The $P_c(4312)^+$ exotic

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Introduction
Minimal quark model

Figure 15.1: SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the u, d, s, and c quarks as a function of isospin $I_Z$, charm C, and hypercharge $Y = B + S - C$. The nonets of light mesons occupy the central planes to which the $c\bar{c}$ states have been added.

The weight diagrams for the ground-state pseudoscalar (0−−+) and vector (1−−) mesons are depicted in Fig. 15.1. The light quark mesons are members of nonets building the middle plane in Fig. 15.1(a) and (b). Isoscalar states with the same $J^{PC}$ will mix, but mixing between the two light quark isoscalar mesons, and the much heavier charmonium or bottomonium states, are generally assumed to be negligible. In the following, we shall use the generic names $a$ for the $I_Z = 1$, $K$ for the $I_Z = 1/2$, and $f$ and $f'$ for the $I_Z = 0$ member of the light quark nonets. Thus, the physical isoscalars are mixtures of the SU(3) wave functions $\psi_8^+$ and $\psi_1^+$:

$$f' = \psi_8^+ \cos \theta - \psi_1^+ \sin \theta, \quad (15.4)$$
$$f = \psi_8^+ \sin \theta + \psi_1^+ \cos \theta, \quad (15.5)$$

Figure 15.4: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

For the “ordinary” baryons (no c or b quark), flavor and spin may be combined in an approximate flavor-spin SU(6), in which the six basic states are $d^\uparrow$, $d^\downarrow$, $\cdots$, $s^\downarrow$ ($\uparrow$, $\downarrow$ = spin up, down). Then the baryons belong to the multiplets on the right side of

$$6 \otimes 6 \otimes 6 = 56_S \oplus 70_M \oplus 70_M \oplus 20_A. \quad (15.24)$$

These SU(6) multiplets decompose into flavor SU(3) multiplets as follows:

$$56 = 4_1^2 \oplus 2_8 \quad (15.25a)$$
Infinite options for color singlets

- Compact pentaquark
- Baryon-meson molecule
- Glueball
- Compact tetraquark
- Meson-meson molecule
- Hybrid

(Multi-baryon molecules are called “nuclei”)
State superposition

\[ |M\rangle = \alpha_0 |q\bar{q}\rangle + \alpha_1 |gg\rangle + \alpha_2 |q\bar{q}g\rangle + \alpha_3 |q\bar{q}gg\rangle + \alpha_4 |q\bar{q}qq\rangle + \ldots \]

\[ |B\rangle = \alpha_0 |qqq\rangle + \alpha_1 |qqqq\bar{q}\rangle + \alpha_2 |qqqg\rangle + \alpha_3 |qqqq\bar{g}\rangle + \ldots \]

\[ \sum_i |\alpha_i|^2 = 1 \]
Example: pentaquark

\[ J/\psi\ p \rightarrow P_c \rightarrow J/\psi\ p \]

\[ [c\bar{c}]\ [uud] \rightarrow [c\bar{c}uud] \rightarrow [c\bar{c}]\ [uud] \]

The minimal quark content is that of a pentaquark.
hadron character of the description of the data, which peaks at processes is implied throughout). A model-dependent six-
the observation of significant analysis of Run 1 data, the LHCb Collaboration reported achieved at the Large Hadron Collider when, from an
Λ
structure peaking at
m
5
P
−
distribution alone, the amplitude analysis also
reflections ( Loosely bound molecular baryon-meson pentaquark states
5
p
−
contribu-
Λ
→
0
J=
4
m
−
P
→
Pc(4312), Pc(4380), Pc(4440), Pc(4450)

Signals
Amplitude analysis of the $P_c(4312)^+$
The main conclusion from these studies is that the dominant systematic uncertainty is due to possible interference between various states from other contributions that vary slowly with cubic quantum-number assignments. These states are described by relativistic Breit–Wigner (BW) amplitudes. These fits are performed to study the robustness of the measured signal, where the angle between the parent. We comment on the results of three-channel fit described in Ref. [1] awaits completion of an amplitude parameterization has often been discussed in the context of the Weinberg compositeness criterion [35] according to cases A (left) and B (right). The latter is hidden from the physical region, a pole here will appear as peaks with Breit-Wigner-like lineshape in the lower half of the III sheet. Poles in these sheets are the thresholds of the two di-pole resonance project into the same partial wave as described in Ref. [1] and (right) when considering various partial waves is added incoherently, and param-eterized with a linear polynomial. The amplitude describes the scattering, i.e., a compact state [1] which is sufficiently far from the physical region between the two thresholds is connected to the lower half of the III sheet. Poles in these sheets are the thresholds of the two di-pole resonance. Therefore, fits alone cannot distinguish between the various partial waves. The events distribution is given by (channel 1) and (channel 2). There are the thresholds of the two di-pole resonance. Therefore, fits alone cannot distinguish between the various partial waves.
Signal interpretation

Triangle singularity
studied in LHCb 1904.03947

Compact pentaquark
Ali, Parkhomenko 1904.00446
Holma, Ohlsson 1906.08499

Virtual state
Burns, Swanson 1908.03528
CFR et al. 1904.10021

Molecule
Wu et al. 1007.0573
Liu et al. 1903.11560
Du et al. 1910.11846

Hadrocharmonium
Eides, Petrov, Polyakov 1904.11616
S-matrix theory

- Probability conservation $\Rightarrow$ Unitarity
- Particle$\leftrightarrow$antiparticle $\Rightarrow$ Crossing symmetry
- Causality $\Rightarrow$ Analyticity and no poles in 1st Riemann sheet
- Additional symmetries: gauge, chiral, etc.
\[ 1 = \sum_n |E_n > < E_n | + \int d\alpha |\alpha > < \alpha | \]

Continuum

Discret
Poles and cuts

- The amplitude is an analytical function in the complex plane
- Singularities determine the amplitude (aka the structure)
  - Poles
  - Cuts
- Singularities are associated to the dynamics

\[
\begin{align*}
A_{11} &= \left| p J/\psi \right> \rightarrow \left| p J/\psi \right> \\
A_{12} &= \left| p J/\psi \right> \rightarrow \left| \Sigma^+_c \bar{D}^0 \right> \\
A_{21} &= \left| \Sigma^+_c \bar{D}^0 \right> \rightarrow \left| p J/\psi \right> \\
A_{22} &= \left| \Sigma^+_c \bar{D}^0 \right> \rightarrow \left| \Sigma^+_c \bar{D}^0 \right>
\end{align*}
\]
Riemann sheets structure

Physical axis
Near-threshold theory: hypotheses

- Hypotheses:
  - Only one partial wave contributes to the signal
  - The threshold drives the physics (tested)
  - Further singularities are irrelevant (tested)

- Caveat:
  - We fit the J/ψ p projection (no info on quantum numbers)
Near-threshold theory: equations

\[ \frac{dN}{d\sqrt{s}} = \rho(s) \left[ |F(s)|^2 + B(s) \right] \]

\[ F(s) = P_1(s)T_{11}(s) \quad \left(T^{-1}\right)_{ij} = M_{ij} - ik_i\delta_{ij} \]

\[ M_{ij}(s) = m_{ij} - c_{ij} s \]

Matrix elements $M_{ij}$ are singularity free and can be Taylor expanded

Frazer, Hendry PR134 (1964) B1307
Near-threshold amplitude

\[ \frac{dN}{d\sqrt{s}} = \rho(s) \left[ |F(s)|^2 + B(s) \right] \]

\[ B(s) = b_0 + b_1 s \]

\[ F(s) = (p_0 + p_1 s) \frac{[m_{22} - c_{22}s - ik_2]}{[m_{22} - c_{22}s - ik_2][m_{11} - c_{11}s - ik_1] - m_{12}^2} \]

Production, hyperons and effects due to further singularities

Scattering length approximation if \( c_{ij} = 0 \)

Only poles on sheets II and IV

If \( c_{ij} \neq 0 \) (effective range approximation); poles in any sheet

Channel coupling
Fits: scattering length vs effective range

2 channel scattering length approximation

$$\chi^2/dof = 48.1/(66 - 7) = 0.82$$

2 channel effective range approximation

$$\chi^2/dof = 43/(66 - 9) = 0.75$$
Poles

Scattering length

Effective range

$M = 4319.7\pm1.6$ MeV  $\Gamma = -0.8\pm2.4$ MeV

$M = 4319.8\pm1.5$ MeV  $\Gamma = 9.2\pm2.9$ MeV
Pole movement: scattering length
Conclusions
Summary of the current consensus

- Universally accepted by the hadron molecule community that the $P_c$s are hadron molecules
- Universally accepted by the quark model community that the $P_c$s are compact pentaquarks
- Universally accepted by the hadrocharmonium community that the $P_c$s are hadrocharmonia
- The triangles community is universally disappointed because LHCb rules them out for two of the states
Conclusions

- Seems that $P_c(4312)$ dynamics is driven by the threshold

- Molecule? Virtual state?

- We favor the virtual state explanation

- We have to wait for the quantum numbers, although a lot of (sensible) speculation is already in the market
Thanks.