The two-step mechanism explaining the dibaryon $d^*(2380)$ peak

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Intro

The dibaryon peak

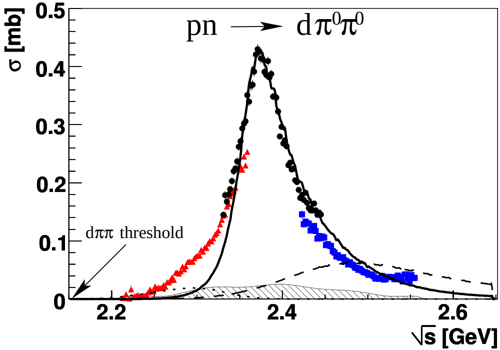


Figure 1: Total cross sections obtained from PRL106,242302(2011) WASA-at-COSY on $pd \to d\pi^0\pi^0 + p_{spectator}$ for the beam energies $T_p = 1.0$ GeV (triangles), 1.2 GeV (dots) and 1.4 GeV (squares). The drawn lines represent the expected cross sections for the Roper excitation process (dotted) and the t-channel $\Delta\Delta$ contribution (dashed) as well as a calculation for a s-channel resonance with m $= 2.37$ GeV and $\Gamma = 68$ MeV (solid).

The dibaryon peak

- The peak is also seen in $pn \to d\pi^+\pi^-$, WASA-at-COSY (2013)

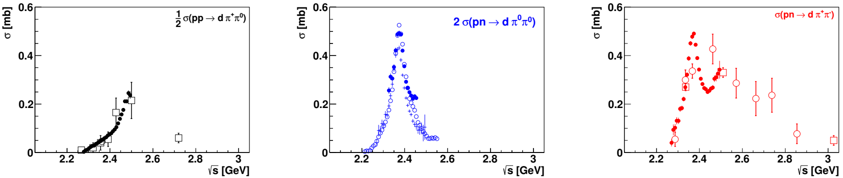


Figure 2: Total cross sections of the basic double-pionic fusion reactions. Left: $pp \to d\pi^+\pi^0$, middle: $pn \to d\pi^0\pi^0$, bottom: $pn \to d\pi^+\pi^-$. Filled symbols denote results from PLB721, 229 (2013), open symbols from previous works. The crosses denote the result for the $pn \to d\pi^0\pi^0$ reaction by using Eq. (1) with the data for the $pn \to d\pi^+\pi^-$ and $pp \to d\pi^+\pi^0$ channels as input.

$$\sigma(pn \to d\pi^+\pi^-) = 2\sigma(pn \to d\pi^0\pi^0) + \frac{1}{2}\sigma(pp \to d\pi^+\pi^0). \qquad (1)$$

- A possible interpretation has been a $\Delta\Delta$ bound state with $I = 0(3^+)$ quantum numbers which are favored. PRL 106, 242302 (2011).

Is this a new 6-quark state OR it is possible another interpretation?

The dibaryon peak

- I. Bar-Nir et al., Nucl. Phys. B 54, 17-28 (1973):

The dominant two pion production plus fusion mechanism comes from a two step single pion production process, $np \to \pi^- pp$ followed by $pp \to \pi^+ d$ (plus $np \to \pi^+ nn$ followed by $nn \to \pi^- d$).

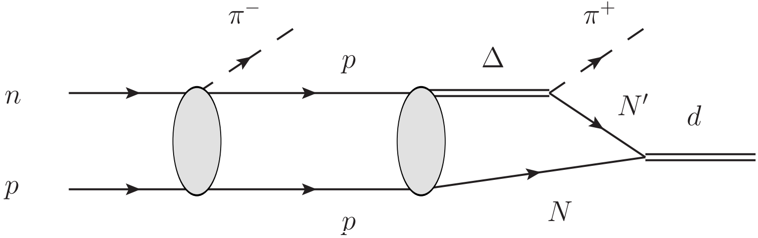


Figure 3: Two step mechanism for $np \to \pi^+\pi^- d$ investigated ($\pi^+ nn$ in the first step is also considered).

The dibaryon peak

$np(I=0) \to \pi^- pp$ H. Clement and T. Skorodko 2010.09217 (2020)

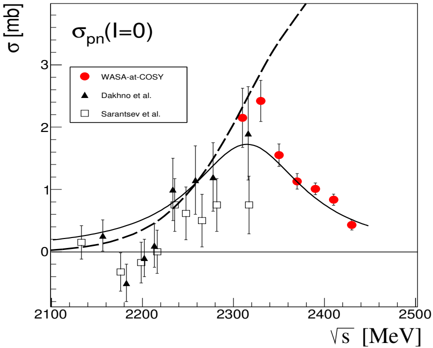
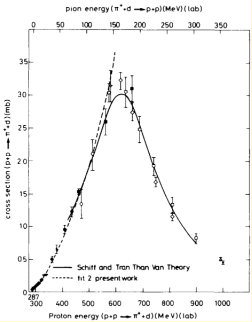


Figure 4: WASA-at-COSY (solid circles), earlier results (squares & triangles), Dakhno, PLBB 114 (1982), Sarantsev, EPJ. A21 (2004). t-channel Roper excitation from Alvarez-Ruso, Oset et al. NPA633(1998) (dashed line). Solid line, Lorentzian fit ($m = 2315$ MeV and $\Gamma = 150$ MeV).

$$\frac{1}{3}\sigma_{np(I=0)\to NN\pi} = \sigma_{np\to pp\pi^-} - \frac{1}{2}\sigma_{pp\to pp\pi^0}$$

Indeed, $\sigma(pp \to \pi^+ d) \sim 3 - 4$ mb. **Too BIG** for a fusion reaction!.

C. Richard-Serre et al., NPB20, 413-440 (1970)
P. W. F. Alons et al., NPA480, 413 (1988)
See also GWU SAID data base for $\pi^+ d \to pp$



Which kind of structure can cause such a large cross section?

Can the two-step mechanism explain the dibaryon peak?

Triangle Singularities

Triangle singularities

Landau, Nucl. Phys. 13, (1959), Coleman, Nuovo Cimento 38 (1965)

The **Coleman-Norton** theorem states that a triangle singularity appears when the process visualized in the triangle-loop diagram can occur at the classical level, the three intermediate particles can be placed simultaneously on shell and they are collinear.

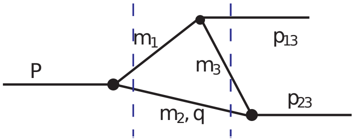


Figure 5: The triangle-loop diagram used in the general discussion of triangle singularities. The two dashed vertical lines correspond to the two relevant cuts.

Let us consider the scalar three-point loop integral, $p_{13} \equiv k$, $p_{23} \equiv P - k$

$$I_1 = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_2^2 + i\,\epsilon)\,[(P-q)^2 - m_1^2 + i\,\epsilon]\,[(P-q-k)^2 - m_3^2 + i\,\epsilon]} \, .$$

After integrating in the q^0 variable, in order to analyze the singularity structure, it is sufficient to focus on the following integral:

$$I(m_{23}) = \int \frac{d^3q}{(P^0 - \omega_1(\vec{q}\,) - \omega_2(\vec{q}\,) + i\,\epsilon)\left(E_{23} - \omega_2(\vec{q}\,) - \omega_3(\vec{k} + \vec{q}\,) + i\,\epsilon\right)}$$

$$= 2\,\pi \int_0^\infty dq \; \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)\,, \qquad (3)$$

where $\omega_{1,2}(q) = \sqrt{m_{1,2}^2 + q^2}$, $\omega_3(\vec{q} + \vec{k}) = \sqrt{m_3^2 + (\vec{q} + \vec{k})^2}$, $E_{23} = P^0 - k^0$, and

$$f(q) = \int_{-1}^{1} dz \, \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 + 2\,q\,k\,z} + i\,\epsilon}\,, \qquad (4)$$

where $q \equiv \vec{q}$, $k \equiv |\vec{k}| = \sqrt{\lambda(M^2, m_{13}^2, m_{23}^2)}/(2M)$, with $M = \sqrt{P^2}$ and $m_{13,23} = \sqrt{p_{13,23}^2}$. The cut crossing particles 1 and 2 provides a pole of the integrand of $I(m_{23})$ given by

$$P^0 - \omega_1(\vec{q}\,) - \omega_2(\vec{q}\,) + i\,\epsilon = 0\,, \qquad (5)$$

Triangle singularities

The solution is

$$q_{\mathrm{on}+} = q_{\mathrm{on}} + i\,\epsilon \quad \text{with} \quad q_{\mathrm{on}} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}\,. \tag{6}$$

The function $f(q)$ has endpoint singularities, which are logarithmic branch points, when

$$E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 \pm 2\,q\,k} + i\,\epsilon = 0\,, \tag{7}$$

$z = \pm 1$, the situations for the momentum of particle 2 to be **anti-parallel and parallel** to the momentum of the (2,3) system in the $\vec{P} = 0$ frame. For $z = -1$, Eq. (7) has two solutions:

$$q_{a+} = \gamma\,(v\,E_2^* + p_2^*) + i\,\epsilon$$

$$q_{a-} = \gamma\,(v\,E_2^* - p_2^*) - i\,\epsilon$$

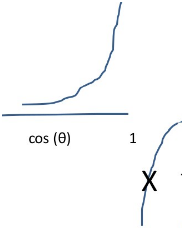
For $z = 1$:

$$q_{b+} = \gamma\,(-v\,E_2^* + p_2^*) + i\,\epsilon$$

$$q_{b-} = -\gamma\,(v\,E_2^* + p_2^*) - i\,\epsilon$$

TS condition:

$$\boxed{\lim_{\epsilon \to 0}\,(q_{\mathrm{on}+} - q_{a-}) = 0}$$



Triangle singularities

This is only possible when all three intermediate particles are on shell and meanwhile $z = -1$, $\omega_1(q_{\rm on}) - p^0_{13} - \sqrt{m_3^2 + (q_{\rm on} - k)^2} = 0$. Physical regions: $m_1 \leq M - m_2$, $m_{23} > m_2 + m_3$.

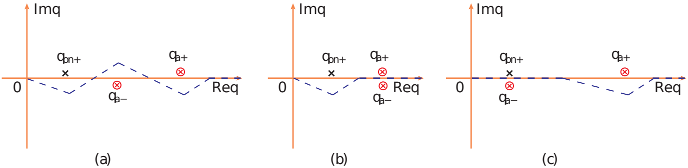


Figure 6: Singularities of the integrand of $I(m_{23})$ when $\lim_{\epsilon\to 0}(q_{a-})$ is positive. (a) no pinching, (b) pinching between q_{a+} and q_{a-}, the two-body threshold singularity, and (c), between $q_{\rm on+}$ and q_{a-}, triangle singularity.

Examples

- $\eta(1405) \to f_0(980)\pi^0$, Wu, Liu, Zhao, Zou, PRL108(2012).
- $a_1(1420)$, Mikhasenko, Ketzer, Sarantsev, PRD91 (2015)
- $X(3872) \to \pi^0\pi^+\pi^-$, Molina, Oset EPJC80 (2020)
- Review on TS, Guo, Liu, Sakai, Prog. Part. Nucl. Phys. 112 (2020)

The $pp \to \pi^+ d$ reaction

The $pp \to \pi^+ d$ reaction

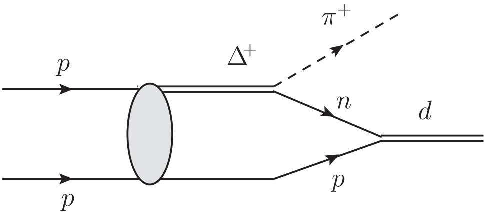


Figure 7: Mechanism for pion production with Δ excitation and *np* fusion in the deuteron.

(Coleman-Norton) The *pp* system produces a Δ and N, back to back in the *pp* rest frame. The Δ decays into $\pi N'$, with the π in the direction of the Δ and N' in its opposite direction (N direction). The N' goes faster than N and catches up with N fusing to give the deuteron.

$$q_{\rm on} = q_{a_-} \longrightarrow \sqrt{s} = 2179\,\mathrm{MeV}\,\mathrm{for}\,\Gamma_\Delta = 0\,(n, p\; on - shell) \tag{8}$$

Broad peak around $\sqrt{s} = 2165$ MeV for $\Gamma_\Delta \simeq 110$ MeV.

Diagrams

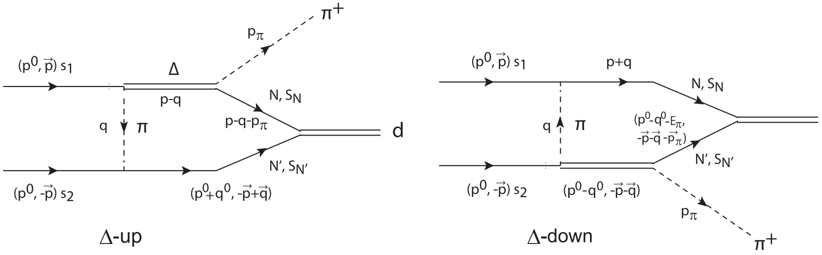


Figure 8: The two different topological structures.

- **Antisymmetrized** *pp* $|pp\rangle \equiv \frac{1}{\sqrt{2}}\left(|\vec{p}, s_1; -\vec{p}, s_2\rangle - |-\vec{p}, s_2; \vec{p}, s_1\rangle\right)$
- **Deuteron wave function** $|d\rangle \equiv \frac{1}{\sqrt{2}}|pn - np\rangle\ \chi_d$

χ_d: three spin 1 states $(\uparrow\uparrow,\ \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow),\ \downarrow\downarrow)$.

Amplitude

πNN & $\pi N\Delta$ vertices

$$-i\delta H_{\pi NN} = \frac{f}{m_\pi}\vec{\sigma}\cdot\vec{q}\;\tau^\lambda; \qquad f = 1.00$$

$$-i\delta H_{\pi N\Delta} = \frac{f^*}{m_\pi}\vec{S}^\dagger\cdot\vec{q}T^{\dagger\lambda}; \qquad f^* = 2.13$$

Isospin factors

$$h_{\Delta-\mathrm{up}} = \frac{4\sqrt{2}}{3}$$

$$h_{\Delta-\mathrm{down}} = -\frac{4\sqrt{2}}{3}$$

$$-it_{\Delta-\mathrm{up}} = \frac{4}{3}\sqrt{2}\left(\frac{f^*}{m_\pi}\right)^2\left(\frac{f}{m_\pi}\right)\int\frac{d^4q}{(2\pi)^4}(-)\vec{S}_1\cdot\vec{p}_\pi(-)\vec{S}_1^\dagger\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}$$
$$\times\frac{2M_\Delta}{2E_\Delta(\vec{p}-\vec{q})}\;\frac{i}{p^0-q^0-E_\Delta(\vec{p}-\vec{q})+i\frac{\Gamma_\Delta}{2}}\;\frac{i}{q^{02}-\vec{q}^{\;2}-m_\pi^2+i\epsilon}$$
$$\times\frac{2M_N}{2E_N(\vec{p}-\vec{q}-\vec{p}_\pi)}\;\frac{i}{p^0-q^0-E_\pi(\vec{p}_\pi)-E_N(\vec{p}-\vec{q}-\vec{p}_\pi)+i\epsilon}$$
$$\times\frac{2M_N}{2E_N(-\vec{p}+\vec{q})}\;\frac{i}{p^0+q^0-E_N(-\vec{p}+\vec{q})+i\epsilon}\;(-i)\;g_d\;\theta(q_{\max}-|\vec{p}_d^{\;\mathrm{CM}}|)$$

with $\vec{p}_d^{\;\mathrm{CM}} = \vec{p}-\vec{q}-\frac{\vec{p}_\pi}{2}$. Similarly for $\Delta-$ *down* but $\vec{p}\to-\vec{p}$.

Amplitude

Momentum dependence

$$F(\vec{p},\vec{q},\vec{p}_\pi) = \frac{M_N}{E_N(-\vec{p}+\vec{q}\,)}\,\frac{M_N}{E_N(\vec{p}-\vec{q}-\vec{p}_\pi)}\,\frac{M_\Delta}{E_\Delta(\vec{p}-\vec{q}\,)}\,\frac{1}{2\omega(q)}$$
$$\times \frac{\theta(q_{\rm max}-|\vec{p}-\vec{q}-\frac{\vec{p}_\pi}{2}|)}{2p^0-E_\pi-E_N(-\vec{p}+\vec{q})-E_N(\vec{p}-\vec{q}-\vec{p}_\pi)+i\epsilon}$$
$$\times\left\{ \frac{1}{p^0-\omega(q)-E_\Delta(\vec{p}-\vec{q}\,)+i\frac{\Gamma_\Delta}{2}}\,\frac{1}{p^0-\omega(q)-E_\pi-E_N(\vec{p}-\vec{q}-\vec{p}_\pi)+i\epsilon}\right.$$
$$+\frac{1}{p^0-\omega(q)-E_\Delta(\vec{p}-\vec{q}\,)+i\frac{\Gamma_\Delta}{2}}\,\frac{1}{2p^0-E_\Delta(\vec{p}-\vec{q}\,)-E_N(-\vec{p}+\vec{q}\,)+i\frac{\Gamma_\Delta}{2}}$$
$$\left.+\frac{1}{p^0-\omega(q)-E_N(-\vec{p}+\vec{q}\,)+i\epsilon}\,\frac{1}{2p^0-E_\Delta(\vec{p}-\vec{q}\,)-E_N(-\vec{p}+\vec{q}\,)+i\frac{\Gamma_\Delta}{2}}\right\}, \tag{9}$$

Explicit cuts (TS): Δ, N on-shell, $\sqrt{s}-E_\Delta-E_N+i\Gamma_\Delta/2=0$, $(2p^0=\sqrt{s})$ and N, N' on-shell, $\sqrt{s}-E_\pi-E_N-E_{N'}+i\epsilon=0$

Amplitude

Spin transitions

Deuteron, $S = 1$

Transitions, (i) $\uparrow\uparrow$ (1), $\uparrow\downarrow$ (2) to (f) $\uparrow\uparrow$ (1), $\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)(2)$, $\downarrow\downarrow$ (3)

$$
\begin{aligned}
Q_{11} &: \uparrow\uparrow \;\rightarrow\; \uparrow\uparrow \\
Q_{12} &: \uparrow\uparrow \;\rightarrow\; \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\
Q_{13} &: \uparrow\uparrow \;\rightarrow\; \downarrow\downarrow \\
Q_{21} &: \uparrow\downarrow \;\rightarrow\; \uparrow\uparrow \\
Q_{22} &: \uparrow\downarrow \;\rightarrow\; \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\
Q_{23} &: \uparrow\downarrow \;\rightarrow\; \downarrow\downarrow
\end{aligned}
$$

$$
\begin{aligned}
Q_{11}^{\mathrm{up}} &= \left(\frac{2}{3}\vec{p}_\pi \cdot \vec{q} - \frac{i}{3} a_z\right) q_z \\
Q_{12}^{\mathrm{up}} &= \frac{1}{\sqrt{2}} \left\{ \left(\frac{2}{3}\vec{p}_\pi \cdot \vec{q} - \frac{i}{3} a_z\right) q_+ - \frac{i}{3} a_+ q_z \right\} \\
Q_{13}^{\mathrm{up}} &= -\frac{i}{3}\, a_+\, q_+ \\
Q_{21}^{\mathrm{up}} &= \left(\frac{2}{3}\vec{p}_\pi \cdot \vec{q} - \frac{i}{3} a_z\right) q_- \\
Q_{22}^{\mathrm{up}} &= \frac{1}{\sqrt{2}} \left\{ - \left(\frac{2}{3}\vec{p}_\pi \cdot \vec{q} - \frac{i}{3} a_z\right) q_z - \frac{i}{3} a_+ q_- \right\} \\
Q_{23}^{\mathrm{up}} &= \frac{i}{3}\, a_+\, q_z
\end{aligned}
$$

Transitions from $\downarrow\uparrow$ and $\downarrow\downarrow$ give a factor 2. And Similarly for the Δ-down diagram.

Amplitude

$$-it^{\pi}_{ij} = -g_d \frac{4\sqrt{2}}{3} \left(\frac{f^*}{m_\pi}\right)^2 \left(\frac{f}{m_\pi}\right) \int \frac{d^3q}{(2\pi)^3} \mathcal{F}_\pi(\vec{q}\,) \left\{ Q^{(\mathrm{u})}_{ij} F(\vec{p}, \vec{q}, \vec{p}_\pi) - Q^{(\mathrm{d})}_{ij} F(-\vec{p}, \vec{q}, \vec{p}_\pi) \right\}$$

Pion form factor: $\mathcal{F}_\pi(\vec{q}\,) = \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + \vec{q}\,^2}\right)^2$, with values of Λ_π around $1 - 1.2$ GeV.

FULL pion propagator. NOT done the static approximation: $((q^{02} - \vec{q}\,^2 - m_\pi^2)^{-1} \to (-\vec{q}\,^2 - m_\pi^2)^{-1})$. Thus,

$$\overline{\sum}\sum |t^\pi|^2 = 2\frac{1}{4}\sum_{i,j} |t^\pi_{ij}|^2 = \frac{1}{2}\sum_{i,j} |t^\pi_{ij}|^2, \qquad (10)$$

and the cross section for $pp \to \pi^+ d$ is then given by

$$\frac{d\sigma}{d\cos\theta_\pi} = \frac{1}{4\pi}\frac{1}{s}(M_N)^2\; M_d\; \frac{p_\pi}{p} \overline{\sum}\sum |t^\pi|^2 \qquad (11)$$

where $\cos\theta_\pi$ is $\frac{\vec{p}\cdot\vec{p}_\pi}{|\vec{p}\,||\vec{p}_\pi|}$.

Deuteron wave function

Gammermann, Nieves, Arriola, Oset, PRD81 (2010)

$$\psi(\vec{p}) \equiv \langle \vec{p}|\psi\rangle = \tilde{g}_d \, \frac{\theta(q_{\max} - |\vec{p}|)}{E_d - E_N(p) - E_N(p)}, \quad \int d^3p \, |\langle \vec{p}|\psi\rangle|^2 = 1. \quad (12)$$

To determine $q_{\max}$, we determine the scattering length from,

$$t(E) = \frac{1}{G(M_d) - G(E)}; \quad a = 2\pi^2 M_N \, \frac{1}{G(M_d) - G(2M_N)}. \quad (13)$$

To get the experimental value, $a = 5.377$ fm, $q_{\max} = 240$ MeV and $\tilde{g}_d^2 = (2.68 \times 10^{-3})^2$ MeV^{-1}, $g_d = (2\pi)^{3/2}\tilde{g}_d$.

Standard formula of Weinberg adapted to our normalization

$$g_d^2 = (2\pi)^3 \tilde{g}_d^2 = \frac{8\pi\gamma}{M_N^2}; \qquad \gamma = \sqrt{M_N B}. \quad (14)$$

which gives $g_d = (2\pi)^{3/2}\, 2.30 \times 10^{-3}$ MeV$^{-1/2}$.

Deuteron wave function Bonn potential

R. Machleid, PRC63, 024001 (2001)

$$\langle \vec{p}|\psi\rangle = \frac{1}{N}\sum_j \frac{C_j}{\vec{p}^{\,2}+m_j^2} \qquad (15)$$

with

$$N^2 = \int d^3p \left(\sum_j \frac{C_j}{\vec{p}^{\,2}+m_j^2}\right)^2 . \qquad (16)$$

In our normalization, where we have $\int d^3p/(2\pi)^3$ integrations instead of $\int d^3p$, and including also the weight factors of Field Theory (M/E close to unity), we have

$$\frac{M_N}{E_N}\,\frac{M_N}{E_{N'}}\,\frac{1}{2p^0-E_\pi-E_N-E_{N'}+i\epsilon}\,g_d\,\theta(q_{\rm max}-|\vec{p}-\vec{q}-\frac{\vec{p}_\pi}{2}|)$$
$$\rightarrow (-)(2\pi)^{3/2}\,\psi(|\vec{p}-\vec{q}-\frac{\vec{p}_\pi}{2}|) \qquad (17)$$

Short range correlations

Oset and Weise, NPA329, 365(1979). In π exchange terms, $S_i^\dagger q_i \sigma_j q_j \frac{1}{q^2-m_\pi^2}$,

$$q_i q_j \frac{1}{q^2 - m_\pi^2} \simeq q_i q_j \frac{1}{-\vec{q}^{\,2}} = (q_i q_j - \frac{1}{3}\vec{q}^{\,2}\delta_{ij})\frac{1}{-\vec{q}^{\,2}} + \frac{1}{3}\frac{\vec{q}^{\,2}}{-\vec{q}^{\,2}}\delta_{ij}. \tag{18}$$

The NN and $N\Delta$ wave functions have a correlation factor at short distances vanishing at $r \to 0$ that kills the δ function. In the realistic case, it is necessary to add the term

$$g'\delta_{ij}, \qquad g' \simeq 0.6 \ \ (\pi \text{ and } \rho \text{ exchange}) \tag{19}$$

We replace

$$\vec{S}\cdot\vec{p}_\pi \ \vec{S}^\dagger\cdot\vec{q} \ \vec{\sigma}\cdot\vec{q} \ \frac{1}{q^2 - m_\pi^2} \to g' \ \vec{S}\cdot\vec{p}_\pi \ \vec{S}^\dagger\cdot\vec{\sigma}. \tag{20}$$

$$-it_{ij}^{\mathrm{corr}} = -\frac{4\sqrt{2}}{3}\left(\frac{f^*}{m_\pi}\right)^2 \frac{f\, g_d\, g'}{m_\pi}\int \frac{d^3q}{(2\pi)^3}\mathcal{F}_\pi(\vec{q}\,)\left\{Q_{ij}'^{(\mathrm{u})}F'(\vec{p},\vec{q},\vec{p}_\pi) - Q_{ij}'^{(\mathrm{d})}F'(-\vec{p},\vec{q},\vec{p}_\pi)\right\}$$

Rho exchange

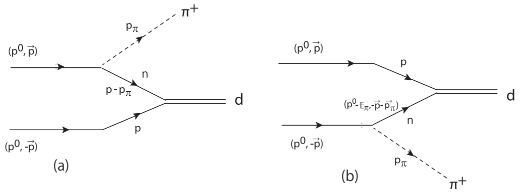
Substitution in π-exchange: Brack, Riska and Weise, NPA287 (1977)

$$\frac{f^*}{m_\pi}\frac{f}{m_\pi} \longrightarrow \frac{f^*_\rho}{m_\rho}\frac{f_\rho}{m_\rho}; \quad \mathcal{F}_\pi(\vec{q}\,) \longrightarrow \mathcal{F}_\rho(\vec{q}\,) = \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 + \vec{q}^{\,2}}\right)^2; \quad Q^\pi_{ij} \longrightarrow Q^\rho_{ij}$$

$$\frac{\vec{S}_1^\dagger \cdot \vec{q}\; \vec{\sigma}_2 \cdot \vec{q}}{q^2 - m_\pi^2 + i\epsilon} \longrightarrow \frac{\vec{S}_1^\dagger \times \vec{q}\; \vec{\sigma}_2 \times \vec{q}}{q^2 - m_\rho^2 + i\epsilon}; \quad F(mom, m_\pi) \longrightarrow F(mom, m_\rho)$$

and $f_\rho = 7.96$; $f^*_\rho = 13.53$; $\Lambda_\pi \sim 1.1$ GeV; $\Lambda_\rho \sim 1.8$ GeV.

Impulse approximation



π^+ **emission vertex**, $-i\delta H_{\pi^+ np} = -\sqrt{2}\frac{f}{m_\pi}\vec{\sigma}\cdot\vec{p}_\pi$.

$$-it_I^{\rm up} = \frac{\sqrt{2}f}{m_\pi}\vec{\sigma}_1 \cdot \vec{p}_\pi \; \frac{M_N}{E_N(\vec{p}-\vec{p}_\pi)} \; \frac{g_d\; \theta(q_{\rm max} - |\vec{p} - \frac{\vec{p}_\pi}{2}|)}{p^0 - E_\pi - E_N(\vec{p}-\vec{p}_\pi) + i\epsilon} \qquad (21)$$

Full amplitude

Impulse approximation with the deuteron wave function

$$-it^I_{11} = -F_I\, p_{\pi,z}[\psi(|\vec{p} - \frac{\vec{p}_\pi}{2}|) - \psi(|-\vec{p} - \frac{\vec{p}_\pi}{2}|)]$$

$$-it^I_{12} = -F_I\, \frac{1}{\sqrt{2}} p_{\pi,+}[\psi(|\vec{p} - \frac{\vec{p}_\pi}{2}|) - \psi(|-\vec{p} - \frac{\vec{p}_\pi}{2}|)]$$

$$-it^I_{13} = 0$$

$$-it^I_{21} = F_I\, p_{\pi,-}\psi(|-\vec{p} - \frac{\vec{p}_\pi}{2}|)$$

$$-it^I_{22} = -F_I\, \frac{1}{\sqrt{2}} p_{\pi,z}[\psi(|\vec{p} - \frac{\vec{p}_\pi}{2}|) + \psi(|-\vec{p} - \frac{\vec{p}_\pi}{2}|)]$$

$$-it^I_{23} = -F_I\, p_{\pi,+}\psi(|\vec{p} - \frac{\vec{p}_\pi}{2}|)\ , \qquad (22)$$

with $F_I = \frac{\sqrt{2}f}{m_\pi} \frac{E(\vec{p})}{M_N} \sqrt{(2\pi)^3}$ and $p_{\pi,+} = p_{\pi,x} + ip_{\pi,y}$ and $p_{\pi,-} = p_{\pi,x} - ip_{\pi,y}$.

$$\sum_{ij} |t^\pi_{ij}|^2 \longrightarrow \sum |t|^2 \equiv \sum_{ij} |t^\pi_{ij} + t^\rho_{ij} + t^{\text{corr.}}_{ij} + t^I_{ij}|^2 \qquad (23)$$

$$\boxed{\frac{d\sigma}{d\cos\theta_\pi} = \frac{1}{4\pi}\frac{1}{s}(M_N)^2\ M_d\ \frac{p_\pi}{p}\ \overline{\sum}\sum |t|^2} \qquad (24)$$

Results I

Results Ikeno, Molina, Oset, Phys. Rev. C104,014614 (2021)

	Range of K_p^{lab} and restrictions [MeV]	Parameters [MeV]	N of data	χ^2	χ^2/dof
(a)	[450:800] $M_\Delta \in$ [1150:1240] $\Gamma_\Delta \in$ [50:200] $\Lambda_\pi \in$ [800:1300] $\Lambda_\rho \in$ [1400:1900]	$M_\Delta = 1216$ $\Gamma_\Delta = 200$ $\Lambda_\pi = 1175$ $\Lambda_\rho = 1400$	50	415.29	9.03
(b)	[450:800] M_Δ, Γ_Δ fixed $\Lambda_\pi \in$ [900:1200] $\Lambda_\rho \in$ [1400:1900]	$M_\Delta = 1232$ $\Gamma_\Delta = 117$ $\Lambda_\pi = 1152$ $\Lambda_\rho = 1900$	50	15883.25	330.90
(c)	[450:800] $M_\Delta \in$ [1200:1250] $\Gamma_\Delta \in$ [100:150] $\Lambda_\pi \in$ [900:1200] $\Lambda_\rho \in$ [1400:1900]	$M_\Delta = 1208$ $\Gamma_\Delta = 150$ $\Lambda_\pi = 1014$ $\Lambda_\rho = 1400$	50	792.50	17.23
(d)	[525:700] $M_\Delta \in$ [1200:1250] $\Gamma_\Delta \in$ [100:150] $\Lambda_\pi \in$ [900:1200] $\Lambda_\rho \in$ [1400:1900]	$M_\Delta = 1215$ $\Gamma_\Delta = 150$ $\Lambda_\pi = 1015$ $\Lambda_\rho = 1428$	29	101.15	4.05
(e)	[550:700] $M_\Delta \in$ [1200:1250] $\Gamma_\Delta \in$ [100:150] $\Lambda_\pi \in$ [900:1200] $\Lambda_\rho \in$ [1400:1900]	$M_\Delta = 1213$ $\Gamma_\Delta = 117$ $\Lambda_\pi = 966$ $\Lambda_\rho = 1552$	25	43.10	2.05

Table 1: We perform fits to Richard-Serre, NPB20 & SAID data, with M_Δ, Γ_Δ, Λ_π, Λ_ρ as free pars. except for (b). $\sigma_{pp\to\pi^+d} = 2\ \frac{3}{4}\left(\frac{p_\pi}{p}\right)^2 \sigma_{\pi^+d\to pp}$

Results

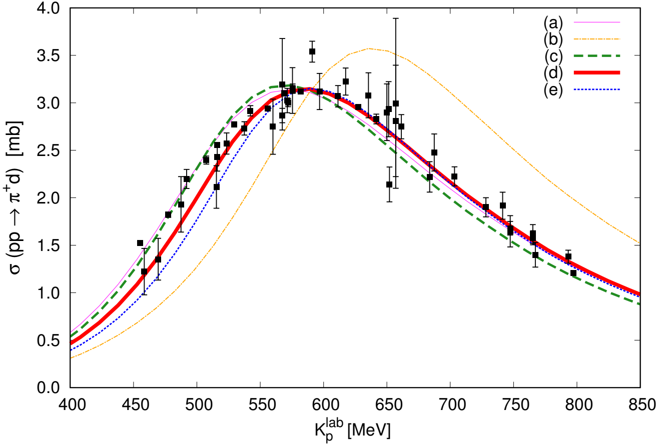
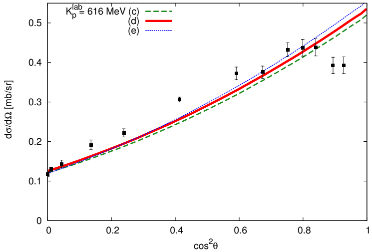
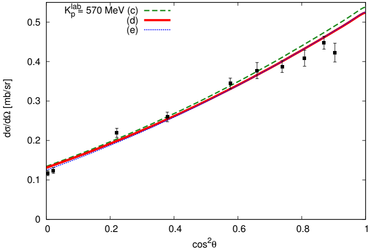


Figure 9: Cross section of $pp \to \pi^+ d$ as a function of the kinetic energy in the lab frame of the proton. The variable s is $s = 4M_N^2 + 2M_N K_p^{\mathrm{lab}}$.

The peak appears around $\sqrt{s} = 2165$ MeV for fit (d).

With the two initial protons antisymmetrical, $d\sigma/d\cos\theta$ gives the same for $\vec{p}_\pi$ or $-\vec{p}_\pi$, thus, it depends on $\cos^2\theta_\pi$.





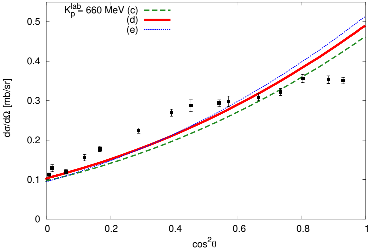


Figure 10: The differential cross section, $d\sigma/d\Omega = \frac{1}{2\pi}\frac{d\sigma}{d\cos\theta_\pi}$, as a function of $\cos^2\theta_\pi$ for $K_p^{\rm lab} = 570, 616$ and 660 MeV.

Contribution of the different terms

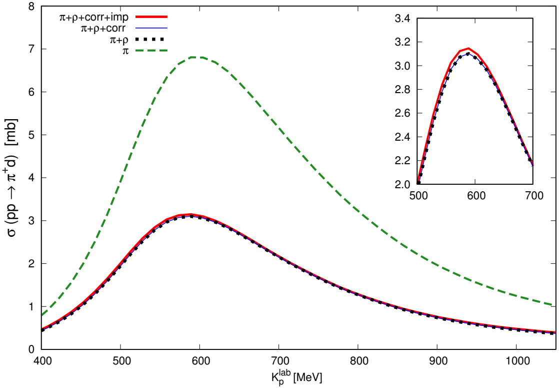


Figure 11: Contribution of the different terms: The π, $\pi + \rho$, $\pi + \rho + \mathrm{corr}$ and $\pi + \rho + \mathrm{corr} + \mathrm{imp}$. Inset: all terms except π-exchange to see the small differences. Pion exchange dominates!. Effect of g' negligible $\Longrightarrow L = 2$.

Results

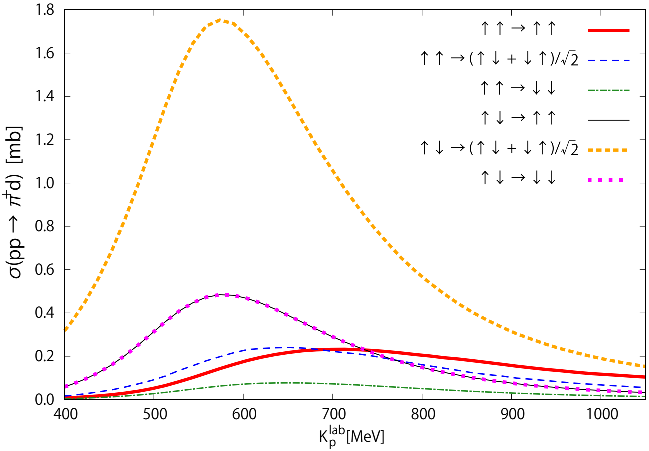
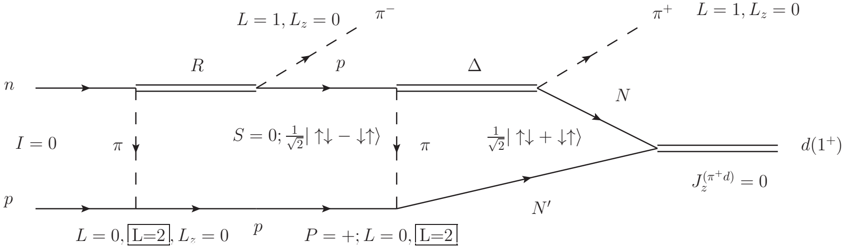


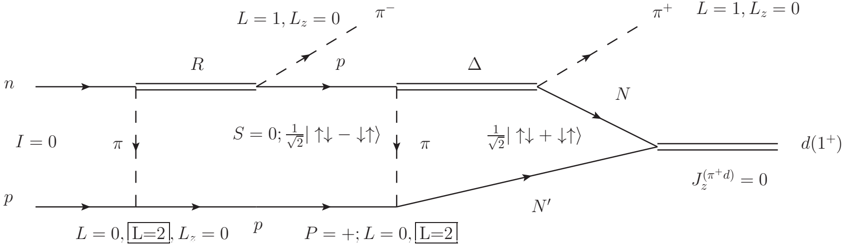
Figure 12: Contribution of the different spin transitions. A factor two is included to account for transitions from $\downarrow\uparrow$ and $\downarrow\downarrow$. $\uparrow\downarrow \to \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$ dominates!. The initial state combination $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$ is the one responsible for the transitions. Altogether, there is dominance of $L = 2, S = 0$ in *pp*!.

The $np \to \pi^+\pi^- d$ reaction

The $np \to \pi^+\pi^- d$ reaction



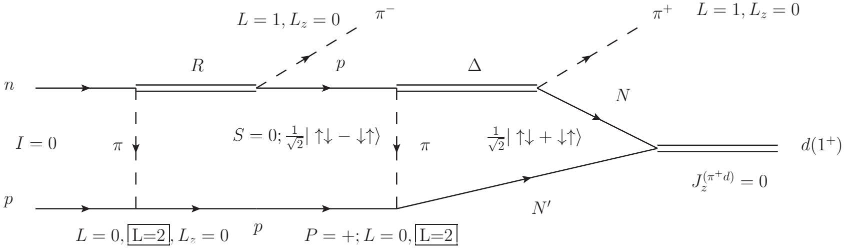
The $np \to \pi^+\pi^- d$ reaction



$$(\pi^-) L = 1, L_z = 0$$

$$J^{(pp)} = 2, J_z^{(pp)} = 0 \quad \longrightarrow J^{(\pi^+ d)} = 2, J_z^{(\pi^+ d)} = 0$$

The $np \to \pi^+\pi^- d$ reaction

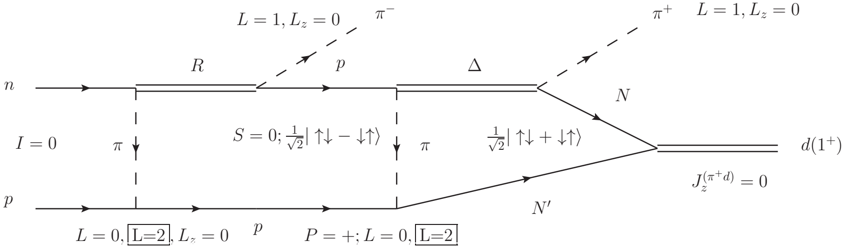


$$(\pi^-) L = 1, L_z = 0$$

$$J^{(pp)} = 2, J_z^{(pp)} = 0 \quad \longrightarrow J^{(\pi^+ d)} = 2, J_z^{(\pi^+ d)} = 0$$

$$\Longrightarrow np^{(I=0)} \equiv |2,0\rangle|1,0\rangle = \sqrt{\tfrac{3}{5}}|(J^{\text{tot}} =)3,0\rangle - \sqrt{\tfrac{2}{5}}|1,0\rangle \equiv |\pi^-\pi^+ d\rangle$$

The $np \to \pi^+\pi^- d$ reaction



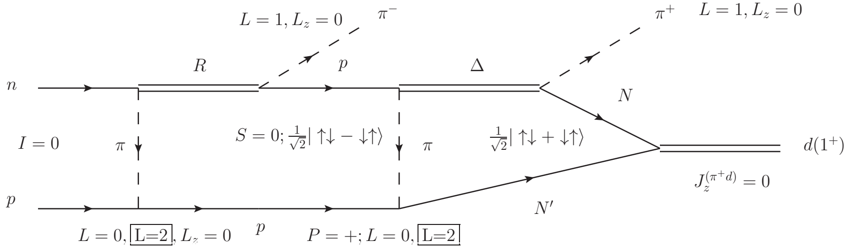
$$(\pi^-) L = 1, L_z = 0$$

$$J^{(pp)} = 2, J_z^{(pp)} = 0 \quad \longrightarrow J^{(\pi^+ d)} = 2, J_z^{(\pi^+ d)} = 0$$

$$\Longrightarrow np^{(I=0)} \equiv |2,0\rangle|1,0\rangle = \sqrt{\tfrac{3}{5}}|(J^{\mathrm{tot}} =)3,0\rangle - \sqrt{\tfrac{2}{5}}|1,0\rangle \equiv |\pi^-\pi^+ d\rangle$$

$$(np)\ I = 0, L = 2 \Longrightarrow S = 1;\ |2,0\rangle|1,0\rangle = \sqrt{\tfrac{3}{5}}|3,0\rangle - \sqrt{\tfrac{2}{5}}|1,0\rangle$$

The $np \to \pi^+\pi^- d$ reaction



$$(\pi^-)L=1, L_z=0$$
$$J^{(pp)}=2, J_z^{(pp)}=0 \quad \longrightarrow J^{(\pi^+d)}=2, J_z^{(\pi^+d)}=0$$

$$\Longrightarrow np^{(I=0)} \equiv |2,0\rangle|1,0\rangle = \sqrt{\tfrac{3}{5}}|(J^{\rm tot}=)3,0\rangle - \sqrt{\tfrac{2}{5}}|1,0\rangle \equiv |\pi^-\pi^+d\rangle$$

$$(np)\ I=0, L=2 \Longrightarrow S=1;\ |2,0\rangle|1,0\rangle = \sqrt{\tfrac{3}{5}}|3,0\rangle - \sqrt{\tfrac{2}{5}}|1,0\rangle$$

$^{2s+1}L_J:\ (np)^3D_3 \qquad (pp)^1D_2 \qquad (\pi^+d)^3P_2 \qquad \boxed{J^{\rm tot,P}=3^+}$

The $np \to \pi^+\pi^- d$ reaction

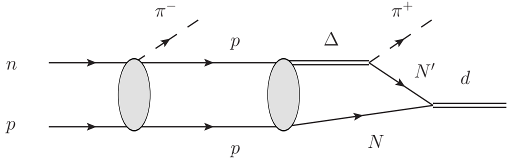


Figure 13: Two step mechanism for $np \to \pi^+\pi^- d$ with explicit Δ excitation in the $pp \to \pi^+ d$. The mechanism with the nn intermediate state is considered in addition.

M. Bashkanov, PRL102(2009), P. Adlarson, PRL106(2011), PLB721(2013) Reaction studied: $NN \to NN\pi\pi$, with double Δ production and subsequent $\Delta \to \pi N$ decay or $N^*(1440)$ production with decay of N^* to $N\pi\pi$, or $N^* \to \pi\Delta(N\pi)$, Alvarez-Ruso, Oset et al, NPA633(1998). Cross sections obtained are too small compared to the peak of the $np \to \pi^0\pi^0 d$ reaction without peak around 2380 MeV.
Previous work on the two step mechanism I. Bar-Nir, NPB54,17(1973)

The $np \to \pi^+\pi^- d$ reaction

- To check whether the two step mechanism can explain the dibaryon peak we want to use experimental data on the $np \to \pi^- pp$ and $pp \to \pi^+ d$ reactions.
- P. Adlarson, PLB721(2013), the same peak visible in the $np \to \pi^0\pi^0 d$ reaction is seen in the $np(\mathrm{I}=0) \to \pi^+\pi^- d$ reaction with about double strength. Thus, np in the first step has $I = 0$.

$np\,(\mathrm{I}=0) \to \pi^- pp$

$$\frac{d\sigma^I_{np\to\pi^- pp}}{dM_{\mathrm{inv}}(p_1 p_1')} = \frac{1}{4ps}(2M_N)^4 \frac{1}{16\pi^3} p_\pi \tilde{p}_1 |\bar{t}|^2 \frac{1}{2} \tag{25}$$

$M_{\mathrm{inv}}(p_1 p_1')$, invariant mass of the two protons, p, CM momentum of the inital $n(p)$, $\tilde{p}_1$, momentum of the final p in the pp rest frame.

$pp \to \pi^+ d$

$$\sigma_{pp\to\pi^+ d} = \frac{1}{16\pi M^2_{\mathrm{inv}}(p_1 p_1')} \frac{p_\pi'}{\tilde{p}_1} |\bar{t}\,'|^2 (2M_N)^2 (2M_d) \tag{26}$$

p_π', π^+ momentum in the pp rest frame, $|\bar{t}'|^2$, angle averaged $|t'|^2$.

Amplitude for the two step process

$$-it'' = \frac{1}{2}\int \frac{d^4 p_1}{(2\pi)^4} \frac{(2M_N)^2}{2E_N(p_1)2E_N(p_1')} \frac{i}{p_1^0 - E_N(p_1) + i\epsilon} \frac{i\,(-i)t\,(-i)t'}{\sqrt{s} - p_1^0 - \omega_\pi - E_N(p_1') + i\epsilon}$$

($\frac{1}{2}$ for the two identical particles). Approximation: take tt' outside the dp_1^0 integration with their on-shell values.

$$t'' \simeq \frac{1}{2}\int \frac{d^3 p_1}{(2\pi)^3} \frac{(2M_N)^2}{2E_N(p_1)2E_N(p_1')} \frac{tt'}{\sqrt{s} - E_N(p_1) - E_N(p_1') - \omega_\pi + i\epsilon}$$

Since

$$\frac{1}{M_{\rm inv}(p_1 p_1') - 2E_N(p_1) + i\epsilon} \equiv$$

$$\mathcal{P}\left[\frac{1}{M_{\rm inv}(p_1 p_1') - 2E_N(p_1)}\right] - i\pi\delta(M_{\rm inv}(p_1 p_1') - 2E_N(p_1))$$

In the on-shell approx. (Bar-Nir), this becomes, $t''_{\rm on} = -i\frac{1}{2}\frac{\tilde{p}_1}{8\pi}\frac{(2M_N)^2}{M_{\rm inv}(p_1 p_1')}t\bar{t}'$.
Off-shell effects can be taken into account by means of,

$$\left(\frac{\tilde{p}_1}{2\pi M_{\rm inv}(p_1 p_1')}\right)^2 \to |G(M_{\rm inv})|^2 \qquad (27)$$

The $np \to \pi^+\pi^- d$ reaction

Thus, for the two-step mechanism,

$$\frac{d\sigma_{np\to\pi^+\pi^- d}}{dM_{\rm inv}(\pi^+\pi^-)} = (2M_N)^2(2M_d)p_d\tilde{p}_\pi \frac{1}{4}\frac{\tilde{p}_1^2}{64\pi^2}\frac{1}{M^2_{\rm inv}(p_1p_1')}\frac{1}{p_\pi\tilde{p}_1}2|\bar{t}'|^2\frac{d\sigma_{np\to\pi^- pp}}{dM_{\rm inv}(p_1p_1')}$$

Approximation: $|t\bar{t}'|^2 \sim |\bar{t}|^2|\bar{t'}^2|$, the amplitudes t, t' have smooth angular structure. Adlarson, PLB774, 599 (2017), Richard-Serre, NPB20, 413 (1970)

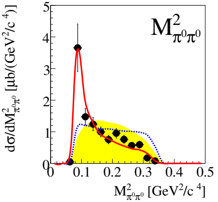
Bose enhancement

For the two pions in $I = 0$, L must be even. Preferred direction for $L = 0$.

$$\frac{d\sigma_{np\to\pi^+\pi^- d}}{dM_{\rm inv(\pi^+\pi^-)}} = \sigma_{np\to\pi^+\pi^- d}\delta(M_{\rm inv}(\pi^+\pi^-) - \bar{M}_{\pi\pi})$$

$$E_{2\pi} = \frac{s + M^2_{\rm inv}(\pi\pi) - M_d^2}{2\sqrt{s}}, E_\pi \simeq E_{2\pi}/2$$

$$M^2_{\rm inv}(p_1p_1') = (P(np) - p_{\pi^-})^2 = s + m_\pi^2 - 2\sqrt{s}E_\pi$$



Bashkanov, PRL102(2009)

The $np \to \pi^+\pi^- d$ reaction

Using

$$2M_{\rm inv}(p_1 p_1')dM_{\rm inv}(p_1 p_1') = -2\sqrt{s}dE_\pi = -M_{\rm inv}(\pi\pi)dM_{\rm inv}(\pi\pi) \ ,$$

we obtain

$$\sigma_{np\to\pi^+\pi^- d} = \frac{M_{\rm inv}(p_1 p_1')}{4\pi} \frac{\sigma_{np\to\pi^- pp}\sigma_{pp\to\pi^+ d}}{M_{\rm inv}(\pi\pi)} \frac{\tilde{p}_1^2}{p_\pi p_\pi'} p_d \tilde{p}_\pi \ . \qquad (28)$$

The amplitudes $np(\mathrm{I}=0) \to \pi^- pp$ and $np(\mathrm{I}=0) \to \pi^+ nn$ are identical up to the phase of π^+ and the same happens for $pp \to \pi^+ d$ and $nn \to \pi^- d$. Isospin symmetry gives $\sigma_{np(I=0)\to pp\pi^-} = \frac{1}{3}\sigma_{np(I=0)\to NN\pi}$.

$$\boxed{\sigma_{np\to\pi^+\pi^- d} = \frac{M_{\rm inv}(p_1 p_1')}{6\pi} \frac{\sigma^I_{np\to NN\pi}\sigma_{pp\to\pi^+ d}}{M_{\rm inv}(\pi\pi)} \frac{\tilde{p}_1^2}{p_\pi p_\pi'} p_d \tilde{p}_\pi}$$

with $\sigma^I_{np\to NN\pi} = \sigma_{np(I=0)\to NN\pi}$.

Adlarson, PLB774(2017), Clement, 2010.09217 (2020).

Results II

The $np \to \pi^+\pi^- d$ reaction

Fit to data

Data for $\sigma_{np(I=0)\to NN\pi}$ from Fig. 1 of Clement, Skorodko, 2010.09217 (2020). Isospin conservation assumed.

$$\sigma_{np(I=0)\to pp\pi^-} = \frac{1}{3}\sigma_{np(I=0)\to NN\pi} = \frac{1}{6}\sigma_{NN(I=0)\to NN\pi} = \frac{1}{6}3(2\sigma_{np\to pp\pi^-} - \sigma_{pp\to pp\pi^0})$$

5 % isospin simmetry violation give systematic errors of 0.5 mb, which we add in quadrature.

The $np(\mathrm{I}=0) \to NN\pi$ cross section is parameterized as,

$$\sigma_i = \left| \frac{\alpha_i}{\sqrt{s} - \tilde{M}_i + i\frac{\tilde{\Gamma}}{2}} \right|^2 \quad (29)$$

- **Set I**: $\tilde{M}_1 = 2326$ MeV, $\tilde{\Gamma}_1 = 70$ MeV, $\alpha_1^2 = 2.6\left(\frac{\tilde{\Gamma}_1}{2}\right)^2$ mb MeV^2.
- **Set II**: $\tilde{M}_2 = 2335$ MeV, $\tilde{\Gamma}_2 = 80$ MeV, $\alpha_2^2 = 2.5\left(\frac{\tilde{\Gamma}_2}{2}\right)^2$ mb MeV^2.

$pp \to \pi^+ d$: $\sigma_3 = \left| \frac{\alpha_3}{M_{\rm inv}(p_1 p_1') - \tilde{M}_3 + i\frac{\tilde{\Gamma}_3}{2}} \right|^2$ with $\tilde{M}_3 = 2165$ MeV, $\tilde{\Gamma}_3 = 123.27$ MeV, $\alpha_3^2 = 3.186\left(\frac{\tilde{\Gamma}_3}{2}\right)^2$ mb MeV^2. Data from Richard-Serre, NPB20 (1970).

The $np \to \pi^+\pi^- d$ reaction

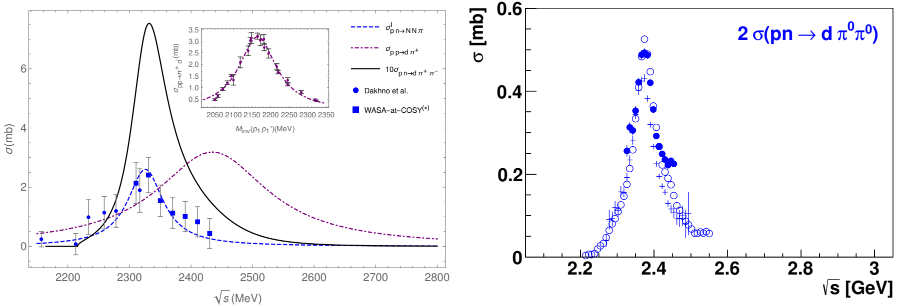


Figure 14: Left: Results with set I for $\sigma_{np\to\pi^- pp}(I=0)$ and $\sigma_{pp\to\pi^+ d}$. The results with $\sigma_{np\to\pi^+\pi^- d}$ in $I=0$ are multiplied by 10 for a better comparison. Data for $np(\mathrm{I}=0)\to\pi NN$ are taken from Dakhno et al., and WASA-at-COSY$^{(*)}$, including systematic errors from isospin violation. Inset: $\sigma_{pp\to\pi^+ d}$ in comparison with data for $pp\to\pi^+ d$ from Richard-Serre. Right: Experimental data $pn\to d\pi^0\pi^0$. Filled symbols denote results from Adlarson (2013), open symbols from previous works, and crosses the result with Eq. (1).

The $np \to \pi^+\pi^- d$ reaction

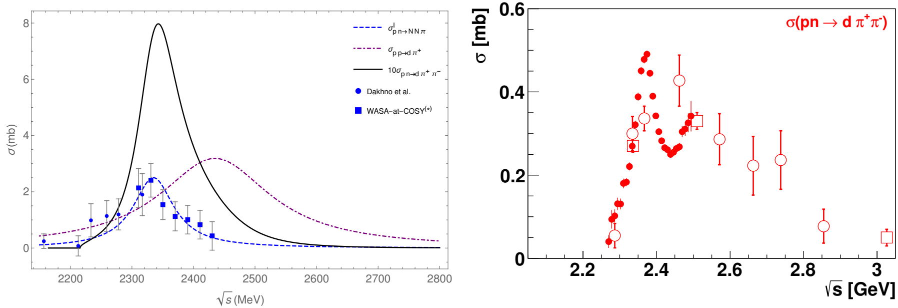


Figure 15: Left: Results for set II; Right: Experimental data $pn \to d\pi^+\pi^-$. Filled symbols denote results from Adlarson, PLB721, 229 (2013), open symbols from previous works.

The $np \to \pi^+\pi^- d$ reaction

	$\delta\bar{M}_{\pi\pi}$ (MeV)			$p_{1,\max}^{\rm o.s.}$ (MeV)	
Set I	40	60	80	700	800
strength (mb)	0.72	0.76	0.75	0.82	0.95
position (MeV)	2332	2332	2332	2332	2332
width (MeV)	76	76	81	75	75
Set II					
strength (mb)	0.75	0.80	0.80	0.85	0.96
position (MeV)	2342	2345	2345	2343	2342
width (MeV)	86	87	88	87	84

Table 2: Peak strength, position, and width, for intermediate particles on shell ($\bar{M}_{\pi\pi} = 2m_\pi + \delta\bar{M}_{\pi\pi}$), and off-shell ("o.s."), $\delta\bar{M}_{\pi\pi} = 60$ MeV.

We obtain the peak strength $\sim 0.76 - 0.96$ mb, the position, $2332 - 2345$ MeV, and the width, $75 - 88$ MeV, to be compared with the experimental one, with strength ~ 0.5 mb, at 2370 MeV.

Conclusions

Conclusions

- We showed that the $pp \to \pi^+ d$ reaction has a **triangle singularity**
- The sequential one pion production in $np(I=0) \to \pi^- pp$ followed by $pp \to \pi^+ d$ offers a natural explanation of the peak so far associated to the dibaryon $d^*(2380)$
- The mechanism naturally gives rise to the dominant **quantum numbers** observed in the experiments (3^+).