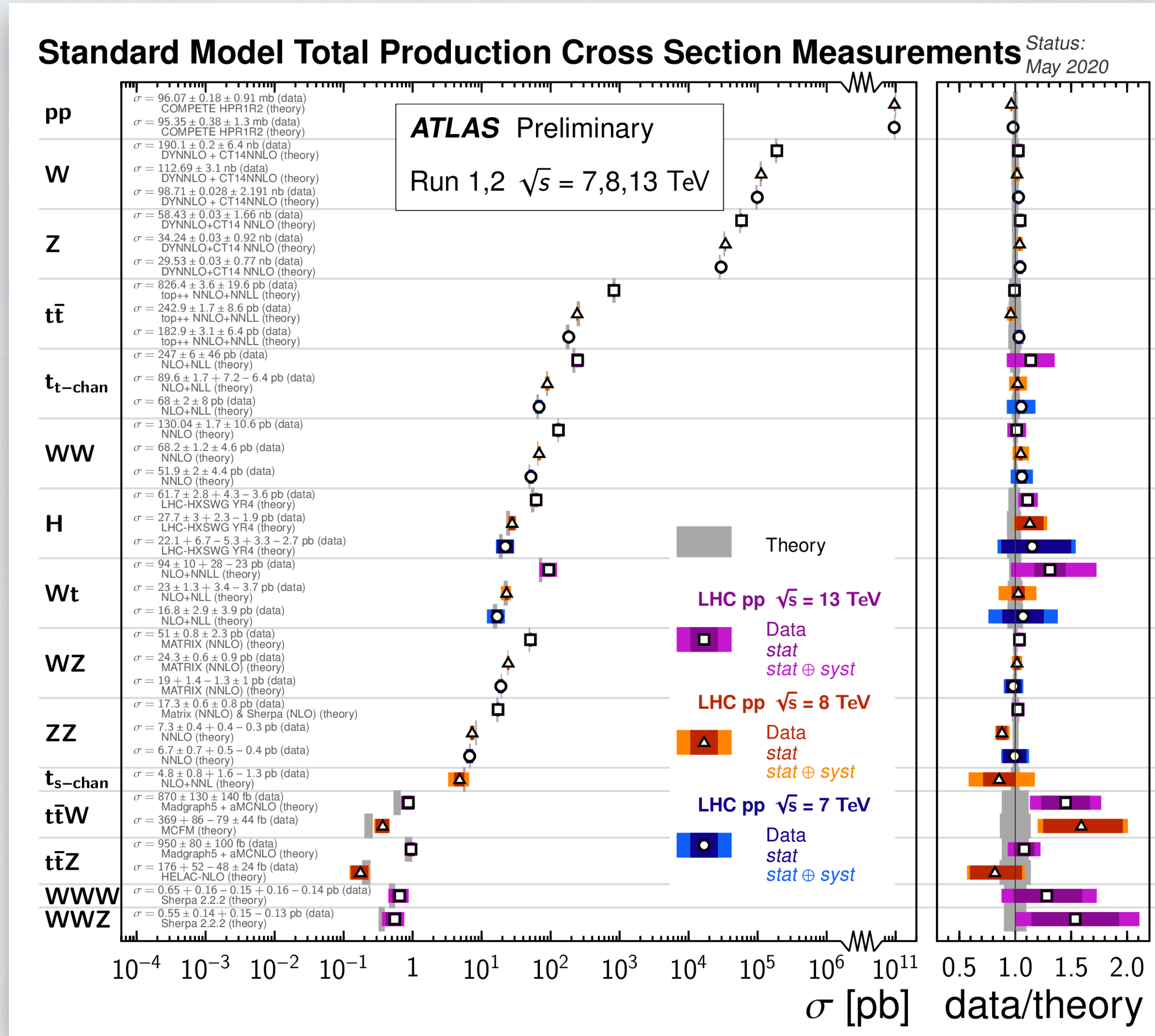
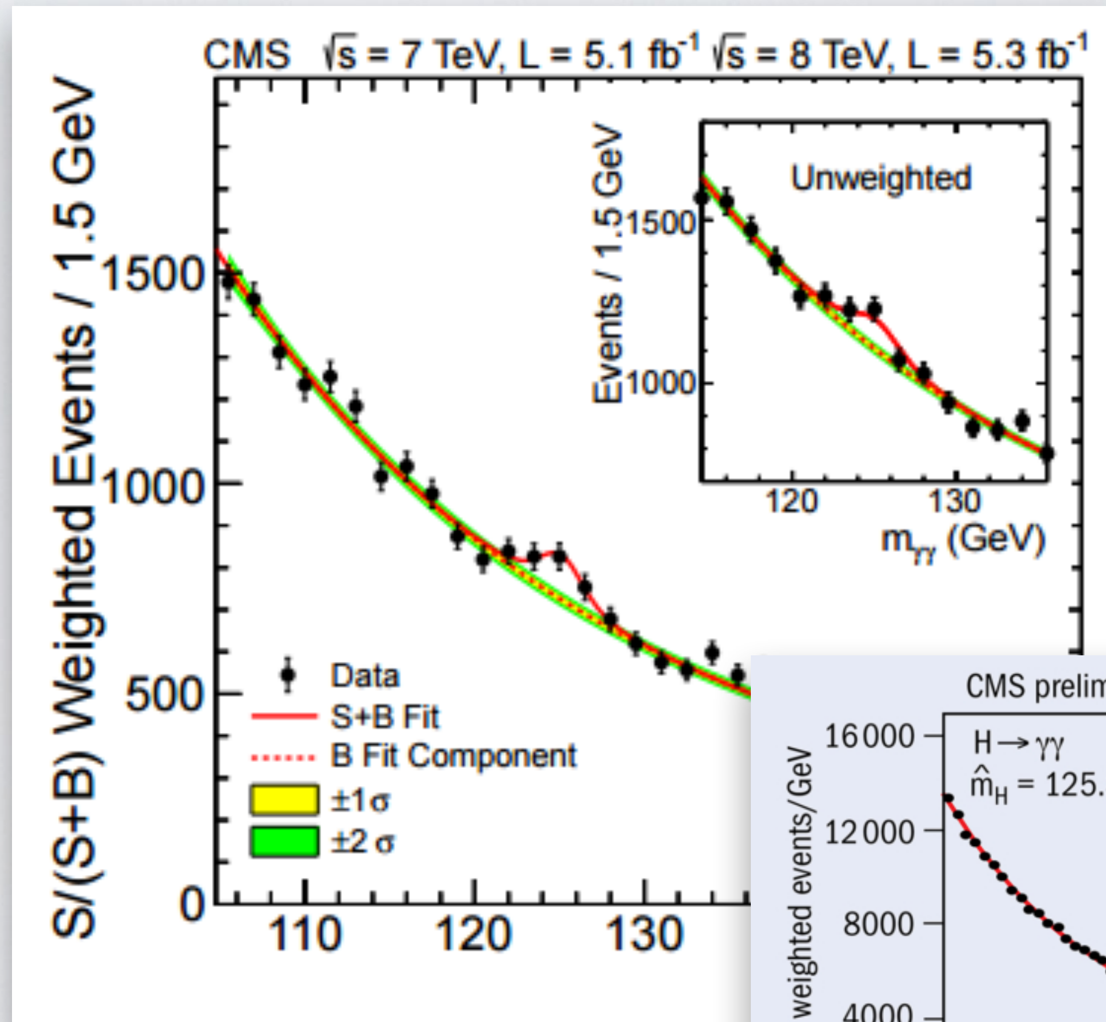
PRECISION MEASUREMENTS

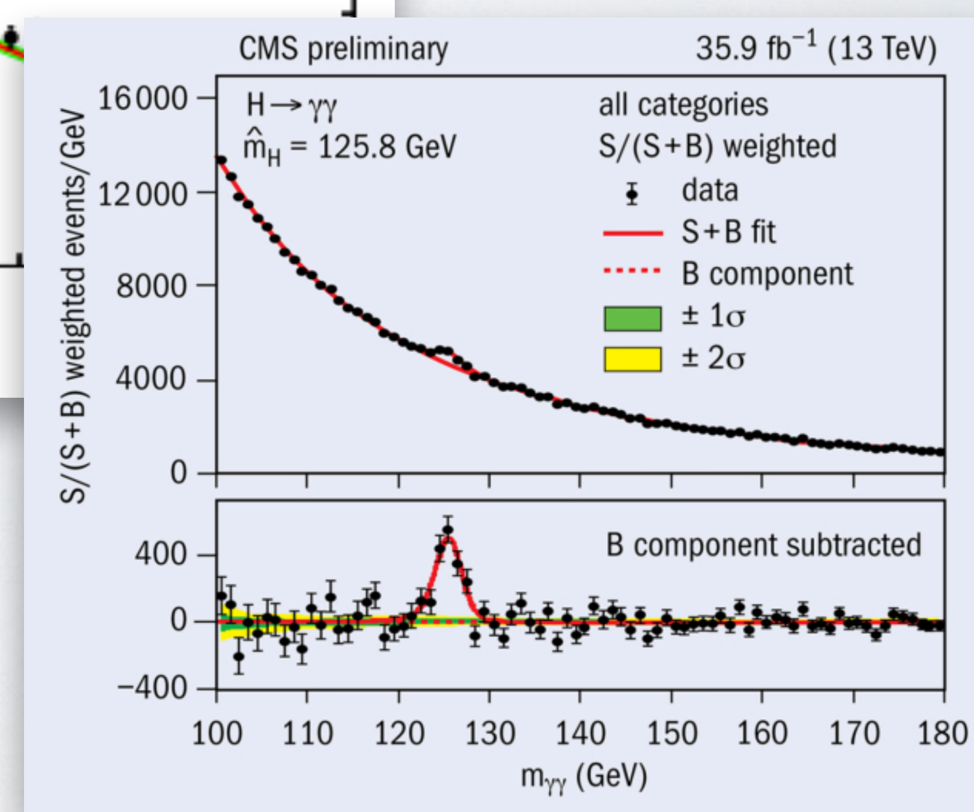
- *High Energy experiments are collecting more data.*



PRECISION MEASUREMENTS

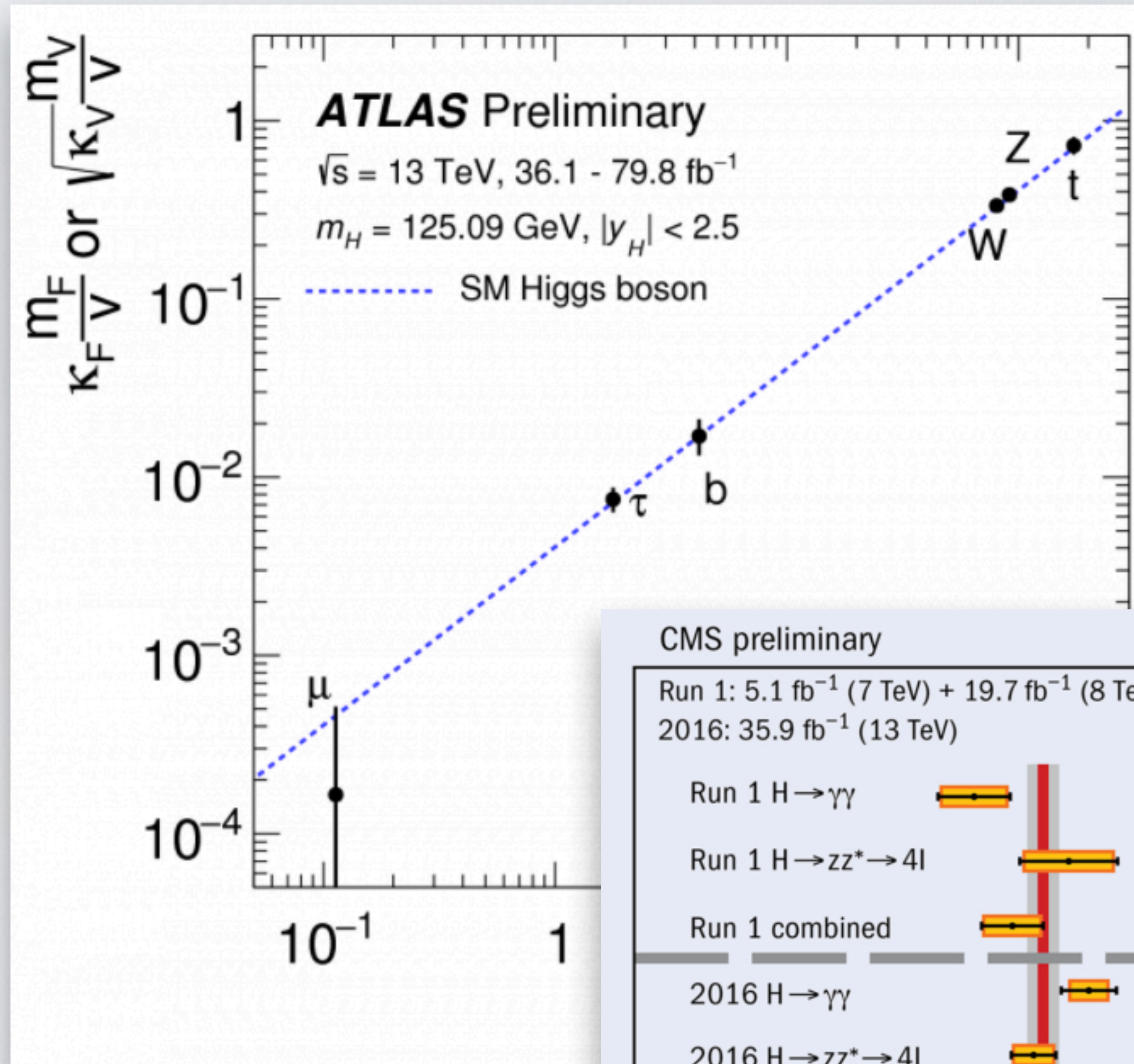
- *High Energy experiments are collecting more data*
- *In 2012, the LHC discovered a resonance compatible with the Standard Model Higgs Boson*

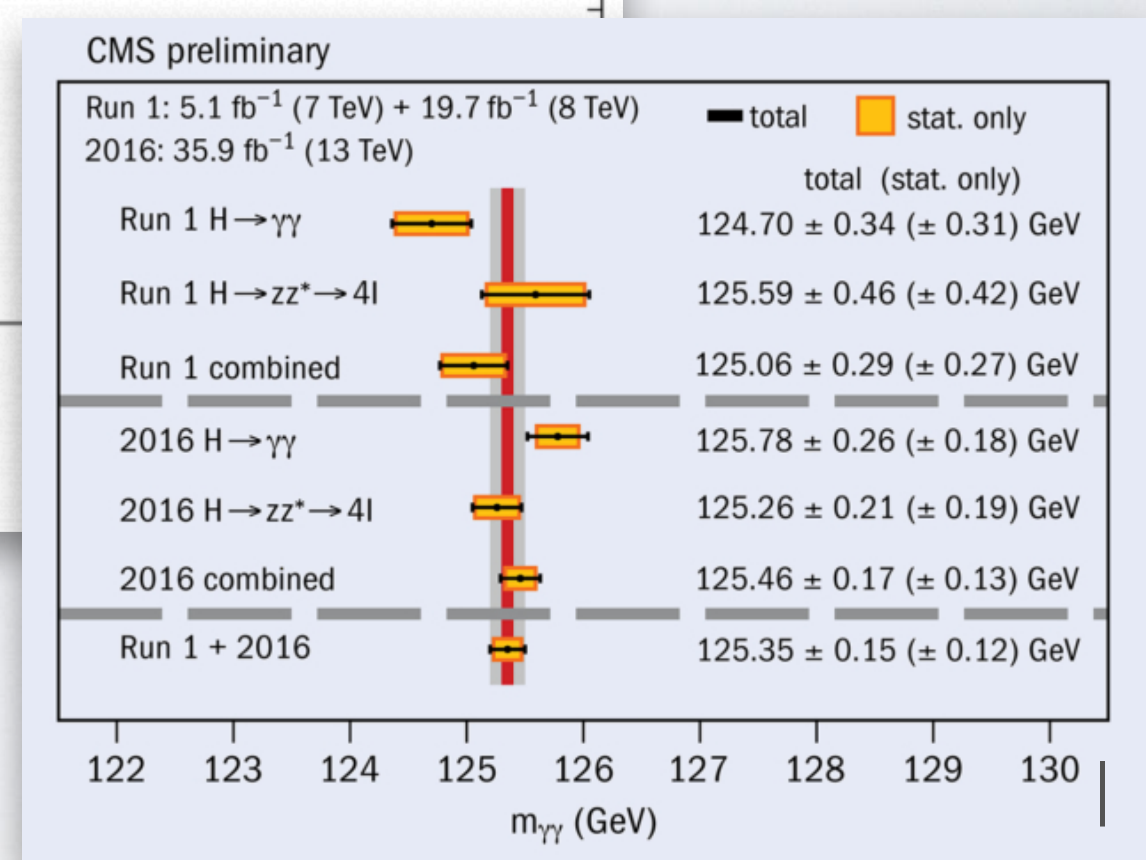




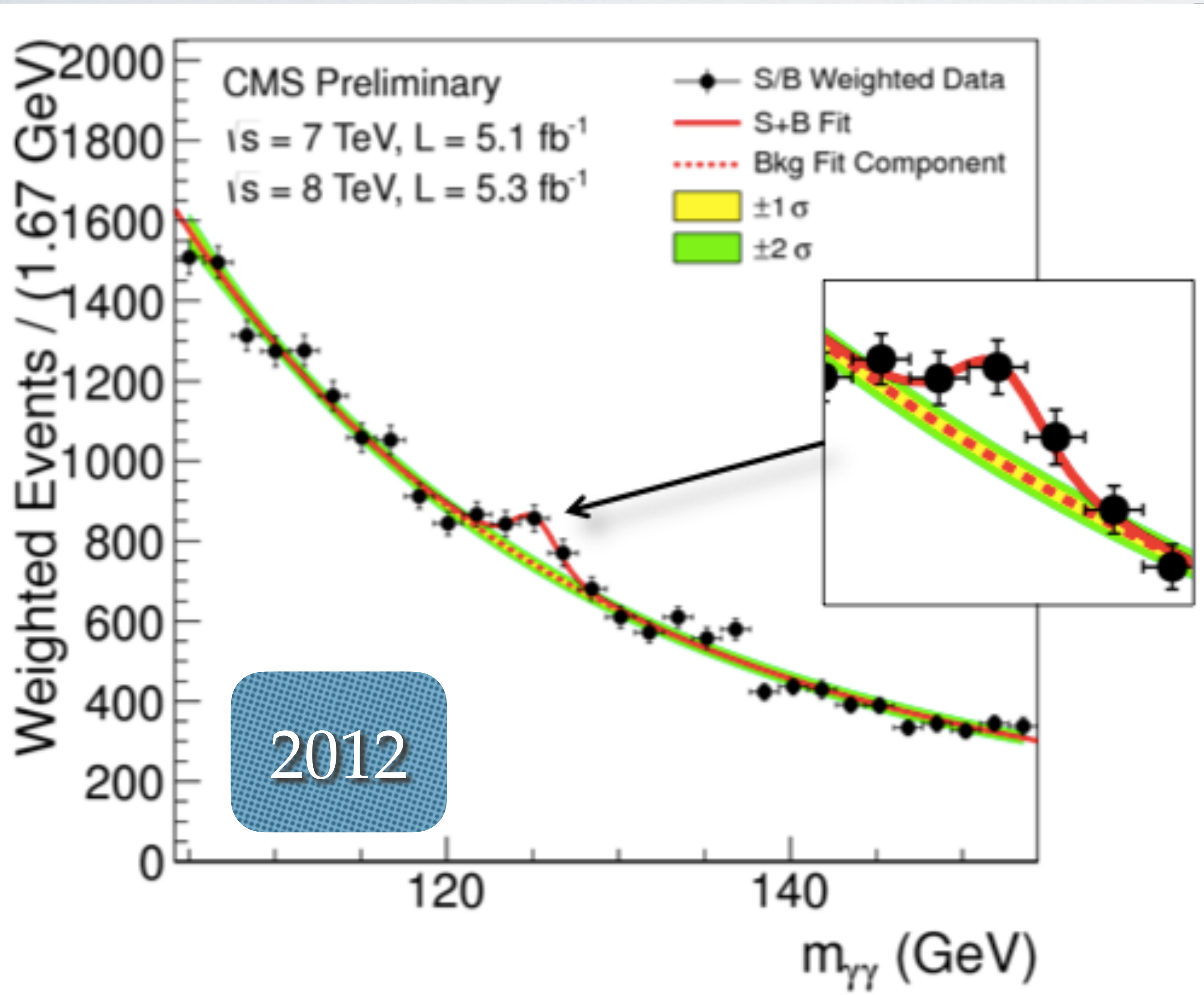
PRECISION MEASUREMENTS

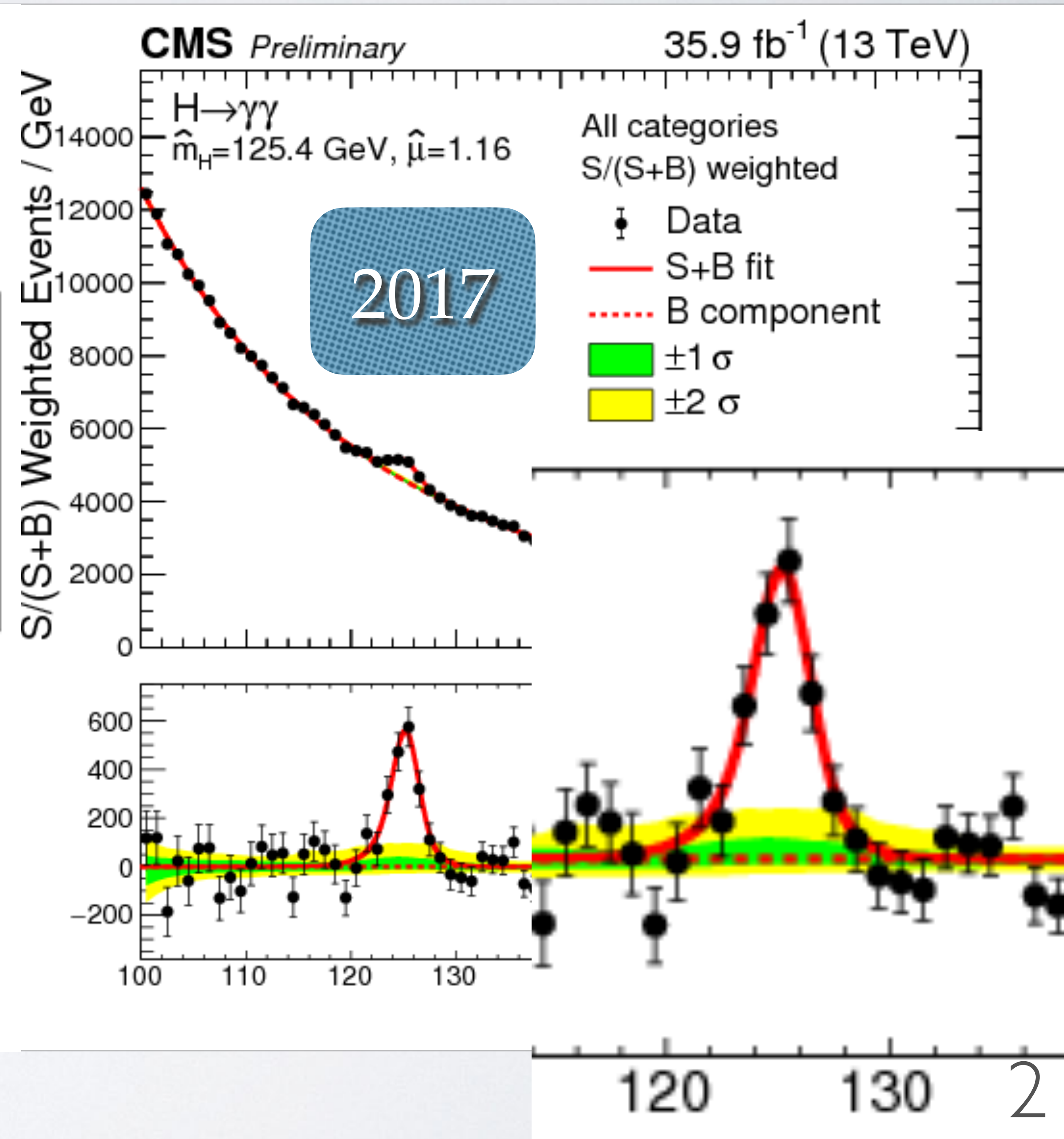
- *High Energy experiments are collecting more data*
- *In 2012, the LHC discovered a resonance compatible with the Standard Model Higgs Boson*
- *The complete determination of the Standard Model Higgs properties requires the computation of theoretical predictions to the highest possible accuracy.*



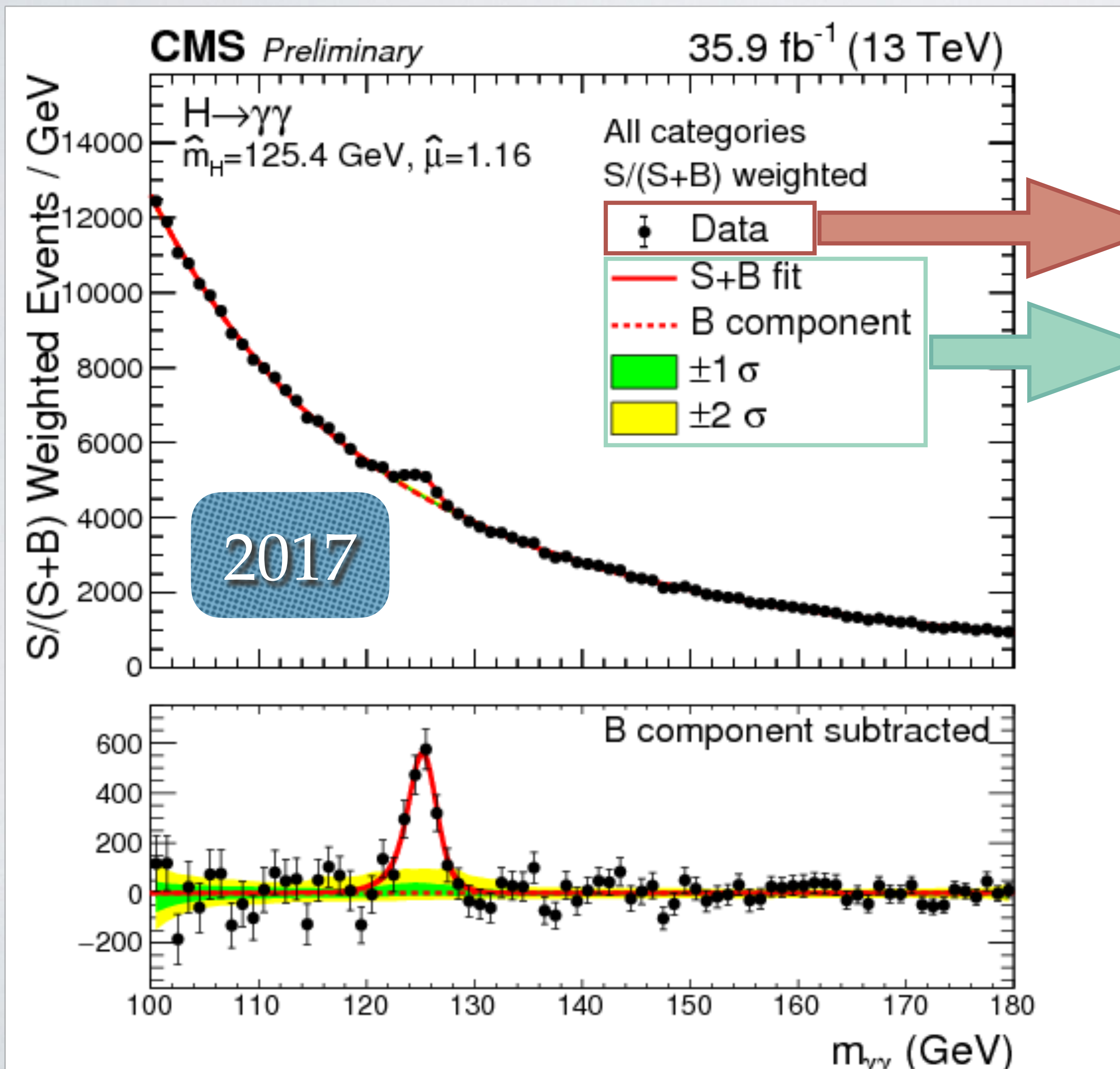


THEORY MEETS EXPERIMENT





THEORY MEETS EXPERIMENT



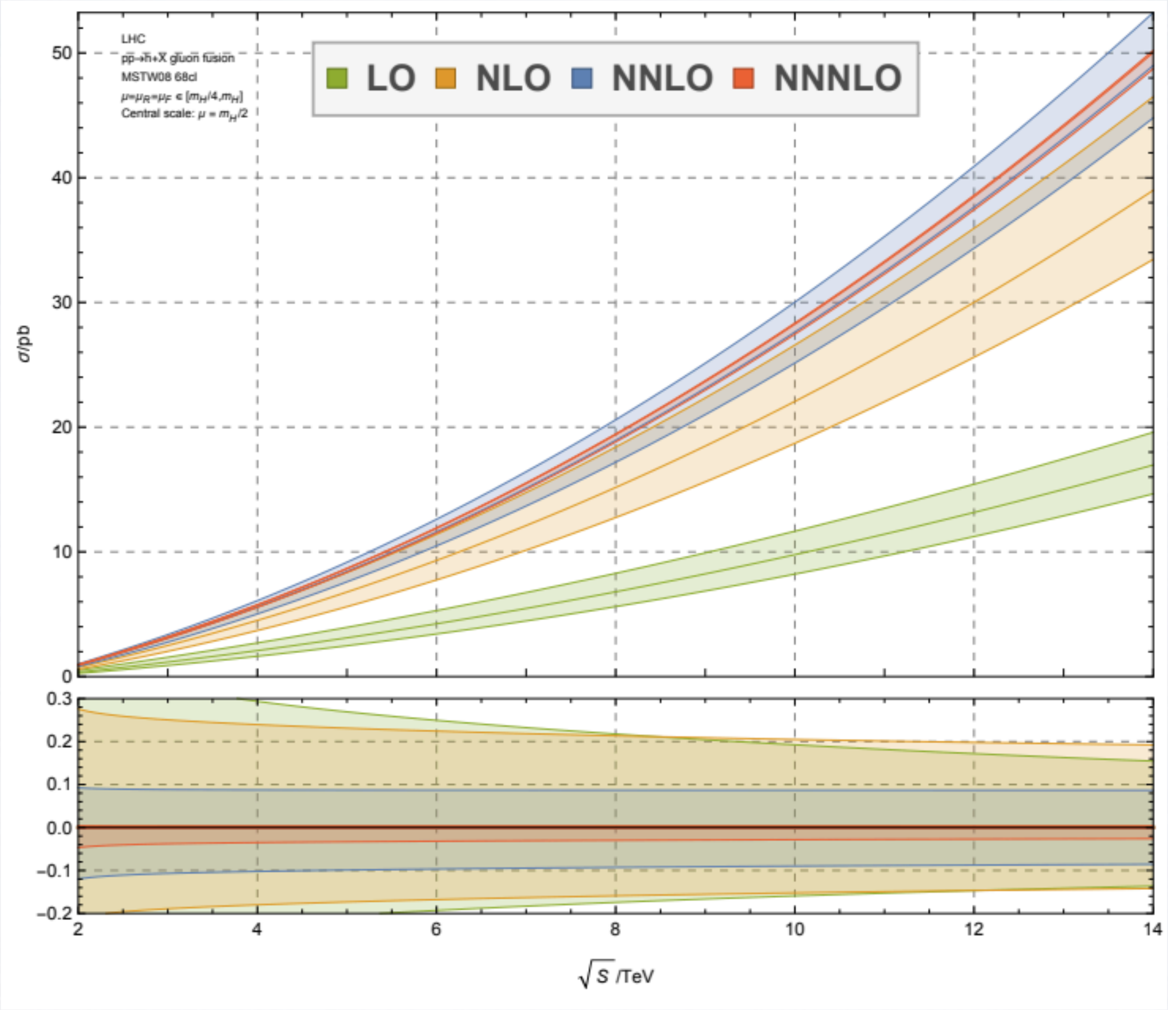
Experiment

Phenomenology

Strong knowledge in:
-Quantum Field Theory,
-Mathematical methods,
-Programming, …

THEORY MEETS EXPERIMENT

- Precision measurements use accurate theoretical predictions.



- Experimental results needs Monte Carlo simulations in order to compare with nature.

THEORETICAL PREDICTIONS

- The total cross section is the theoretical prediction computed to compare with the experiment. It has a series expansion as,

$$\sigma = \sigma^{(LO)} + \alpha_S(\mu)\sigma^{(NLO)} + \alpha_S^2(\mu)\sigma^{(NNLO)} + \cdots$$

- where all the elements are given in terms of integrals, as

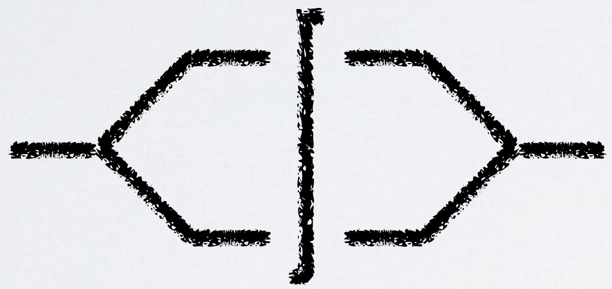
$$\sigma^{(NLO)} = \int_{\Omega} d\sigma^{V} + \int_{\Omega+1} d\sigma^{R}$$

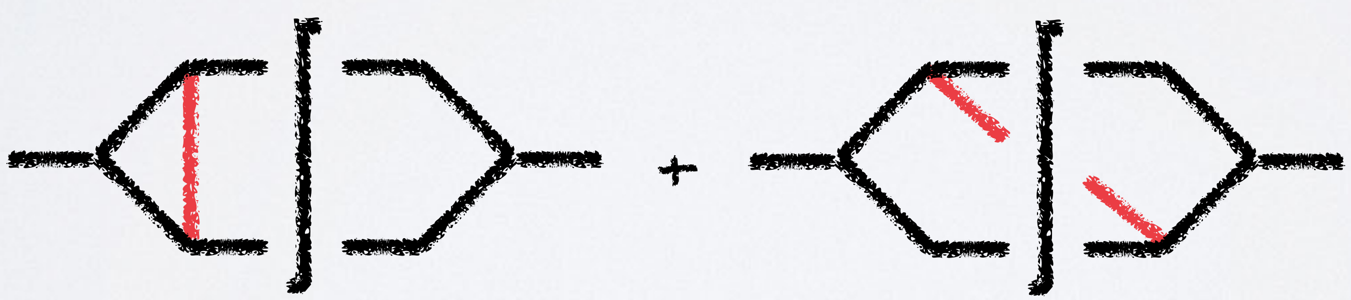
$$\sigma^{(NNLO)} = \int_{\Omega} d\sigma^{VV} + \int_{\Omega+1} d\sigma^{RV} + \int_{\Omega+2} d\sigma^{RR}$$

THEORETICAL PREDICTIONS

- However, we must keep in mind that a cross section is always finite at all orders in the series.

- The integrands are known using diagrams, Feynman diagrams. For instance,

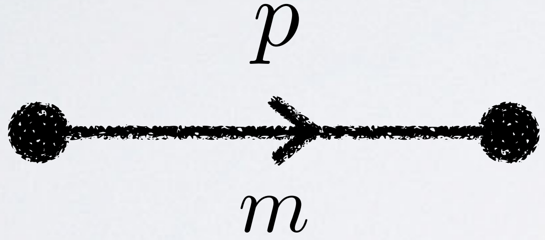
$\sigma^{\mathrm{LO}} =$ 

$\sigma^{\mathrm{NLO}} =$ 

$\sigma^{\mathrm{NNLO}} =$ 

THEORETICAL PREDICTIONS

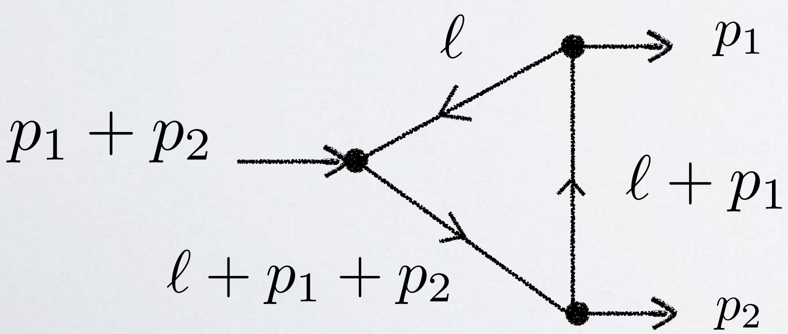
- The cross section is made, diagrammatically, by dots and lines. A line between two dots is called the propagator and has the functional form, for simplicity we consider no numerator,



$$: \qquad f(p,m) = \frac{1}{p^2 - m^2}$$

- where $p^2 = p_0^2 - \vec{p}^{\,2}$.

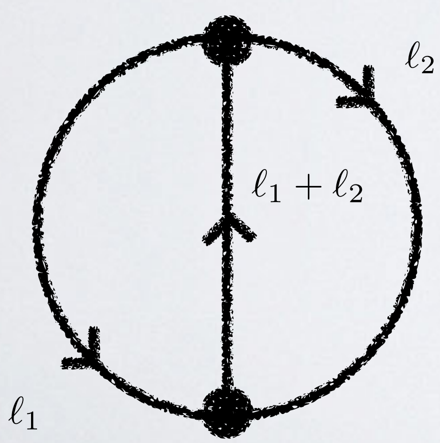
- Considering the triangle diagram, it has the structure,

- 

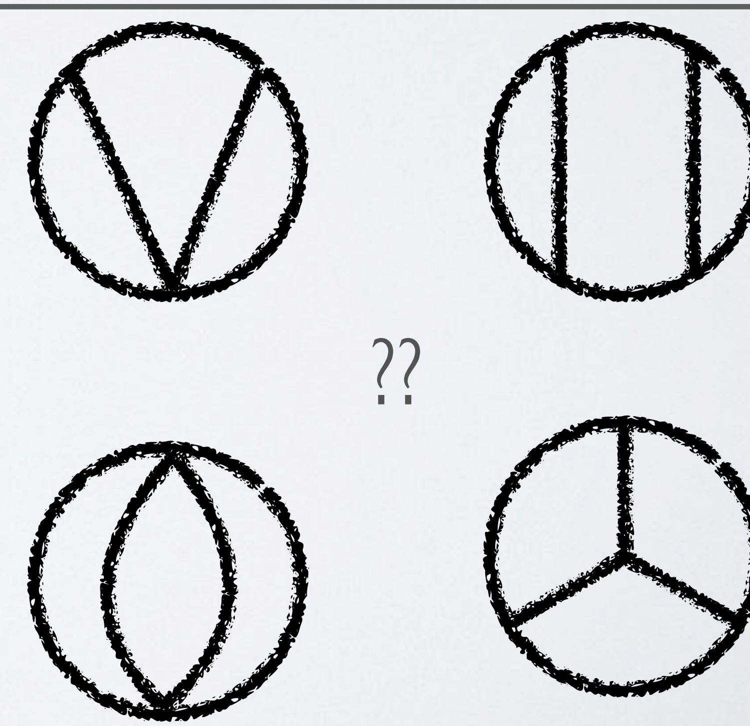
$$: \qquad \int d\ell_0 d\ell_x d\ell_y d\ell_z \frac{1}{\ell^2 - m^2} \times \frac{1}{(\ell + p_1 + p_2)^2 - m^2} \frac{1}{(\ell + p_1)^2 - m^2}$$

THEORETICAL PREDICTIONS

- The simplest two loop diagram involves a different complexity in the integrands.
- With more loops and legs, you need to impose momentum conservation at each vertex and apply the corresponding propagators and vertices "recipe" to build the integrand. Then, integrate.
- How to compute the complete set of integrals ?



$$: \int d\ell_1^4 d\ell_2^4 \frac{1}{\ell_1^2 - m^2} \times \frac{1}{\ell_2^2 - m^2} \frac{1}{(\ell_1 + \ell_2)^2 - m^2}$$

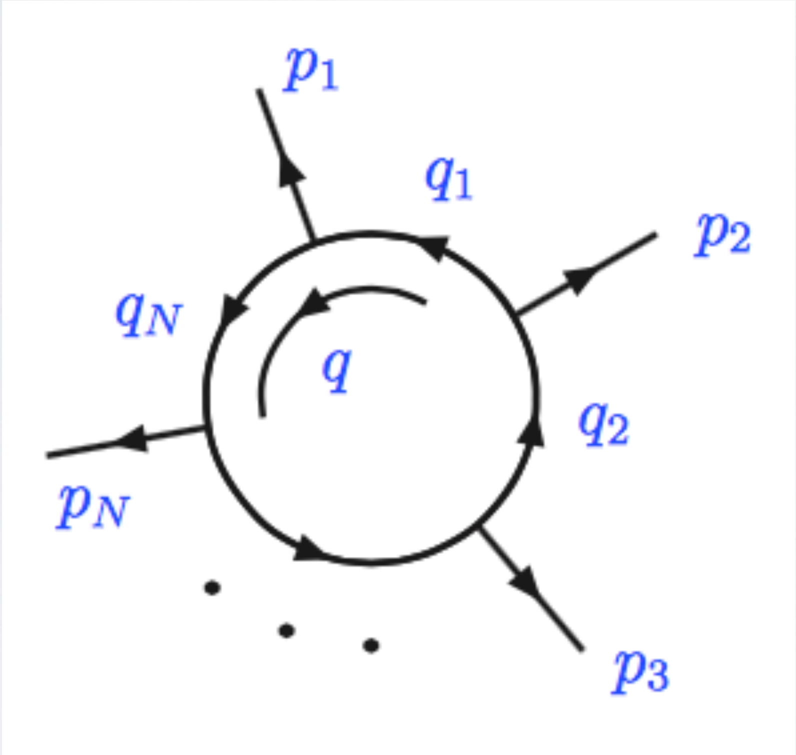


THEORETICAL ISSUES

- In a physical problem, integrands are usually lengthy.
- In addition, integrands are divergent. It is necessary to work in (4-2e)-space-time dimensions.
- The number of Feynman diagrams increase enormously when high accuracy is required.
- Time consuming Monte Carlo simulations.
- New methods for higher order calculations are extremely important.

LOOP-TREE DUALITY

- Massive one-loop scalar integrals are,



$$= -\imath \int \frac{d^d q}{(2\pi)^d} \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + \imath 0}$$

- where the +i0 prescription establishes that particles are going forward in time.

- LTD at one loop establishes then

$$L^{(1)}(p_1, \cdots, p_N) = -\sum \int_{\ell_1} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} G_D(q_i; q_j)$$

- where Feynman propagators are transformed to dual propagators.

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot (q_j - q_i)}$$

- $\tilde{\delta}(q_i) = 2\pi i \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$ and sets internal lines on-shell and in the positive energy mode.

- LTD modify the +i0 prescription, instead of having multiple cuts like in the Feynman Tree Theorem.

- η^μ is a future-like vector, for simplicity we take $\eta^\mu = (1, \mathbf{0})$. In fact, the only relevance is the sign in the prescription.

NUMERICAL IMPLEMENTATION

- Faster computations are needed for the Montecarlo simulations for the LHC observables.
- Using LTD the standard methods become time consuming

	Rank	Tensor Pentagon	Real Part	Imaginary Part	Time [s]
P16	2	LoopTools	-1.86472×10^{-8}		
		SecDec	$-1.86471(2) \times 10^{-8}$		45
		LTD	$-1.86462(26) \times 10^{-8}$		1
P17	3	LoopTools	1.74828×10^{-3}		
		SecDec	$1.74828(17) \times 10^{-3}$		550
		LTD	$1.74808(283) \times 10^{-3}$		1
P18	2	LoopTools	-1.68298×10^{-6}	$+i\ 1.98303 \times 10^{-6}$	
		SecDec	$-1.68307(56) \times 10^{-6}$	$+i\ 1.98279(90) \times 10^{-6}$	66
		LTD	$-1.68298(74) \times 10^{-6}$	$+i\ 1.98299(74) \times 10^{-6}$	36
P19	3	LoopTools	-8.34718×10^{-2}	$+i\ 1.10217 \times 10^{-2}$	
		SecDec	$-8.33284(829) \times 10^{-2}$	$+i\ 1.10232(107) \times 10^{-2}$	1501
		LTD	$-8.34829(757) \times 10^{-2}$	$+i\ 1.10119(757) \times 10^{-2}$	38

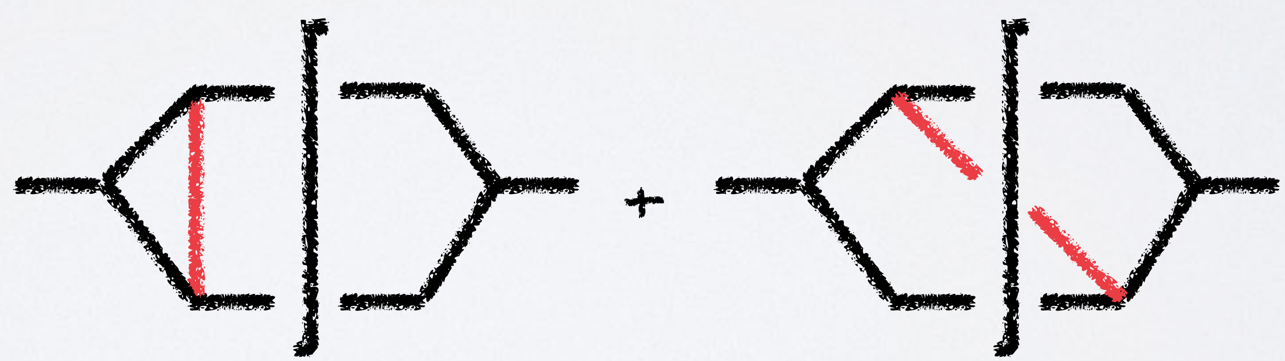
	Rank	Tensor Hexagon	Real Part	Imaginary Part	Time[s]
P20	1	SecDec	$-1.21585(12) \times 10^{-15}$		36
		LTD	$-1.21552(354) \times 10^{-15}$		6
P21	3	SecDec	$4.46117(37) \times 10^{-9}$		5498
		LTD	$4.461369(3) \times 10^{-9}$		11
P22	1	SecDec	$1.01359(23) \times 10^{-15}$	$+i\, 2.68657(26) \times 10^{-15}$	33
		LTD	$1.01345(130) \times 10^{-15}$	$+i\, 2.68633(130) \times 10^{-15}$	72
P23	2	SecDec	$2.45315(24) \times 10^{-12}$	$-i\, 2.06087(20) \times 10^{-12}$	337
		LTD	$2.45273(727) \times 10^{-12}$	$-i\, 2.06202(727) \times 10^{-12}$	75
P24	3	SecDec	$-2.07531(19) \times 10^{-6}$	$+i\, 6.97158(56) \times 10^{-7}$	14280
		LTD	$-2.07526(8) \times 10^{-6}$	$+i\, 6.97192(8) \times 10^{-7}$	85

- This results shows have been implemented for several data points for tensor pentagons and hexagons.
- Integrals considering massive internal lines were computed numerically.
- The results using LTD are, in some cases, four order of magnitudes faster than SecDec.

FDU

- The Four-Dimensional Unsubtraction relies on the fact that cross sections are always finite.
- Poles at the integrand level should cancel at the integral level, by adding proper counter-terms.
- Loop-Tree Duality sets virtual particles on-shell. Therefore, merging the variables must cancel divergences at integral level.

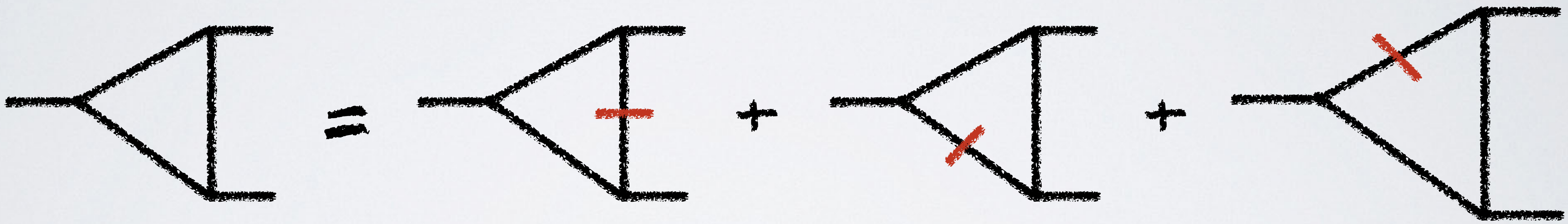
$$\sigma^{\mathrm{NLO}} =$$



- OBJECTIVE: Apply the LTD for matching the virtual and the real contributions at integrand level at NLO where the integrand should not have divergences.

IR DIVERGENCES

- Let's apply the LTD to the triangle:



- It means that the full integrals is the sum over three phase-space integrals,

$$L^{(1)}(p_1, p_2, -p_3) = -\sum_{i=1}^{3} I_i$$

IR REGULARISATION

- Let's define real and virtual cross sections as,

$$\tilde{\sigma}_{i,R} = \sigma_0^{-1} 2\mathrm{Re} \int d\Phi_{1\to 3} \langle \mathcal{M}_{2r}^{(0)} | \mathcal{M}_{1r}^{(0)} \rangle \theta(y'_{jr} - y'_{ir})$$

$$\tilde{\sigma}_{i,V} = \sigma_0^{-1} 2\mathrm{Re} \int d\Phi_{1\to 2} \langle \mathcal{M}^{(0)} | \mathcal{M}_{i}^{(1)} \rangle \theta(y'_{jr} - y'_{ir})$$

- where

$$\langle \mathcal{M}^{(0)} | \mathcal{M}_i^{(1)} \rangle = -g^4 s_{12} I_i \qquad y'_{ir} = \frac{s_{12}}{s'_{ir}}$$

$$\langle \mathcal{M}_{2r}^{(0)} | \mathcal{M}_{1r}^{(0)} \rangle = g^4 s_{12} / (s'_{1r} s'_{2r})$$

- Momentum conservation: $p_1 + p_2 = p'_1 + p'_2 + p'_r$

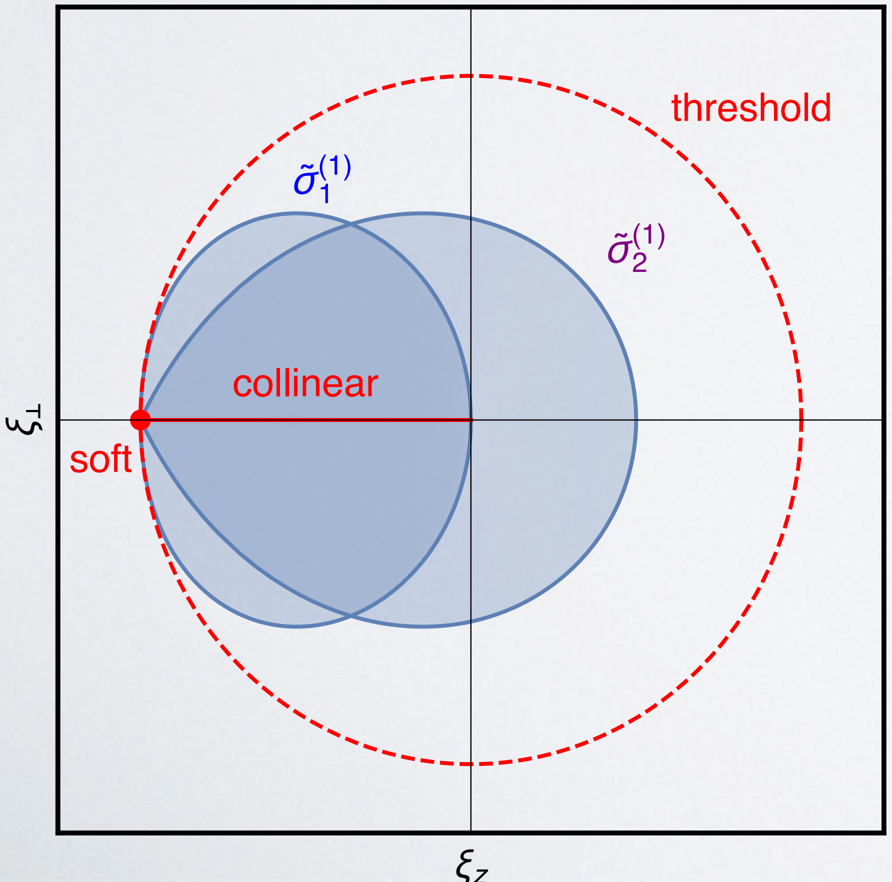
- Claim: $\tilde{\sigma}_i = \tilde{\sigma}_{i,V} + \tilde{\sigma}_{i,R}$ allows a 4-dimensional representation at the integrand level.

- Building the mapping for the condition $y'_{1r} < y'_{2r}$:

$$p'^{\mu}_r = q^{\mu}_1$$
$$p'^{\mu}_1 = p^{\mu}_1 - q^{\mu}_1 + \alpha_1 p^{\mu}_2$$
$$p'^{\mu}_2 = (1-\alpha_1)p^{\mu}_2$$

$$\alpha_1 = \frac{q_3^2}{2q_3 \cdot p_2}$$
$$q_1 = \ell + p_1$$

- and a similar mapping for $y'_{1r} > y'_{2r}$. The integral regions are



$$\tilde{\sigma}_1 = \mathcal{O}(\epsilon)$$
$$\tilde{\sigma}_2 = -c_\Gamma \frac{g^2}{s_{12}} \frac{\pi^2}{6} + \mathcal{O}(\epsilon)$$
$$\bar{\sigma}_V = c_\Gamma \frac{g^2}{s_{12}} \frac{\pi^2}{6} + \mathcal{O}(\epsilon)$$

UV RENORMALISATION

- UV renormalisation requires local cancellation of divergences.
- In general, counterterms are obtained by expanding the propagator around a UV propagator

$$G_F(q_i) = \frac{1}{q_{UV}^2 - \mu_{UV}^2 + \imath 0} + \cdots \,, \qquad q_{UV} = \ell + k_{UV}$$

- For the bubble integral, the counterterm is

$$I_{UV}^{cnt} = \int_\ell \frac{1}{(q_{UV}^2 - \mu_{UV}^2 + \imath 0)^2}$$

LTD

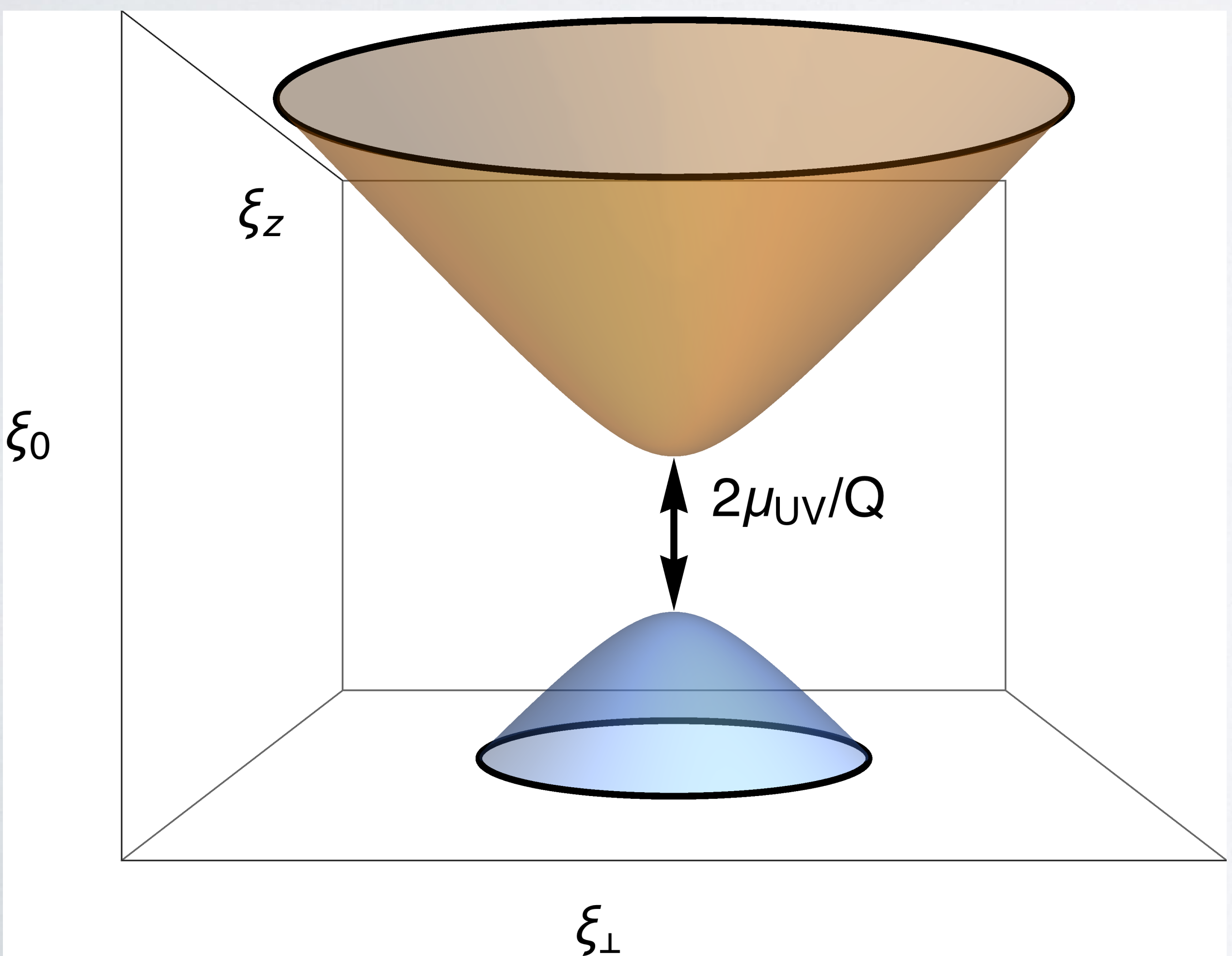
$$I_{UV}^{cnt} = \int_\ell \frac{\tilde{\delta}(q_{UV})}{2(q_{UV,0}^{(+)})^2}$$

$$q_{UV,0}^{(+)} = \sqrt{\mathbf{q}_{UV}^2 + \mu_{UV}^2 - \imath 0}$$

- 4 dimensional representation of the renormalised bubble integral is,

$$
\begin{aligned}
L^{(1,R)} &= L^{(1)}(p,-p) - I_{UV}^{cnt} \\
&= -4\int d[\xi]d[v]\left[\frac{\xi}{1-2\xi+\imath 0}+\frac{\xi}{1+2\xi}+\frac{\xi^2}{2(\xi^2+m_{UV}^2)^{3/2}}\right] \\
&= \frac{1}{4\pi^2}\left[-\log\left(-\frac{p^2}{\mu_{UV}^2}-\imath 0\right)+2\right]+\mathcal{O}(\epsilon)
\end{aligned}
$$

- the integration regions corresponds to hyperboloids

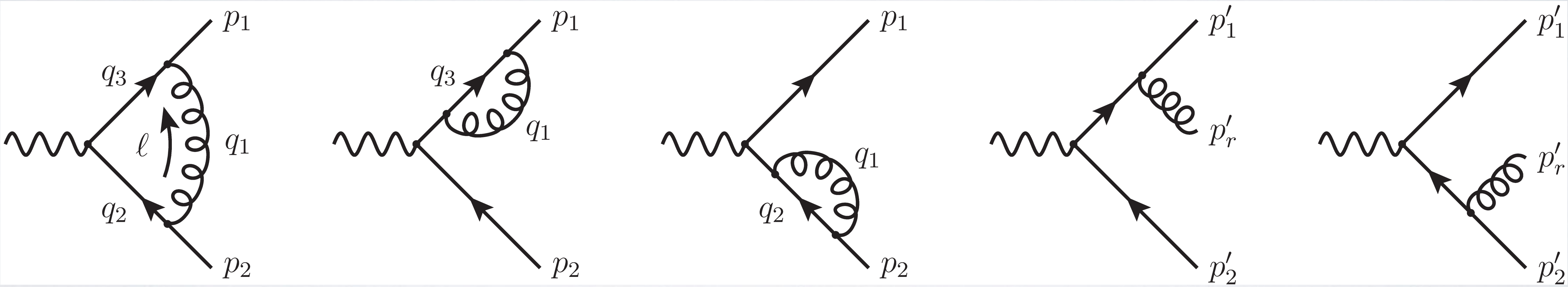


- Physical interpretation of renormalisation scale: Avoid the intersection of hyperboloids. Thus

$$\mu_{UV} = Q/2$$

$\gamma^* \to q\bar{q}$ AT NLO IN QCD

- In this well known process, the Feynman diagrams are



- Using the LTD we find,

$$\tilde{\sigma}_1^{(1)} = \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \left(19 - 32\log(2)\right),$$

$$\tilde{\sigma}_2^{(1)} = \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \left(-\frac{11}{2} + 8\log(2) - \frac{\pi^2}{3}\right),$$

-

$$\bar{\sigma}_V^{(1)} = \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \left(-\frac{21}{2} + 24\log(2) + \frac{\pi^2}{3}\right).$$

The sum coincides with the result in 4-dimensions.

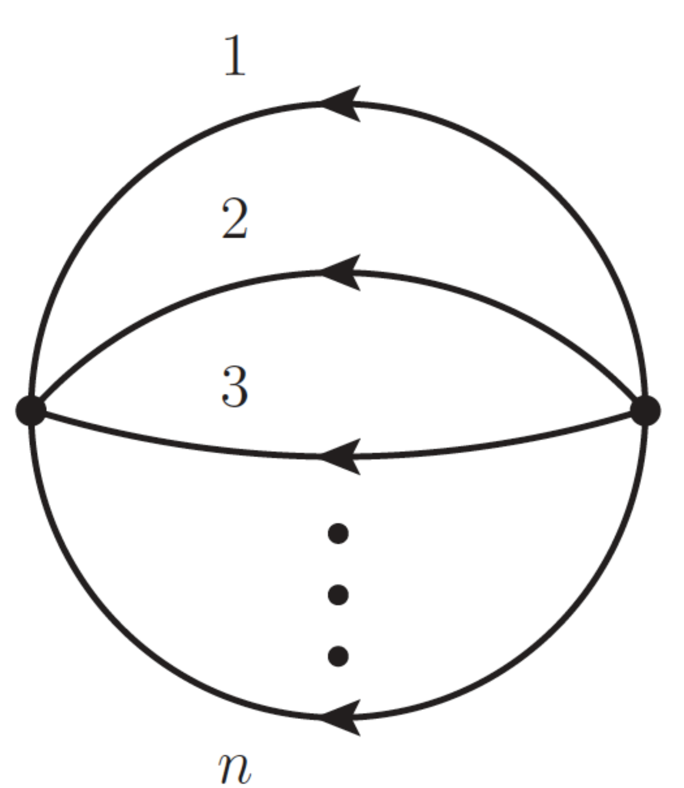
It is local !

INTEGRAL REPRESENTATIONS

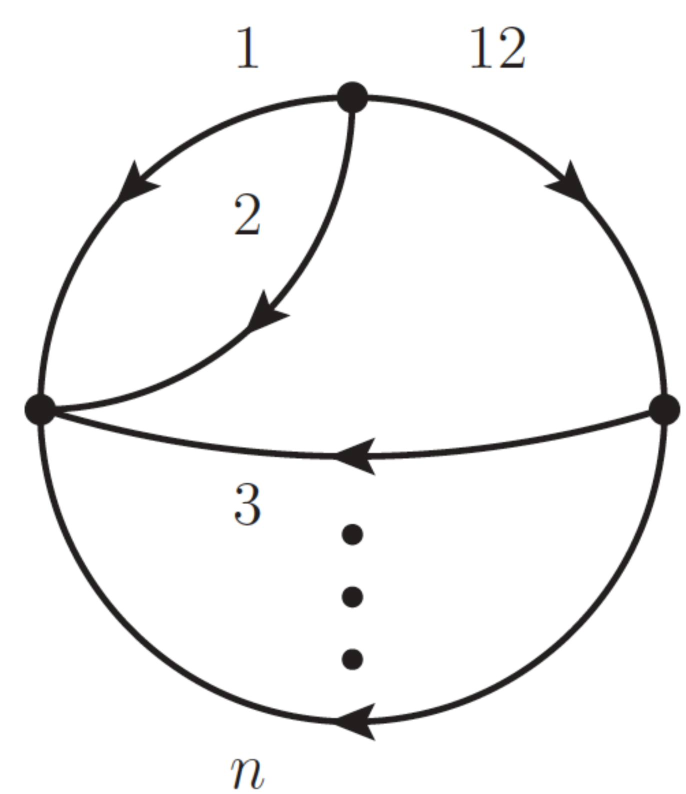
- Integrals (sums) can be written in many ways.
- We call the Feynman representation to the integrals that are obtained by the Feynman rules, without making any treatment.
- The dual representation is the one obtained by the Loop-Tree Duality theorem.
- The causal representation is obtained when partial fractioning is made to the dual representation, and only physical singularities remain.

INTEGRAL REPRESENTATIONS

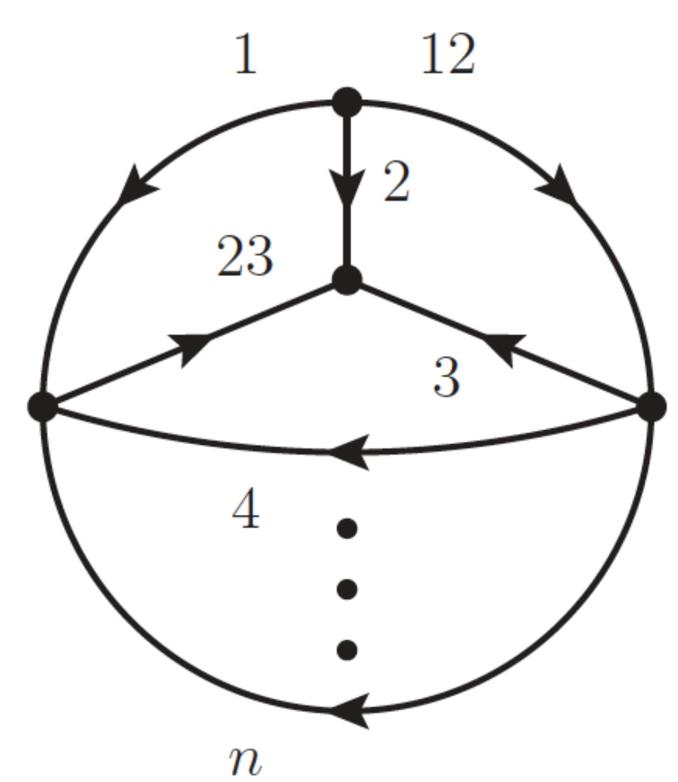
- We were interested in the application of the LTD to some topologies.
- The arrangement most simplest are,



Maximal Loop Topology
MLT



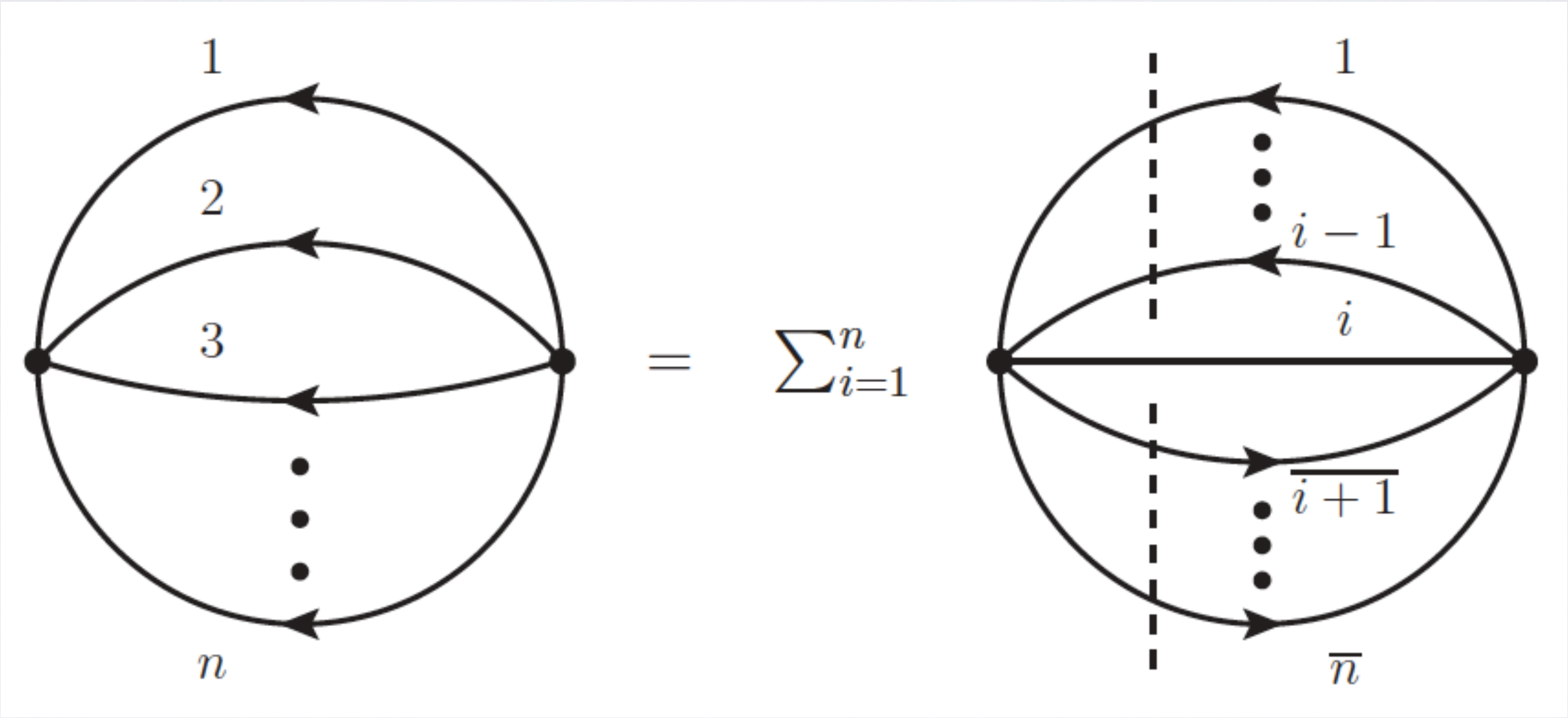
Next-to-Maximal Loop Topology
NMLT



Next-to-Next-Maximal Loop Topology
NNMLT

INTEGRAL REPRESENTATIONS

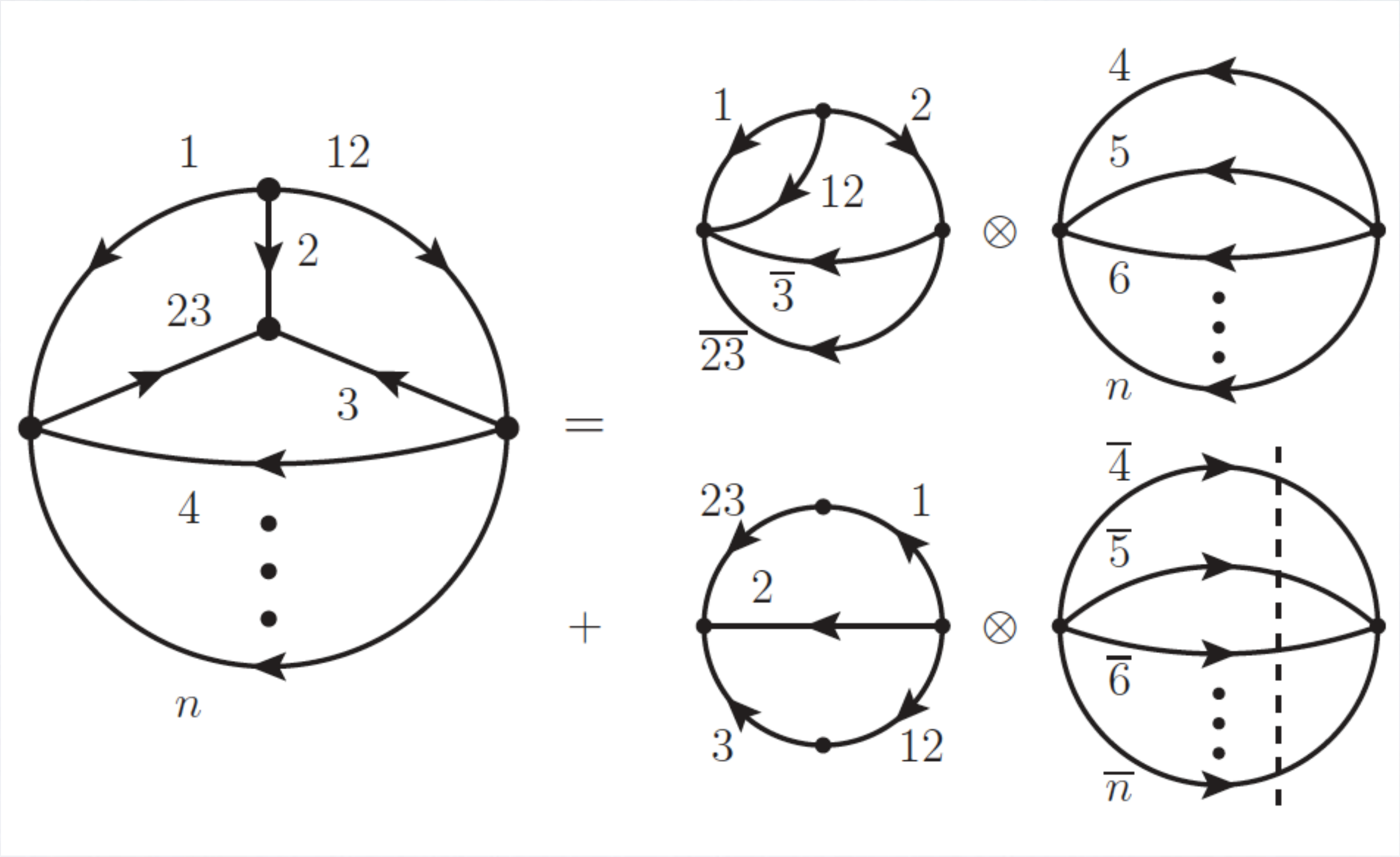
- It is possible to find, dual representations for each topology. For instance,



$$\mathcal{A}_{\mathrm{MLT}}^{(L)}(1,\cdots,n)=\int_{\ell_1,\cdots,\ell_L}\sum_{i=1}^{n}\mathcal{A}_D^{L}(1,\cdots,i-1,\overline{i+1},\cdots,\overline{n};i)$$

INTEGRAL REPRESENTATIONS

- It is possible to recicle the results in order to build more complicated topolgies, such as,



$$\mathcal{A}^{(L)}_{\rm NNMLT}(1,\cdots,n,12,23) = \mathcal{A}^{(3)}_{\rm NMLT}(1,12,\bar{3},\overline{23},2)\otimes\mathcal{A}^{(L-3)}_{\rm MLT}(4,\cdots,n)$$
$$+\mathcal{A}^{(2)}_{\rm MLT}(1\cup 23,2,3\cup 12)\otimes\mathcal{A}^{(L-2)}_{\rm MLT}(\bar{4},\cdots,\bar{n})$$

INTEGRAL REPRESENTATIONS

- Denominators are important in the representations since the complexity of the integrals are explicitly there.

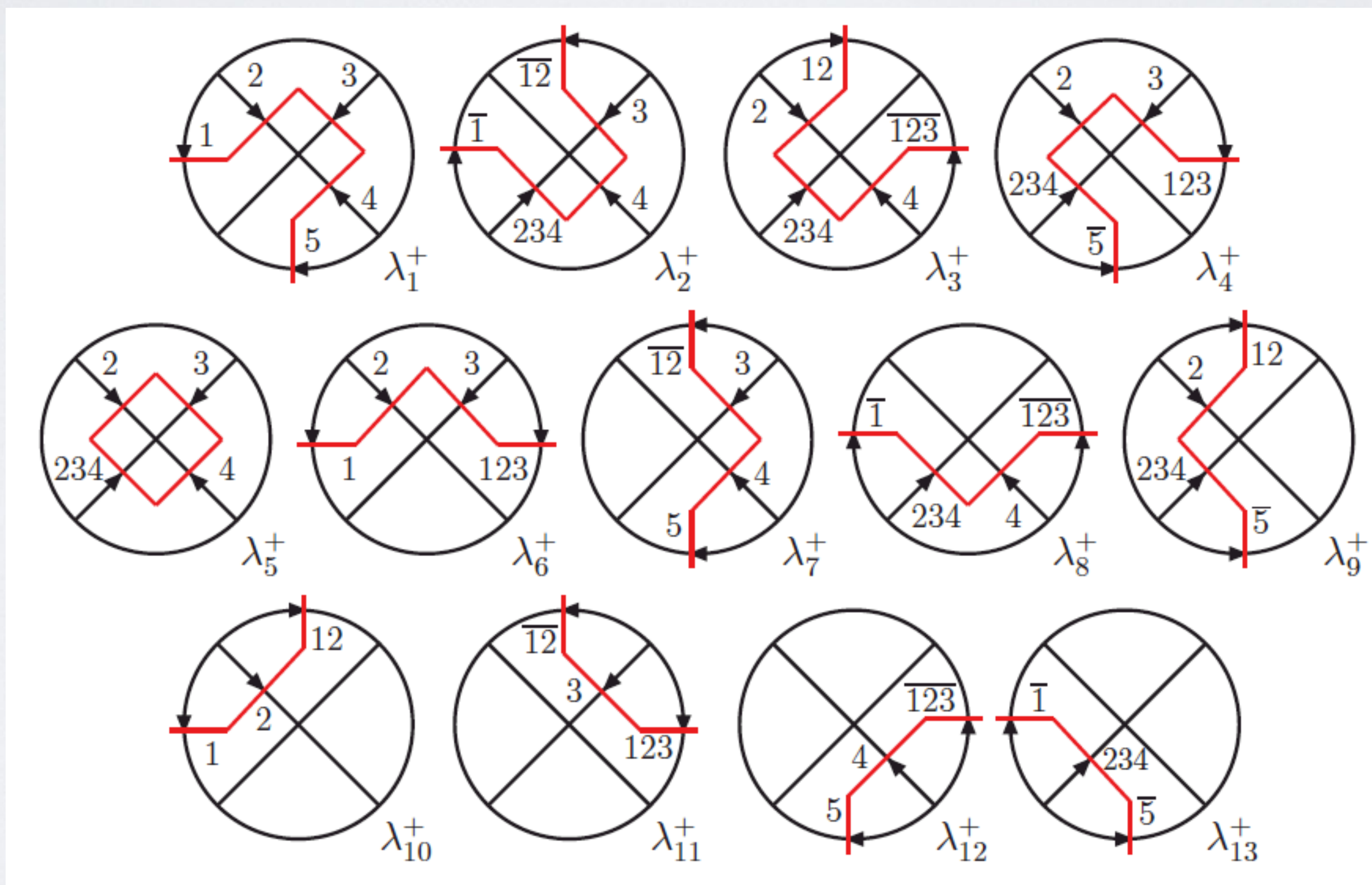
- Combinatorics in the dual representations shows that there are denominators that could present a non-physical singularity, such as,

$$q_{1,0}^{(+)} + \cdots + q_{i-1,0}^{(+)} - q_{i,0}^{(+)} + q_{i+1,0}^{(+)} + \cdots + q_{N,0}^{(+)}$$

- However, causal representation do not have such non-physical divergences.

INTEGRAL REPRESENTATIONS

- For instance, the N^3MLT requires 13 causal propagators to describe the full amplitude.



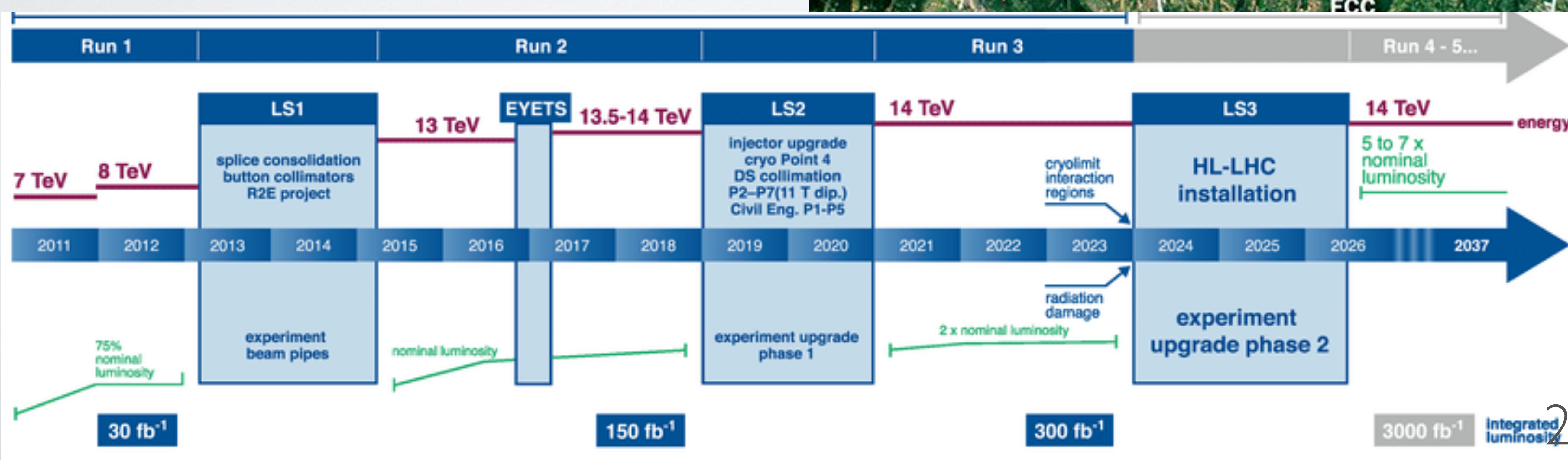
- How about more loops or different complexities ??

CONCLUSIONS

- New methods for computing higher order corrections are needed for the upcoming LHC observables.
- The Loop-Tree Duality is a new method for computing efficiently divergent amplitudes and its application through the Four-Dimensional Unsubtraction opens a new window for the high precision era.
- Causal representation can plays an important role when numerical integration is performed for a large number of diagrams.

- The Standard Model cannot be the end of the road.
- New accelerators are searching for new particles and interactions.
- The discovery of New Physics will be driven by the understanding small deviations between the theoretical predictions and the experimental data.

- **Future Circular Collider (FCC)**
  Circumference: 90 -100 km
  Energy: 100 TeV (pp) 90-350 GeV (e^+e^-)
- **Large Hadron Collider (LHC)**
  Large Electron-Positron Collider (LEP)
  Circumference: 27 km
  Energy: 14 TeV (pp) 209 GeV (e^+e^-)

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THANK YOU !