

July 30, 2021

# Mesonic excitations of magnetized and hot quark matter within effective models of QCD

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# Outline

- Motivation
- Thermo-magnetic effects on the coupling constants in NJL model - IMC
- The importance of implementing a proper regularization procedure in order to treat thermo and magnetic contributions within non renormalizable theories
- Neutral meson pole mass in a magnetized and thermal medium within the  $SU(2)$  NJL model
- Neutral meson pole mass in a magnetized medium within the Linear Sigma model with quarks
- Conclusions and perspectives

# Strong magnetic fields may be produced in off-central heavy ion collisions

K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008). D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 80, 0304028 (2009). D. E. Kharzeev, Nucl. Phys. A 830, 543c (2009).

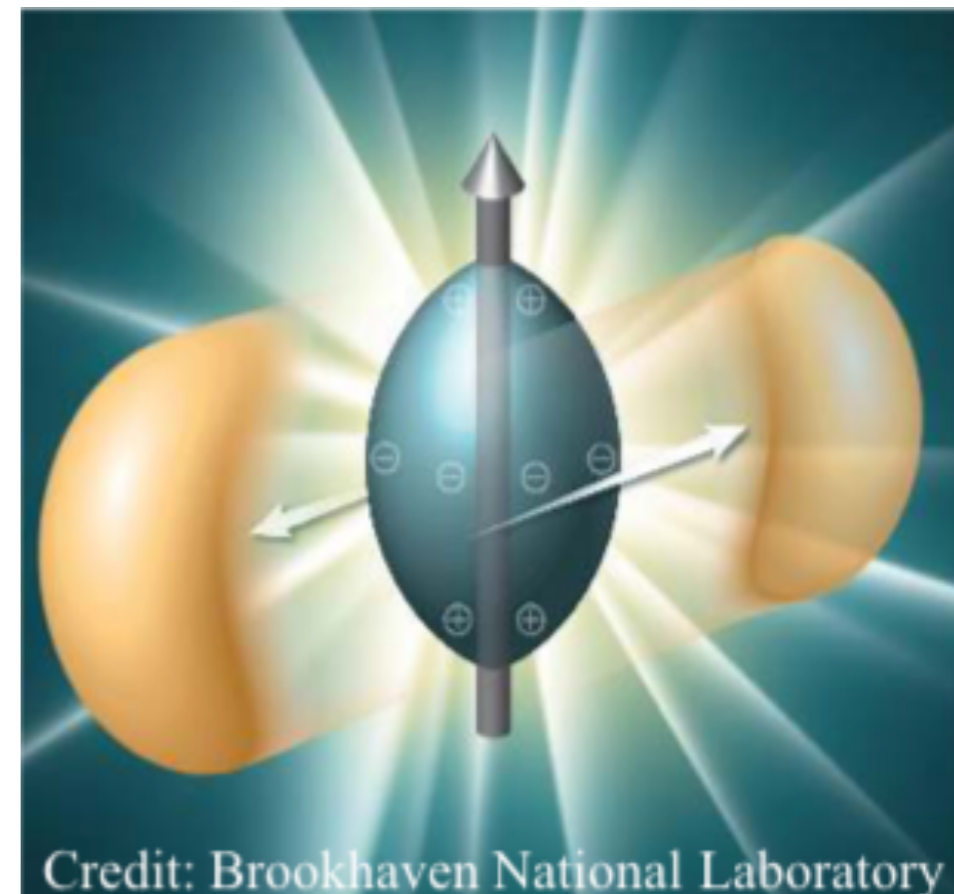
- Chiral magnetic effect
- Effects in EoS
- Anisotropies

heavy-ion collisions:

temporarily  $B \lesssim 10^{19}$  G

Skokov, Illarionov, Toneev,

Int. J. Mod. Phys. A 24, 5925 (2009)



# Neutron Stars

- Strong magnetic fields are also present in magnetars:

C. Kouveliotou et al., *Nature* 393, 235 (1998).

magnetars:

at surface  $B \lesssim 10^{15}$  G

Duncan, Thompson, *Astrophys.J.* 392, L9 (1992)

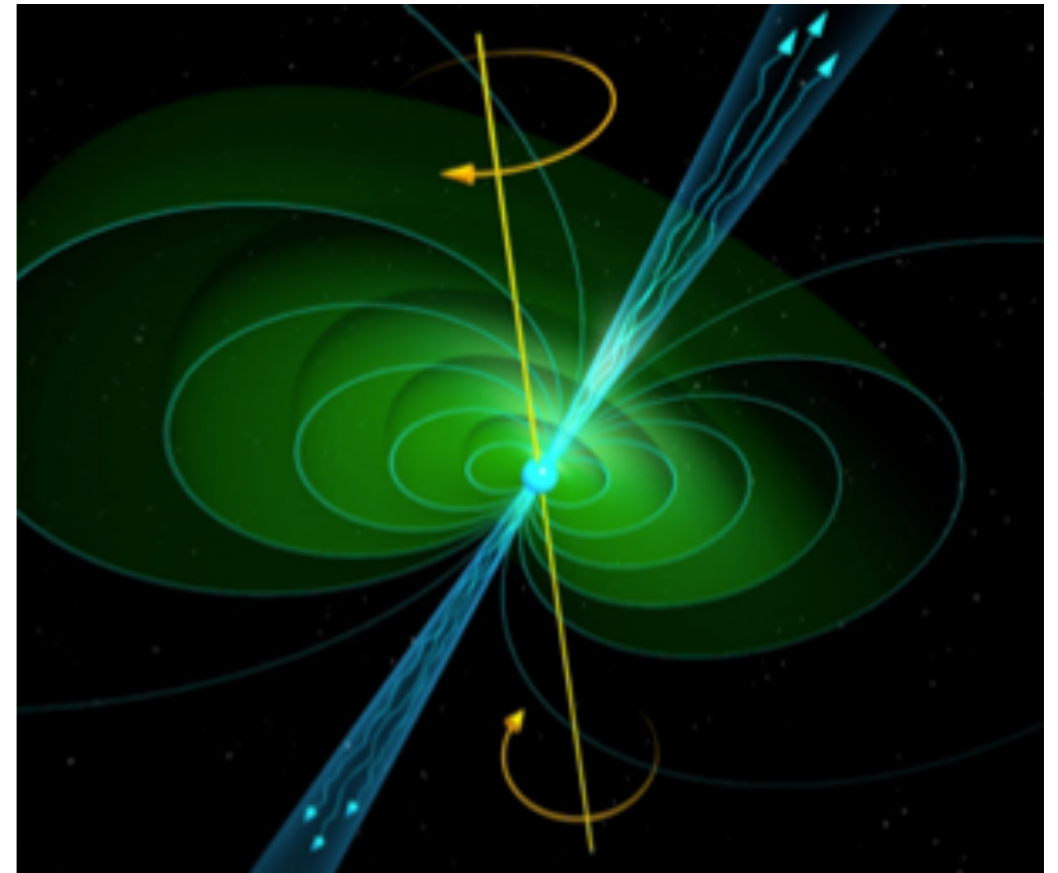
larger in the interior,

$B \sim 10^{18-20}$  G?

Lai, Shapiro, *Astrophys.J.* 383, 745 (1991)

E. J. Ferrer *et al.*, *PRC* 82, 065802 (2010)

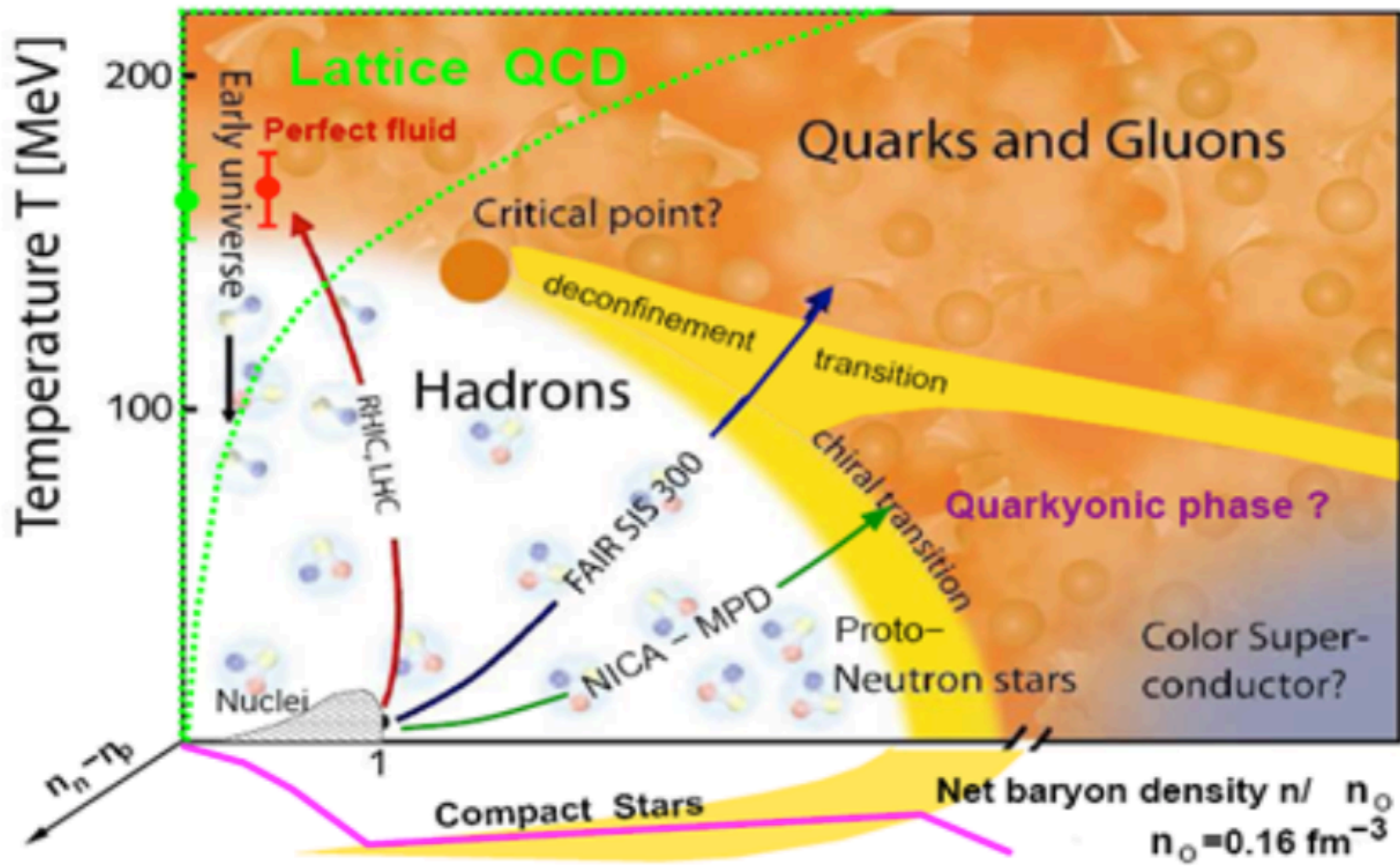
- and might have played an important role in the physics of the early universe. T. Vaschupati, *Phys. Lett. B* 265, 258 (1991).



A. K. Harding, D. Lai, *Rept. Prog. Phys.* 69, 2631 (2006)

D. Grasso and H.R. Rubinstein, *Phys. Rep.* 348, 163 (2001).

# B effects in QCD Phase Diagram



NICA - Nuclotron-based  
Ion Collider Facility

FAIR - Facility for Antiproton and  
Ion Research

# Lattice Results: Sign Problem

- fermion determinant is complex

$$[\det M(\mu)]^* = \det M(-\mu^*) \in \mathbb{C}$$

- no positive weight in path integral

$$Z = \int \mathcal{D}U e^{-S_{YM}} \det M(\mu)$$

- standard lattice methods base on importance sampling cannot be used!

# To make progress

- We use Quantum Field Theory (in medium)
- DSE (beyond RL truncation...)
- Holographic models
- EOS + susceptibilities calculated on the lattice + Taylor series (limitations...)
- Effective models (just a few degrees of freedom): NJL/PNJL, Linear  $\sigma$  model, MIT,...

QCD at finite T and B :

NO

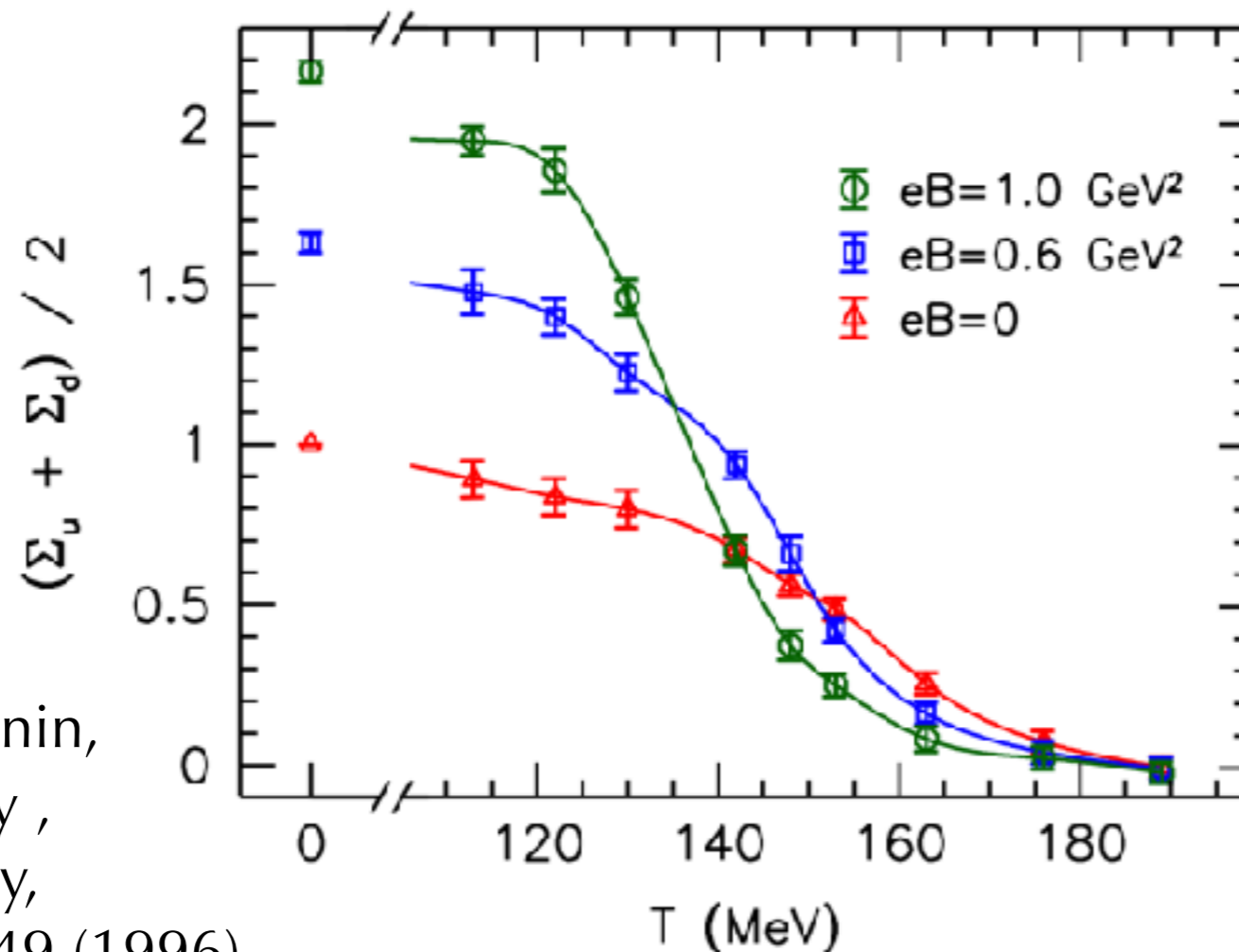
Sign Problem!

There is hope that lattice simulations of QCD with B can be used as a benchmark platform for comparing different effective models used in the literature.



# B Effects on QCD phase transitions?

$$\Lambda_{\text{QCD}}^2 \sim (200 \text{ MeV})^2 \sim 2 \times 10^{18} \text{ G}$$

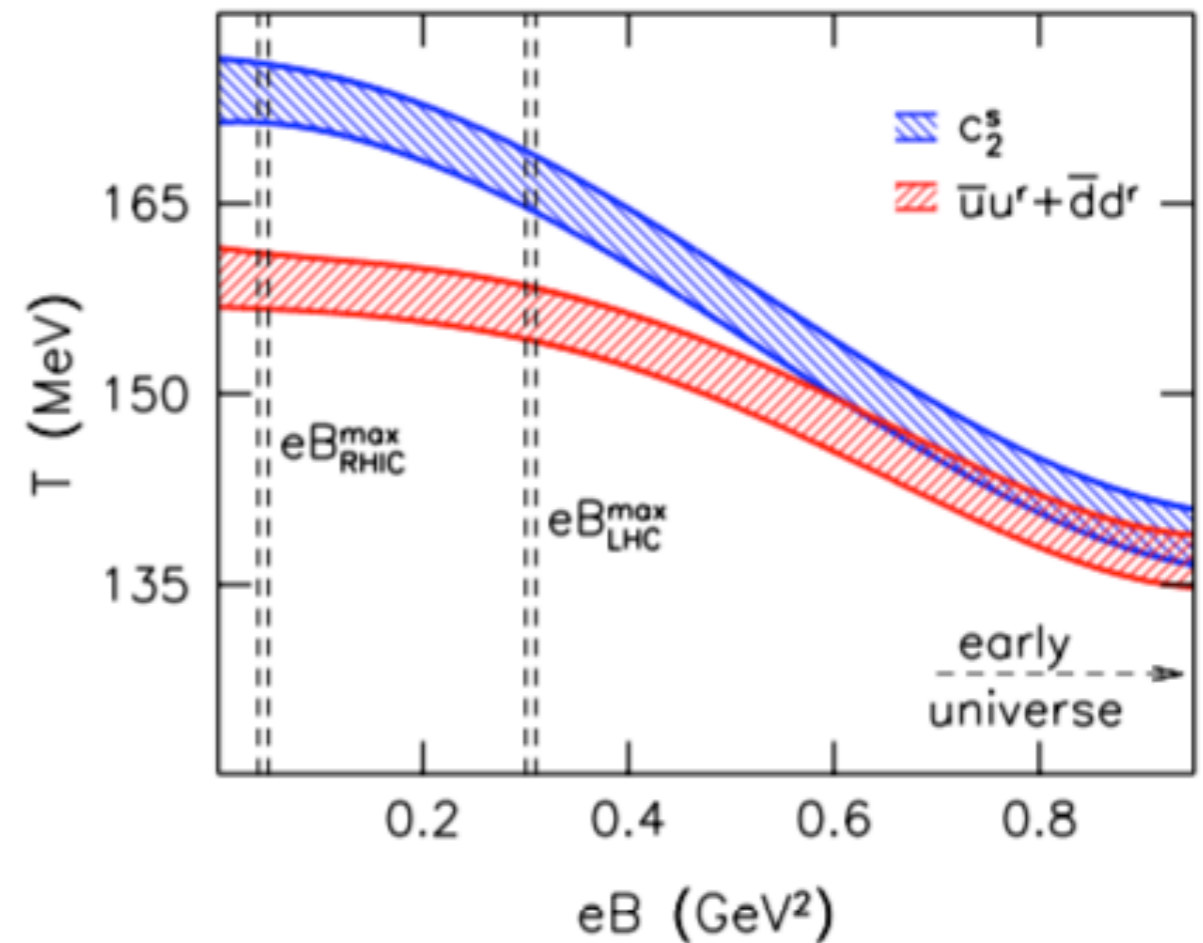
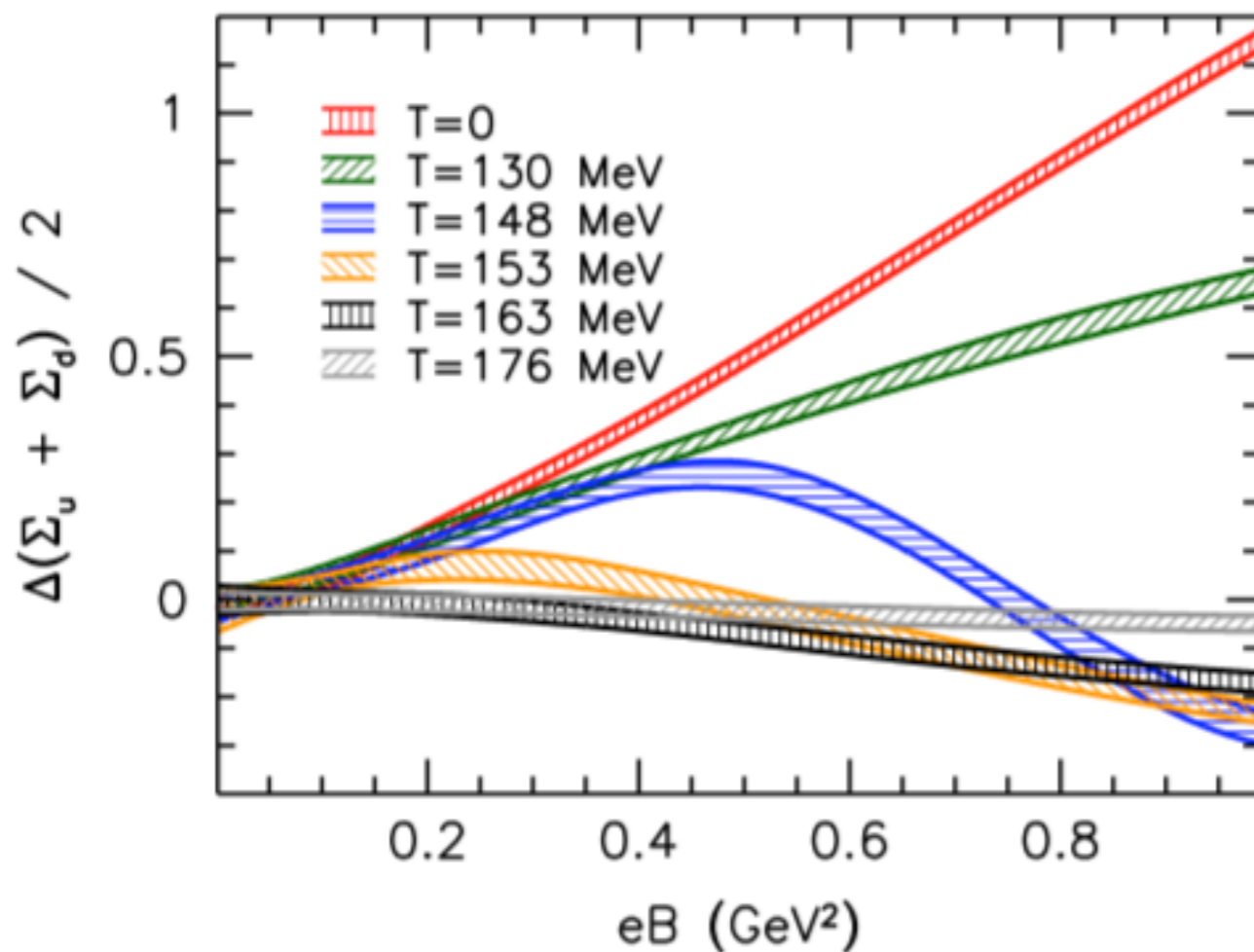


**MC:** V.P. Gusynin,  
V.A. Miransky,  
I.A. Shovkovy,  
*Nucl. Phys. B* **462** 249 (1996)

**IMC:** Bali, Bruckmann,  
Endrodi, Fodor,  
Katz et al.  
*JHEP* 02 (2012) 044  
*Phys.Rev.D* 86 (2012)  
071502

# B Effects on QCD phase transitions?

MC and IMC

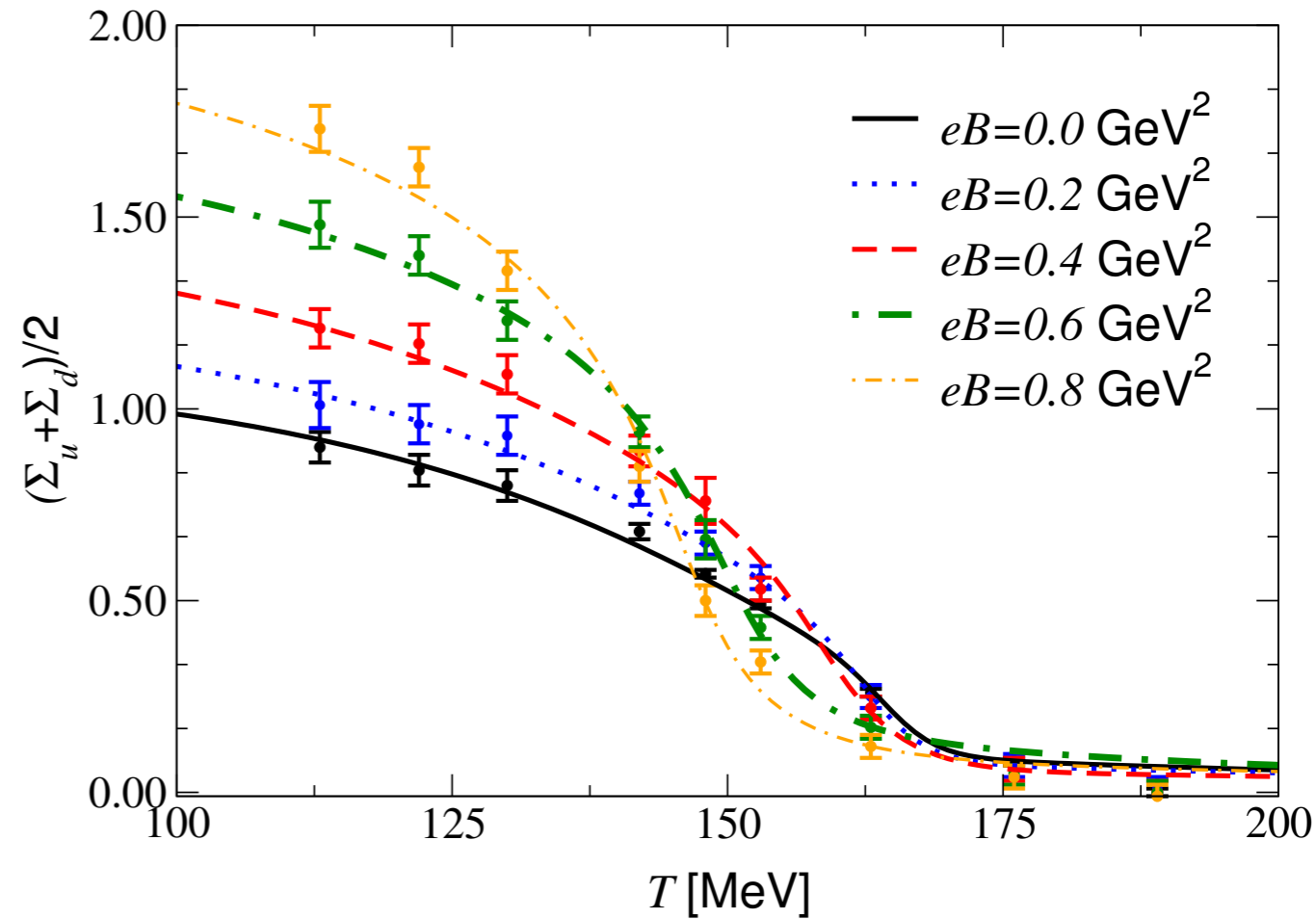
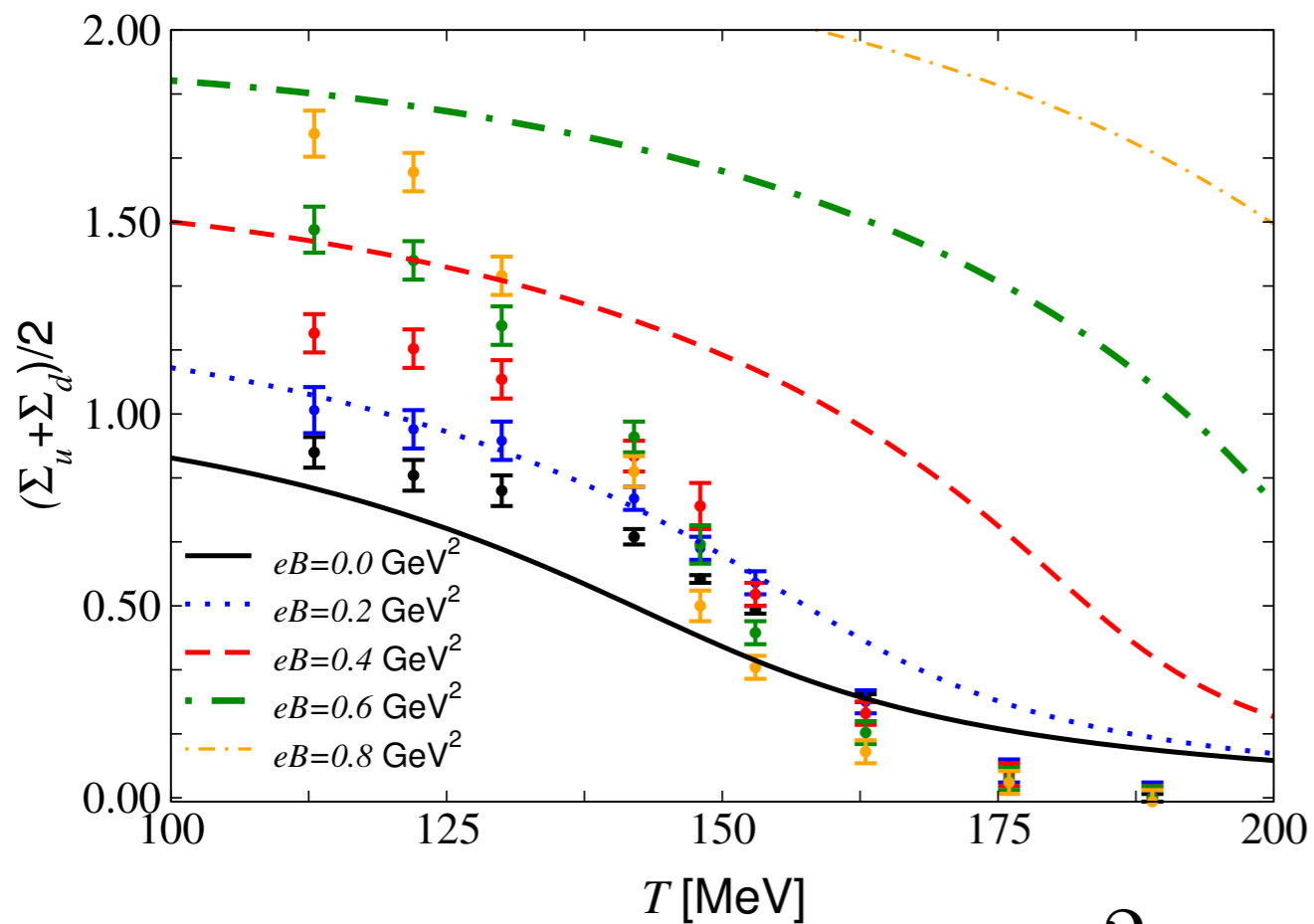


**Failure** of ALL effective models in providing inverse magnetic catalysis!

# SU(2) NJL + Thermo- Magnetic effects $G(B,T)$

$G(0,0)$

$G(B,T)$



$$\Sigma_f(B, T) = \frac{2m_f}{m_\pi^2 f_\pi^2} \left[ \langle \bar{\psi}_f \psi_f \rangle - \langle \bar{\psi}_f \psi_f \rangle_0 \right] + 1$$

**RLSF**, K.P. Gomes, M.B.Pinto, G. Krein, Phys. Rev. C **90**, 025203 (2014).

**RLSF**, V.S. Timoteo, S.S.Avancini, M.B.Pinto and G.Krein Eur. Phys. J. A (2017) **53**: 101

# Thermo-Magnetic effects $G(B,T)$

$$G(B, T) = c(B) \left[ 1 - \frac{1}{1 + e^{\beta(B)[T_a(B) - T]}} \right] + s(B)$$

$eB$ [GeV <sup>2</sup> ]	$c$ [GeV <sup>-2</sup> ]	$T_a$ [MeV]	$s$ [GeV <sup>-2</sup> ]	$\beta$ [MeV <sup>-1</sup> ]
0.0	0.9000	168.000	3.73110	0.40000
0.2	1.2256	167.922	3.2621	0.34117
0.4	1.7693	169.176	2.2942	0.22988
0.6	0.7412	155.609	2.8638	0.14401
0.8	1.2887	157.816	1.8040	0.11506

[SU\(2\) \*\*RLSF\*\*, V.S. Timoteo, S.S.Avancini, M.B.Pinto and G.Krein Eur. Phys. J. A \(2017\) 53: 101](#)

[SU\(3\) \*\*RLSF\*\*, W. Tavares, S.S.Avancini, V.S. Timoteo, G.Krein and M.B. Pinto, e-Print: 2104.11117 \[hep-ph\]](#)

# B Effects on QCD phase transitions?

Inverse magnetic catalysis: how much do we know about?

A. Bandyopadhyay, R.L.S. Farias, *Eur. Phys. J. ST* 230 (2021) 3, 719-728,  
B. e-Print: 2003.11054 [hep-ph]

## Possible explanations for IMC:

- Competition of B effects on sea and valence quarks, F. Bruckmann, G. Endrodi, T. G. Kovacs, *JHEP* 04 (2013) 112
- Inclusion of plasma screening effects that capture the physics of collective, long-wave modes, and thus describe a prime property of plasmas near transition lines, namely, long distance correlations. e-Print: 2104.05854 [hep-ph]

**IMC  $\neq$  Tc decreasing with eB**

# SU(2) Nambu—Jona-Lasinio model (NJL)

$$\mathcal{L}_{NJL} = \bar{\psi} (\not{D} - m) \psi + G [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$D^\mu = (i\partial^\mu - QA^\mu)$$

good **chiral** physics, pions,...

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

**BUT** no confinement

$$Q = \text{diag}(q_u = 2e/3, q_d = -e/3)$$

✓ **strong magnetic field background that is constant and homogeneous!**

$$G, \Lambda \text{ and } m_c \longrightarrow m_\pi, f_\pi \text{ and } \langle \bar{\psi}\psi \rangle$$

natural units:  $1\text{GeV}^2 \approx 5.34 \times 10^{19} \text{ G}$  and  $e = \sqrt{\frac{4\pi}{137}}$

# NJL at finite B

$$\text{At } B=0 \quad \mathcal{F} = \frac{(M - m_c)^2}{4G} - N_c \sum_{f,s} \int \frac{d^4 p}{(2\pi)^4} \ln [p^2 + M^2]$$

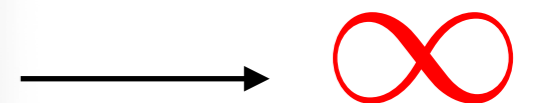
By using the replacement  $\vec{p}^2 \rightarrow p_3^2 + 2k|q_f|B$

$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \sum_{k=0}^{\infty} \alpha_k$$

$$\alpha_k = 2 - \delta_{k0}$$

$$\mathcal{F} = \frac{(M - m_c)^2}{4G} - N_c \sum_f \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \ln [p_4^2 + p_3^2 + 2k|q_f|B + M^2]$$

And the gap equation:  $\partial\mathcal{F}/\partial M = 0$



We need a regularization  
procedure!

Which procedure/method  
is more appropriate?

**Is there any criteria?**



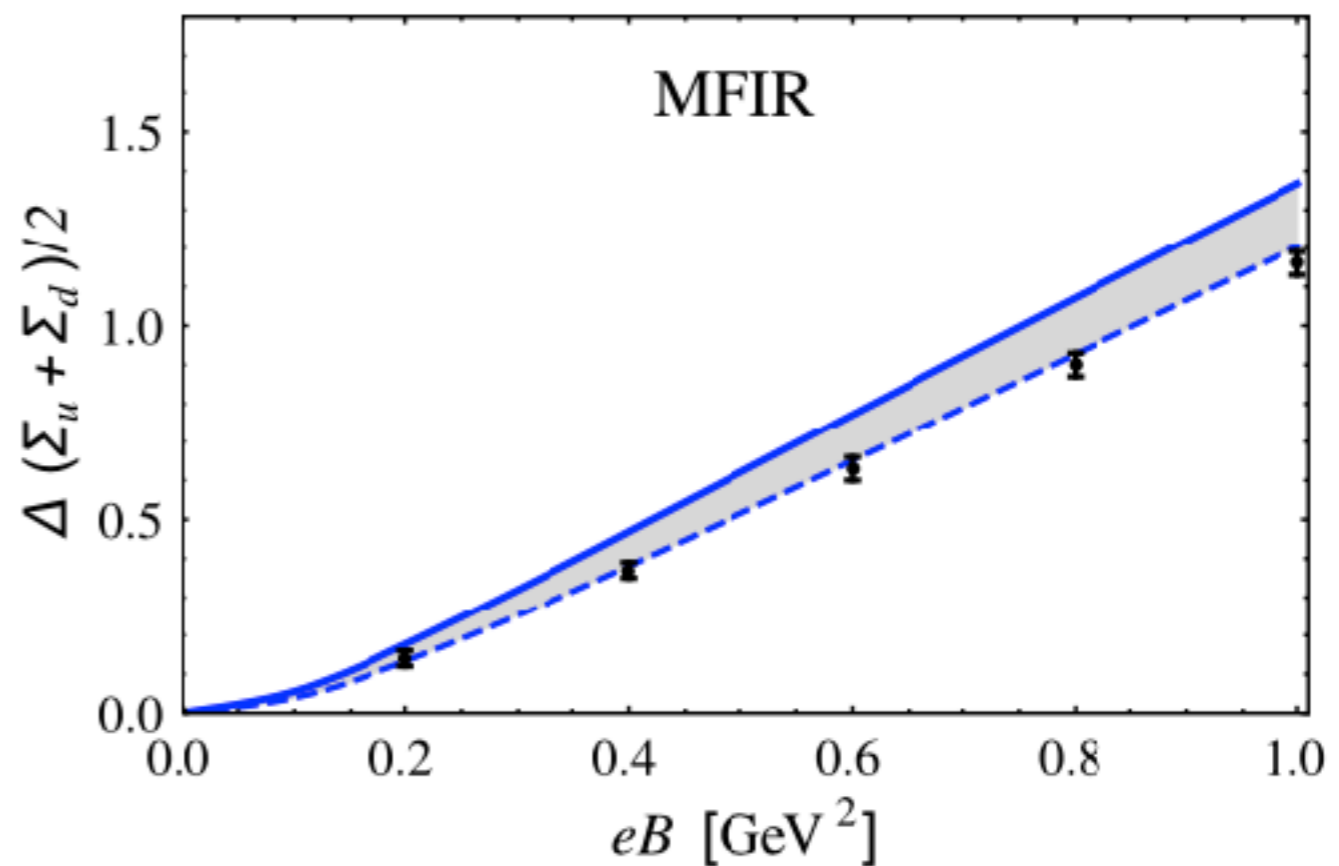
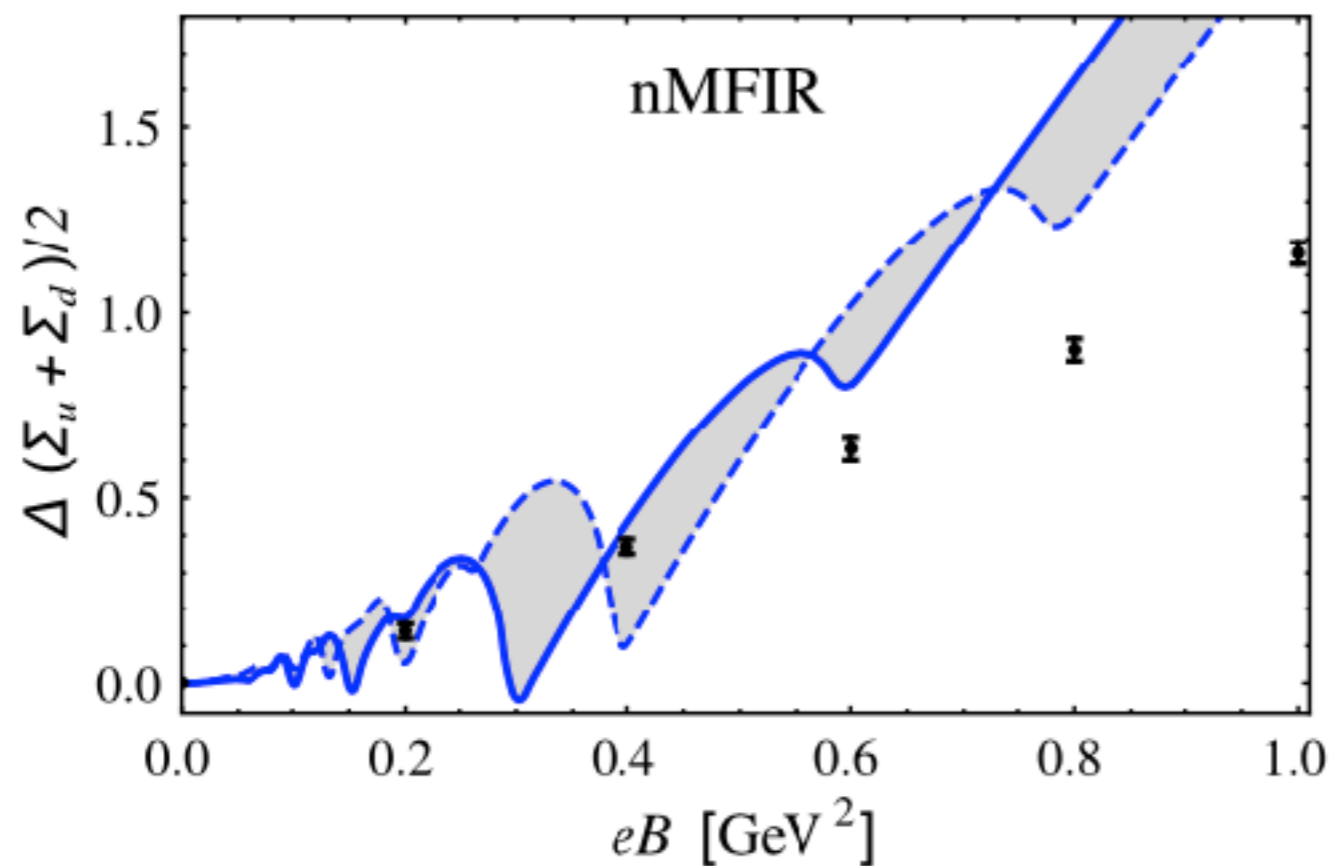
# MFIR - Magnetic Field Independent Regularization

- ✓ D. Ebert and K.G. Klimenko, *Nucl. Phys.* **A728**, 203 (2003).
- ✓ D. P. Menezes, M. B. Pinto, S. S. Avancini, A. P. Martínez, and C. Providência, *Phys. Rev. C* **79**, 035807 (2009).
- ✓ P. G. Allen, A. G. Grunfeld, and N. N. Scoccola, *Phys. Rev. D* **92**, 074041 (2015).
- ✓ D.C.Duarte, P.G.Allen, R.L.S.Farias, P.H.A.Manso,R.O.Ramos,and N. N. Scoccola, *Phys. Rev. D* **93**, 025017 (2016).
- ✓ S. S. Avancini, W. R. Tavares, and M. B. Pinto, *Phys. Rev. D* **93**, 014010 (2016).
- ✓ ...

# Fermi-Dirac Form Factor

$$U_{\Lambda}^{\text{FD}}(x) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{\frac{x}{\Lambda} - 1}{\alpha} \right) \right]$$

$$245 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$$

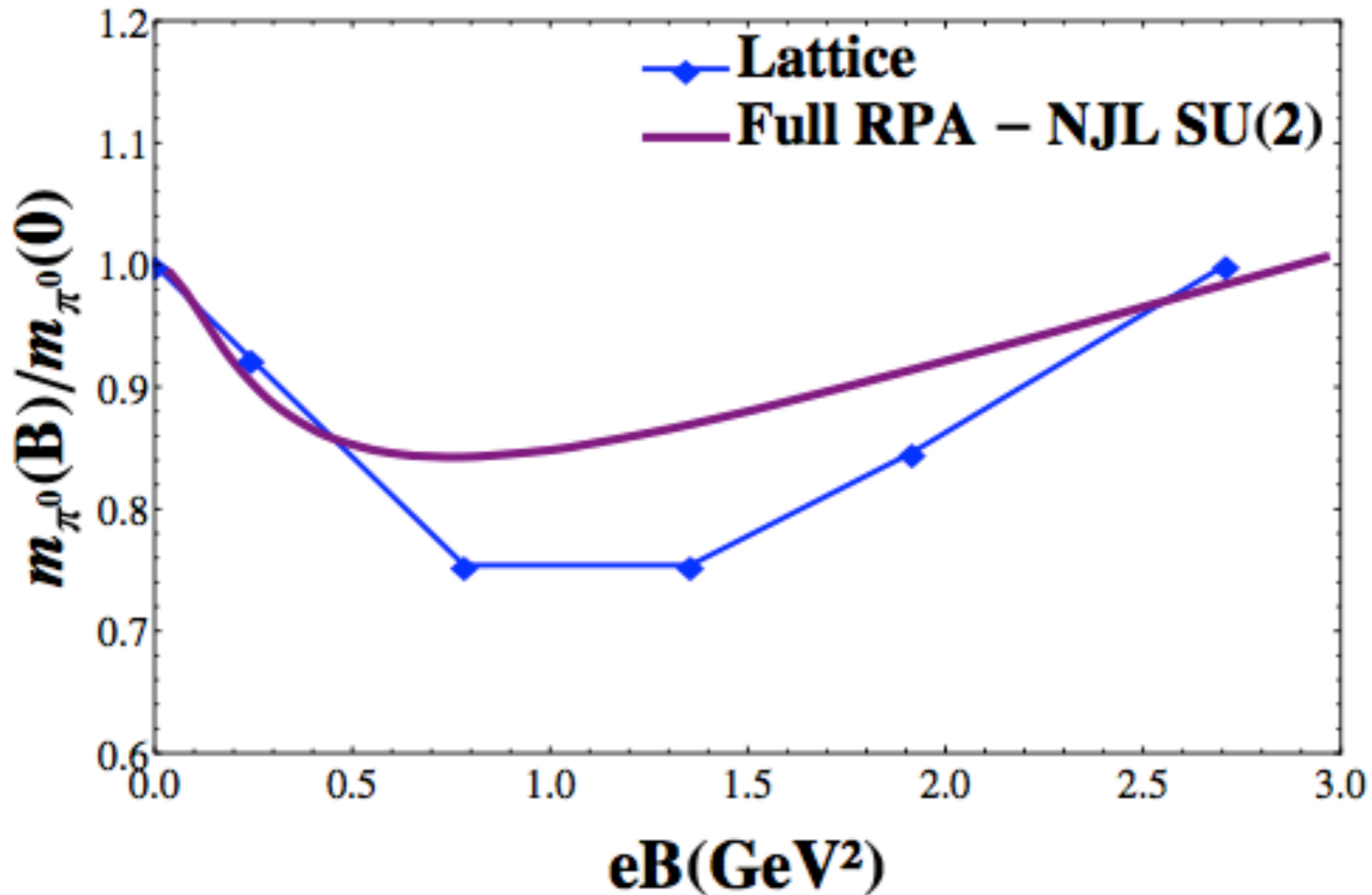


Lattice data: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502(R) (2012)

**RLSF**, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

# Meson masses

# Lattice Results (2013)

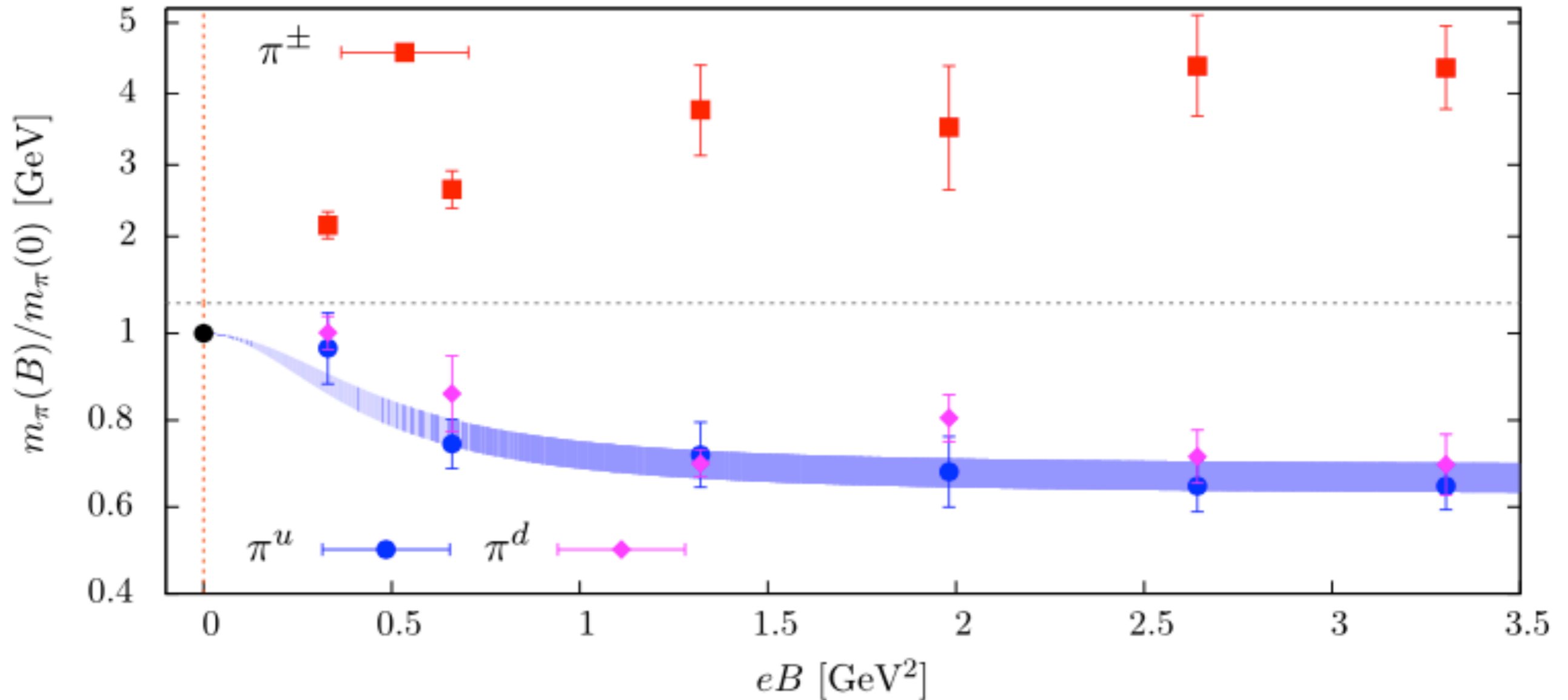


**NJL + MFIR**

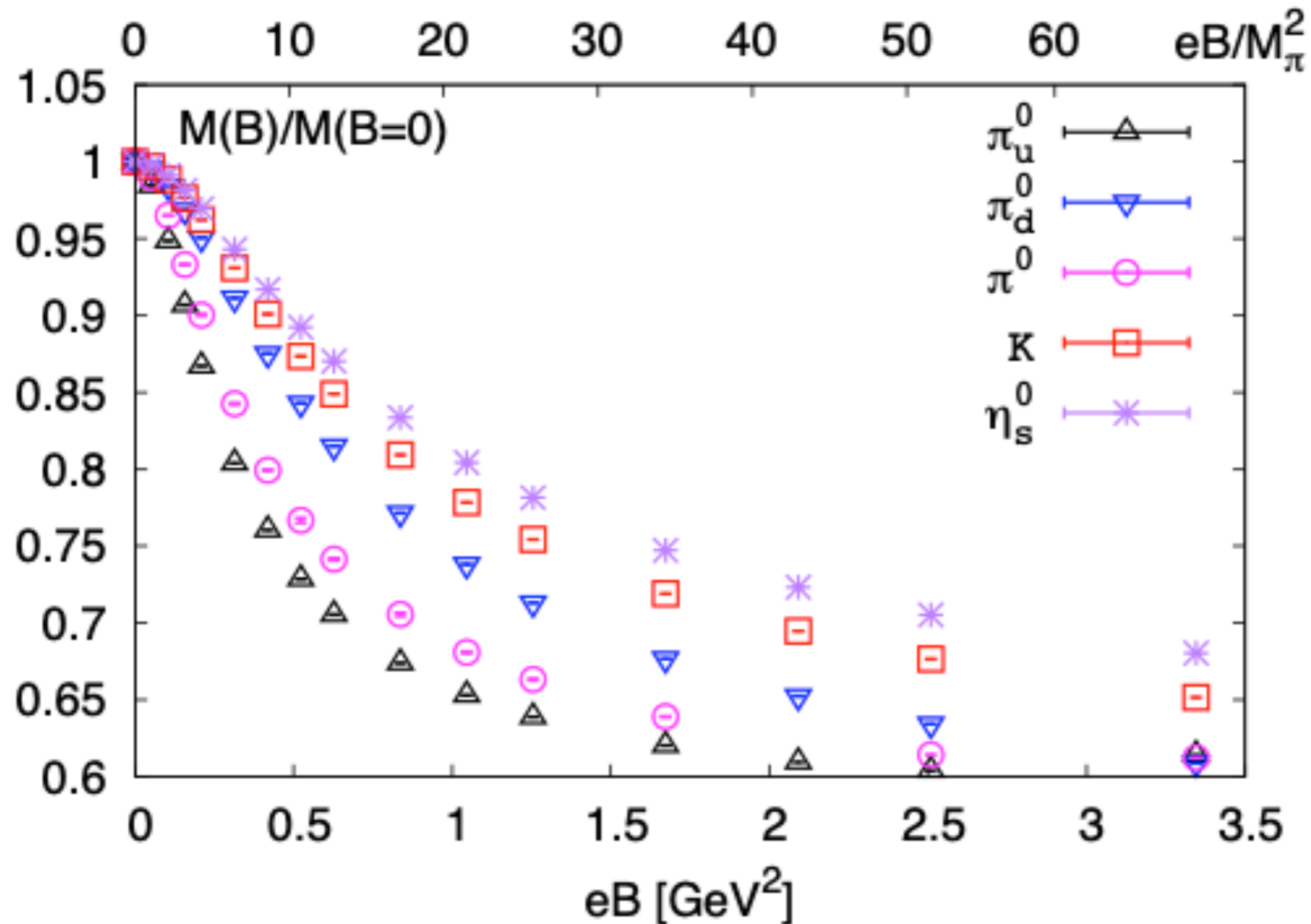
Y. Hidaka and A. Yamamoto,  
Phys. Rev. D 87, 094502 (2013)

S.S. Avancini, W. R. Tavares, M.B. Pinto,  
Phys.Rev D 93, 014010 (2016)

# Lattice Results (2018)



# Lattice Results (2020)



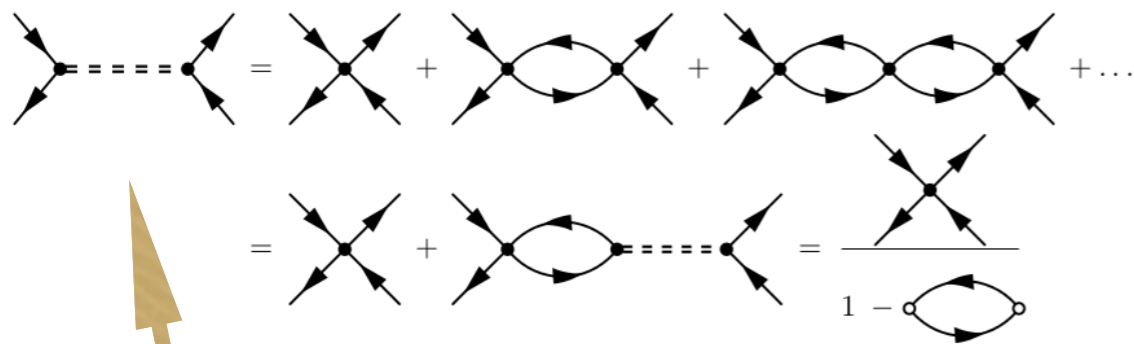
H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, Phys. Rev. D 104, 014505 (2021)

# NJL - Meson properties at finite B

T matrix for the scattering of pairs of quarks,  $q_1 q_2 \rightarrow q_1' q_2'$   
 can be calculated  $\rightarrow$  solving the Bethe-Salpeter equation in the  
 ladder or random phase approximation (RPA)

$$m_{\pi_0}(B, T = 0)$$

only pionic degrees of freedom:



$$(ig_{\pi^0 qq})^2 iD_{\pi^0}(k^2) = \frac{2iG}{1 - 2G\Pi_{ps}(k^2)}$$

$$D_{\pi^0}(k^2) = \frac{1}{k^2 - m_{\pi^0}^2}$$

$$\mathcal{L}_{\pi qq} = ig_{\pi qq} \bar{\psi} \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi$$

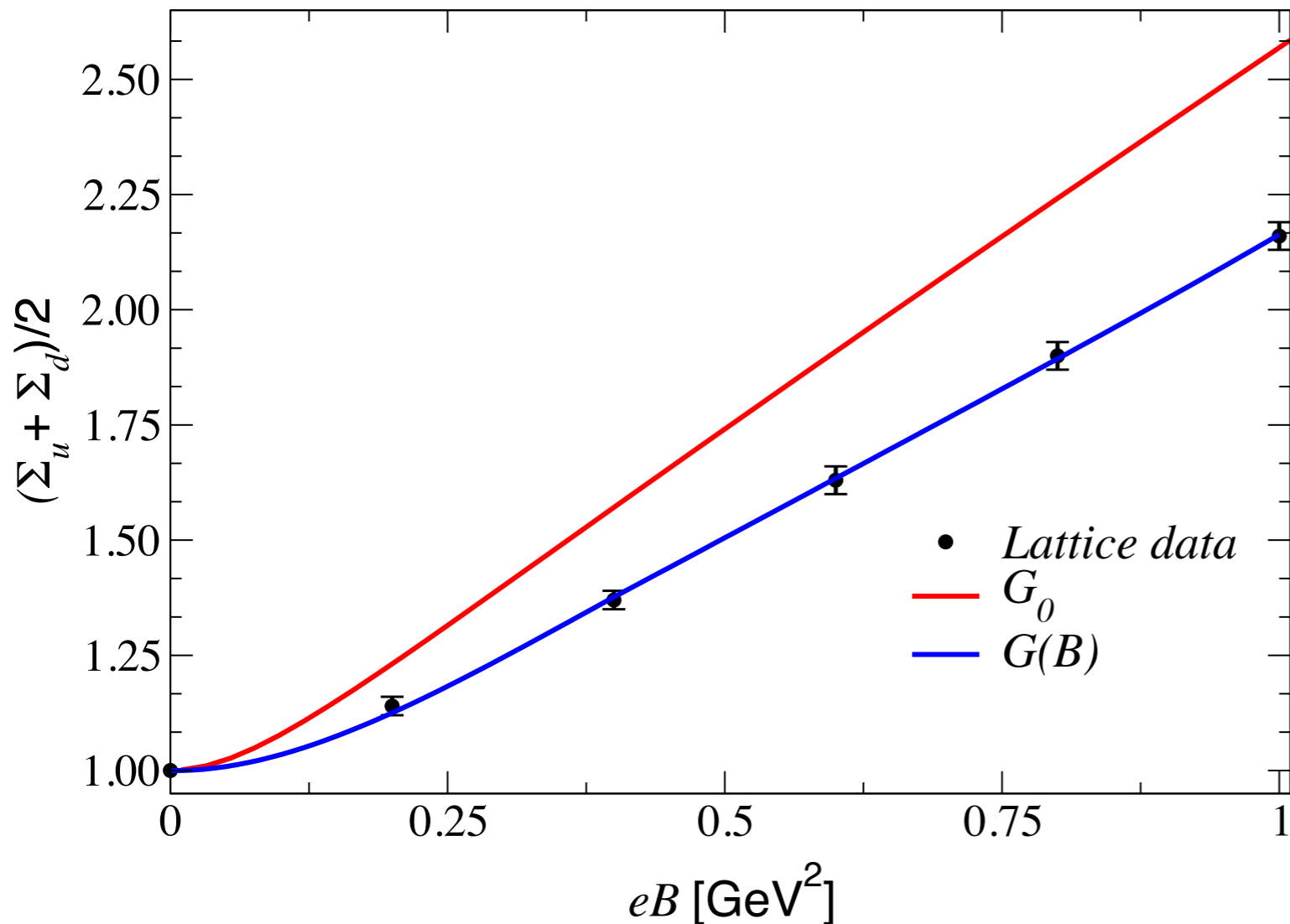
$$1 - 2G\Pi_{ps}(k^2)|_{k^2 = m_{\pi^0}^2} = 0$$

✓ Magnetic Field Independent Regularization (MFIR)

# Field dependent coupling

## $G(B, T=0)$

$$\Sigma_f(B) = \frac{2m}{m_\pi^2 f_\pi^2} [\langle \bar{\psi}_f \psi_f \rangle_B - \langle \bar{\psi}_f \psi_f \rangle_{00}] + 1$$



$$G(eB) = \alpha + \beta e^{-\gamma (eB)^2}$$

$$\alpha = 1.44373 \text{ GeV}^{-2}$$

$$\beta = 3.06 \text{ GeV}^{-2}$$

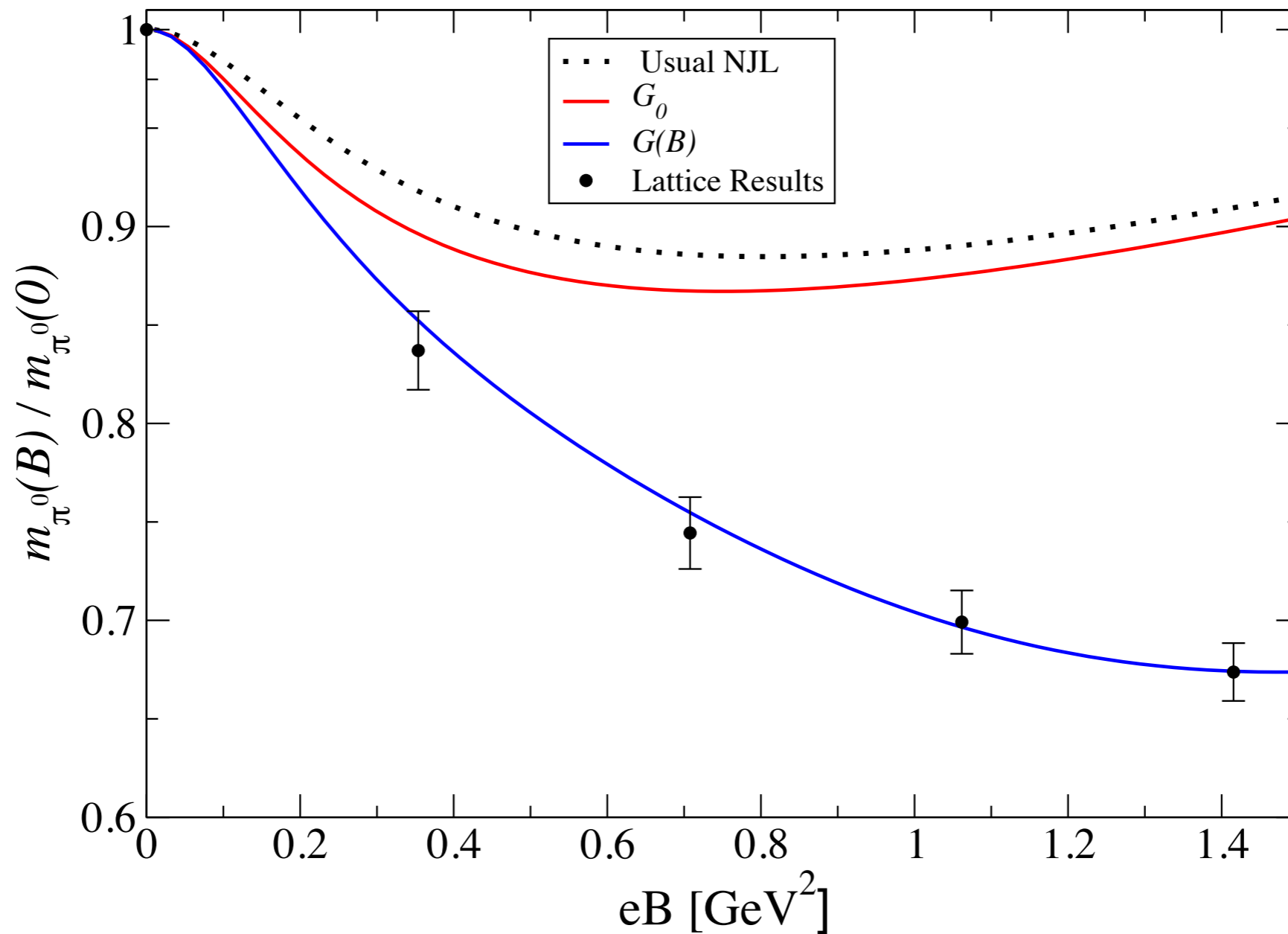
$$\gamma = 1.31 \text{ GeV}^{-4}$$

lattice results: Phys. Rev. D 86, 071502(R) (2012)

$G(B)$  in Farias et al., Phys. Lett. B 767 (2017) 247–252



# Meson masses at finite B



Farias et al., Phys. Lett. B 767 (2017) 247–252

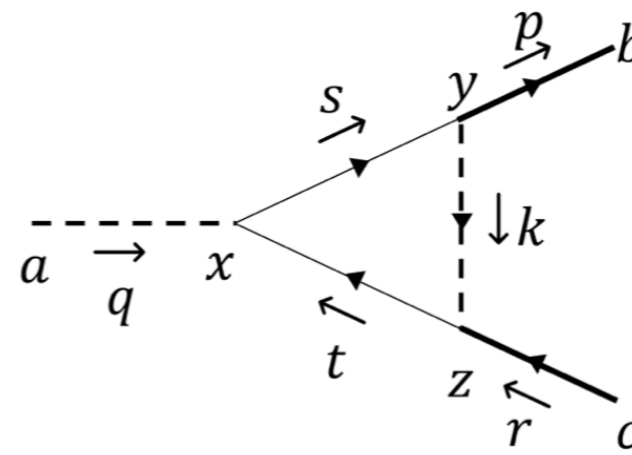
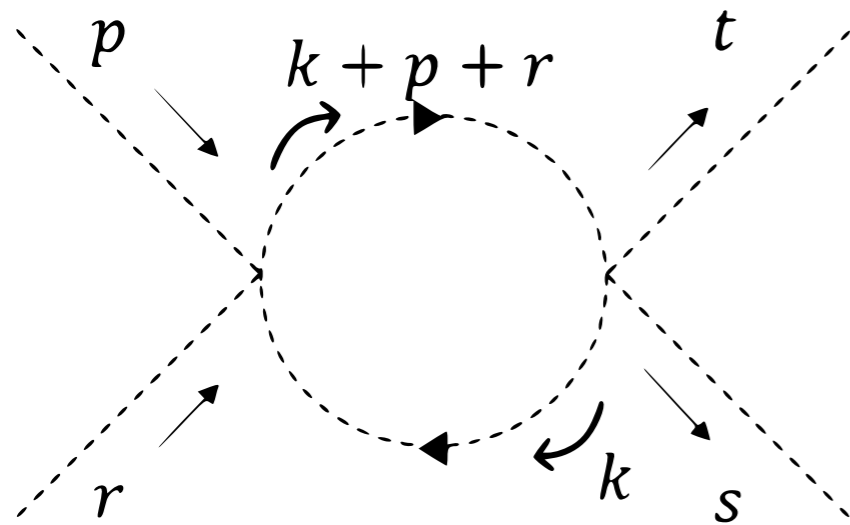
Lattice data: G. Bali, B.B. Brandt, G. Endrodi, B. Glaesle, arXiv:1510.03899 [hep-lat]

# Linear Sigma model with quarks - LSMq

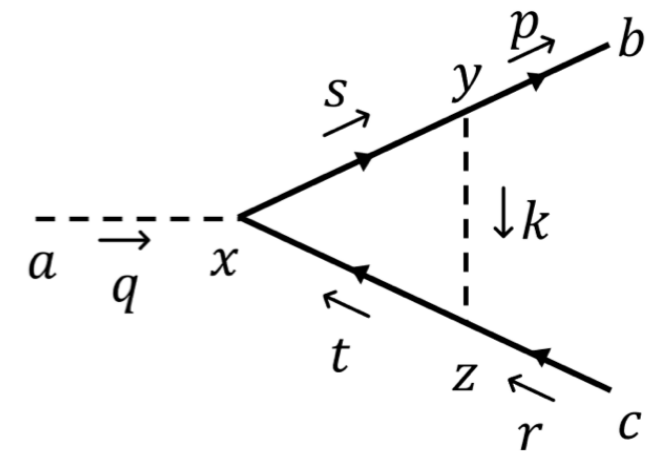
$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 \\ &+ i\bar{\psi}\gamma^\mu\partial_\mu\psi - ig\gamma^5\bar{\psi}\vec{\tau}\cdot\vec{\pi}\psi - g\bar{\psi}\psi\sigma\end{aligned}$$

- Pions are described by an isospin triplet,  $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ ;
- Two species of quarks are represented by an  $SU(2)$  isospin doublet,  $\psi$ ;
- The  $\sigma$  scalar is included by means of an isospin singlet;
- $\lambda$  is the boson self-coupling;
- $g$  is the fermion-boson coupling;
- $a^2 > 0$  is the mass parameter.

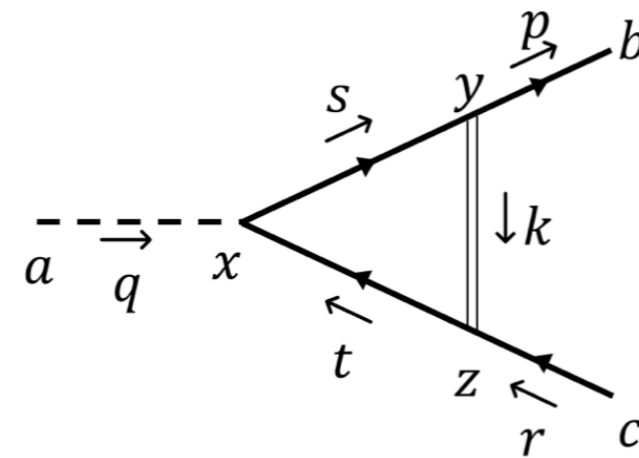
# Magnetic corrections to the boson self-coupling and boson-fermion coupling in the LSMq



(a)

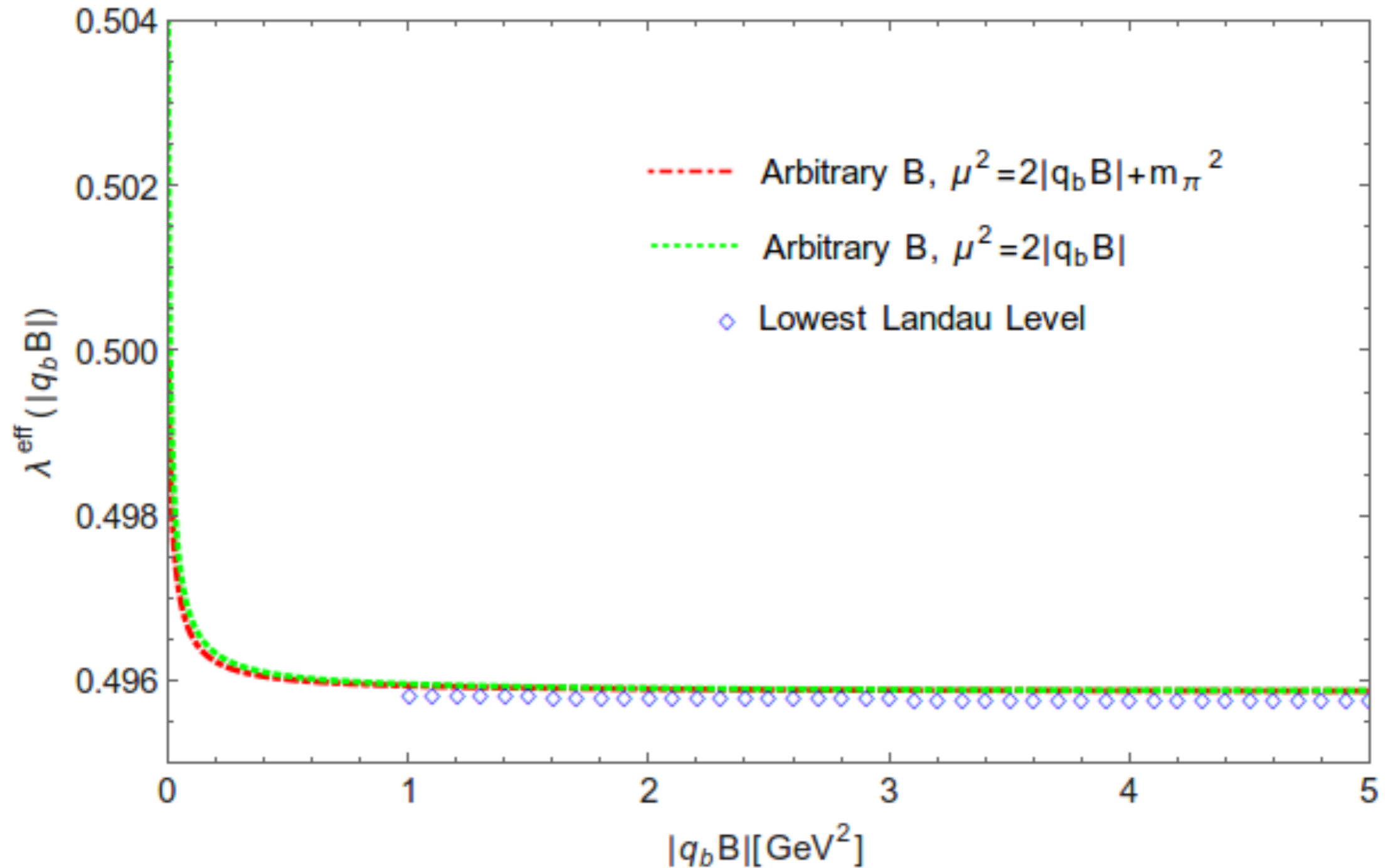


(b)



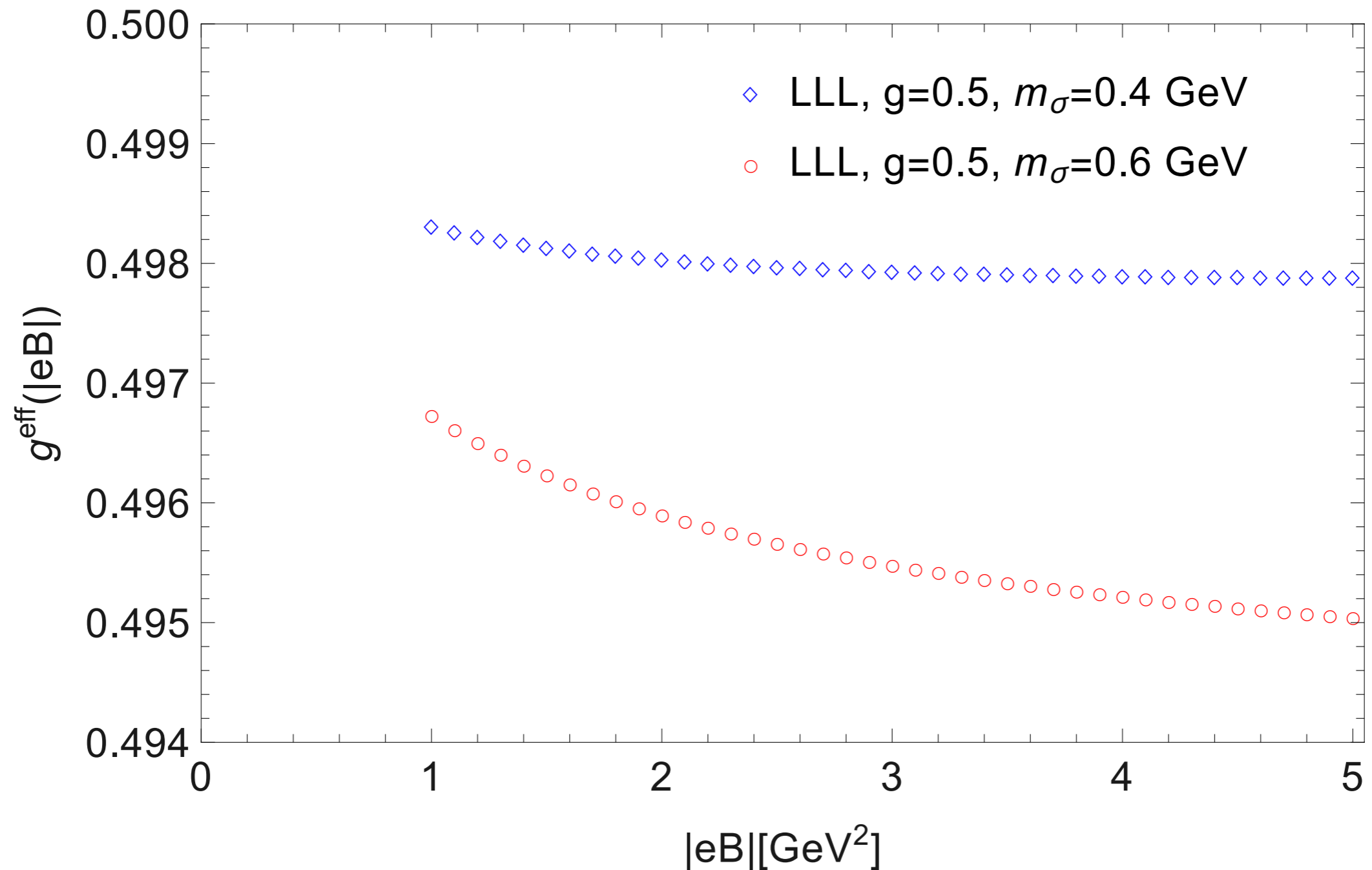
(c)

# Magnetic corrections to the boson self-coupling and boson-fermion coupling in the LSMq



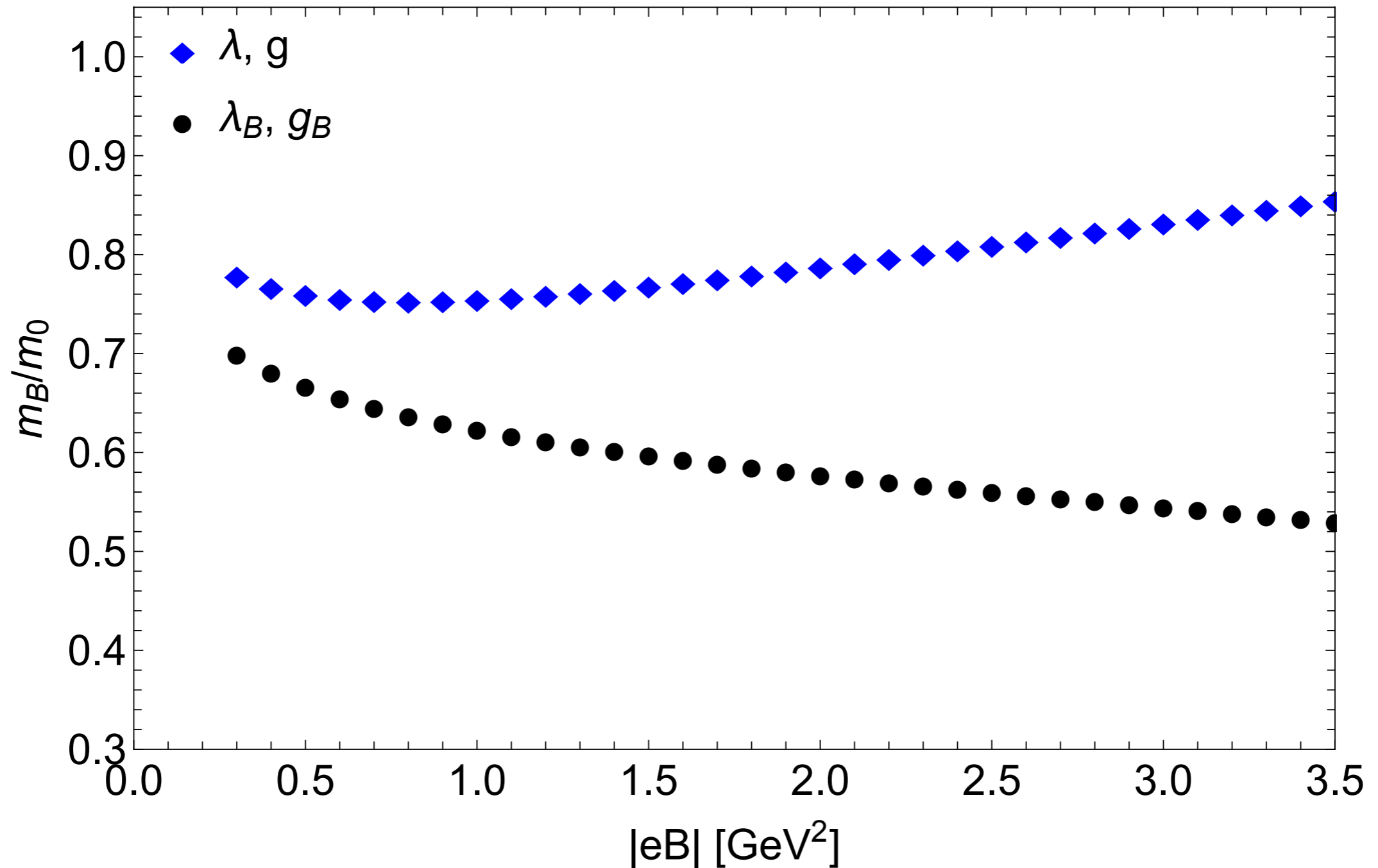
A. Ayala, J.L. Hernández, L. A. Hernández, R.L. S. Farias, R. Zamora, *Phys.Rev.D* 102 (2020) 11, 114038, arXiv:2009.13740 [hep-ph],

# Magnetic corrections to the boson self-coupling and boson-fermion coupling in the LSMq



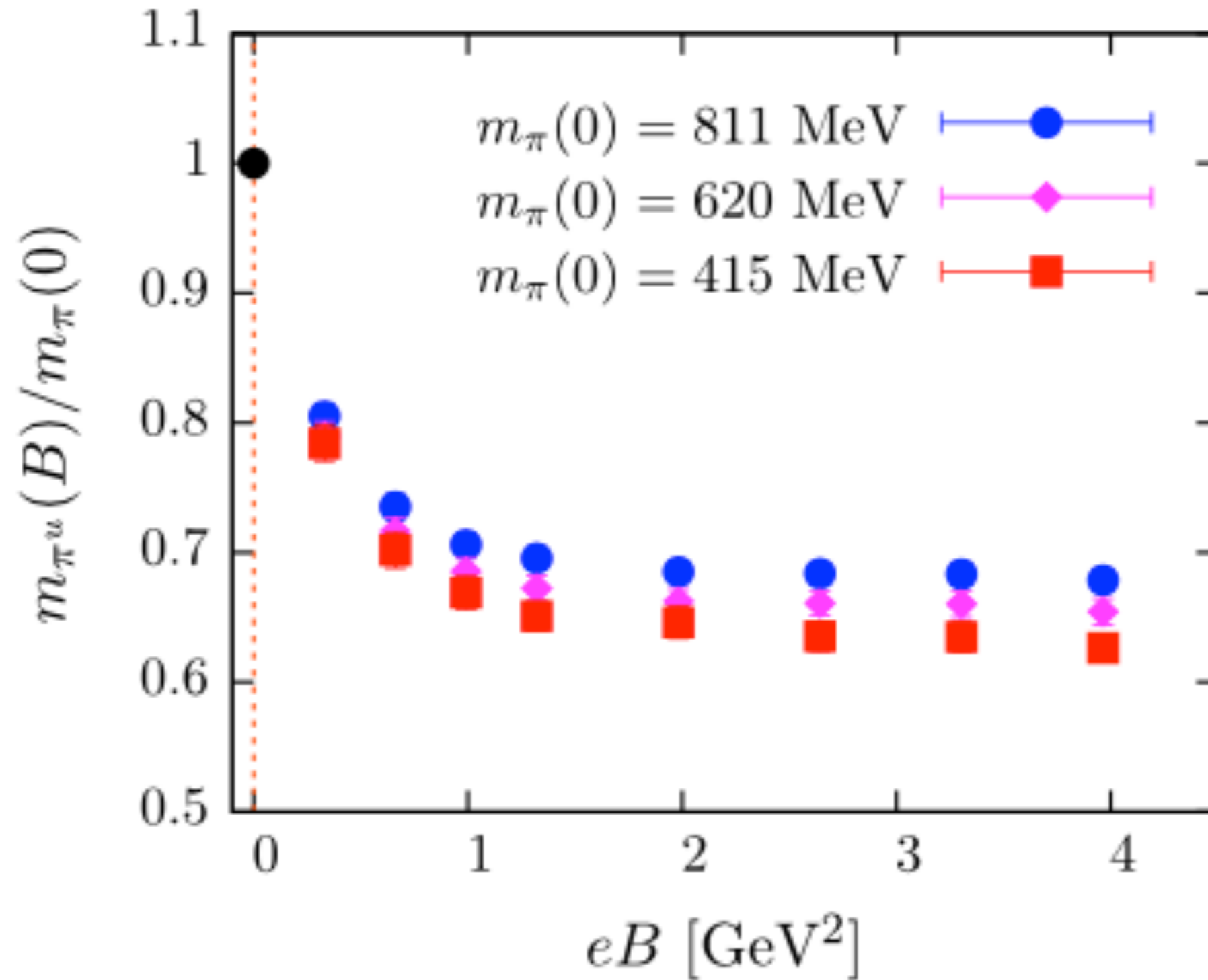
A.Ayala, J.L.Hernández, L. A. Hernández, R.L. S. Farias, R. Zamora, *Phys.Rev.D* 102 (2020) 11, 114038, arXiv:2009.13740 [hep-ph],

# Neutral pion mass $\chi$ eB

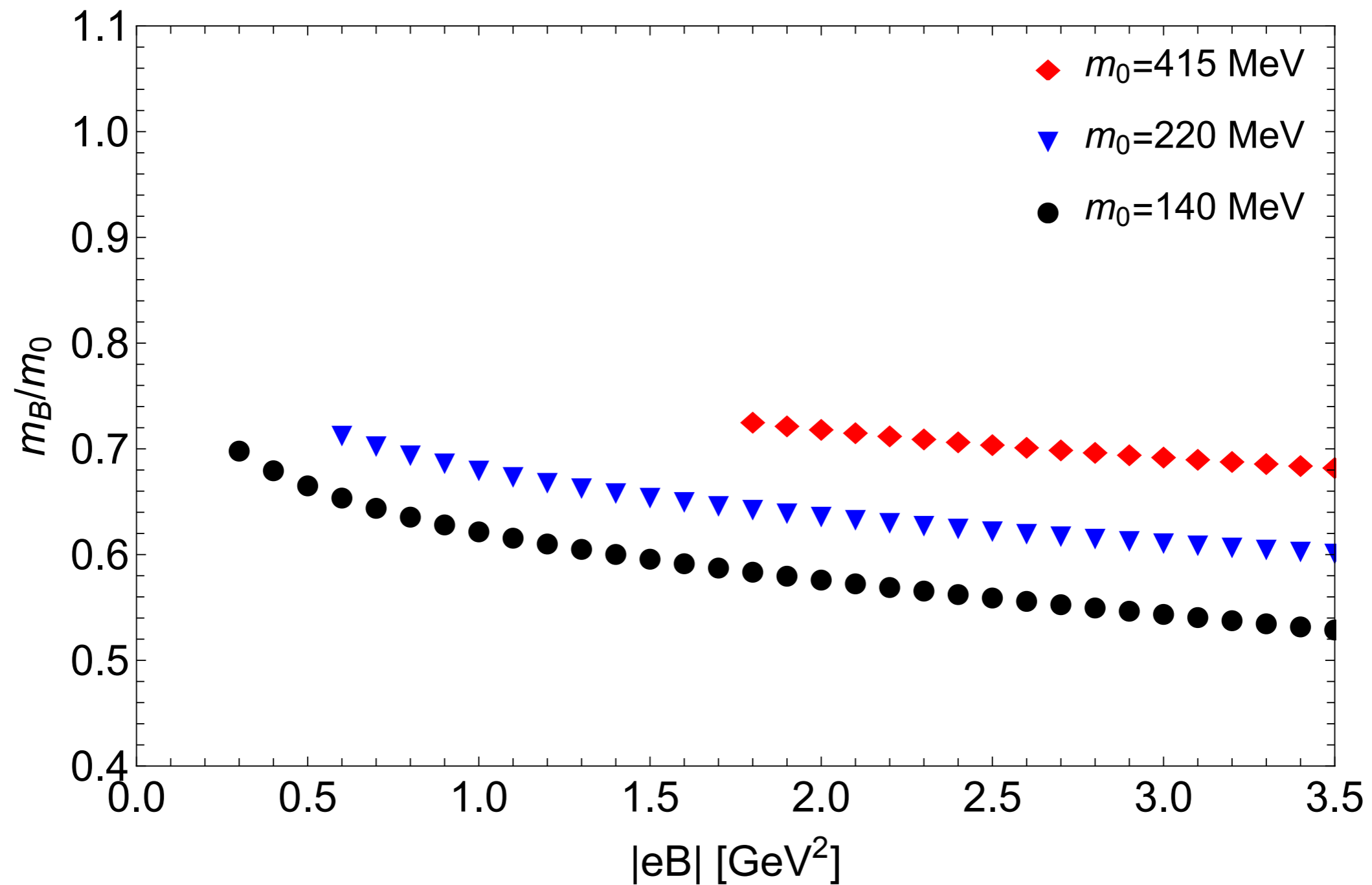


A. Ayala, J.L. Hernández, L. A. Hernández, R.L. S. Farias, R. Zamora, *Phys.Rev.D* 103 (2021) 5, 054038, e-Print: [2011.03673](#) [hep-ph].

# Lattice



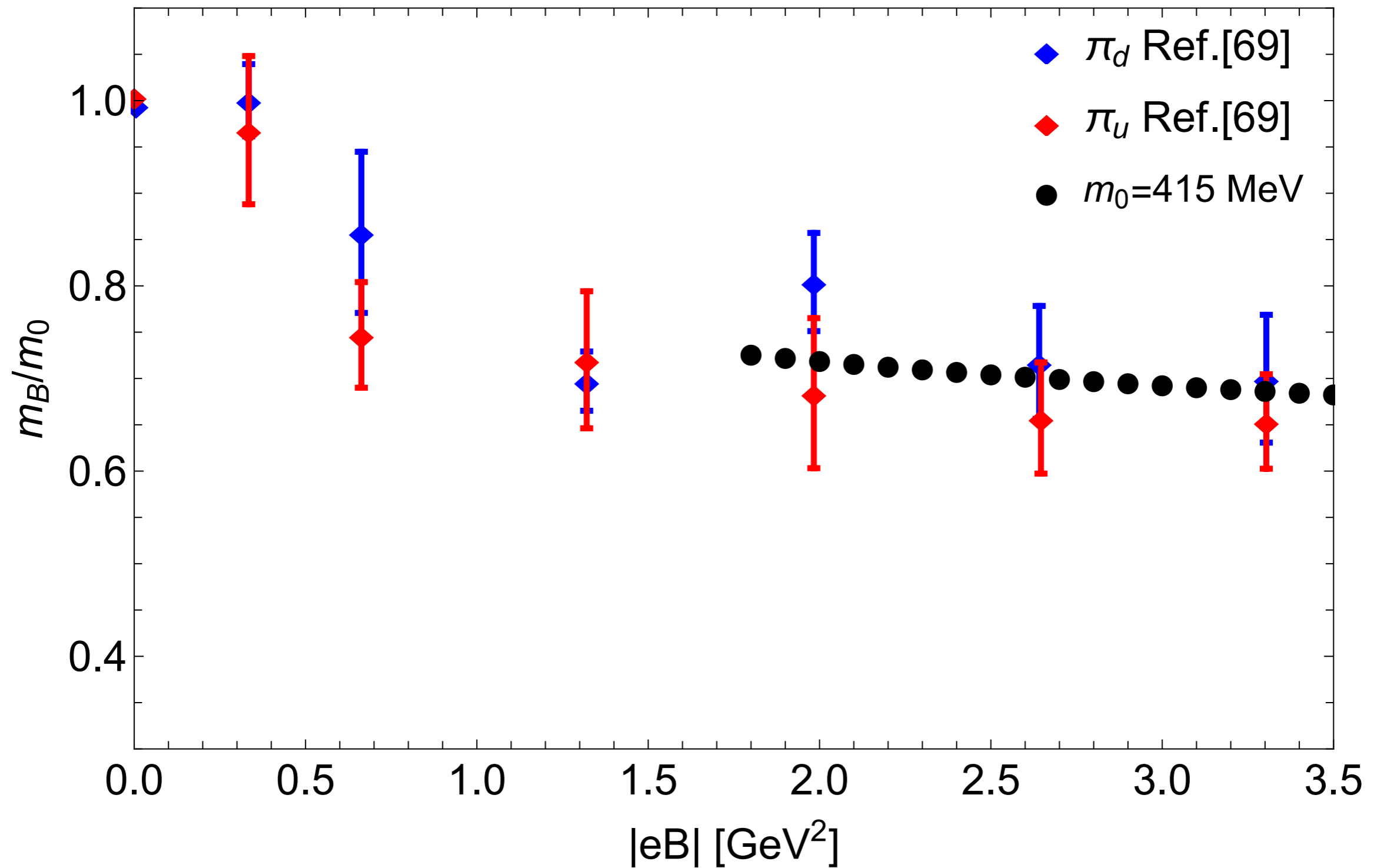
# LSMq strong field



A. Ayala, J.L. Hernández, L. A. Hernández, R.L. S. Farias, R. Zamora, *Phys.Rev.D* 103 (2021) 5, 054038, e-Print: [2011.03673](https://arxiv.org/abs/2011.03673) [hep-ph].



# LSMq strong field

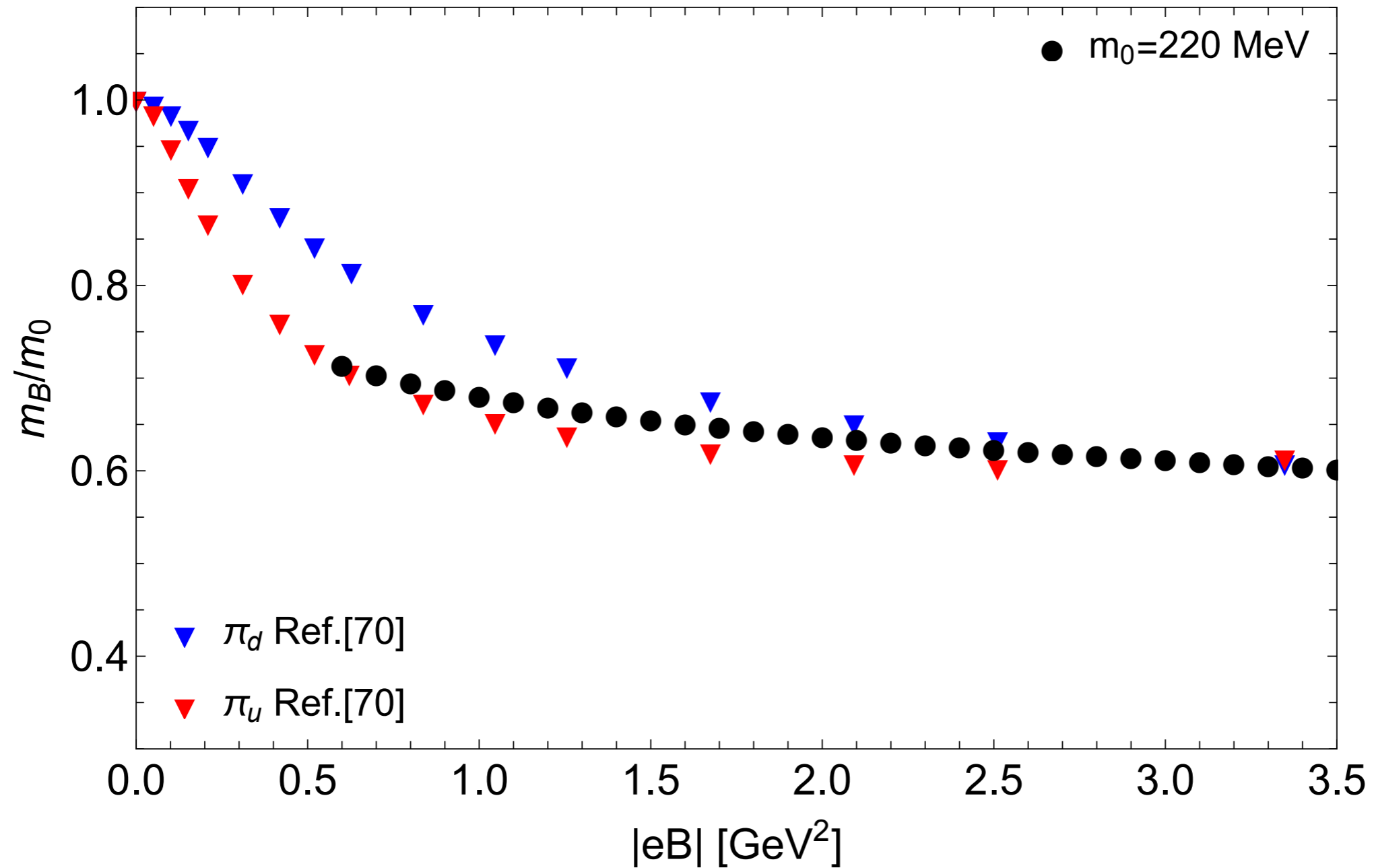


Lattice data

A. Ayala, J.L. Hernández, L. A. Hernández,  
R.L. S. Farias, R. Zamora, *Phys.Rev.D* 103 (2021)  
5, 054038, e-Print: [2011.03673](https://arxiv.org/abs/2011.03673) [hep-ph].

G.S. Bali, B.B. Brandt, G. Endrodi and  
B.Glaessle, *Phys. Rev D* 97, 034505 (2018)

# LSMq strong field



Lattice data

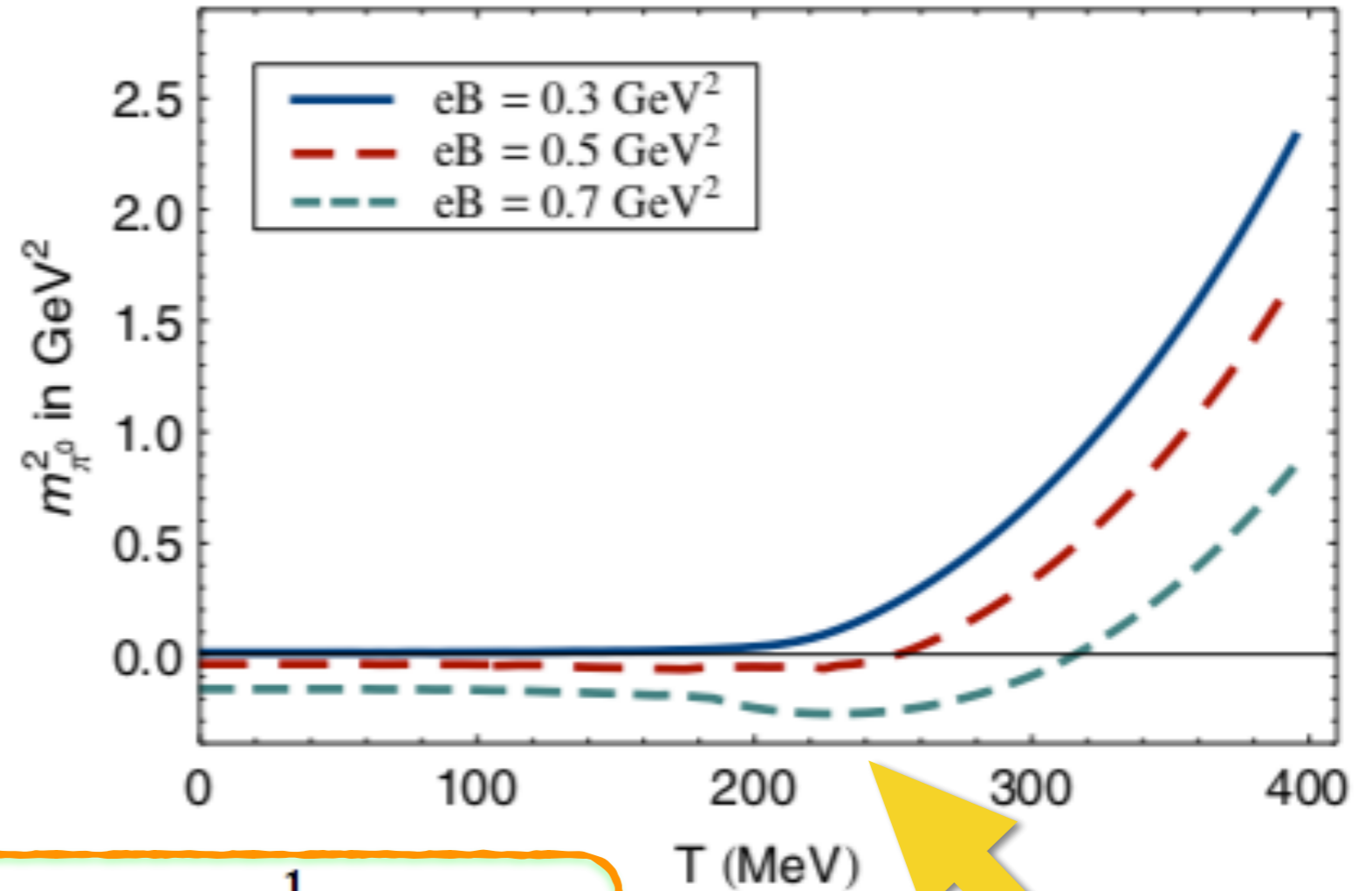
A. Ayala, J.L. Hernández, L. A. Hernández,  
R.L. S. Farias, R. Zamora, *Phys.Rev.D* 103 (2021)  
5, 054038, e-Print: [2011.03673](https://arxiv.org/abs/2011.03673) [hep-ph].

H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang  
and Y. Zhang, *Phys. Rev. D* 104, 014505 (2021).

Meson masses + T + B  
using NJL

# Neutral pion mass + B + T

**SU(2) NJL  
+ B + T**



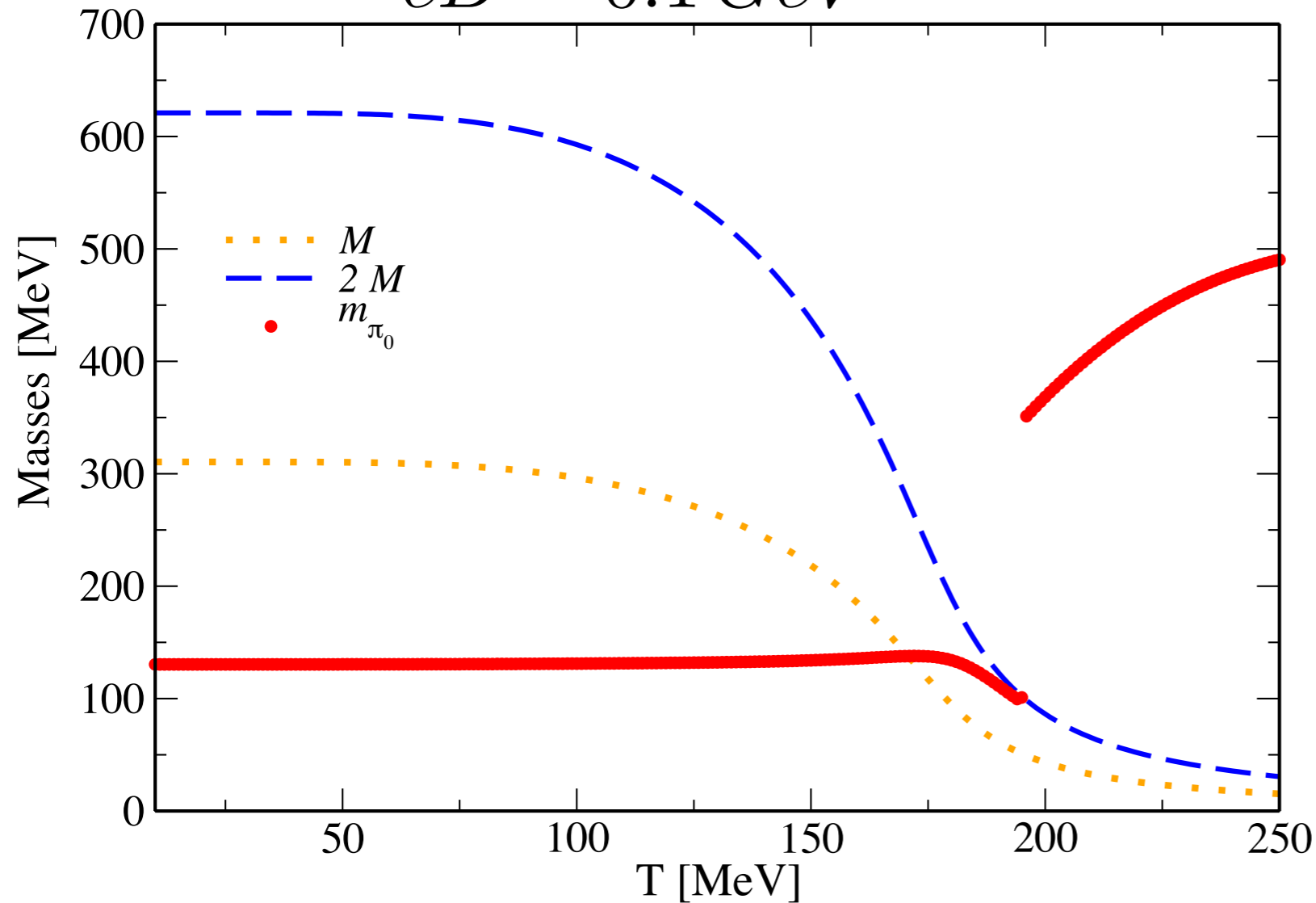
$$f_{\Lambda} = \frac{1}{1 + \exp\left(\frac{|\mathbf{p}| - \Lambda}{A}\right)},$$

$$f_{\Lambda, B}^p = \frac{1}{1 + \exp\left(\frac{\sqrt{p_3^2 + 2|qeB|p} - \Lambda}{A}\right)}$$

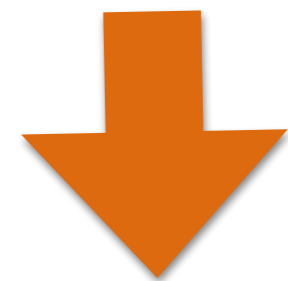
**Tachyonic Instabilities**

# Neutral pion mass $\times$ $eB$ + MFIR

$$eB = 0.1 \text{ GeV}^2$$



**More energetic resonances that we obtain as we increase the magnetic field**



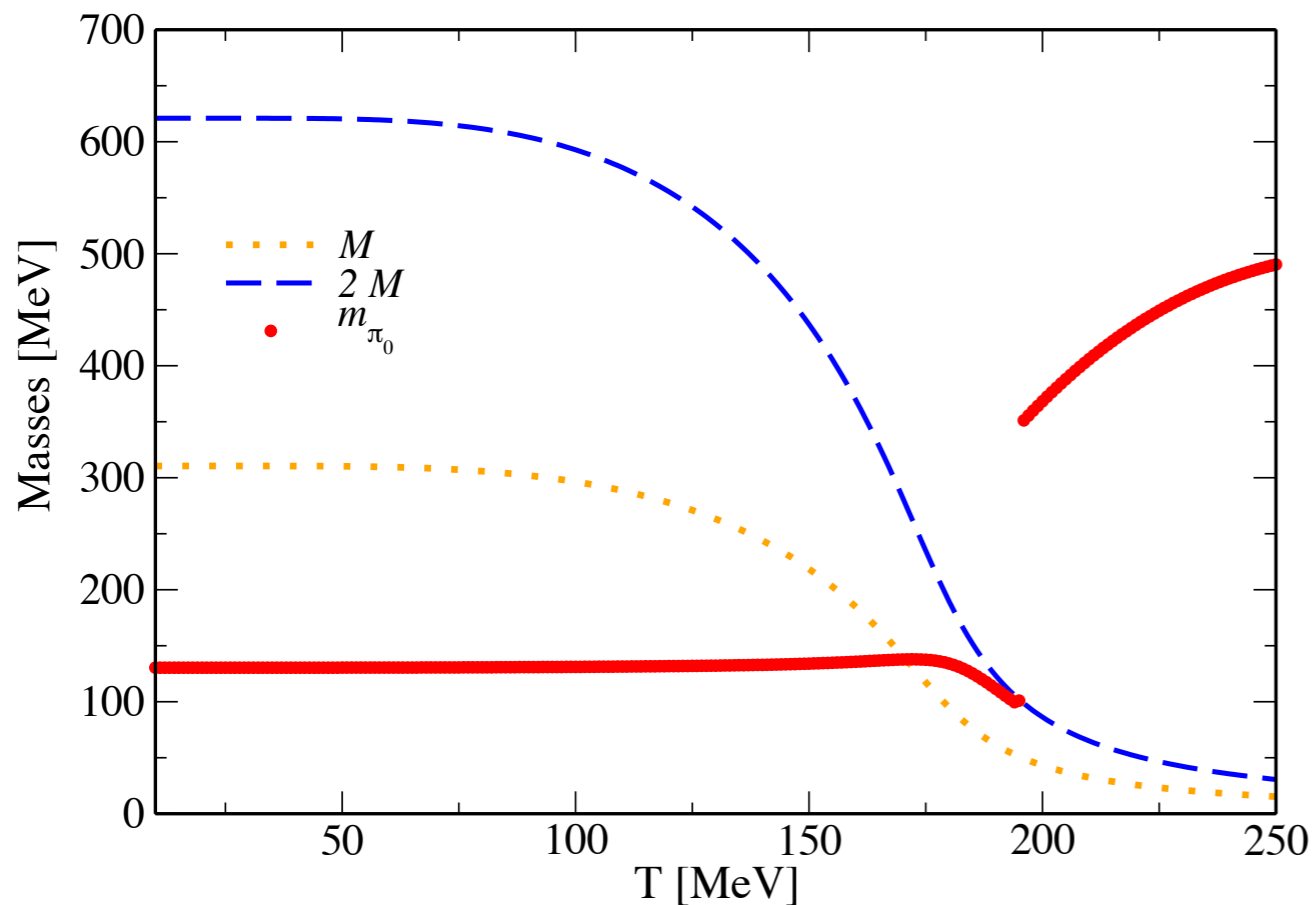
**dimensional reduction**

**No Tachyonic Instabilities**

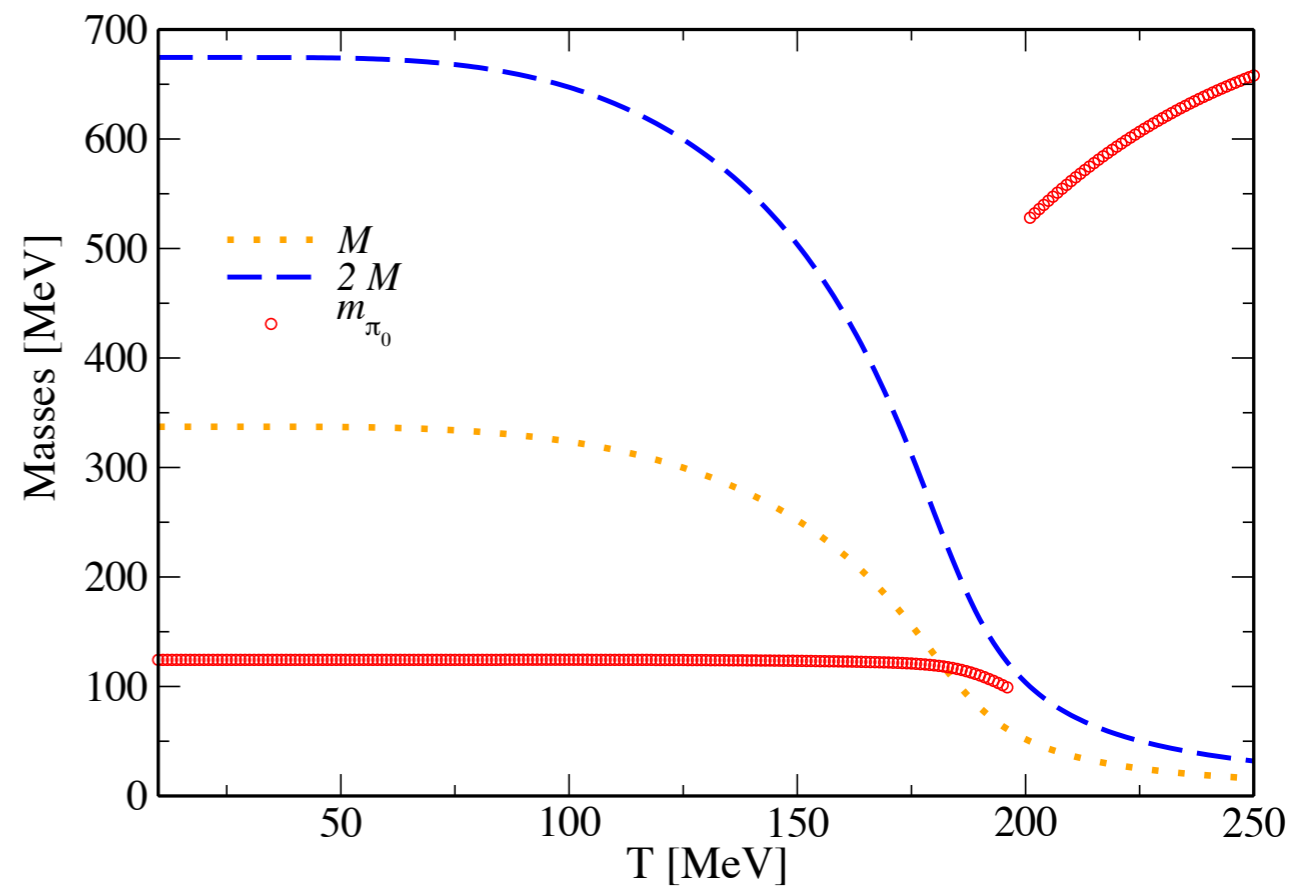
# Neutral pion mass $\times eB$

**No Tachyonic Instabilities!!!**

$$eB = 0.1 \text{ GeV}^2$$



$$eB = 0.2 \text{ GeV}^2$$



**T Mott increase with B using MFIR approach!**

# Conclusions

- ✓ We use results from lattice simulations of QCD in the presence of intense magnetic fields as a benchmark platform for comparing different regularization procedures used in the literature for the NJL type models MFIR X nMFIR
- ✓ MFIR scheme avoid some unphysical results, and this choice of regularization provide to us some different results from most of the regularizations prescriptions of the current literature.
- ✓ At  $T=0$  meson masses evaluate using NJL and LSMq are in agreement with lattice when their coupling constants depend on  $B$ ;

# Conclusions

- ✓ Mott dissociation temperature is catalyzed with the increase of  $B$
- ✓ The dramatic result is the more energetic resonances that we obtain as we increase the magnetic field. The  $\pi$  meson at the Mott dissociation temperature jumps to a resonance in a degenerate state with the  $\sigma$  meson.
- ✓ This is a direct result from the dimensional reduction of the system at strong magnetic fields that enforces the system to go to another state, since we have less states to the creation of the thermal  $q - q^-$  excitation.



# Perspectives

- ✓ Inclusion of thermo-magnetic effects in LSMq
- ✓ Charged mesons: NJL and LSMq

**Thank you for your attention!**