The molecular picture of the $\Omega(2012)$ and the $X_0(2866)$

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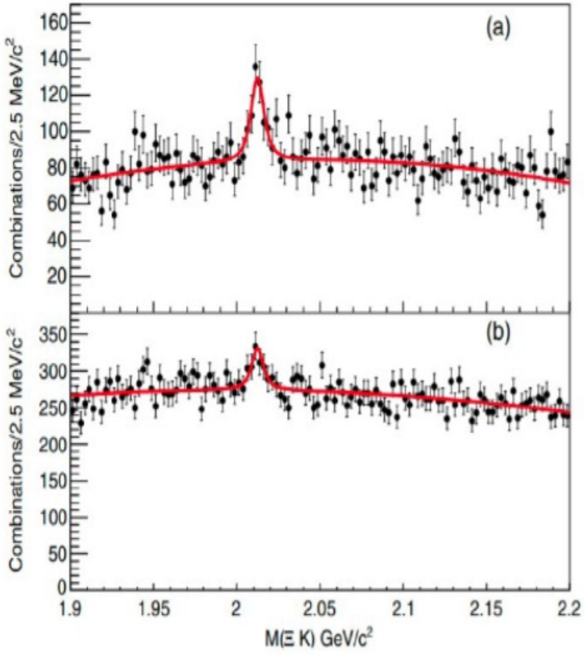
Discovery of Ω(2012): Strangeness =-3

- In 2018, Belle reported an Ω(2012) state: [Phys. Rev. Lett. 121, 052003 (2018)]

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\ \mathrm{MeV}$$
$$\Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\ \mathrm{MeV}$$



- This prompted many theoretical studies on the issue
  – quark model pictures
  – molecular pictures based on the meson-baryon interaction

■ Coupled channels such as $\bar{K}\Xi^*(1530)$, ηΩ, $\bar{K}\Xi$

Single channel

- Y. H. Lin and B. S. Zou, Phys. Rev. D 98, 056013 (2018).

- M. P. Valderrama, Phys. Rev. D 98, 054009 (2018).

- Y. Huang, M. Z. Liu, J. X. Lu, J. J. Xie, and L. S. Geng, Phys. Rev. D 98, 076012 (2018).

- R. Pavao and E. Oset, Eur. Phys. J. C 78, 857 (2018).

- M. V. Polyakov, H. D. Son, B. D. Sun, and A. Tandogan, Phys. Lett. B 792, 315 (2019).

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If single channel Kbar Ξ* it will decay to Kbar pi Ξ

Update of the Belle data

- In 2019, Belle showed a new result: [Phys. Rev. D 100, 032006(2019)]
  - Ratio of the $\Xi\pi K$ width to the ΞK width is smaller than 11.9%.

$$\mathcal{R}^{\Xi\pi K}_{\Xi K} = \frac{\mathcal{B}(\Omega(2012) \to \Xi(1530)(\to \Xi\pi)K)}{\mathcal{B}(\Omega(2012) \to \Xi K)} < 11.9\%$$

  - Experimental cut: $1.49\,\mathrm{GeV} < M_{\mathrm{inv}}(\pi\Xi) < 1.53\,\mathrm{GeV}$

This paper concludes:

``The result strongly disfavors the molecular interpretation"

=> In order to see to which extend the data rule out the molecular picture or not, we reanalyzed the work of R. Pavao and E. Oset, EPJC78(2018)

Calculation reproduces the mass and width

New results:

Molecular picture for the Ω(2012) revisited

Natsumi Ikeno,[1,2,*] Genaro Toledo,[2,3,†] and Eulogio Oset[2,‡]

PHYSICAL REVIEW D 101, 094016 (2020)

Formalism: Coupled channels approach

R. Pavao and E. Oset, EPJC78(2018)

$\boxed{\bar{K}\Xi^*, \eta\Omega \text{ (s-wave)}, \ \bar{K}\Xi \text{ (d-wave)}}$

- Bethe-Salpeter equation: $T = [1 - VG]^{-1} V$
- Transition potential: Ω^* $J^P=3/2^-$

$$
\begin{array}{cc}
\begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} & \\
V = \begin{pmatrix} 0 & 3F & \alpha q_{\rm on}^2 \\ 3F & 0 & \beta q_{\rm on}^2 \\ \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 & 0 \end{pmatrix} & \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array}
\end{array}
$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0) \qquad q_{\rm on} = \frac{\lambda^{1/2}\left(s, m_{\bar{K}}^2, m_{\Xi}^2\right)}{2\sqrt{s}}$$

- s-wave potentials between $\bar{K}\Xi^*$ and $\eta\Omega$:
Taken from chiral Lagrangian,

S. Sarkar, E. Oset, M.J. Vicente Vacas, NPA750(2005) 294

- d-wave potential between $\bar{K}\Xi$ and $\bar{K}\Xi^*$or $\eta\Omega$:
described in terms of α, β: free parameters

- Meson-Baryon loop function G:

$$G(\sqrt{s}) = \begin{pmatrix} G_{\bar{K}\Xi^*}(\sqrt{s}) & 0 & 0 \\ 0 & G_{\eta\Omega}(\sqrt{s}) & 0 \\ 0 & 0 & G_{\bar{K}\Xi}(\sqrt{s}) \end{pmatrix}$$

For s-wave channel

$$G_i(\sqrt{s}) = \int_{|\boldsymbol{q}|<q_{\rm max}} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_i(\boldsymbol{q})} \frac{M_i}{E_i(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_i(\boldsymbol{q}) - E_i(\boldsymbol{q}) + i\epsilon}$$

for $i = \bar{K}\Xi^*, \eta\Omega$ with $\omega_i(\boldsymbol{q}) = \sqrt{m_i^2 + \boldsymbol{q}^2}$, $E_i(\boldsymbol{q}) = \sqrt{M_i^2 + \boldsymbol{q}^2}$

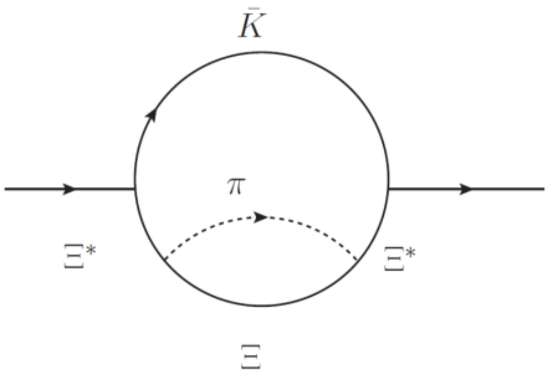
For d-wave channel

$$G_{\bar{K}\Xi}(\sqrt{s}) = \int_{|\boldsymbol{q}|<q'_{\rm max}} \frac{d^3q}{(2\pi)^3} \frac{(q/q_{\rm on})^4}{2\omega_{\bar{K}}(\boldsymbol{q})} \frac{M_\Xi}{E_\Xi(\boldsymbol{q})} \frac{1}{\sqrt{s} - \omega_{\bar{K}}(\boldsymbol{q}) - E_\Xi(\boldsymbol{q}) + i\epsilon}$$

$q_{\rm max}$: cut off parameters

$G_{K^- \Xi*}$ function accounting for $\Xi^* \to \pi\Xi$ decay

- $\Omega(2012)$ is close to $\bar{K}\Xi^*$ threshold
- We take into account the Ξ^* mass distribution due to its width for $\Xi^* \to \pi\Xi$ decay.



- $G_{K^-\Xi*}$ is convolved with the Ξ^* mass distribution

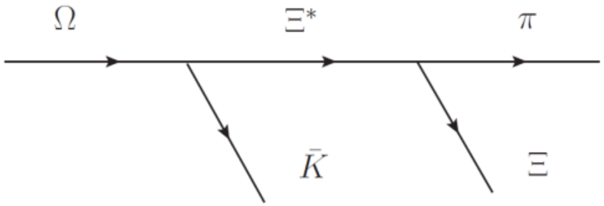
$$\tilde{G}_{\bar{K}\Xi^*}(\sqrt{s}) = \frac{1}{N}\int_{M_{\Xi^*}-\Delta M_{\Xi^*}}^{M_{\Xi^*}+\Delta M_{\Xi^*}} d\tilde{M}\left(-\frac{1}{\pi}\right)\mathrm{Im}\left(\frac{1}{\tilde{M}-M_{\Xi^*}+i\frac{\Gamma_{\Xi^*}}{2}}\right) G_{\bar{K}\Xi^*}(\sqrt{s}, m_{\bar{K}}, \tilde{M})$$

R. Pavao and E. Oset, EPJC78(2018)

  – Energy dependence of Ξ^* width (New)

$$\Gamma_{\Xi^*} = \Gamma_{\Xi^*,\mathrm{on}}\frac{\tilde{p}_\pi^3}{\tilde{p}_{\pi,\mathrm{on}}^3}\,\theta(M_{\mathrm{inv}}(\pi\Xi) - m_\pi - M_\Xi)$$ Ikeno, Toledo and E. Oset, PRD(2020)

✓ $\Omega(2012) \to \pi K\Xi$ decay in the coupled channels approach



- Γ_{con} <– $G_{K^-\Xi*}$ with convolution (accounts for $K\Xi$ and $\pi K\Xi$ decays)
- Γ_{non} <– $G_{K^-\Xi*}$ without convolution (only for $K\Xi$ decay)

-> Comparison of the $\Omega(2012)$ with/without convolution gives us the estimate of $\Omega(2012)$ decay width into $K\Xi$ and $\pi K\Xi$ decay channels : $\dfrac{\Gamma_{\Omega^*,\mathrm{con(Edep)}} - \Gamma_{\Omega^*,\mathrm{non}}}{\Gamma_{\Omega^*,\mathrm{non}}} = \dfrac{\Gamma_{\Omega^*\to\pi\bar{K}\Xi}}{\Gamma_{\Omega^*\to\bar{K}\Xi}}$

Calculated mass, width and ratio

We make a fit to the experimental data by changing the q_{max}, α, β parameters.

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6\,\text{MeV} \qquad \Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6\,\text{MeV} \qquad \frac{\Gamma_{\Omega}(\pi \bar{K} \Xi)}{\Gamma_{\Omega, \bar{K}\Xi}} < 11.9\,\%$$

- M. P. Valderrama, Phys. Rev. D 98, 054009 (2018).

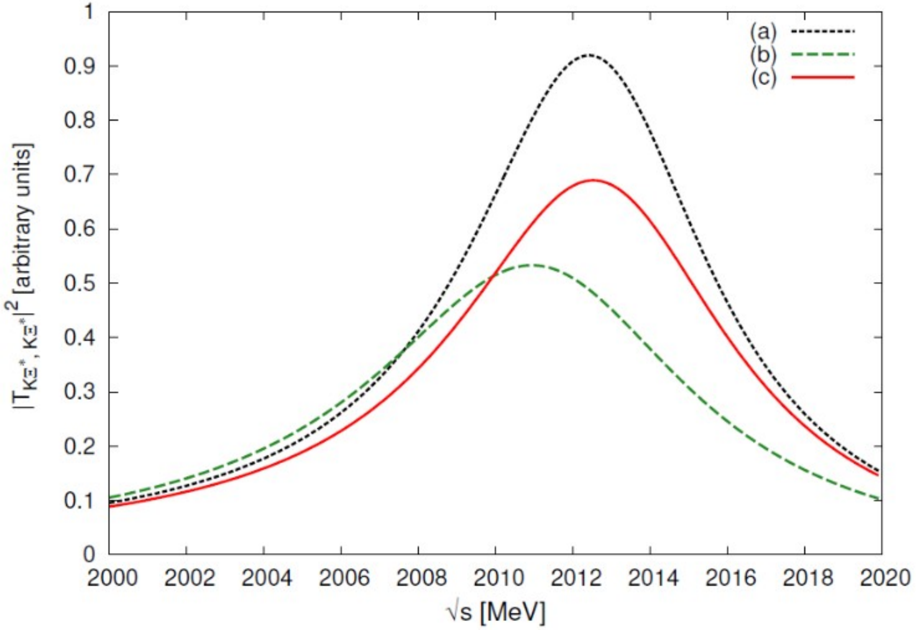


FIG. 3. $|T|^2$ for the diagonal $\bar{K}\Xi^*$ channel with three options: (a) $G_{\bar{K}\Xi^*}$ without convolution; (b) $G_{\bar{K}\Xi^*}$ with convolution and Γ_{Ξ^*} fixed; (c) $G_{\bar{K}\Xi^*}$ with convolution and Γ_{Ξ^*} energy dependent.

We find an acceptable solution in terms of natural values for the parameters which reproduce fairly well the experimental data.

$$q_{\max} = 735\,\text{MeV}; \quad q_{\max}(\eta\Omega) = 750\,\text{MeV};$$
$$\alpha = -11.0 \times 10^{-8}\,\text{MeV}^{-3}; \quad \beta = 20.0 \times 10^{-8}\,\text{MeV}^{-3},$$

$$M_{\Omega^*} = 2012.6\,\text{MeV}$$
$$\text{(a) } \Gamma_{\Omega^*,\text{non}} = 8.2\,\text{MeV}$$
$$\text{(c) } \Gamma_{\Omega^*,\text{con(Edep)}} = 9.1\,\text{MeV}$$
$$\frac{\Gamma_{\Omega^*,\text{con(Edep)}} - \Gamma_{\Omega^*,\text{non}}}{\Gamma_{\Omega^*,\text{non}}} = 10.9\,\%$$

Similar results in
J. Lu, C. Zeng, E. Wang, J. Xie, and L. Geng
Eur. Phys. J. C, 361 (2020)

Couplings g_i of different channels

- The couplings g_i of the $\Omega(2012)$ to the different channels are obtained from the residue of the T-matrix of the pole in the second Riemann sheet. Close to the pole

$$T_{ij} = \frac{g_i g_j}{z - z_R} \; (z, \text{complex energy}; \; z_R, \text{complex pole position})$$

$$g_i^2 = \lim_{z \to z_R} (z - z_R) T_{ii}; \;\; g_j = g_i \frac{T_{ij}}{T_{ii}}|_{z=z_R}.$$

R. Pavao and E. Oset, EPJC78(2018)

	$\bar{K}\Xi^*$ (2027)	$\eta\Omega$ (2220)	$\bar{K}\Xi$ (1812)
g_i	$1.88 + i0.04$	$3.55 - i0.67$	$-0.42 + i0.22$
$\lvert\tilde{g}_i\rvert$	1.77	3.42	0.44
$\lvert g_{i,\mathrm{conv}}\rvert$	1.75	3.38	0.45
$\mathrm{wf}_i(g_i G_i)$	$-34.37 - i2.42$	$-31.99 + i5.63$	...
$-g_i^2 \frac{\partial G_i}{\partial\sqrt{s}}$	$0.57 + i0.16$	$0.26 - i0.09$	...

We also show the wave function at the origin for the s-wave states, wf(g_iG_i), calculated at the peak, and the probability of each channel $-g_i^2 \frac{\partial G_i}{\partial\sqrt{s}}$

-D. Gamermann, J. Nieves, E. Oset, and E. R. Arriola, PRD 81, 014029 (2010).
-F. Aceti, L.R. Dai, L.S. Geng, E. Oset and Y. Zhang, EPJA50, 57 (2014)

The strength of the *wf* and the probability dominates for the $\bar{K}\Xi^*$ state.
Note, however, that the $\eta\Omega$ channel is required to bind Ω^* state since the diagonal potential of the $\bar{K}\Xi^*$ channel is null and hence cannot produce any bound state by itself.
We find that all data including the recent Belle experiment are still compatible with the molecular picture.

Actually it is not possible to go with the ratio R much below 10% and still claim to have a molecule.

A ratio R of the order of 3% would definitely invalidate the molecular picture.

Aparently there are new results from Belle, with a definite number, not a boundary.

Hence, this information will be very valuable to settle this problem.

Molecular picture for the $X_0(2866)$ as a $D^*\bar{K}^*$ $J^P = 0^+$ state and related $1^+, 2^+$ states

R. Molina *, E. Oset *

Physics Letters B 811 (2020) 135870

Amplitude analysis of the $B^+ \to D^+ D^- K^+$ decay

R. Aaij *et al.*[*]
(LHCb Collaboration)

PHYSICAL REVIEW D **102,** 112003 (2020)

$$X_0(2900):\ M = 2.866 \pm 0.007 \pm 0.002\ \mathrm{GeV}/c^2,$$
$$\Gamma = 57 \pm 12 \pm 4\ \mathrm{MeV},$$
$$X_1(2900):\ M = 2.904 \pm 0.005 \pm 0.001\ \mathrm{GeV}/c^2,$$
$$\Gamma = 110 \pm 11 \pm 4\ \mathrm{MeV},$$

Observed in
the $D^- K^+$ spectrum.
(it has cbar and sbar)
This is exotic unlike
the D K states which
lead to the $D_{s0}^*(2317)$

D. Johnson (CERN), LHC Seminar, B → DDh decays: A new (virtual) laboratory for exotic particle searches at LHCb p. August 11 (2020)

LHCb finds two states of $J^P = 0^+, 1^-$ decaying to DKbar that offers us the first clear example of an exotic hadron with open heavy flavor, of type cs ubar dbar. The states found are

$$X_0(2866): M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9\,\text{MeV},$$
$$X_1(2900): M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5\,\text{MeV}$$

R. Molina, T. Branz, and E. Oset PHYSICAL REVIEW D 82, 014010 (2010)

New interpretation for the $D_{s2}(2573)$ and the prediction of novel exotic charmed mesons

TABLE VI. $C = 1$; $S = -1$; $I = 0$. Mass and width for the states with $J = 0$ and 2.

Molecular state of D* K*bar

$I[J^P]$	$\sqrt{s}_{\text{pole}}$ (MeV)	Model	Γ (MeV)
$0[0^+]$	2848	A, $\Lambda = 1400$ MeV	23
		A, $\Lambda = 1500$ MeV	30
		B, $\Lambda = 1000$ MeV	25
		B, $\Lambda = 1200$ MeV	59
$0[1^+]$	2839	Convolution	3
$0[2^+]$	2733	A, $\Lambda = 1400$ MeV	11
		A, $\Lambda = 1500$ MeV	14
		B, $\Lambda = 1000$ MeV	22
		B, $\Lambda = 1200$ MeV	36

X_0 as molecular state of D* Kbar*

M.-Z. Liu, J.-J. Xie, and L.-S. Geng (2020), 2008.07389

Y. Huang, J.-X. Lu, J.-J. Xie, and L.-S. Geng (2020), 2008.07959

Y. Dong, A. Faessler, and V. E. Lyubovitskij, Prog. Part. Nucl. Phys. 94, 282 (2017)

M.-W. Hu, X.-Y. Lao, P. Ling, and Q. Wang (2020), 2008.06894 HQSS, 0^+,1^+,2^+ degenerate

J. He and D.-Y. Chen (2020), 2008.07782

H.-X. Chen, W. Chen, R.-R. Dong, and N. Su (2020), 2008.07516

J.-R. Zhang (2020), 2008.07295 tetraquark

Z.-G. Wang (2020), 2008.07833

Y. Xue, X. Jin, H. Huang, and J. Ping (2020), 2008.09516 Quark model favoring molecule

M. Karliner and J. L. Rosner (2020), 2008.05993

X.-G. He, W. Wang, and R.-L. Zhu, J. Phys. G 44, 014003 (2017), 1606.00097.

X.-H. Liu, M.-J. Yan, H.-W. Ke, G. Li, and J.-J. Xie (2020), 2008.07190 Triangle singularity

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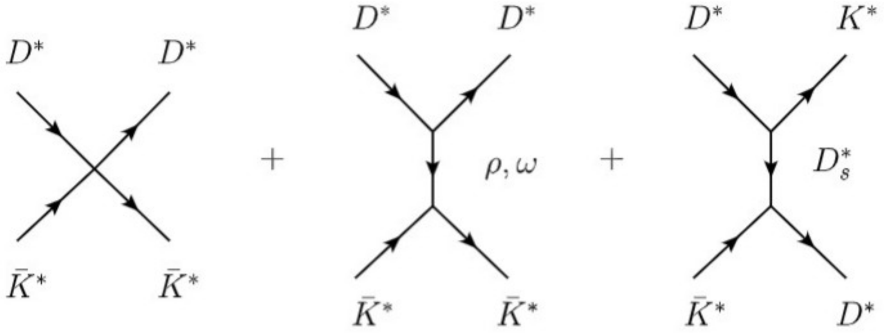
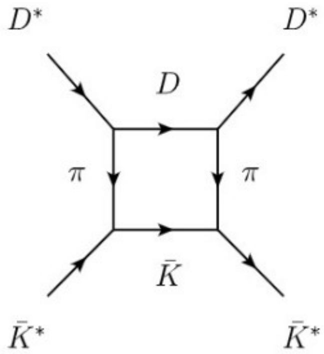
Revision to the light of experimental results R. Molina, E. O. Phys.Lett.B 811 (2020) 135870



Decay mode



$$\mathcal{L}_{VVVV} = \frac{1}{2} g^2 \langle [V_\mu, V_\nu] V^\mu V^\nu \rangle,$$
$$\mathcal{L}_{VVV} = ig \langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu) \rangle \qquad (1)$$

where $g = M_V/2f_\pi$ ($M_V = 800$ MeV, $f_\pi = 93$ MeV) and V_μ is given by

$$V_\mu = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu .$$

$$F(q) = e^{((p_1^0 - q^0)^2 - \vec{q}^{\,2})/\Lambda^2}$$

$$\mathcal{P}^{(0)} = \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu$$
$$\mathcal{P}^{(1)} = \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu - \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu)$$
$$\mathcal{P}^{(2)} = \{ \frac{1}{2} (\epsilon_\mu \epsilon_\nu \epsilon^\mu \epsilon^\nu + \epsilon_\mu \epsilon_\nu \epsilon^\nu \epsilon^\mu) - \frac{1}{3} \epsilon_\mu \epsilon^\mu \epsilon_\nu \epsilon^\nu \}$$

The potential is projected over the three orthogonal spin states and the Bethe-Salpeter equation is solved by iteration of the potential

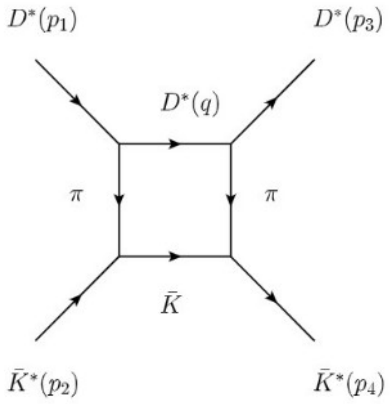
$$T=(1-VG)^{-1}V$$

where V accounts for the contact term, tree level vector exchange and box diagram

The contact term is peculiar of the local hidden gauge approach and breaks the degeneracy of the spins states (the exchange of heavy vectors also breaks it).
The dominant terms come from exchange of light vectors and respect HQSS (degenerate states $0^+,1^+,2^+$ with only these terms)

Tree level amplitudes for $D^*\bar{K}^*$ in $I=0$. The last column shows the value of V at threshold.

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-9.9g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-10.2g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-15.9g^2$



$$\mathcal{L} = \frac{iG'}{\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\langle \delta_\mu V_\nu \delta_\alpha V_\beta P\rangle$$

with $G' = \frac{3g'}{4\pi^2 f}$; $g' = -\frac{G_V m_\rho}{\sqrt{2}f^2}$, $G_V \simeq 55$ MeV,

$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$X_0(2866)$

No D Kbar decay $\quad 0^- \, 0^-$ needs L=1 , parity

No D* Kbar decay $\quad 1^- \, 0^-$ needs L=1, parity

Conclusions:

The $\Omega(2012)$ is favored as a molecular state of the $\bar{K}\Xi^*$, $\eta\Omega$(s-wave), and $\bar{K}\Xi$(d-wave) channels
This is so even with strict limits of Belle on the Kbar π Ξ decay channel, now relaxed.

The $X_0(2866)$ finds also much support as a D* Kbar* molecular state.
The picture is also appealing since it led to prediction of this state 10 years before observation.

This picture also predicts partner states with $0^+,1^+,2^+$, with particular decay channels. The observation of the $1^+,2^+$ states would help much to unravel the nature of the X_0 state.