The status and prospects of the muon g-2

Pere Masjuan
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The theory of (g_μ-2) is particle physics in a nutshell. It is an interesting, exciting and difficult subject [...] at the cutting edge of current research in particle physics, and any deviation [...] might be interpreted as a signal of an as-yet-unknown new physics.

- A. Vainshtein

EXCELENCIA
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BIST Barcelona Institute of Science and Technology

RADPyC20
Ciudad de México

Universitat Autònoma
de Barcelona

The anomalous magnetic moment of the muon

Outline

- Introduction
- News about the anomalous magnetic moment of the muon from:
  - theory (White Paper: https://arxiv.org/abs/2006.04822)
  - experiment (https://muon-g-2.fnal.gov/)

- Videoreference
  Cátedra Augusto García González (https://www.fis.cinvestav.mx/~gae/catedra.html): The muon g-2: the gate to New Physics

The anomalous magnetic moment of the muon

Introduction

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} \sim 30 \times 10^{-10} \sim (3...4)\sigma$$

Jegerlehner-Nyffeler '09 review

$$\Delta a_\mu^{\rm SM} \sim 6.2\sigma$$

Bennet et al, PRD73,072003 (2006) (~2200 citations)

$$a_\mu^{\rm exp} = 11\,659\,209.1\,\underbrace{(5.4)(3.3)}_{(6.3)}\times 10^{-10}$$

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$$a_\mu^{\rm exp} = 11\,659\,209.1\,\underbrace{(5.4)(3.3)}_{(6.3)} \times 10^{-10}$$

FNAL (2020-): $a_\mu^{\rm exp} = 11\,659\,???(??) \times 10^{-10}$

Goal: $\sim 1.6 \times 10^{-10}$

Th. White Paper (06/2020): $a_\mu^{\rm exp} = 11\,659\,???(??) \times 10^{-10}$

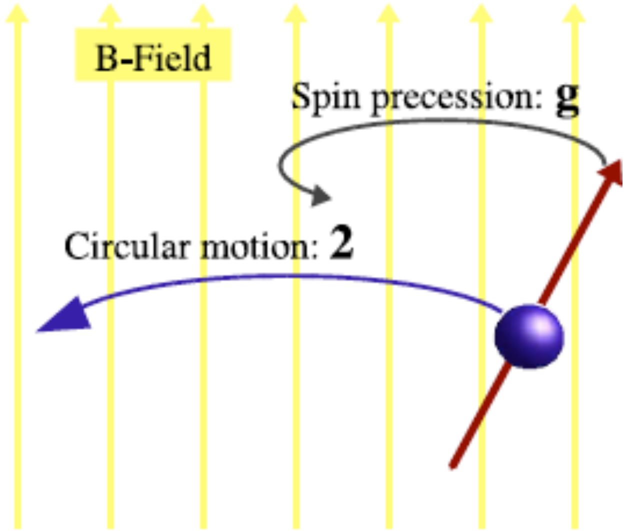
Goal: from 6.2 to $\sim 1.6 \times 10^{-10}$

The anomalous magnetic moment of the muon

- gyromagnetic ratio: g

$$\vec{\mu} = g\frac{e}{2m}\cdot\vec{S}$$

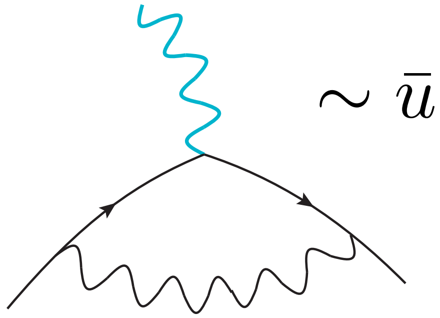
spin $\frac{1}{2}$ → Dirac theory: $g = 2$
QFT (Rad. Corr): $g \neq 2$



$$H_{\text{magnetic}} = -2(1+a_\mu)\frac{e}{2m}\vec{S}\cdot\vec{B}$$

- Deviation from the Dirac value $g = 2$ for muon is:

$$a_\mu = \frac{g_\mu - 2}{2}$$



$$\sim \bar{u}(p')\left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\nu}q^\nu F_M(q^2)\right]u(p)$$

$$a_\mu = -2m_\mu F_M(0)$$

$$a_\mu^{\text{QED,LO}} = \alpha/2\pi \sim 1.16\times 10^{-3}$$

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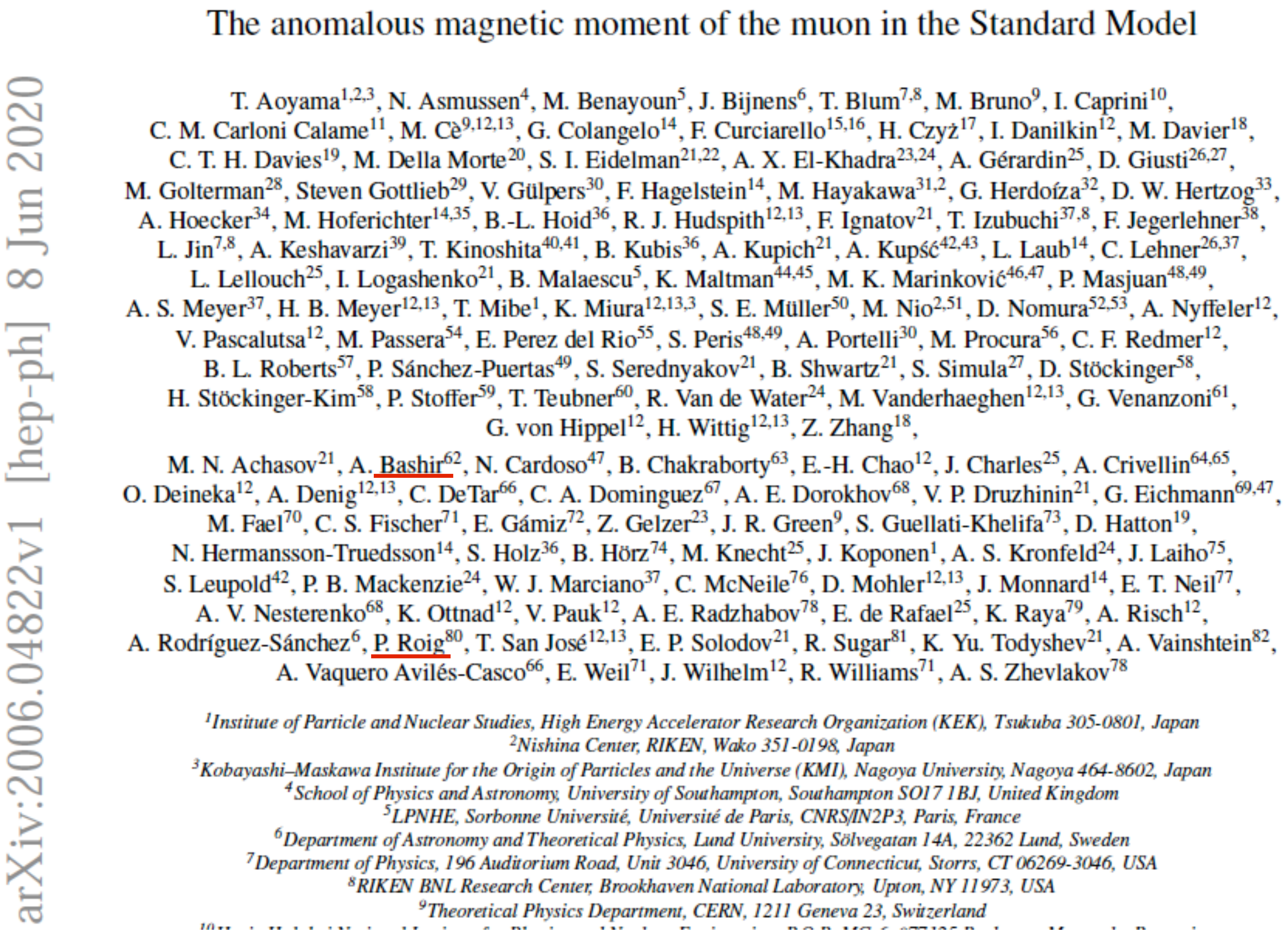

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[18], C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[21,22], A. X. El-Khadra[23,24], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33], A. Hoecker[34], M. Hoferichter[14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[26,37], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49], A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C. F. Redmer[12], B. L. Roberts[57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[60], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77], A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

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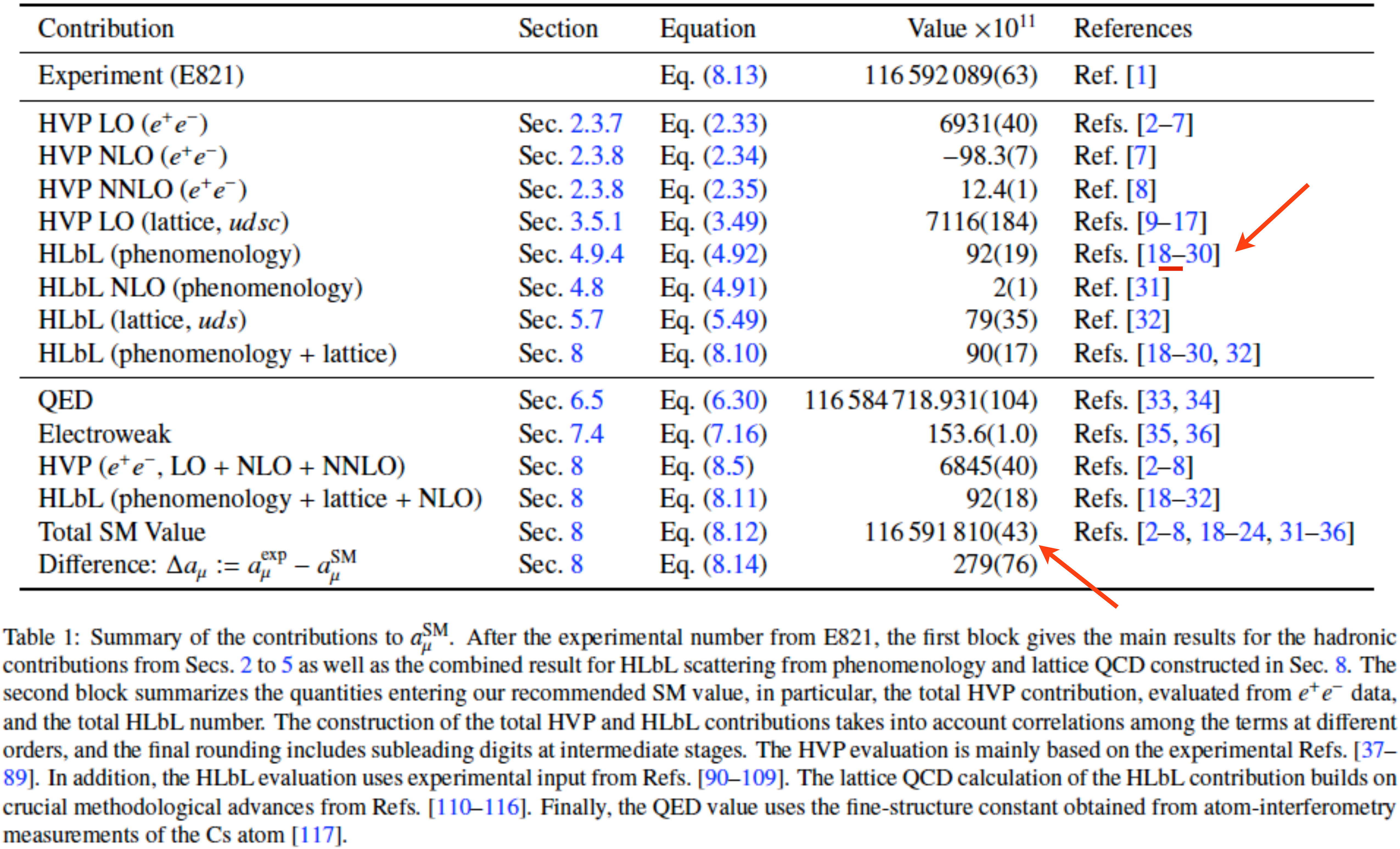


Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 8	Eq. (8.14)	279(76)	

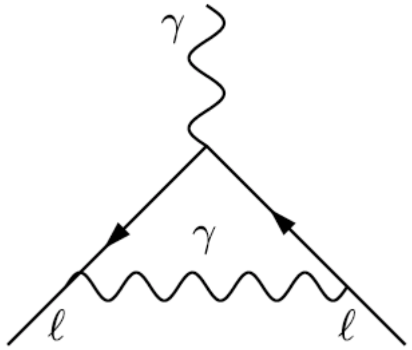
Table 1: Summary of the contributions to $a_\mu^{\rm SM}$. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

All numbers revised, few are new!

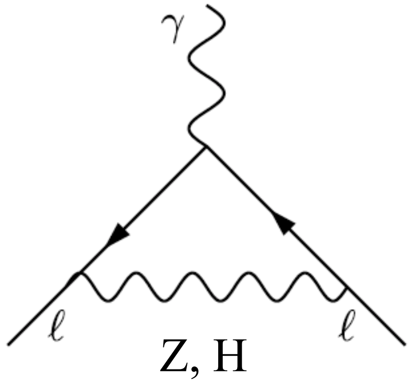
goal not reached yet, 4.3 instead of $\sim 1.6 \times 10^{-10}$

The anomalous magnetic moment of the muon

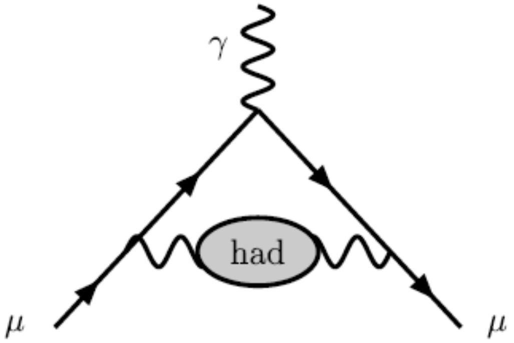
$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$



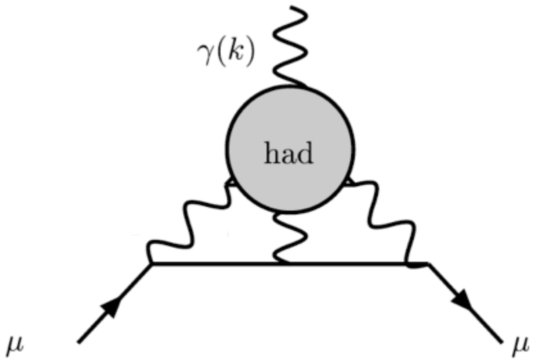
QED: $116\,584\,718.931(104) \times 10^{-11}$



EW: $153.6(1.0) \times 10^{-11}$



Hadronic VP: $6\,845(40) \times 10^{-11}$



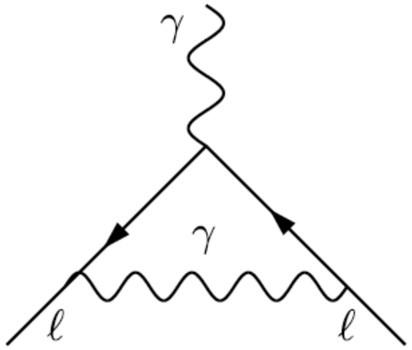
Hadronic LBL: $92(18) \times 10^{-11}$

[White Paper: 2006.04822]

SM: $116\,591\,810(43) \times 10^{-11}$ | $\Delta a_\mu = 279(76) \times 10^{-11}\,(3.7)\sigma$

The anomalous magnetic moment of the muon

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$



QED: $116\,584\,718.931(104) \times 10^{-11}$

- Complete 5-loop result

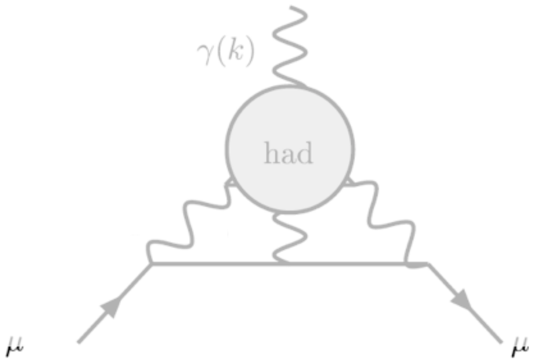
[Aoyama, Hayakawa, Kinoshita, Nio ’12]

- Analytic checks at 4-loop

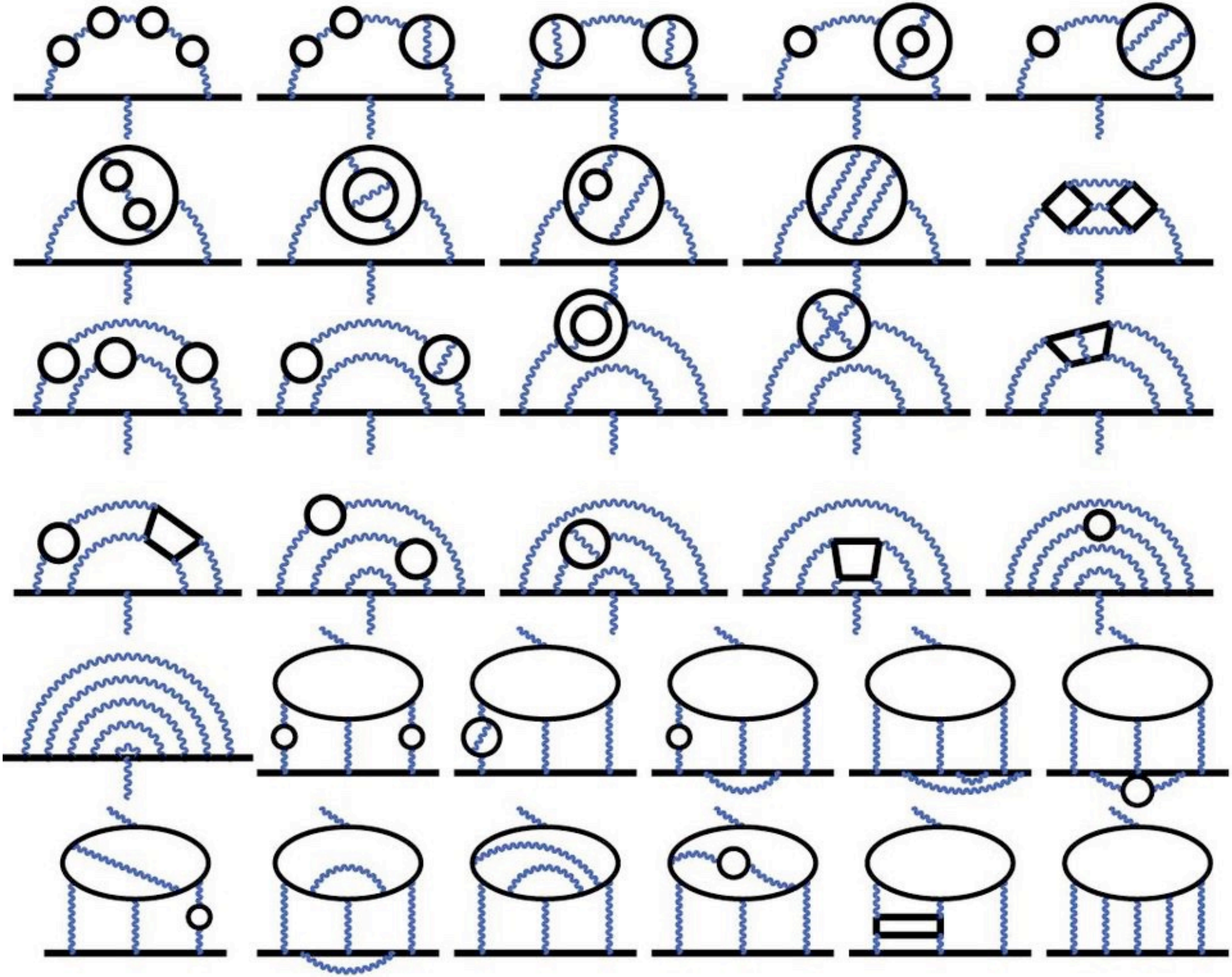
[Marquard, Steinhauser et al ’13-’16]
[Laporta ’17]

EW:

Hadronic VP:



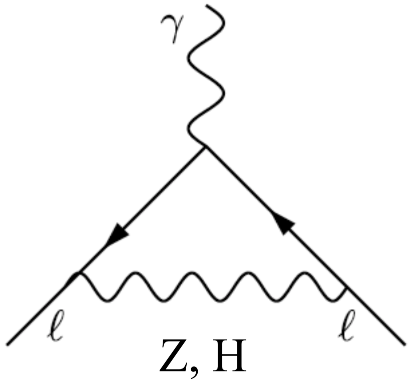
Hadronic LBL:



The anomalous magnetic moment of the muon

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

QED:



EW: $153.6(1.0) \times 10^{-11}$

- Purely EW/bosonic 2-loop result

[Czarnecki, Krause, Marciano '95]
[Heinemeyer, Stöckinger, Weiglein '04]
[Czarnecki, Gribouk '05]

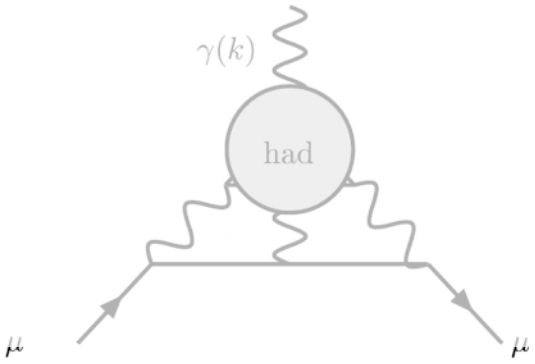
- EW+quarks

[Peris, Perrottet, de Rafael '95][Knecht+PPdR '02]
[Czarnecki, Marciano, Vainsthein '03]

- Including Higgs Mass

[Gnendiger, Stöckinger, Stöckinger-Kim '13]

Hadronic VP:



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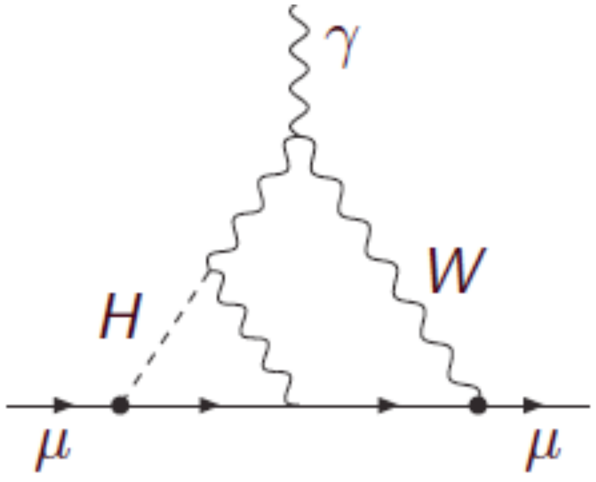

The anomalous magnetic moment of the muon

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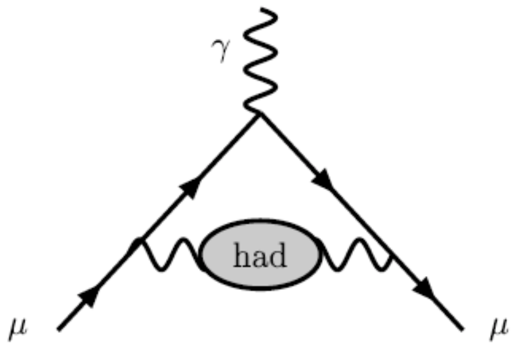
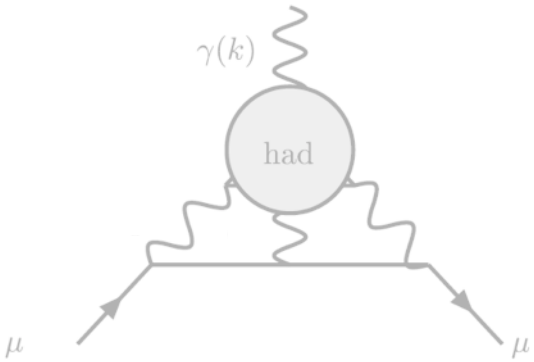
QED:

EW:

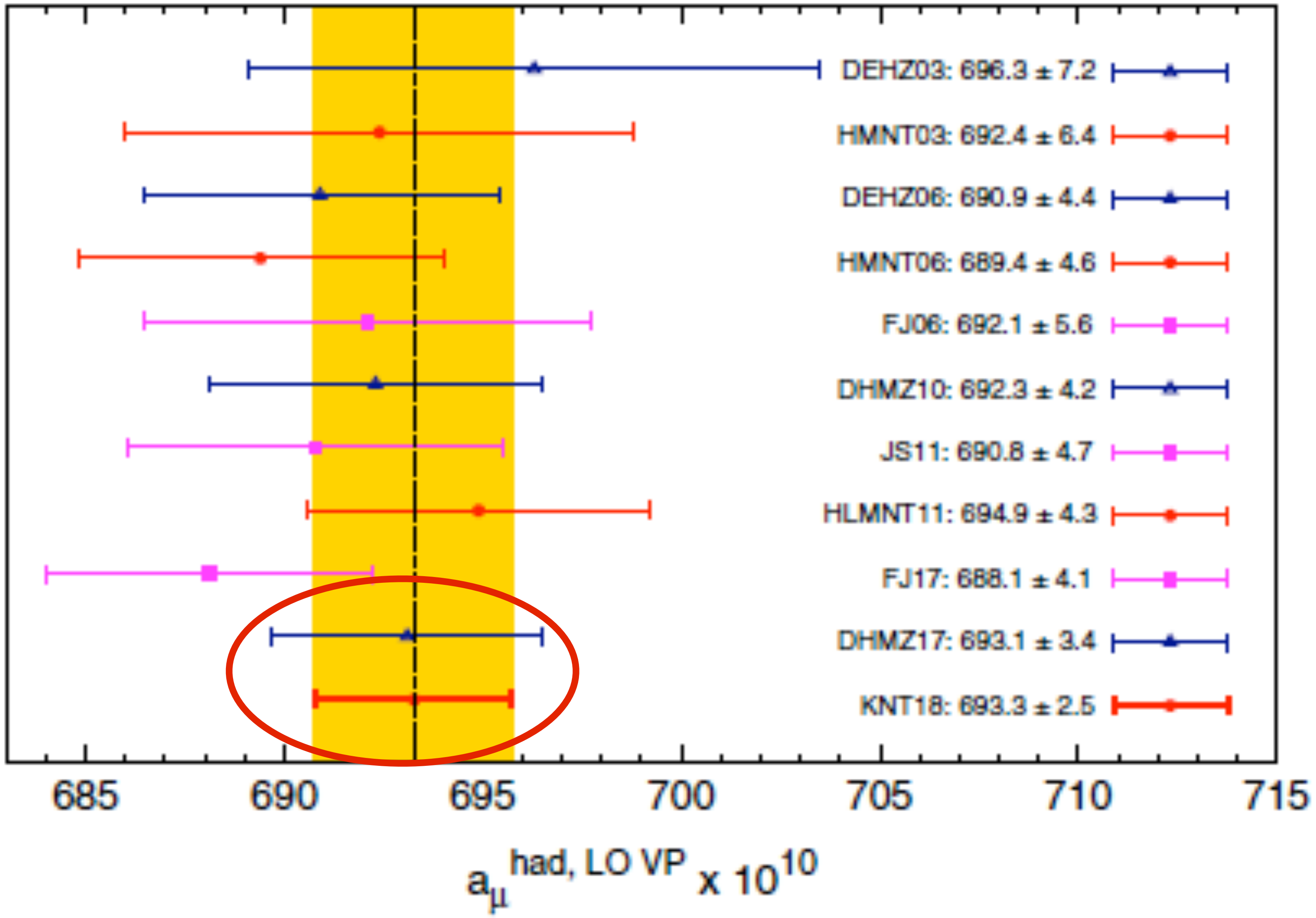


Hadronic VP:

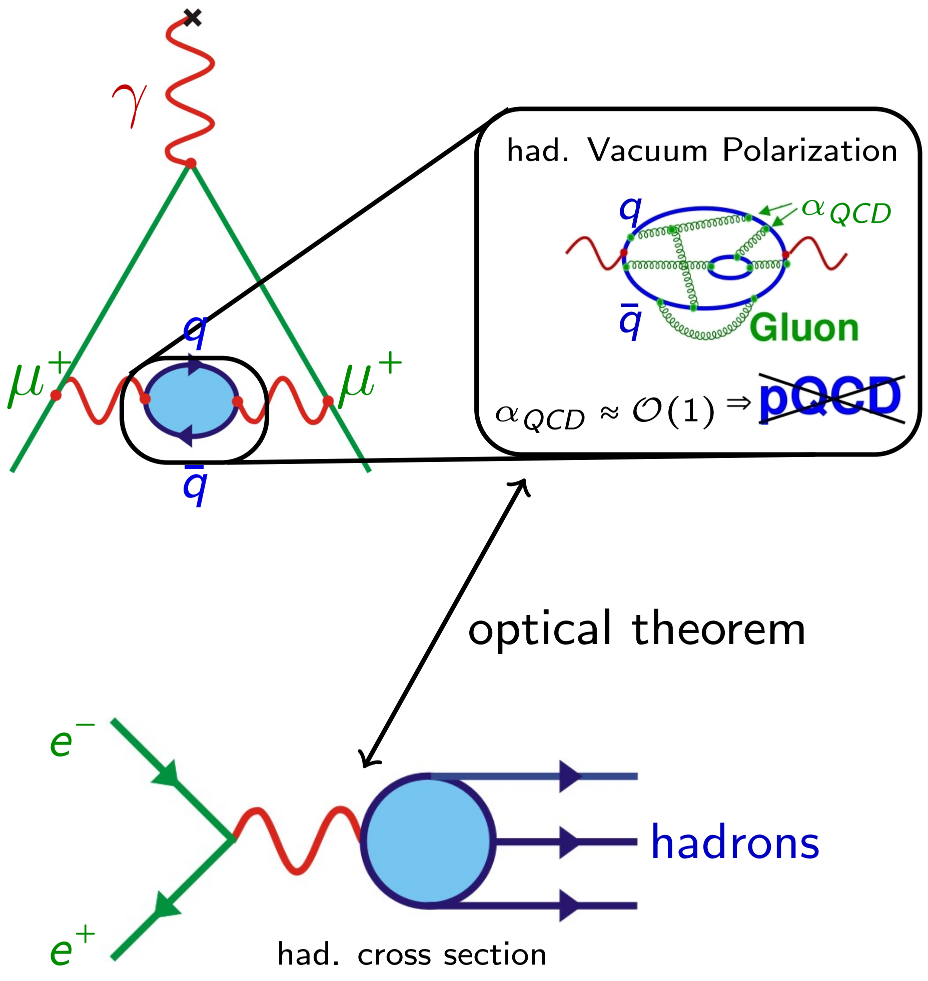


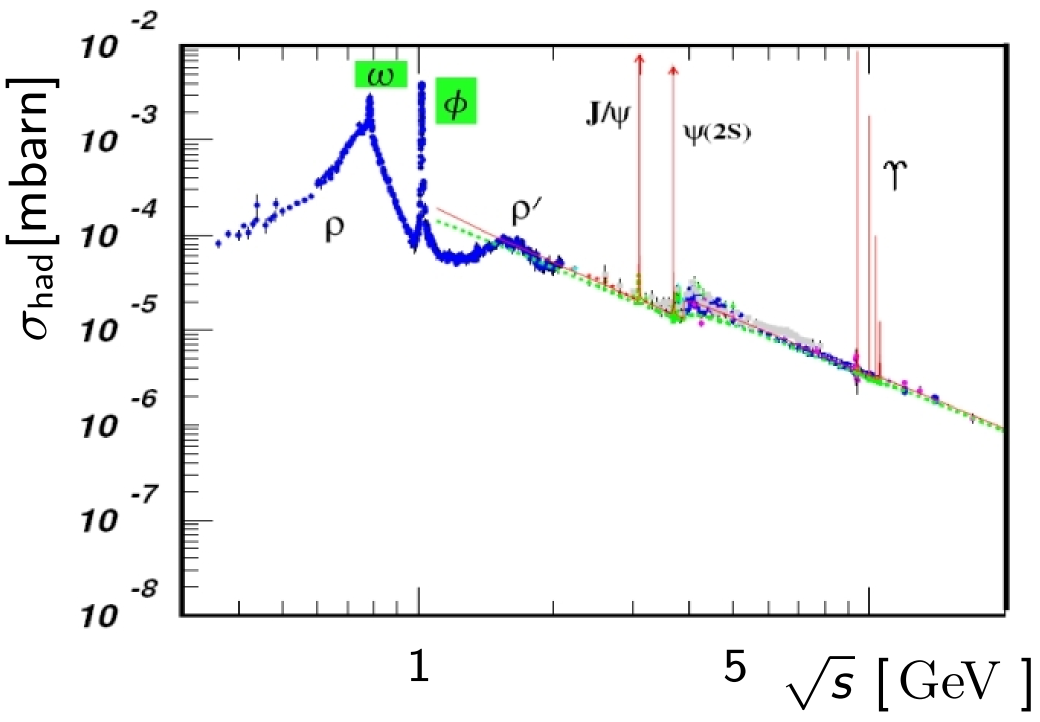
Hadronic LBL:

Progress on Hadronic Vacuum Polarization:



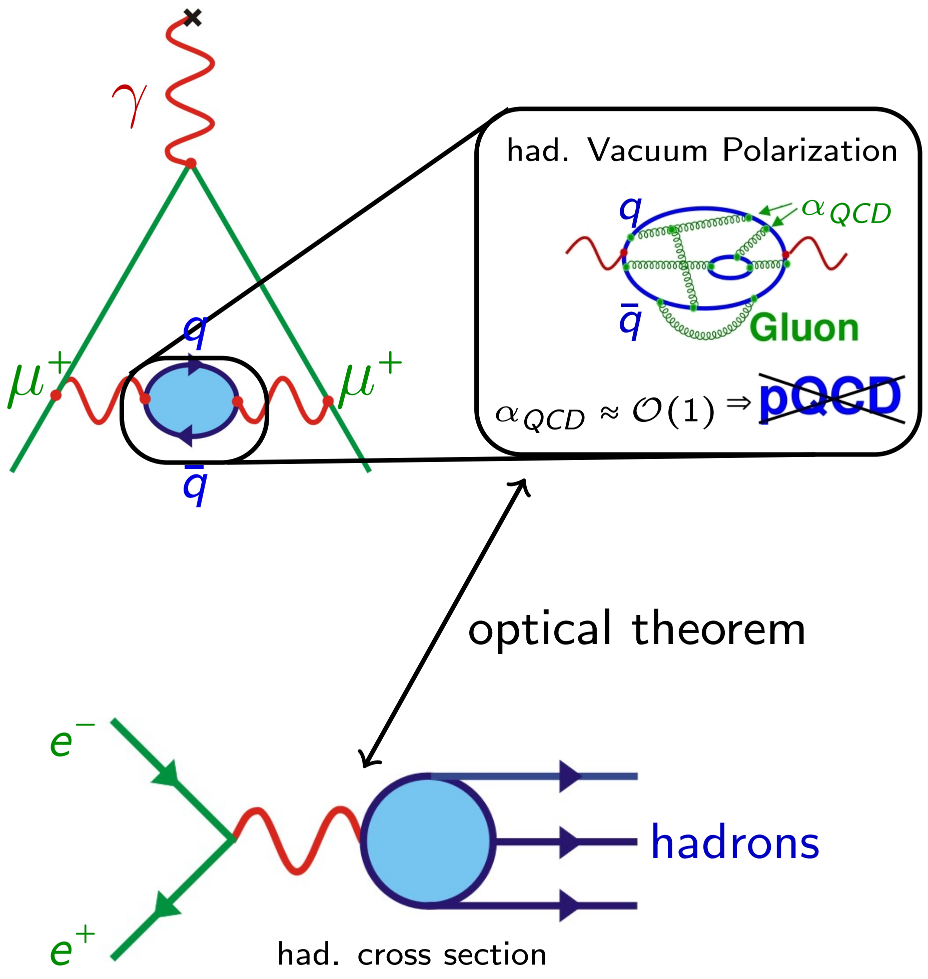
Hadronic Vacuum Polarization

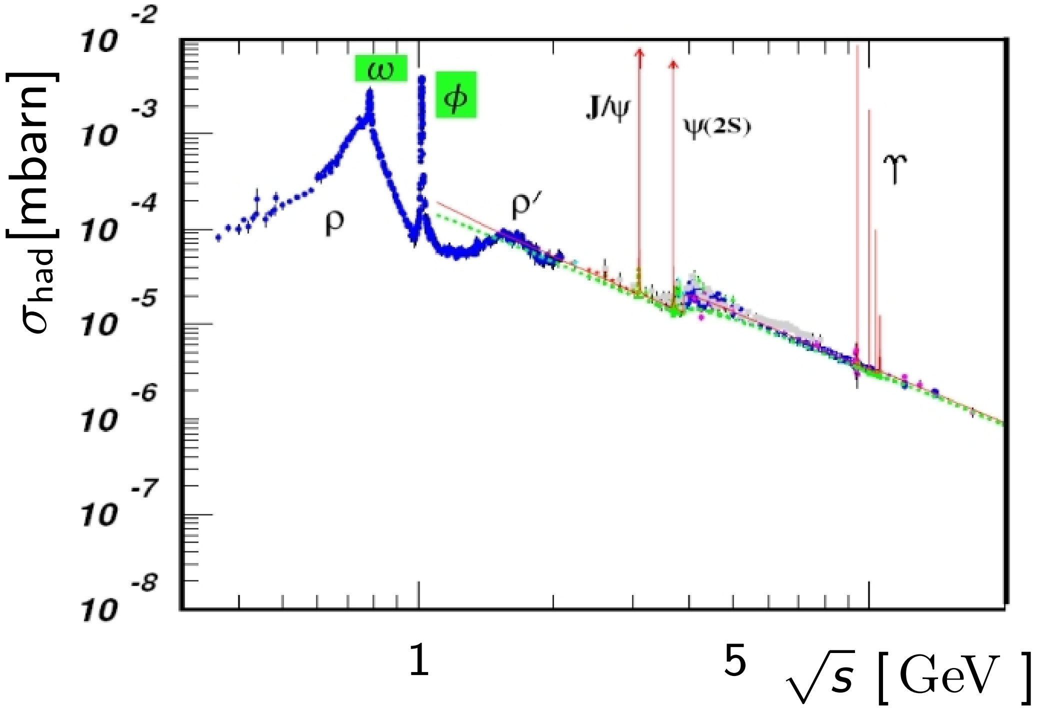




$$a^{\text{had}}_{\mu,LO} = \frac{1}{4\pi^3}\int_{m^2_{\pi^0}}^{\infty} ds\; K(s)\;\sigma_{\text{had}}(s)$$

Hadronic Vacuum Polarization





$$a_{\mu,LO}^{\text{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\, \sigma_{\text{had}}(s)$$

$$\sigma_{\text{had}}(s) \sim 1/s \quad \& \quad K(s) \sim 1/s$$

$$\Downarrow$$

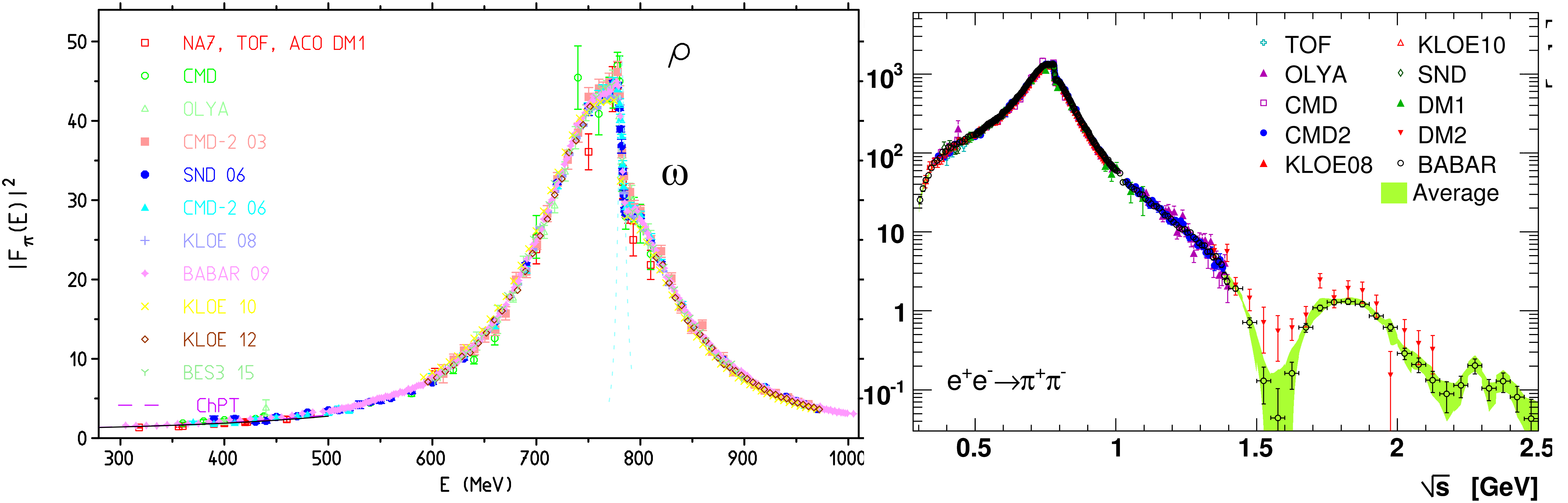
Low energy region important! $\sim 1/s^2$

Sum of exclusive σ_{had}

$$\Downarrow$$

Hadronic contribution of a_μ

Hadronic Vacuum Polarization

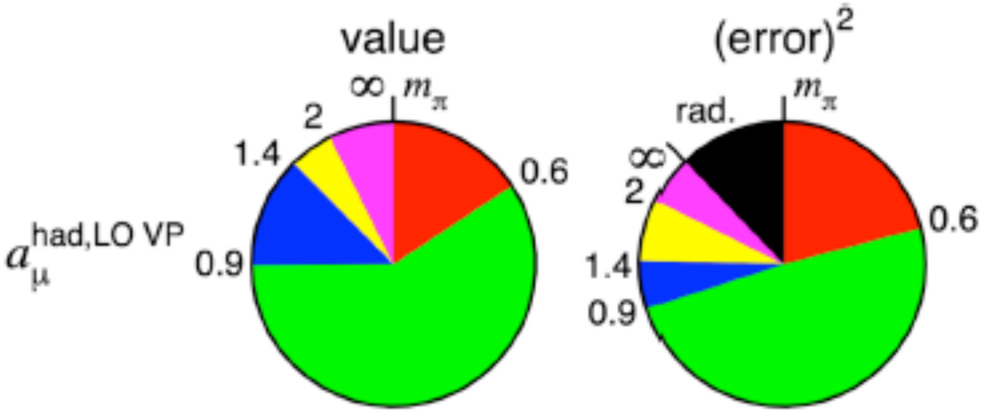


- ρ peak
- ρ-ω interference
- Contribution to a_μ(VP): 75%
- Largest error from 1-2GeV

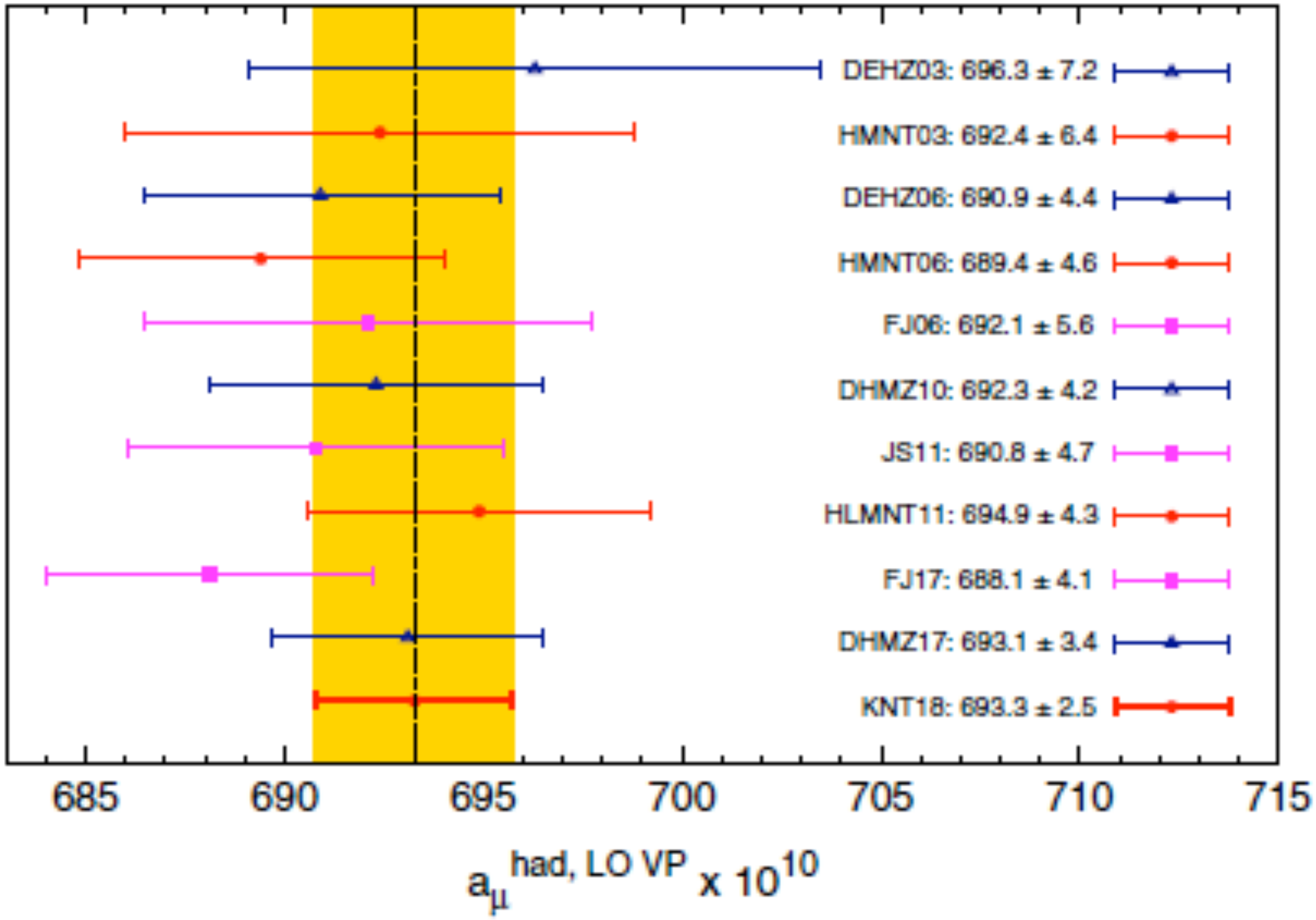
Hadronic Vacuum Polarization

[Keshavarzi et al 18]

Channel	Energy range [GeV]	$a_\mu^{\text{had,LO VP}} \times 10^{10}$
	Chiral perturbation theory (ChPT) threshold contributions	
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00
	Data based channels ($\sqrt{s} \le 1.937$ GeV)	
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78
$(2\pi^+2\pi^-\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01
$(2\pi^+2\pi^-2\pi^0)_{\text{no}\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06
$(\eta\pi^+\pi^-\pi^0)_{\text{no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09
$\eta(\to \text{npp})K\bar{K}_{\text{no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01
	Estimated contributions ($\sqrt{s} \le 1.937$ GeV)	
$(\pi^+\pi^-3\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04
$(\pi^+\pi^-4\pi^0)_{\text{no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02
$\omega(\to \text{npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02
$\omega(\to \text{npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03
$\omega(\to \text{npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04
	Other contributions ($\sqrt{s} > 1.937$ GeV)	
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67
J/ψ	$\cdots$	6.26 ± 0.19
ψ'	$\cdots$	1.58 ± 0.04
$\Upsilon(1S-4S)$	$\cdots$	0.09 ± 0.00
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46



Hadronic Vacuum Polarization



[Keshavarzi et al 18]

$$a_\mu^{\text{had, LO VP}} = 693.26(2.46) \times 10^{-10}$$

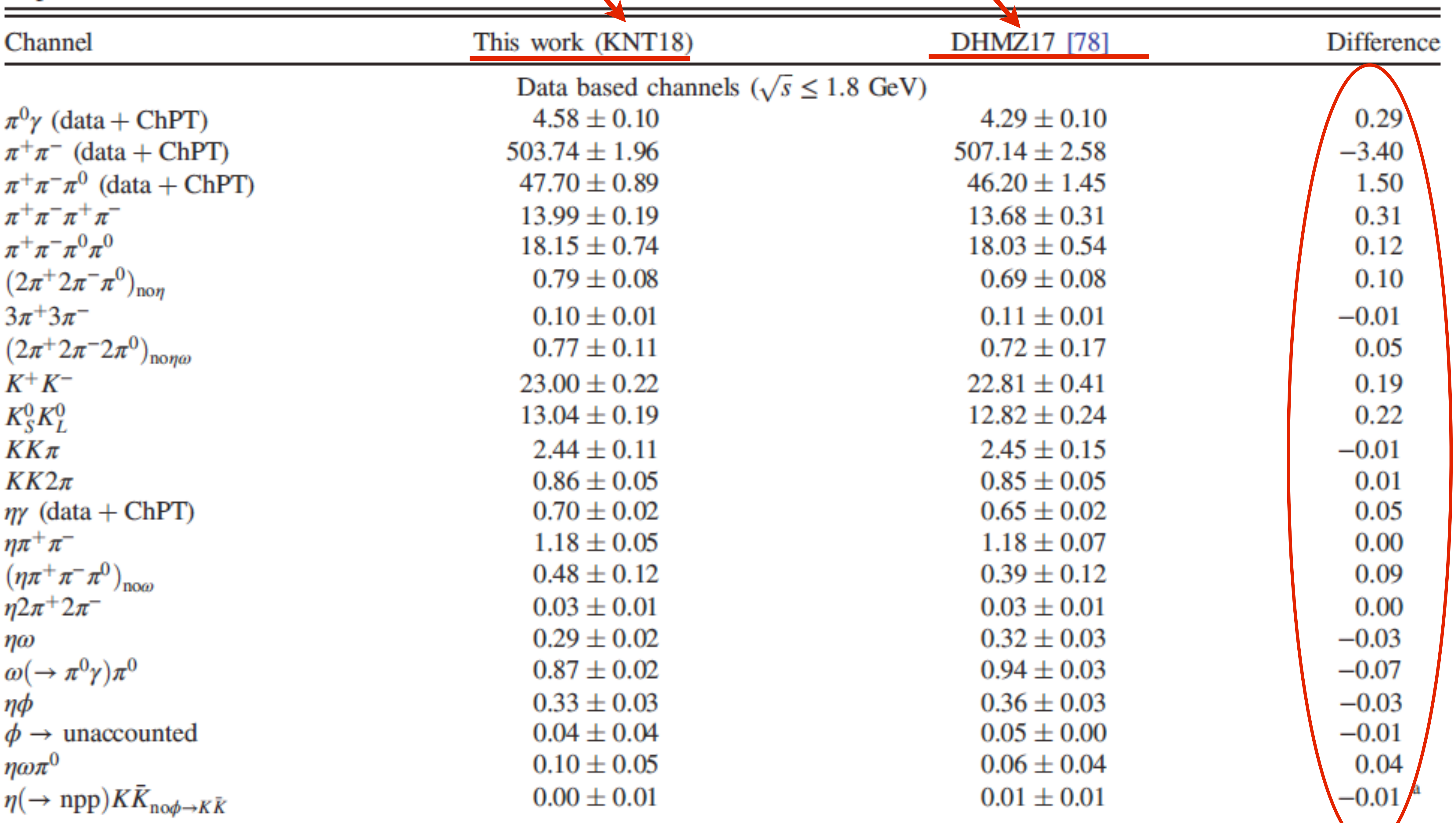
$$a_\mu^{\text{had, NLO VP}} = -9.82(0.04) \times 10^{-10}$$

$$a_\mu^{\text{had, NNLO VP}} = 1.24(0.01) \times 10^{-10}$$

$$\boxed{a_\mu^{\text{had, VP}} = 684.68(\underline{2.42}) \times 10^{-10}}$$

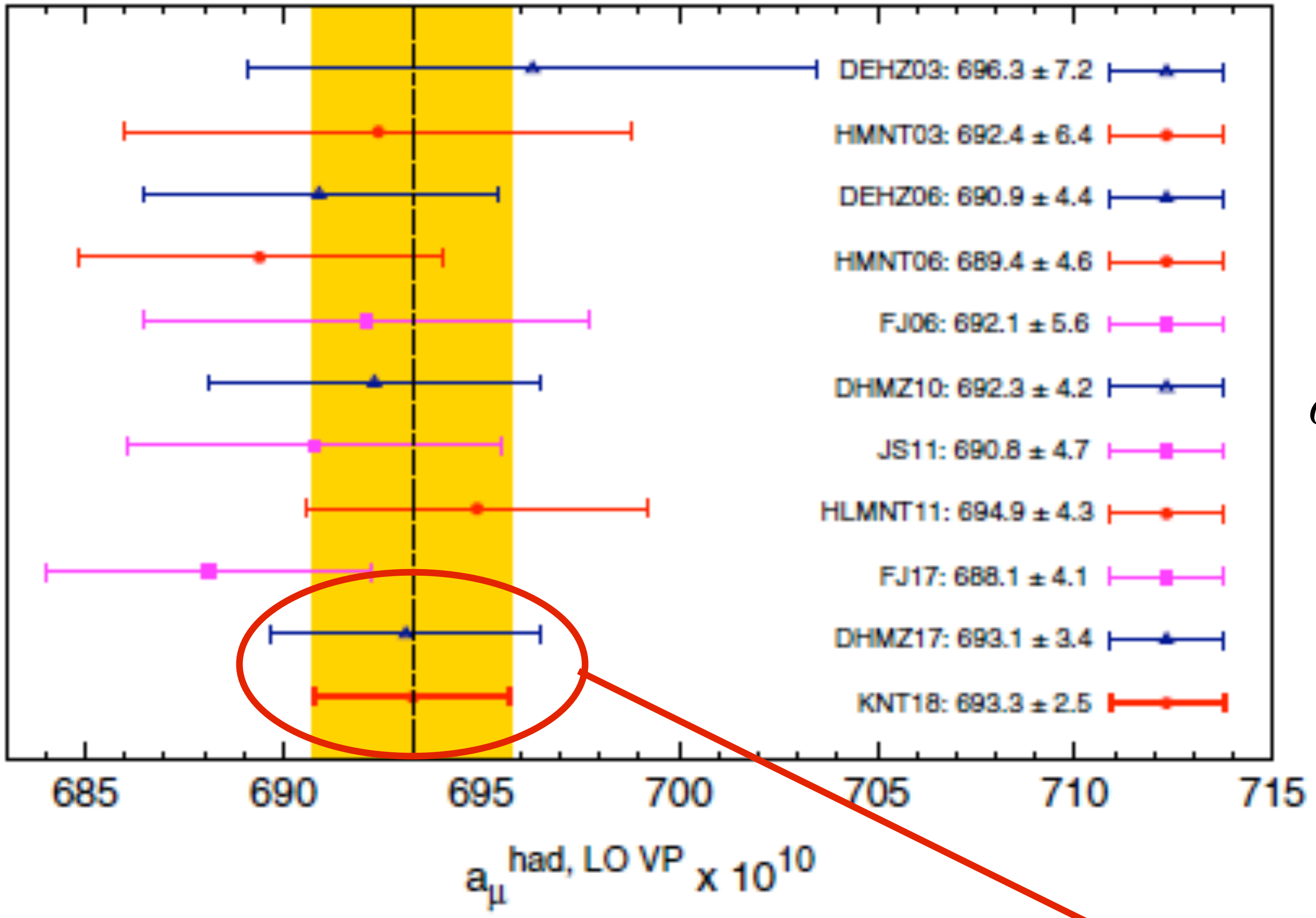
Hadronic Vacuum Polarization

[Keshavarzi et al 18]



Channel	This work (KNT18)	DHMZ17 [78]	Difference
	Data based channels ($\sqrt{s} \leq 1.8$ GeV)		
$\pi^0\gamma$ (data + ChPT)	4.58 ± 0.10	4.29 ± 0.10	0.29
$\pi^+\pi^-$ (data + ChPT)	503.74 ± 1.96	507.14 ± 2.58	−3.40
$\pi^+\pi^-\pi^0$ (data + ChPT)	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
$(2\pi^+2\pi^-\pi^0)_{\mathrm{no}\eta}$	0.79 ± 0.08	0.69 ± 0.08	0.10
$3\pi^+3\pi^-$	0.10 ± 0.01	0.11 ± 0.01	−0.01
$(2\pi^+2\pi^-2\pi^0)_{\mathrm{no}\eta\omega}$	0.77 ± 0.11	0.72 ± 0.17	0.05
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22
$KK\pi$	2.44 ± 0.11	2.45 ± 0.15	−0.01
$KK2\pi$	0.86 ± 0.05	0.85 ± 0.05	0.01
$\eta\gamma$ (data + ChPT)	0.70 ± 0.02	0.65 ± 0.02	0.05
$\eta\pi^+\pi^-$	1.18 ± 0.05	1.18 ± 0.07	0.00
$(\eta\pi^+\pi^-\pi^0)_{\mathrm{no}\omega}$	0.48 ± 0.12	0.39 ± 0.12	0.09
$\eta 2\pi^+2\pi^-$	0.03 ± 0.01	0.03 ± 0.01	0.00
$\eta\omega$	0.29 ± 0.02	0.32 ± 0.03	−0.03
$\omega(\rightarrow \pi^0\gamma)\pi^0$	0.87 ± 0.02	0.94 ± 0.03	−0.07
$\eta\phi$	0.33 ± 0.03	0.36 ± 0.03	−0.03
$\phi \rightarrow$ unaccounted	0.04 ± 0.04	0.05 ± 0.00	−0.01
$\eta\omega\pi^0$	0.10 ± 0.05	0.06 ± 0.04	0.04
$\eta(\rightarrow \mathrm{npp})K\bar{K}_{\mathrm{no}\phi\rightarrow K\bar{K}}$	0.00 ± 0.01	0.01 ± 0.01	−0.01[a]

Hadronic Vacuum Polarization



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$$a_\mu^{\text{had, NNLO VP}} = 1.24(0.01) \times 10^{-10}$$

$$a_\mu^{\text{HVP}} = 6\,845(40) \times 10^{-11}$$

[White Paper: 2006.04822]

[Masjuan,'19]

$$a_\mu^{\text{HVP}} = 6\,849(43) \times 10^{-11}$$

The anomalous magnetic moment of the muon

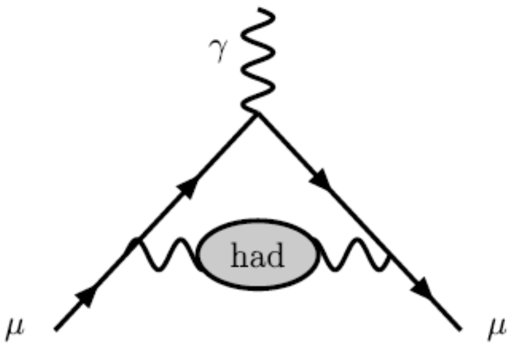
$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Progress on Hadronic Vacuum Polarization:

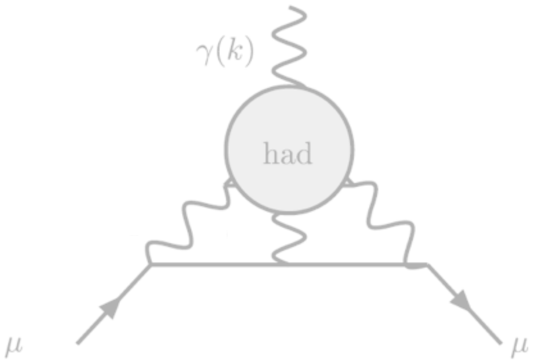
Lattice simulation!

QED:

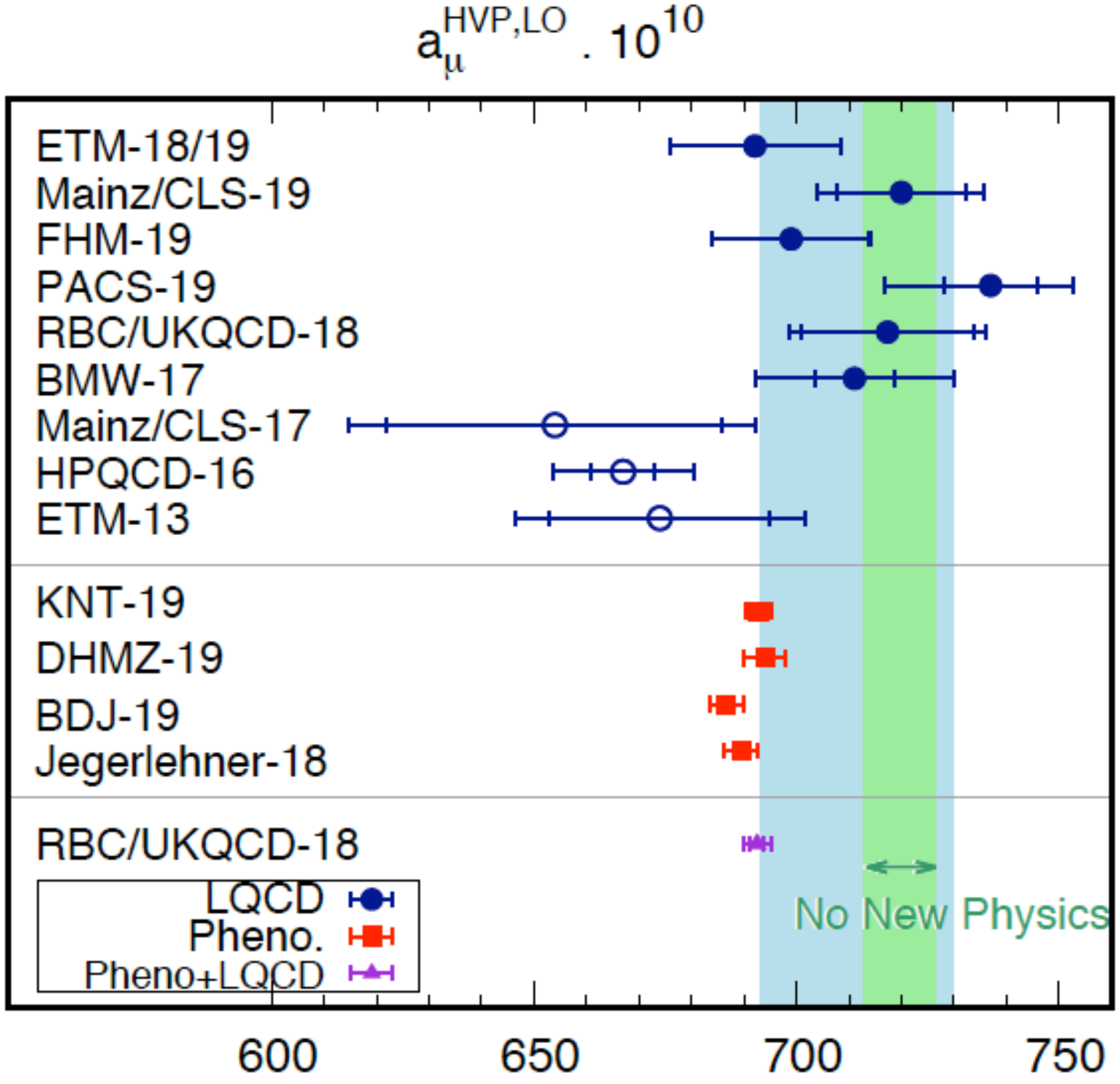
EW:



Hadronic VP:



Hadronic LBL:



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Progress on Hadronic Vacuum Polarization:

Lattice simulation!

QED:

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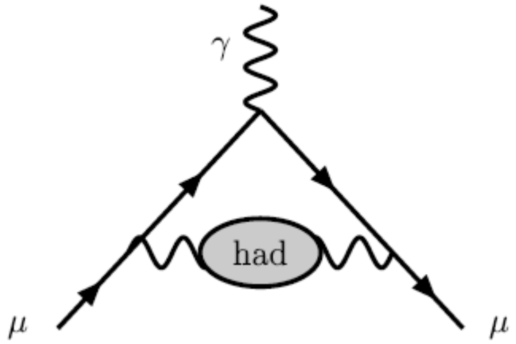
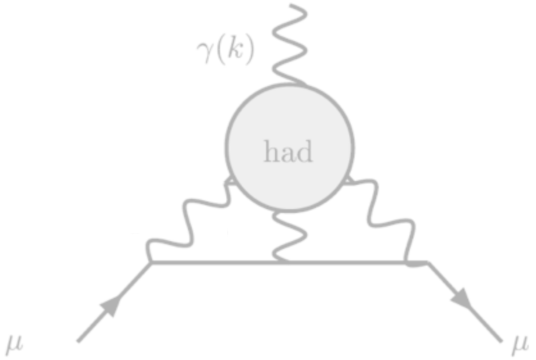
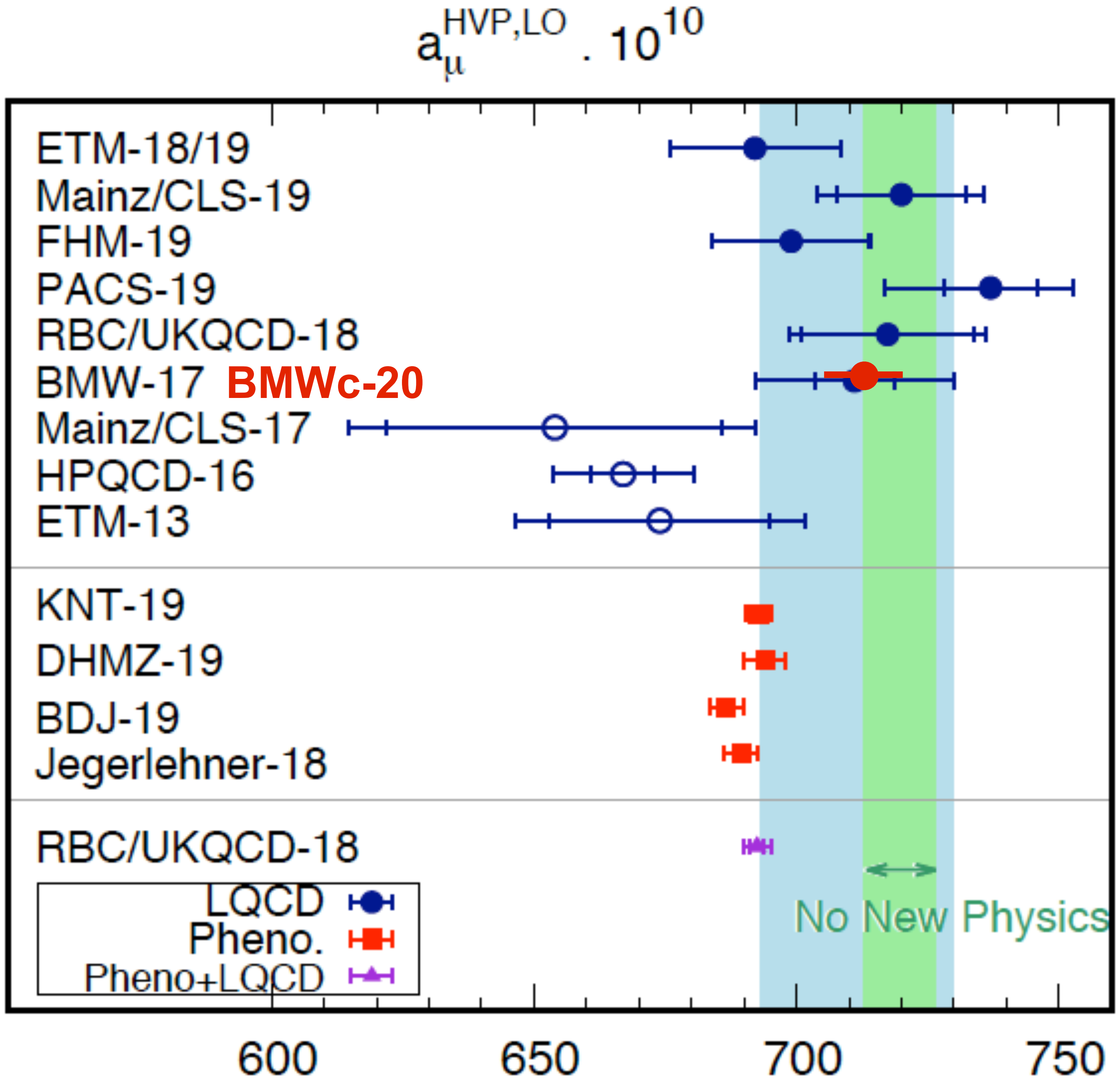


Hadronic VP:



Hadronic LBL:



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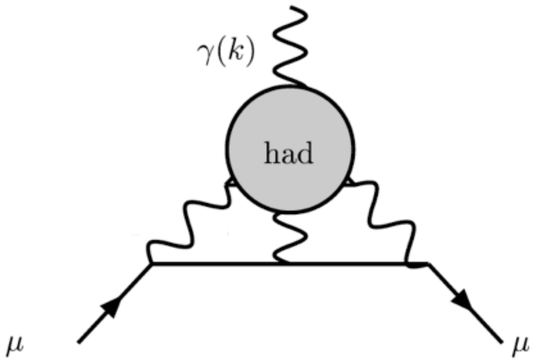
QED:

EW:

Hadronic VP:

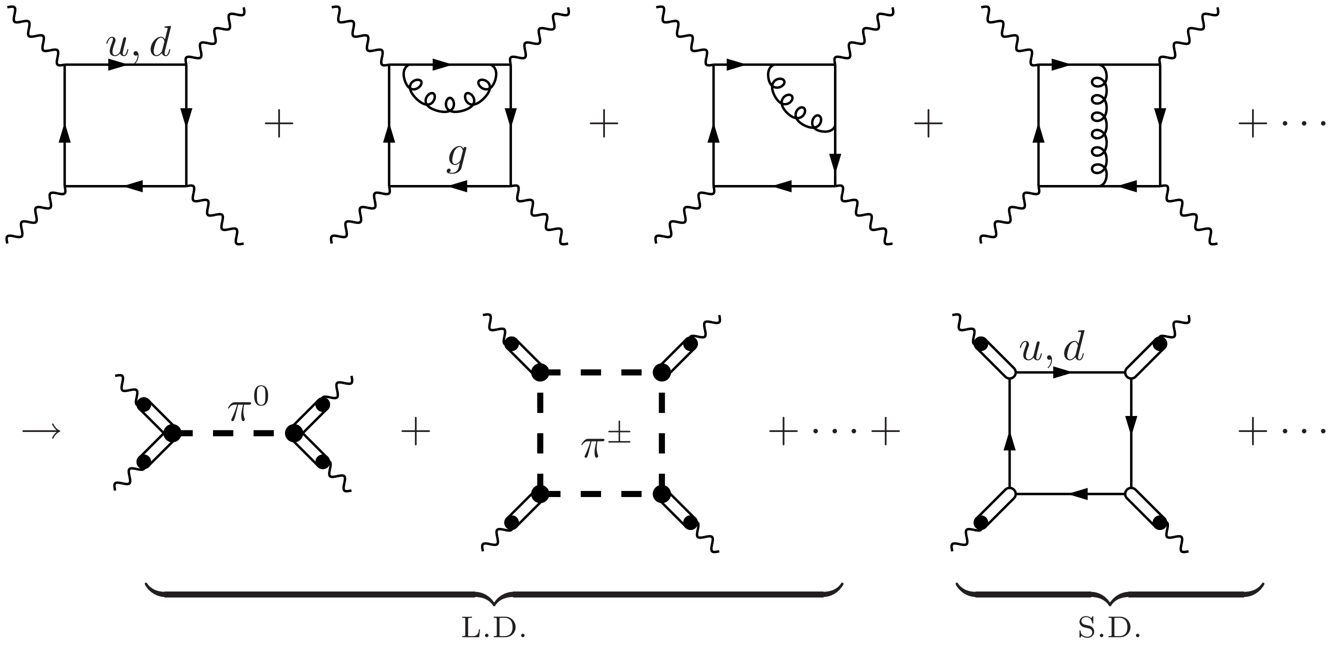


Hadronic LBL:

Progress on Hadronic Light-by-Light:

order O(α³) hadronic contribution

Multiscale problem



Model at low energies
(with exchange of resonances)

Model at high energies
(quark-loop)

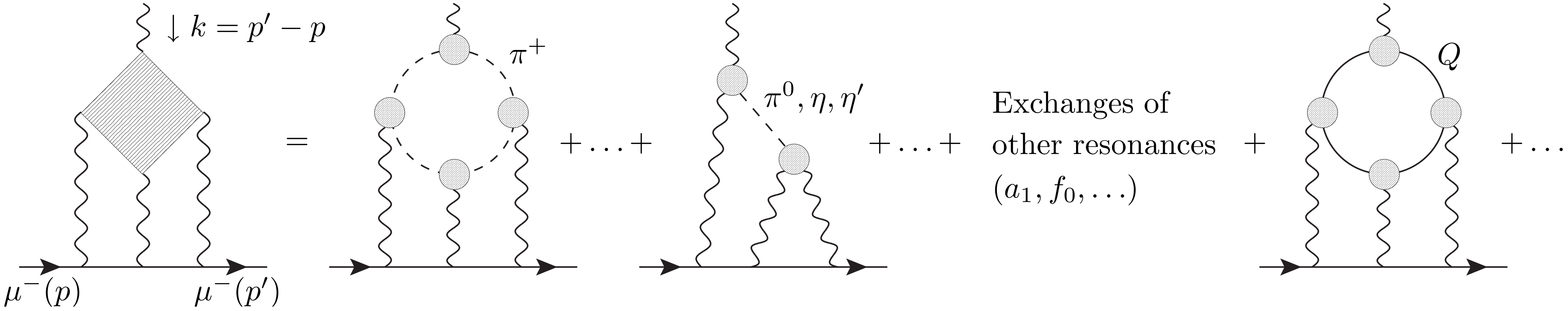
The anomalous magnetic moment of the muon

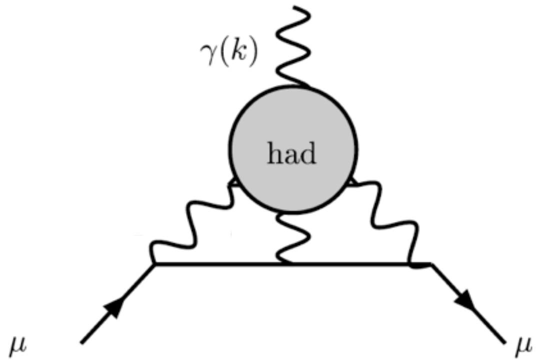
$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Progress on Hadronic Light-by-Light:

Traditionally, low-energy models:

$$a_\mu^{\rm HLBL}({\rm Jegerlehner}\,'18) = [(6.47+1.49+1.59)(1.24)^{\pi,\eta,\eta'} + 0.76(27)^{\rm axial} - 0.60(12)^{\pi\pi} - 0.5(5)^{\pi,Kloop+SD}]\times 10^{-10}$$





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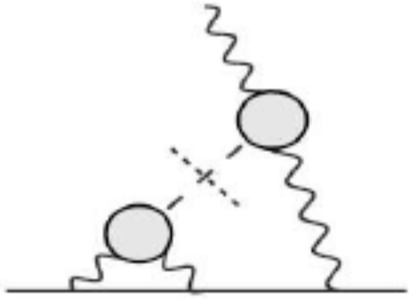


Now, dispersion relations: huge theoretical progress:
- QFT definition of pion-pole/pion-box ...
- Allows a data driven, with controlled errors estimates:

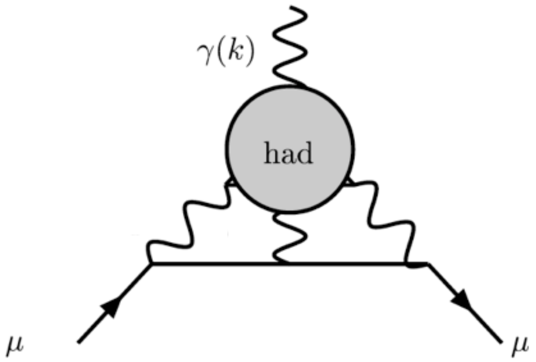
[Colangelo, Hoferichter, Procura, Stoffer ’15][Pauk, Vanderhaeghen ’14]
[Masjuan, Sanchez-Puertas ’17][Hoferichter et. al ’18] [Colangelo et al ’18][Bijnens et al ’19][Roig, Sanchez-Puertas ’20]

$$a_\mu^{HLBL;\pi^0} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

$$\times \left(\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, (q_1+q_2)^2) F_{\pi^0\gamma^*\gamma^*}(q_2^2, 0)}{q_2^2 - M_\pi^2} T_1(q_1, q_2; p) \right.$$

$$\left. + \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_\pi^2} T_2(q_1, q_2; p) \right)$$

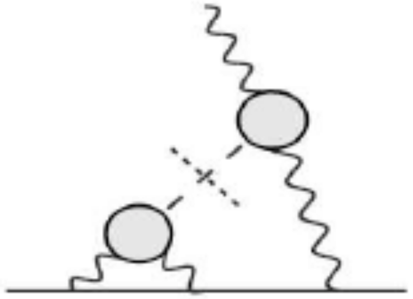
Use data from the pion
Transition Form Factor



The anomalous magnetic moment of the muon

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Progress on Hadronic Light-by-Light:



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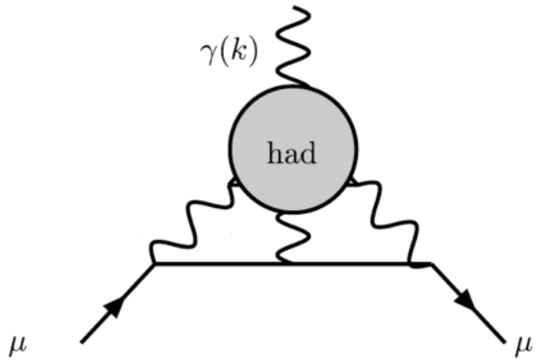
$$a_\mu^{\rm HLBL}({\rm WhitePaper}\,'20) = [\underline{6.36(2.7)^{\pi} + 1.63(1.4)^{\eta} + 1.45(1.9)^{\eta'}}\ \ \underline{+0.6(6)^{\rm axial}} - 0.8(1)^{\pi\pi}\ \ +5.4(5)^{\pi,Kloop+SD+charm}] \times 10^{-10}$$

[Masjuan, Sanchez-Puertas '17] [Roig, Sanchez-Puertas '20]



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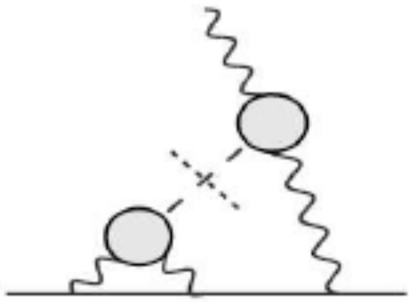

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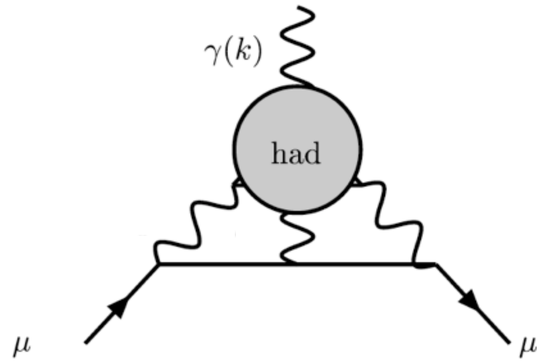
$$a_\mu^{\mathrm{HLBL}}(\mathrm{WhitePaper}\,{}'20) = 92(19) \times 10^{-11}$$

In summary:

- via using exp. data (CLEO, BES, BABAR, BELLE), we realize that the Transition Form Factor is more important than expected
- systematic errors are difficult to account but important



The anomalous magnetic moment of the muon

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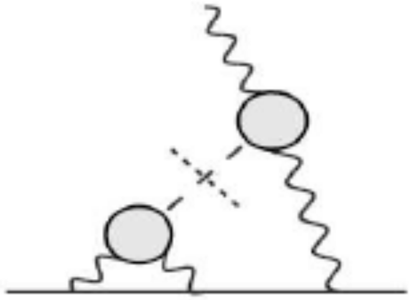

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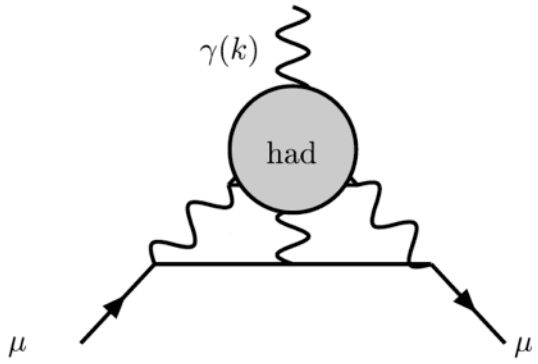
$$a_\mu^{\mathrm{HLBL}}(\mathrm{WhitePaper}\,'20) = 92(19) \times 10^{-11}$$

- Nice confirmation from Lattice: $a_\mu^{\mathrm{HLBL}}(\mathrm{Lattice}\,'19) = 7.87(3.06)_{\mathrm{stat}}(1.77)_{\mathrm{sys}} \times 10^{-10}$

[Blum et al, '19]



The anomalous magnetic moment of the muon

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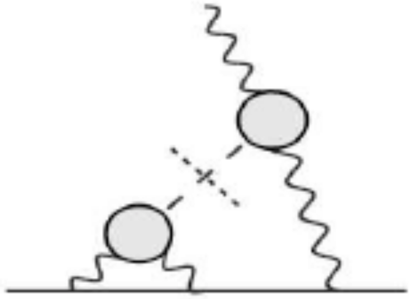

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$$a_\mu^{\rm HLBL}({\rm WhitePaper\,'20}) = 92(19)\times 10^{-11}$$

- Still contrasting with ballpark estimates based on pQCD quark loop:

$$a_\mu^{HLBL}(M(Q)) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left[\left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right)\frac{m_\mu^2}{M(Q)^2} + \mathcal{O}\left(\frac{m_\mu^4}{M(Q)^4}\log^2\frac{m_\mu^2}{M(Q)^2}\right)\right]$$

[Laporta and Remiddi '96]

$$\boxed{105\times 10^{-11} < a_\mu^{\rm HLBL}({\rm Ballparks}) < 150\times 10^{-11}}$$

$$M_q \sim 0.160 - 0.180 {\rm GeV}$$

[Pivovarov '03][Erler and Toledo Sanchez '06]
[Boughezal and Melnikov, '11]
[Masjuan, Vanderhaeghen 2012]

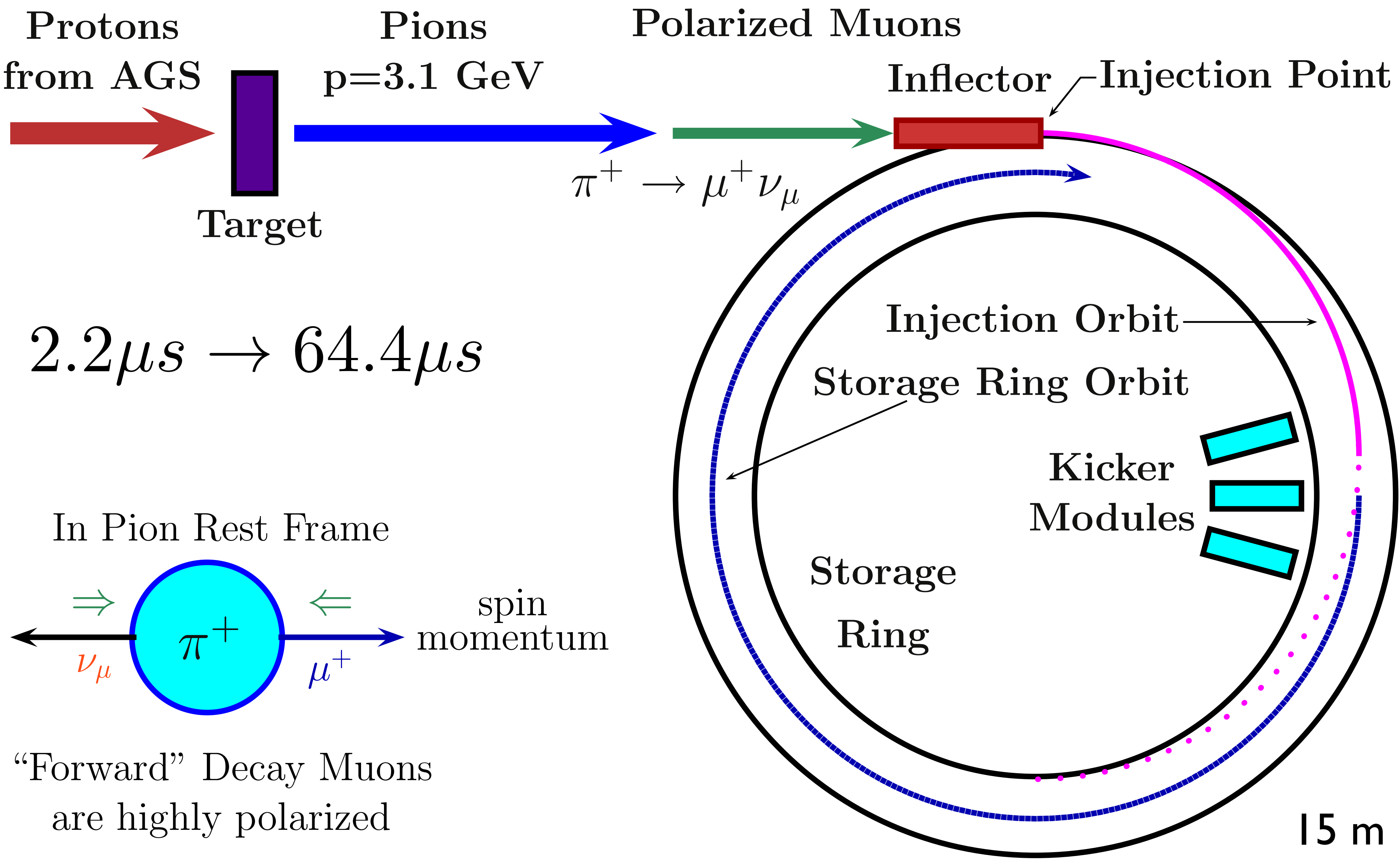
Still something to understand: link low-high energies

The anomalous magnetic moment of the muon

Outline

- Introduction
- News about the anomalous magnetic moment of the muon from:
  - theory (White Paper: https://arxiv.org/abs/2006.04822)
  - experiment (https://muon-g-2.fnal.gov/)

The anomalous magnetic moment of the muon: the experiment



Protons from AGS

Target

Pions p=3.1 GeV

$\pi^+ \rightarrow \mu^+ \nu_\mu$

Polarized Muons

Inflector

Injection Point

Injection Orbit

Storage Ring Orbit

Kicker Modules

Storage Ring

$$2.2\mu s \rightarrow 64.4\mu s$$

In Pion Rest Frame

$\Rightarrow$ $\Leftarrow$

ν_μ π^+ μ^+

spin
momentum

"Forward" Decay Muons
are highly polarized

15 m diameter

$$1.45T$$

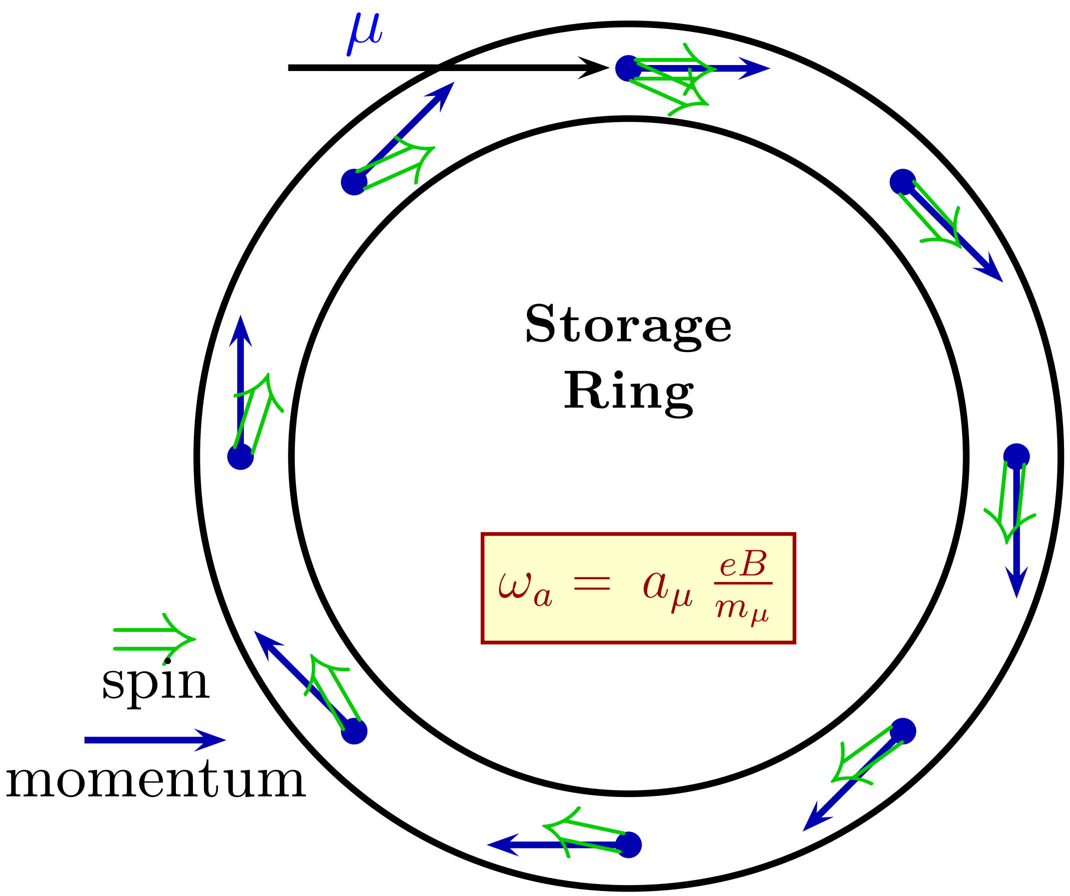
The anomalous magnetic moment of the muon: the experiment

$$\omega_c = \frac{eB}{m\gamma}$$

cyclotron precession

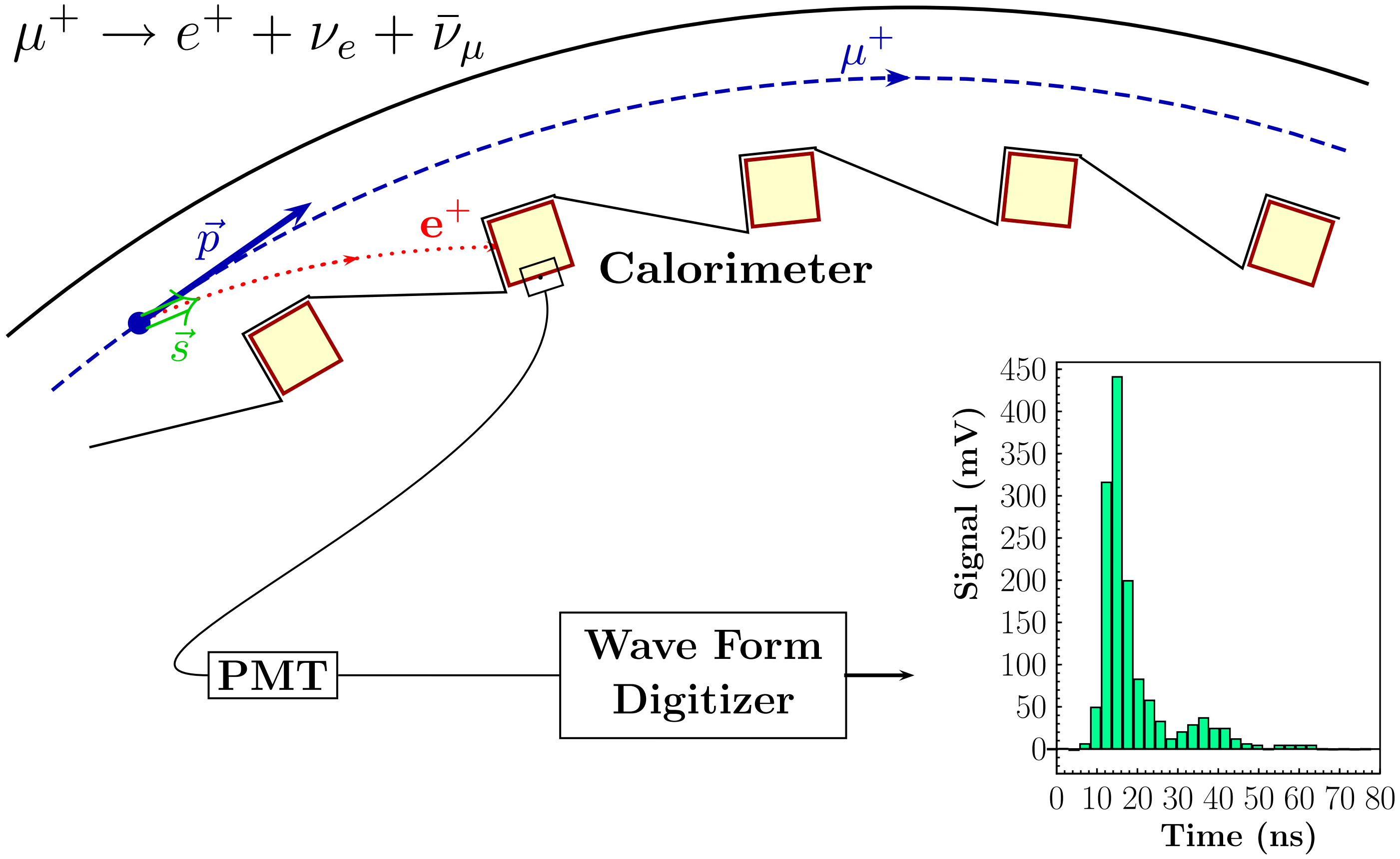
$$\omega_s = \frac{geB}{2m} + (1-\gamma)\frac{eB}{m\gamma}$$

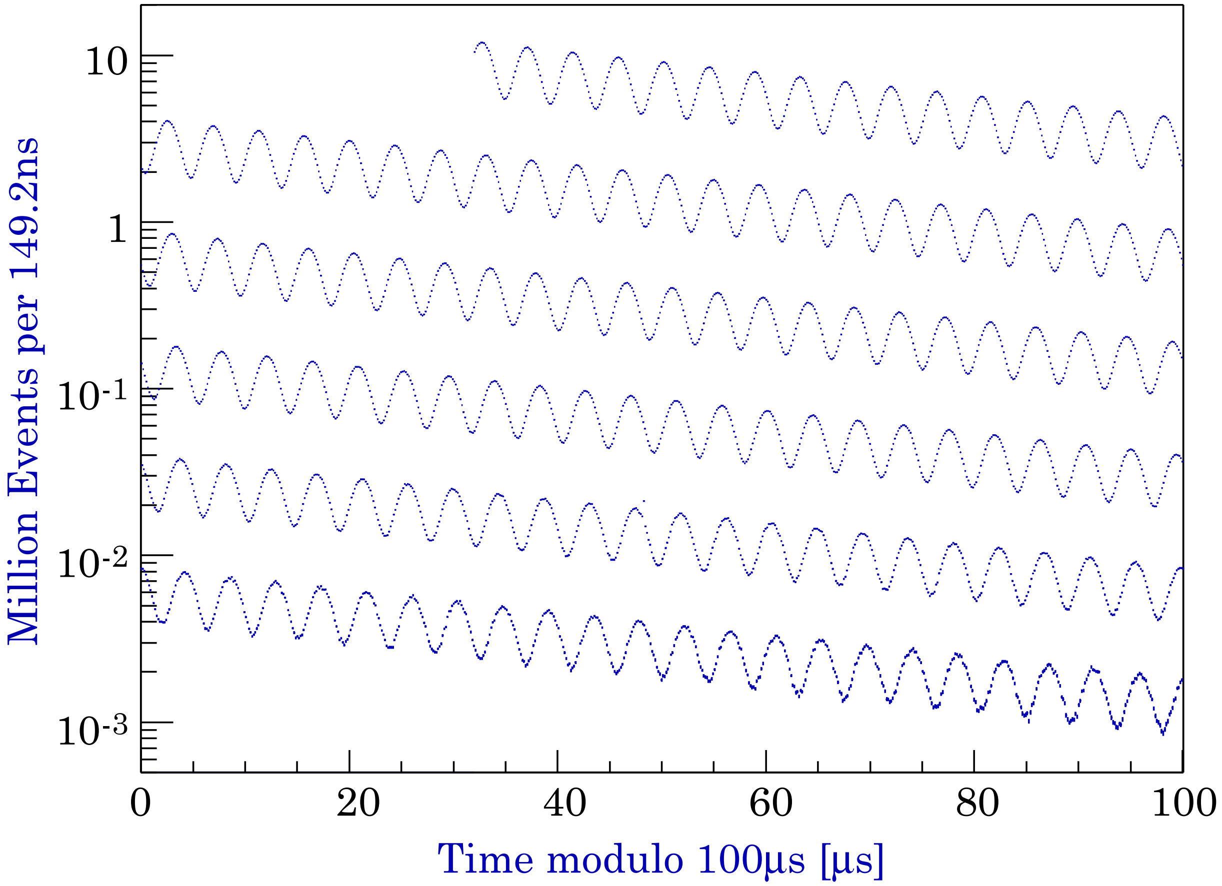
spin precession (Larmor)



$$\boxed{\omega_a = \omega_s - \omega_c = \left(\frac{g-2}{2}\right)\frac{eB}{m} = a_\mu \frac{eB}{m}}$$

The anomalous magnetic moment of the muon: the experiment

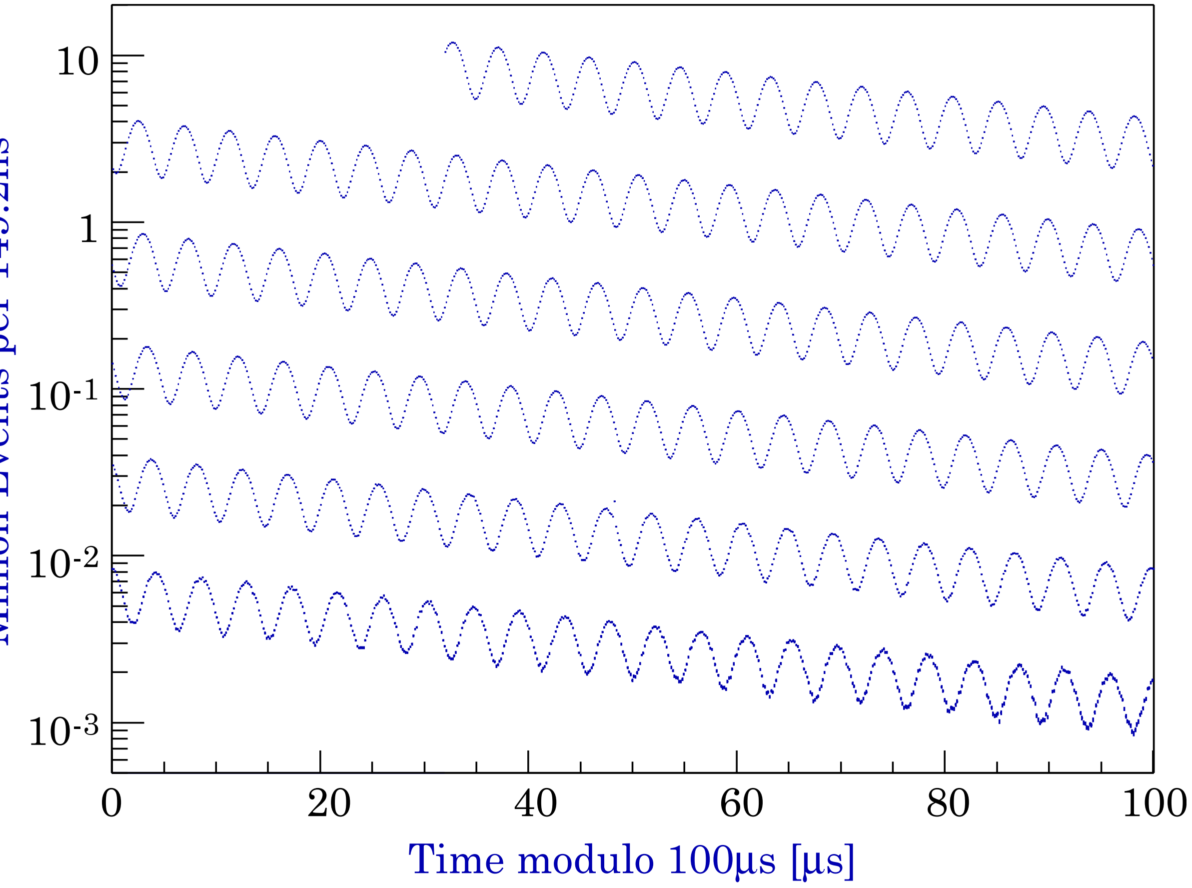




$$N(t) = N_0(E) exp\left(\frac{-t}{\gamma\tau_\mu}\right)\left[1 + A(E)\sin(\omega_a t + \phi(E))\right]$$

The anomalous magnetic moment of the muon: the experiment

$$N(t) = N_0(E) exp\left(\frac{-t}{\gamma\tau_\mu}\right)[1 + A(E)\sin(\omega_a t + \phi(E))] \quad \Rightarrow \quad \omega_a$$



Magnetic field B?

$$a_\mu = \frac{\omega_a/\omega_p}{\lambda_+ - \omega_a/\omega_p} = \frac{R}{\lambda_+ - R}$$

(free-proton precession frequency)

$$R = 0.0037072047(26)$$

$$\lambda_+ = \mu_{\mu^+}/\mu_p = 3.183345137(85)$$

(muonium $\mu^+ e^-$ hyperfine level structure measurements)

The anomalous magnetic moment of the muon

- The E821 experiment at BNL

Bennet et al, PRD73,072003 (2006)

$$a_{\mu+}^{\text{exp}} = 11\,659\,204(6)(5) \times 10^{-10}$$ [2000]

$$a_{\mu-}^{\text{exp}} = 11\,659\,215(8)(3) \times 10^{-10}$$ [2001]

- Assuming CPT invariance

Bennet et al, PRD73,072003 (2006)

(~2200 citations)

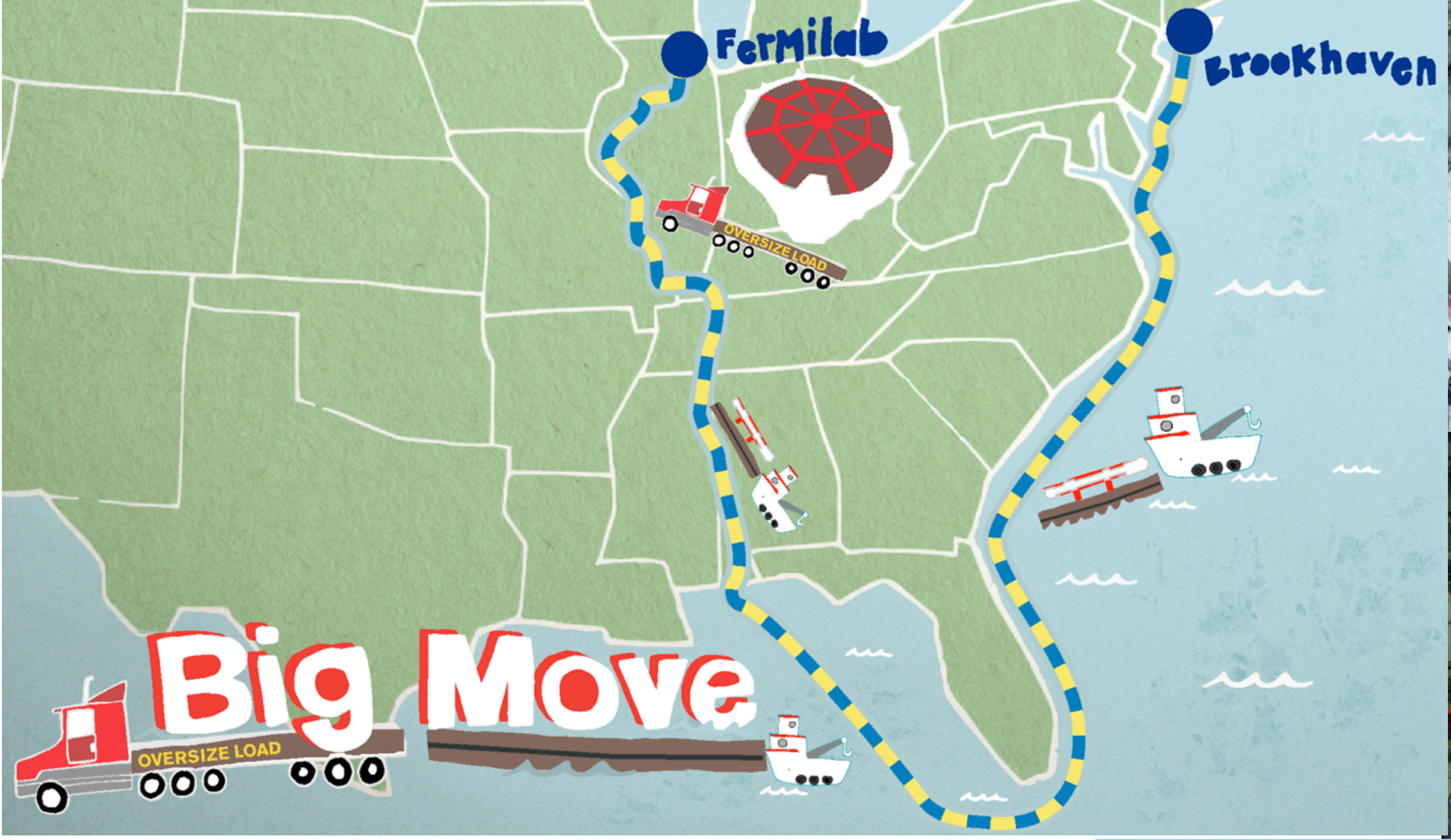
$$a_{\mu}^{\text{exp}} = 11\,659\,209.1\,\underbrace{\overset{\text{stat}}{(5.4)}\overset{\text{sys}}{(3.3)}}_{(6.3)} \times 10^{-10}$$

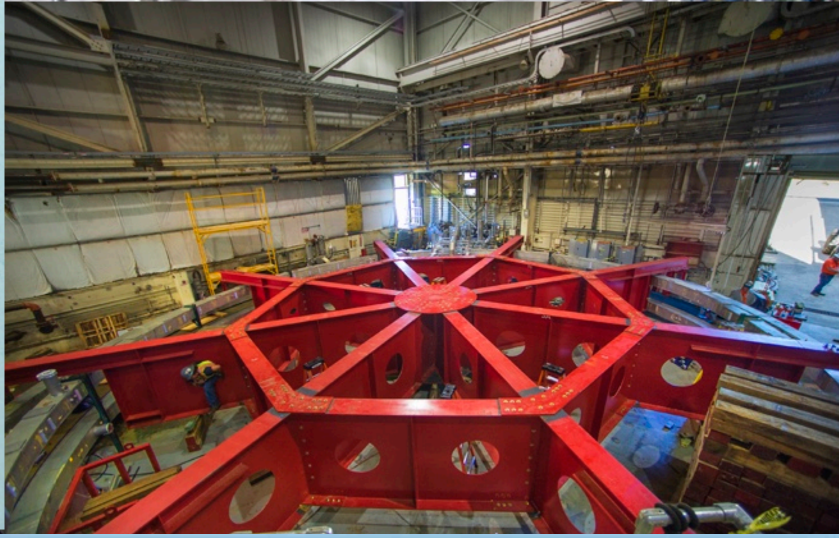
The anomalous magnetic moment of the muon

Bennet et al, PRD73,072003 (2006)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11450000(220000)	4300
CERN II	1962–1968	μ^+	11661600(3100)	270
CERN III	1974–1976	μ^+	11659100(110)	10
CERN III	1975–1976	μ^-	11659360(120)	10
BNL	1997	μ^+	11659251(150)	13
BNL	1998	μ^+	11659191(59)	5
BNL	1999	μ^+	11659202(15)	1.3
BNL	2000	μ^+	11659204(9)	0.73
BNL	2001	μ^-	11659214(9)	0.72
Average			$\boxed{11659208.0(6.3)}$	0.54

Forthcoming exp: FNAL & J-PARC$\sim 1.6 \times 10^{-10}$

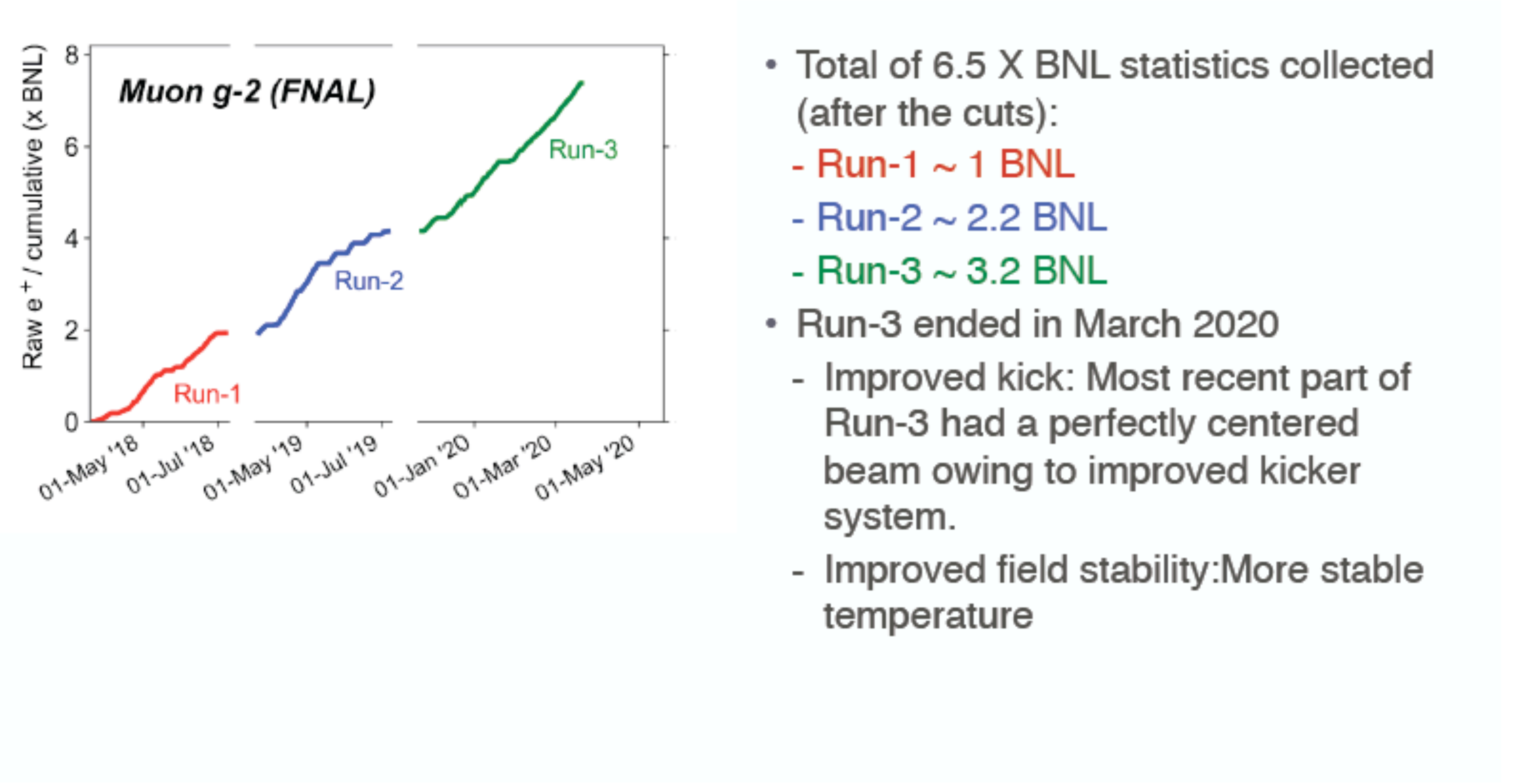




http://muon-g-2.fnal.gov/bigmove/gallery.shtml

The anomalous magnetic moment of the muon

Experimental data collection status @ FNAL: [from the talk by Esra Barlas-Yucel@FPCP 2020]



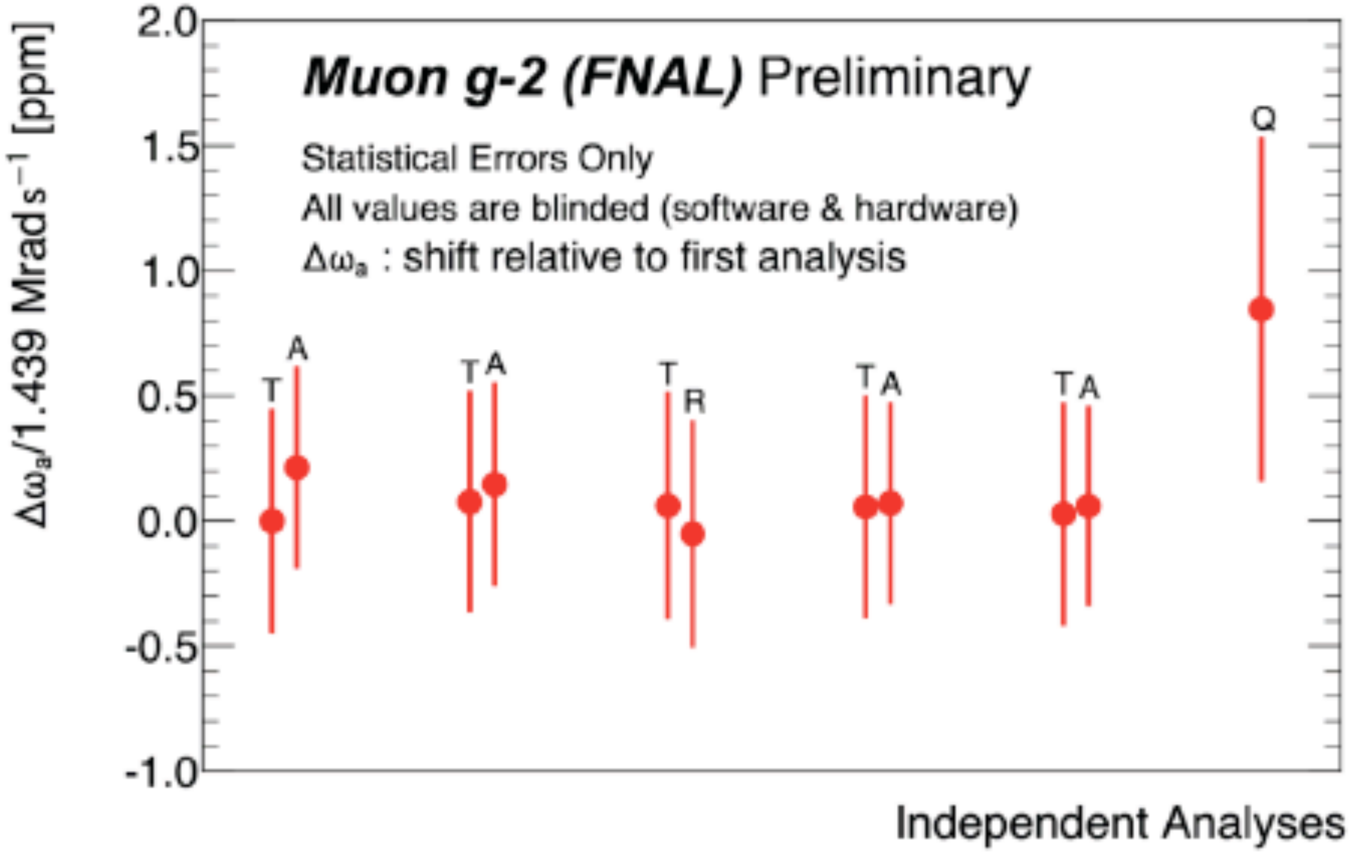
- Total of 6.5 X BNL statistics collected (after the cuts):
  - Run-1 ~ 1 BNL
  - Run-2 ~ 2.2 BNL
  - Run-3 ~ 3.2 BNL
- Run-3 ended in March 2020
  - Improved kick: Most recent part of Run-3 had a perfectly centered beam owing to improved kicker system.
  - Improved field stability:More stable temperature

The anomalous magnetic moment of the muon

Experimental data collection status @ FNAL:

[from the talk by Esra Barlas-Yucel@FPCP 2020]

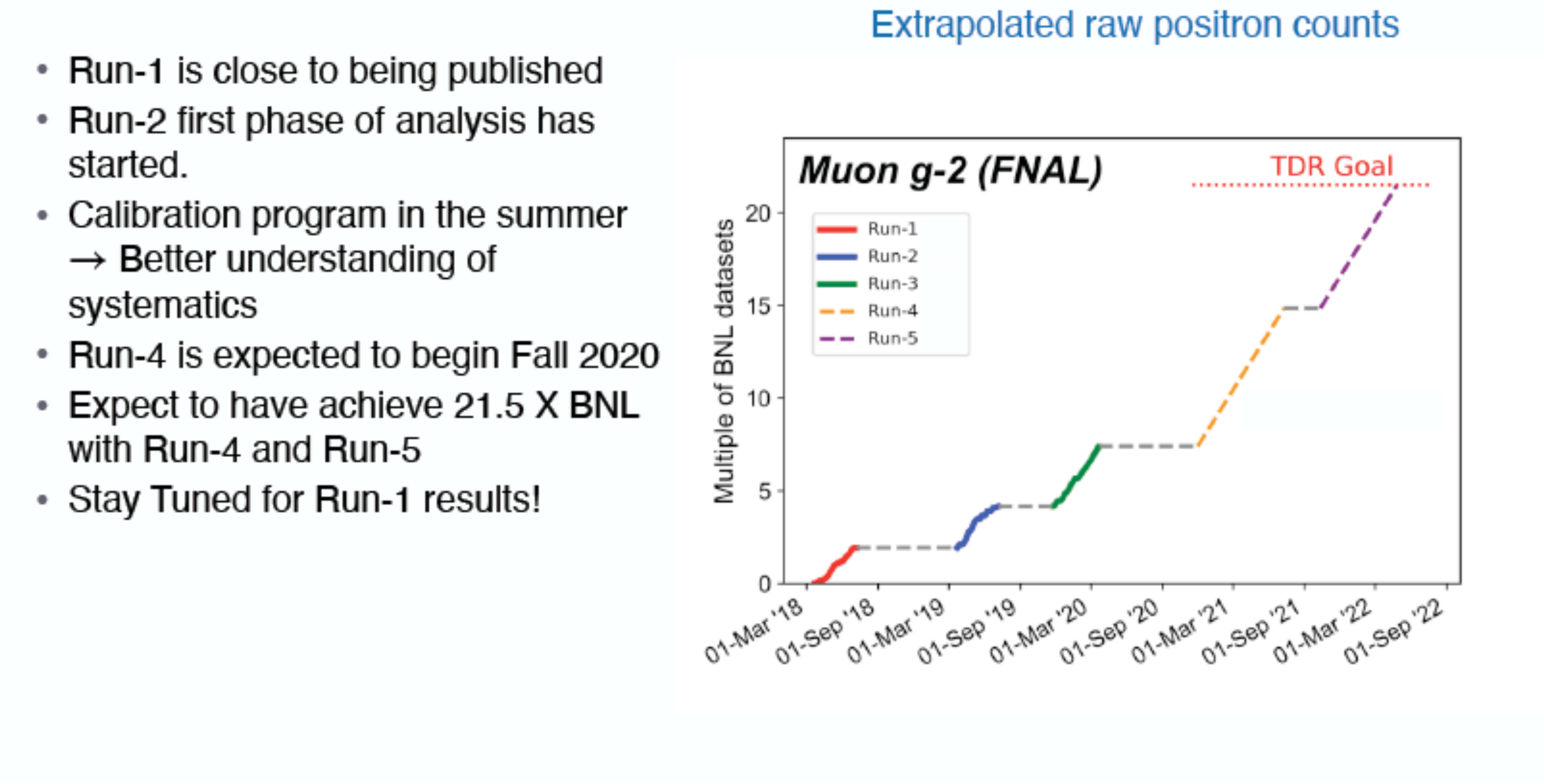
- During Run 1:
  - Collected nearly 1xBNL equivalent of data
  - Total statistical error ~ 450 ppb
  - Systematic Error is still being worked through. Major systematics studies are almost completed.
- 6 different ω_a analysis team look at the precession data and performed a commonly software blinded analysis
  - Still blinded on hardware and software
  - 4 different fitting methods
  - 2 independent reconstruction algorithms
  - All agree with each other
- 2 different ω_p analysis team is looking at the field data
  - Blinded on software
  - Have just done a relative unblinding
  - Both agree on each other on all relative trends for all Run-1 sub-datasets.



The anomalous magnetic moment of the muon

[from the talk by Esra Barlas-Yucel@FPCP 2020]

Experimental data collection prospects @ FNAL:



- Run-1 is close to being published
- Run-2 first phase of analysis has started.
- Calibration program in the summer → Better understanding of systematics
- Run-4 is expected to begin Fall 2020
- Expect to have achieve 21.5 X BNL with Run-4 and Run-5
- Stay Tuned for Run-1 results!

Extrapolated raw positron counts

Pessimist view

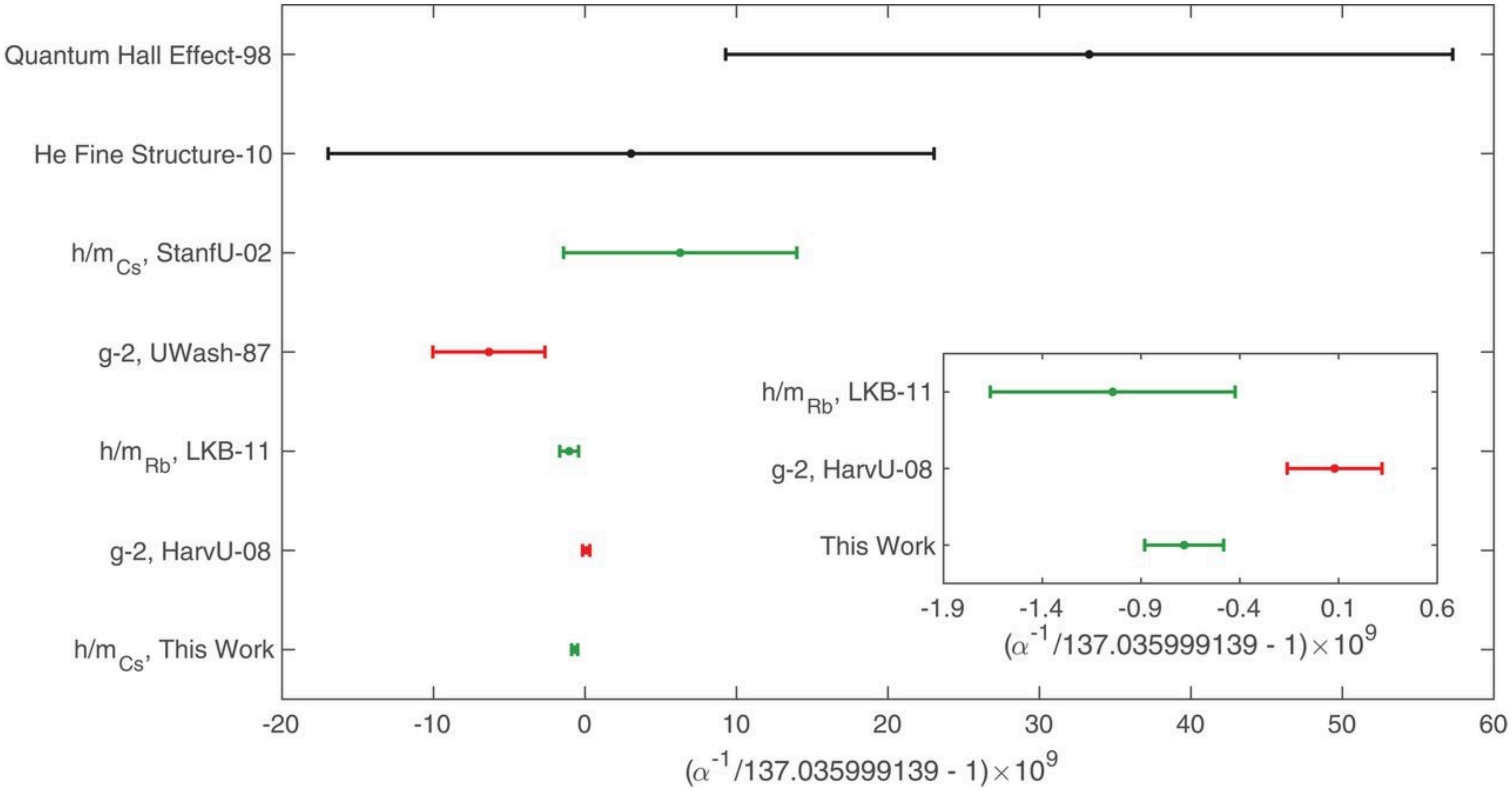
The anomalous magnetic moment of the electron

New measurement of α_{em}

[Parker et al, Science 360 (2018) 191]

$$\alpha^{-1}(Cs) = 137.035\,999\,046\,(27)$$

2.0×10^{-10} accuracy



[Davoudiasl-Marciano, 1806.10252]

$$\Delta a_e \equiv a_e^{\text{exp}} - a_e^{\text{SM}} = (-87 \pm 36) \times 10^{-14} \longrightarrow 2.4\sigma$$

↑ (almost no SM error!)

(negative deviation)

New physics contributions to g-2

Generic contributions:

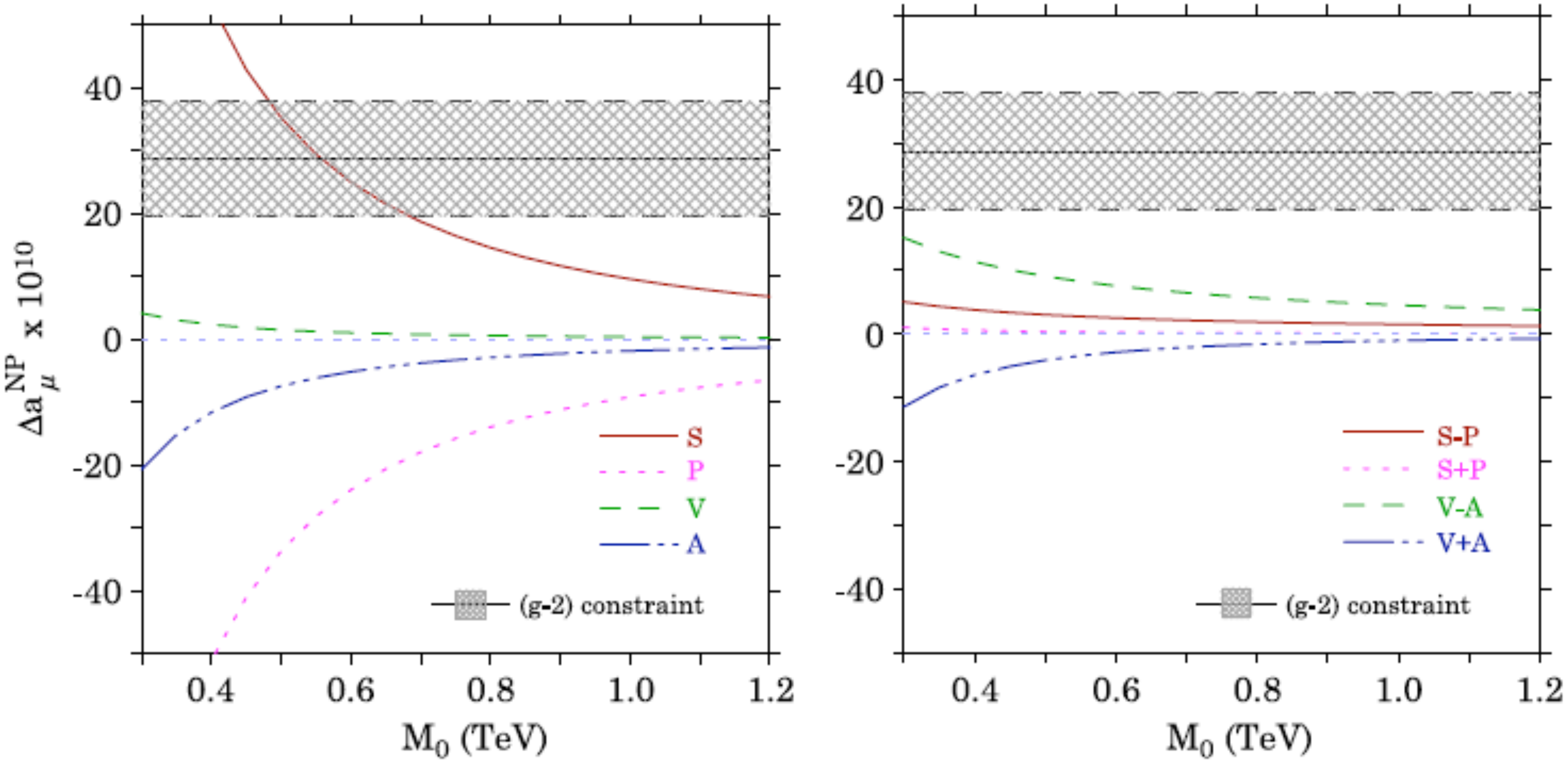
[Jegerlehner, Nyffeler 09]

$$\Delta a_\mu^{\mathrm{NP}} = \frac{f^2}{4\pi^2}\frac{m_\mu^2}{M_0^2}L, \quad L = \frac{1}{2}\int_0^1 \mathrm{d}x \frac{Q(x)}{(1-x)(1-\lambda^2 x) + (\epsilon\lambda)^2 x},$$

Scalar: $Q_S = x^2(1+\epsilon-x)$
Pseudoscalar: $Q_P = x^2(1-\epsilon-x)$
Vector: $Q_V = 2x(1-x)(x-2(1-\epsilon)) + \lambda^2(1-\epsilon)^2 Q_S$
Axialvector: $Q_A = 2x(1-x)(x-2(1+\epsilon)) + \lambda^2(1+\epsilon)^2 Q_P$

$\epsilon = M/m_\mu$

$\lambda = m_\mu/M_0$



(a) Case: $m_\mu = M \ll M_0$.

(b) Case: $m_\mu \ll M_0 = M$.

The anomalous magnetic moment of the muon

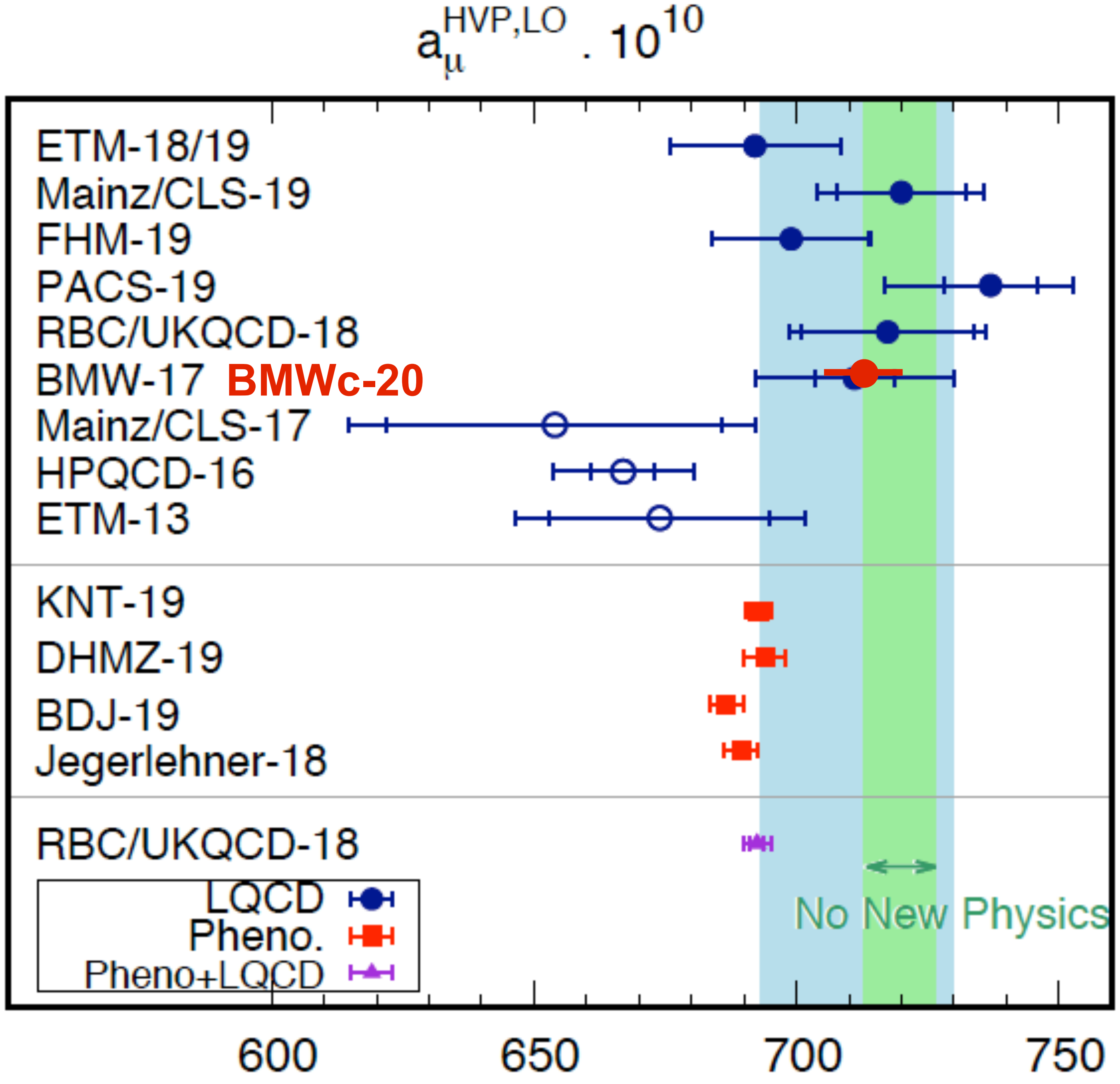
Bennet et al, PRD73,072003 (2006)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11450000(220000)	4300
CERN II	1962–1968	μ^+	11661600(3100)	270
CERN III	1974–1976	μ^+	11659100(110)	10
CERN III	1975–1976	μ^-	11659360(120)	10
BNL	1997	μ^+	11659251(150)	13
BNL	1998	μ^+	11659191(59)	5
BNL	1999	μ^+	11659202(15)	1.3
BNL	2000	μ^+	$\boxed{11659204(9)}$	0.73
BNL	2001	μ^-	11659214(9)	0.72
Average			11659208.0(6.3)	0.54

Forthcoming exp @FNAL with μ^+ ! → 2σ !

The anomalous magnetic moment of the muon

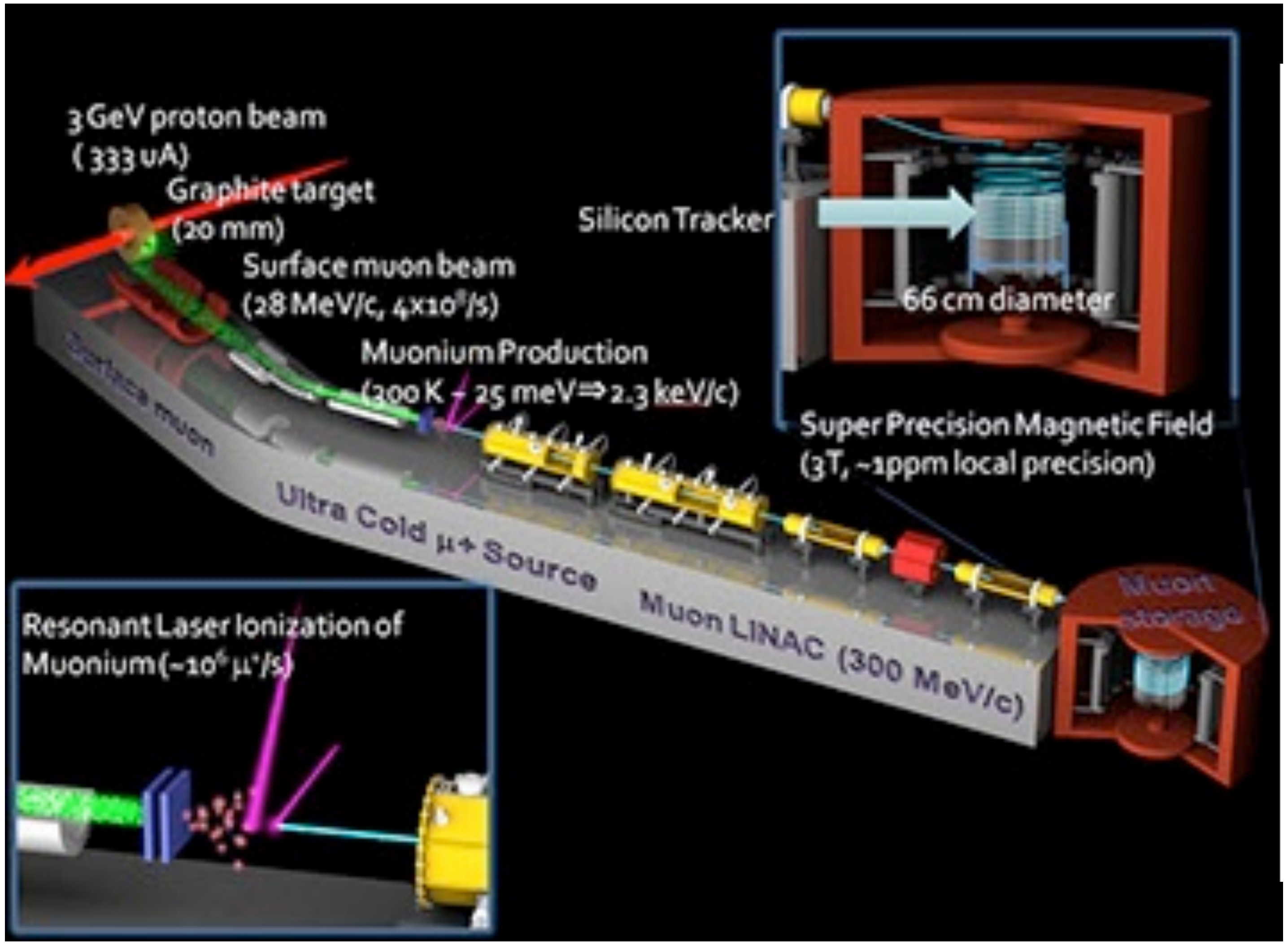
Real consensus in theory number for HVP?

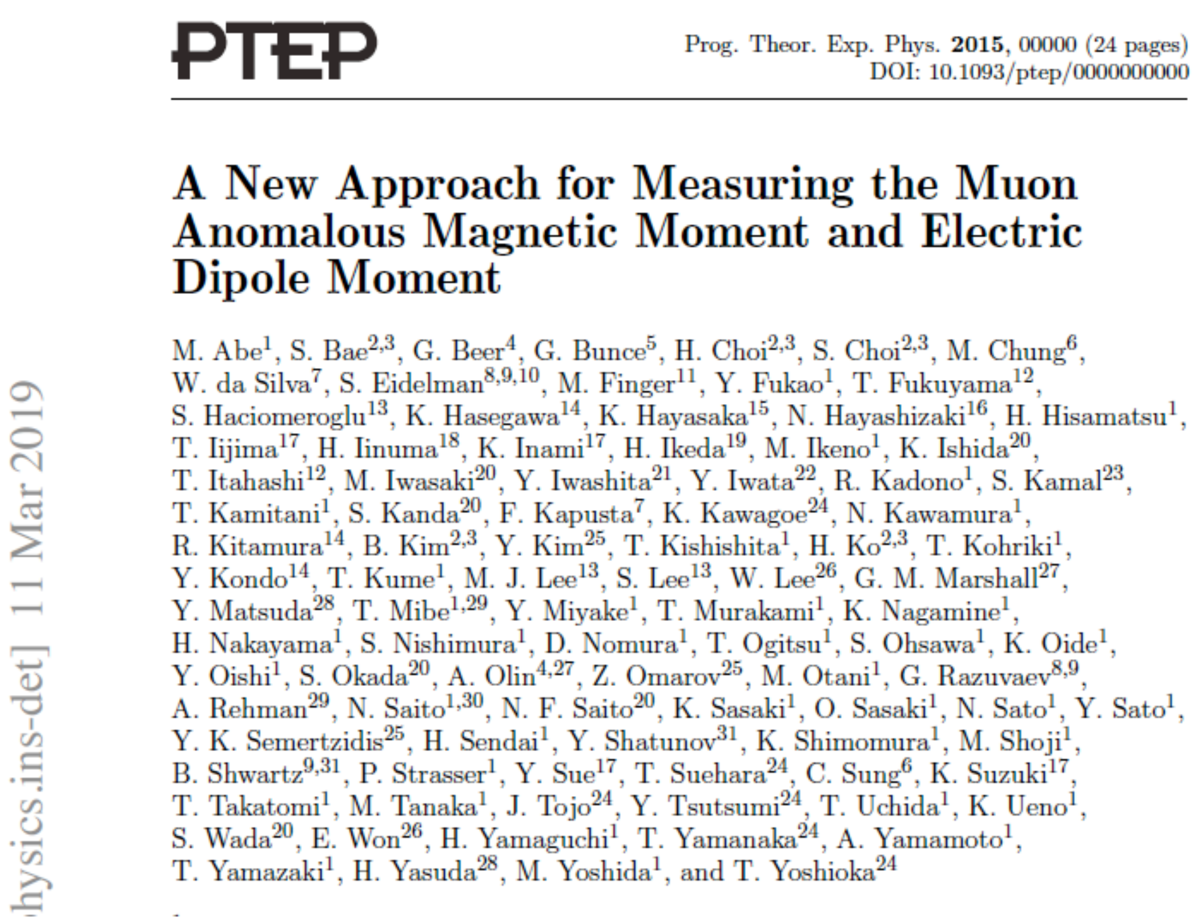


Optimist view

The anomalous magnetic moment of the muon

Second completely different experiment with cold muons @ JPARC!





PTEP

Prog. Theor. Exp. Phys. **2015**, 00000 (24 pages)
DOI: 10.1093/ptep/0000000000

A New Approach for Measuring the Muon Anomalous Magnetic Moment and Electric Dipole Moment

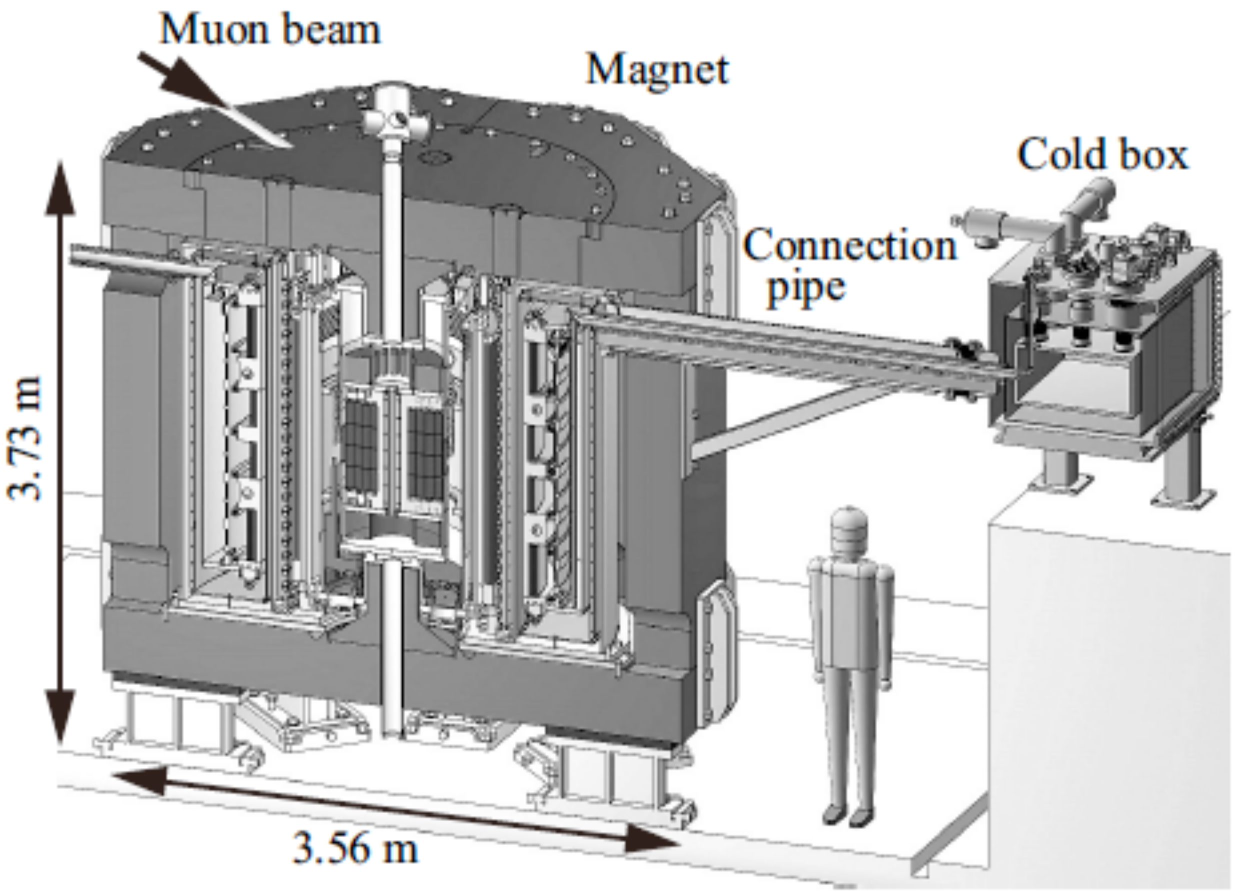
M. Abe[1], S. Bae[2,3], G. Beer[4], G. Bunce[5], H. Choi[2,3], S. Choi[2,3], M. Chung[6], W. da Silva[7], S. Eidelman[8,9,10], M. Finger[11], Y. Fukao[1], T. Fukuyama[12], S. Haciomeroglu[13], K. Hasegawa[14], K. Hayasaka[15], N. Hayashizaki[16], H. Hisamatsu[1], T. Iijima[17], H. Iinuma[18], K. Inami[17], H. Ikeda[19], M. Ikeno[1], K. Ishida[20], T. Itahashi[12], M. Iwasaki[20], Y. Iwashita[21], Y. Iwata[22], R. Kadono[1], S. Kamal[23], T. Kamitani[1], S. Kanda[20], F. Kapusta[7], K. Kawagoe[24], N. Kawamura[1], R. Kitamura[14], B. Kim[2,3], Y. Kim[25], T. Kishishita[1], H. Ko[2,3], T. Kohriki[1], Y. Kondo[14], T. Kume[1], M. J. Lee[13], S. Lee[13], W. Lee[26], G. M. Marshall[27], Y. Matsuda[28], T. Mibe[1,29], Y. Miyake[1], T. Murakami[1], K. Nagamine[1], H. Nakayama[1], S. Nishimura[1], D. Nomura[1], T. Ogitsu[1], S. Ohsawa[1], K. Oide[1], Y. Oishi[1], S. Okada[20], A. Olin[4,27], Z. Omarov[25], M. Otani[1], G. Razuvaev[8,9], A. Rehman[29], N. Saito[1,30], N. F. Saito[20], K. Sasaki[1], O. Sasaki[1], N. Sato[1], Y. Sato[1], Y. K. Semertzidis[25], H. Sendai[1], Y. Shatunov[31], K. Shimomura[1], M. Shoji[1], B. Shwartz[9,31], P. Strasser[1], Y. Sue[17], T. Suehara[24], C. Sung[6], K. Suzuki[17], T. Takatomi[1], M. Tanaka[1], J. Tojo[24], Y. Tsutsumi[24], T. Uchida[1], K. Ueno[1], S. Wada[20], E. Won[26], H. Yamaguchi[1], T. Yamanaka[24], A. Yamamoto[1], T. Yamazaki[1], H. Yasuda[28], M. Yoshida[1], and T. Yoshioka[24]

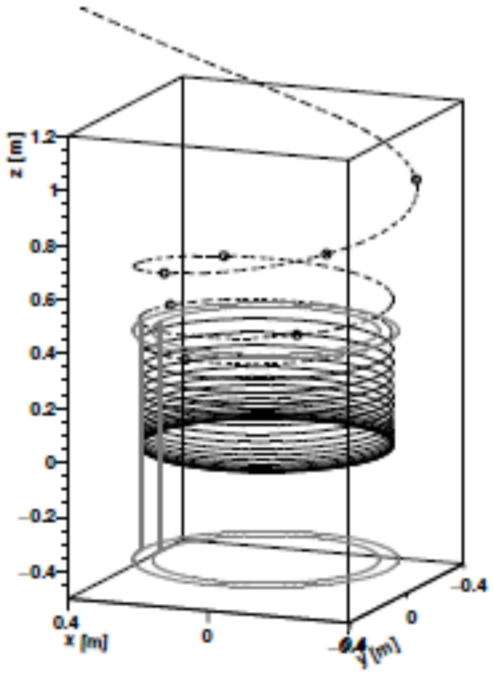
[arXiv:1901.03047]

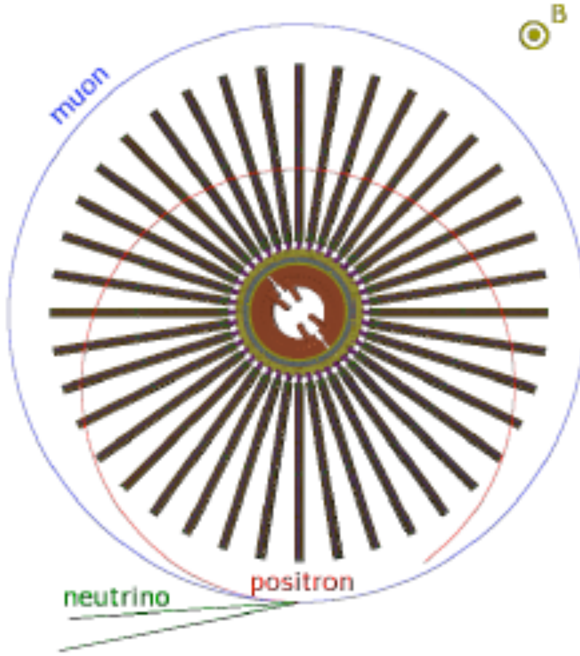
The anomalous magnetic moment of the muon

Second completely different experiment with cold muons @ JPARC!

Goal: $\sim 1.6 \times 10^{-10}$







Outlook

- New data required:

  FNAL (Run-1 soon, this year)!

  JPARC as well necessary!

- (my opinion) Theory errors underestimated in HVP:
  - Lattice QCD seems to demonstrate that... (meaning?)
- (my opinion) Theory issues still to be understood in HLBL
  - Missing contributions:

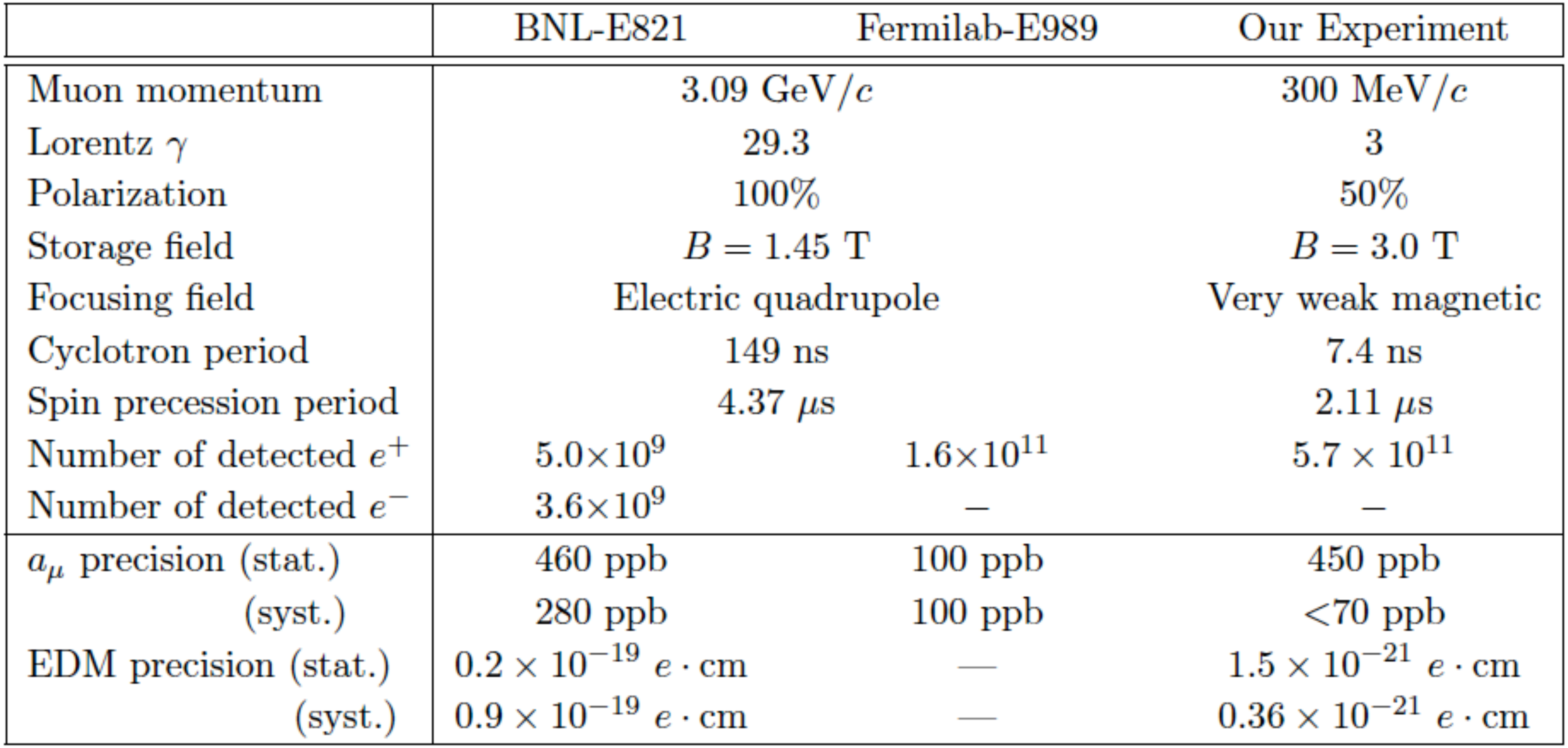
    more $\gamma\gamma \to$ hadrons (t-channel), and @mid-large energies

- New Physics effects very constrained by LHC, not easy either!
  - But nice synergy with flavor anomalies in B decays

- Summary: News by the end of the year! Discrepancy still alive?

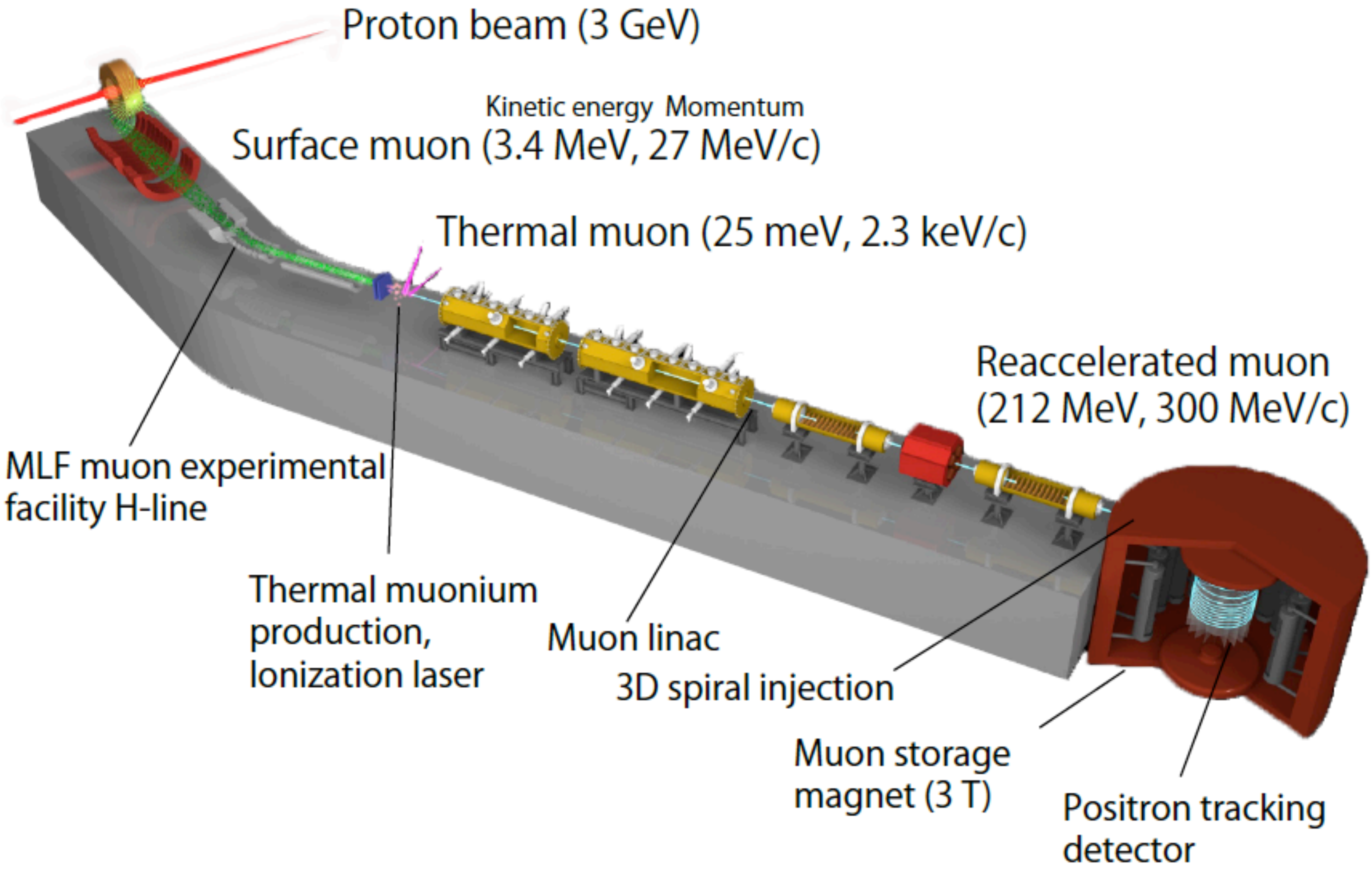
The anomalous magnetic moment of the muon

Second completely different experiment with cold muons @ JPARC!



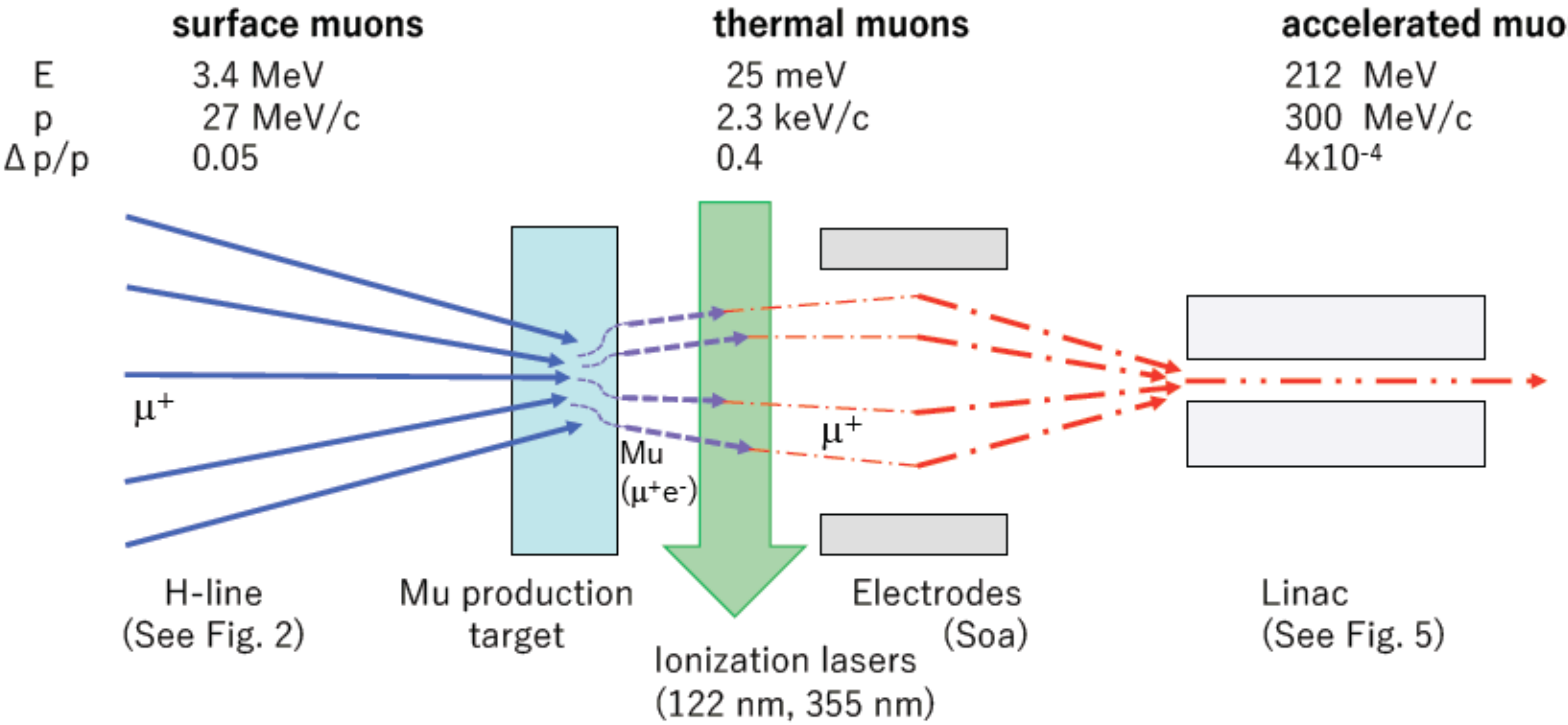
<table>
<tr><th></th><th>BNL-E821</th><th>Fermilab-E989</th><th>Our Experiment</th></tr>
<tr><td>Muon momentum</td><td colspan="2">3.09 GeV/c</td><td>300 MeV/c</td></tr>
<tr><td>Lorentz γ</td><td colspan="2">29.3</td><td>3</td></tr>
<tr><td>Polarization</td><td colspan="2">100%</td><td>50%</td></tr>
<tr><td>Storage field</td><td colspan="2">$B = 1.45$ T</td><td>$B = 3.0$ T</td></tr>
<tr><td>Focusing field</td><td colspan="2">Electric quadrupole</td><td>Very weak magnetic</td></tr>
<tr><td>Cyclotron period</td><td colspan="2">149 ns</td><td>7.4 ns</td></tr>
<tr><td>Spin precession period</td><td colspan="2">4.37 μs</td><td>2.11 μs</td></tr>
<tr><td>Number of detected e^+</td><td>5.0×10^9</td><td>1.6×10^{11}</td><td>5.7×10^{11}</td></tr>
<tr><td>Number of detected e^-</td><td>3.6×10^9</td><td>–</td><td>–</td></tr>
<tr><td>a_μ precision (stat.)</td><td>460 ppb</td><td>100 ppb</td><td>450 ppb</td></tr>
<tr><td>(syst.)</td><td>280 ppb</td><td>100 ppb</td><td><70 ppb</td></tr>
<tr><td>EDM precision (stat.)</td><td>0.2×10^{-19} $e\cdot$cm</td><td>—</td><td>1.5×10^{-21} $e\cdot$cm</td></tr>
<tr><td>(syst.)</td><td>0.9×10^{-19} $e\cdot$cm</td><td>—</td><td>0.36×10^{-21} $e\cdot$cm</td></tr>
</table>

The anomalous magnetic moment of the muon



The anomalous magnetic moment of the muon

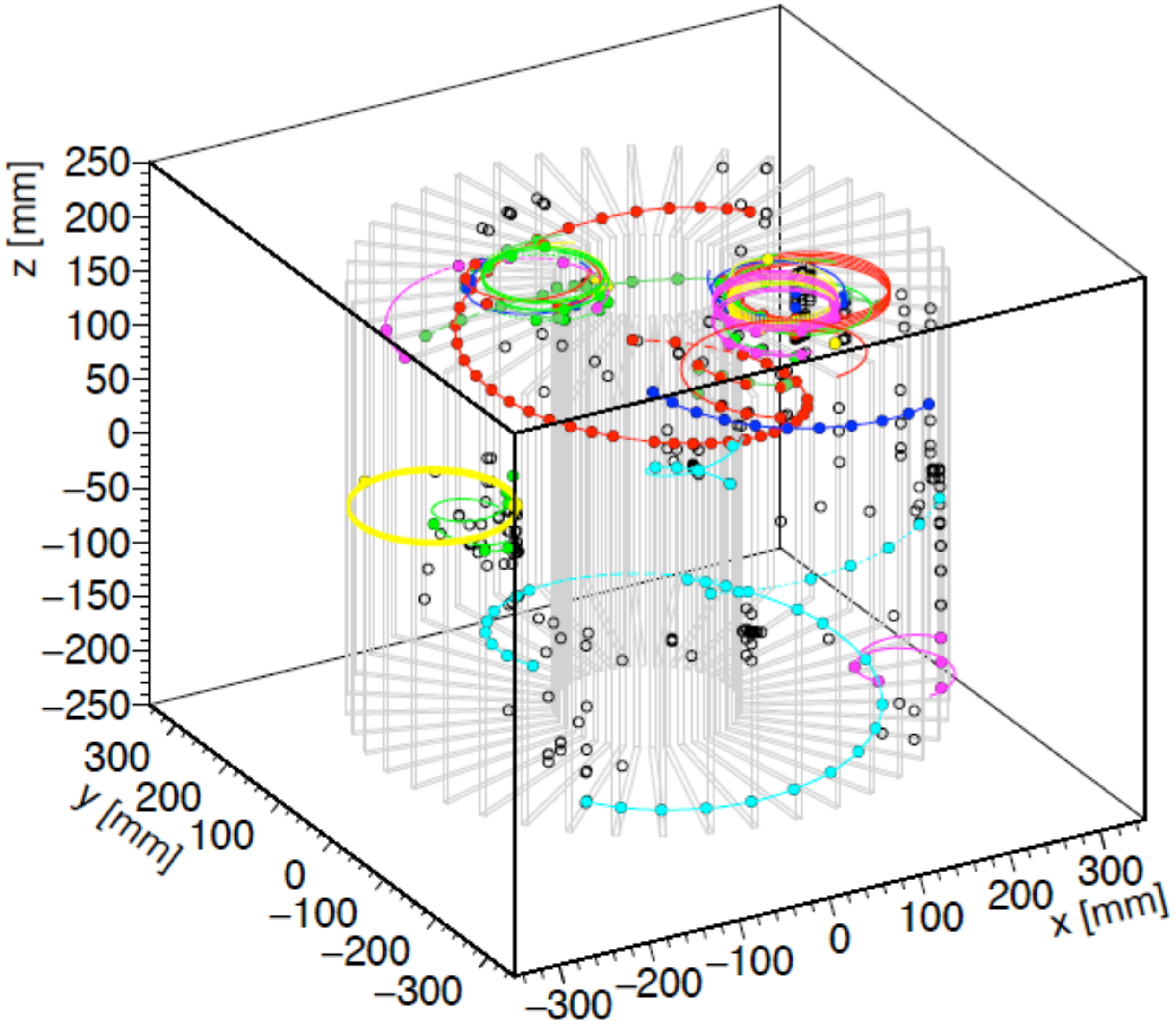
Second completely different experiment with cold muons @ JPARC!

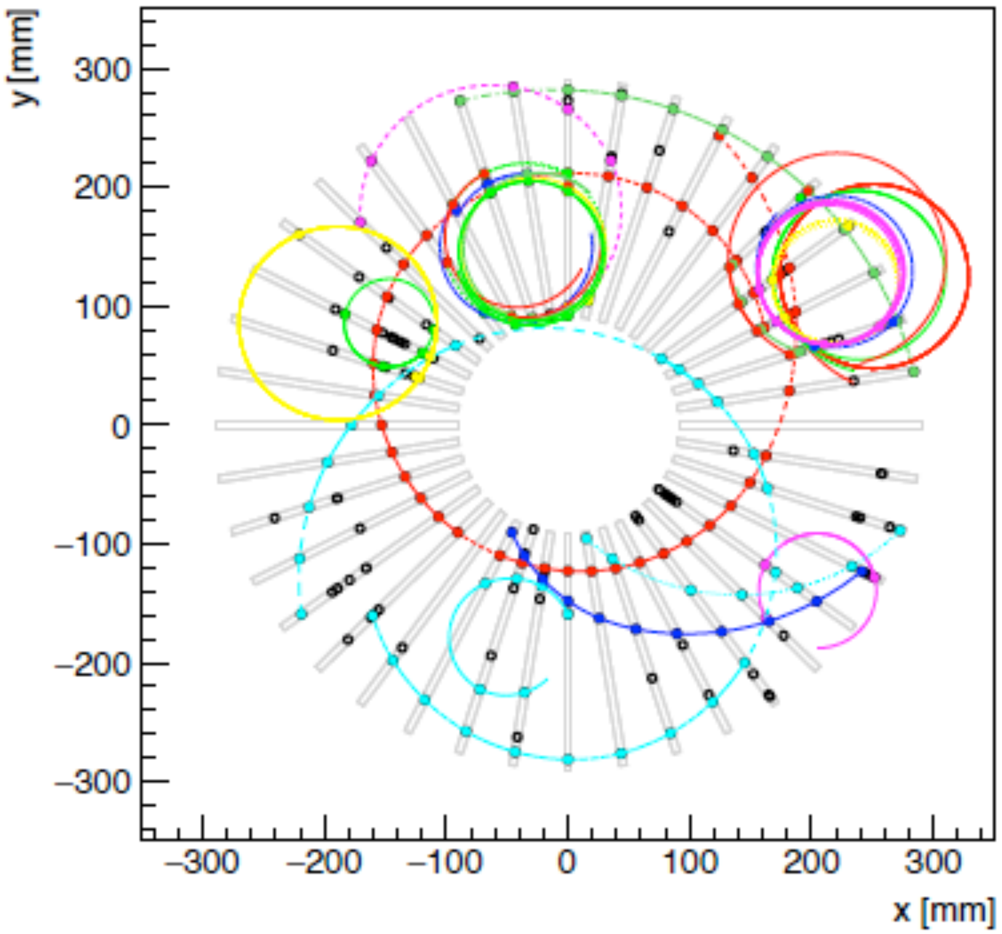


	surface muons	**thermal muons**	**accelerated muons**
E	3.4 MeV	25 meV	212 MeV
p	27 MeV/c	2.3 keV/c	300 MeV/c
Δ p/p	0.05	0.4	4×10^{-4}

The anomalous magnetic moment of the muon

Second completely different experiment with cold muons @ JPARC!

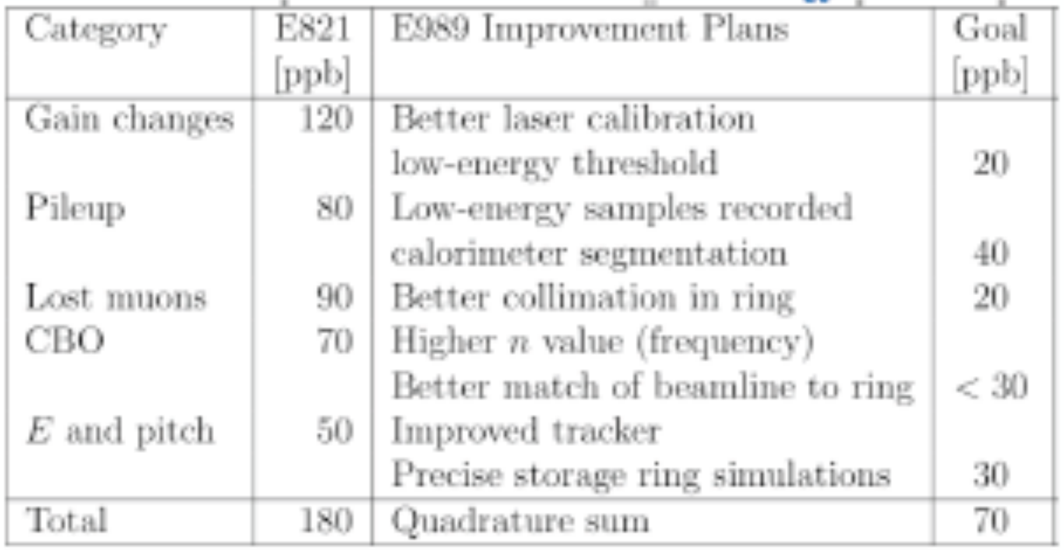




The anomalous magnetic moment of the muon

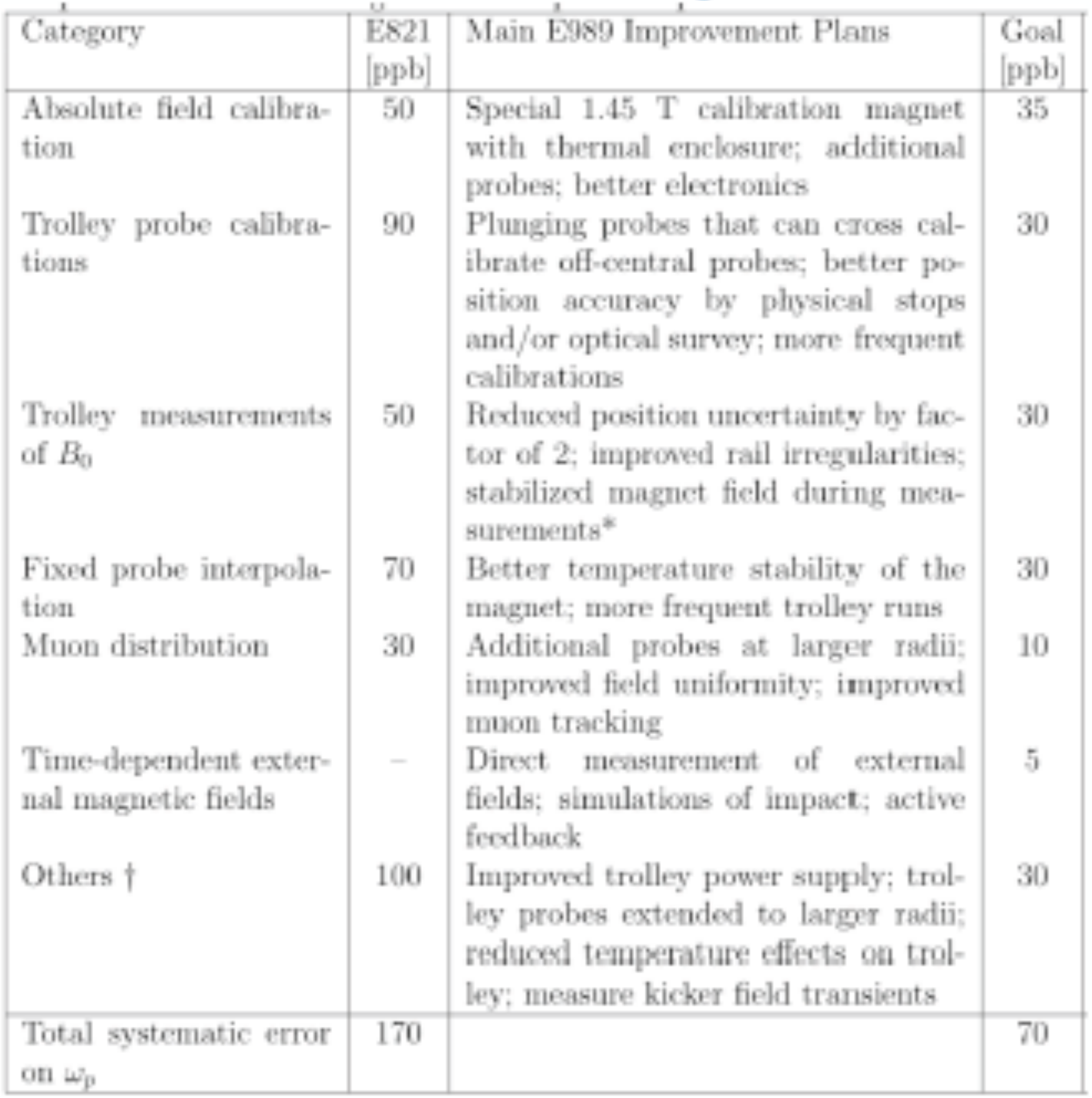
Experimental data collection status @ FNAL: [from the talk by Esra Barlas-Yucel@FPCP 2020]

Systematics on ω_a



Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]
Gain changes	120	Better laser calibration low-energy threshold	20
Pileup	80	Low-energy samples recorded calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher n value (frequency) Better match of beamline to ring	< 30
E and pitch	50	Improved tracker Precise storage ring simulations	30
Total	180	Quadrature sum	70

Systematics on ω_p



Category	E821 [ppb]	Main E989 Improvement Plans	Goal [ppb]
Absolute field calibration	50	Special 1.45 T calibration magnet with thermal enclosure; additional probes; better electronics	35
Trolley probe calibrations	90	Plunging probes that can cross calibrate off-central probes; better position accuracy by physical stops and/or optical survey; more frequent calibrations	30
Trolley measurements of B_0	50	Reduced position uncertainty by factor of 2; improved rail irregularities; stabilized magnet field during measurements*	30
Fixed probe interpolation	70	Better temperature stability of the magnet; more frequent trolley runs	30
Muon distribution	30	Additional probes at larger radii; improved field uniformity; improved muon tracking	10
Time-dependent external magnetic fields	–	Direct measurement of external fields; simulations of impact; active feedback	5
Others †	100	Improved trolley power supply; trolley probes extended to larger radii; reduced temperature effects on trolley; measure kicker field transients	30
Total systematic error on ω_p	170		70