# Flavor Symmetry from String Theory

Alexander Baur

Instituto de Física UNAM & Technical University of Munich

Reunión Anual de la División de Partículas y Campos de la SMF – 10.07.2020

# Flavor Symmetry from String Theory

based on: A. B. , H.P. Nilles, A. Trautner, P.K.S. Vaudrevange – 1901.03251
A. B. , H.P. Nilles, A. Trautner, P.K.S. Vaudrevange – 1908.00805
A. B. , M. Kade, H.P. Nilles, S. Ramos-Sánchez, P.K.S. Vaudrevange – 2008.xxxxx

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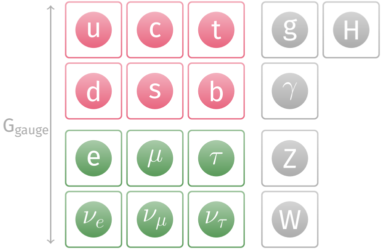
# Motivation

We observe a flavor structure:

- three flavors of SM particles
- mass patterns (quarks, leptons, neutrinos)
- mass mixings (CKM, PMNS)



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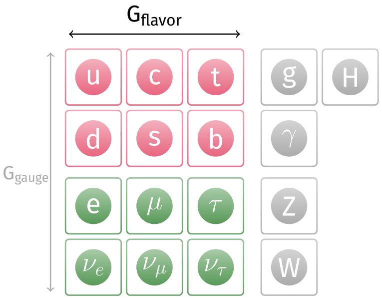


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- mass mixings (CKM, PMNS)

A simple (symmetry driven) explanation:

$$G_{\text{gauge}} \times G_{\text{flavor}}$$

# Goals

1. To try to convince you that flavor symmetries arise naturally from extra dimensions

2. To show that orbifold compactifications give rise to eclectic flavor symmetries

# Bottom up

## Traditional Way

Review: [S. T. Petcov: 1711.10806]
Example: $A_4$ model for $q$,$e$,$\nu$:
[F. Anda, N. Nath, J. Valle, C. Vaquera-Araujo: 2004.06735]

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## Traditional Way

- Discrete flavor symmetry $G_f$
- Extra particles 'flavons' break $G_f$

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$$\Psi \to \rho(g)\,\Psi\;, \qquad g \in G_f$$

- Yukawa couplings are constants

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## Modular Way

Seminal: [F. Feruglio: 1706.08749]
Example: modular $A_4$ model for $q$,$e$,$\nu$:
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## Modular Way

- Modular flavor symmetry $G_f$
- Add a modulus $T$ to your theory

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## Modular Way

- Modular flavor symmetry $G_f$
- Add a modulus $T$ to your theory
- SM-particles transform with an automorphy factor under $G_f$

$$\Psi \to \varphi(g,T)\,\rho(g)\,\Psi\;, \qquad g \in G_f$$

- Yukawa couplings are modular forms (functions of $T$)

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- Yukawa couplings are modular forms (functions of $T$)
- Give a VEV to the modulus $T$

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# Extra dimensions from string theory

What you need to know about string theory (for this talk):

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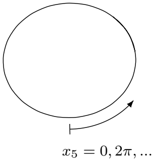
We can roll up 1 extra dimension to




$\sim (10^{-35}\ \text{cm})$

Circle $S^1$

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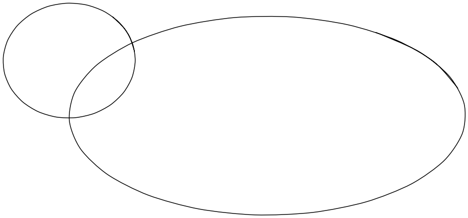
We can roll up 2 extra dimension to



2 circles $S^1 \times S^1$

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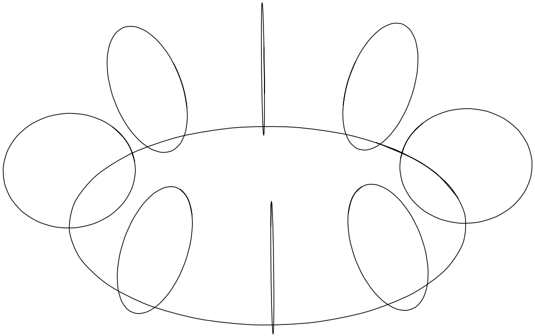

Torus $\mathbb{T}^2$

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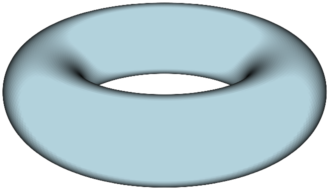

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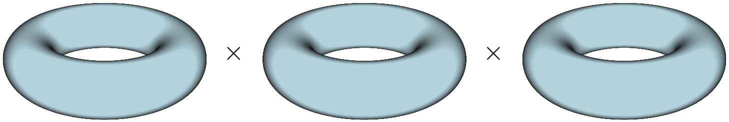

3 Tori $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$

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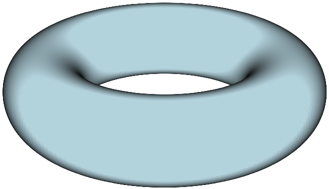

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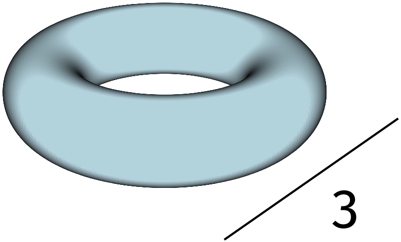


Torus $\mathbb{T}^2$ mod $\mathbb{Z}_3$

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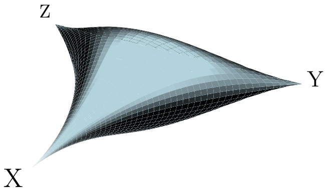


Orbifold $\mathbb{T}^2/\mathbb{Z}_3$

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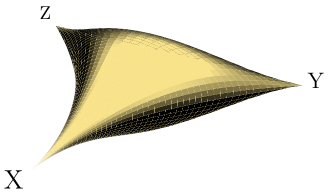


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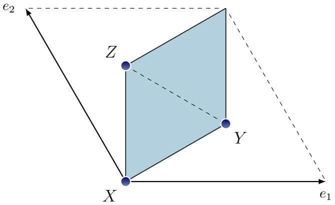
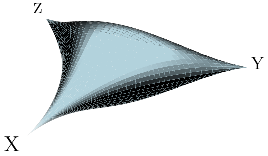



$\simeq$




- Singularities $X$,$Y$,$Z$
- Winded strings
- Twisted strings

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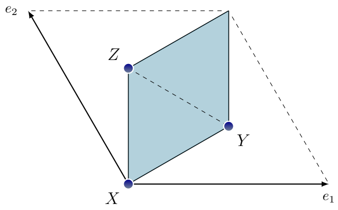


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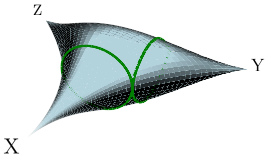


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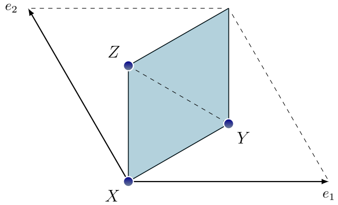


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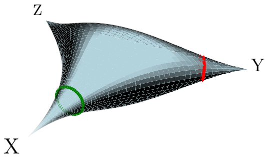


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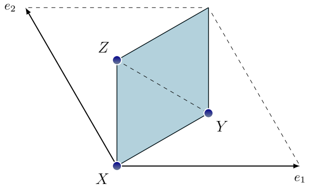
# How to obtain the flavor symmetry

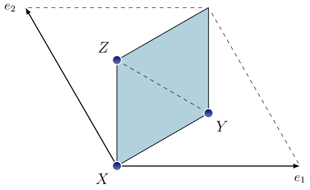
Traditional approach



# How to obtain the flavor symmetry

Traditional approach

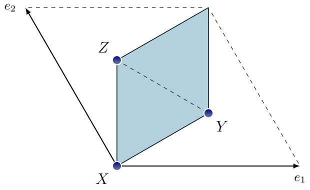
Geometrical symmetries $S_3$



# How to obtain the flavor symmetry

Traditional approach

| | |
|---|---|
| Geometrical symmetries | $S_3$ |
| String selection rules | $\mathbb{Z}_3 \times \mathbb{Z}_3$ |



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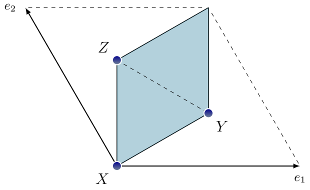
Traditional approach

Geometrical symmetries $S_3$

String selection rules $\mathbb{Z}_3 \times \mathbb{Z}_3$

$\Delta(54)$

"Traditional flavor symmetry"




Classification traditional appro.: [Olguin-Trejo, Pérez-Martínez, Ramos-Sánchez:1808.06622 ][Kobayashi et al.: hep-ph/0611020]
Pheno with $\Delta(54)$ flavor from orbifolds: [Carballo-Pérez, Peinado, Ramos-Sánchez: 1607.06812]

# How to obtain the flavor symmetry

**New approach**

1. Mathematically describe the orbifold, i.e. space group
2. Derive the symmetries of this space group, i.e. automorphisms

[A. B. , H. P. Nilles, A. Trautner, P. Vaudrevange: 1901.03251, 1908.00805]

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**Traditional approach**

| | |
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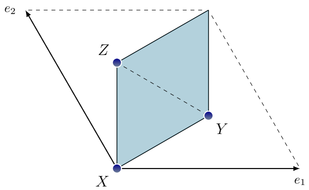
| "Traditional flavor symmetry" | $\Delta(54)$ |

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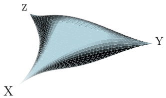
# Resulting flavor symmetries

$\mathbb{T}^2/\mathbb{Z}_3$




| flavor symmetry | |
|---|---|
| traditional | ??? |
| $\Delta(54)$ | $A_4 \rtimes \mathbb{Z}_2$ |

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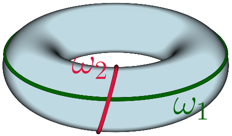
# MODULAR SYMMETRIES

Our Orbifolds are based on a torus!

Parameterization of a torus




modulus: $T = \frac{\omega_2}{\omega_1}$

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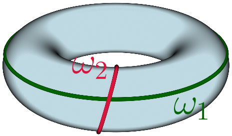
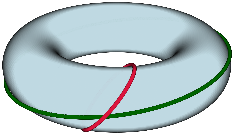


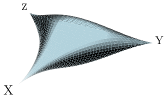
modulus: $T = \frac{\omega_2}{\omega_1}$

$$T' = \frac{a\,T + b}{c\,T + d}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})$ generate a modular symmetry.

# Resulting flavor symmetries

$$\mathbb{T}^2/\mathbb{Z}_3$$




| flavor symmetry | |
|---|---|
| traditional | modular |
| $\Delta(54)$ | $A_4 \rtimes \mathbb{Z}_2$ |

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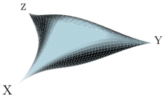


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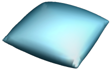

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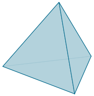

| flavor symmetry | |
|---|---|
| traditional | modular |
| $\dfrac{D_8 \times D_8}{\mathbb{Z}_2}$ | $(S_3 \times S_3) \rtimes \mathbb{Z}_2$ |

14 more ...

[A. B. , H. P. Nilles, A. Trautner, P. Vaudrevange: 1901.03251, 1908.00805]

[A. B. , M. Kade, H. P. Nilles, S. Ramos-Sánchez, P. Vaudrevange: 2008.xxxxx]

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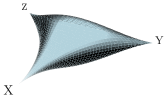
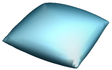



| flavor symmetry | |
|---|---|
| traditional | modular |
| $\Delta(54)$ | $A_4 \rtimes \mathbb{Z}_2$ |
| $\mathbf{3}$ | $\mathbf{2}' \oplus \mathbf{1}$ |

$\mathbb{T}^2/\mathbb{Z}_2$



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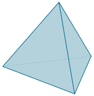

| flavor symmetry | |
|---|---|
| traditional | modular |
| $\dfrac{D_8 \times D_8}{\mathbb{Z}_2}$ | $(S_3 \times S_3) \rtimes \mathbb{Z}_2$ |
| $\mathbf{4}$ | $\mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1}$ |

14 more …

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# Eclectic flavor groups

$G_{\text{traditional}}$ $G_{\text{modular}}$

# Eclectic flavor groups

$$G_{\text{traditional}} \qquad \rtimes \qquad G_{\text{modular}}$$

# Eclectic flavor groups

$$G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}}$$

[H. P. Nilles, S. Ramos-Sánchez, P. Vaudrevange: 2001.01736, 2004.05200 , 2006.03059]

# Eclectic flavor groups

Normal subgroup of $G_{\text{ecl.}}$

↑

$$G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}}$$

[H. P. Nilles, S. Ramos-Sánchez, P. Vaudrevange: 2001.01736, 2004.05200 , 2006.03059]

# Eclectic flavor groups

Normal subgroup of $G_{\text{ecl.}}$

Consists only of outer automorphisms of $G_{\text{trad.}}$

$$G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}}$$

[H. P. Nilles, S. Ramos-Sánchez, P. Vaudrevange: 2001.01736, 2004.05200 , 2006.03059]

# Eclectic flavor groups

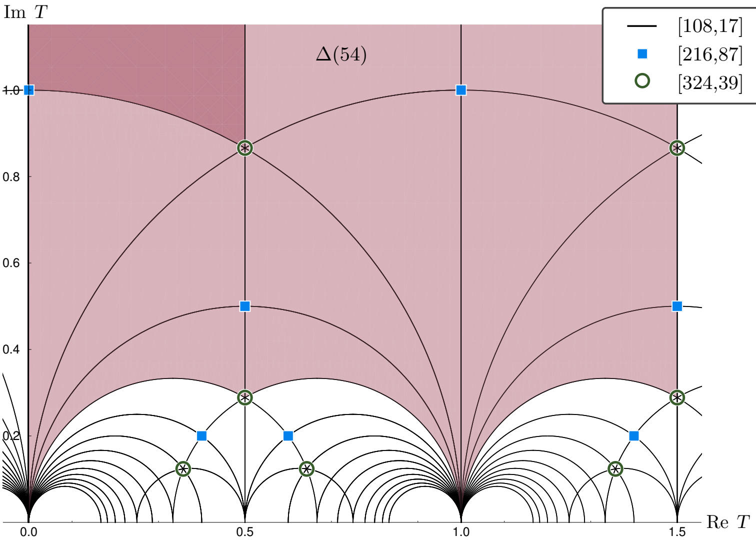
$$G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}}$$

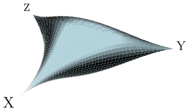
$G_{\text{traditional}}$: ↑ Normal subgroup of $G_{\text{ecl.}}$ ; ↓ Broken by flavons

$G_{\text{modular}}$: ↑ Consists only of outer automorphisms of $G_{\text{trad.}}$ ; ↓ Broken by $< T >$

[H. P. Nilles, S. Ramos-Sánchez, P. Vaudrevange: 2001.01736, 2004.05200 , 2006.03059]

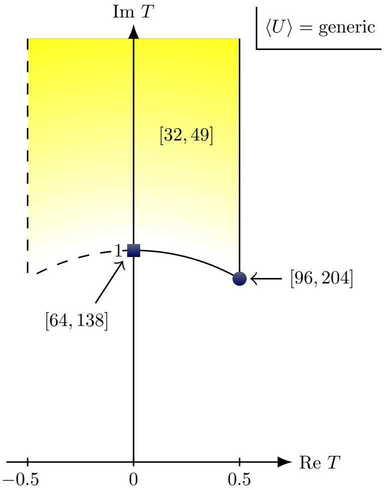
# Flavor symmetries in moduli space – $\mathbb{T}^2/\mathbb{Z}_3$



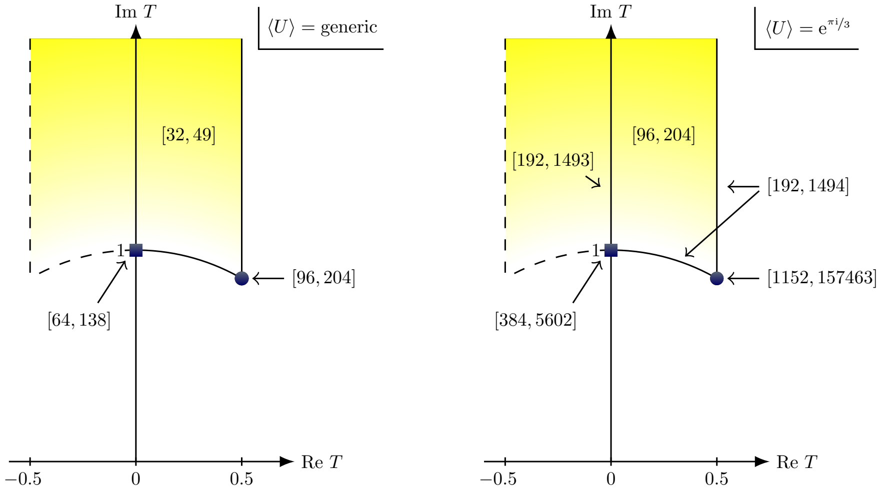

Moduli space of the $\mathbb{T}^2/\mathbb{Z}_3$ Orbifold. $[108, 17]$ referrs to SmallGroup$(108, 17)$ of the SmallGroups Library of GAP.

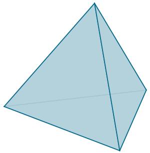
# Flavor symmetries in moduli space – $\mathbb{T}^2/\mathbb{Z}_2$




Moduli space of the $\mathbb{T}^2/\mathbb{Z}_2$ Orbifold. $[32, 49]$ referrs to SmallGroup$(32, 49)$ of the SmallGroups Library of GAP.

# Flavor symmetries in moduli space – $\mathbb{T}^2/\mathbb{Z}_2$




Moduli space of the $\mathbb{T}^2/\mathbb{Z}_2$ Orbifold. $[32, 49]$ referrs to SmallGroup$(32, 49)$ of the SmallGroups Library of GAP.

# Conclusion

- Flavor symmetries naturally arise from compactification of extra dimensions
- Orbifold compactifications give rise to eclectic flavor symmetries consisting of traditional and modular symmetries
- How would this influence $\nu$-physics and DM?

  $\longrightarrow$ talk by E. Garcés N. Nath, L. Flores,

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Thank you!