

**ROUND TABLE ON**

# Open questions in Quarkonium Suppression

Moderator: Nora Brambilla (TUM)

Panel members :

Chris Allton (Swansea U.): Lattice

Mike Strickland (Kent U.): Hydro, Effective Field theory  
and Open Quantum Systems. Linblad equation

Xiaojun Yao (MIT): Effective field theory and Open Quantum System,  
Boltzmann equation, Quantum computing

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In this panel we will discuss the state of the art and after that the related open  
questions!

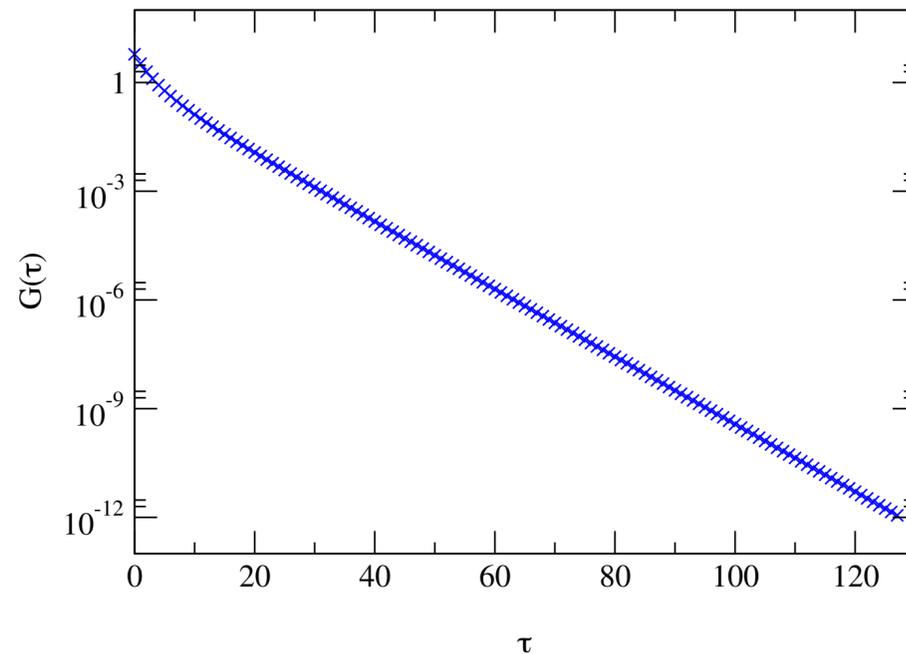
# Lattice Determinations of Quarkonia Width

Extracting Spectral F'ns

Euclidean Lattice Correlator  $\rightarrow G(\tau) = \int d\omega K(\omega, \tau) \rho(\omega) \leftarrow$  Spectral F'n

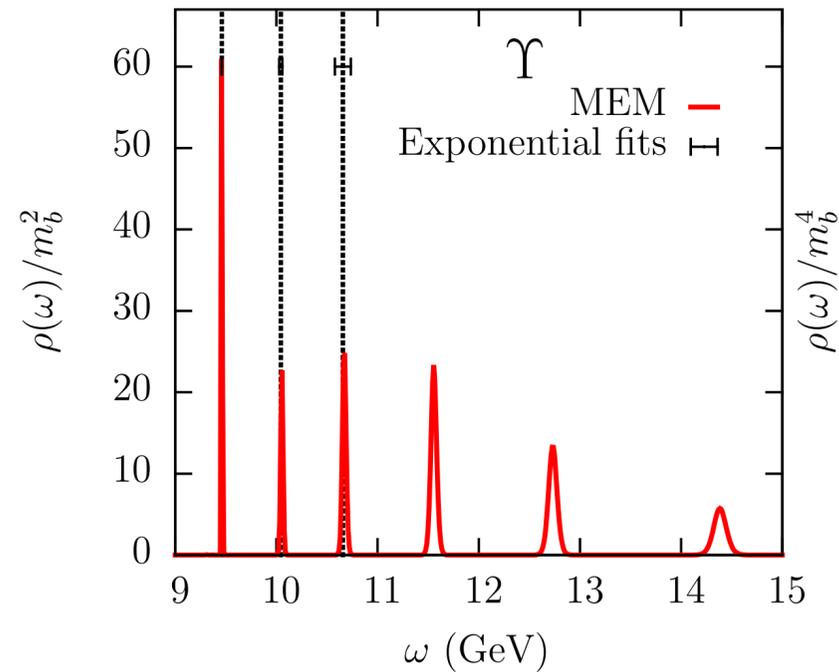
Input Data: **AND they are correlated!**

$G_{\pm}(\tau), \tau = 1, \dots, \mathcal{O}(10 - 100)$



Output Data:

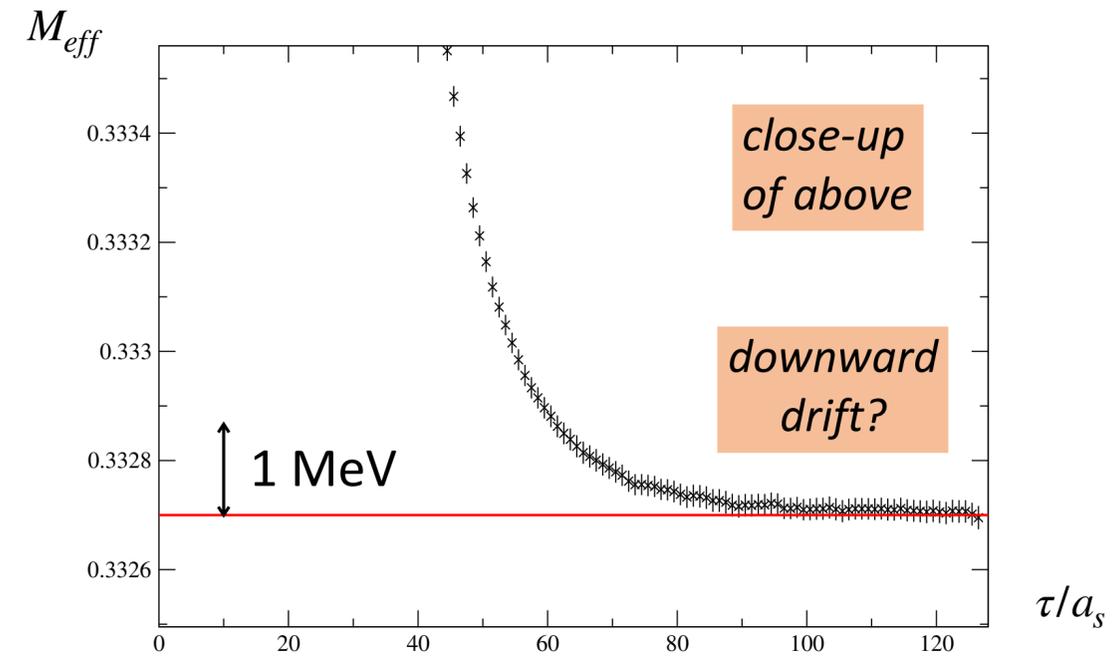
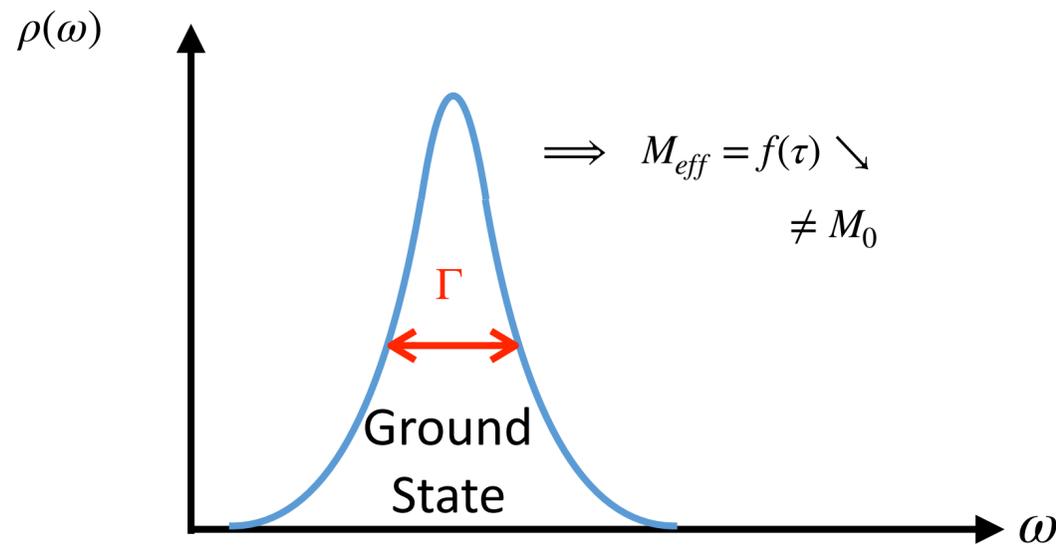
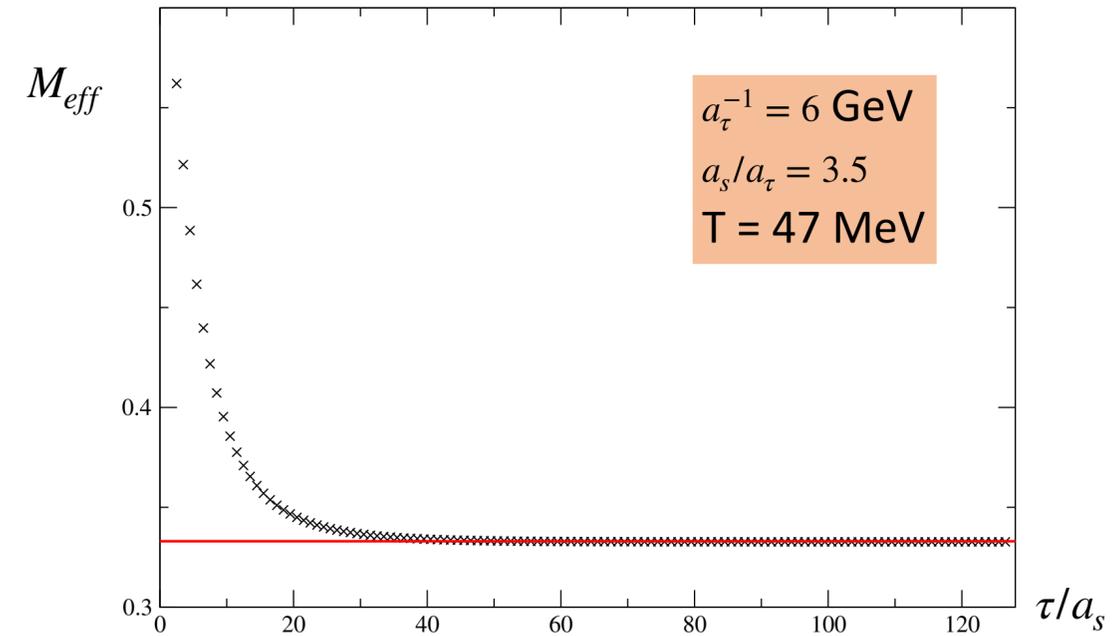
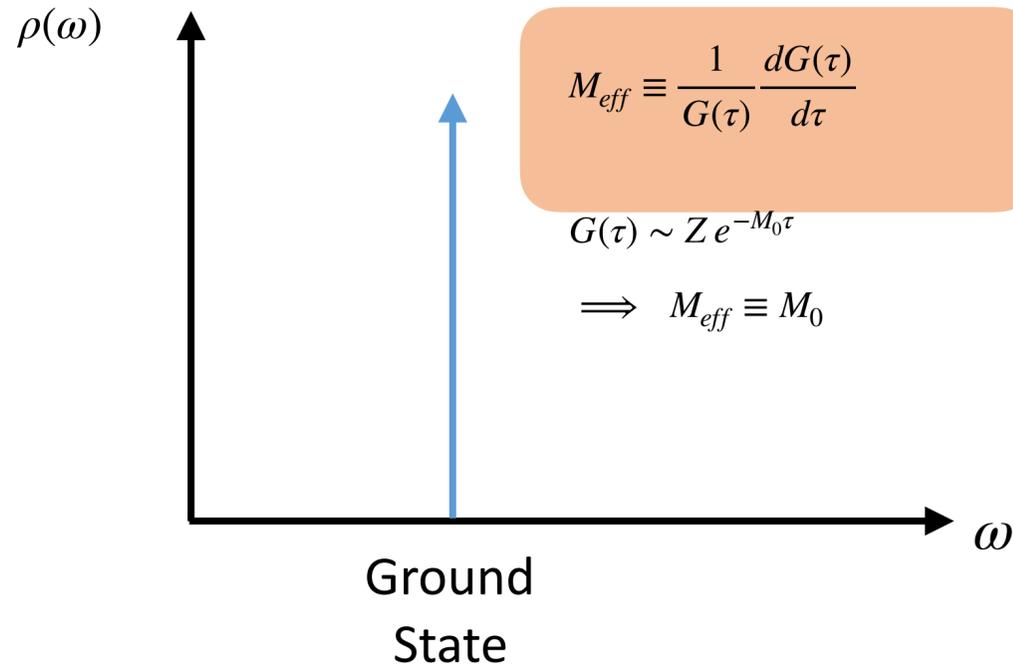
$\rho_{\pm}(\omega), \omega \sim 1, \dots, \mathcal{O}(1000)$



**ill-posed! i.e.  $\infty$  solutions with  $\chi^2 = 0$**

An allegory of life: You can't get more out than you put in.

# Correlation Functions

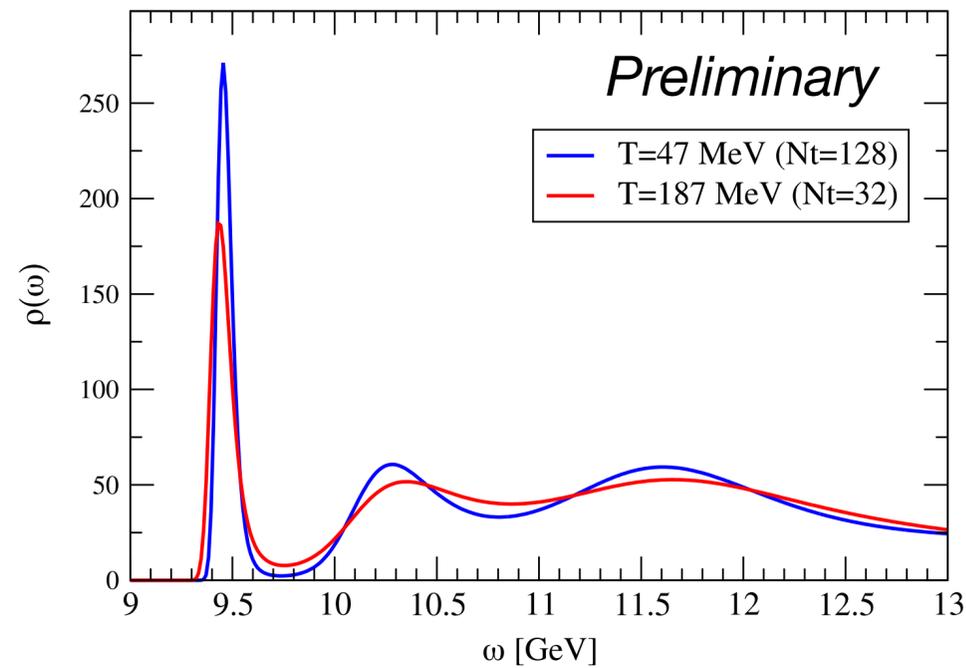


Finite  $N_\tau \Rightarrow$  Finite Resolution  $\Rightarrow \Gamma$  is upper bound

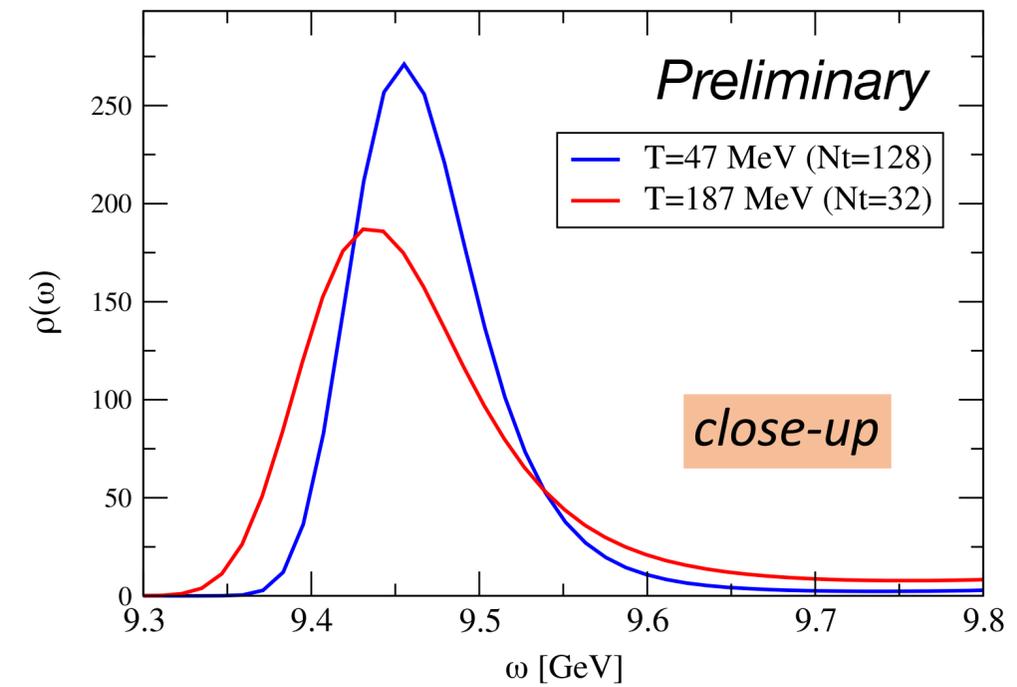
# Apples and Apples

Systematic effects in T

Fitting:  $\tau = [2,30]$  for *both* T



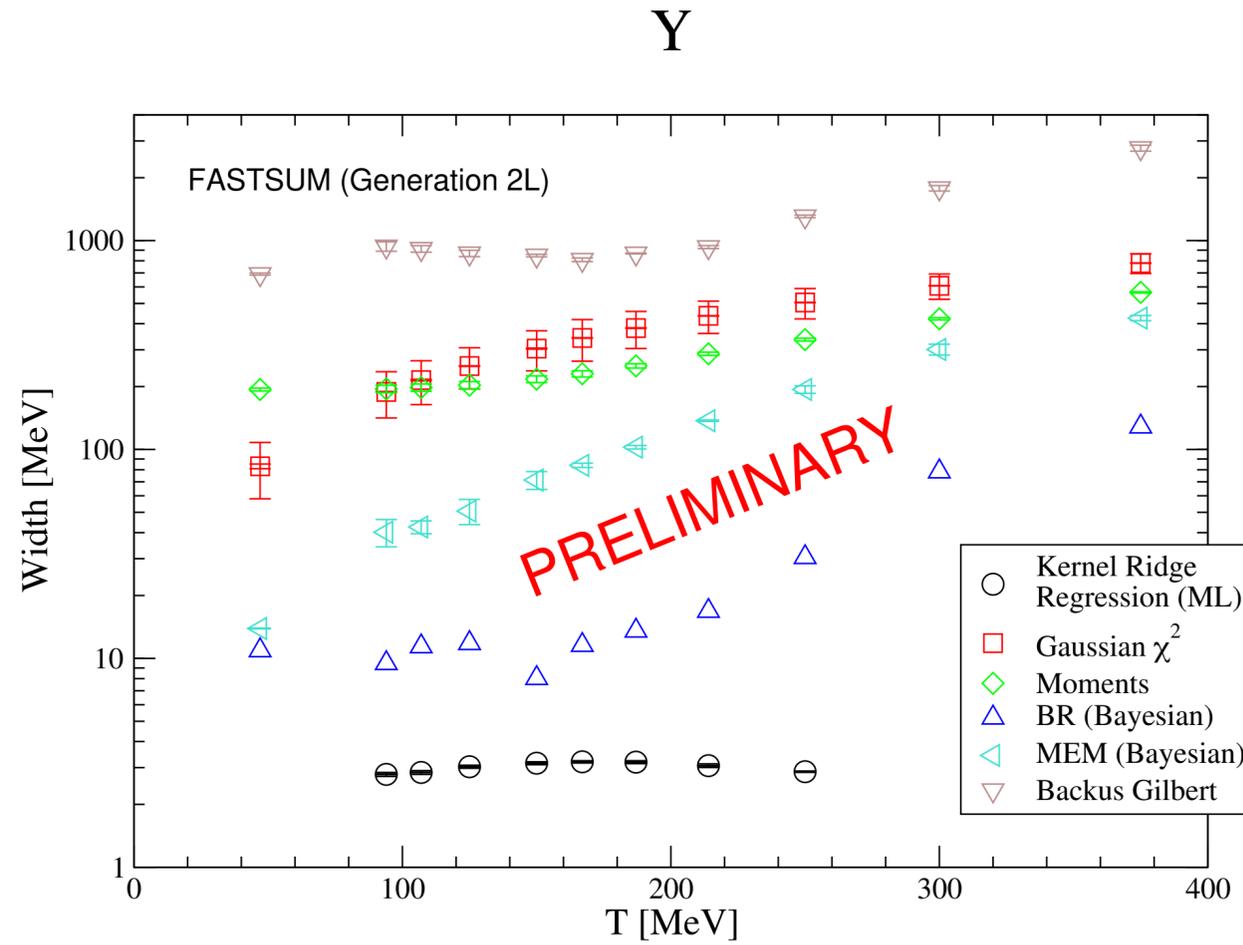
Sequential suppression



$\Gamma \nearrow$  as  $T \nearrow$

Although  $\Gamma$  is *upper bound*, we can resolve thermal trends

# Comprehensive Study of Systematics from Analysis Techniques

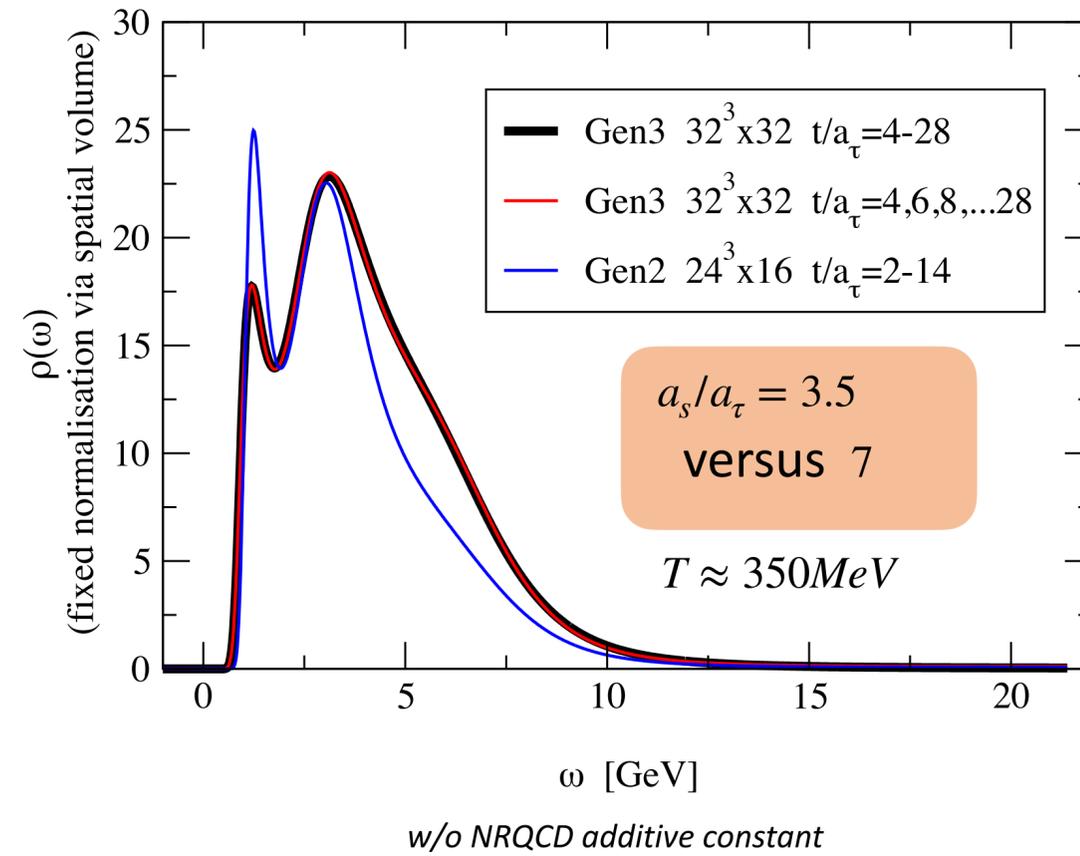
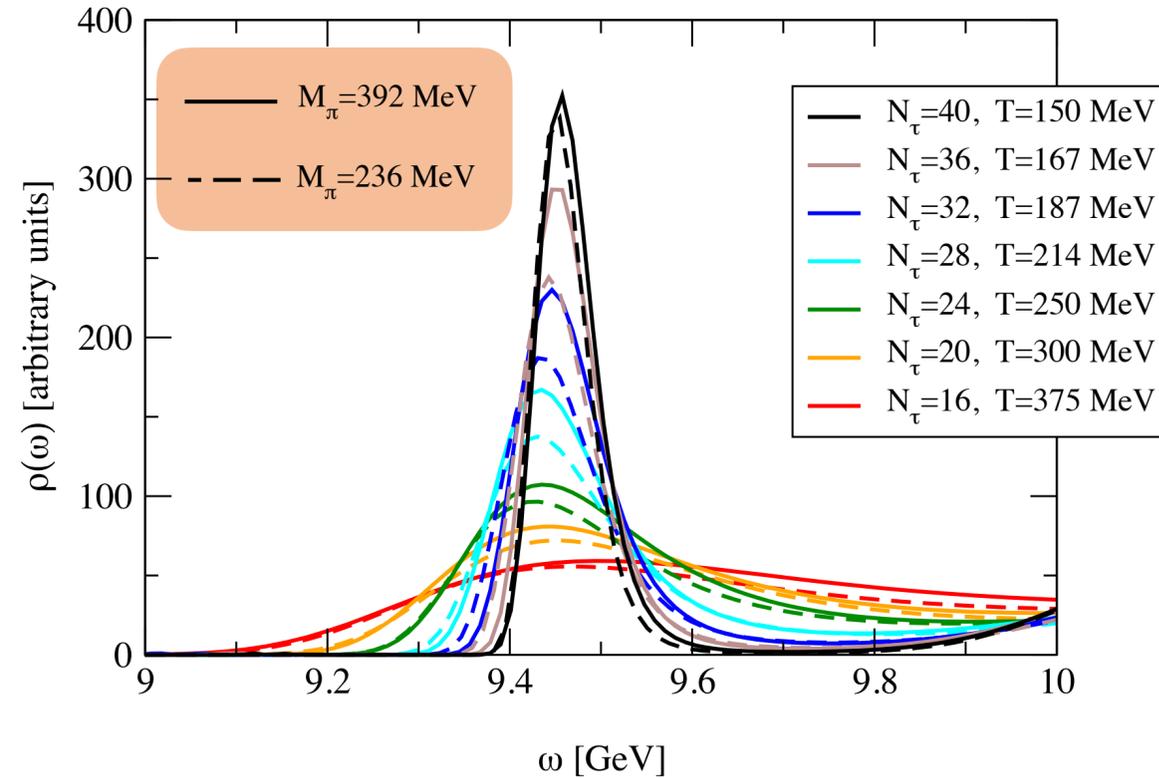


Quantity	Order of Difficulty
$M_0$	Easy $O(1)$
$\Gamma$	Difficult $O(2)$
Line Shape	Very Difficult! $O(3)$

# Lattice systematics - are “small”

Going lighter  $m_q \searrow$

Going finer  $a_\tau \searrow$



Preliminary

# Open quantum system (OQS) approach



**Probe** = heavy-quarkonium state

**Medium** = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\text{tot}} = H_{\text{probe}} \otimes I_{\text{medium}} + I_{\text{probe}} \otimes H_{\text{medium}} + H_{\text{int}}$$

- Total density matrix

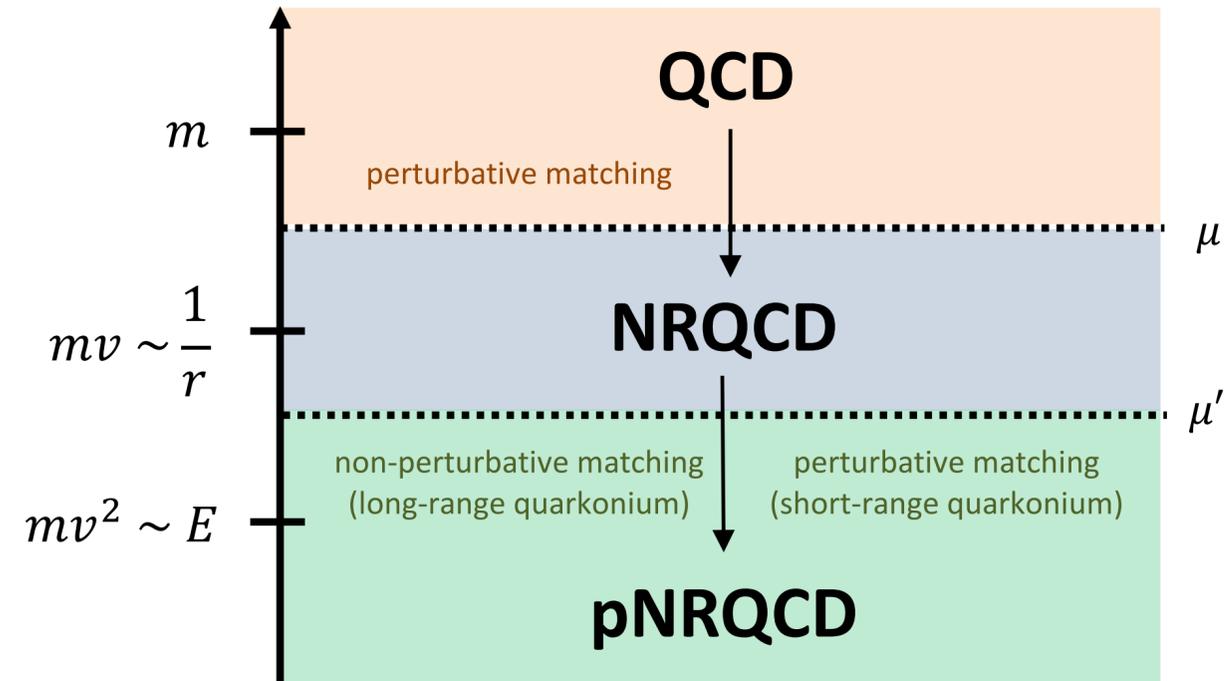
$$\rho_{\text{tot}} = \sum_j p_j |\psi_j\rangle \langle \psi_j| \quad \longrightarrow \quad \frac{d}{dt} \rho_{\text{tot}} = -i[H_{\text{tot}}, \rho_{\text{tot}}]$$

- Reduced density matrix

$$\rho_{\text{probe}} = \text{Tr}_{\text{medium}}[\rho_{\text{tot}}] \quad \longrightarrow \quad \text{“Master equation”}$$

# OQS + pNRQCD → Lindblad equation

- What are the relevant scales?
  - Temperature  $T$
  - Bound state mass  $m \gg T$
  - Bound state size  $r \sim 1/mv \sim a_0$  (Bohr radius)
  - Debye mass  $m_D$
  - Binding energy  $E \sim mv^2$



- Separation of time scales

- Medium relaxation time scale  $\langle \hat{O}_M(t) \hat{O}_M(0) \rangle \sim e^{-t/t_M} \rightarrow \frac{1}{T}$
- Intrinsic probe time scale  $t_P \sim \frac{1}{\omega_i - \omega_j} \rightarrow \frac{1}{E}$
- Probe relaxation time scale  $\langle p(t) \rangle \sim e^{-t/t_{rel}} \rightarrow \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3} \quad \Lambda = T, E$

$$1/r \gg T \sim m_D \gg E$$

→

$$t_{rel}, t_P \gg t_M$$

$$\frac{d\rho_{\text{probe}}}{dt} = -i[H_{\text{probe}}, \rho_{\text{probe}}] + \sum_n \left( C_n \rho_{\text{probe}} C_n^\dagger - \frac{1}{2} \{C_n^\dagger C_n, \rho_{\text{probe}}\} \right)$$

# OQS + pNRQCD Hamiltonian and collapse operators

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

STRICKLAND

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^\dagger + \sum_n C_n \rho_{\text{probe}} C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H_{\text{probe}} = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2 - 2}{2(N_c^2 - 1)} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2 - 1}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix},$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2 - 2}{2(N_c^2 - 1)} \end{pmatrix} r^i$$

Total width  $\rightarrow \text{Im}[V]$   
 $H_{\text{eff}} = H_{\text{probe}} - \frac{i}{2}\Gamma$

$$C_i^1 = \sqrt{\frac{(N_c^2 - 4)\kappa}{2(N_c^2 - 1)}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

**Six collapse operators cover**

- singlet  $\rightarrow$  octet,
- octet  $\rightarrow$  singlet
- octet  $\rightarrow$  octet

$$\gamma \equiv \frac{g^2}{6 N_c} \text{Im} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

$$\kappa \equiv \frac{g^2}{6 N_c} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, \mathbf{0}) E^{a,i}(0, \mathbf{0}) \rangle$$

# A parallelizable approach: Quantum trajectories

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2012.01240

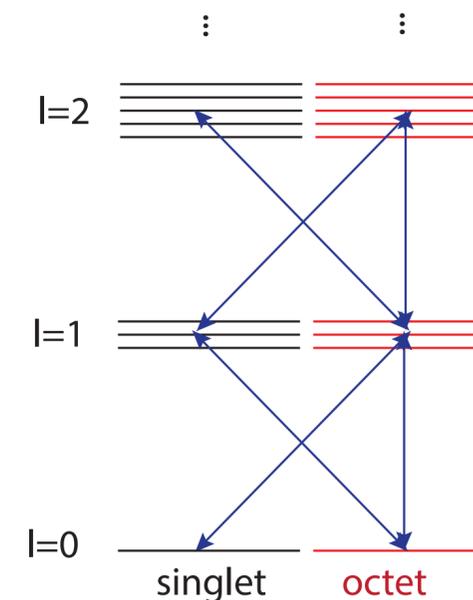
STRICKLAND

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^\dagger + \sum_n C_n \rho_{\text{probe}} C_n^\dagger$$

Non-unitary “no jump” evolution

Can treat this “quantum jump” term stochastically

- Can be reduced to the solution of a large set of “quantum trajectories” in which we solve a 1D Schrödinger equation with a **non-Hermitian Hamiltonian  $H_{\text{eff}}$** , subject to **stochastic quantum jumps**.
- The evolution with the non-Hermitian  $H_{\text{eff}}$  preserves the color and angular momentum state of the system (but not norm).
- Collapse/jump operators encode transitions between different color/angular momentum states (subject to selection rules).
- For each **physical trajectory** (path through the QGP) we average over a large set of **independent quantum trajectories** → **Embarrassingly parallel**
- **Added benefit: Can describe all angular momentum states (no cutoff) .**



# Coupled Boltzmann Equations of Heavy Flavors

open heavy quark antiquark

$$\left(\frac{\partial}{\partial t} + \dot{x}_Q \cdot \nabla_{x_Q} + \dot{x}_{\bar{Q}} \cdot \nabla_{x_{\bar{Q}}}\right) f_{Q\bar{Q}}(\mathbf{x}_Q, \mathbf{p}_Q, \mathbf{x}_{\bar{Q}}, \mathbf{p}_{\bar{Q}}, t) = C_{Q\bar{Q}} - C_{Q\bar{Q}}^+ + C_{Q\bar{Q}}^-$$

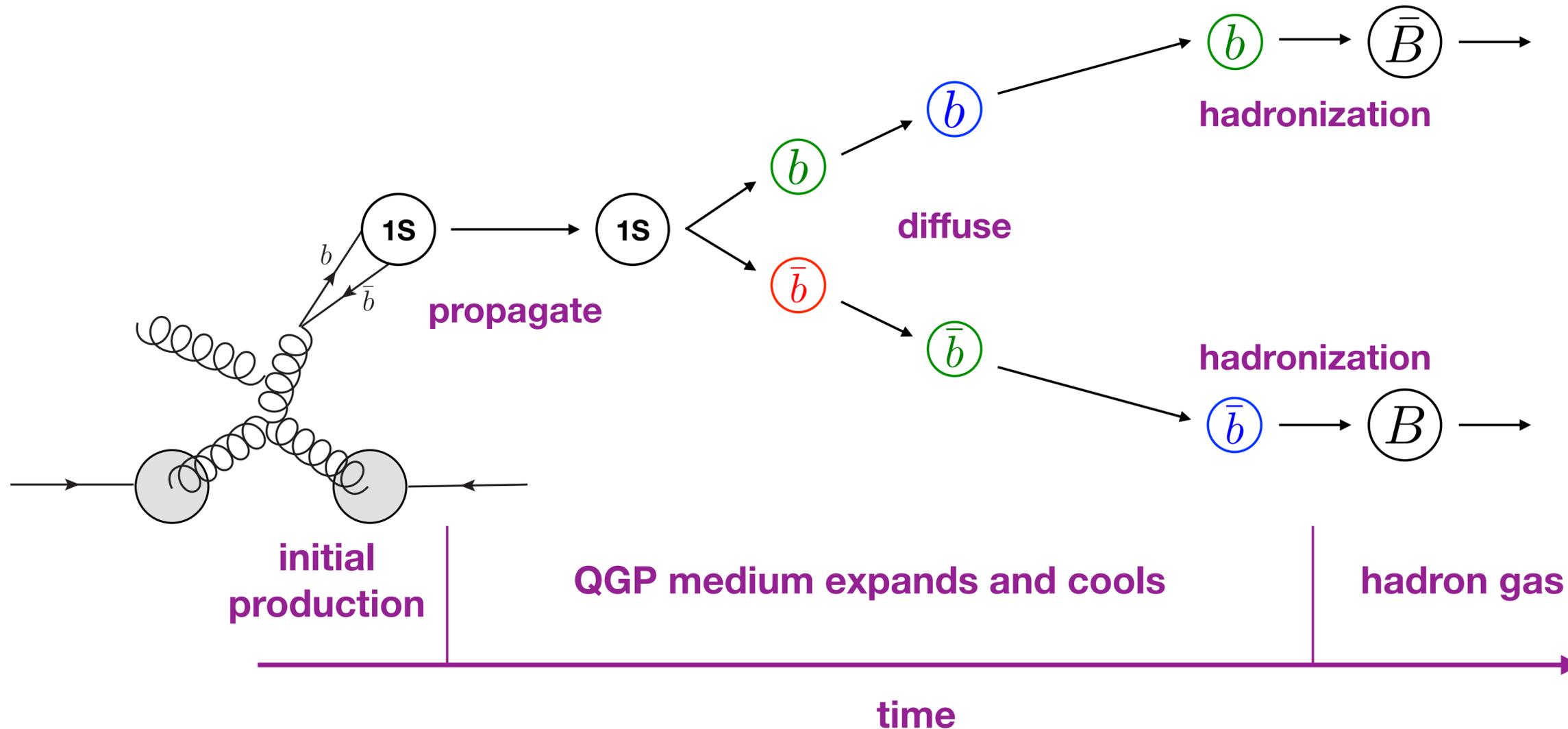
each quarkonium state  
nl = 1S, 2S, 1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x\right) f_{nls}(\mathbf{x}, \mathbf{p}, t) = C_{nls}^+ - C_{nls}^- \longrightarrow \text{Dissociation}$$

X.Yao, W.Ke, Y.Xu, S.A.Bass, B.Müller 2004.06746

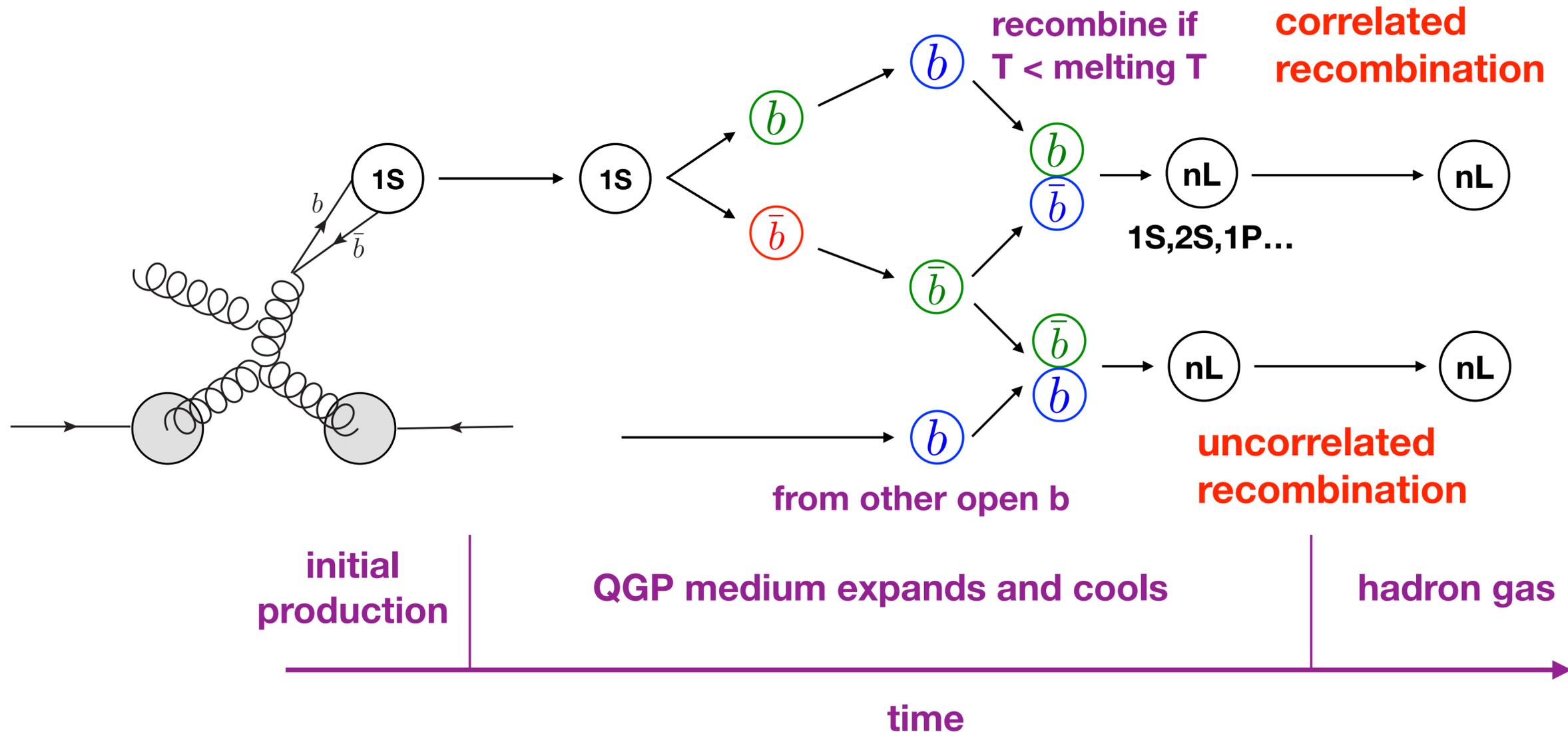
**Recombination**

**Diffusion of HQ**



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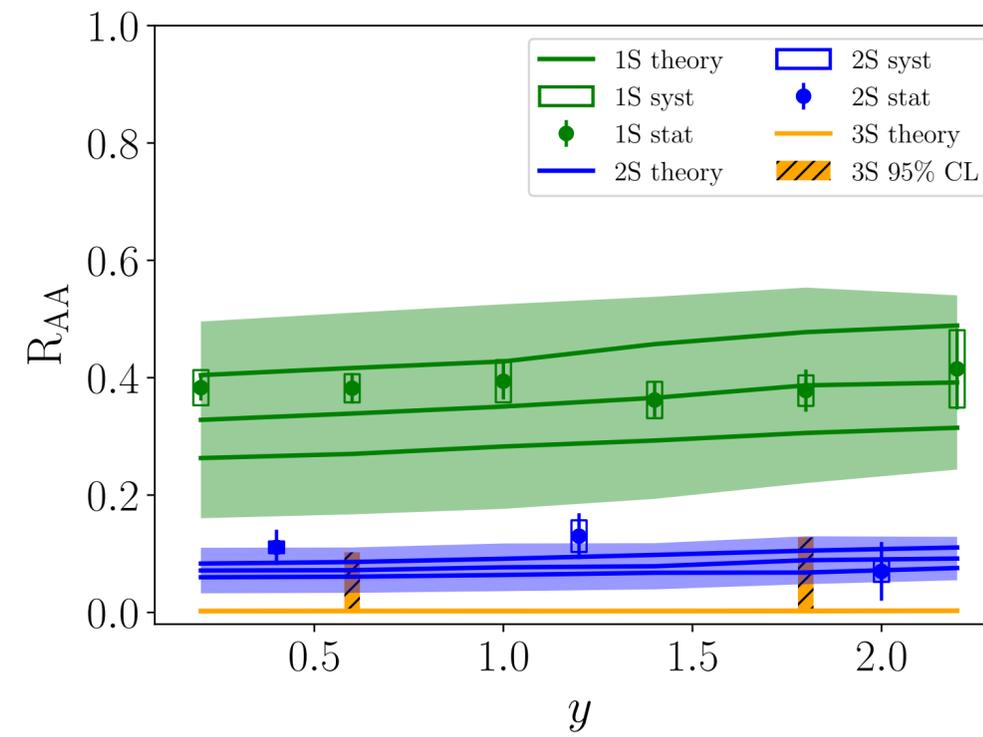
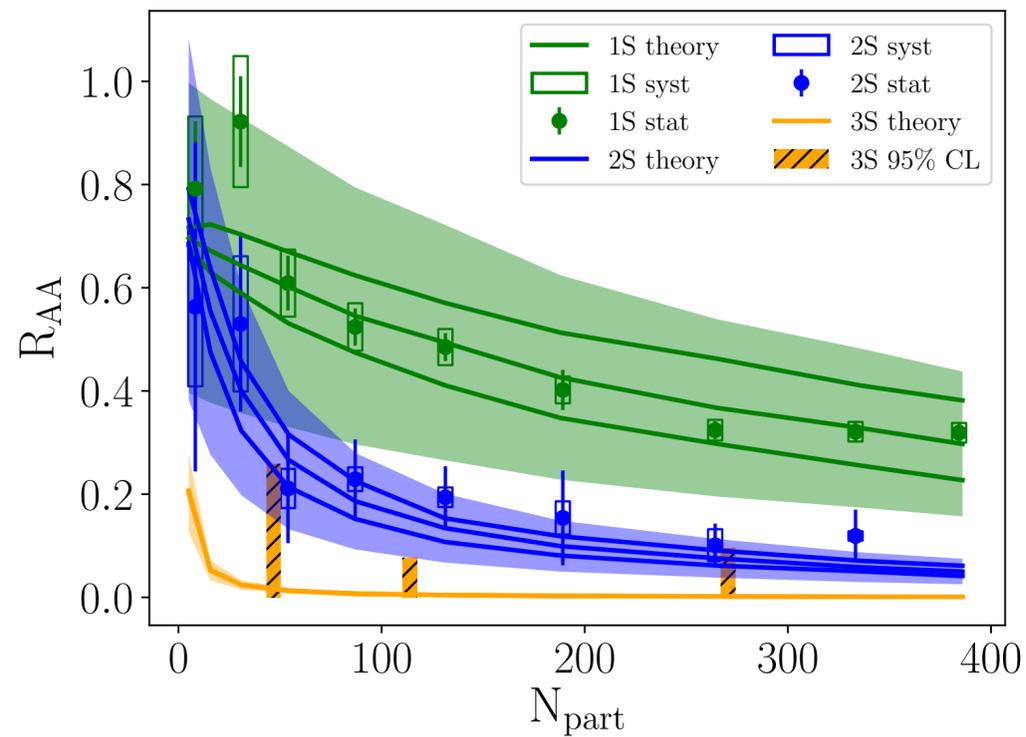
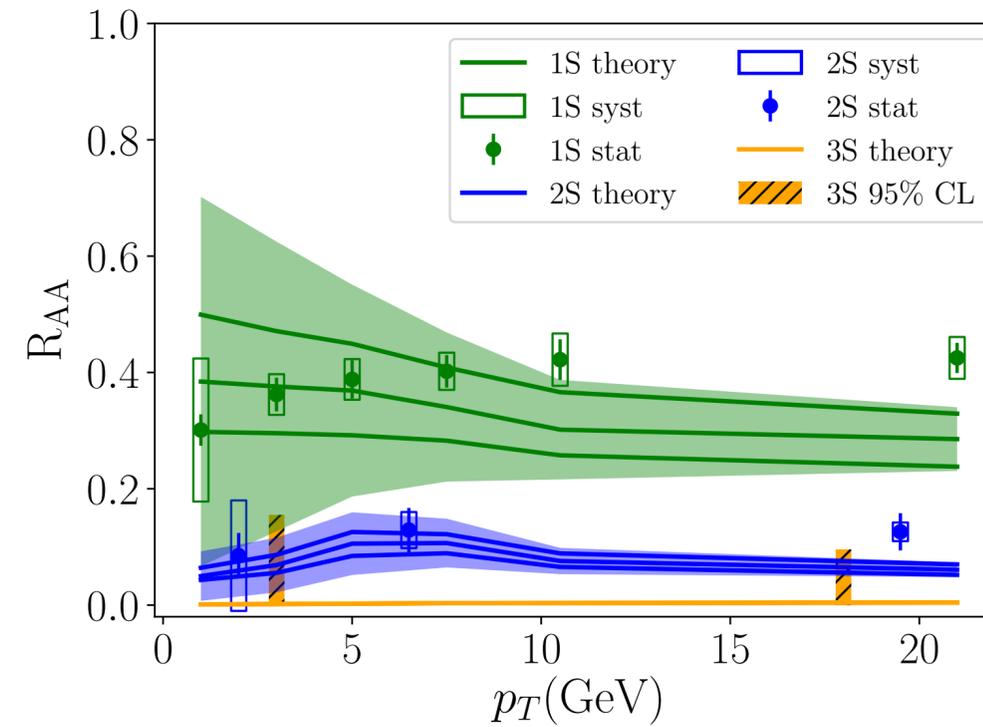
Boltzmann equation for quarkonium derived from first principles by using **open quantum systems in the quantum optical limit** + **pNRQCD** (T.Mehen, X.Yao 1811.07027, 2009.02408), includes dissociation and recombination (both correlated and uncorrelated), HQ diffusion drives system to kinetic equilibrium, dissociation and recombination drive system to chemical equilibrium (X.Yao, B.Müller 1709.03529)



# Coupled Boltzmann Equations: Phenomenology

Initial production: Trento (position distribution) +  
Pythia with EPPS16 nPDF (momentum  
distribution)

QGP evolution in spacetime: calibrated  
2+1D viscous hydro  
Hadronic phase: feed-down



**QUESTIONS!**

Q1. What can we learn from quarkonium suppression on the QGP? Can we learn something directly from experiment? Do we always need to pass through a model? How can we connect the quarkonium suppression data to information on QCD at finite T (or in medium)? What experimental observables can provide the most constraining power for models? Should we fix a set of possible background evolutions that could serve as a reference point for removing this variation?

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**X. YAO**

**Q1:** We are studying electric correlator of QGP by measuring quarkonium suppression

$$D_{i_1 i_2}(q^0, \mathbf{q}) = g^2 \int dt d^3 R e^{iq^0(t_1 - t_2) - i\mathbf{q} \cdot (\mathbf{R}_1 - \mathbf{R}_2)} \langle E_{i_1}(t_1, \mathbf{R}_1) \mathcal{W} E_{i_2}(t_2, \mathbf{R}_2) \rangle_T$$

Dissociation & recombination rates in Boltzmann equations depend on it (T.Mehen, X.Yao 2009.02408), extract from data + transport calculations

Due to nPDF uncertainty, Raa ratios are better observables

Medium description calibrated with observables of light particles (pions, kaons, protons)

# Questions/Answers – M. Strickland

***What can we learn from quarkonium suppression in the QGP? Can we learn something directly from experiment? Do we always need to pass through a model?***

- I would say that we have already learned something that is valuable by comparing the suppression in pPb and PbPb, with the former being much smaller than the latter. Of course, one could argue that this is not clear evidence of the creation of a QGP since one could create a model with very strong final state interactions (e.g. co-mover models) and mock-up the effect.
- I'm afraid we will always need a model, if for nothing else, for the medium evolution to be treated properly. We certainly need realistic medium evolution for observables like  $v_2$ .

***How can we connect the quarkonium suppression data to information on QCD at finite  $T$  (or in medium)?***

- At least in the equilibrium case, we strive to formulate things in terms of mathematical objects (e.g. EE correlators) that are computable on the lattice. Out of equilibrium this becomes more difficult (see subsequent question)

***What experimental observables can provide the most constraining power for models?***

- At this moment I would say that the the double-ratios as a function of  $N_{part}$  and  $p_T$  provide the strongest constraints at present.
- If we could obtain experimental measurements of the suppression of P-wave states such as the  $\chi_{c1}$ , this would also be great; however, I think this is a very difficult observable since  $\chi_{c1}$  does not decay to dimuons and instead makes an E&M transition to  $1S$ . As a result, experimentalists must detect the emitted photon and the subsequent  $1S$  dimuon decay.

***Should we fix a set of possible background evolutions that could serve as a reference point for removing this variation?***

- Yes. These should be made publicly available in an easy-to-read format.

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## Questions/Answers – M. Strickland

***Quarkonium suppression is the result of a nonequilibrium process. Which techniques do we have to address the non-equilibrium evolution of a nonrelativistic system in a hot medium***

- Here I would point to my recent work with B. Kasmaei and K. Boguslavski (2102.12587) on the extraction of the imaginary part of the heavy-quark potential using real-time Yang Mills (classical statistical approach). There we demonstrated that this is possible, and that one can obtain a continuum extrapolated despite the Rayleigh-Jeans divergence. We found that this method can be used in EQ.
- The next step is to turn on the dynamical expansion which makes everything time-dependent. This will include the development of momentum-space anisotropies due to the rapid longitudinal expansion of the QGP. Using such studies, we can assess how large of the effects of momentum-anisotropy on  $\text{Im}[V]$

***For example, how can one incorporate large local rest frame momentum-space anisotropies expected in high-energy heavy-ion collisions in a real-time non-equilibrium approach ??***

- As mentioned above, we can extract a momentum-anisotropic potential. These could be used in Heff simulations, however, since the rotation symmetry is broken 1D evolution no longer suffices. We are looking into ways to obtain effective isotropic screening masses and  $\text{Im}[V]$  which can be used in 1D simulations. Also, can directly solve the 3D problem, but this quite computationally demanding.

***How can lattice inform our efforts on this front? What formulation of lattice could address non equilibrium evolution? Can classical Yang-Mills simulations provide some input?***

- Yes, I think so (see above). There have also been recent works on Hamiltonian formalism for lattice application. Perhaps Chris can comment on this.

**X. YAO**

**Q2:** Open quantum system approach: Lindblad equation, quantum Brownian motion v.s. quantum optical limit (review: X.Yao 2102.01736)

We only know how to derive the evolution equation when the system and thermal bath are weakly coupled

**Q5+Q3:** Quantum computing may be able to solve Lindblad equation efficiently, at least the number of qubits needed  $\sim 3 \cdot \log_2(\# \text{ of states})$  (W.A.de Jong, M.Metcalf, J.Mulligan, M.Płoskoń, F.Ringer, X.Yao, 2010.03571)

Quantum computing may also provide ways to efficiently calculate real-time quantities that are difficult to calculate on Euclidean lattice; For applications at finite temperature, we need to prepare thermal states. (E.g. prepare thermal states for QED in 1+1D, W.A.de Jong, K.Lee, J.Mulligan, M.Płoskoń, F.Ringer, X.Yao, 2106.08394)

**C.ALLTON Q3 Q5**

Hamiltonian Approach circumvents sign problem... and has been overlooked (relatively speaking) for decades(!)

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Q4 **STRICKLAND** There is probably a continuous connection between high-multiplicity pp and pA into AA. Hydro seems to describe the spectra and anisotropic flow coefficients reasonably well suggesting that a small drop of QGP can be created. As increased the multiplicity, we will have stronger and stronger final state interactions. I don't expect (but could be wrong) that there would be some "step" in this dependence

Q5 **STRICKLAND** These are all excellent questions for open discussion among the panelists and participants.

Q6. What are the theoretical limits to the extraction of the (equilibrium) spectral features of quarkonium from the lattice at finite temperature? What can be expected from the lattice in the next few years for predictions of widths given the likely quality of data that will be available?

**Q6 ALLTON** There are theoretical limits to the extraction of the widths from a given “quality” of data, where “quality” is a measure of the size of  $N_\tau$  and statistics. Understanding the merits of different analysis techniques is work in progress.