HADRON 2021

19th INTERNATIONAL
CONFERENCE ON HADRON
SPECTROSCOPY AND STRUCTURE

Meson & Glueball spectroscopy within the graviton soft-wall model

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and
Vicente Vento

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M.R. and V. Vento EPJA 54 (18)
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THE MIXING PROBLEM IN ADS/QCD

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1

Open questions in Glueball Physics

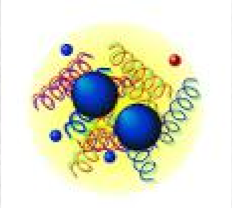
1 Open questions in Glueball Physics

QCD, the gauge theory describing strong interactions

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum \bar{\Psi}\left(i\gamma \cdot D - m\right)\Psi \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abs} A^b_\mu A^c_\nu$$

Mesons

$$\mathbf{3} \otimes \bar{\mathbf{3}}$$



Baryons

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$$

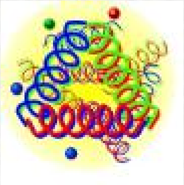
Exotic states

$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$$

HYBRIDS

$$\left.\begin{array}{l}\mathbf{8} \otimes \mathbf{8} \\ \mathbf{8} \otimes \cdots\cdots \otimes \mathbf{8}\end{array}\right\}$$

GLUEBALLS



Why Glueballs?

1 Open questions in Glueball Physics

QCD, the gauge theory describing strong interactions

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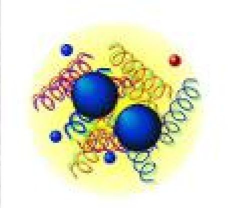

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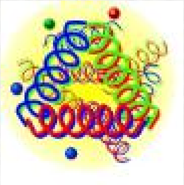

Glueball spectroscopy is a unique laboratory to test non perturbative QCD and CONFINEMENT

However :

1) several mesons have similar mass and quantum number ⟹ MIXING
2) Their characterization is not clear
3) Lattice calculations of decay are difficult! Models could help!

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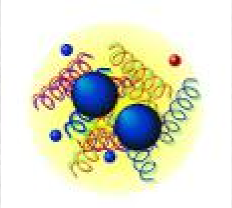

Baryons

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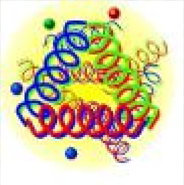

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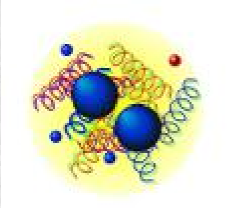

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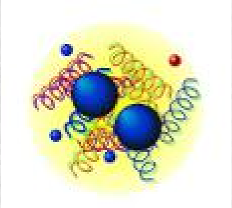

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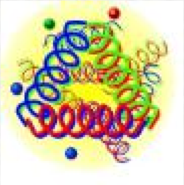
GLUEBALLS



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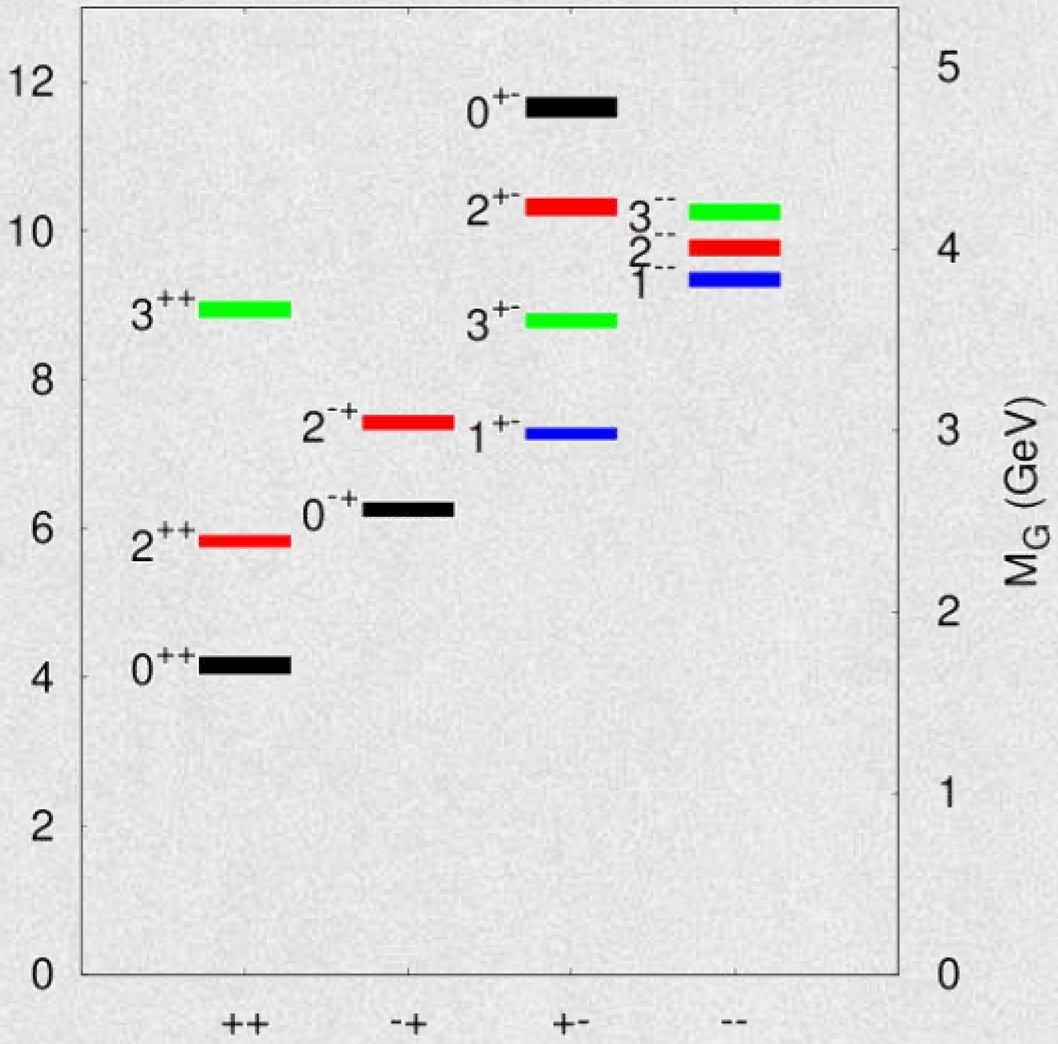
1 Open questions in Glueball Physics

Data have been obtained from Lattice QCD!



A recent result for the scalar ground state from the J/ψ decay:

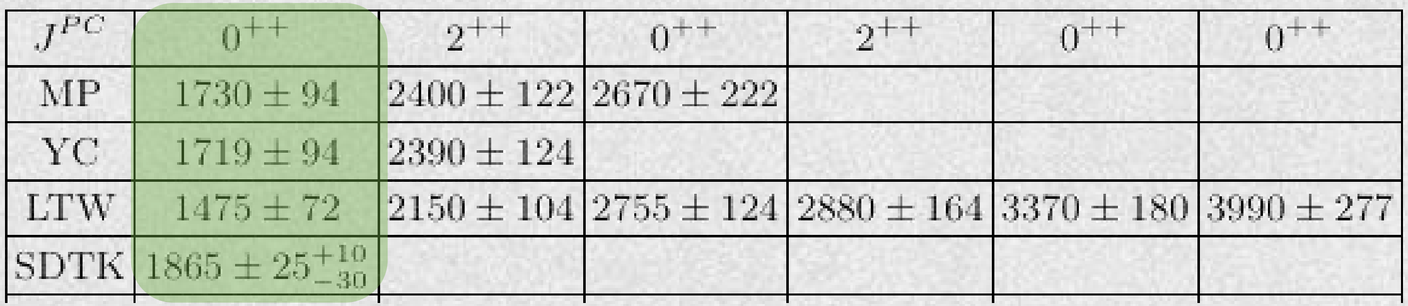
$$M_0 \sim 1865 \pm 25^{+10}_{-30}\ \mathrm{MeV}$$

E. Klempt et al PLB 816, 136227 (2021)
see Klempt's talk

1 Open questions in Glueball Physics

So far their properties have been obtained by Lattice QCD!
BUT also in this framework we have problems:

MP: **C.J. Morningstar et al, PRD 60, 034509 (1999)** YC: Y**. Chen et al, PRD 73, 014516 (2006)** LTW: **B. Lucini et al, JHEP 06, 012 (2004)**



J^{PC}	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277
SDTK	$1865 \pm 25^{+10}_{-30}$					

SDTK**: E. Klempt et al PLB 816, 136227 (2021)**

The mass of the lightest state is very hard to estimate

Could model help in this scenario?
We used AdS/QCD models!

1 Open questions in Glueball Physics

One of the main difficulties in the observation of glueballs is related to their mixing with mesons!

For example:

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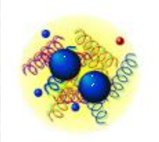
Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	1992 ± 16	2101 ± 7	2189 ± 13



Mixing?

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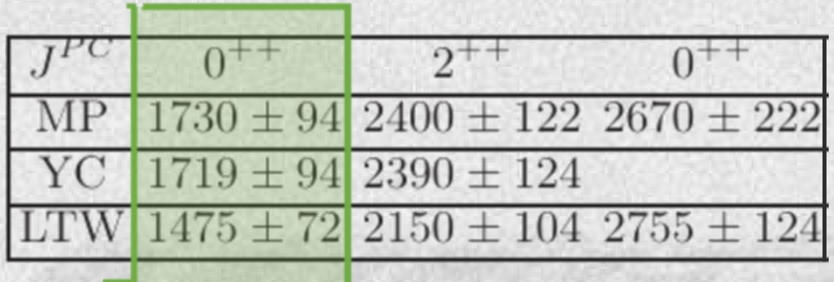

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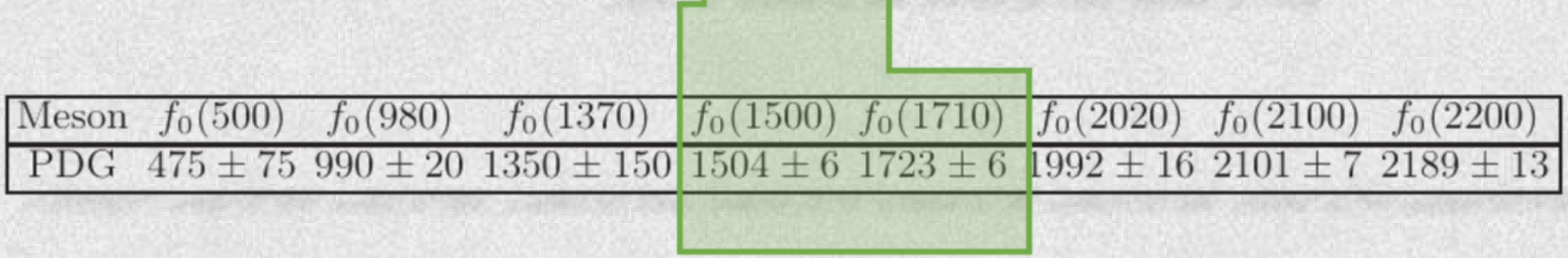

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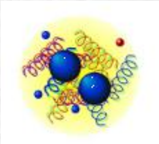


Mixing?

We use AdS/QCD models to study the MIXING problems and "predict" the kinematic conditions where pure glueball states could be observed.

2

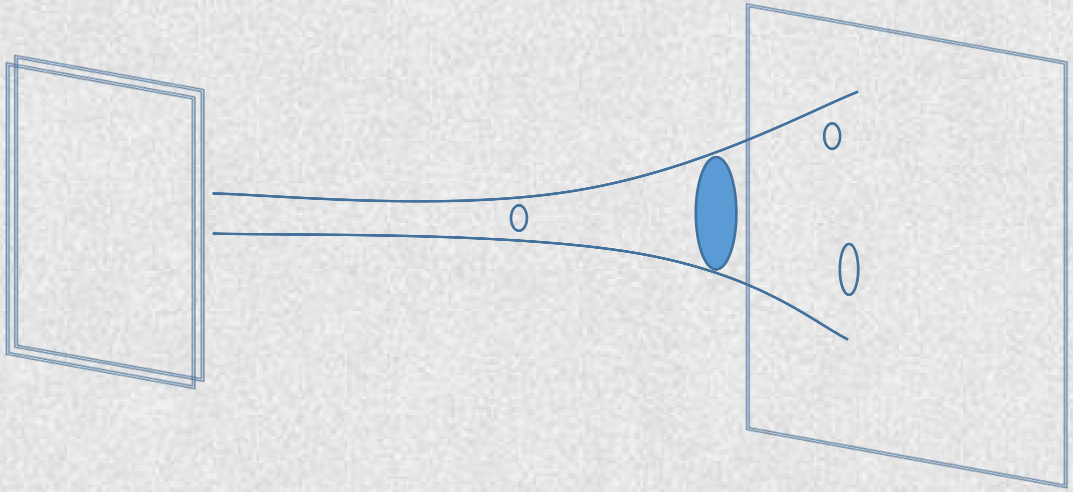
Introduction to AdS/QCD

2 Introduction to AdS/QCD

From Maldacena conjecture: AdS/CFT

N=4 SU(N) SYM ⟷ String theory on AdS_5 x S_5

Isometries between group symmetries



$$g_{YM}^2 N \underset{N\to\infty}{=} \frac{R^4}{l^4}$$

R= radius of the manifold

l = length

2 Introduction to AdS/QCD

From Maldacena conjecture: AdS/CFT

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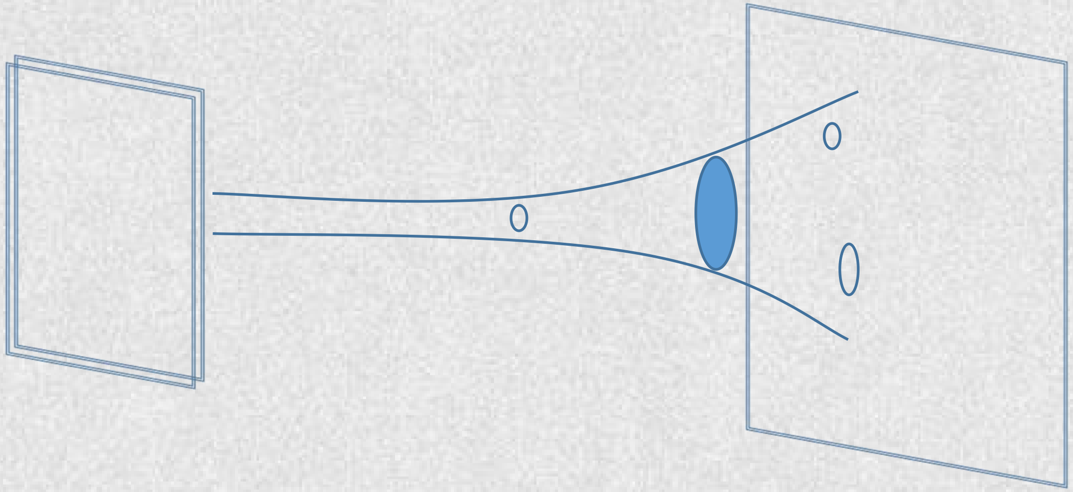
Isometries between group symmetries

This is not QCD!
No supersymmetry
Confinement
Conformal symmetry broken
N is finite



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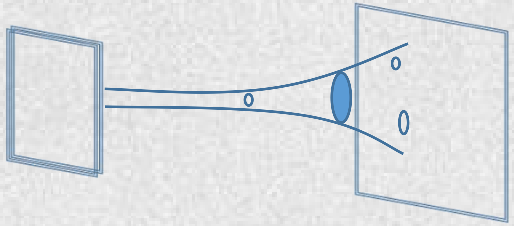
Introduction to AdS/QCD

The dream is to understand QCD using its dual gravity theory!



No supersymmetry
Confinement
Conformal symmetry broken
N is finite

Top-down Approach:

Find a gravitational theory dual to QCD

Advantages: duality is exact and well understanding of the theory

Disadvantages: a dual of QCD with fundamental flavors even at large *N* has not been found

Bottom-up Approach:

Starts from QCD and attempts to construct a five dimensional holographic dual

Advantages: some freedom in matching the model to features of QCD

Disadvantages: some discrepancies with data have been found

Witten:

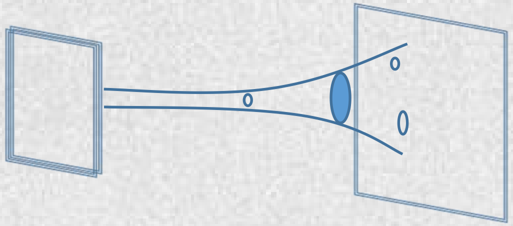
Supersymmetry could be neglected by compactifying one of the spatial directions and imposing antiperiodic boundary conditions.

⬇

Gauge fields at low energies

SUSY partners at the compactification scale

2 Introduction to AdS/QCD



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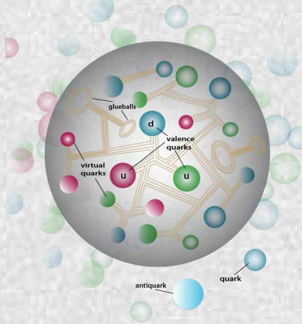


Hard-wall model

Compactification of the 5° dimension by hand. AdS geometry cut by two branes: UV and IR.

Soft-wall model

Soft cutoff of AdS space by introducing a dilaton field.

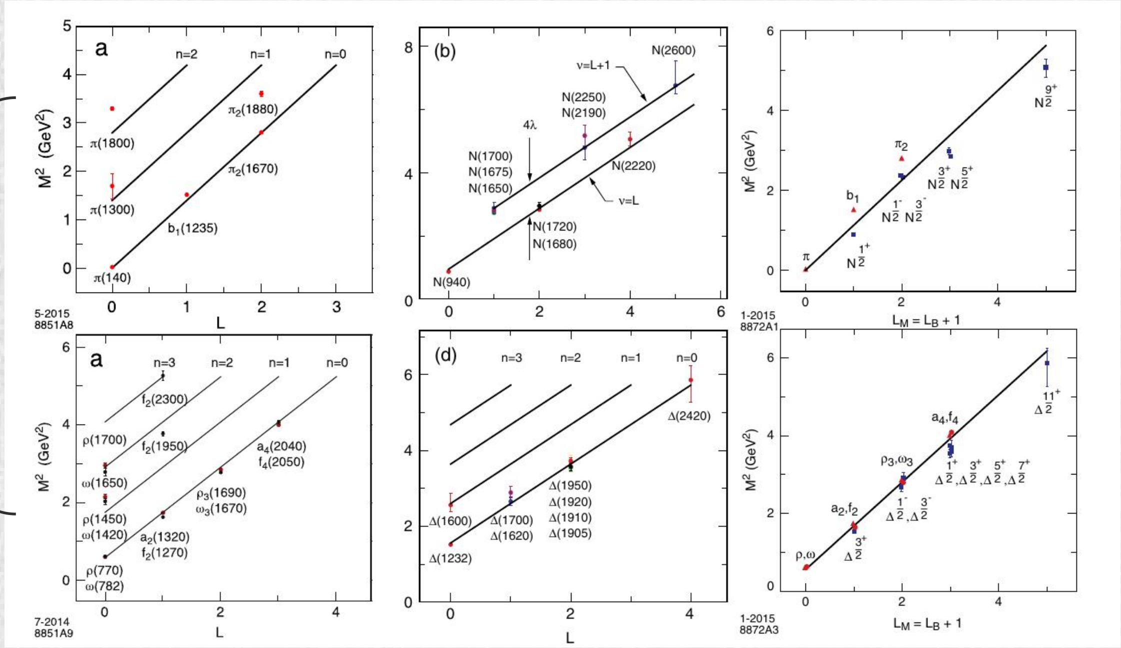
$$\mathrm{e}^{-\varphi}$$

2

Introduction to AdS/QCD: applications

HADRON SPECTRUM:

S.J. Brodsky et al, Phys. Rep. 584 (2015)
H.G. Dosh et al PRD 91, 045040 (2015), 085016 (2015)



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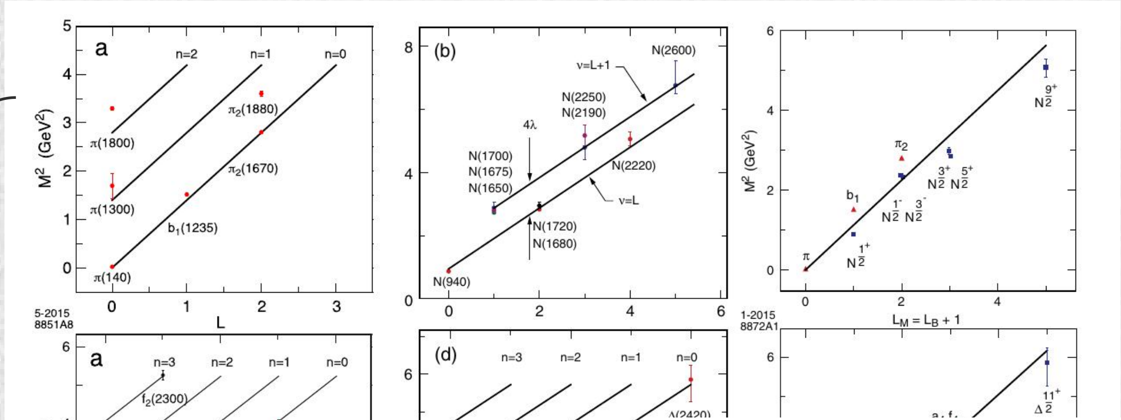

see Brodsky's talk on Thursday

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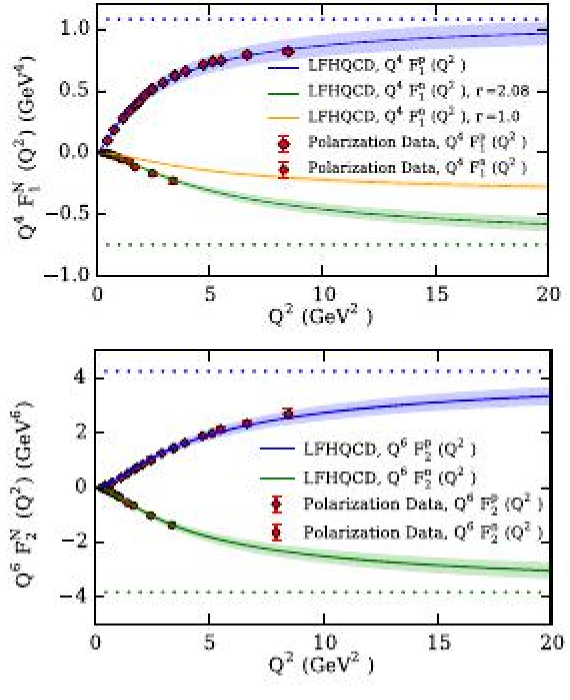
R.S. Sufian et al PRD 95, 014011 (2017)



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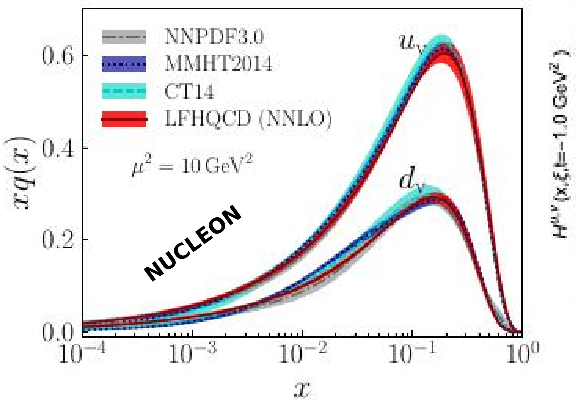
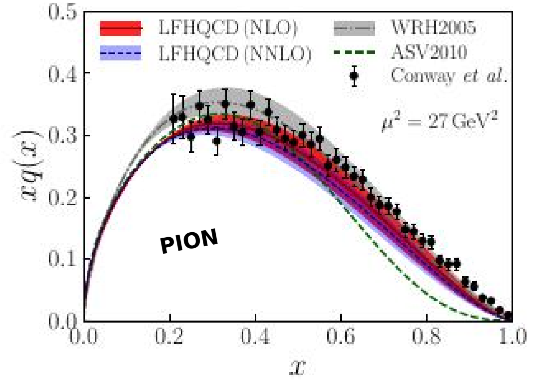
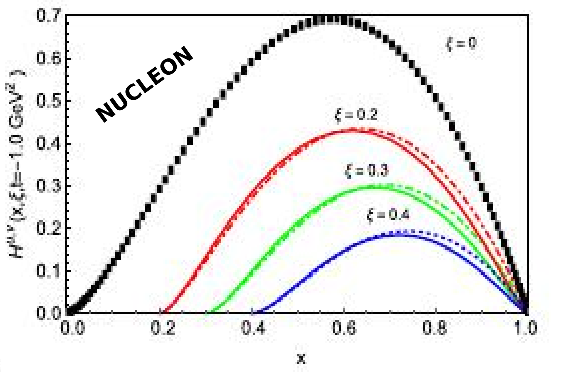
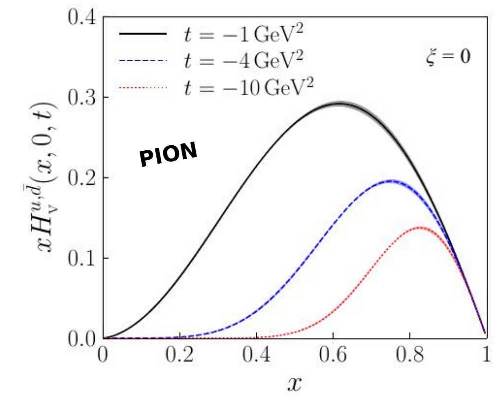




M. Rinaldi, PLB 771 (2017)





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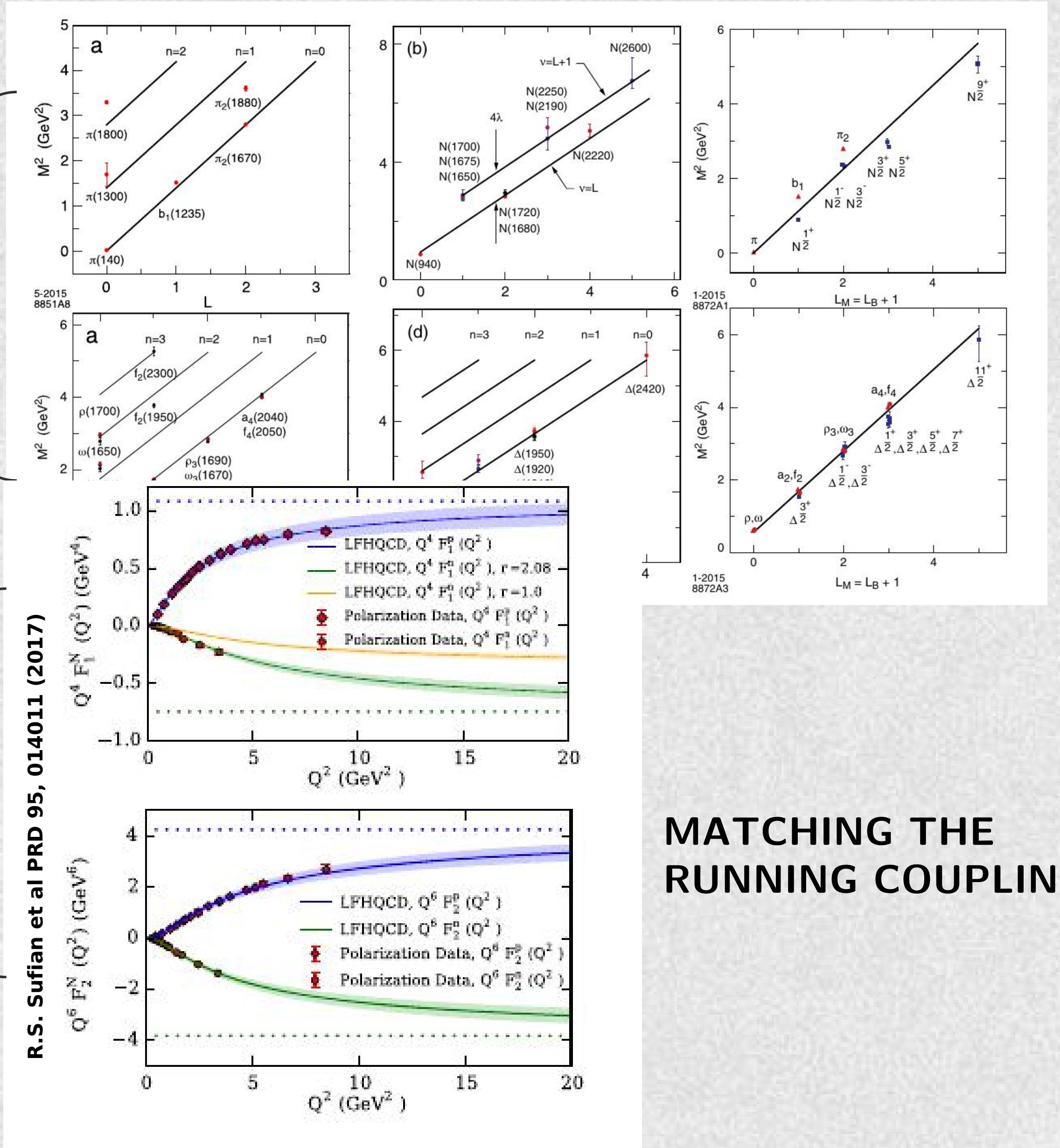

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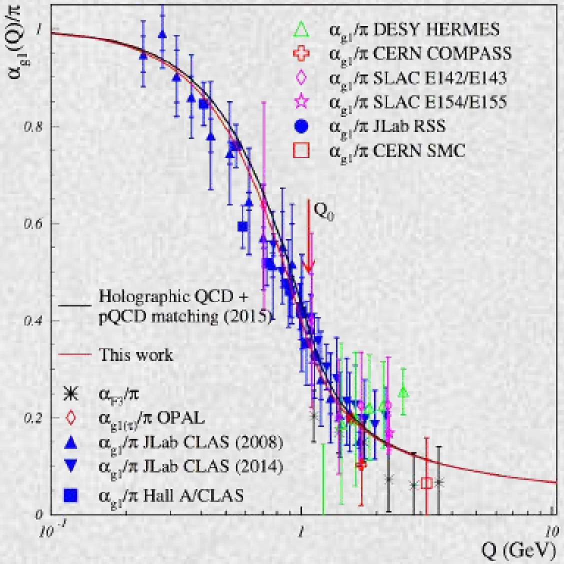
MATCHING THE RUNNING COUPLING



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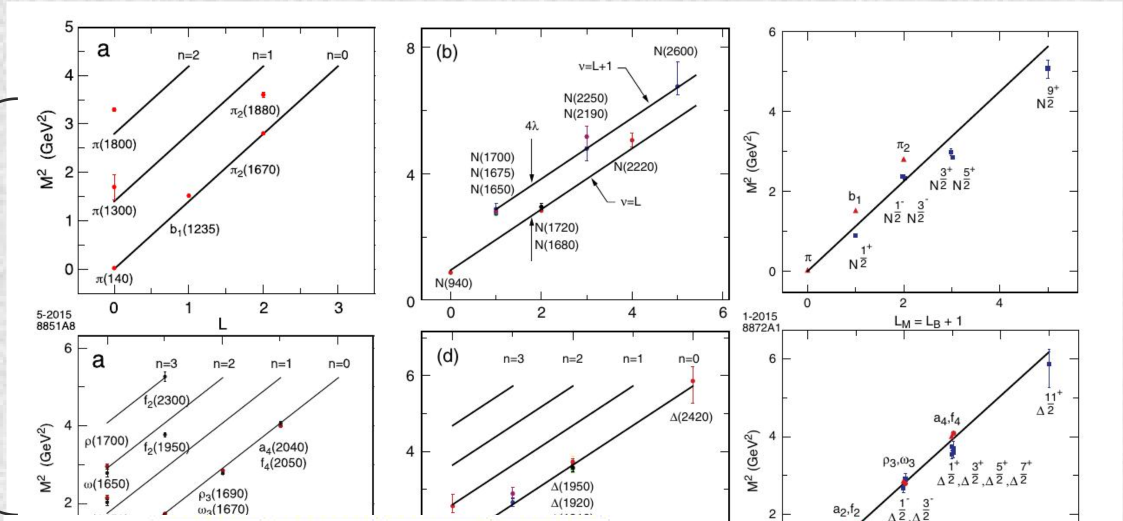

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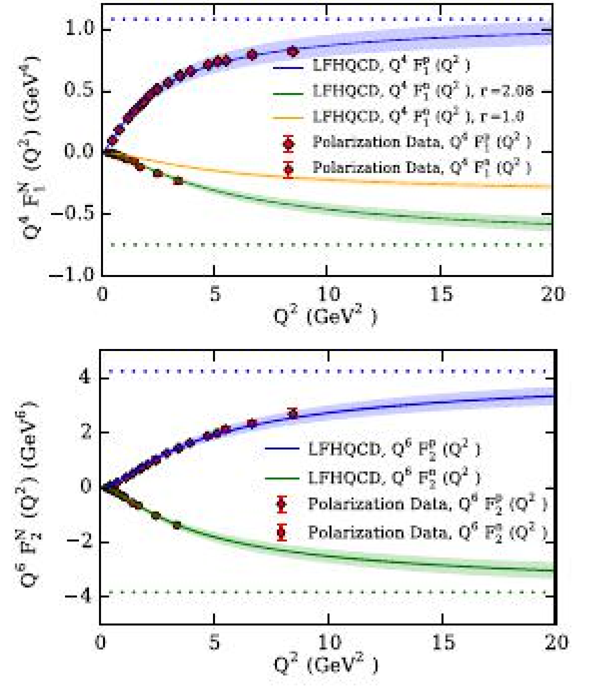

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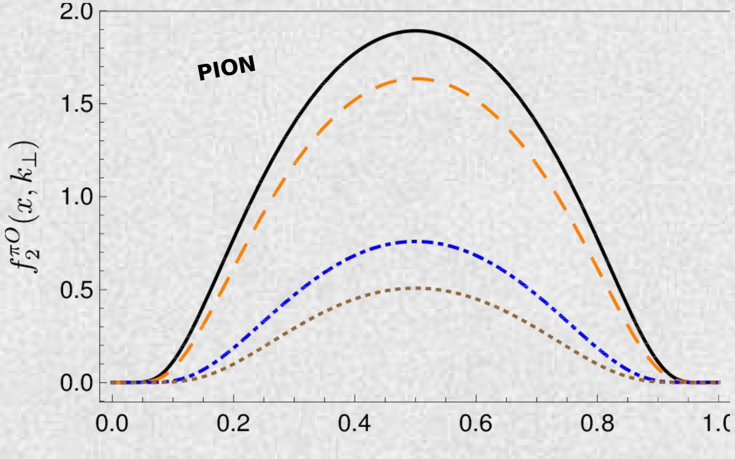
APPLICATIONS TO DOUBLE PARTON SCATTERING



M. Rinaldi et al, EPJC 78, 781 (2018)

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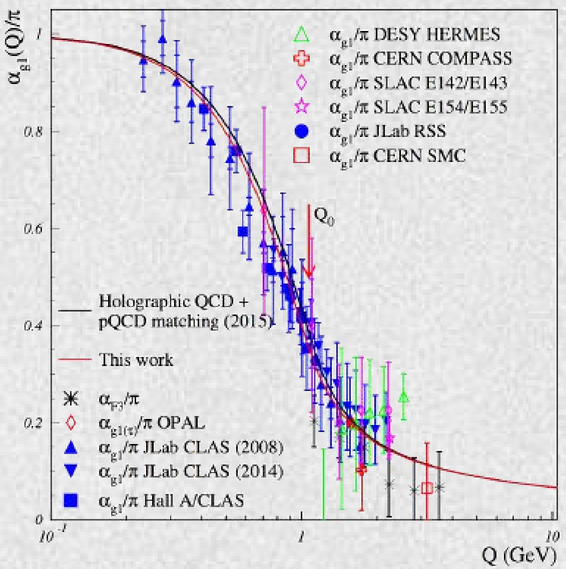

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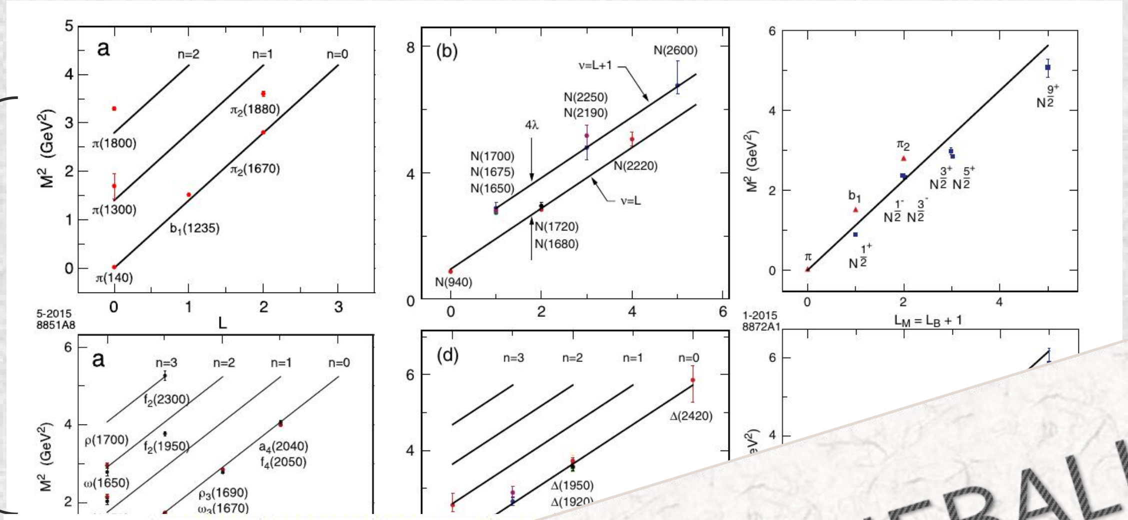

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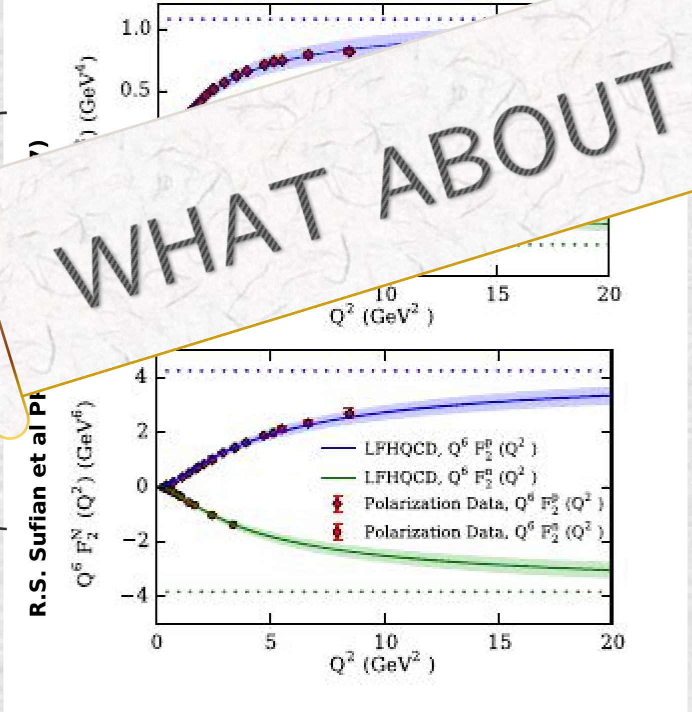

R.S. Sufian et al Ph

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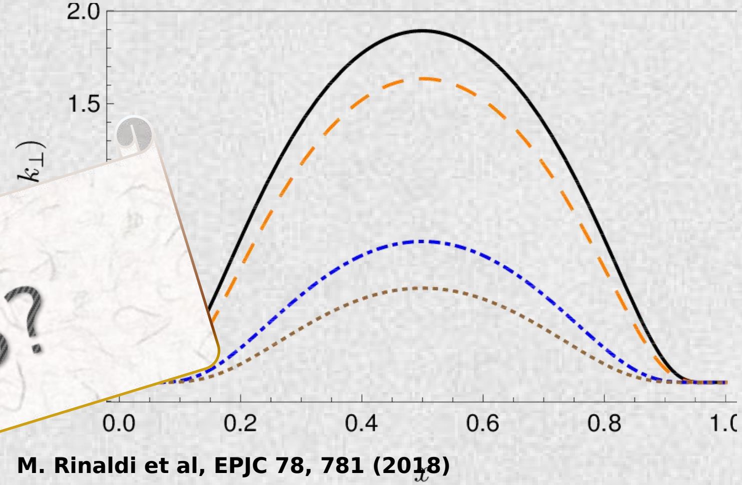

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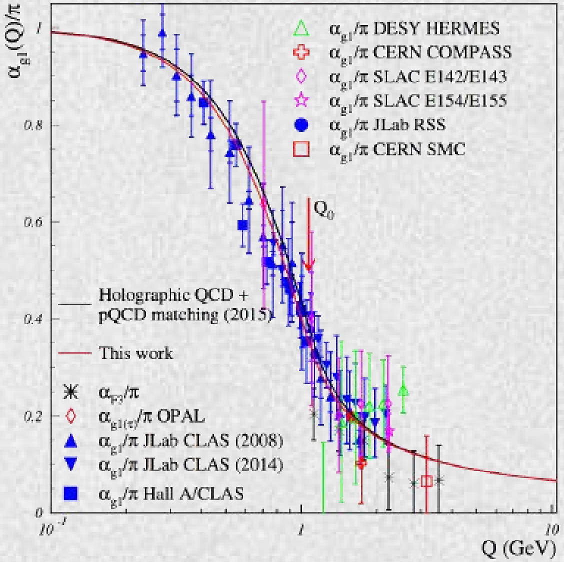


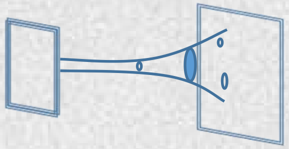
de Teramond et al, PRL 120,182001 (2018)

WHAT ABOUT GLUEBALLS?

3

Glueballs in AdS/QCD

3 Glueballs in AdS/QCD: Hard-Wall model



In this case we have the following $AdS_5 \times S_5$ metric :

$$ds^2 = g_{MN}dx^M dx^N + R^2 d\Omega_5 = \frac{R^2}{z^2}\left(dz^2 + \underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{\text{Minkowski space}}\right) + R^2 d\Omega_5$$

Holographic 5° dimension

Radius of the AdS space

In the hard-wall (HW) model confinement is implemented by imposing the following IR cutoff:

$$0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$$

WHAT ABOUT GLUEBALLS?

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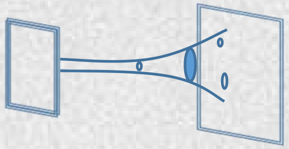

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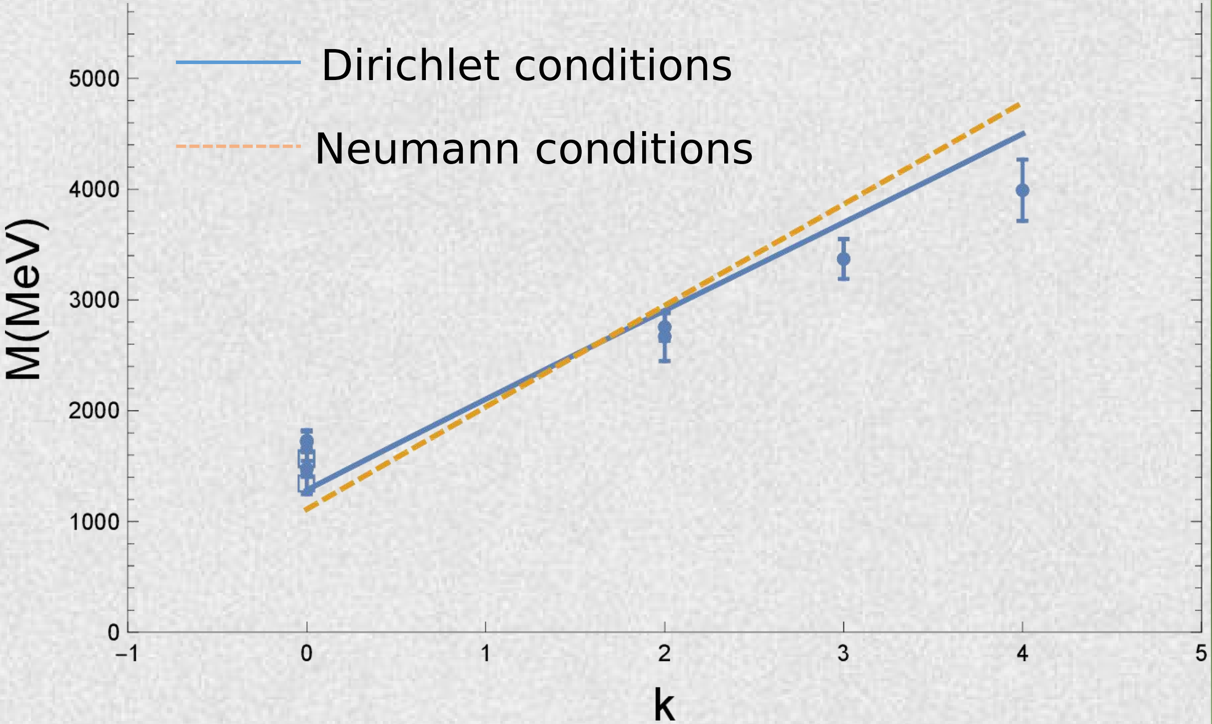
H. Boschi-Filho et al, PRD 73, 047901 (2006)

0++ GLUEBALL SPECTRUM

M.Rinaldi and V. Vento EPJA 54 (2018)



2++ GLUEBALL SPECTRUM

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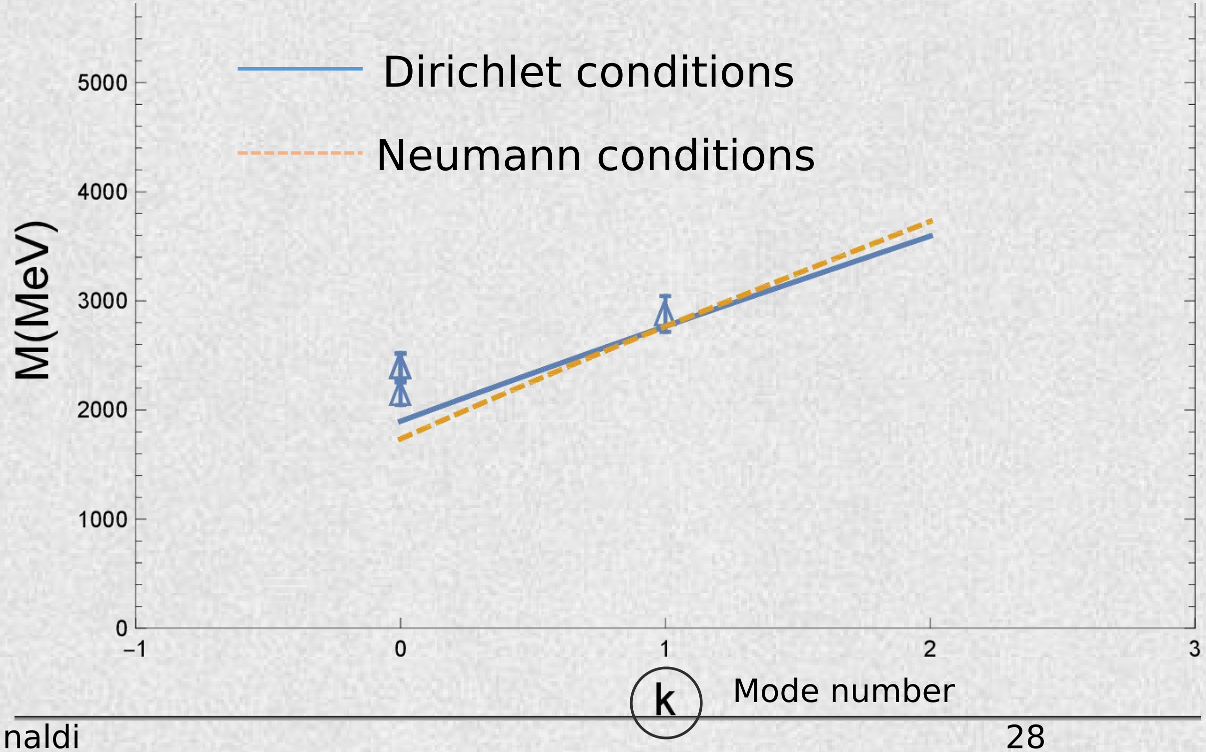

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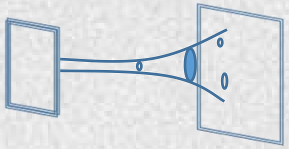

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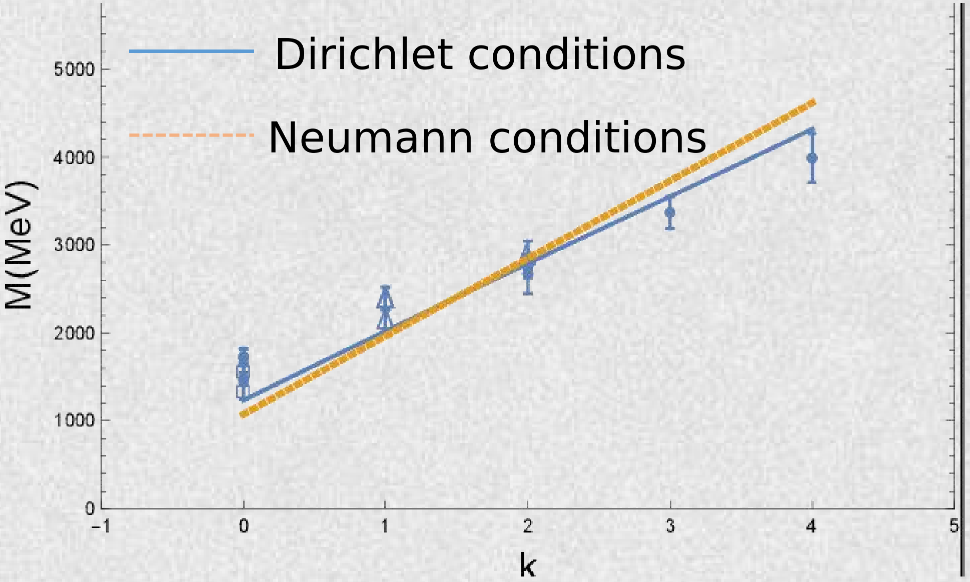
0^{++} $\otimes$ 2^{++} GLUEBALL SPECTRA

M.Rinaldi and V. Vento EPJA 54 (2018)



Good agreement!

However the HW model does not reproduce the meson spectrum.

$$M_n^2 \sim n^2$$

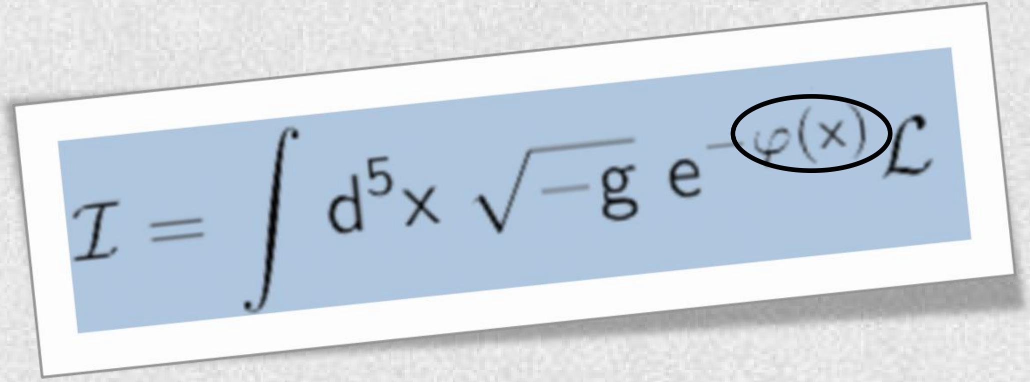
In order to have a unified view we need another model, i.e.: the Soft-wall model?

3 Glueballs in AdS/QCD: The Soft-Wall

karch et al, PRD 74, 015005 (2006)

In the original model we have: $g_{MN}dx^M dx^N = \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$

but a soft **cutoff** to space-time is obtained by adding a **dilaton** field in the action:



$$\mathcal{I} = \int d^5x \sqrt{-g}\, e^{-\varphi(x)} \mathcal{L}$$

Successful in describing the Regge behavior of the spectrum: $M^2_{n,J} \sim n + j, \qquad j \geq 0$

WHAT ABOUT GLUEBALLS?

3 Glueballs in AdS/QCD: Soft-Wall model

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We consider the profile function: $\varphi(z) = k^2 z^2$

SCALR FIELD EQUATION:

Equation of motion of the scalar glueball can be obtained:

$$I = \int d^5x \,\sqrt{g}\; e^{-\varphi(z)}\left[g^{MN}\partial_M\mathcal{G}\partial_N\mathcal{G} + M_5^2\mathcal{G}^2\right]$$

Dilaton field

$\Delta =$ conformal dimension

$$\Delta = 2 + \sqrt{4 + M_5^2 R^2}$$

The equation of motion for the scalar is:

$$-\Psi''(z) + \left[z^2 + \frac{15}{4z^2} + 2\right]\Psi(z) = M^2\Psi(z)$$

where:

$$\mathcal{G}(x,z) = e^{iP_\mu x^\mu}\left(\frac{z}{R}\right)^{3/2} e^{\kappa^2 z^2/2}\Psi(z), \qquad P^2 = -M^2$$

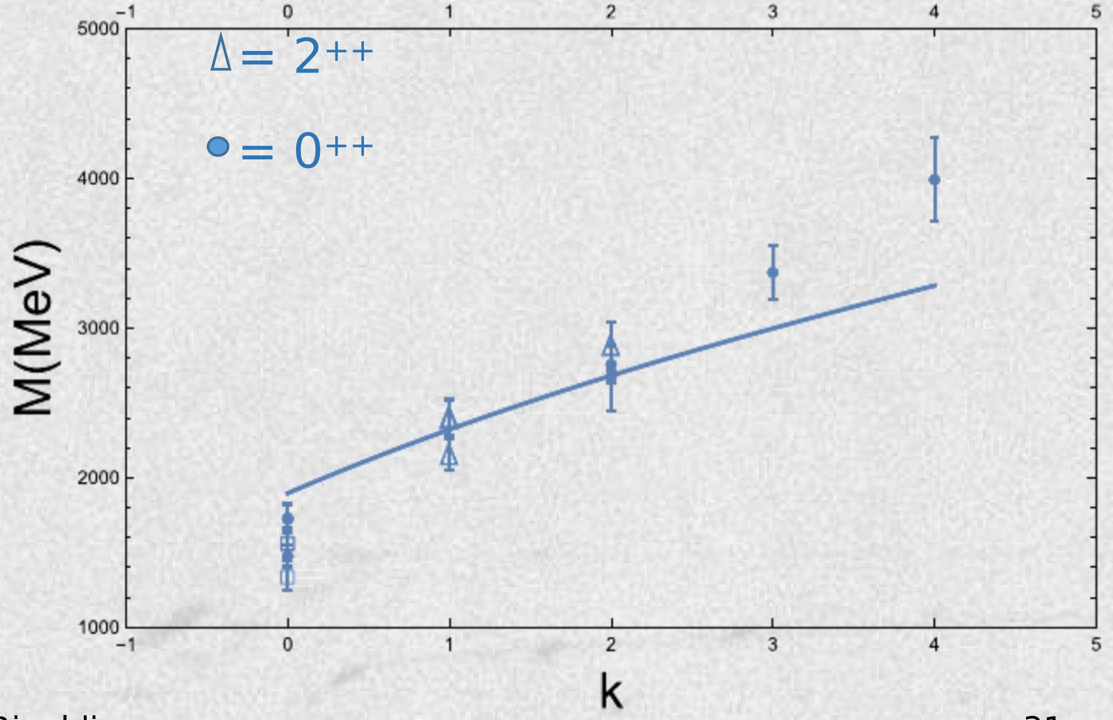
Eduardo Folco Capossoli et al, PLB 753 (2019) 419-423

SCALAR GLUEBALL SPECTRUM:

$$M_J^2 = 4k + 4 + 2\sqrt{4 + J(J+4)} = 4k + 8$$

$\rightarrow k = 0, 1....$ **scalar**

$\rightarrow k = 1, 2....$ **tensor**



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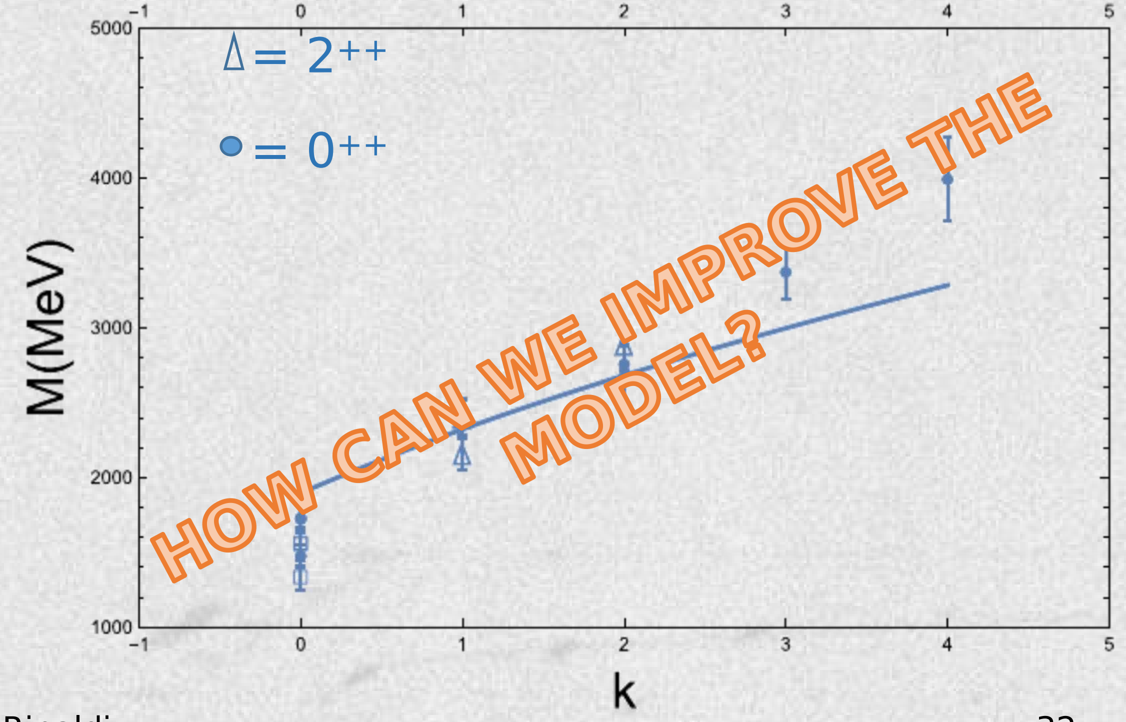
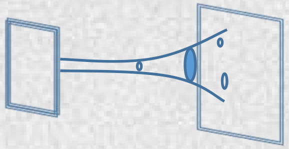


HOW CAN WE IMPROVE THE MODEL?

3 Glueballs in AdS/QCD: The GSW model



In **M.Rinaldi and V. Vento EPJA 54 (2018)** we propose to use a soft-wall graviton (GSW) model.

In this case a dilatonic cutoff is incorporated in the metric:

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

IR deformation → **QCD scale**

However, a dilatonic contribution in the action can still be kept:

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted
M.R. and V. Vento J. P. G 47 (20), 12, 125003

$$\tilde{\mathcal{I}} = \int d^5x \, \sqrt{-\tilde{g}} \, e^{-\beta\varphi(x)} \mathcal{L}$$

In order to preserve the good description of the hadronic spectrum we require:

$$\int d^5x \, \sqrt{-\tilde{g}} \, e^{-\beta\varphi(x)} \mathcal{L} \sim \int d^5x \, \sqrt{-g} \, e^{-\varphi(x)} \mathcal{L}$$

kinetic term for a scalar

$$\frac{3\alpha}{2} + \beta = 1$$

Modified Soft-Wall model in e.g.:

E. F. Capossoli et al, PLB 753, 419-423 (2006)
O. Andreev, PRD 100 (2019) 2, 026013
E. F. Capossoli et al, Chin. Phys. C 44 (2020) 6, 064194
W. de Paula et al, PRD 79, 075019 (2009)

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In **M.Rinaldi and V. Vento EPJA 54 (2018)** $\qquad \alpha k^2$ is the **unique parameter!**

GRAVITON EoM and SPECTRUM

EoM for metric perturbation is obtained from the Einstein's equation: $-\frac{1}{2}\tilde{h}^{;c}_{ab;c} - \frac{1}{2}\tilde{h}^{c}_{c;ab} + \frac{1}{2}\tilde{h}^{;c}_{ac;b} + \frac{1}{2}\tilde{h}^{;c}_{bc;a} + 4\tilde{h}_{ab} = 0$

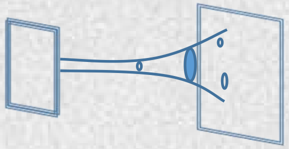
R.C. Brower et al, Nucl. Phys. B 587, 249 (2000)

By choosing the gauge:

$$\begin{cases} \tilde{h}_{tt} = (z^{-2} - z^2)\phi(z)e^{-Mx_3} & \text{Scalar component} \\ \tilde{h}_{ij} = q_{ij}T(z)e^{-Mx_3} & \text{Tensor component} \end{cases}$$

"Tensor" wave-function

3 Glueballs in AdS/QCD: The GSW model



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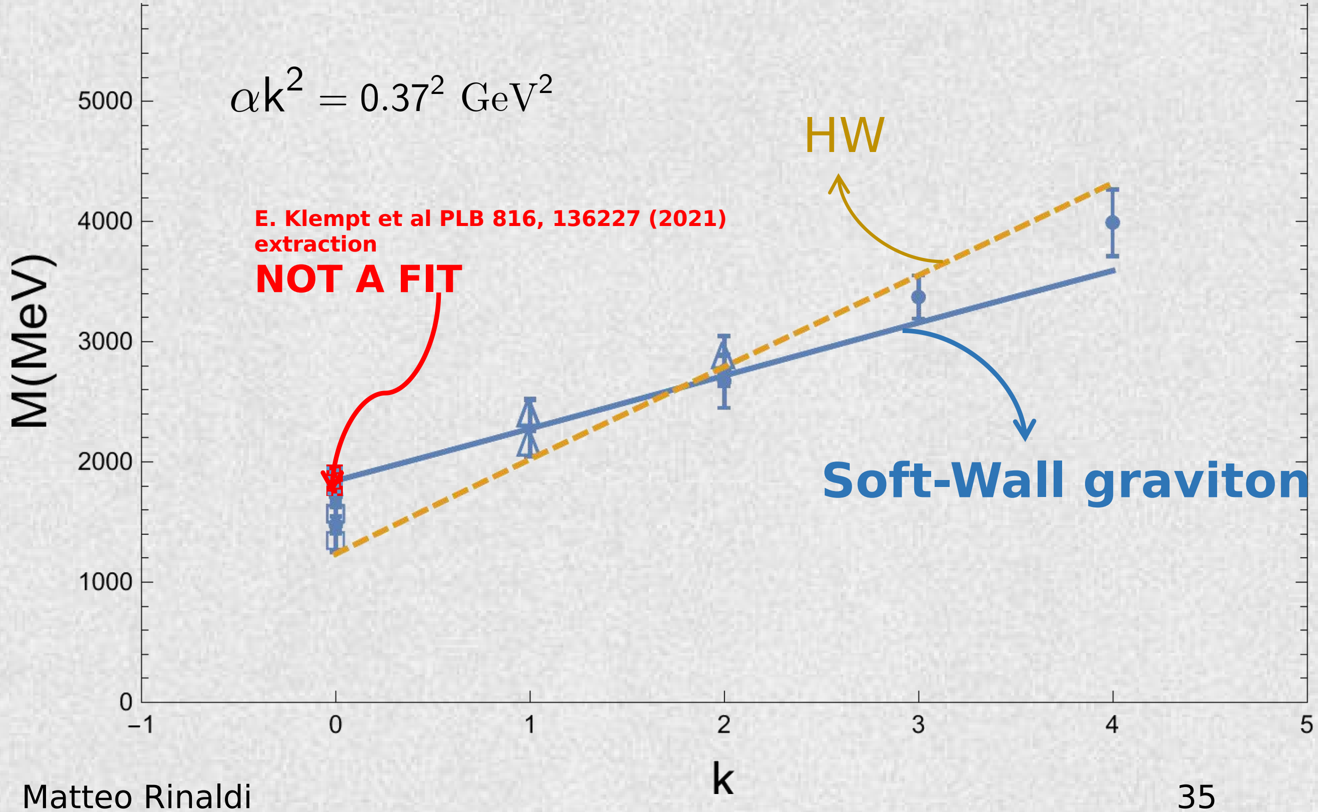
$$-\frac{1}{2}\tilde{h}^{;c}_{ab;c} - \frac{1}{2}\tilde{h}^{c}_{c;ab} + \frac{1}{2}\tilde{h}^{;c}_{ac;b} + \frac{1}{2}\tilde{h}^{;c}_{bc;a} + 4\tilde{h}_{ab} = 0$$

$$\Psi''(t) + V_G(t)\Psi(t) = \Lambda^2\Psi(t)$$

with:

$$\begin{cases} t = i\alpha z/\sqrt{2} \\ \Lambda^2 = \dfrac{M^2}{\alpha^2} \\ V_G(t) = \dfrac{e^{2t^2}}{t^2} - \dfrac{17}{4t^2} + 14 - 15t^2 \end{cases}$$



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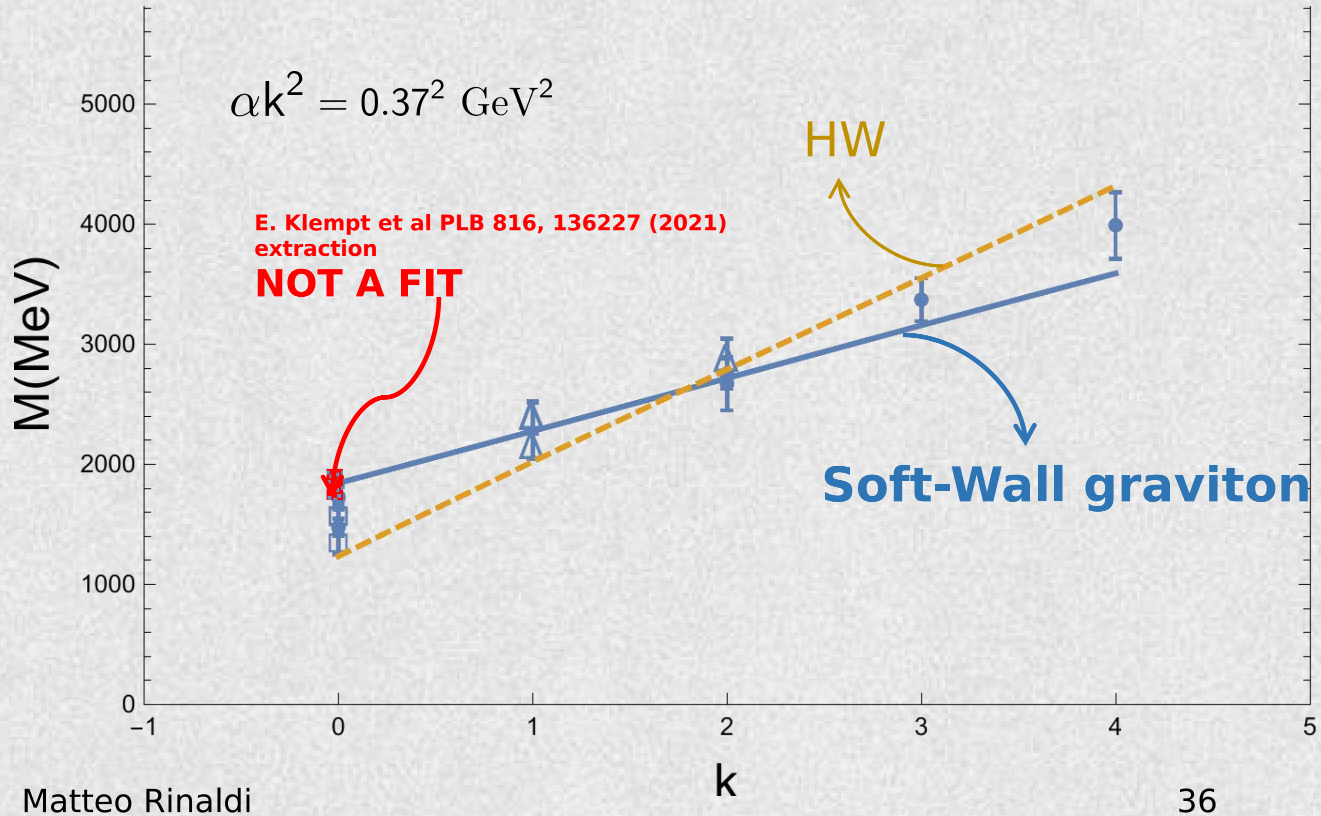
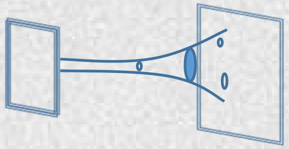
Also in this case we have a good description of data, but now (w.r.t. the HW model):

1) We have a model describing glueball and mesons spectra at the same time with (2 parameters)-LATER

2) we describe the extracted the low scalar mass



3 High Spin Glueballs in the GSW model



In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}_{\alpha k^2 = 0.37^2\ \mathrm{GeV}^2}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\qquad \varphi(z) = k^2 z^2$

Equation of motion of the scalar glueball can be obtained:

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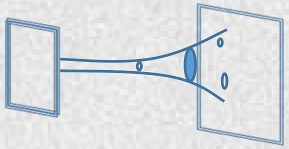

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Metric effects!!

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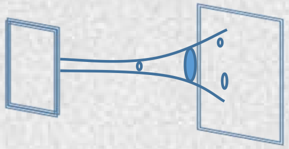

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EVEN SPIN:

$$M_5^2 R^2 = J(J+4) \text{ for even } J$$

ODD SPIN:

$$M_5^2 R^2 = (J+2)(J+6) \text{ for odd } J$$

E.F. Capossoli et al, PLB 753, 419 (2016)

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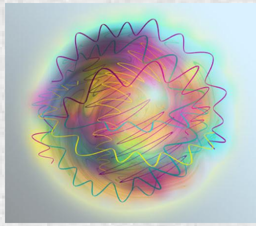

SINCE THE CONFORMAL IS POSITIVE, THE POTENTIAL **IS BINDING**

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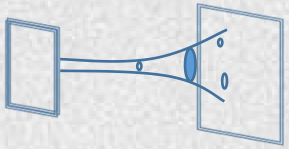

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M.R. and V. Vento, arXiv: 2101,02616, PRD accepted

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$M_5^2R^2 = J(J+4)$ for even J

J^{PC}	M&P	Ky	My	Gy	Sk	Mtb	This work
2^{++}	2400 ± 145	2390 ± 150	2150 ± 130	2620 ± 50	2420	2590	2695 ± 21
4^{++}			3640 ± 150		3990	3770	3920 ± 14
6^{++}			4360 ± 460			4600	5141 ± 12

ODD SPIN:

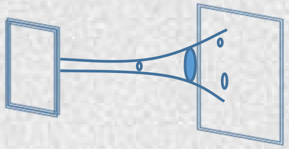
$M_5^2R^2 = (J+2)(J+6)$ for odd J

J^{PC}	M&P	Ky	My	Ll	Mta	Sz	This work
1^{--}	3850 ± 140	3830 ± 130	3240 ± 480	3950	3990	3001	3308 ± 15
3^{--}	4130 ± 290	4200 ± 245	4330 ± 460	4150	4160	4416	4451 ± 12
5^{--}				5050	5260	5498	5752 ± 10
7^{--}				5900			$6972 \pm\ 8$

Regge trajectory: $J \sim (0.21 \pm 0.01)M^2 + 0.58 \pm 0.34$ $J \sim 0.18 \pm 0.01M^2 - 0.75 \pm 0.28$

Lattice slope: 0.25 0.18

3 Mesons in the GSW model



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SCLARS: f$_0$ family

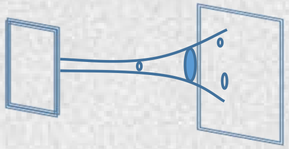
$$M_5^2R^2 = -3$$

PSEUDO-SCALARS: η family

$$M_5^2R^2 = -4$$

A.Vega et al, Chin. J. Phys. 66, 715 (2020)

3 Mesons in the GSW model



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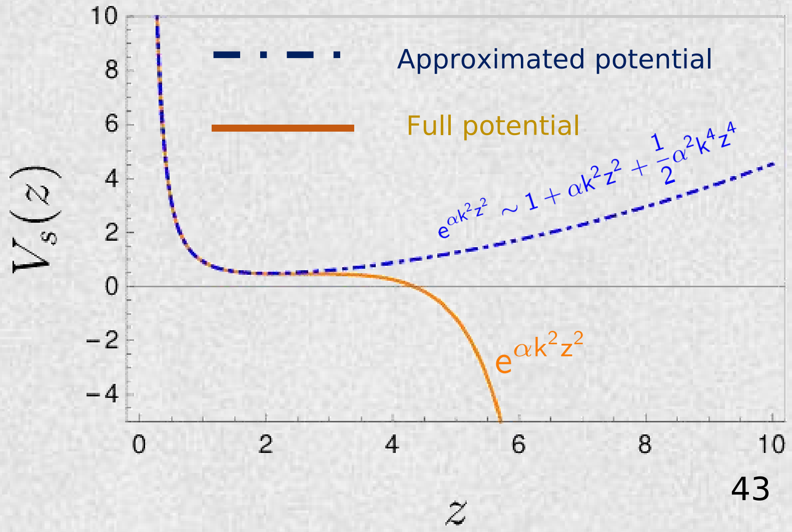
NEGATIVE CONFORMAL MASSES!

POTENTIAL NOT BINDING!

The equation of motion for the scalar is:

$$-\psi''(z) + \left[k^4 z^2 - 2k^2 + \frac{15}{4z^2} + \frac{M_5^2 R^2}{z^2} e^{\alpha k^2 z^2} \right] \psi(z) = M^2 \psi(z)$$



SCLARS: f_0 family

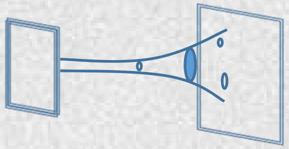
$$M_5^2 R^2 = -3$$

PSEUDO-SCALARS: η family

$$M_5^2 R^2 = -4$$

A.Vega et al, Chin. J. Phys. 66, 715 (2020)

3 Mesons in the GSW model



In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = \underbrace{e^{-\alpha\varphi(z)}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\varphi(z) = k^2 z^2$

$\alpha k^2 = 0.37^2 \ \mathrm{GeV}^2 \ \ 0.51 \leq \alpha \leq 0.59$

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted

Equation of motion of the scalar glueball can be obtained:

$$\tilde{I} = \int d^5x \sqrt{g}\ e^{-\varphi(z)} \left[g^{MN}\partial_M \mathcal{S} \partial_N \mathcal{S} + e^{\alpha\varphi(z)} M_5^2 \mathcal{S}^2\right]$$

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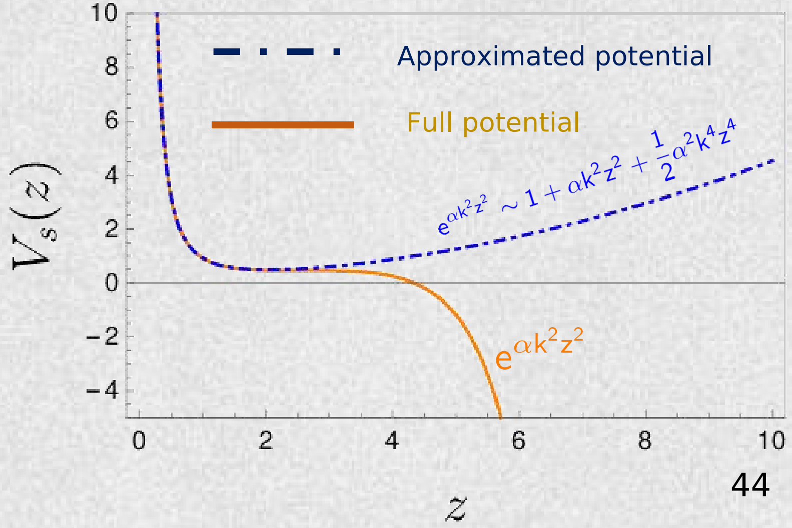
NEGATIVE CONFORMAL MASSES!

$M_5^2 R^2 = -3$ f$_0$

$\downarrow$

$M_5^2 R^2 = -4$ η

POTENTIAL NOT BINDING!

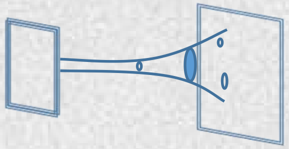


$$e^{\alpha k^2 z^2} \sim 1 + \alpha k^2 z^2 + \frac{1}{2}\alpha^2 k^4 z^4$$

Appealing approximation:

1) leads to a binding potential
2) contains gluo dynamics described through the the metric deformation

3 Mesons in the GSW model



In this case we have the following $AdS_5 \times S_5$ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\qquad \varphi(z) = k^2z^2$

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M.R. and V. Vento, arXiv: 2101,02616, PRD accepted

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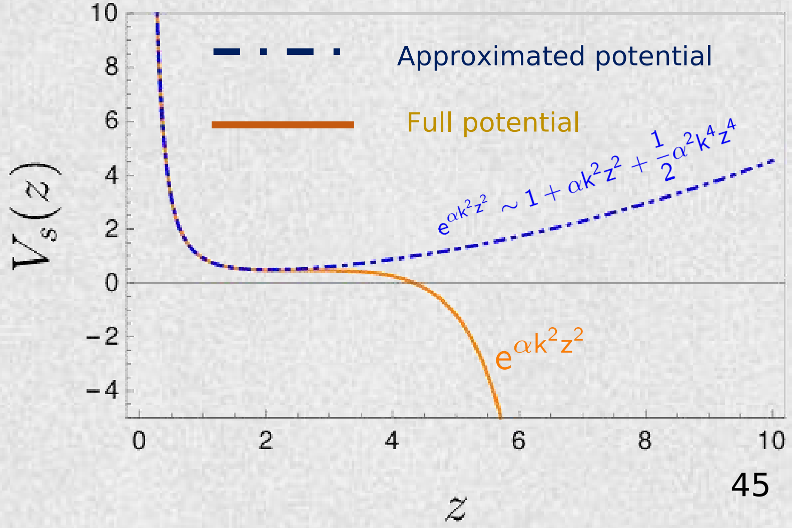
$\downarrow$

$M_5^2R^2 = -4$ η

POTENTIAL NOT BINDING!



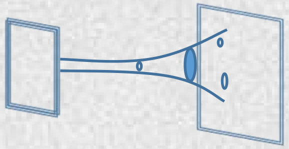
$$e^{\alpha k^2 z^2} \sim 1 + \alpha k^2 z^2 + \frac{1}{2}\alpha^2 k^4 z^4$$

HOW CAN WE MOTIVATE IT?

Appealing approximation:

1) leads to a binding potential
2) contains gluo dynamics described through the the metric deformation

3 Mesons in the GSW model



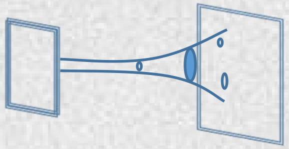
In this case we have the following AdS_5 x S_5 metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}_{\alpha k^2 = 0.37^2 \ \mathrm{GeV}^2 \ \ 0.51 \leq \alpha \leq 0.59}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\quad \varphi(z) = k^2 z^2$

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted

We need a correction to the dilaton profile function:

$$\tilde{I} = \int d^5x \ \sqrt{g} \ e^{-\varphi(z) - \varphi_n(z)} \left[g^{MN}\partial_M \mathcal{S} \partial_N \mathcal{S} + e^{\alpha\varphi(z)} M_5^2 \mathcal{S}^2 \right]$$

3 Mesons in the GSW model



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The potential in the EoM:

$$V_s(z) = \frac{15}{4z^2} + M_5^2 R^2 \frac{e^{\alpha k^2 z^2}}{z^2} + 2k^2 + k^4 z^2 + \varphi_n'(z)\left(\frac{3}{2z} + k^2 z\right) + \frac{\varphi_n'(z)^2}{4} - \frac{\varphi_n''(z)}{2}$$

The approximated potential (expanding the exponential) is:

$$V_s^A(z) = \frac{15}{4z^2} + \frac{M_5^2 R^2}{z^2}\left[1 + \alpha k^2 z^2 + \frac{1}{2}\alpha^2 k^4 z^4\right] + 2k^2 + k^4$$

3 Mesons in the GSW model

In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\varphi(z) = k^2z^2$

$$\alpha k^2 = 0.37^2 \ \mathrm{GeV}^2 \quad 0.51 \leq \alpha \leq 0.59$$

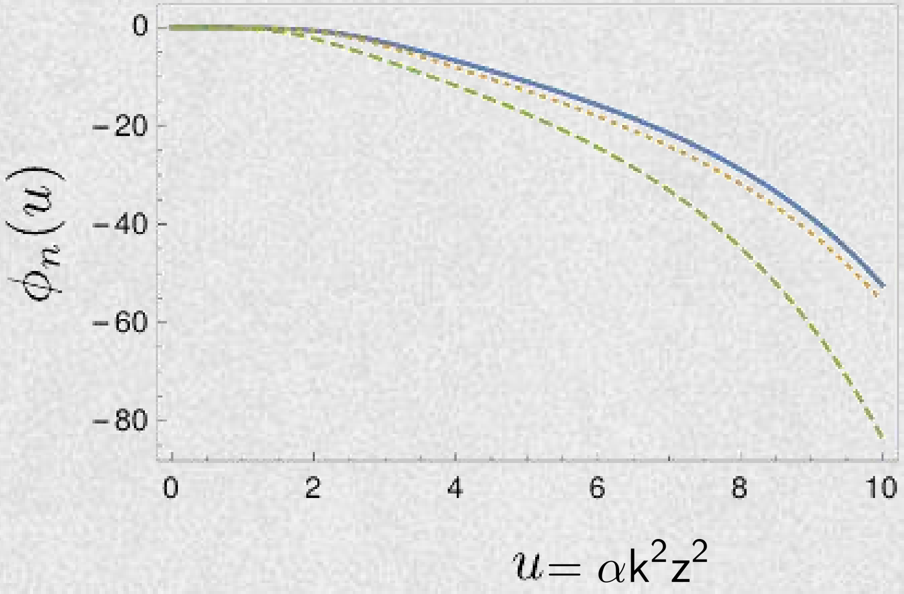
M.R. and V. Vento, arXiv: 2101,02616, PRD accepted

An equation for the correction can be found:

$$V_s(z) - V_s^A(z) = -\frac{\varphi_n''(z)}{2} + \varphi_n'(z)\left(\frac{3}{2z} + k^2z\right) + \frac{\varphi_n'(z)^2}{4} + \frac{M_5^2R^2}{z^2}\left[e^{\alpha k^2z^2} - 1 - \alpha k^2z^2 - \frac{1}{2}\alpha^2k^4z^4\right] = 0$$

We are able to find a correction for the dilaton,
e.g., the scalars (f_0), the pseudo-scalars (η, π).

There is a dependence on: the kind of field (scalar, vector...) and on M_5 .
HOWEVER THERE ARE NO FREE PARAMETERS!



PHENOMENOLOGICAL RESULTS

3 Phenomenology: SCALARS (light & heavy)

In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = \underbrace{e^{-\alpha\varphi(z)}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$

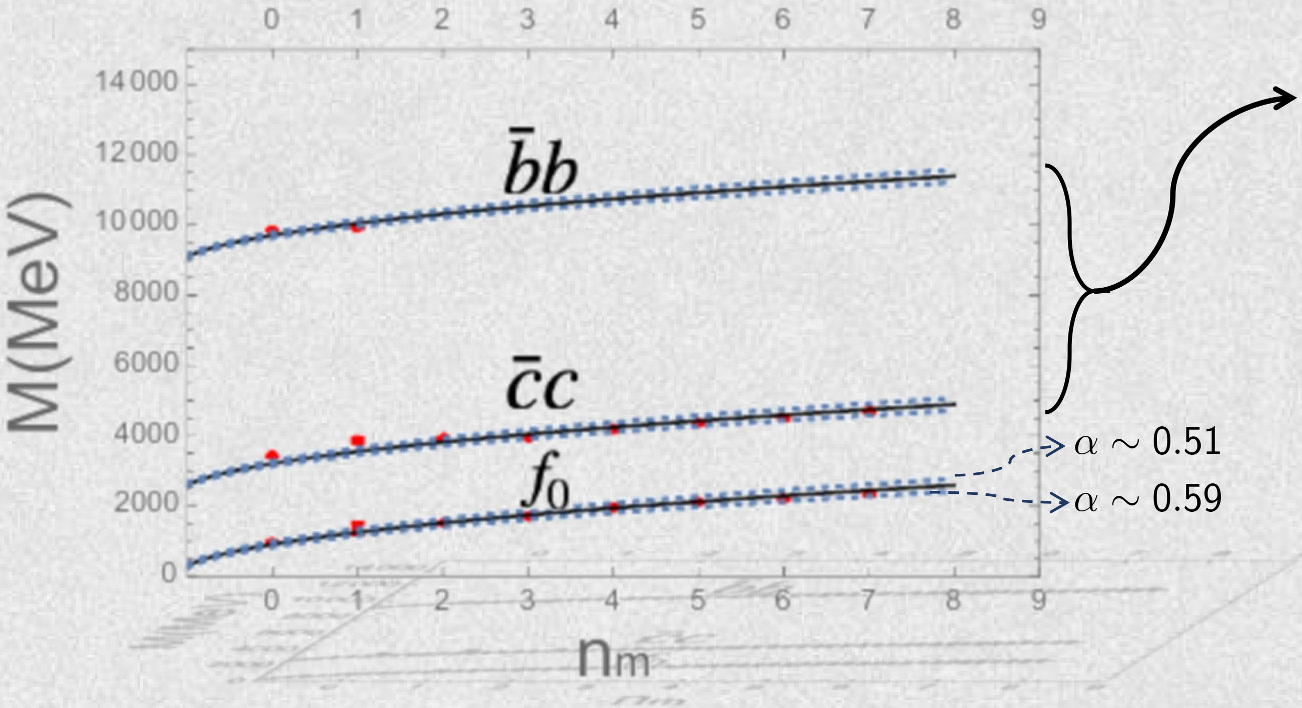
$$\varphi(z) = k^2 z^2$$

$$\tilde{I} = \int d^5x\,\sqrt{g}\,e^{-\varphi(z)-\varphi_n(z)}\left[g^{MN}\partial_M\mathcal{S}\partial_N\mathcal{S} + e^{\alpha\varphi(z)}M_5^2\mathcal{S}^2\right]$$

$$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2 \quad 0.51 \leq \alpha \leq 0.59$$

$$M_5^2R^2 = -3$$

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted
M.R. and V. Vento J. P. G 47 (20), 12, 125003

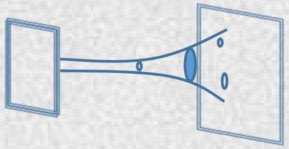


In order to describe heavy scalar mesons. we considered the following approach:

S. S. Afonin et al, Phys. Lett. B726, 283 (2013)
A. Vega et al, Phys. Rev. D82, 074022 (2010)
Y. Kim, J.-P. Lee et al, Phys. Rev. D75, 114008 (2007)

$$\begin{tikzcd} & C_{b\bar{b}} \sim 2m_b \\ M_{q\bar{q}} \sim M_{f_0} + C_{q\bar{q}} \arrow[ur] \arrow[dr] & \\ & C_{c\bar{c}} \sim 2m_c \end{tikzcd}$$

3 Phenomenology: PSEUDO-SCALARS η



In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}}\frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\varphi(z) = k^2z^2$

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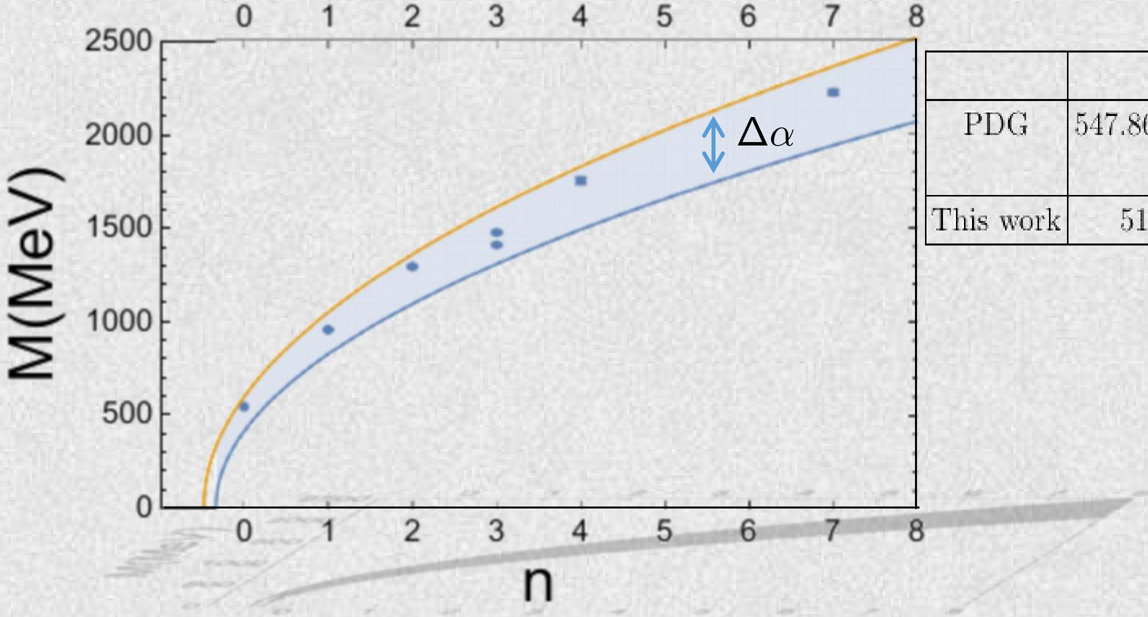
$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$ $0.51 \le \alpha \le 0.59$

$$M_5^2R^2 = -4$$

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted



	η	η'	$\eta(1295)$	$\eta(1405)$ - $\eta(1475)$	$\eta(1760)$	$\eta(????)$	$\eta(????)$	$\eta(2225)$
PDG	547.862 ± 0.017	957.78 ± 0.06	1295 ± 4	1408.8 ± 2.0 1475 ± 4	1751 ± 15			2221 ± 12
This work	513 ± 92	943 ± 111	1231 ± 133	1463 ± 151	1663 ± 168	1842 ± 183	2005 ± 198	2155 ± 210

The GSW model predicts this 2 new states

3 Phenomenology: VECTOR ρ

In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$

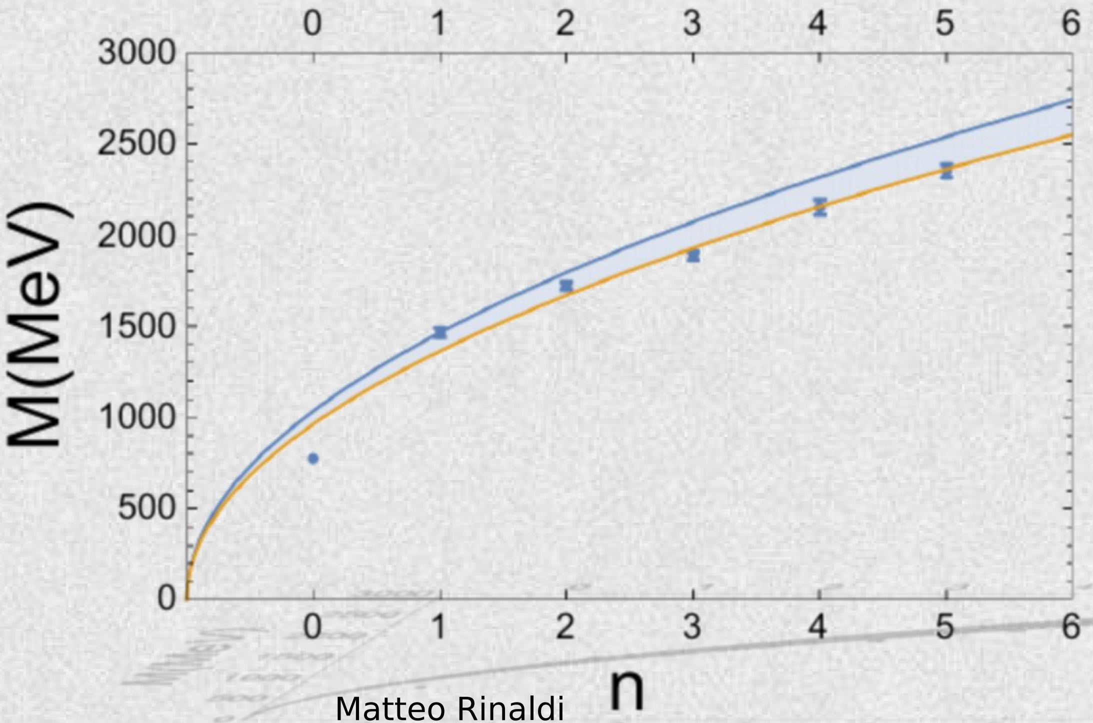
$\varphi(z) = k^2 z^2$

$$\bar{S} = -\frac{1}{2}\int d^5x\, \sqrt{-g}\, e^{-k^2z^2}\left[\frac{1}{2}g^{MP}g^{QN}F_{MN}F_{PQ}\right]$$

$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$ $\quad 0.51 \leq \alpha \leq 0.59$

$$M_5^2 = 0 \Longrightarrow \varphi_n(z) = 0$$

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted



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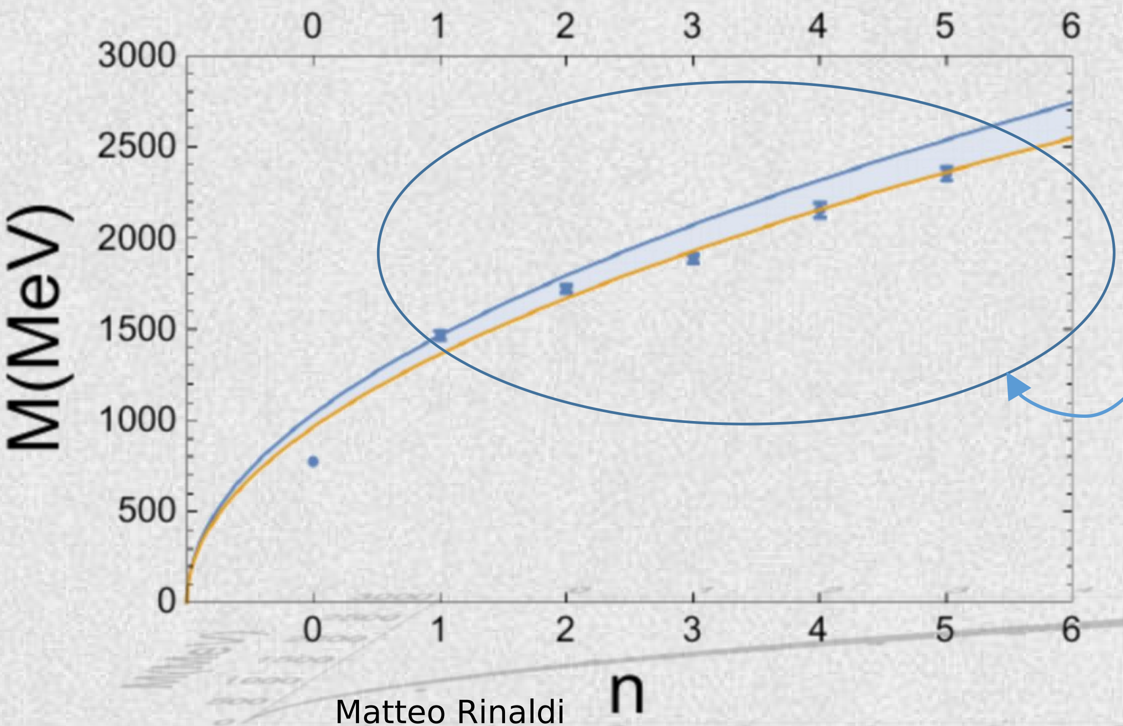
M.R. and V. Vento, arXiv: 2101,02616, PRD accepted



Good agreement

3 Phenomenology: VECTOR ρ

In this case we have the following AdS$_5$ x S$_5$ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$

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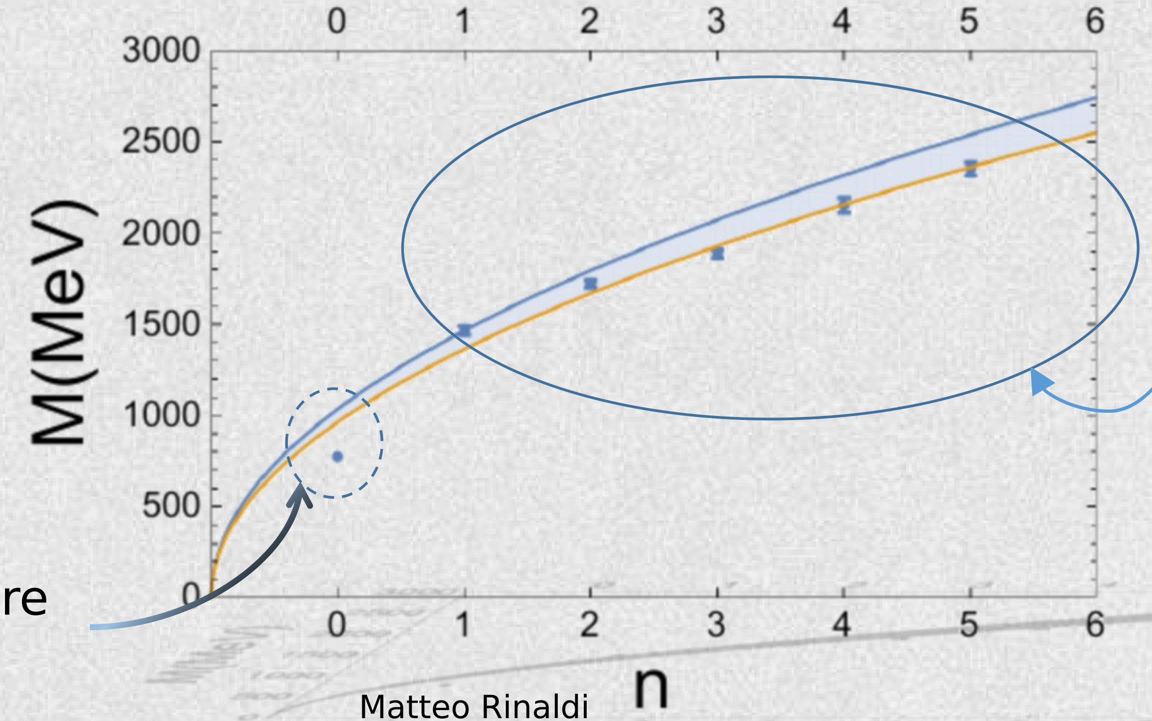


Good agreement

Improvements are required!

3 Phenomenology: VECTOR (axial) a_1

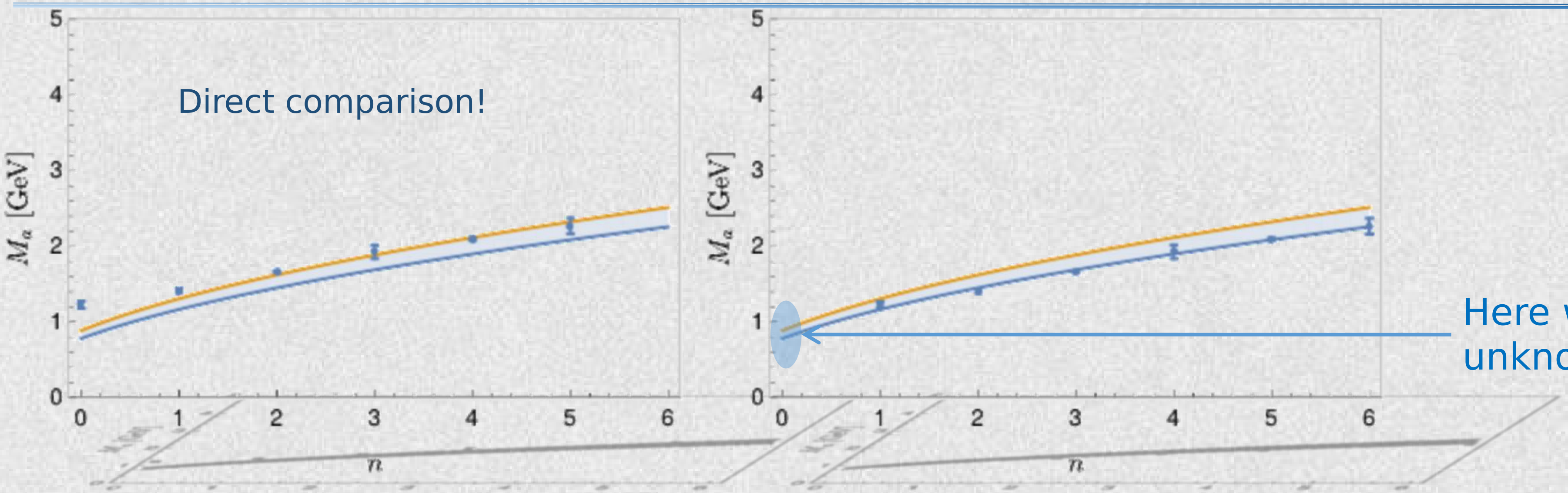
In this case we have the following AdS₅ x S₅ metric: $\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}}\frac{R^2}{z^2}\left(dz^2+\eta_{\mu\nu}dx^\mu dx^\nu\right)$ $\qquad \varphi(z)=k^2z^2$

$$\bar{S} = -\frac{1}{2}\int d^5x\sqrt{-g}e^{-k^2z^2-\varphi_n}\left[\frac{1}{2}g^{MP}g^{QN}F_{MN}F^{PQ} + M_5^2R^2g^{PM}A_PA_Me^{\alpha k^2z^2}\right]$$

$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2 \quad 0.51 \leq \alpha \leq 0.59$

$M_5^2 = -1$

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted

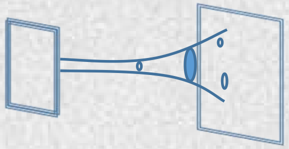


GOOD AGREEMENT!

Here we guess the existence of an unknown lightest grand state...

	$a_1(1260)$	$a_1(1420)$	$a_1(1640)$	$a_1(1930)$	$a_1(2095)$	$a_1(2270)$
PDG & Av	1230 ± 40	1411^{+15}_{-13}	1655 ± 16	1930^{+19}_{-70}	2096^{+17}_{-121}	2270^{+55}_{-40}
This work	833 ± 53	1235 ± 72	1535 ± 87	1785 ± 100	2005 ± 111	2202 ± 122

3 Phenomenology: PSEUDO-SCALARS π



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$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$ $0.51 \leq \alpha \leq 0.59$

$$M_5^2R^2 = -4$$

Must be corrected in order to encode chiral-symmetry breaking, for details:

M.R. and V. Vento, arXiv: 2101,02616, PRD accepted **T. Gherghetta et al, Phys. Rev. D79, 076003 (2009)**

VERY GOOD AGREEMENT

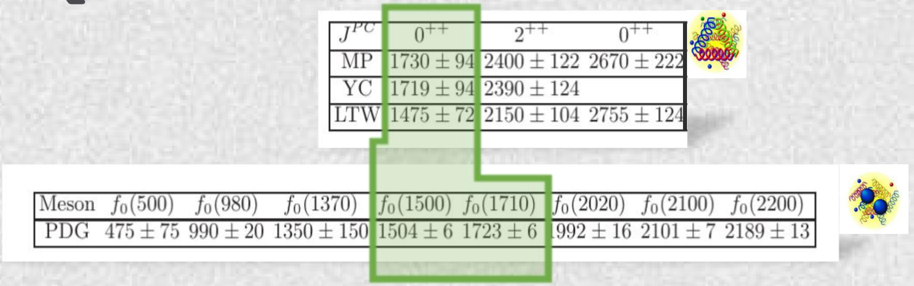
	π^0		$\pi(1300)$			$\pi(1800)$
PDG	134.9768 ± 0.0005		1300 ± 100			1819 ± 10
This work	135	943 ± 111	1231 ± 133	1463 ± 151	1663 ± 168	1842 ± 183

INTERMEDIATE STATES PREDICTED

4

The mixing problem in AdS/QCD

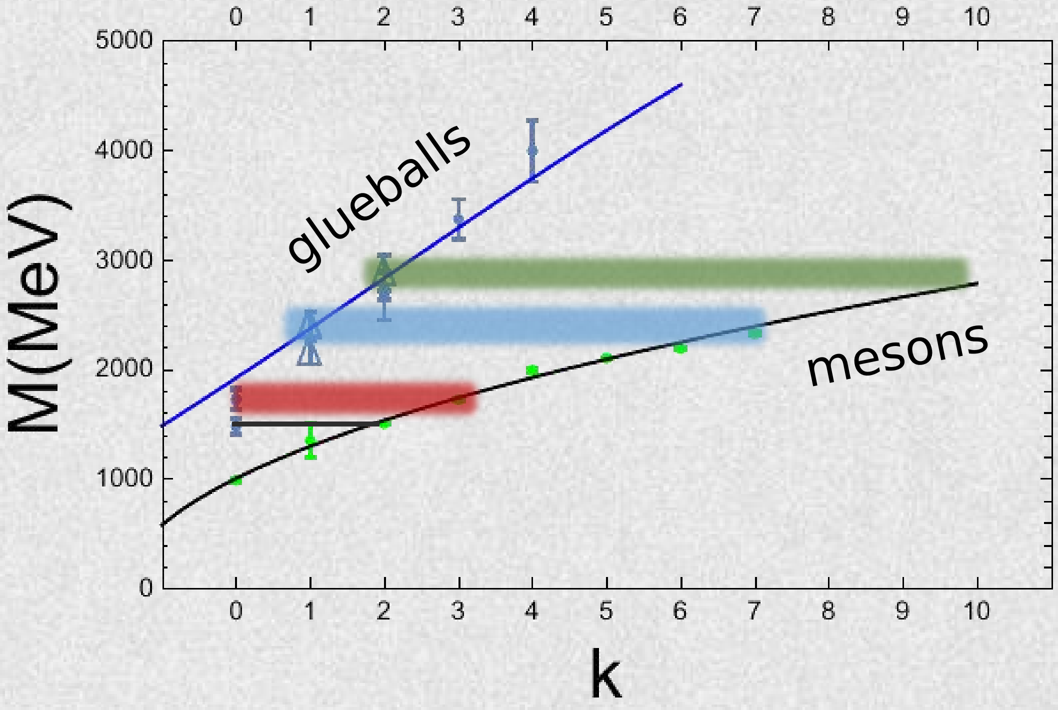
4 The mixing problem in AdS/QCD



J^{PC}	0^{++}	2^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222
YC	1719 ± 94	2390 ± 124	
LTW	1475 ± 72	2150 ± 104	2755 ± 124

Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	992 ± 16	2101 ± 7	2189 ± 13

In terms of modes numbers:



Glueball masses for: $\mathrm{k} = 0, 1, 2...$
are similar to
meson masses for: $\mathrm{k} = 4, 7, 10...$

4 The mixing problem in AdS/QCD

We consider the Light-Front formulation of the EoM in therms of the Hamiltonian. Within this framework the latter would be defined by the AdS/QCD model.

M.R. and V. Vento J. P. G 47 (20), 5, 055104

$$H_{LC}|\Psi_k\rangle = M^2|\Psi_k\rangle$$

We consider its representation in a 2-D meson-glueball subspace: $\{\ |\Psi^m\rangle, |\Psi^g\rangle\ \}$

$$[H] = \begin{pmatrix} m_m & \alpha \\ \alpha & m_g \end{pmatrix}$$

$$m_g = \langle\Psi^g|H|\Psi^g\rangle$$

$$m_m = \langle\Psi^m|H|\Psi^m\rangle$$

$$\alpha = \langle\Psi^m|H|\Psi^g\rangle \propto \langle\Psi^m|\Psi^g\rangle$$

OVERLAP

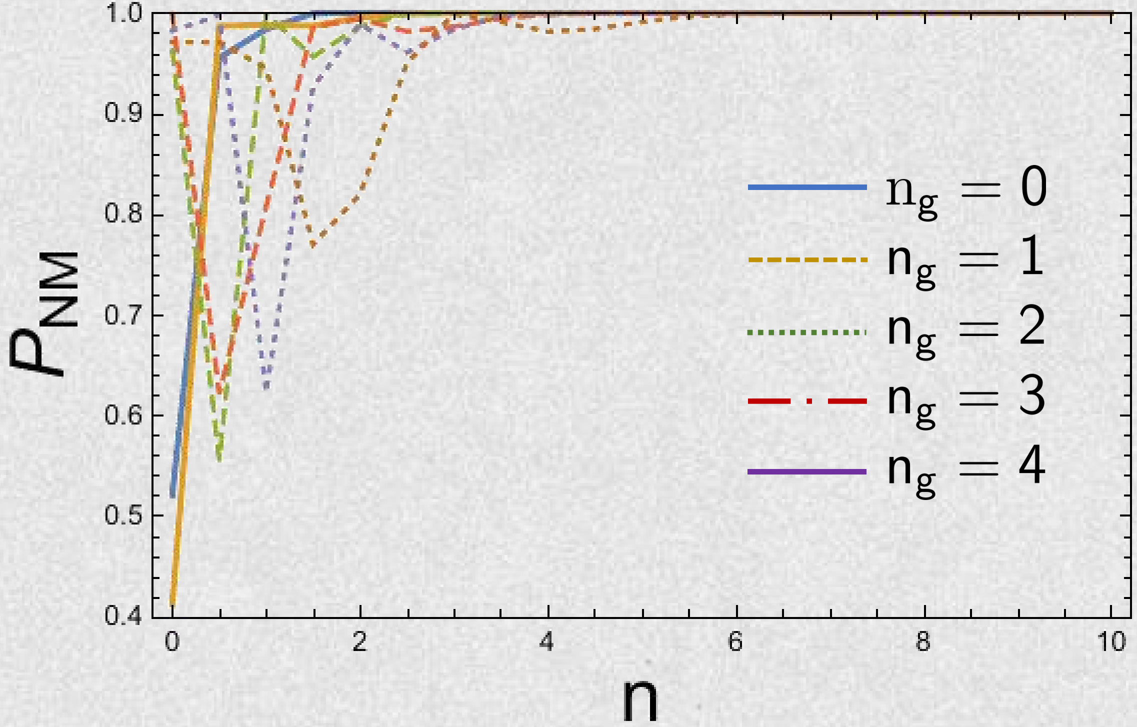
Mixing parameter!

4

The mixing problem in AdS/QCD

We define the probability for NO MIXING as: $P_{mg} \equiv 1 - |\langle \Psi^g | \Psi^m \rangle|^2$

M.R. and V. Vento J. P. G 47 (20), 5, 055104
M.R. and V. Vento J. P. G 47 (20), 12, 125003



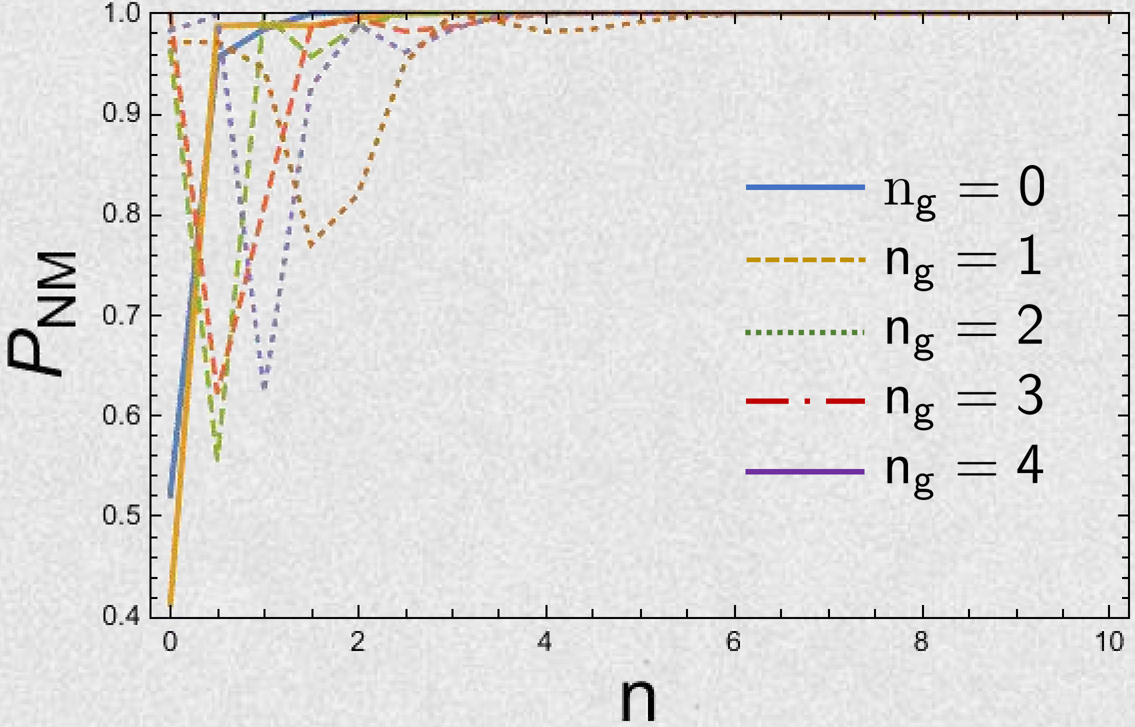
Meson mode number

For heavy glueballs (e.g. n_g =2,3...) which would have similar mass of meson (e.g. n_m =10,13...) the probability of mixing is **small**!!

4 The mixing problem in AdS/QCD

We define the probability for NO MIXING as: $P_{mg} \equiv 1 - |\langle \Psi^{g} | \Psi^{m} \rangle|^2$

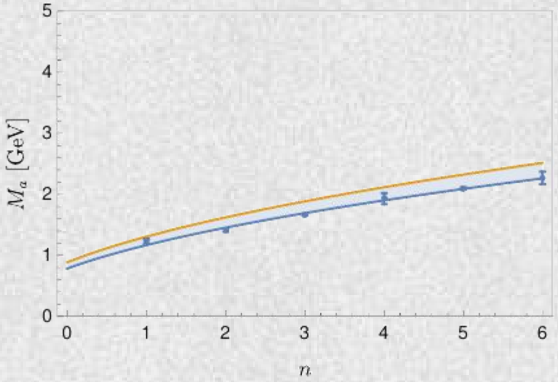
M.R. and V. Vento J. P. G 47 (20), 5, 055104
M.R. and V. Vento J. P. G 47 (20), 12, 125003



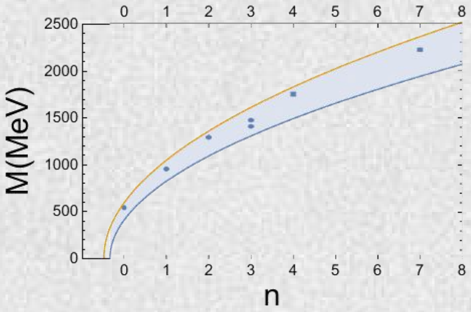
Meson mode number

Within the GSW AdS/QCD models (standard and with graviton) **pure glueballs in the scalar sector exist in the mass range above 2 GeV**!

CONCLUSIONS

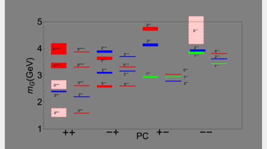


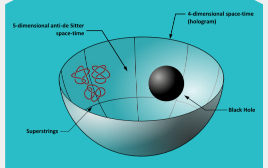
J^{PC}	M&P	Ky	My	Ll	Mta	Sz	This work
1^{--}	3850 ± 140	3830 ± 130	3240 ± 480	3950	3990	3001	3308 ± 15
3^{--}	4130 ± 290	4200 ± 245	4330 ± 460	4150	4160	4416	4451 ± 12
5^{--}				5050	5260	5498	5752 ± 10
7^{--}				5900			6972 ± 8



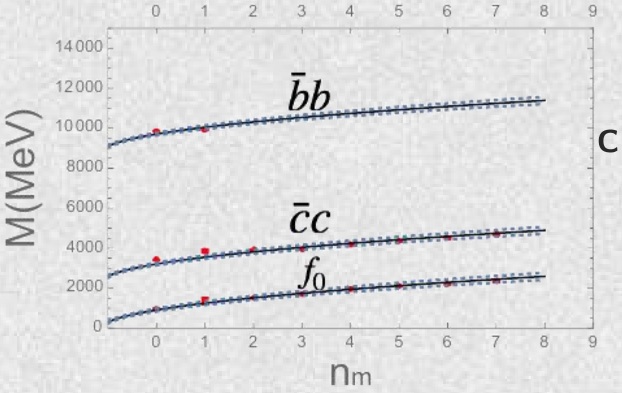
J^{PC}	M&P	Ky	My	Gy	Sk	Mtb	This work
2^{++}	2400 ± 145	2390 ± 150	2150 ± 130	2620 ± 50	2420	2590	2695 ± 21
4^{++}			3640 ± 150		3990	3770	3920 ± 14
6^{++}			4360 ± 460			4600	5141 ± 12

WE CONSIDER THE GLUEBALL & MESON SPECTRA

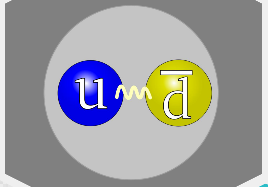


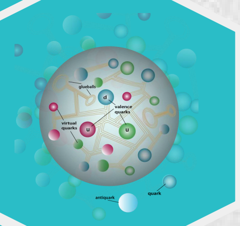


WE DEVELOPEPD THE GSW AdS/QCD MODELS



WE INCLUDED CHIRAL-SYMMETRY BREAKING: π





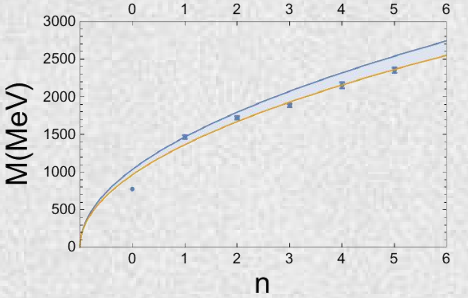
WE DESCRIBED QUITE WELL GLUBALL & MESON SPECTRA WITH

2 PARAMETERS

	π^0		$\pi(1300)$			$\pi(1800)$
PDG	134.9768 ± 0.0005		1300 ± 100			1819 ± 10
This work	135	943 ± 111	1231 ± 133	1463 ± 151	1663 ± 168	1842 ± 183

$\langle \Psi^m | \Psi^g \rangle$

WE FOUND THAT PURE SCALAR GLUEBALLS COULD BE FOUND ABOVE 2 GeV



4 The mixing problem in AdS/QCD

We consider the Light-Front formulation of the EoM in therms of the Hamiltonian. Within this framework the latter would be defined by the AdS/QCD model.

M.R. and V. Vento J. P. G 47 (20), 5, 055104

$$H_{LC}|\Psi_k\rangle = M^2|\Psi_k\rangle$$

We consider its representation in a 2-D meson-glueball subspace: $\{\ |\Psi^m\rangle, |\Psi^g\rangle\ \}$

$$[H] = \begin{pmatrix} m_m & \alpha \\ \alpha & m_g \end{pmatrix}$$

$$m_g = \langle\Psi^g|H|\Psi^g\rangle \qquad m_m = \langle\Psi^m|H|\Psi^m\rangle$$

$$\alpha = \langle\Psi^m|H|\Psi^g\rangle$$

Mixing parameter!

3 Glueballs in AdS/QCD: The GSW model

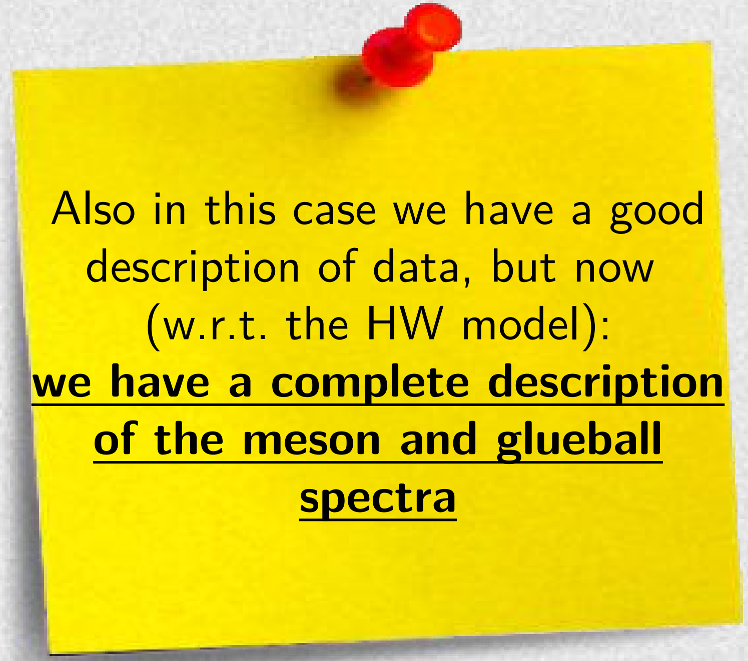
In this case we have the following AdS_5 metric :

$$\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}}\frac{R^2}{z^2}\left(dz^2+\eta_{\mu\nu}dx^\mu dx^\nu\right)$$

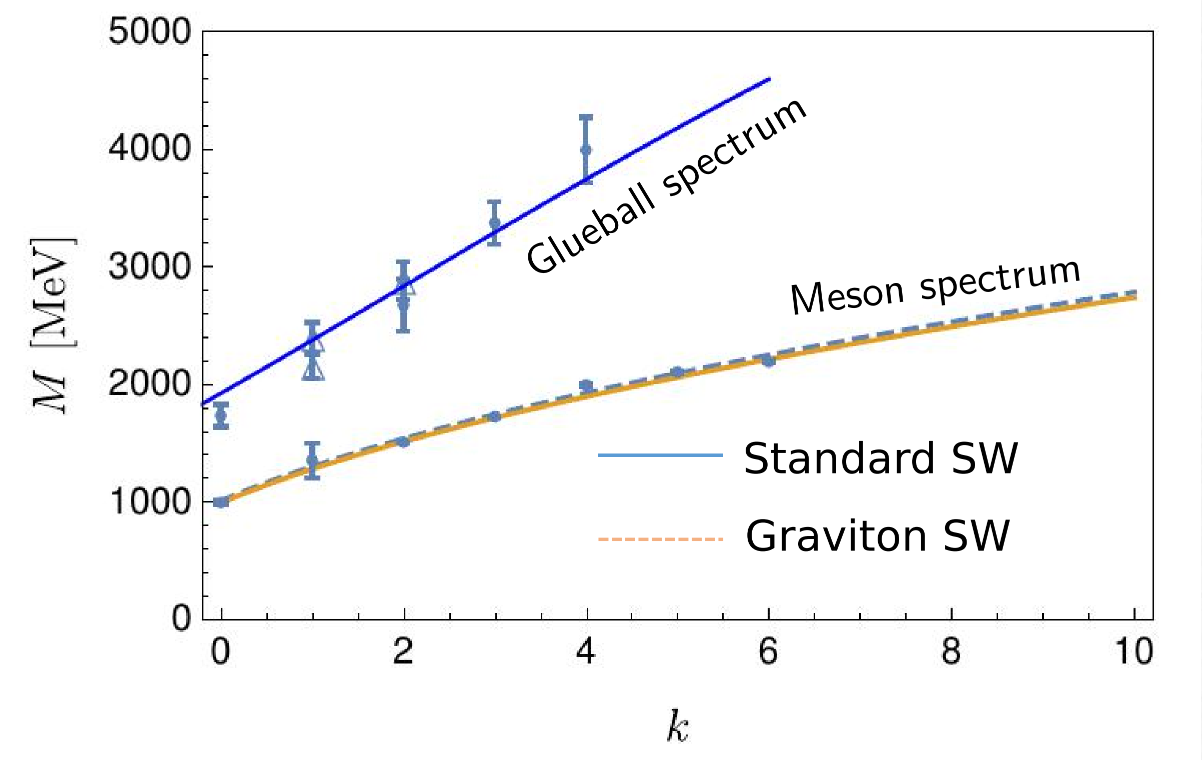
In **M.Rinaldi and V. Vento EPJA 54 (2018)** we consider $\alpha\kappa^2$ as the only **one parameter!**

GRAVITON EoM and SPECTRUM

$$-\frac{1}{2}\tilde{h}^{;c}_{ab;c}-\frac{1}{2}\tilde{h}^{c}_{c;ab}+\frac{1}{2}\tilde{h}^{;c}_{ac;b}+\frac{1}{2}\tilde{h}^{;c}_{bc;a}+4\tilde{h}_{ab}=0$$



Also in this case we have a good description of data, but now (w.r.t. the HW model): **we have a complete description of the meson and glueball spectra**

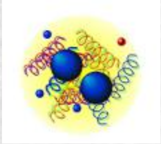


4 The mixing problem in AdS/QCD

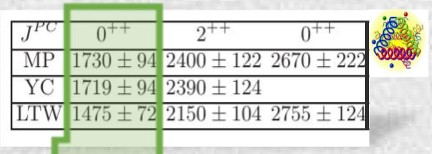
Glueball and meson states could mix!

J^{PC}	0^{++}	2^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222
YC	1719 ± 94	2390 ± 124	
LTW	1475 ± 72	2150 ± 104	2755 ± 124

Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	1992 ± 16	2101 ± 7	2189 ± 13



4 The mixing problem in AdS/QCD



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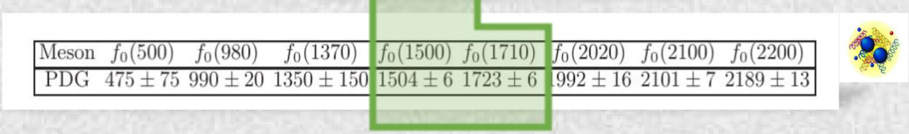
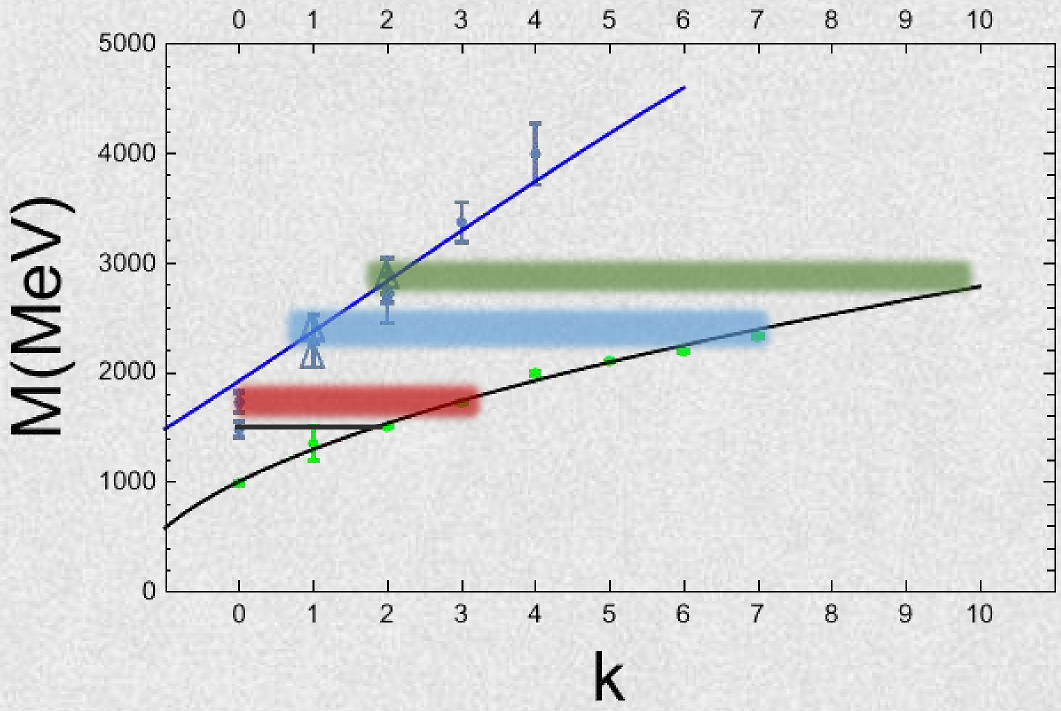


Meson	$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(2020)$	$f_0(2100)$	$f_0(2200)$
PDG	475 ± 75	990 ± 20	1350 ± 150	1504 ± 6	1723 ± 6	992 ± 16	2101 ± 7	2189 ± 13

In terms of modes numbers:



Glueball masses for: $\mathrm{k} = 0, 1, 2...$ **are similar to**
meson masses for: $\mathrm{k} = 4, 7, 10...$

Since the soft-wall model reproduces both the glueball and meson spectra, we can use it to study the mixing condition!

M.Rinaldi and V.Vento arXiv:1803.05738

THANKS

3 Glueballs in AdS/QCD: The Soft-Wall II

In this case we have the following AdS₅ metric :

$$\tilde{g}_{MN}dx^M dx^N = e^{\underbrace{-\alpha\varphi(z)}_{\alpha\kappa^2}} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

In **M.Rinaldi and V. Vento EPJA 54 (2018)** we consider $\alpha\kappa^2$

SCALR FIELD EQUATION:

Equation of motion of the scalar glueball can be obtained:

$$\tilde{I} = \int d^5x\, \sqrt{g}\, e^{-\varphi(z)}\left[g^{MN}\partial_M\mathcal{S}\partial_N\mathcal{S} + e^{-\alpha\varphi(z)}M_5^2\mathcal{S}^2\right]$$

Dilaton field

Graviton contribution

1) scalar glueball state 0^{++} is represented by: $\mathcal{O}_{\Delta=4} = \mathrm{Tr}\left(F^{\mu\nu}F_{\mu\nu}\right)$

2) For example for even spin j: $\mathcal{O}_{\Delta=4+j} = FD_{\{\mu 1\ldots}D_{\mu j\}}F$

The equation of motion for the scalar is (for small α):

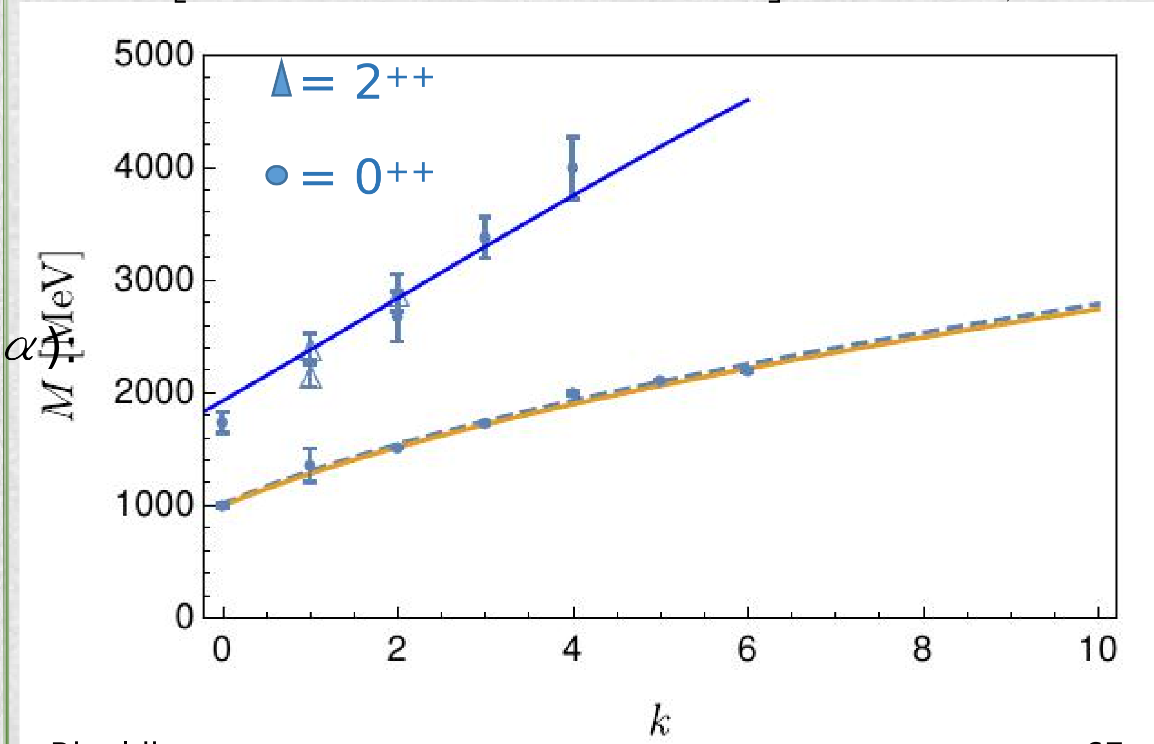
$$-\psi''(z) + \left[\kappa^2 z^2 + \frac{15}{4z^2} + 2\kappa + M_5^2\left(\frac{R^2}{z^2}\right) - M_5^2R^2\alpha\kappa\right]\psi(z) = M^2\psi(z)$$

SCALAR GLUEBALL SPECTRUM:

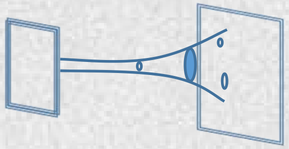
$$M_n = \left[4n + 4 + 2\sqrt{M_5^2R^2 + 4} - \alpha M_5^2R^2\right]$$

→ $k = 0, 1....$ **scalar**

→ $k = 1, 2....$ **tensor**



3 Glueballs in AdS/QCD: Hard-Wall model



In this case we have the following $AdS_5 \, x \, S_5$ metric : $ds^2 = g_{MN}dx^M dx^N + R^2 d\Omega_5 = \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right) + R^2 d\Omega_5$

In the hard-wall (HW) model confinement is implemented by imposing an IR cutoff: $0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$

SCALR FIELD EQUATION:

Equation of motion of the scalar glueball can be obtained:

$$I = \int d^5x \sqrt{g}\left[g^{MN}\partial_M \mathcal{G} \partial_N \mathcal{G} + M_5^2 \mathcal{G}^2\right] \begin{cases} \Delta = \text{conformal dimension} \\ \Delta = 2 + \sqrt{4 + M_5^2 R^2} \end{cases}$$

Mass in AdS space

1) scalar glueball state 0^{++} is dual $\mathcal{O}_{\Delta=4} = Tr\left(F^{\mu\nu}F_{\mu\nu}\right)$
2) For example for even spin J: $\mathcal{O}_{\Delta=4+j} = FD_{\{\mu 1}...D_{\mu j\}}F$

The equation of motion for the scalar is:

$$\frac{d^2\phi(z)}{dz^2} - \frac{3}{z}\frac{d\phi(z)}{dz} + M^2\phi(z) = 0$$

where:

$\mathcal{G}(x,z) \sim \phi(z)e^{-iP_\mu x^\mu}, \ P^2 = -M^2$

H. Boschi-Filho et al, JHEP 05, 009 (2003)
H. Boschi-Filho et al, PRD 73, 047901 (2006)
P. Colangelo et al, PLB 652, 73 (2007)

GRAVITON SPECTRUM:

Equation of motion for metric perturbation h_{MN} obtained fro
linearized Einstein's equation :
R.C. Brower et al, Nucl. Phys. B 587, 249 (2000)

$$-\frac{1}{2}h^{;c}_{ab;c} - \frac{1}{2}h^{c}_{c;ab} + \frac{1}{2}h^{;c}_{ac;b} + \frac{1}{2}h^{;c}_{bc;a} + 4h_{ab} = 0$$

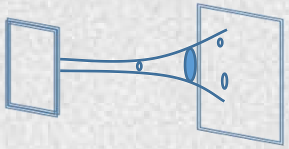
By choosing the gauge:

$$\begin{cases} h_{tt} = \left(z^{-2} - z^2\right)\phi(z)e^{-Mx_3} & \text{Scalar component} \\ h_{ij} = q_{ij}T(z)e^{-Mx_3} & \text{Tensor component} \end{cases}$$

"Tensor" wave-function

Same equation of motion of the scalar field for the sc
component.

3 Glueballs in AdS/QCD: Hard-Wall model



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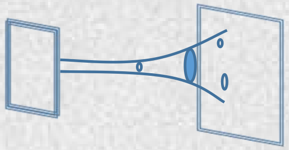
“Tensor” wave-function

Same equation of motion for the scalar field for the scalar component of the graviton.

3

Glueballs in AdS/QCD: **Hard-Wall** model



In this case we have the following $AdS_5 \, x \, S_5$ metric : $ds^2 = g_{MN} dx^M dx^N + R^2 d\Omega_5 = \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right) + R^2 d\Omega_5$

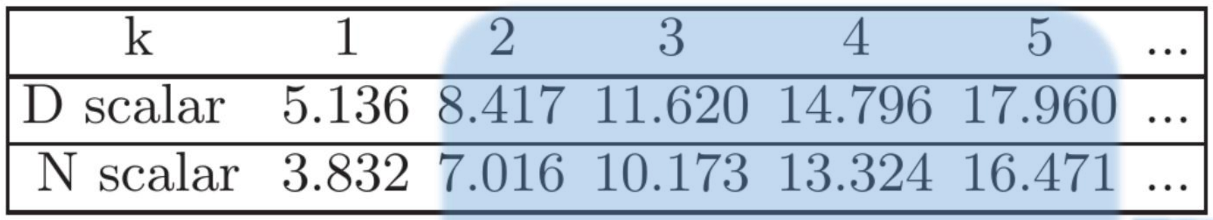
In the hard-wall (HW) model confinement is implemented by imposing an IR cutoff: $0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$

SCALR FIELD EQUATION:

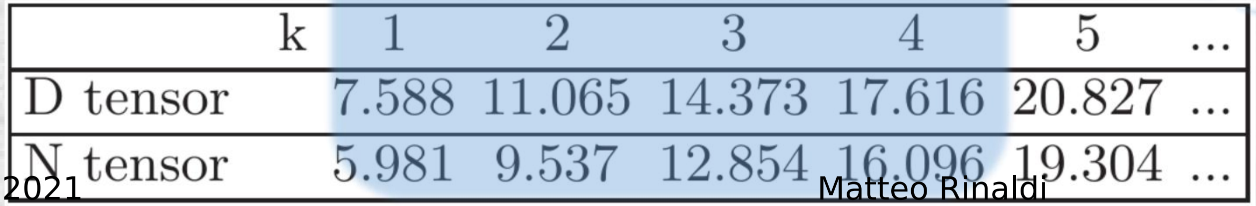
GRAVITON SPECTRUM:

Within this model the spectrum of the scalar field is the same of that of the scalar component of the graviton!

What about the tensor component?



k	1	2	3	4	5	...
D scalar	5.136	8.417	11.620	14.796	17.960	...
N scalar	3.832	7.016	10.173	13.324	16.471	...

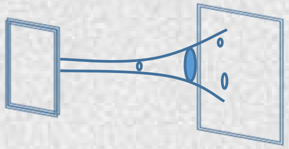


k	1	2	3	4	5	...
D tensor	7.588	11.065	14.373	17.616	20.827	...
N tensor	5.981	9.537	12.854	16.096	19.304	...

Almost degeneracy!

The skip in the mode number is **equivalent to a mass contribution in the tensor sector!**

3 Glueballs in AdS/QCD: Hard-Wall model



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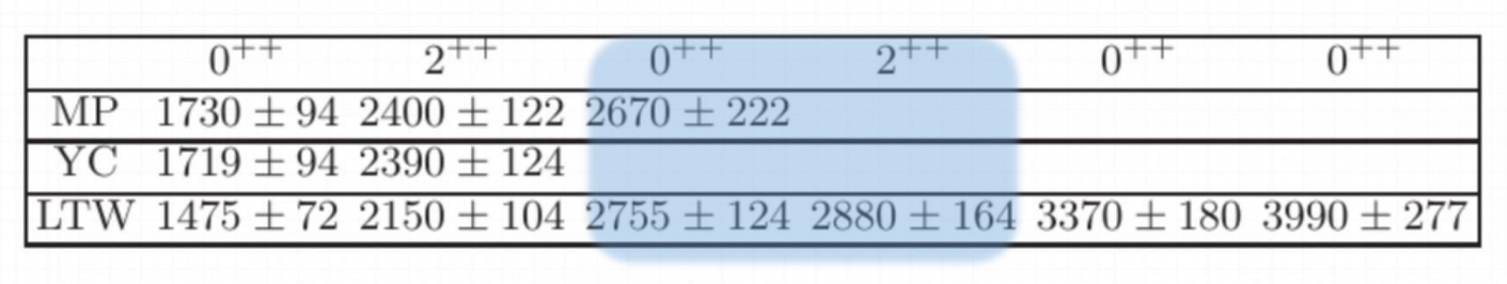
In the hard-wall (HW) model confinement is implemented by imposing an IR cutoff: $0 \leq z \leq z_{max} = \frac{1}{\Lambda_{QCD}}$

0^{++} ⊗ 2^{++} GLUEBALL SPECTRA

M.Rinaldi and V. Vento EPJA 54 (2018)

MP: **C.J. Morningstar et al, PRD 60, 034509 (1999)** YC: **Y. Chen et al, PRD 73, 014516 (2006)** LTW: **B. Lucini et al, JHEP 06, 012 (2004)**

LATTICE DATA:



	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
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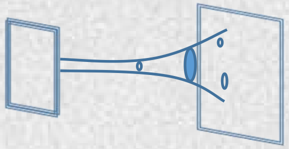
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277

These two states are almost **degenerate**

3 Glueballs in AdS/QCD: The GSW model



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In **M.Rinaldi and V. Vento EPJA 54 (2018)** $\alpha\kappa^2$ is the **unique parameter!**

GRAVITON EoM and SPECTRUM

$$-\frac{1}{2}\tilde{h}^{;c}_{ab;c} - \frac{1}{2}\tilde{h}^{c}_{c;ab} + \frac{1}{2}\tilde{h}^{;c}_{ac;b} + \frac{1}{2}\tilde{h}^{;c}_{bc;a} + 4\tilde{h}_{ab} = 0$$

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$$\Psi''(t) + V_G(t)\Psi(t) = \Lambda^2\Psi(t)$$

with:

$$\begin{cases} t = i\alpha z/\sqrt{2} \\ \Lambda^2 = \dfrac{M^2}{\alpha^2} \\ V_G(t) = \dfrac{e^{2t^2}}{t^2} - \dfrac{17}{4t^2} + 14 - 15t^2 \end{cases}$$

1) The scalar and tensor components have the same EoM

2) Bound states are found for $\alpha<0$

3) From the fitting procedure we found that: $\alpha \leq \kappa \leq \beta$

3 Glueballs in AdS/QCD: The GSW model

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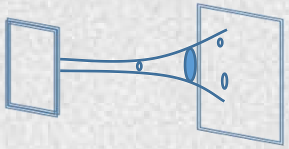

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GRAVITON EoM and SPECTRUM

$$-\frac{1}{2}\tilde{h}^{;c}_{ab;c} - \frac{1}{2}\tilde{h}^{c}_{c;ab} + \frac{1}{2}\tilde{h}^{;c}_{ac;b} + \frac{1}{2}\tilde{h}^{;c}_{bc;a} + 4\tilde{h}_{ab} = 0$$

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2) Bound states are found for $\alpha<0$

3) From the fitting procedure we found that: $\alpha \leq \underset{\substack{\| \\ \frac{M_\rho}{\sqrt{2}}}}{\kappa} \leq \beta$

Guy F. de Teramond et al, PRL 120, 182001 (2018)

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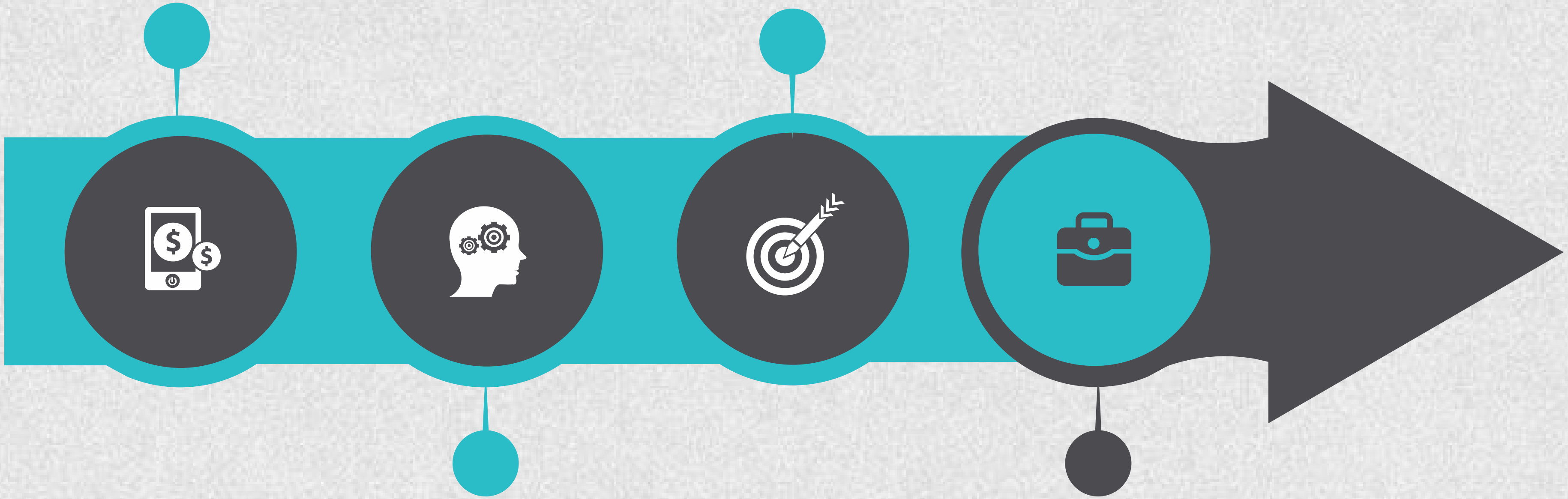

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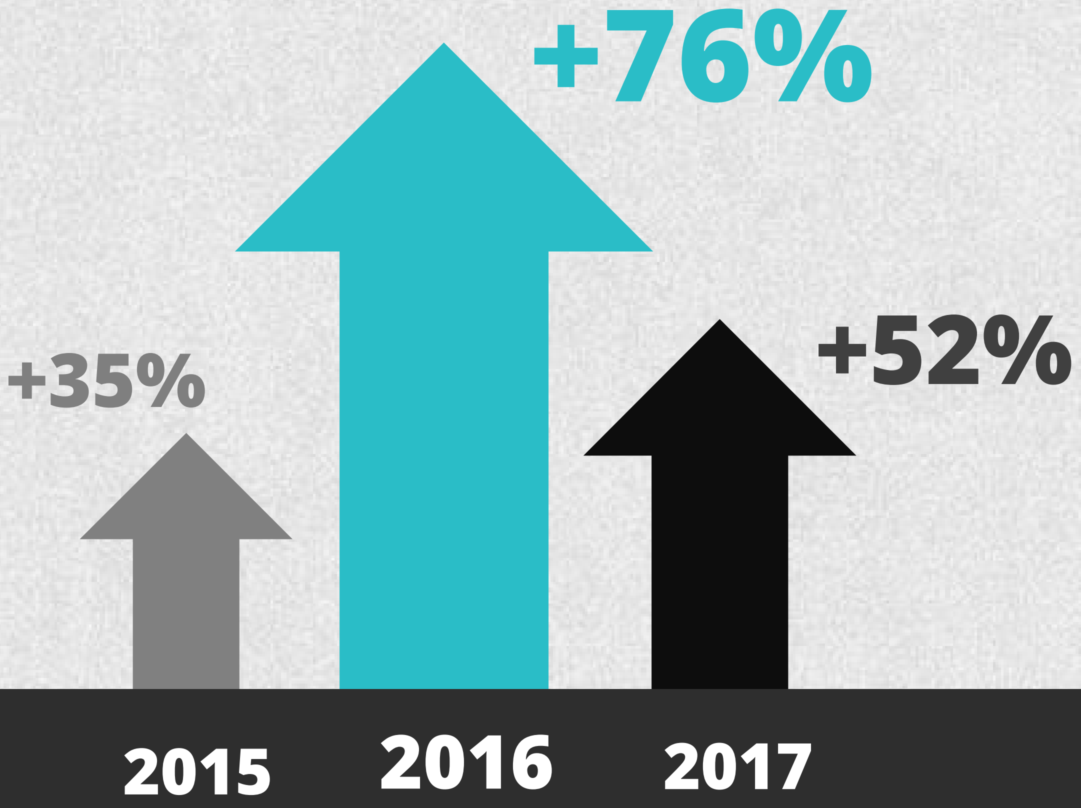

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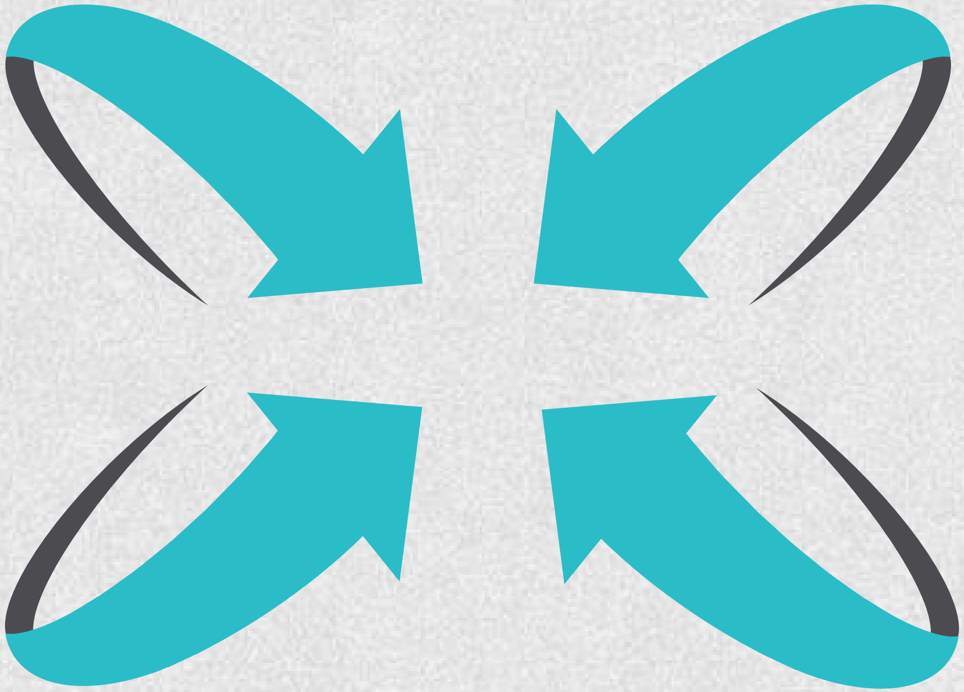

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