The phase-diagram of the NJL model

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MexNICA Collaboration Winter Meeting 2020

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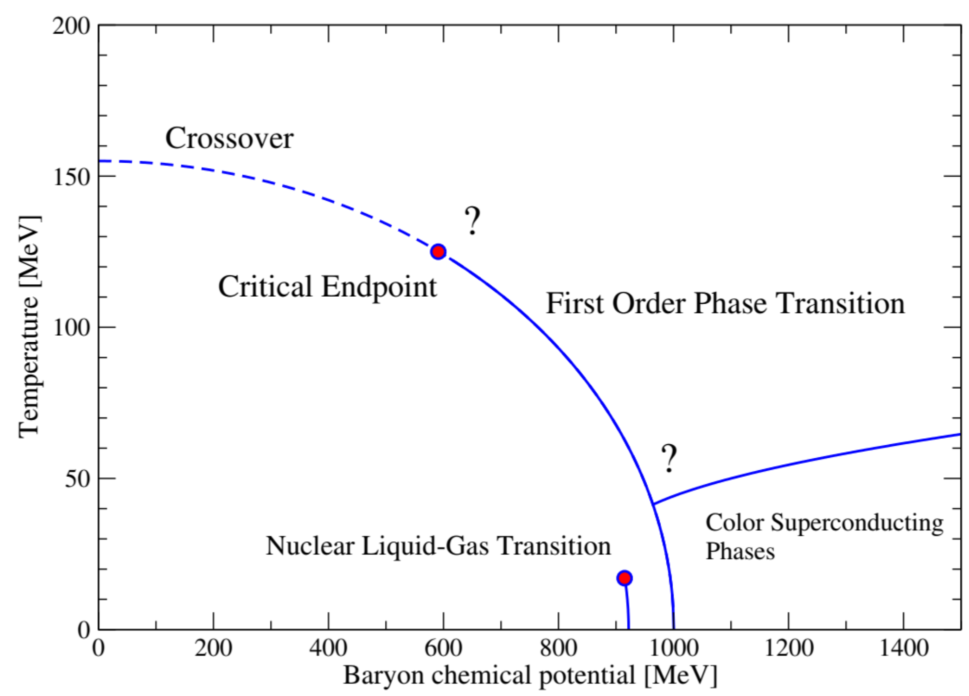
Introduction

As we know, the drawing of the QCD phase-diagram it's a hard task due the complexity of QCD itself.

On the other hand, the interest in drawing the phase diagram in a more precise manner has risen given the new experiments in the **RHIC** and **NICA** facilities that will let us explore the phase diagram close to the **Critical End Point** (CEP).

In our work, we focus on exploring the QCD phase-diagram using the simpler NJL model that will allow us to draw the more robust characteristics of QCD.



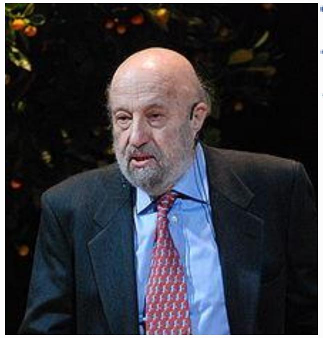
NJL model

The NJL model was proposed by Yoichiro Nambu and Giovanni Jona-Lasinio in 1961. It was one of the first efforts of modelling the strong interactions before the introduction of QCD.

One of the characteristics of the NJL model is that describes the breaking of the chiral symmetry, in parallel to the Cooper pairs formation in the BCS theory of superconductivity.

As it was proposed before the discovery of the quarks it does not contemplate confinement.

Also it is not renormalizable.



Giovanni Jona-Lasinio

Yoichiro Nambu

NJL model

The lagrangian of the NJL model reads

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m_q)\psi + G\left\{\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi}i\gamma^5\vec{\tau}\psi\right)^2\right\}$$

Where

- ψ is the quark field.
- G is the coupling constant.
- $\vec{\tau}$ are the Pauli matrices acting on the isospin space.
- m_q is the quark current mass.

Gap equation

Performing a Hartree-Fock transformation to the lagrangian we get the gap so called gap equation of the NJL model

$$m = m_q - 2G \left\langle \bar{\psi}\psi \right\rangle$$

Where $\langle \bar{\psi}\psi \rangle$ is the chiral condensate, defined as:

$$\langle \bar{\psi}\psi \rangle = -\int \frac{d^4k}{(2\pi)^4} Tr[iS(k)]$$

Gap equation

As the model is non-renormalizable it is necessary to apply a regularization scheme. We will use a hard 3D cut-off on the momentum.

Taking the chiral limit, the gap equation takes the form:

$$m = \frac{GN_f N_c}{\pi^2} m \left[\Lambda_{3D} \sqrt{\Lambda_{3D}^2 + m^2} - m^2 \ln \left(\frac{\Lambda_{3D} + \sqrt{\Lambda_{3D}^2 + m^2}}{m} \right) \right]$$

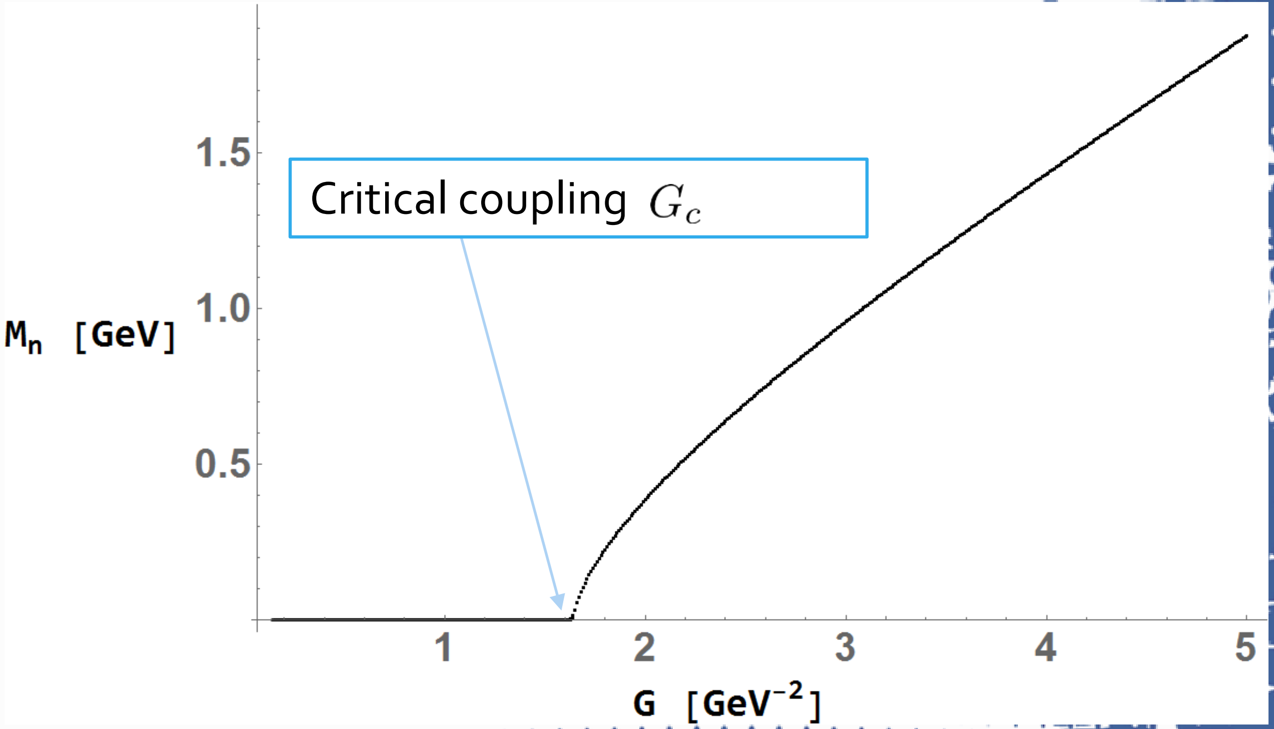
Where Λ_{3D} is the regularization parameter and N_c and N_f are the color and flavors species respectively.

We will use $N_c = 3$ and $N_f = 2$.

Gap equation

Solution $\Lambda_{3D} = 1$

Is there an analytical way to obtain the critical coupling G_c as a function of the regularization parameter?



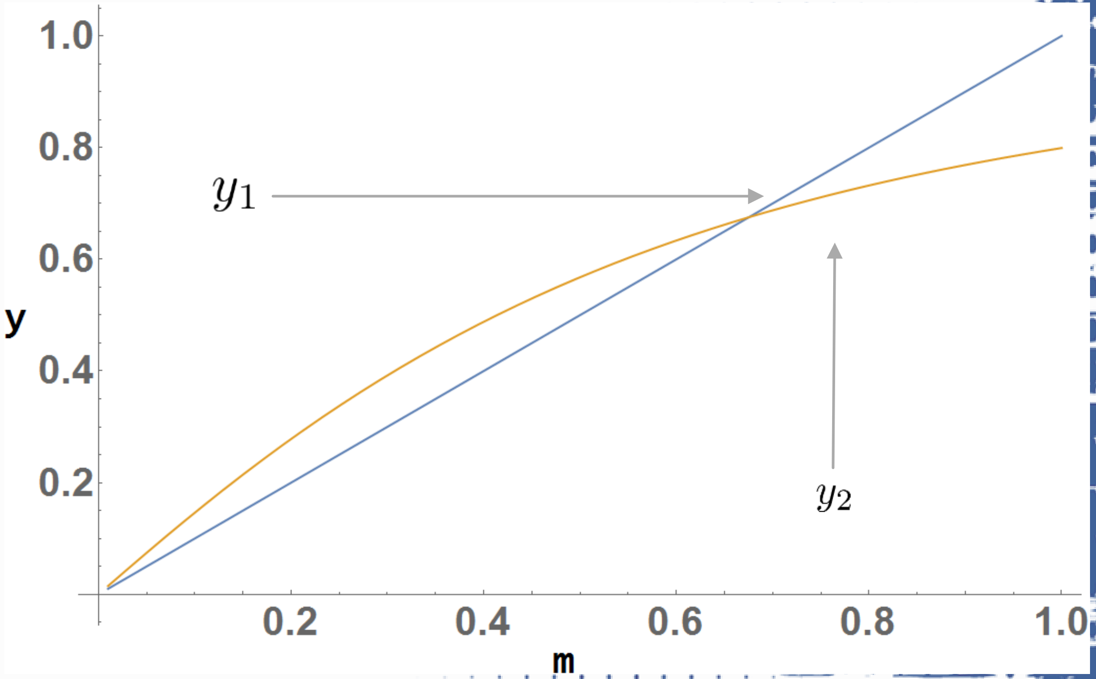
Critical coupling

Splitting the right and left hand of the gap equation:

$$y_1 = m$$

$$y_2 = \frac{GN_fN_c}{\pi^2} m \left[\Lambda_{3D}\sqrt{\Lambda_{3D}^2 + m^2} - m^2 \ln\left(\frac{\Lambda_{3D} + \sqrt{\Lambda_{3D}^2 + m^2}}{m} \right) \right]$$

And then we plot.



Critical coupling

Imposing equal derivatives:

$$\left.\frac{d}{dm}m\right|_{m=0} = G_c \left.\frac{d}{dm}mI(m)\right|_{m=0}$$

In our case

$$I(m) = \frac{N_f N_c}{\pi^2}\left[\Lambda_{3D}\sqrt{\Lambda_{3D}^2 + m^2} - m^2 \ln\left(\frac{\Lambda_{3D} + \sqrt{\Lambda_{3D}^2 + m^2}}{m}\right)\right]$$

Critical coupling

After this we get the condition

$$1 = -\frac{6G_c\left(3m^2\left(m^2+\sqrt{m^2+1}+1\right)\log\left(\frac{\sqrt{m^2+1}+1}{m}\right)-\left(3m^2+1\right)\left(\sqrt{m^2+1}+1\right)\right)}{\pi^2\sqrt{m^2+1}\left(\sqrt{m^2+1}+1\right)}$$

Taking $m \to 0$

$$1 = \frac{6}{\pi^2}G_c \quad \Rightarrow \quad G_c = \frac{\pi^2}{6} \approx 1.644$$

Critical coupling: Thermal bath

Furthermore, we can add the effects of the temperature. Using the Matsubara formalism.

$$\int \frac{dp_0}{2\pi} f(p_0) \quad \longrightarrow \quad T \sum_{n=-\infty}^{\infty} f(\omega_n)$$

$$\omega_n = 2(n + \frac{1}{2})\pi T$$

$$E = \sqrt{m^2 + p^2}$$

$$m = \frac{12}{\pi^2} G m \int_0^1 \frac{p^2}{E} \left(1 - \frac{2}{e^{\frac{E}{T}} + 1}\right) dp$$

Critical coupling: Thermal bath

Imposing the our critical condition and evaluating at $m \to 0$

$$1 = G_c \frac{12}{\pi^2} \int_0^1 p \left(1 - \frac{2}{e^{\frac{p}{T}} + 1}\right) dp$$

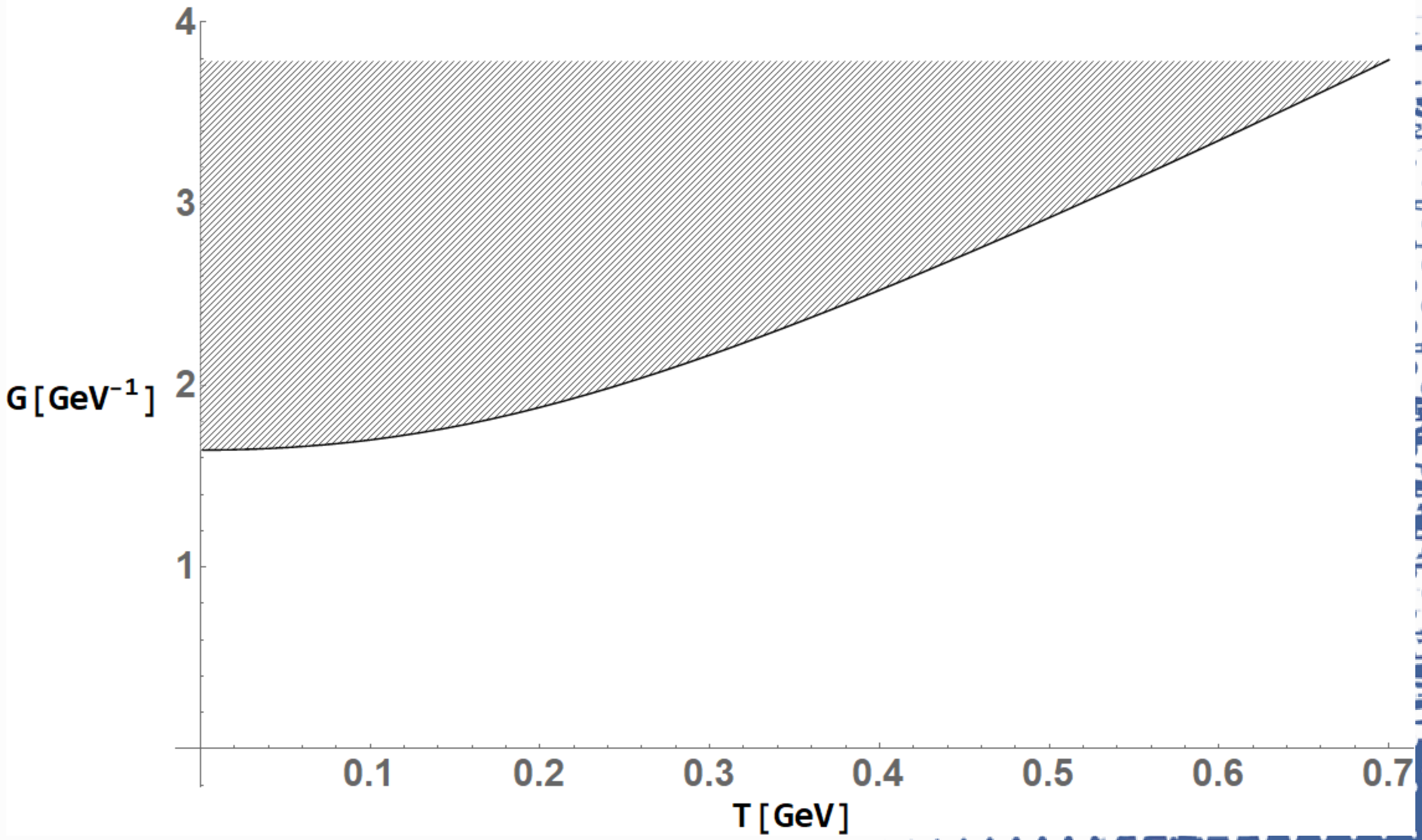
$$= \frac{12}{\pi^2} G_c \left(\frac{1}{2} - \frac{\pi^2 T^2}{12} + T \log(1 + e^{-\frac{1}{T}}) - T^2 \mathrm{Li}_2(-e^{-\frac{1}{T}}) \right)$$

Here Li_2 is the Polylogarithm function defined as

$$\mathrm{Li}_{s+1}(x) = x \frac{(-1)^s}{s!} \int_0^1 \frac{\log^s(t)}{1-t\,x} dt$$

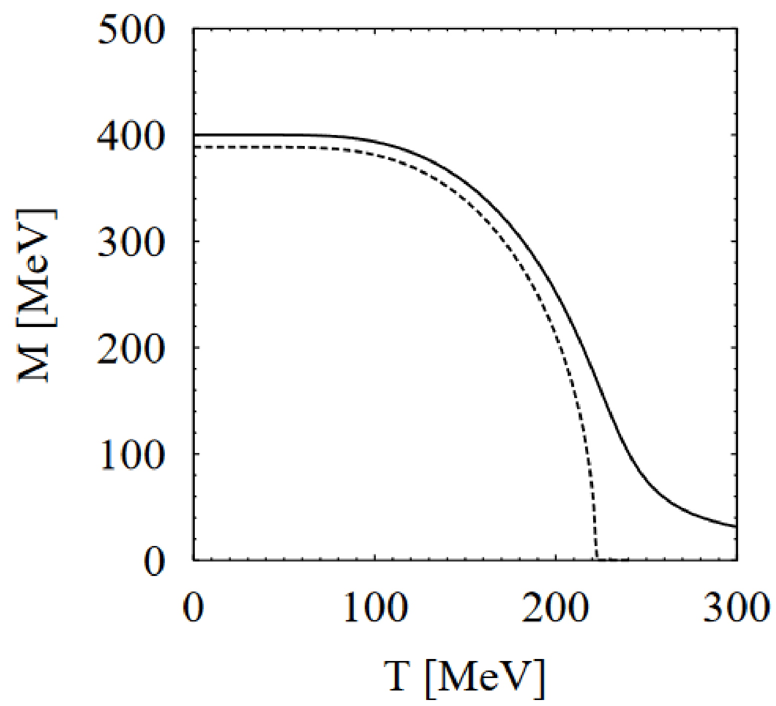
Critical coupling: Thermal bath

$\Lambda_{3D} = 1$



Transition temperature

How we can use this critical condition to get the transition temperature T_c ?



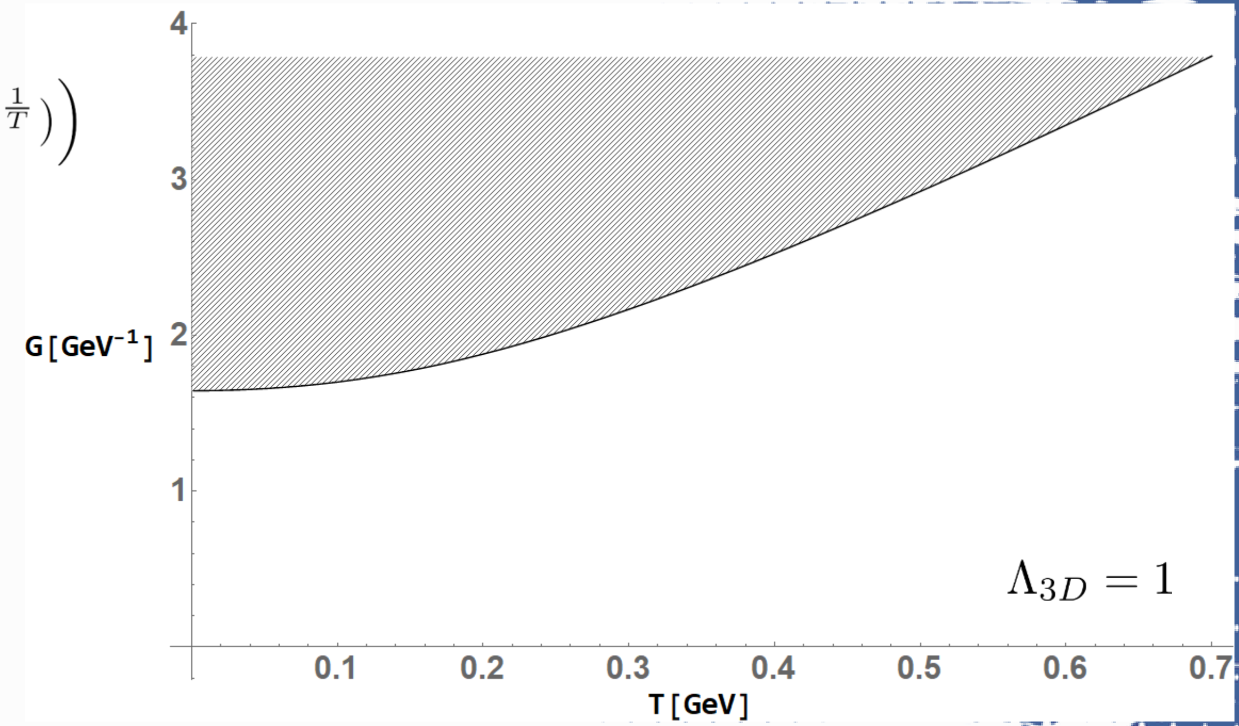
NJL model analysis of quark matter at large density - Buballa, Michael Phys.Rept. 407 (2005) 205-376 hep-ph/0402234

Transition temperature

$$1 = \frac{12}{\pi^2} G_c \left(\frac{1}{2} - \frac{\pi^2 T^2}{12} + T \log(1 + e^{-\frac{1}{T}}) - T^2 \mathrm{Li}_2(-e^{-\frac{1}{T}}) \right)$$

$$G_0 = 3$$

$$T \approx 0.518$$



Phase-diagram $\mu - T$

The more interesting case is when the chemical potential μ is taken into account.

We can add the chemical potential with the substitution:

$$w_n \rightarrow w_n + \mu$$

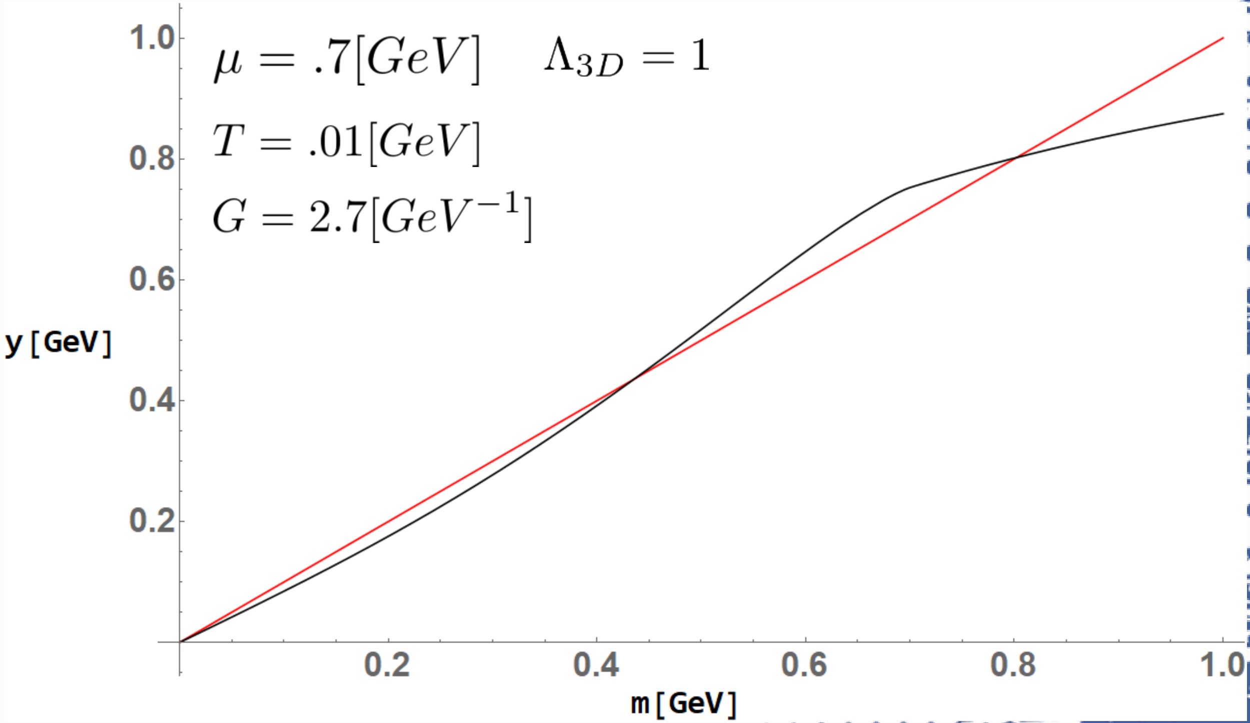
Then the gap equation becomes

$$m = \frac{12}{\pi^2} G m \left(\int_0^1 dp \frac{p^2}{\sqrt{m^2 + p^2}} - \int_0^1 dp \frac{p^2}{\sqrt{m^2 + p^2}} \left(\frac{1}{e^{\frac{\sqrt{m^2+p^2}+\mu}{T}} + 1} + \frac{1}{e^{\frac{\sqrt{m^2+p^2}-\mu}{T}} + 1} \right) \right)$$

Phase-diagram $\mu - T$

We can see the effects of the chemical potential right away, the kernel (right-hand side of the gap equation) now has a “bump”. This is highly significant because It adds the possibility of discontinuous transitions.

Beware we can not longer evaluate the critical condition at $m = 0$.



Phase-diagram $\mu - T$

To solve this issue consider the problem of maximizing a curve with a restriction, is this framework:

- The right hand side of the gap equation becomes the curve to maximize.
- The curve $y = m$ becomes the restriction.
- The coupling constant G is the Lagrange multiplier that we want to know for each value of T, μ and Λ_{3D}.

Phase-diagram $\mu - T$

Then the equation system becomes:

$$m_0 = m_0 G_c I(m_0; \mu, T) \tag{1}$$

$$1 = G_c \Big[I(m_0; \mu, T) + m_0 I'(m_0; \mu, T) \Big] \tag{2}$$

Where

- m_0 is the value of the mass that maximizes the curve $m_0 G_c I(m_0; \mu, T)$ restricted by $y = m$ for a set of fixed values of T, μ and Λ_{3D}.
- G_c is the critical coupling.

Phase-diagram $\mu - T$

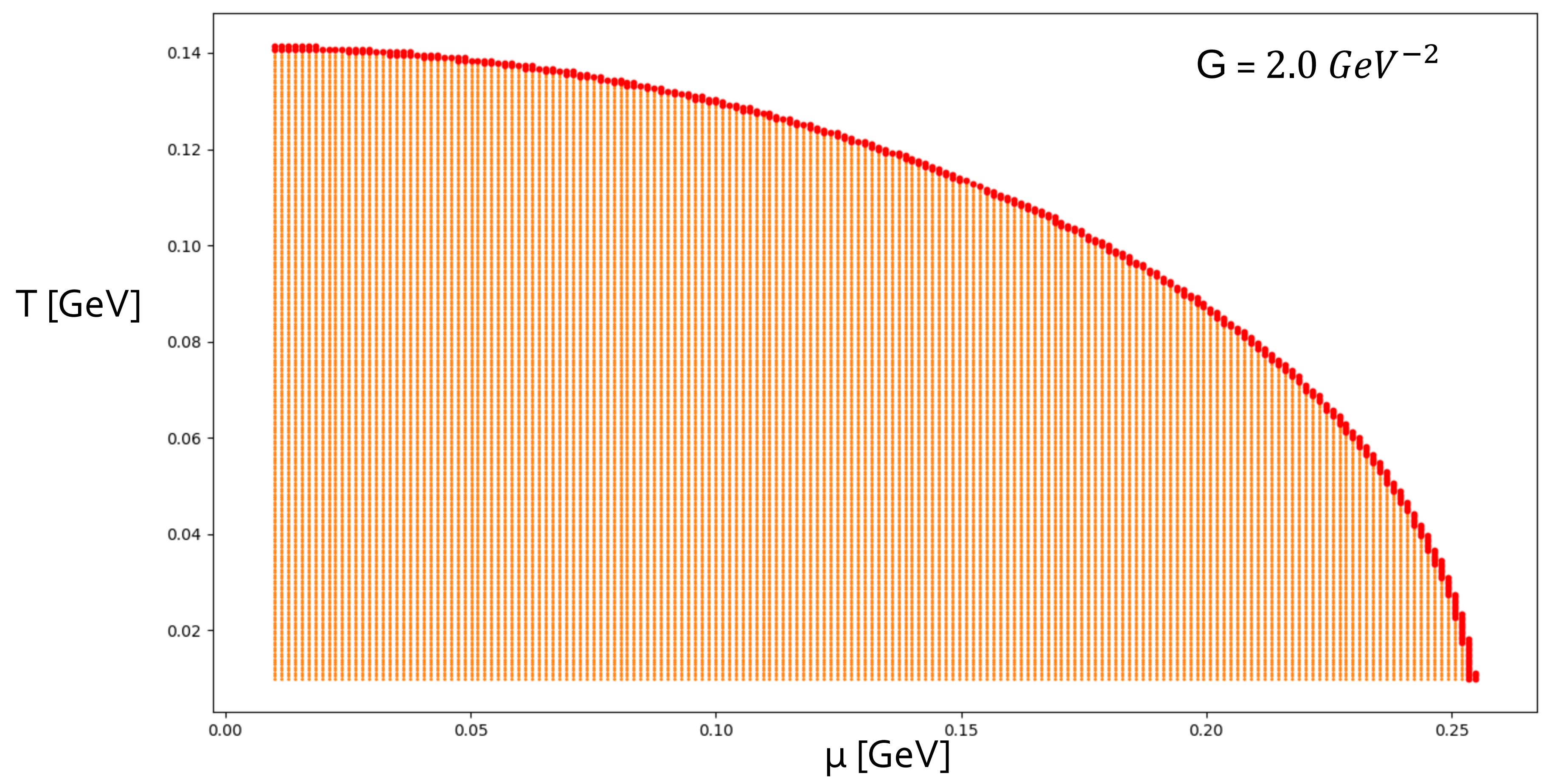
Combining equation (1) and (2) yields a new condition for m_0.

$$m_0^2 I'(m_0; \mu, T) = 0$$

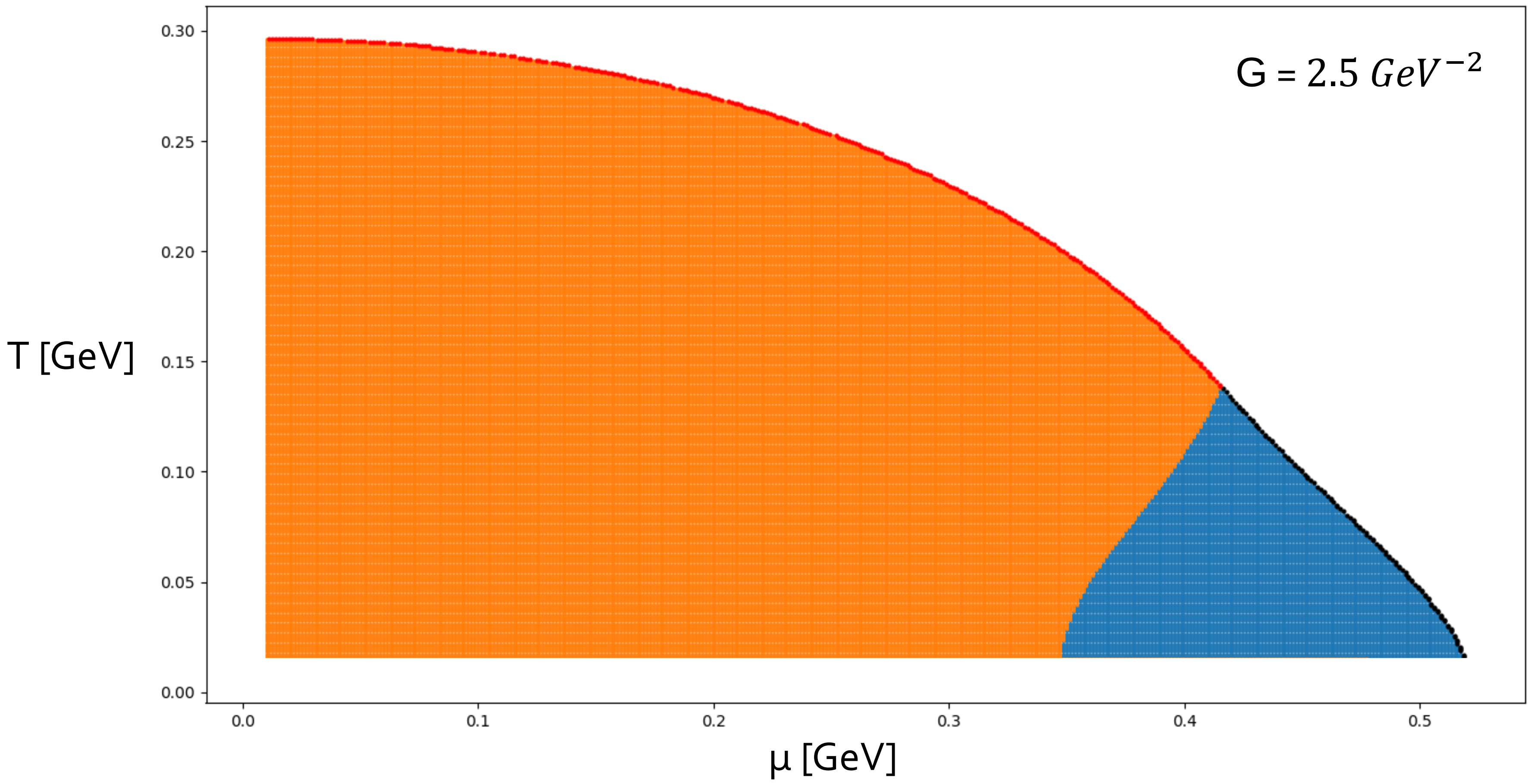
This equation has trivial solutions $m_0 = 0$ thus returning to the last case. This values define a second order phase-transition.

In the other hand now there is a set of values μ, T and Λ_{3D} from which the condition has $m_0 \neq 0$ solutions. This set of values define the first order phase-transitions.

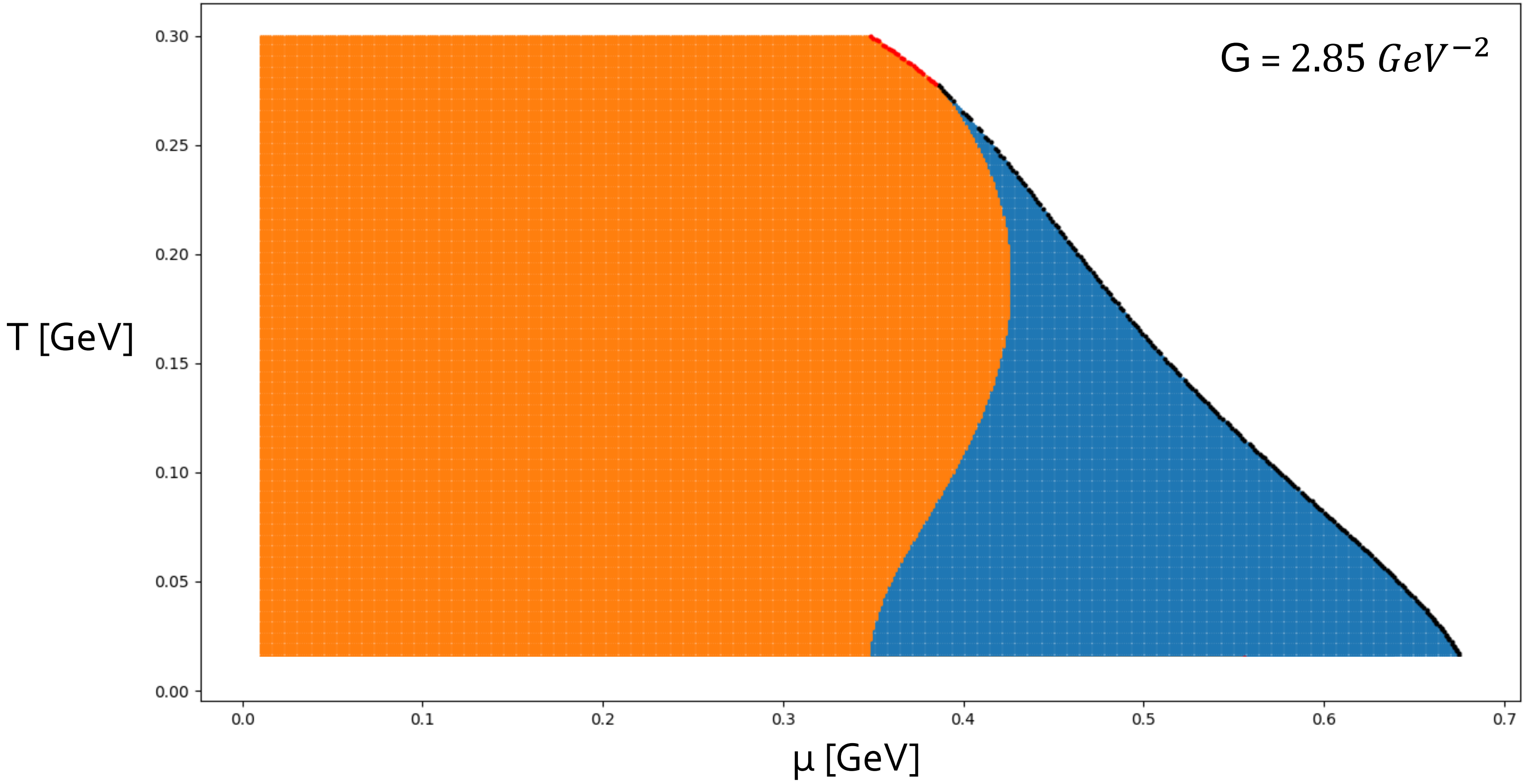
Phase-diagram $\mu - T$



Phase-diagram $\mu - T$



Phase-diagram $\mu - T$



Conclusions

- We were able to draw the phase diagram for any range of values of the temperature and chemical potential.
- There is the need to search further and see it the method will be applicable in more complex models where the mass depends on the momentum.

Thank you