Tau physics

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S. Gonzàlez-Solís, A. Miranda, J. Rendón, P. Roig; Phys.Lett.B 804 (2020) 135371

S. Gonzàlez-Solís and P. Roig; Eur. Phys. J. C79 (2019) 436

R. Escribano, S. Gonzàlez-Solís, M. Jamin and P. Roig; JHEP 1409 (2014) 042

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Outline

1. Introduction
2. Hadronic tau decays: form factors
3. Hadronic tau decays as probes of non-SM interactions
4. Prospects for tau physics at Belle-II
5. Outlook

Introduction

Evidence for Anomalous Lepton Production in e^+–e^- Annihilation*

M. L. Perl, G. S. Abrams, A. M. Boyarski, M. Breidenbach, D. D. Briggs, F. Bulos, W. Chinowsky, J. T. Dakin,† G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber, G. Hanson, F. B. Heile, B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke, D. Lüke,‡ B. A. Lulu, V. Lüth, D. Lyon, C. C. Morehouse, J. M. Paterson, F. M. Pierre,§ T. P. Pun, P. A. Rapidis, B. Richter, B. Sadoulet, R. F. Schwitters, W. Tanenbaum, G. H. Trilling, F. Vannucci,‖ J. S. Whitaker, F. C. Winkelmann, and J. E. Wiss

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(Received 18 August 1975)

We have found events of the form $e^+ + e^- \rightarrow e^\pm + \mu^\mp$ + missing energy, in which no other charged particles or photons are detected. Most of these events are detected at or above a center-of-mass energy of 4 GeV. The missing-energy and missing-momentum spectra require that at least two additional particles be produced in each event. We have no conventional explanation for these events.

We have found 64 events of the form

$$e^+ + e^- \rightarrow e^\pm + \mu^\mp + \geq 2 \text{ undetected particles} \quad (1)$$

for which we have no conventional explanation. The undetected particles are charged particles or photons which escape the 2.6π sr solid angle of the detector, or particles very difficult to detect such as neutrons, $K_L{}^0$ mesons, or neutrinos. Most of these events are observed at center-of-mass energies at, or above, 4 GeV. These events were found using the Stanford Linear Accelerator Center–Lawrence Berkeley Laboratory (SLAC-

Events corresponding to (1) are the signature for new types of particles or interactions. For example, pair production of heavy charged leptons[1-4] having the decay modes $l^- \rightarrow \nu_l + e^- + \bar{\nu}_e$, $l^+ \rightarrow \bar{\nu}_l + e^+ + \nu_e$, $l^- \rightarrow \nu_l + \mu^- + \bar{\nu}_\mu$, and $l^+ \rightarrow \bar{\nu}_l + \mu^+ + \nu_\mu$ would appear as such events. Another possi-

SLAC-PUB-1923
April 1977
(T/E)

EVIDENCE FOR, AND PROPERTIES OF, THE NEW CHARGED HEAVY LEPTON* †

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ABSTRACT

This paper summarizes the evidence for, and the properties of, the mass $1.9 \pm .1\ \mathrm{GeV}/c^2$ charged heavy lepton recently found in e^+e^- annihilation.

3. EVIDENCE FOR EXISTENCE OF THE τ: $e\mu$ EVENTS

The reaction

$$e^+ + e^- \to e^\pm + \mu^\mp + \text{no other particles detected}$$

produced thru

$$e^+ + e^- \to \underset{\displaystyle \bar{\nu}_\tau e^+ \nu_e}{\underset{\downarrow}{\tau^+}} \qquad + \underset{\displaystyle \nu_\tau \mu^- \bar{\nu}_\mu}{\underset{\downarrow}{\tau^-}}$$

Test of lepton universality

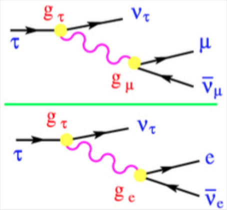
- Purely leptonic

$$\Gamma(\ell' \to \ell \bar{\nu}_\ell \nu_{\ell'}) = \frac{G_{\ell'} G_\ell m_{\ell'}^5}{192\pi^3} f\left(\frac{m_\ell^2}{m_{\ell'}^2}\right) R_W^{\ell'} R_\gamma^{\ell'} ,$$

$$G_x = \frac{g_x^2}{4\sqrt{2} M_W^2} , \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x ,$$

  - Experimental input: $BR(\tau \to e \bar{\nu}_e \nu_\tau) = 17.82(4)\%$, $BR(\tau \to \mu \bar{\nu}_\mu \nu_\tau) = 17.39(4)\%$
  - Theory: rad. corrections $R_W^\tau R_\gamma^\tau = 0.992$ (Marciano'88)

    - $\frac{\Gamma(\tau \to \mu \bar{\nu}_\mu \nu_\tau)}{\Gamma(\tau \to e \bar{\nu}_e \nu_\tau)} \Rightarrow \left(\frac{g_\mu}{g_e}\right) = 1.0018(14)$ $\longleftarrow$
    - $\frac{\Gamma(\tau \to e \bar{\nu}_e \nu_\tau)}{\Gamma(\mu \to e \bar{\nu}_e \nu_\mu)} \Rightarrow \left(\frac{g_\tau}{g_\mu}\right) = 1.0010(14)$
    - $\frac{\Gamma(\tau \to \mu \bar{\nu}_\mu \nu_\tau)}{\Gamma(\mu \to e \bar{\nu}_e \nu_\mu)} \Rightarrow \left(\frac{g_\tau}{g_e}\right) = 1.0029(14)$

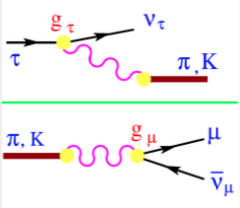


  - Universality i.e. $g_e = g_\mu = g_\tau \equiv g$ tested at 0.14% from tau leptonic Brs

- Semi-Leptonic decays ($h = \pi$ or K)

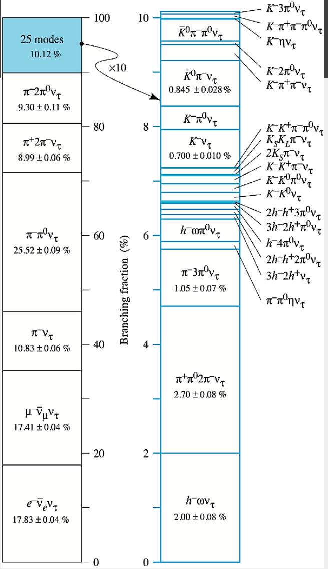
$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta R_{\tau/h}) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 ,$$

  - We measure:
    - $\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9958(26)(14)$
    - $\left(\frac{g_\tau}{g_\mu}\right)_K = 0.9879(63)$
  - Averaging the three g_τ/g_μ
    - $\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 0.9999(14)$
  - Similar tests with electrons but less precise



Hadronic tau decays

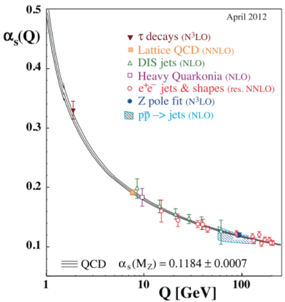
- Tau properties:
  - Mass: $m_\tau = 1.77686(12)$ GeV
  - Lifetime: $\tau_\tau = 2.903(15) \times 10^{-13}$ s
- The only lepton heavy enough to decay into hadrons:
  - Very rich phenomenology
  - Test of QCD and EW interactions
- For the test:
  - Precise measurements needed
  - Hadronic uncertainties under control
- Tau decays: tool to search for New Physics



Test of QCD and ElectroWeak Interactions

- Inclusive decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$

  Full hadron spectra (precision physics)

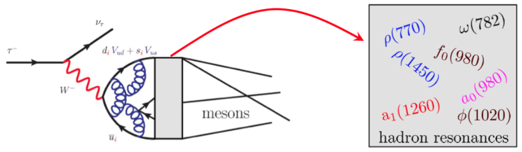


  Fundamental SM parameters: $\alpha_s(m_\tau), m_s, |V_{us}|$

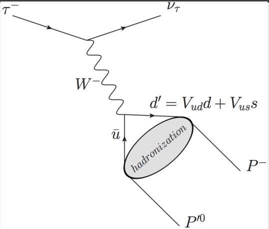
- Exclusive decays: $\tau^- \to (PP, PPP, \ldots)\nu_\tau$

  specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)



- $\tau^- \to P^- \nu_\tau$: $F_{\pi,K}$ (decay constants)
- $\tau^- \to (2P)^- \nu_\tau$: good control
- $\tau^- \to (3P)^- \nu_\tau$: reasonable good control
- $\tau^- \to (> 3P)^- \nu_\tau$: poor knowledge



$$\mathcal{M}\left(\tau^- \to P^- P'^0 \nu_\tau\right) = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}} \bar{u}(p_{\nu_\tau}) \gamma^\mu (1-\gamma^5) u(p_\tau) \langle P^- P'^0 | d' \gamma^\mu u | 0 \rangle \,,$$

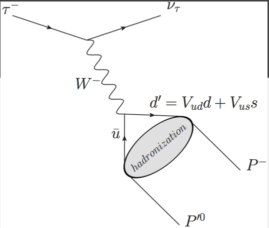
$$\langle P^- P'^0 | d' \gamma^\mu u | 0 \rangle = \mathcal{C}_{P^- P'^0} \left\{ \left(p_- - p_0 - \frac{\Delta_{P^- P'^0}}{s} q \right)^\mu F_V^{P^- P'^0}(s) + \frac{\Delta_{P^- P'^0}}{s} q^\mu F_S^{P^- P'^0}(s) \right\}$$

$$\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\mathrm{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

$$\times \left\{ \left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^- P^0}^{3/2}(s) |F_V^{P^- P^0}(s)|^2 + 3 \frac{\Delta_{P^- P^0}^2}{s^2} \lambda_{P^- P^0}^{1/2}(s) |F_S^{P^- P^0}(s)|^2 \right\}$$

$$\Delta_{P^- P^0} = m_{P^-}^2 - m_{P^0}^2 \,, \quad C_{P' P^0} : \mathrm{Clebsch-Gordon}$$

τ DECAYS INTO TWO MESONS



$$\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

$$\left\{ \left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^-P^0}^{3/2}(s) |F_V^{P^-P^0}(s)|^2 + 3\frac{\Delta_{P^-P^0}^2}{s^2} \lambda_{P^-P^0}^{1/2}(s) |F_S^{P^-P^0}(s)|^2 \right\}$$

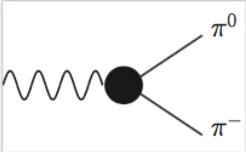
$$\Delta_{P^-P^0} = m_{P^-}^2 - m_{P^0}^2$$

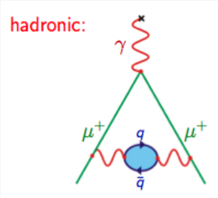
- $\tau^- \to \pi^- \pi^0 \nu_\tau$: Pion vector form factor, $\rho(770), \rho(1450), \rho(1700)$
- $\tau^- \to K^- K_S \nu_\tau$: Kaon vector form factor, $\rho(770), \rho(1450), \rho(1700)$
- $\tau^- \to K_S \pi^- \nu_\tau$: $K\pi$ form factor, $K^*(892), K^*(1410)$, $K_{\ell 3}$, V_{us}
- $\tau^- \to K^- \eta^{(\prime)} \nu_\tau$: $K^*(1410)$, V_{us}
- $\tau^- \to \pi^- \eta^{(\prime)} \nu_\tau$: isospin violation

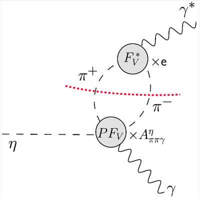
Hadronic tau decays: form factors

The pion vector form factor: Motivation

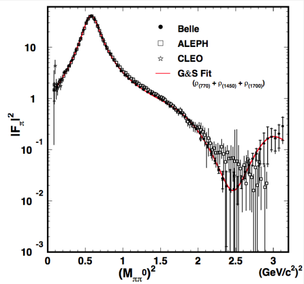
- Enters the description of many physical processes







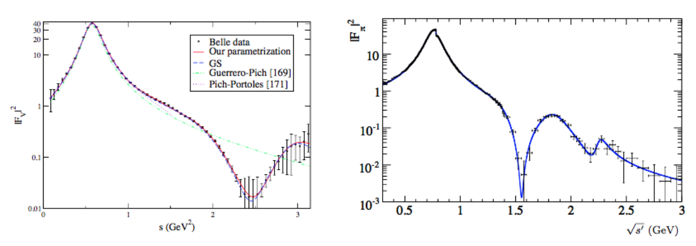
- Belle measurement of the pion form factor (PRD 78 (2008) 072006)



- high-statistics data until de τ mass
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

THE PION VECTOR FORM FACTOR $F_V^\pi(s)$

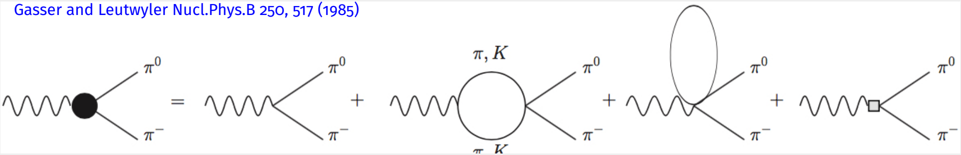
- How to determine $F_V^\pi(s)$ experimentally?
  - $\tau^- \to \pi^-\pi^0\nu_\tau$ (Belle PRD 78 (2008) 072006) and $e^+e^- \to \pi^+\pi^-$ (BaBar PRD 86 (2012) 032013)



- What do we know theoretically on the form factor?
  - Its low-energy behaviour: given by ChPT (Gasser&Leutwyler'85)
  - Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage'79)
  - For the intermediate energy region: models

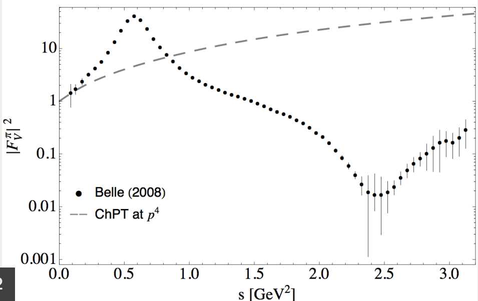
Pion vector form factor: ChPT $\mathcal{O}(p^4)$

Gasser and Leutwyler Nucl.Phys.B 250, 517 (1985)

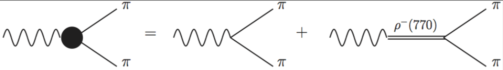


$$F_V^\pi(s)|_{\rm ChPT} = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right),$$

$$A_P(s,\mu^2) = \log\frac{m_P^2}{\mu^2} + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3(s)\log\left(\frac{\sigma_P(s)+1}{\sigma_P(s)-1}\right), \sigma_P(s) = \sqrt{1-4\frac{m_P^2}{s}}$$



Pion vector Form Factor: ChPT with resonances

- Resonace Chiral Th. (RχT) 

$$F_V^\pi(s) = 1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_\rho^2 - s} \overset{F_V G_V = F_\pi^2}{\Longrightarrow} \frac{M_\rho^2}{M_\rho^2 - s}\,,$$

- Expansion in s and comparing ChPT and RχT

$$F_V^\pi(s) = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right)$$

$$F_V^\pi(s) = 1 + \left(\frac{s}{M_\rho^2}\right) + \left(\frac{s}{M_\rho^2}\right)^2 + \cdots$$

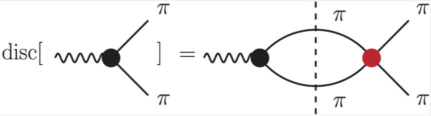
- Chiral coupling estimate: $L_9^r(M_\rho) = \frac{F_V G_V}{2M_\rho^2} = \frac{F_\pi^2}{2M_\rho^2} \simeq 7.2 \times 10^{-3}$

- Combining ChPT and RχT

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]\,,$$

Dispersive representation

- Dispersive approach

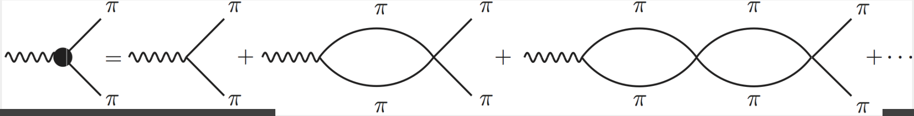


$$\mathrm{disc}F_V(s) = 2i\sigma_\pi(s)F_V(s)T_1^{1*}(s) = 2iF_V(s)\sin\delta_1^1(s)e^{-i\delta_1^1(s)}\,,$$

$$F_V(s) = \frac{1}{2i\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\mathrm{disc}F_V(s')}{s'-s-i\varepsilon}\,,$$

- Analytic solution (Omnès equation)

$$F_V(s) = P(s)\Omega(s)\,, \quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\delta_1^1(s')}{s'(s'-s-i\varepsilon)}\right\}\,,$$



- Resummation of final-state interactions to all orders (Omnès)

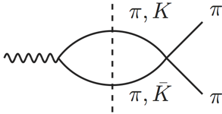
$$F_V^\pi(s) = P_n(s)\exp\left\{\frac{s^n}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{(s')^n}\frac{\delta_1^1(s')}{s'-s-i\varepsilon}\right\},$$

- Get a model for the phase from $\pi\pi\to\pi\pi$ scattering at $\mathcal{O}(p^2)$

$$T(s) = \frac{s-m_\pi^2}{F_\pi^2} \to T_1^1(s) = \frac{s\sigma_\pi^2(s)}{96\pi F_\pi^2} \to \delta_1^1(s) = \sigma_\pi(s)T_1^1(s) = \frac{s\sigma_\pi^3(s)}{96\pi F_\pi^2},$$

- Omnès exponentiation of the full loop function

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2-s}\exp\left\{-\frac{s}{96\pi^2F_\pi^2}A_\pi(s,\mu^2)\right\}.$$

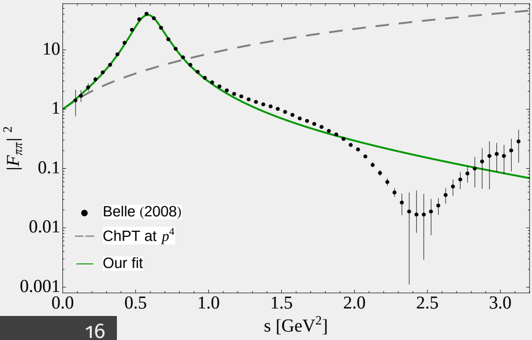


- Incorporatation of the (off-shell) ρ width

$$\Gamma_\rho(s) = -\frac{M_\rho s}{96\pi^2 F_\pi^2}\mathrm{Im}\left[A_\pi(s) + \frac{1}{2}A_K(s)\right] = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_\pi(s)^3\theta(s-4m_\pi^2) + \sigma_K(s)^3\theta(s-4m_K^2)\right] \,.$$

Guerrero, Pich PLB 412, 382 (1997)

$$F_V^\pi(s)\big|_{\mathrm{expo}}^{\mathrm{1\,res}} = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{-\frac{s}{96\pi^2F_\pi^2}\mathrm{Re}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]\right\} .$$



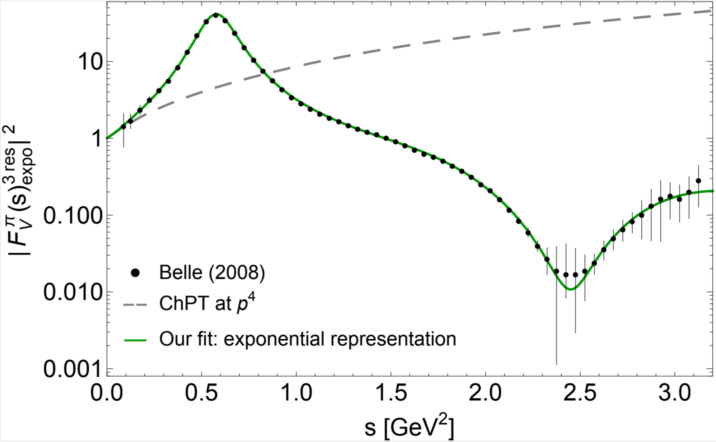
- Incorporation of the $\rho'(1450), \rho''(1700)$

$$F_V^\pi(s)|^{3\,\mathrm{res}}_{\mathrm{expo}} = \frac{M_\rho^2 + s\left(\gamma e^{i\phi_1} + \delta e^{i\phi_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\}$$
$$-\gamma\frac{s\,e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3\sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\}$$
$$-\delta\frac{s\,e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3\sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\},$$

where

$$\Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''}\frac{M_{\rho',\rho''}}{\sqrt{s}}\frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)}.$$

RχT + Omnès: Exponential representation



$$M_\rho = 775.2(4)\,\text{MeV}\,, \quad \gamma = 0.15(4)\,, \quad \phi_1 = -0.36(24)\,,$$

$$M_{\rho'} = 1438(39)\,\text{MeV}\,, \quad \Gamma_{\rho'} = 535(63)\,\text{MeV}\,, \quad \delta = -0.12(4)\,, \phi_2\,, = -0.02(45)\,,$$

$$M_{\rho''} = 1754(91)\,\text{MeV}\,, \quad \Gamma_{\rho''} = 412(102)\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 0.92$$

- Drawbacks: Constraints from analyticity, unitarity and chiral symmetry not fully respected
- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right] \,,$$

  - Low-energy observables

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \cdots \,,$$

$$\langle r^2\rangle_V^\pi\Big|_{\rm ChPT}^{\mathcal{O}(p^4)} = \frac{12L_9^r(\mu)}{F_\pi^2} - \frac{1}{32\pi^2F_\pi^2}\left[2\log\left(\frac{M_\pi^2}{\mu^2}\right) + \log\left(\frac{M_K^2}{\mu^2}\right) + 3\right] \,,$$

$$\langle r^2\rangle_V^\pi = 6\alpha_1 \,, \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right) \,, \quad \alpha_k = \frac{k!}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{s'^{k+1}} \,.$$

  - $s_{\rm cut}$: cut-off to check stability

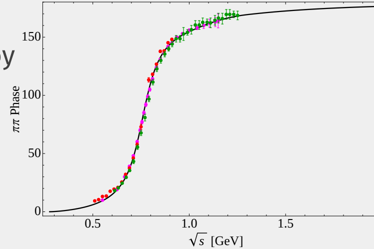
- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right],$$

- Form Factor phase $\phi(s)$
  - $4m_\pi^2 < s < 1$ GeV: $\pi\pi$ phase from Roy (García-Martín et.al PRD 83, 074004 (2011))
  - $1 < s < m_\tau^2$: "Pheno" phase shift

$$\tan\phi(s) = \frac{{\rm Im}F_V^\pi(s)|^{3\,\rm res}_{\rm expo}}{{\rm Re}F_V^\pi(s)|^{3\,\rm res}_{\rm expo}},$$

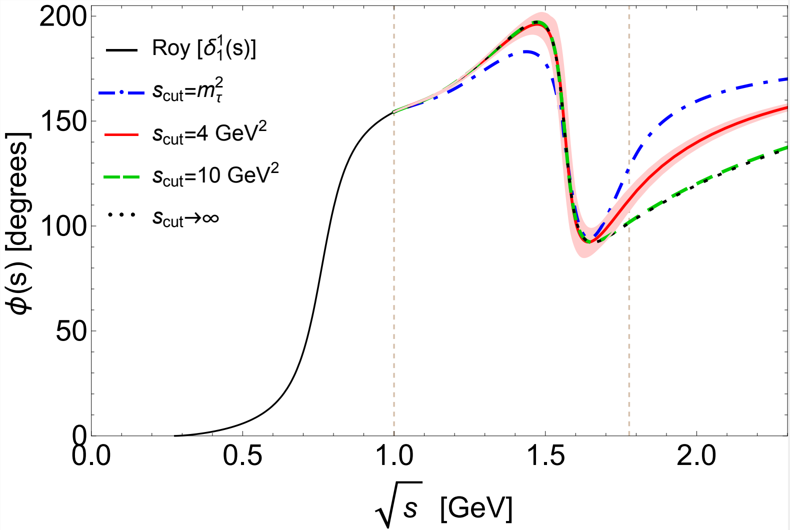
  - $m_\tau^2 < s$: phase guided smoothly to π



- Fits for different values of $s_{\rm cut}$ and matching at 1 GeV

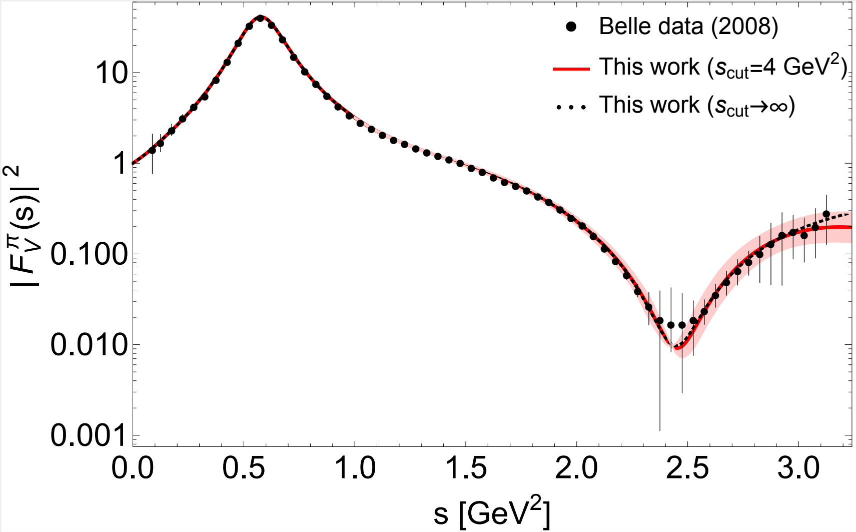
Fits	Parameter	$s_{\rm cut}$ [GeV²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1	α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.89(1)	1.89(1)
	α_2 [GeV^{-4}]	4.40(1)	4.34(1)	4.32(1)	4.32(1)
	m_ρ [MeV]	$=$ 773.6(9)	$=$ 773.6(9)	$=$ 773.6(9)	$=$ 773.6(9)
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1365(15)	1376(6)	1313(15)	1311(5)
	$\Gamma_{\rho'}$ [MeV]	562(55)	603(22)	700(6)	701(28)
	$M_{\rho''}$[MeV]	1727(12)	1718(4)	1660(9)	1658(1)
	$\Gamma_{\rho''}$ [MeV]	278(1)	465(9)	601(39)	602(3)
	γ	0.12(2)	0.15(1)	0.16(1)	0.16(1)
	ϕ_1	$-$0.69(1)	$-$0.66(1)	$-$1.36(10)	$-$1.39(1)
	δ	$-$0.09(1)	$-$0.13(1)	$-$0.16(1)	$-$0.17(1)
	ϕ_2	$-$0.17(5)	$-$0.44(3)	$-$1.01(5)	$-$1.03(2)
	χ^2/d.o.f	1.47	0.70	0.64	0.64

- Form Factor phase shift for different values of s_{cut}



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

- Modulus squared of the pion vector form factor



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

Central results

- Fit results (central value $\pm$ stat fit error $\pm$ syst th. error)

$$\alpha_1 = 1.88(1)(1)\ \mathrm{GeV}^{-2},\ \alpha_2 = 4.34(1)(3)\ \mathrm{GeV}^{-4},$$

$$M_\rho \doteq 773.6 \pm 0.9 \pm 0.3\ \mathrm{MeV}\,,$$

$$M_{\rho'} = 1376 \pm 6^{+18}_{-73}\ \mathrm{MeV}\,, \quad \Gamma_{\rho'} = 603 \pm 22^{+236}_{-141}\ \mathrm{MeV}\,,$$

$$M_{\rho''} = 1718 \pm 4^{+57}_{-94}\ \mathrm{MeV}\,, \quad \Gamma_{\rho''} = 465 \pm 9^{+137}_{-53}\ \mathrm{MeV}\,,$$

$$\gamma = 0.15 \pm 0.01^{+0.07}_{-0.03}\,, \quad \phi_1 = -0.66 \pm 0.01^{+0.22}_{-0.99}\,,$$

$$\delta = -0.13 \pm 0.01^{+0.00}_{-0.05}\,, \quad \phi_2 = -0.44 \pm 0.03^{+0.06}_{-0.90}\,,$$

- Physical pole mass and width

$$M_\rho^{\mathrm{pole}} = 760.6 \pm 0.8\ \mathrm{MeV}\,, \quad \Gamma_\rho^{\mathrm{pole}} = 142.0 \pm 0.4\ \mathrm{MeV}\,,$$

$$M_{\rho'}^{\mathrm{pole}} = 1289 \pm 8^{+52}_{-143}\ \mathrm{MeV}\,, \quad \Gamma_{\rho'}^{\mathrm{pole}} = 540 \pm 16^{+151}_{-111}\ \mathrm{MeV}\,,$$

$$M_{\rho''}^{\mathrm{pole}} = 1673 \pm 4^{+68}_{-125}\ \mathrm{MeV}\,, \quad \Gamma_{\rho''}^{\mathrm{pole}} = 445 \pm 8^{+117}_{-49}\ \mathrm{MeV}\,,$$

$\rho(1450)$ AND $\rho(1700)$ RESONANCE PARAMETERS

Reference	Model parameters $M_{\rho'}, \Gamma_{\rho'}$ [MeV]	Pole parameters $M^{\rm pole}_{\rho'}, \Gamma^{\rm pole}_{\rho'}$ [MeV]	Data
ALEPH	$1328 \pm 15, 468 \pm 41$	$1268 \pm 19, 429 \pm 31$	τ
ALEPH	$1409 \pm 12, 501 \pm 37$	$1345 \pm 15, 459 \pm 28$	τ & e
Belle (fixed $\lvert F_V^\pi(0)\rvert^2$)	$1446 \pm 7 \pm 28, 434 \pm 16 \pm 60$	$1398 \pm 8 \pm 31, 408 \pm 13 \pm 50$	τ
Belle (all free)	$1428 \pm 15 \pm 26, 413 \pm 12 \pm 57$	$1384 \pm 16 \pm 29, 390 \pm 10 \pm 48$	τ
Dumm et. al.	—	$1440 \pm 80, 320 \pm 80$	τ
Celis et. al.	$1497 \pm 7, 785 \pm 51$	$1278 \pm 18, 525 \pm 16$	τ
Bartos et. al.	—	$1342 \pm 47, 492 \pm 138$	e^+e^-
Bartos et. al.	—	$1374 \pm 11, 341 \pm 24$	τ
This work	$1376 \pm 6^{+18}_{-73}, 603 \pm 22^{+236}_{-141}$	$1289 \pm 8^{+52}_{-143}, 540 \pm 16^{+151}_{-111}$	τ
Reference	Model parameters $(M_{\rho''}, \Gamma_{\rho''})$ [MeV]	Pole parameters $(M^{\rm pole}_{\rho''}, \Gamma^{\rm pole}_{\rho''})$ [MeV]	Data
ALEPH	$= 1713, = 235$	$1700, 232$	τ
ALEPH	$1740 \pm 20, = 235$	$1728 \pm 20, 232$	τ & e
Belle (fixed $\lvert F_V^\pi(0)\rvert^2$)	$1728 \pm 17 \pm 89, 164 \pm 21^{+89}_{-26}$	$1722 \pm 18, 163 \pm 21^{+88}_{-27}$	τ
Belle (all free)	$1694 \pm 41, 135 \pm 36^{+50}_{-26}$	$1690 \pm 94, 134 \pm 36^{+49}_{-28}$	τ
Dumm et. al.	—	$1720 \pm 90, 180 \pm 90$	τ
Celis et. al.	$1685 \pm 30, 800 \pm 31$	$1494 \pm 37, 600 \pm 17$	τ
Bartos et. al.	—	$1719 \pm 65, 490 \pm 17$	e^+e^-
Bartos et. al.	—	$1767 \pm 52, 415 \pm 120$	τ
This work	$1718 \pm 4^{+57}_{-94}, 465 \pm 9^{+137}_{-53}$	$1673 \pm 4^{+68}_{-125}, 445 \pm 8^{+117}_{-49}$	τ

Low-energy observables

References	$\langle r^2\rangle_V^\pi$ (GeV^{-2})	c_V^π (GeV^{-4})
Colangelo et al. [55]	11.07 ± 0.66	3.2 ± 1.03
Bijnens et al. [32]	11.22 ± 0.41	3.85 ± 0.60
Pich et al. [6]	11.04 ± 0.30	3.79 ± 0.04
Bijnens et al. [33]	11.61 ± 0.33	4.49 ± 0.28
de Troconiz et al. [56]	11.10 ± 0.03	3.84 ± 0.02
Masjuan et al. [57]	11.43 ± 0.19	3.30 ± 0.33
Guo et al. [58]	–	4.00 ± 0.50
Lattice [59]	10.50 ± 1.12	3.22 ± 0.40
Ananthanarayan et al. [60]	11.17 ± 0.53	[3.75, 3.98]
Ananthanarayan et al. [61]	[10.79, 11.3]	[3.79, 4.00]
Schneider et al. [48]	10.6	3.84 ± 0.03
Dumm et al. [7]	10.86 ± 0.14	3.84 ± 0.03
Celis et al. [8]	11.30 ± 0.07	4.11 ± 0.09
Ananthanarayan et al. [62]	11.10 ± 0.11	–
Hanhart et al. [63]	$11.34 \pm 0.01 \pm 0.01$	–
Colangelo et al. [39]	11.02 ± 0.10	–
PDG [42]	11.61 ± 0.28	–
This work	11.28 ± 0.08	3.94 ± 0.04

Sum rule	s_{cut} (GeV^2)			Fit Eq. (42)
	4	10	∞	
α_1	1.52	1.66	1.75	$1.88 \pm 0.01 \pm 0.01$
α_2	4.26	4.30	4.31	$4.34 \pm 0.01 \pm 0.03$

Kaon vector Form Factor

$$\frac{d\Gamma(\tau^- \to K^- K^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 |V_{ud}|^2}{768\pi^3} M_\tau^3 \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \sigma_K^3(s) |F_V^K(s)|^2 \,,$$

- Chiral Perturbation Theory $\mathcal{O}(p^4)$

$$\begin{aligned} F_{K^+K^-}(s)|_{\rm ChPT} &= 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{192\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + 2A_K(s,\mu^2)\right] \,, \\ F_{K^0\bar{K}^0}(s)|_{\rm ChPT} &= -\frac{s}{192\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) - A_K(s,\mu^2)\right] \,. \end{aligned}$$

- Extract the $I = 1$ component

$$F_V^K(s)|_{\rm ChPT} = F_{K^+K^-}(s) - F_{K^0\bar{K}^0}(s) = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]$$

- At $\mathcal{O}(p^4)$, the pion and kaon vector form factor are the same
- Assumption: we consider that both are also the same at higher energies

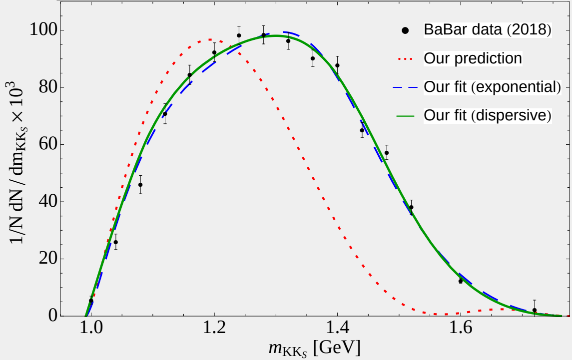
- Different resonance mixing contribution than $F_V^\pi(s)$

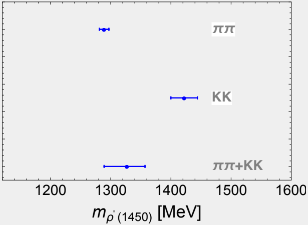
$$
\begin{aligned}
F_V^K(s) &= \frac{M_\rho^2 + s\left(\tilde{\gamma} e^{i\tilde{\phi}_1} + \tilde{\delta} e^{i\tilde{\phi}_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\} \\
&\quad -\tilde{\gamma}\frac{s\, e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\} \\
&\quad -\tilde{\delta}\frac{s\, e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\}, \\
\Gamma_{\rho',\rho''}(s) &= \Gamma_{\rho',\rho''}\frac{s}{M_{\rho',\rho''}^2}\frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)}\theta(s - 4m_\pi^2)\,.
\end{aligned}
$$

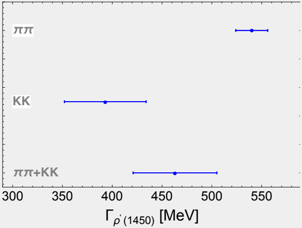
- Extract the phase $\tan\phi_{KK}(s) = \mathrm{Im}F_V^K(s)/\mathrm{Re}F_V^K(s)$
- Use a three-times subtracted dispersion relation

FIT RESULTS TO BABAR $\tau^- \to K^- K_S \nu_\tau$ DATA

Parameter	Fit dispersive	Fit exponential
$\tilde{\alpha}_1$	$= 1.88(1)$	—
$\tilde{\alpha}_2$	$= 4.34(1)$	—
$M_{\rho'}$ [MeV]	1467(24)	1411(12)
$\Gamma_{\rho'}$ [MeV]	415(48)	394(35)
$\tilde{\gamma}$	0.10(2)	0.09(1)
$\tilde{\phi}_1$	$-1.19(16)$	$-1.88(9)$
χ^2/d.o.f.	2.9	3.3



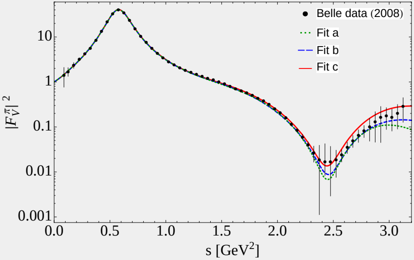


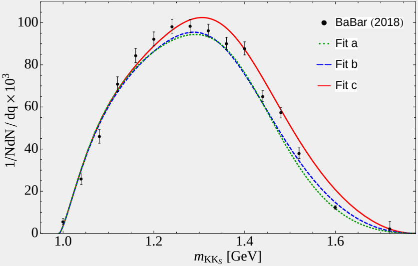


1902.02273 [hep-ph]

Combined analysis of $F_V^\pi(s)$ and $\tau^- \to K^- K_S \nu_\tau$

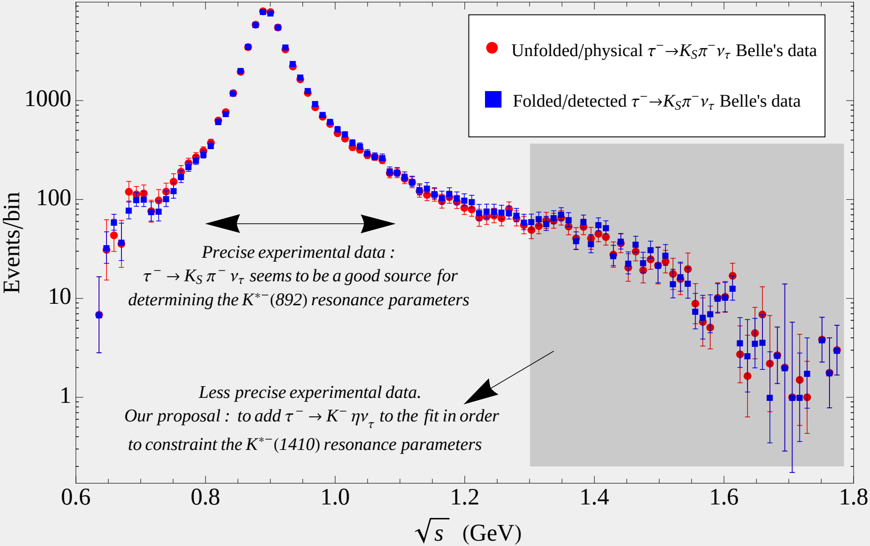
Parameter	$s_{\rm cut} = 4$ [GeV^2]		
	Fit *a*	Fit *b*	Fit *c*
α_1	1.88(1)	1.89(1)	1.87(1)
α_2	4.34(2)	4.31(2)	4.38(3)
$\tilde\alpha_1$	$=\alpha_1$	$=\alpha_1$	1.88(24)
$\tilde\alpha_2$	$=\alpha_2$	$=\alpha_2$	4.38(29)
m_ρ [MeV]	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$
M_ρ [MeV]	$=m_\rho$	$=m_\rho$	$=m_\rho$
$M_{\rho'}$ [MeV]	1396(19)	1453(19)	1406(61)
$\Gamma_{\rho'}$ [MeV]	507(31)	499(51)	524(149)
$M_{\rho''}$[MeV]	1724(41)	1712(32)	1746(1)
$\Gamma_{\rho''}$ [MeV]	399(126)	284(72)	413(362)
γ	0.12(3)	0.15(3)	0.11(11)
$\tilde\gamma$	0.11(2)	$=\gamma$	0.11(5)
ϕ_1	$-0.23(26)$	0.29(21)	$-0.27(42)$
$\tilde\phi_1$	$-1.83(14)$	$-1.48(13)$	$-1.90(67)$
δ	$-0.09(2)$	$-0.07(2)$	$-0.10(5)$
$\tilde\delta$	$=0$	$=0$	$-0.01(4)$
ϕ_2	$-0.20(31)$	0.27(29)	$-1.15(71)$
$\tilde\phi_2$	$=0$	$=0$	0.40(3)
χ^2/d.o.f	1.52	1.19	1.25





Belle $\tau^- \to K_S\pi^-\nu_\tau$ measurement

$\tau^- \to K_S\pi^-\nu_\tau$ Belle's data Phys. Lett. **B 654 (2007) 65** [arXiv:0706.2231]



- RχT with two resonances: $K^*(892)$ and $K^*(1410)$

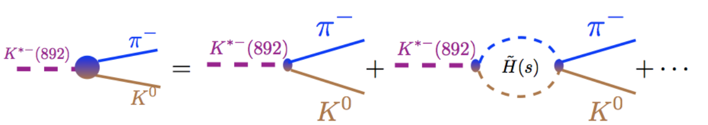


figure courtesy of D. Boito

$$\widetilde{F}_V^{K\pi}(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star}, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}}, \gamma_{K^{\star\prime}})},$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \mathrm{Re}\left[H_{K\pi}(s)\right] - i m_n \Gamma_n(s),$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$

- We then have a phase with two resonances

$$\delta^{K\pi}(s) = \tan^{-1}\left[\frac{\mathrm{Im}F_V^{K\pi}(s)}{\mathrm{Re}F_V^{K\pi}(s)}\right]$$

- Three subtractions: helps the convergence of the form factor and suppresses the the high-energy region of the integral

$$F_V^{K\pi}(s) \;=\; P(s)\exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta^{K\pi}(s')}{(s')^3(s'-s-i0)}\right]$$

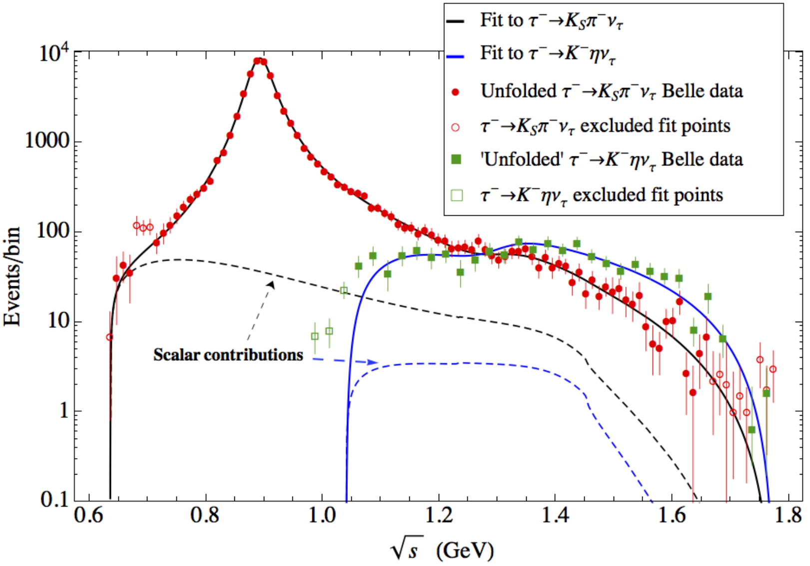
- $\alpha_1 = \lambda'_+$ and $\alpha_1^2 + \alpha_2 = \lambda''_+$ low energies parameters

$$F_V^{K\pi}(t) = 1 + \frac{\lambda'_+}{M_{\pi^-}^2} t + \frac{1}{2}\frac{\lambda''_+}{M_{\pi^-}^4} t^2$$

- $s_{\rm cut}$: cut-off to check stability
- Parameters to Fit: λ'_+ , λ''_+, m_{K^*}, γ_{K^*}, $m_{K^{*\prime}}$, $\gamma_{K^{*\prime}}$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

Escribano, Gonzàlez-Solís, Jamin, Roig JHEP 1409 (2014) 042



- Different choices regarding linear slopes and resonance mixing parameters ($s_{cut} = 4$ GeV^2)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{th}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
$\chi^2/\text{n.d.f.}$	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

- Reference fit results obtained for different values of s_{cut}

Parameter	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
$\chi^2/\text{n.d.f.}$	$105.8/105$	$108.1/105$	$111.0/105$	$111.1/105$

- Central results including the largest variation of s_{cut}

$M_{K^*(892)} = 892.03 \pm 0.19$ MeV

$\Gamma_{K^*(892)} = 46.18 \pm 0.44$ MeV

$M_{K^*(1410)} = 1305^{+16}_{-18}$ MeV

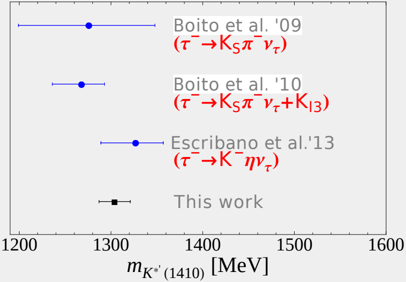
$\Gamma_{K^*(1410)} = 168^{+65}_{-59}$ MeV

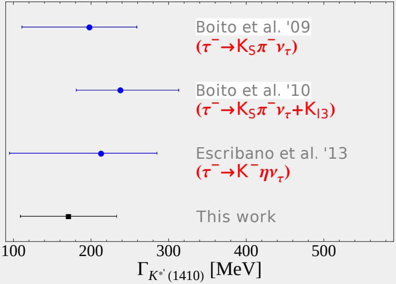
$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$

$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$

$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$

$\chi^2/d.o.f = 108.1/105 = 1.03$





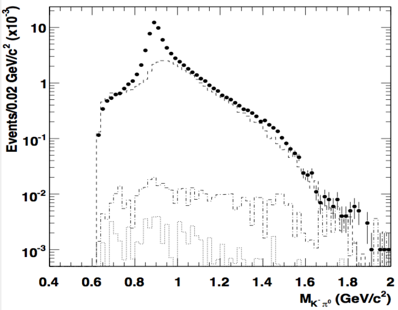
Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

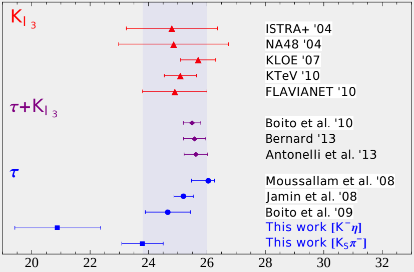
- Central results including the largest variation of s_{cut}

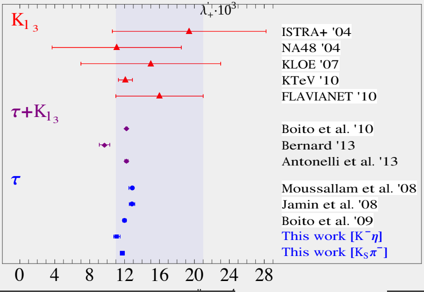
$$\left.\begin{aligned}\lambda'_{K\pi} &= (23.9 \pm 0.9)\cdot 10^{-3}\\ \lambda'_{K\eta} &= (20.9 \pm 2.7)\cdot 10^{-3}\end{aligned}\right\}\text{isospin violation?}$$

$$\left.\begin{aligned}\lambda''_{K\pi} &= (11.8 \pm 0.2)\cdot 10^{-4}\\ \lambda''_{K\eta} &= (11.1 \pm 0.5)\cdot 10^{-4}\end{aligned}\right\}\text{isospin violation?}$$

$\tau^- \to K^-\pi^0\nu_\tau$ (PRD 76 (2007) 051104)







Hadronic tau decays as probes of non-SM interactions

- $2.6\sigma(2.4\sigma)$ LFU deviation from $|g_\tau/g_\mu|(|g_\tau/g_e|)$ in $W^- \to \tau^-\bar{\nu}_\tau$
- 2.8σ deviation CP asymmetry in $\tau^- \to K_S\pi^-\nu_\tau$:

  $A_{CP} = -3.6(2.3)(1.1) \times 10^{-3}$ (exp) vs $A_{CP} = 3.6(1) \times 10^{-3}$ (th)
- $\tau^- \to \nu_\tau \bar{u} D$ $(D =, d, s)$ as probes on non-SM interactions

$$\mathcal{L}_{CC} = -\frac{G_F V_{uD}}{\sqrt{2}}\Big[(1+\epsilon_L^\tau)\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1-\gamma^5)D$$
$$+\epsilon_R^\tau\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1+\gamma^5)D + \bar{\tau}(1-\gamma^5)\nu_\tau \cdot \bar{u}(\epsilon_S^\tau - \epsilon_P^\tau\gamma^5)D$$
$$+\epsilon_T^\tau\bar{\tau}\sigma_{\mu\nu}(1-\gamma^5)\nu_\tau\, \bar{u}\sigma^{\mu\nu}(1-\gamma^5)D\Big] + h.c.\,,$$

  - E. A. Garcés, et.al. [JHEP **1712**, 027 (2017)]; J. A. Miranda et.al. [JHEP **1811**, 038 (2018)]; V. Cirigliano et.al. [Phys. Rev. Lett. **122** (2019) no.22, 221801]; J. Rendón et.al. [Phys. Rev. D **99**, no. 9, 093005 (2019)]; S. Gonzàlez-Solís et.al. [Phys.Lett.B 804 (2020) 135371]

- Measured partial widths and decay spectra
  - Strangeness-conserving: $\tau \to \pi\nu_\tau$, $\tau \to \pi\pi^0\nu_\tau$, $\tau \to KK^0\nu_\tau$
  - Strangeness-changing: $\tau \to K\nu_\tau$, $\tau \to K_S\pi\nu_\tau$, $\tau \to K\eta\nu_\tau$
- $f_\pi = 130.2(8)$ MeV, $f_K = 155.7(7)$ MeV (FLAG 1902.08191)

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2},$$

For more details: see poster by Javier Rendón

GLOBAL FIT TO $\Delta S = 0$ AND $|\Delta S| = 1$ TRANSITIONS

- Combination to **one and two meson decays**

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2},$$

- Comparison with other bounds (assuming LFU):
  - Semileptonic kaon decays: $\epsilon_S^\mu = -0.039(49) \cdot 10^{-2}$, $\epsilon_T^\mu = 0.05(52) \cdot 10^{-2}$ [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]
  - (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_S^\tau \\ \epsilon_P^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 1.0 \pm 1.1 \\ 0.2 \pm 1.3 \\ -0.6 \pm 1.5 \\ 0.5 \pm 1.2 \\ -0.04 \pm 0.46 \end{pmatrix} \times 10^{-2},$$

Prospects for tau physics at Belle-II

Prospects for tau physics at Belle-II

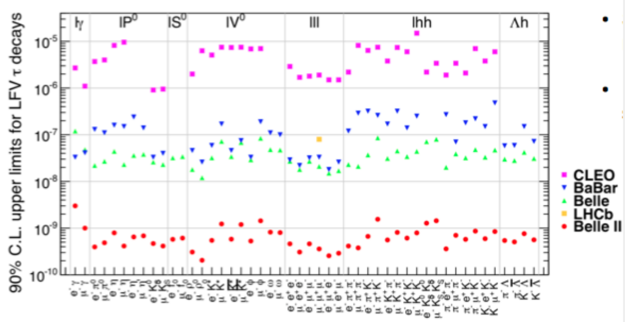
- Huge amount of data to be delivered

Experiment	Number of τ pairs
LEP	~3x10^5
CLEO	~1x10^7
BaBar	~5x10^8
Belle	~9x10^8
Belle II	~10^{12}

- Broad program of tau lepton physics:
  - Searches for Lepton Flavor Violation (LFV)
  - CP violation
  - Second Class Currents
  - and much more (Michel parameters, precision m_τ, EDM, ...)
- See "The Belle II Physics Book" (1808.10567) and (next) talk by Marcela García Hernández

Searches for charged LFV

- Tau as a tool to probe non-SM interactions:
  - radiative: $\tau^- \to \ell^- \gamma$
  - leptonic: $\tau^- \to \ell^- \ell^+ \ell^-$
  - semi-leptonic: $\tau^- \to \ell^- h(h)$ $(h = P, S, V \ldots)$



- Belle-II will push the current bound forward by at least one order of magnitude!

- Observation of charged LFV would be a clear signal of New Physics

CP VIOLATION IN $\tau \to K_S \pi^\pm \nu_\tau$

$$A_\tau = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}$$

- SM prediction: $A_\tau \approx 2\mathrm{Re}(\epsilon) \approx (3.6 \pm 0.1) \times 10^{-3}$ (Bigi, Sanda'05, Grossman, Nir'11)
- Exp. measurement: $(-3.6 \pm 2.3 \pm 1.1) \times 10^{-3}$ (BaBar 2011) 2.8σ from the SM
- New physics? Very difficult to explain
  - Charged Higgs, $W_L - W_R$ mixings (Devi, Dhargyal, Sinha' 2014)

  - Tensor interactions (Rendón, Roig, Toledo 2019)
- An improved A_τ measurement is a priority for Belle II

Second class currents (SCC) in $\tau \to \pi\eta\nu_\tau$

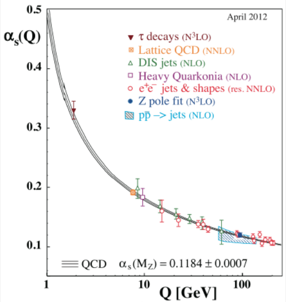
- SCC: $J^{PG} = 0^{+-}, 0^{-+}, 1^{++}, 1^{--}$ not yet observed!
- In the SM, $\tau \to \pi\eta\nu_\tau$ decays proceed via SCC with tiny BRs $\leq \mathcal{O}(10^{-5})$ (Escribano'16, Moussallam'14)
- Searched for at last-generation B-factories
  - $BR < 7.3 \times 10^{-5}$ (Belle), $BR < 9.9 \times 10^{-5}$ (BaBar)
- The observation of SCC via $\tau \to \pi\eta\nu_\tau$ is a priority at Belle-II
- Clear signal could suggest New Physics

Outlook

- Hadronic τ physics as a privileged tool for the investigation of QCD...
- ...but also as a laboratory of NP
- Important experimental activities: Belle-II, BaBar, LHCb, BESIII
- A lot of interesting physics to be done in the tau sector

QUANTUM CHROMODYNAMICS

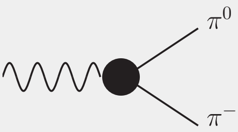
- **asymptotic freedom**:
  "like QED", but only at high energies
- **confinement**:
  at low energies the gluons bind the quarks together to form the hadrons

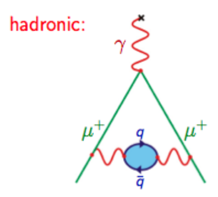


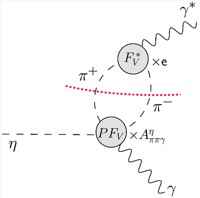
- Approaches to describe the Low-energy regime of QCD:

  1. Lattice QCD simulations: determination of SM fundamental parameters from first principles (quark masses, α_s)
  2. Chiral Perturbation theory
  3. *S*-matrix theory: based on analyticity and unitarity arguments (dispersion relations)

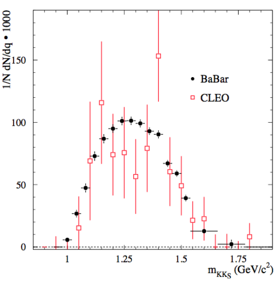
- Enters the description of many physical processes





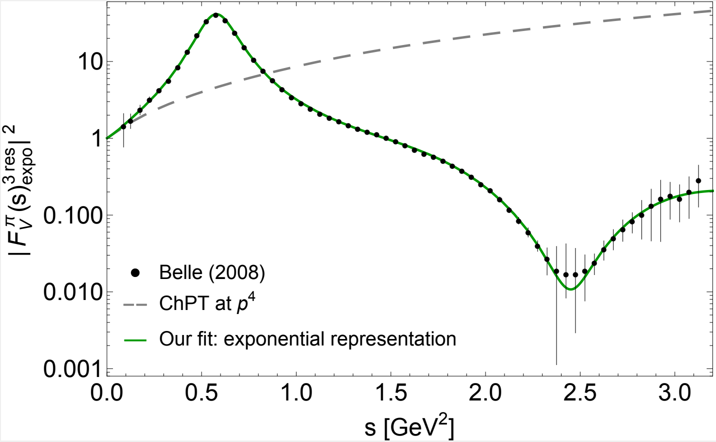


- BaBar measurement of $\tau^- \to K^- K_S \nu_\tau$ (PRD 98 (2018) no.3, 032010)



- good quality data
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

RχT + Omnès: Exponential representation



$$M_\rho^{\text{pole}} = 762.0(3)\,\text{MeV}\,, \quad \Gamma_\rho^{\text{pole}} = 143.0(2)\,\text{MeV}\,,$$

$$M_{\rho'}^{\text{pole}} = 1366(38)\,\text{MeV}\,, \quad \Gamma_{\rho'}^{\text{pole}} = 488(48)\,\text{MeV}\,,$$

$$M_{\rho''}^{\text{pole}} = 1718(82)\,\text{MeV}\,, \quad \Gamma_{\rho''}^{\text{pole}} = 397(88)\,\text{MeV}\,,$$

Variant (I)

- Fits for different matching point and with $s_{\text{cut}} = 4$ GeV

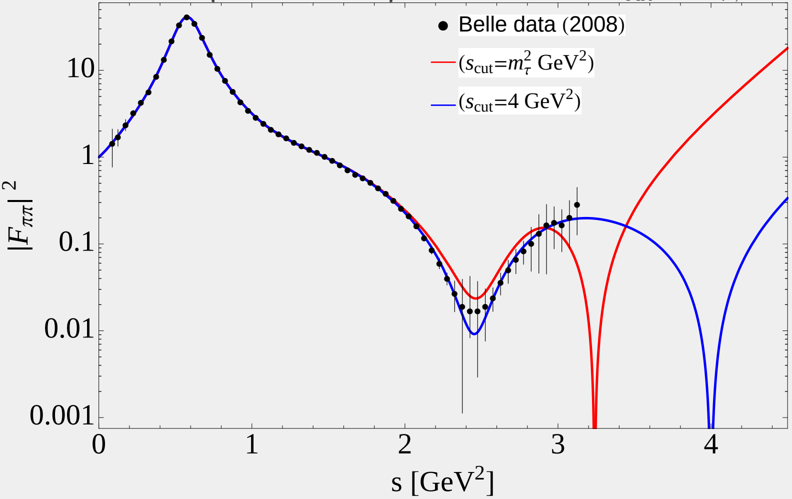
Fits	Parameter	Matching point [GeV]			
		0.85	0.9	0.95	1 (reference fit)
Fit I	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.88(1)	1.88(1)
	α_2 [GeV^{-4}]	4.35(1)	4.35(1)	4.34(1)	4.34(1)
	m_ρ [MeV]	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$
	M_ρ [MeV]	$=m_\rho$	$=m_\rho$	$=m_\rho$	$=m_\rho$
	$M_{\rho'}$ [MeV]	1394(6)	1374(8)	1351(5)	1376(6)
	$\Gamma_{\rho'}$ [MeV]	592(19)	583(27)	592(2)	603(22)
	$M_{\rho''}$[MeV]	1733(9)	1715(1)	1697(3)	1718(4)
	$\Gamma_{\rho''}$ [MeV]	562(3)	541(45)	486(7)	465(9)
	γ	0.12(1)	0.12(1)	0.13(1)	0.15(1)
	ϕ_1	$-0.44(3)$	$-0.60(1)$	$-0.80(1)$	$-0.66(1)$
	δ	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$
	ϕ_2	$-0.38(3)$	$-0.51(2)$	$-0.62(1)$	$-0.44(3)$
	χ^2/d.o.f	0.75	0.74	0.68	0.70

Variant (II): intermediate states other than $\pi\pi$

- Fit A: $\rho' \to K\bar{K}$ and $\rho'' \to K\bar{K}$
- Fit B: $\rho' \to K\bar{K}$ + $\rho' \to \omega\pi$

Parameter	$s_{\mathrm{cut}} = 4$ GeV^2		
	Fit A	Fit B	reference fit
α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.88(1)
α_2 [GeV^{-4}]	4.37(1)	4.35(1)	4.34(1)
m_ρ [MeV]	$=$ 773.6(9)	$=$ 773.6(9)	$=$ 773.6(9)
M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'}$ [MeV]	1373(5)	1441(3)	1376(6)
$\Gamma_{\rho'}$ [MeV]	462(14)	576(33)	603(22)
$M_{\rho''}$[MeV]	1775(1)	1733(9)	1718(4)
$\Gamma_{\rho''}$ [MeV]	412(27)	349(52)	465(9)
γ	0.13(1)	0.15(3)	0.15(1)
ϕ_1	$-$0.80(1)	$-$0.53(5)	$-$0.66(1)
δ	$-$0.14(1)	$-$0.14(1)	$-$0.13(1)
ϕ_2	$-$0.44(2)	$-$0.46(3)	$-$0.44(3)
χ^2/d.o.f	0.93	0.70	0.70

- Modulus squared of the pion form factor $s_{\rm cut} = m_\tau, 4$ GeV²



Variant (III)

- Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i\varepsilon)} + \frac{s}{\pi}\int_{s_{\rm cut}}^{\infty} ds' \frac{\delta_{\rm eff}(s')}{(s')(s'-s-i\varepsilon)}\right]\Sigma(s)$$

- Properties for $\delta_{\rm eff}(s)$
  - $\delta_{\rm eff}(s_{\rm cut}) = \delta_1^1(s_{\rm cut})$ and $\delta_{\rm eff}(s) \to \pi$ for large s to recover $1/s$

$$\delta_{\rm eff}(s) = \pi + \left(\delta_1^1(s_{\rm cut}) - \pi\right)\frac{s_{\rm cut}}{s}$$

  - Integrating the piece with $\delta_{\rm eff}(s)$

$$\begin{aligned} F_V^\pi(s) &= e^{1-\frac{\delta_1^1(s_{\rm cut})}{\pi}}\left(1-\frac{s}{s_{\rm cut}}\right)^{\left(1-\frac{\delta_1^1(s_{\rm cut})}{\pi}\right)\frac{s_{\rm cut}}{s}}\left(1-\frac{s}{s_{\rm cut}}\right)^{-1} \\ &\times \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i\varepsilon)}\right]\Sigma(s) \\ \Sigma(s) &= \sum_{i=0}^{\infty} a_i\omega^i(s)\,, \quad \omega(s) = \frac{\sqrt{s_{\rm cut}}-\sqrt{s_{\rm cut}-s}}{\sqrt{s_{\rm cut}}+\sqrt{s_{\rm cut}-s}} \end{aligned}$$

Variant (III)

The resulting fit parameters are found to be

$$
\begin{aligned}
a_1 &= 2.99(12)\,,\\
M_{\rho'} &= 1261(7)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'} = 855(15)\,\mathrm{MeV}\,,\\
M_{\rho''} &= 1600(1)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''} = 486(26)\,\mathrm{MeV}\,,\\
\gamma &= 0.25(2)\,, \quad \phi_1 = -1.90(6)\,,\\
\delta &= -0.15(1)\,, \quad \phi_2 = -1.60(4)\,,
\end{aligned}
$$

with a $\chi^2/\text{d.o.f} = 32.3/53 \sim 0.61$ for the one-parameter fit, and

$$
\begin{aligned}
a_1 &= 3.03(20)\,, \quad a_2 = 1.04(2.10)\,,\\
M_{\rho'} &= 1303(19)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'} = 839(102)\,\mathrm{MeV}\,,\\
M_{\rho''} &= 1624(1)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''} = 570(99)\,\mathrm{MeV}\\
\gamma &= 0.22(10)\,, \quad \phi_1 = -1.65(4)\,,\\
\delta &= -0.18(1)\,, \quad \phi_2 = -1.34(14)\,,
\end{aligned}
$$

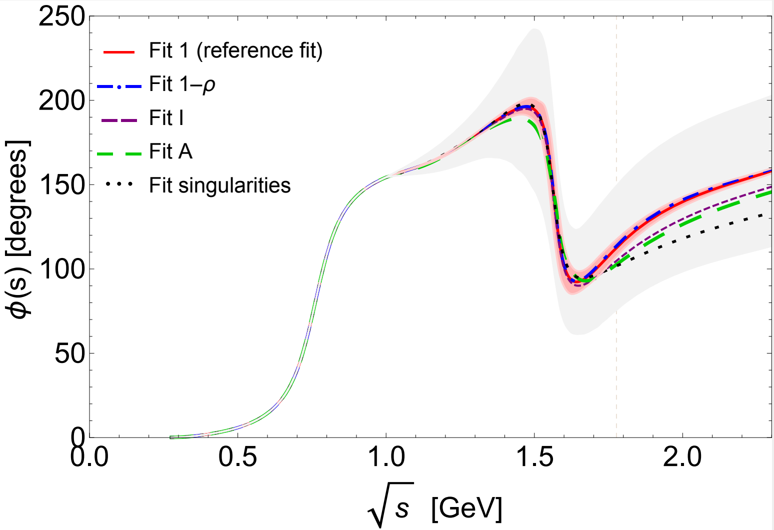
with a $\chi^2/\text{d.o.f} = 35.6/52 \sim 0.63$ for the two-parameter fit.

Variant (IV)

- Fits for different s_{cut} and allowing the ρ-mass to float

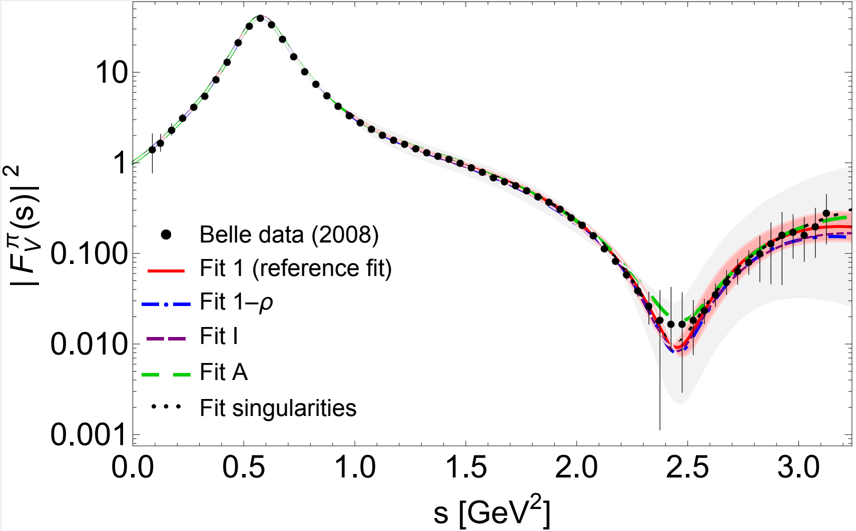
Fits	Parameter	s_{cut} [GeV²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1-ρ	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.89(1)	1.88(1)
	α_2 [GeV^{-4}]	4.37(3)	4.34(1)	4.31(3)	4.34(1)
	m_ρ [MeV]	773.9(3)	773.8(3)	773.9(3)	773.9(3)
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1382(71)	1375(11)	1316(9)	1312(8)
	$\Gamma_{\rho'}$ [MeV]	516(165)	608(35)	728(92)	726(26)
	$M_{\rho''}$[MeV]	1723(1)	1715(22)	1655(1)	1656(8)
	$\Gamma_{\rho''}$ [MeV]	315(271)	455(16)	569(160)	571(13)
	γ	0.12(13)	0.16(1)	0.18(2)	0.17(1)
	ϕ_1	−0.56(35)	−0.69(1)	−1.40(19)	−1.41(8)
	δ	−0.09(3)	−0.13(1)	−0.17(4)	−0.17(3)
	ϕ_2	−0.19(69)	−0.45(12)	−1.06(10)	−1.05(11)
	χ^2/d.o.f	1.09	0.70	0.63	0.66

- Form Factor phase shift for different parametrizations



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

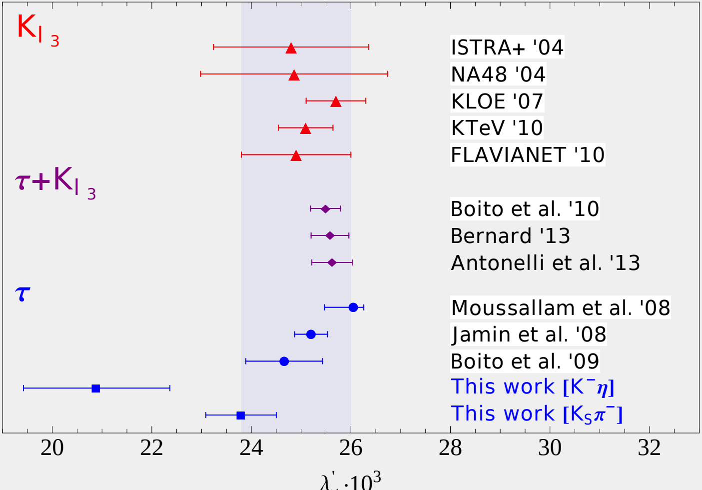
- Modulus squared of the pion vector form factor



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

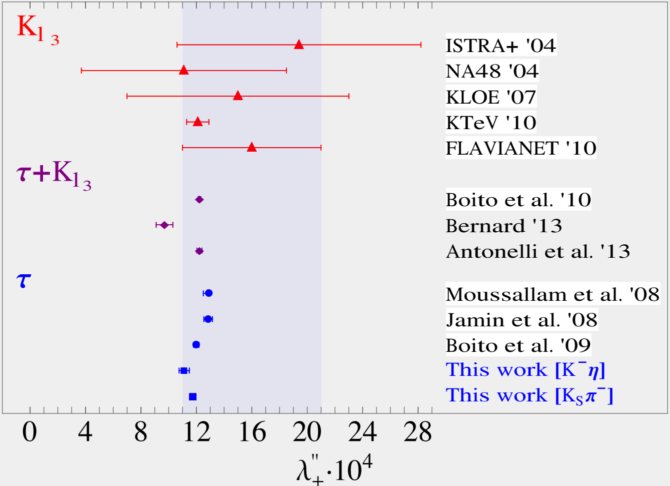
$$\left.\begin{aligned}\lambda'_{K\pi} &= (23.9 \pm 0.9)\cdot 10^{-3}\\ \lambda'_{K\eta} &= (20.9 \pm 2.7)\cdot 10^{-3}\end{aligned}\right\}\text{isospin violation?}$$



Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

$\lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4}$

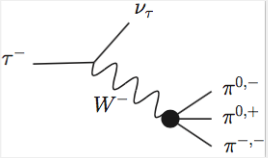
$\lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4}$

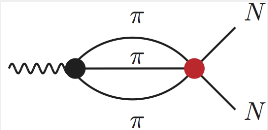


$\tau \to 3\pi\nu_\tau$ (PRELIMINARY RESULTS)

$\tau \to 3\pi\nu_\tau$ DECAYS

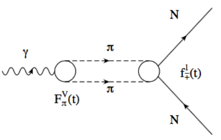
- Motivation:
  - To investigate the axial-vector weak hadronic current (J_A^μ)
  - Dynamics of the 3π system
  - Properties of the $a_1(1260)$
- Applications: input for the axial-vector form factor of the nucleon





Alarcón, Weiss, Phys.Lett. B784 (2018)

- Dispersive parametrizations
  (Belushkin'06, Lorenz'12, Hoferichter'16, Leupold'17, Alarcon'18)



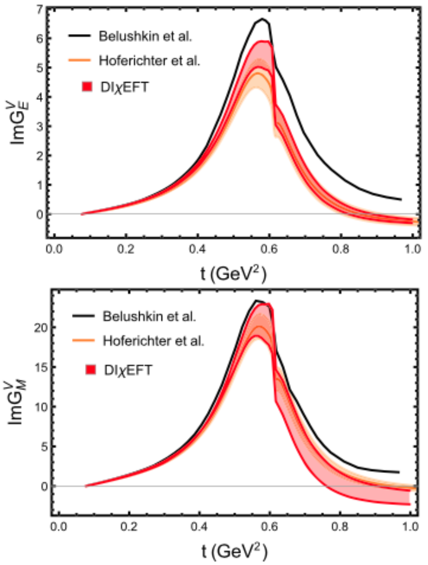
$$G_{E,M}^{p,n}(t) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds \frac{\mathrm{Im}G_{E,M}^{p,n}(s)}{s-t-i\epsilon},$$

$$G_{E,M}^{V,S} = \frac{1}{2}\left(G_{E,M}^{p} - G_{E,M}^{n}\right),$$

$$\mathrm{Im}G_E^V(t) = \frac{k_{\rm cm}^3}{m_N\sqrt{t}} f_+^1(t) F_\pi^{V*}(t)$$

$$\mathrm{Im}G_M^V(t) = \frac{k_{\rm cm}^3}{\sqrt{2t}} f_-^1(t) F_\pi^{V*}(t)$$

where $k_{\rm cm} = \sqrt{t/4 - m_\pi^2}$



$\tau^- \to (PPP)^-\nu_\tau$: Basics

- Generic Amplitude for a 3-meson decay of the τ

$$\mathcal{M}(\tau^- \to (PPP)^-\nu_\tau) = \frac{G_F}{\sqrt{2}}|V_{ij}|\bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau\langle(PPP)^-|(V-A)^\mu|0\rangle \,,$$

- Hadronic matrix element in terms of four form factors

$$\begin{aligned}\langle(P(p_1)P(p_2)P(p_3))^-|(V-A)^\mu|0\rangle &= V_1^\mu F_1^A(Q^2,s_1,s_2) + V_2^\mu F_2^A(Q^2,s_1,s_2)\,,\\ &+ Q^\mu F_3^A(Q^2,s_1,s_2) + iV_4^\mu F_4^V(Q^2,s_1,s_2)\,,\end{aligned}$$

where

$$\begin{aligned}V_1^\mu &= \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right)(p_1-p_3)_\nu\,, \quad V_2^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right)(p_2-p_3)_\nu\,,\\ V_4^\mu &= \varepsilon^{\mu\alpha\beta\gamma}p_{1\alpha}p_{2\beta}p_{3\gamma}\,, \quad Q^\mu = (p_1+p_2+p_3)^\mu\,, \quad s_i = (Q-p_i)^2\,,\end{aligned}$$

$\tau^- \to (PPP)^- \nu_\tau$: Basics

- Generic Amplitude for a 3-meson decay of the τ

$$\mathcal{M}(\tau^- \to (PPP)^- \nu_\tau) = \frac{G_F}{\sqrt{2}} |V_{ij}| \bar{u}_{\nu_\tau} \gamma_\mu (1-\gamma_5) u_\tau \langle (PPP)^- | (V-A)^\mu | 0 \rangle \,,$$

- hadronic matrix element in terms of four form factors

$$\begin{aligned} \langle (P(p_1)P(p_2)P(p_3))^- | (V-A)^\mu | 0 \rangle &= V_1^\mu F_1^A(Q^2, s_1, s_2) + V_2^\mu F_2^A(Q^2, s_1, s_2)\,, \\ &+ Q^\mu F_3^A(Q^2, s_1, s_2) + i V_4^\mu F_4^V(Q^2, s_1, s_2)\,, \end{aligned}$$

  - $F_{1,2}^A(Q^2, s_1, s_2)$: $J^P = 1^+$ transition (axial-vector form factors)
  - $F_3^A(Q^2, s_1, s_2)$: $J^P = 0^-$ transition (pseudoscalar form factor)
  - $F_4^V(Q^2, s_1, s_2)$: $J^P = 1^-$ transition (vector form factor)

$\tau \to \pi^-\pi^+\pi^-\nu_\tau$

- Bose symmetry: $F_1^A(Q^2, s_1, s_2) = F_2^A(Q^2, s_2, s_1) \equiv F_A(Q^2, s_1, s_2)$
- Conservation of J_A^μ in the chiral limit: $F_3^A(Q^2, s_1, s_2)$ must vanish with m_π^2
- G-parity conservation: $F_4^V(Q^2, s_1, s_2) = 0$
- The axial-vector hadronic current takes the form

$$J_A^\mu = F_A(Q^2, s_1, s_2)V_1^\mu + F_A(Q^2, s_2, s_1)V_2^\mu \,,$$

- Decay rate

$$\frac{d\Gamma(\tau^- \to \pi^-\pi^-\pi^+\nu_\tau)}{dQ^2} = \frac{G_F^2|V_{ud}|^2}{32\pi^2 M_\tau}\left(M_\tau^2 - Q^2\right)^2\left(1 + \frac{2Q^2}{M_\tau^2}\right) a_1(Q^2) \,,$$

$\tau \to \pi^-\pi^+\pi^-\nu_\tau$

- Spectral function

$$a_1(Q^2) \;=\; \frac{1}{768\pi^3}\frac{1}{Q^4}\int_{s_{1,\min}}^{s_1^{\max}} ds_1 \int_{s_{2,\min}}^{s_2^{\max}} ds_2\, W_A\,.$$

where

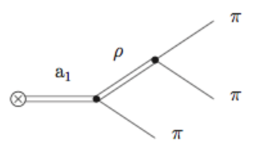
$$\begin{aligned} W_A \;&=\; -\left[V_1^{\mu} F_A(Q^2,s_1,s_2) + V_2^{\mu} F_A(Q^2,s_2,s_1)\right] \times \\ &\quad \left[V_{1\mu} F_A(Q^2,s_1,s_2) + V_{2\mu} F_A(Q^2,s_2,s_1)\right]\;, \end{aligned}$$

- For comparison of theory and experiment

$$\frac{\Gamma(\tau^- \to (3\pi)^-\nu_\tau)}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}\frac{1}{N_{\rm events}}\frac{dN_{\rm events}}{dQ^2} = \frac{6\pi|V_{ub}|^2 S_{EW}}{M_\tau^2}\left(1-\frac{Q^2}{M_\tau^2}\right)^2\left(1+2\frac{Q^2}{M_\tau^2}\right) a_1(Q^2)$$

Evaluation of the axial-vector form factor

Kühn, Santamaria, Z.Phys.C 48 (1990)



- Factorization ansatz: $\tau \to \nu_\tau a_1 \to \nu_\tau \rho\pi \to 3\pi$

$$F_A(Q^2, s_1, s_2) = F_{a_1}(Q^2) F_\rho(s_2)\,,$$

  - $F_{a_1}(Q^2)$ accounts for the $a_1(1260)$-resonance production
  - $F_\rho(s_i) = \Omega(s_i)$ (Omnès): line-shape of the ρ-meson ($\rho \to \pi\pi$)

$$F_\rho(s_i) \to 1\,, \quad s_i \to 0\,.$$

  - Axial-vector current

$$J^\mu_A = F_{a_1}(Q^2)\left[F_\rho(s_2)V_{1\mu} + F_\rho(s_1)V_{2\mu}\right]\,,$$

  - Isospin relation: same predictions for both modes (in the isospin limit)

$$F^A(Q^2, s_1, s_2) = -F^A_{\pi^0\pi^0\pi^-}(Q^2, s_1, s_2)\,,$$

Evaluation of the axial-vector form factor

- ChPT prediction at $\mathcal{O}(p^2)$ (Fischer, Wess, Z. Phys. C 3 (1980))

$$J_A^\mu\big|_{\rm ChPT}^{\mathcal{O}(p^2)} = -\frac{2\sqrt{2}}{3F_\pi}\left(V_{1\mu} + V_{2\mu}\right) ,$$

- Normalization of $F_{a_1}(Q^2)$

$$F_{a_1}(Q^2) = -\frac{2\sqrt{2}}{3F_\pi} f_{a_1}(Q^2) , \quad f_{a_1}(Q^2 \to 0) \to 1 ,$$

- In this framework, the axial spectral function reads:

$$a_1(Q^2) = \frac{1}{768\pi^3}\left(-\frac{2\sqrt{2}}{3F_\pi}\right)^2 |f_{a_1}(Q^2)|^2 \frac{g(Q^2)}{Q^2} ,$$

where

$$g(Q^2) = \frac{1}{Q^2}\int_{s_{1,\rm min}}^{s_1^{\rm max}} ds_1 \int_{s_{2,\rm min}}^{s_2^{\rm max}} ds_2 \Big\{ - V_1^2|F_\rho(s_2)|^2 - V_2^2|F_\rho(s_1)|^2$$

$$- 2V_1V_2 {\rm Re}\left[F_\rho(s_1)(F_\rho(s_2))^*\right] \Big\} ,$$

Breit-Wigner evaluation of the axial-vector f.f.

- Breit-Wigner with $a_1(1260)$:

$$f_{a_1}(Q^2)|^{1\,\mathrm{res}}_{\mathrm{BW}} = \frac{m^2_{a_1} + \mathrm{Re}\Pi_{a_1}(0)}{m^2_{a_1} - Q^2 + \mathrm{Re}\Pi_{a_1}(Q^2) - i m_{a_1}\Gamma_{a_1}(Q^2)}\,, \quad \Gamma_{a_1}(Q^2) = \gamma_{a_1}\frac{g(Q^2)}{g(m^2_{a_1})}$$

$$\mathrm{Re}\Pi_{a_1}(Q^2) = \mathcal{H}_{a_1}(Q^2) - \mathcal{H}_{a_1}(m^2_{a_1})\,, \quad \mathcal{H}_{a_1}(Q^2) = -\frac{Q^2}{\pi}\int_{9m^2_\pi}^{s_{\mathrm{cut}}} ds'\,\frac{m_{a_1}\Gamma_{a_1}(s')}{(s')(s'-Q^2)}\,,$$

- Breit-Wigner with $a_1(1260) + a_1(1640)$:

$$\begin{aligned} f_{a_1}(Q^2)|^{2\,\mathrm{res}}_{\mathrm{BW}} &= \frac{1}{1+|\kappa|e^{i\phi}}\left[\frac{m^2_{a_1} + \mathrm{Re}\Pi_{a_1}(0)}{m^2_{a_1} - Q^2 + \mathrm{Re}\Pi_{a_1}(Q^2) - i m_{a_1}\Gamma_{a_1}(Q^2)}\right. \\ &+ \left.|\kappa|e^{i\phi}\frac{m^2_{a_1'} + \mathrm{Re}\Pi_{a_1'}(0)}{m^2_{a_1'} - Q^2 + \mathrm{Re}\Pi_{a_1'}(Q^2) - i m_{a_1'}\Gamma_{a_1'}(Q^2)}\right], \end{aligned}$$

Dispersive evaluation of the axial-vector f.f.

- Dispersive description:

$$f_{a_1}(Q^2) = \exp\left[\alpha_1 Q^2 + \frac{Q^4}{\pi}\int_{9m_\pi^2}^{\infty} ds' \frac{\delta(s')}{(s')^2(s'-Q^2-i\varepsilon)}\right],$$

$$\tan\delta(Q^2) = \frac{\mathrm{Im} f_{a_1}(Q^2)\big|_{\mathrm{BW}}^{1(2)\ \mathrm{res}}}{\mathrm{Re} f_{a_1}(Q^2)\big|_{\mathrm{BW}}^{1(2)\ \mathrm{res}}},$$

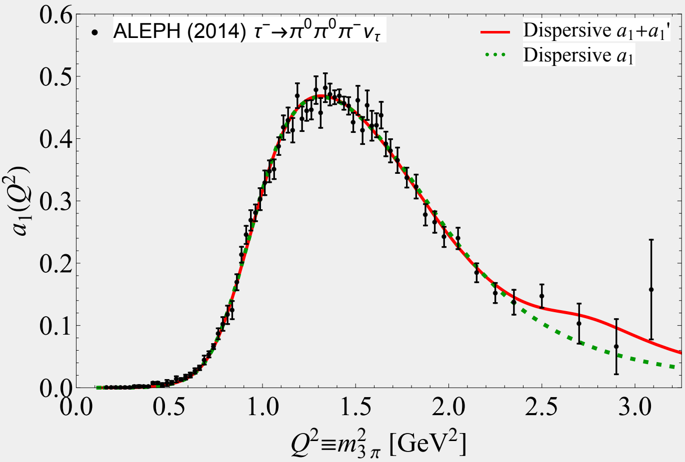
$$\alpha_k^{\mathrm{s.r.}} = \frac{k!}{\pi}\int_{9m_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'^{k+1}}.$$

  - α_1: fit parameter that absorbs other production mechanism

FITS TO THE $\tau \to \pi^0\pi^0\pi^-\nu_\tau$ AXIAL SPECTRAL FUNCTION

Fit results: $m_{a_1} = 1302(8)$ MeV, $\gamma_{a_1} = 493(11)$ MeV, $\alpha_1 = 0.59(1)$, $\alpha_1^{\text{s.r.}} = 0.64(1)$, $\chi^2_{\text{dof}} = 0.96$

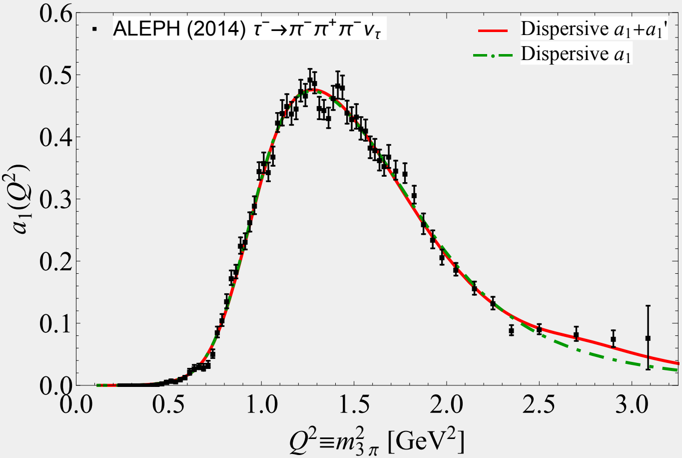
$m_{a_1} = 1296(6)$ MeV, $\gamma_{a_1} = 483(10)$ MeV, $\alpha_1 = 0.60(1)$, $\alpha_1^{\text{s.r.}} = 0.62(1)$, $|\kappa| = 0.10(4)$, $\chi^2_{\text{dof}} = 0.88$



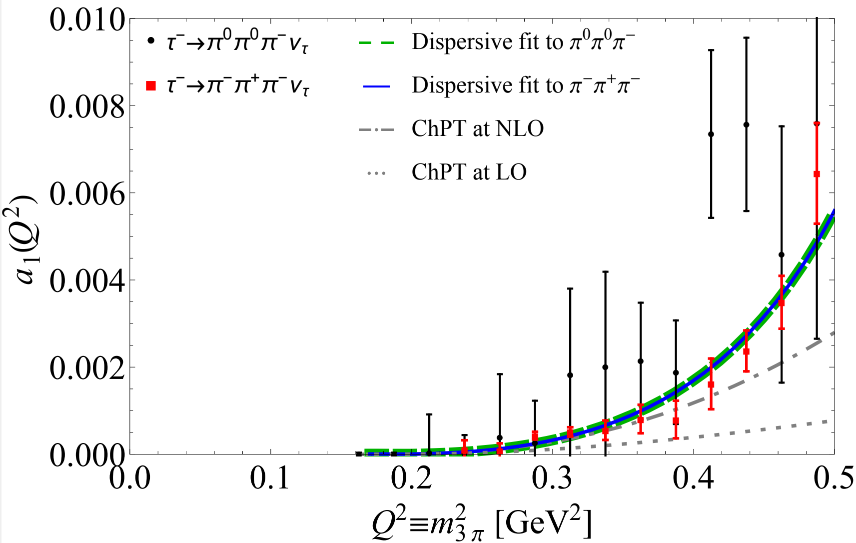
Fits to the $\tau \to \pi^+\pi^-\pi^-\nu_\tau$ axial spectral function

Fit results: $m_{a_1} = 1277(5)$ MeV , $\gamma_{a_1} = 475(8)$ MeV , $\alpha_1 = 0.58(1)$, $\alpha_1 = 0.66(1)$, $\chi^2_{\rm dof} = 1.49$

$m_{a_1} = 1273(5)$ MeV , $\gamma_{a_1} = 466(8)$ MeV , $\alpha_1 = 0.59(1)$, $\alpha_1^{\rm s.r.} = 0.64(1)$, $|\kappa| = 0.06(2)$, $\chi^2_{\rm dof} = 1.39$

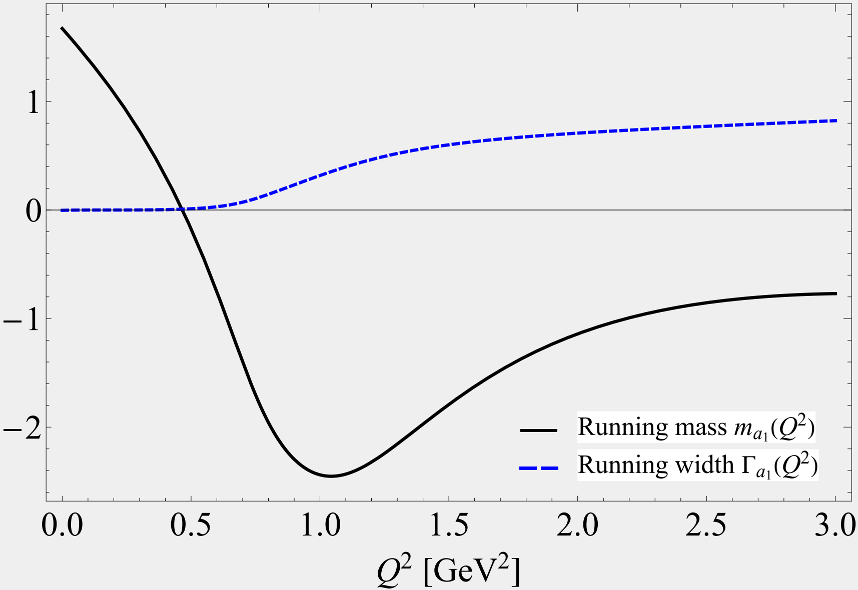


Fits to $\tau \to 3\pi\nu_\tau$: Low-Q^2 region

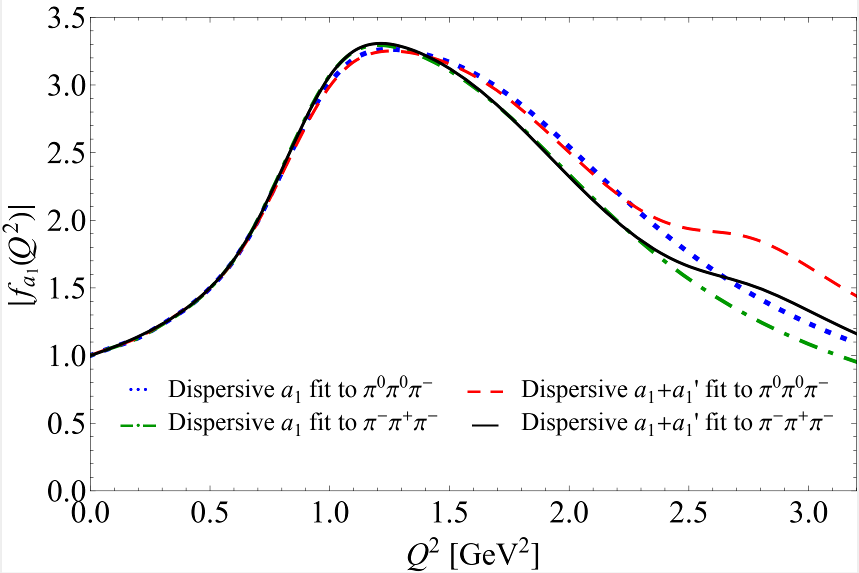


ChPT: Colangelo, Finkemeir, Urech, PRD 54 (1996)

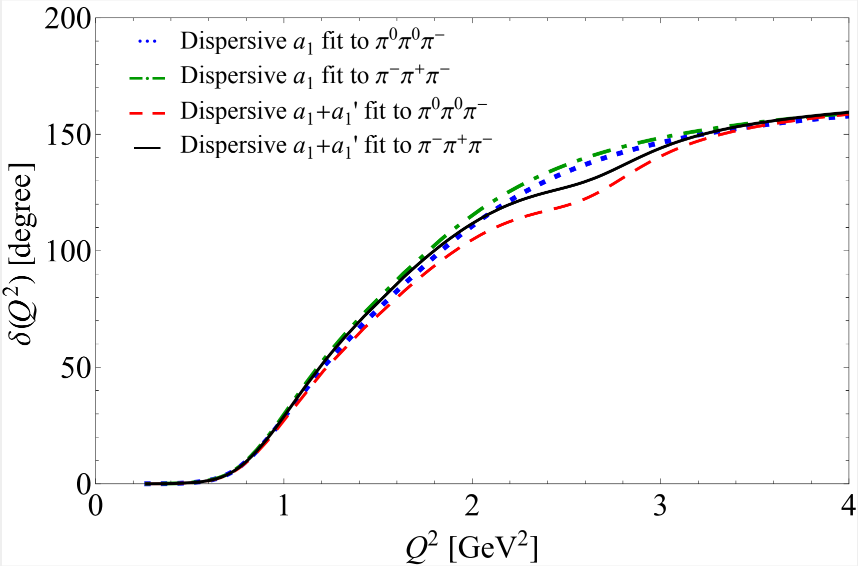
Running $a_1(1260)$ mass and width



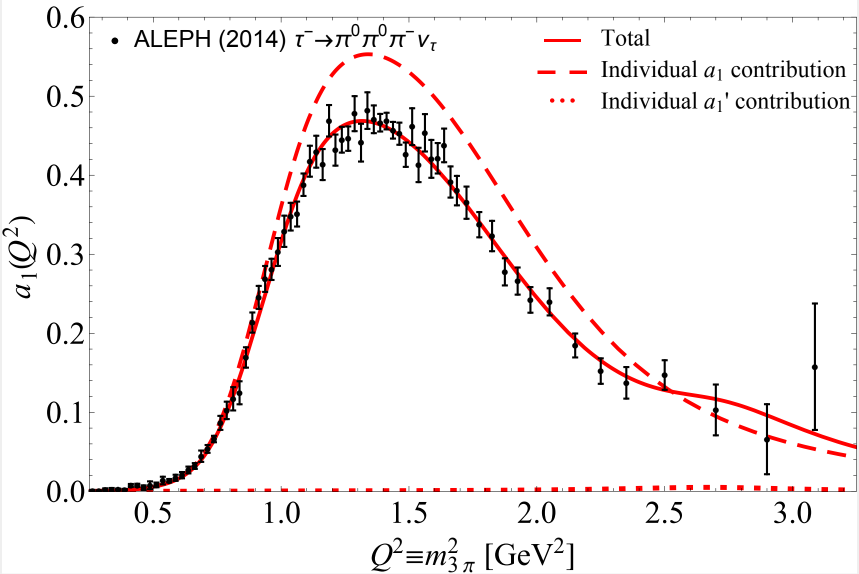
Axial-Vector Form Factor



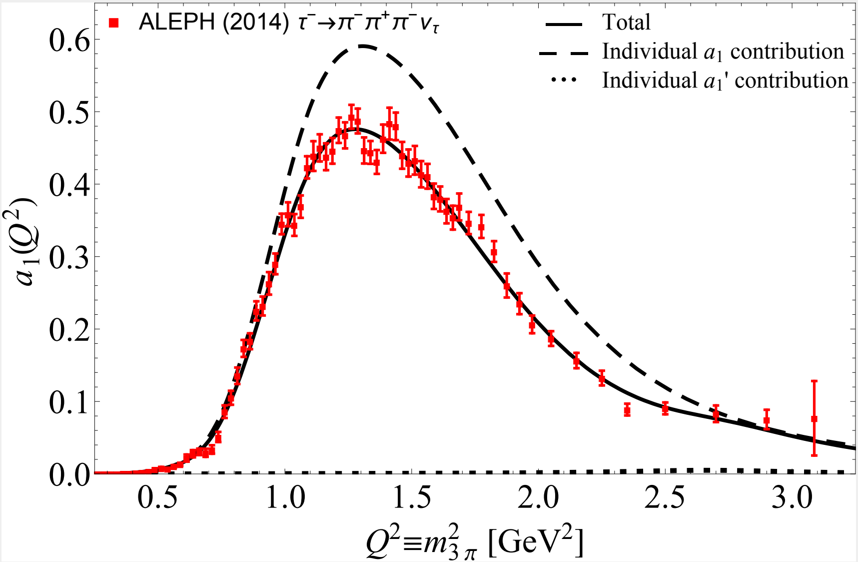
Axial-Vector Form Factor



Spectral function: individual contributions



Spectral function: individual contributions



Form Factors of the Nucleon

- Nucleon matrix element

$$\langle N(p')|J^{\mu}_{em}|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu}F_1(t) + \frac{i}{2m_N}\sigma^{\mu\nu}(p'-p)_{\nu}F_2(t)\right]u(p)\,,$$

$$\langle N(p')|J^{\mu a}_{A}|N(p)\rangle = \bar{u}(p')\frac{\tau^a}{2}\gamma_5\left[\gamma^{\mu}F_A(t) + \frac{(p'-p)^{\mu}}{2m_N}F_P(t)\right]u(p)\,,$$

- Four Form Factor to determine ($t = (p'-p)^2$)
  - $F_1(t)$ and $F_2(t)$: Dirac and Pauli Form Factors
  - $G_E(t)$ and $G_M(t)$: electric and magnetic (Sachs) FFs, well-known

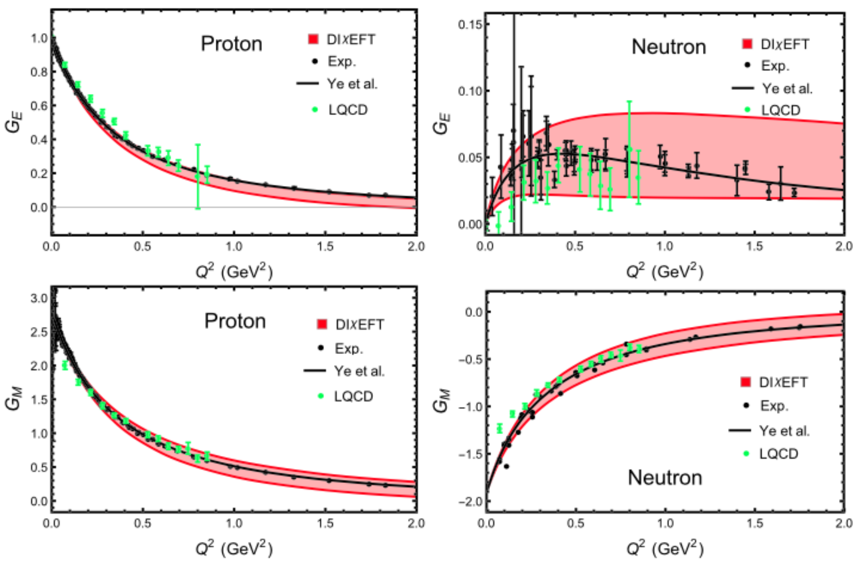
$$G_E(t) = F_1(t) + \frac{t}{4m_N^2}F_2(t)\,, \quad G_M(t) = F_1(t) + F_2(t)$$

  - $F_A(t)$: Axial Form Factor: main unkown
  - $F_P(t)$: Pseudo-scalar Form Factor. It can be related to the Axial FF using PCAC or pion-pole approximation

$$F_P(t) = \frac{2m_N^2}{m_\pi^2 - t}F_A(t)$$

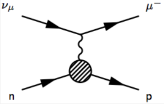
Electromagnetic Form Factors: Space-like

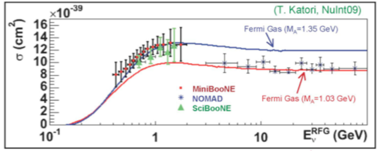
$Q^2 \equiv -t \geq 0$



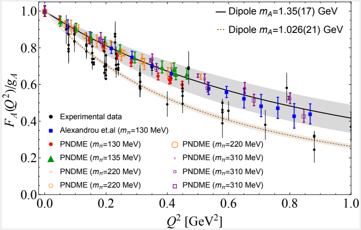
Axial Vector Form Factor

- $F_A(t)$: Main Unknown
- Motivation: How to determine $F_A(t)$ experimentally?
  - $\nu_\mu n \to \mu p$: $F_A(Q^2) = \frac{F_A(0)}{(1+Q^2/M_A^2)^2}$

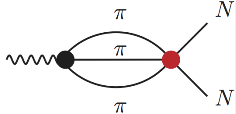




- Objective: Theoretical effort to improve $F_A(t)$
  - Lattice



  - Analytically: 3π intermediate state

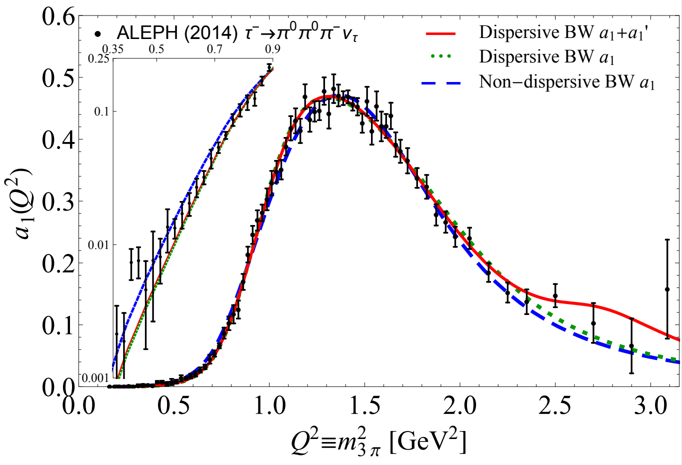


FITS TO THE $\tau \to \pi^0\pi^0\pi^-\nu_\tau$ AXIAL SPECTRAL FUNCTION

Breit-Wigner $m_{a_1} = 1271(13)$ MeV, $\gamma_{a_1} = 523(18)$ MeV, $\mathcal{N} = 1.59(7)$, $\chi^2_{\rm dof} = 1.16$

$m_{a_1} = 1293(10)$ MeV, $\gamma_{a_1} = 501(11)$ MeV, $\mathcal{N} = 0.88(2)$, $\chi^2_{\rm dof} = 1.01$

$m_{a_1} = 1293(6)$ MeV, $\gamma_{a_1} = 485(10)$ MeV, $|\kappa| = 0.12(4)$, $\mathcal{N} = 0.97(5)$, $\chi^2_{\rm dof} = 0.86$

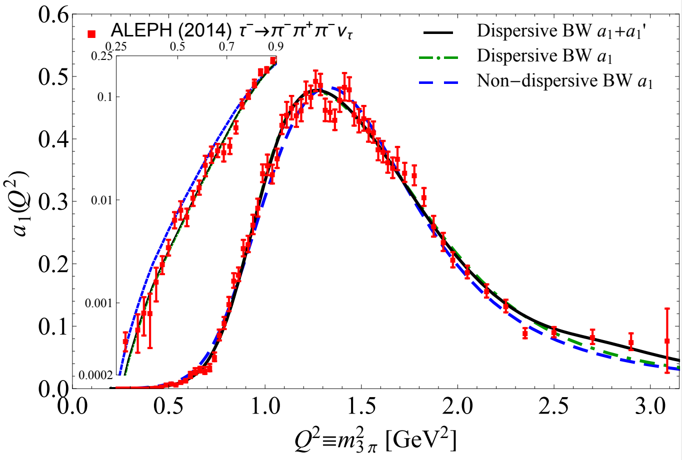


Fits to the $\tau \to \pi^+\pi^-\pi^-\nu_\tau$ axial spectral function

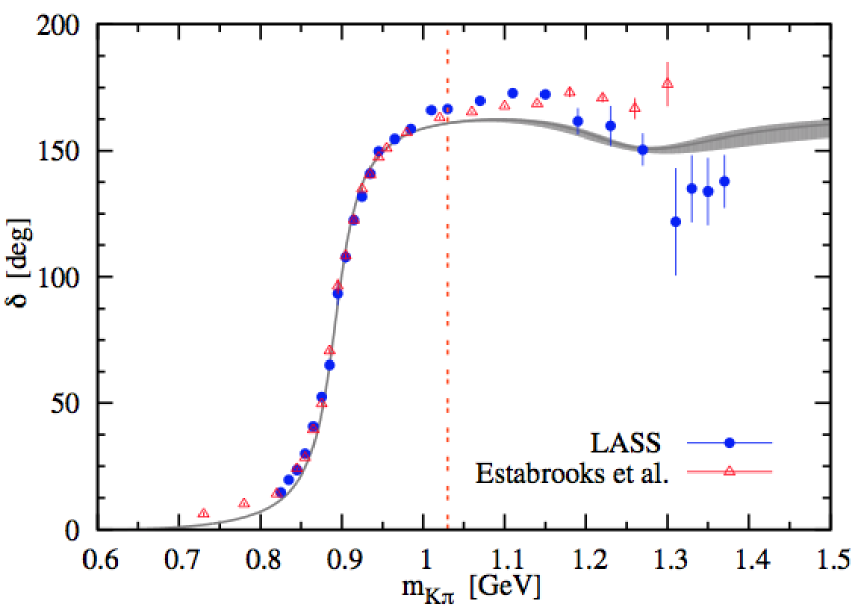
Breit-Wigner: $m_{a_1} = 1243(6)$ MeV, $\gamma_{a_1} = 480(8)$ MeV, $\mathcal{N} = 1.43(3)$, $\chi^2_{\rm dof} = 3.11$

$m_{a_1} = 1259(6)$ MeV, $\gamma_{a_1} = 474(8)$ MeV, $\mathcal{N} = 0.81(2)$, $\chi^2_{\rm dof} = 1.51$

$m_{a_1} = 1260(5)$ MeV, $\gamma_{a_1} = 467(8)$ MeV, $|\kappa| = 0.05(2)$, $\mathcal{N} = 0.85(2)$, $\chi^2_{\rm dof} = 1.43$



$K\pi$ PHASE



200
150
100
50
0
δ [deg]
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
m_Kπ [GeV]
LASS
Estabrooks et al.

Strangeness-conserving transitions ($\Delta S = 0$)

- **One meson** decay $\tau^- \to \pi^- \nu_\tau$ ($G_F \tilde{V}^e_{uD} = G_F \left(1 + \epsilon^e_L + \epsilon^e_R\right) V_{uD}$)

$$\Gamma(\tau^- \to \pi^- \nu_\tau) = \frac{G_F^2 |\tilde{V}^e_{ud}|^2 f_\pi^2 m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \left(1 + \delta^{\tau\pi}_{\rm em} + 2\Delta^{\tau\pi} + \mathcal{O}(\epsilon^\tau_i)^2 + \mathcal{O}(\delta^{\tau\pi}_{\rm em}\epsilon^\tau_i)\right),$$

- Inputs: $f_\pi = 130.2(8)$ MeV (FLAG 1902.08191); $\delta^{\tau\pi}_{\rm em} = 1.92(24)\%$; $|\tilde{V}^e_{ud}| = 0.97420(21)$ (β decays, PDG);
- Constraint for the NP effective couplings (this work):

$$\Delta^{\tau\pi} \equiv \epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m_\pi^2}{m_\tau(m_u + m_d)} \epsilon^\tau_P = (-0.12 \pm 0.68) \times 10^{-2},$$

  Errors (hierarchy): $f_\pi, BR, \delta^{\tau\pi}_{\rm em}$

- $\Gamma(\tau \to \pi\nu)/\Gamma(\pi \to \mu\nu)$: tighter constraints (not used in this work)

$$\epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m_\pi^2}{m_\tau(m_u + m_d)} \epsilon^\tau_P + \frac{m_\pi^2}{m_\mu(m_u + m_d)} \epsilon^\mu_P = (-0.38 \pm 0.27) \times 10^{-2},$$

Strangeness-conserving transitions ($\Delta S = 0$)

- Partial decay width for **two-meson** decays

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |\tilde{V}_{uD}^e|^2 m_\tau^3 S_{EW}}{384\pi^3 s} \left(1 - \frac{s}{m_\tau^2}\right)^2 \lambda^{1/2}(s, m_P^2, m_{P'}^2)$$

$$\times \left[(1 + 2(\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e)) X_{VA} + \epsilon_S^\tau X_S + \epsilon_T^\tau X_T + (\epsilon_S^\tau)^2 X_{S^2} + (\epsilon_T^\tau)^2 X_{T^2} \right],$$

$$X_{VA} = \frac{1}{2s^2} \left\{ 3 \left(C_{PP'}^S\right)^2 |F_0^{PP'}(s)|^2 \Delta_{PP'}^2 + \left(C_{PP'}^V\right)^2 |F_+^{PP'}(s)|^2 \left(1 + \frac{2s}{m_\tau^2}\right) \lambda(s, m_P^2, m_{P'}^2) \right\},$$

$$X_S = \frac{3}{s\, m_\tau} \left(C_{PP'}^S\right)^2 |F_0^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{m_d - m_u},$$

$$X_T = \frac{6}{s\, m_\tau} C_{PP'}^V \, \mathrm{Re}\left[F_T^{PP'}(s) (F_+^{PP'}(s))^*\right] \lambda(s, m_P^2, m_{P'}^2),$$

$$X_{S^2} = \frac{3}{2\, m_\tau^2} \left(C_{PP'}^S\right)^2 |F_0^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{(m_d - m_u)^2},$$

$$X_{T^2} = \frac{4}{s} |F_T^{PP'}(s)|^2 \left(1 + \frac{s}{2\, m_\tau^2}\right) \lambda(s, m_P^2, m_{P'}^2),$$

Tensor Form Factors

- No experimental data
- Theoretical assumptions only

$$\mathrm{Im}F_T^{PP'}(s) = \sigma_{PP'}(s)t_+^*(s)F_T^{PP'}(s)\,,$$

$$F_T^{PP'}(s) = F_T^{PP'}(0)\exp\left[\frac{s}{\pi}\int_{s_{\rm th}}^{s_{\rm cut}}\frac{ds'}{s'}\frac{\delta_T^{PP'}(s')}{(s'-s-i0)}\right]\,,$$

- $s_{\rm th} = (m_P + m_{P'})^2$: two-meson production threshold
- In the elastic region: $\delta_T^{PP'}(s) = \delta_+^{PP'}(s)$
- We guide the phase to $\pi \Rightarrow$ asymptotic $1/s$ dictated by pQCD
- $F_T^{PP'}(0)$: ChPT with tensor fields+lattice

Strangeness-conserving transitions ($\Delta S = 0$)

- Global fit to one and two meson decays

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\text{th}} - \bar{N}_k^{\text{exp}}}{\sigma_{\bar{N}_k^{\text{exp}}}} \right)^2 + \left(\frac{BR_{\pi\pi}^{\text{th}} - BR_{\pi\pi}^{\text{exp}}}{\sigma_{BR_{\pi\pi}^{\text{exp}}}} \right)^2 + \left(\frac{BR_{KK}^{\text{th}} - BR_{KK}^{\text{exp}}}{\sigma_{BR_{KK}^{\text{exp}}}} \right)^2 + \left(\frac{BR_{\tau\pi}^{\text{th}} - BR_{\tau\pi}^{\text{exp}}}{\sigma_{BR_{\tau\pi}^{\text{exp}}}} \right)^2$$

- $\bar{N}_k^{\text{th}}$: normalized distribution for $\tau^- \to \pi^- \pi^0 \nu_\tau$

$$\bar{N}^{\text{th}} \equiv \frac{1}{N_{\text{events}}} \frac{dN_{\text{events}}}{ds} = \frac{1}{\Gamma(\epsilon_i^\tau, \epsilon_j^e)} \frac{d\Gamma(s, \epsilon_i^\tau, \epsilon_j^e)}{ds} \Delta^{\text{bin}}$$

- Data: unfolded distribution measured by Belle (0805.3773)
- Constraints:
  - $BR(\tau^- \to \pi^- \pi^0 \nu_\tau)^{\text{exp}} = 25.49(9)\%$
  - $BR(\tau^- \to K^- K^0 \nu_\tau)^{\text{exp}} = 1.486(34) \times 10^{-3}$
  - $BR(\tau^- \to \pi^- \nu_\tau)^{\text{exp}} = 10.82(5)\%$

Strangeness-conserving transitions ($\Delta S = 0$)

- Bounds for the non-SM effective couplings

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 0.6^{+2.3}_{-1.8}{}^{+0.2}_{-0.1} \pm 0.4 \\ 0.3 \pm 0.5^{+1.1}_{-0.9}{}^{+0.1}_{-0.0} \pm 0.2 \\ 9.7^{+0.5}_{-0.6} \pm 21.5^{+0.0}_{-0.1} \pm 0.2 \\ -0.1 \pm 0.2^{+1.1}_{-1.4}{}^{+0.0}_{-0.1} \pm 0.2 \end{pmatrix} \times 10^{-2},$$

- Errors:
  - *i*) Statistic (1st)
  - *ii*) Systematic: pion vector form factor (2nd), quark masses (3rd) and tensor form factor (4th)

$$\rho_{ij} = \begin{pmatrix} 1 & 0.684 & -0.493 & -0.545 \\ & 1 & -0.337 & -0.372 \\ & & 1 & 0.463 \\ & & & 1 \end{pmatrix},$$

Strangeness-conserving transitions ($\Delta S = 0$)

- Bounds for the non-SM effective couplings

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 0.6^{+2.3}_{-1.8}{}^{+0.2}_{-0.1} \pm 0.4 \\ 0.3 \pm 0.5^{+1.1}_{-0.9}{}^{+0.1}_{-0.0} \pm 0.2 \\ 9.7^{+0.5}_{-0.6} \pm 21.5^{+0.0}_{-0.1} \pm 0.2 \\ -0.1 \pm 0.2^{+1.1}_{-1.4}{}^{+0.0}_{-0.1} \pm 0.2 \end{pmatrix} \times 10^{-2},$$

- Comparison with other bounds (assuming LFU):
  - Semileptonic kaon decays: $\epsilon_S^\mu = -0.039(49) \cdot 10^{-2}$, $\epsilon_T^\mu = 0.05(52) \cdot 10^{-2}$ [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]
  - (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_S^\tau \\ \epsilon_P^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 1.0 \pm 1.1 \\ 0.2 \pm 1.3 \\ -0.6 \pm 1.5 \\ 0.5 \pm 1.2 \\ -0.04 \pm 0.46 \end{pmatrix} \times 10^{-2},$$

Strangeness-changing transitions ($|\Delta S| = 1$)

- One meson decay $\tau^- \to K^- \nu_\tau$

$$\Gamma(\tau^- \to K^- \nu_\tau) = \frac{G_F^2 |\tilde{V}_{us}^e|^2 f_K^2 m_\tau^3}{16\pi} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 \times \left(1 + \delta_{\rm em}^{\tau K} + 2\Delta^{\tau K} + \mathcal{O}(\epsilon_i^\tau)^2 + \mathcal{O}(\delta_{\rm em}^{\tau K} \epsilon_i^\tau)\right),$$

- Inputs: $f_K = 155.7(7)$ MeV (FLAG 1902.08191); $\delta_{\rm em}^{\tau\pi} = 1.98(31)\%$; $|\tilde{V}_{us}^e| = 0.2231(7)$ (PDG)
- Constraint for the NP effective couplings (this work):

$$\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u + m_d)} \epsilon_P^\tau = (-0.41 \pm 0.93) \times 10^{-2},$$

- Errors (hierarchy): $f_K, |V_{us}|, BR, \delta_{\rm em}^{\tau K}$

Strangeness-conserving transitions ($\Delta S = 0$)

- Global fit to one and two meson decays

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\rm th} - \bar{N}_k^{\rm exp}}{\sigma_{\bar{N}_k^{\rm exp}}} \right)^2 + \left(\frac{BR_{K\pi}^{\rm th} - BR_{K\pi}^{\rm exp}}{\sigma_{BR_{K\pi}^{\rm exp}}} \right)^2 + \left(\frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left(\frac{BR_{\tau K}^{\rm th} - BR_{\tau K}^{\rm exp}}{\sigma_{BR_{\tau K}^{\rm exp}}} \right)^2$$

- $\bar{N}_k^{\rm th}$: distribution for $\tau^- \to K_S \pi^- \nu_\tau$

$$\bar{N}^{\rm th} \equiv \frac{dN_{\rm events}}{ds} = \frac{N_{\rm events}}{\Gamma(\epsilon_i^\tau, \epsilon_j^e)} \frac{d\Gamma(s, \epsilon_i^\tau, \epsilon_j^e)}{ds} \Delta^{\rm bin}$$

- Data: unfolded distribution measured by Belle (0706.2231)
- Constraints:
  - $BR(\tau^- \to K_S \pi^- \nu_\tau)^{\rm exp} = 0.404(2)\%$ (Belle)
  - $BR(\tau^- \to K^- \eta \nu_\tau)^{\rm exp} = 1.55(8) \times 10^{-4}$ (PDG)
  - $BR(\tau^- \to K^- \nu_\tau)^{\rm exp} = 6.96(10) \times 10^{-3}$ (PDG)

Strangeness-conserving transitions ($|\Delta S| = 1$)

- Bounds for the non-SM effective couplings

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 1.5 \pm 0.3 \\ 0.4 \pm 0.9 \pm 0.2 \\ 0.8^{+0.8}_{-0.9} \pm 0.3 \\ 0.9 \pm 0.7 \pm 0.3 \end{pmatrix} \times 10^{-2},$$

- Errors: Statistic (fit)+systematic (tensor form factor).

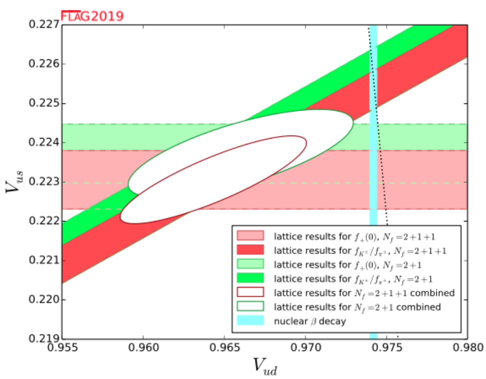
$$\rho_{ij} = \begin{pmatrix} 1 & 0.854 & -0.147 & 0.437 \\ & 1 & -0.125 & 0.373 \\ & & 1 & -0.055 \\ & & & 1 \end{pmatrix},$$

Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Precision experimental data on kaon decays (FLAG'19, 1902.08191):

$$|V_{us}| f_+^{K\pi}(0) = 0.2165(4)\,, \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} = 0.2760(4)\,,$$

- Correlation between $|V_{us}|$ and $|V_{ud}|$



Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Combination to **one and two meson decays**

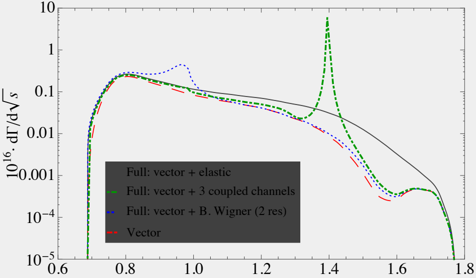
$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2},$$

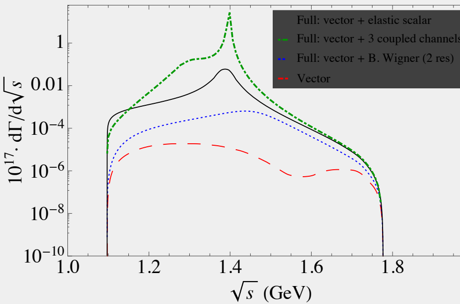
- Errors: Statistic$\pm V_{CKM} \pm \delta_{\rm em}^{\tau\pi(K)}$$\pm$tensor form factor$\pm$quark masses

$$\mathcal{A} = \begin{pmatrix} 1 & 0.055 & 0.000 & -0.279 & -0.394 \\ & 1 & -0.997 & -0.015 & -0.022 \\ & & 1 & 0.000 & 0.000 \\ & & & 1 & 0.243 \\ & & & & 1 \end{pmatrix},$$

$\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$: Invariant mass distribution and Branching Ratio

Escribano, Gonzàlez-Solís, Roig PRD 94 (2016





- $\tau^- \to \pi^-\eta\nu_\tau$
  - Theory predictions: $BR \sim 1 \times 10^{-5}$ (Escribano'16, Moussallam'14)
  - BaBar: $BR < 9.9 \cdot 10^{-5}$ 95% CL , Belle: $BR < 7.3 \cdot 10^{-5}$ 90% CL
- $\tau^- \to \pi^-\eta'\nu_\tau$
  - Theory predictions: $BR \sim [10^{-7}, 10^{-6}]$ (Escribano'16)
  - BaBar: $BR < 4 \cdot 10^{-6}$ 90% CL

Challenging for Belle I