Transition Form Factor calculation for B_s meson in Covariant Confined Quark Model

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Motivation

- Semileptonic bottom meson decays are the ideal tools to explore the various aspects of heavy quark decays.
- To provide insights into the origin of flavor and CP-violation.
- To extract the information about the CKM matrix elements.
- To look for new physics beyond the standard model.

Covariant Confined Quark Model (CCQM)

- Effective Quantum Field approach for hadronic interaction based on Effective Lagrangian of hadrons interacting with constituent quarks.
- Built-in Infrared confinement which removes all possible thresholds in quark loop diagrams.
- Form factors computation in the entire range of momentum transfer.

Covariant Confined Quark Model of Hadrons

- Main assumption: hadrons interact via quark exchange only
- Interaction Lagrangian

$$\mathcal{L}_{int} = g_H \cdot H(x) \cdot J_H(x)$$

- Quark Currents

$$J_{meson} = \int dx_1 \int dx_2 F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \Gamma_{meson} q^a_{f_2}(x_2)$$

- Interaction Lagrangian

$$\mathcal{L}_{int} = g_M M(x) \int dx_1 \int dx_2 F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \Gamma_M q^a_{f_2}(x_2) + H.c.$$

Compositeness Condition [1,2]

- A composite field and its constituents are introduced as elementary particles.
- The transition of a composite field to its constituents is provided by the interaction Lagrangian.
- The renormalization constant $Z^{1/2}$ is the matrix element between a physical state and the corresponding bare state

$$Z_H^{1/2} = \langle H_{bare} | H_{dressed} \rangle = 0$$

- We use the compositeness condition to determine the mesons-quark coupling constant

$$Z_M = 1 - \tilde{\Pi}'(m_M^2) = 0$$

where $\tilde{\Pi}(p^2)$ is the meson mass operator

[1] S. Weinberg, Phys. Rev. **130**, 776 (1963).
[2] A. Salam, Nuovo Cim. **25**, 224 (1962).

The vertex function and quark propagator

- Translation invariance for the vertex function

$$F_M(x+a, x_1+a, x_2+a) = F_M(x, x_1, x_2), \qquad \forall a$$

- Our choice

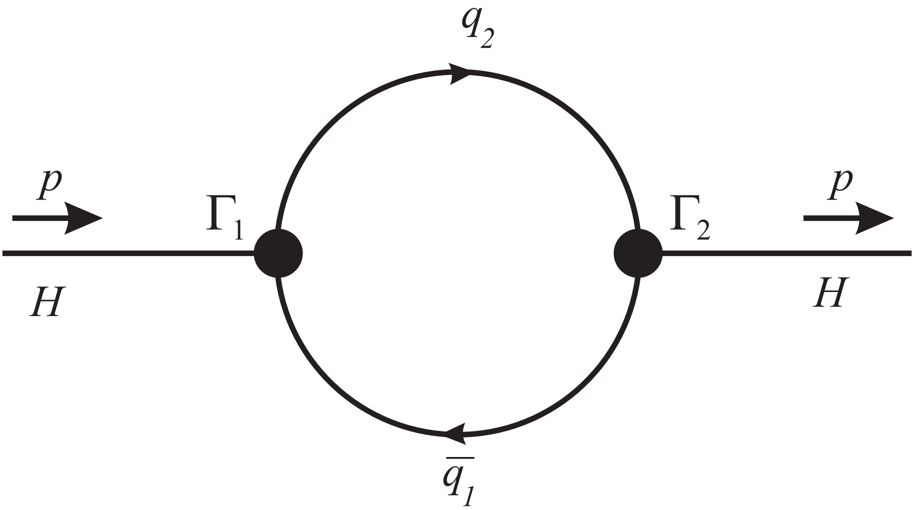
$$F_M(x, x_1, x_2) = \delta^{(4)}\left(x - \sum_{i=1}^{2} w_i x_i\right) \Phi_M\left((x_1 - x_2)^2\right)$$

where Φ is the correlation function of two constituent quarks with masses m_1 and m_2; $w_i = m_i/(m_i + m_j)$.

- The quark propagator

$$S_q(k) = \frac{1}{m_q - \not k}$$

Mass operator



The matrix element

- The matrix elements are described by a set of the Feynman diagrams which are convolution of the quark propagators and vertex functions.
- Let Π be the matrix element corresponding to the Feynman diagram
  j $\rightarrow$ external momenta; n $\rightarrow$ quark propagators;
  ℓ $\rightarrow$ loop integrations; m $\rightarrow$ vertices
  in the momentum space it will be represented as

$$\prod(p_1, ...p_j) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}(-K^2_{i_1+n}) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$

$$K^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$

where
  $\tilde{k}_i$ linear combination of the loop momenta k_i
  $\tilde{p}_i$ linear combination of the external momenta p_i

Infrared confinement

- Use the Schwinger representation of the propagator:

$$\frac{m + \not{k}}{m^2 - k^2} = (m + \not{k}) \int_0^\infty d\alpha \ \ exp[-\alpha(m^2 - k^2)]$$

- Choose a simple Gaussian form factor for the vertex function

$$\Phi(-K^2) = exp\left(k^2/\Lambda^2\right)$$

where Λ characterizes the finite size of hadron

- We imply that the loop integration k proceed over Eucliden space

$$k^0 \to e^{i\pi/2} = ik_4, \quad k^2 = (k^0)^2 - \vec{k}^2 \to -k_E^2 \leq 0.$$

$$p^0 \to e^{i\pi/2} p_4 = ip_4, \quad p^2 = (p^0)^2 - \vec{p}^2 \to -p_E^2 \leq 0$$

so that the quadratic momentum form in the exponent becomes negative-definite and the loop integrals are absolutely convergent.

Infrared confinement

- Convert the loop momenta in the numerator into derivatives over external momenta:

$$k_i^{\mu} e^{2kr} = \frac{1}{2} \frac{\partial}{\partial r_{i\mu}} e^{2kr}$$

- Calculate the scalar loop integral:

$$\prod_{i=1}^{n} \int \frac{d^4 k}{i\pi^2} e^{kAk+2kr} = \frac{1}{|A|^2} e^{-rA^{-1}r}$$

  where a symmetric $n \times n$ real matrix A is positive definite

- Use the identity

$$P\left(\frac{1}{2}\frac{\partial}{\partial r}\right) e^{-rA^{-1}r} = e^{-rA^{-1}r} P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1}r\right)$$

Infrared confinement

- Employ the commutator

$$\left[\frac{\partial}{\partial r_{i\mu}}, r_{j\nu}\right] = \delta_{ij} g_{\mu\nu}$$

to make differentation in

$$P\left(\frac{1}{2}\frac{\partial}{\partial r} - A^{-1} r\right)$$

for any polynomial in P.

- We will have

$$\Pi = \int_0^\infty d^n\alpha \; F(\alpha_1, ..., \alpha_n)$$

where F stands for the whole structure of a given diagram.

Infrared confinement

- The set of Schwinger parameters α_i can be turned into a simplex by introducing an additional t – integration via the identity

$$1 = \int_0^\infty dt\delta\left(1 - \sum_{i=1}^n \alpha_i\right)$$

leading to

$$\Pi = \int_0^\infty t^{n-1}dt \int_0^1 \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, ..., t\alpha_n)$$

- Cut off the upper integration at $1/\lambda^2$

$$\Pi^c = \int_0^{1/\lambda^2} dtt^{n-1} \int_0^1 \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, ..., t\alpha_n)$$

The infrared cut-off [3] will remove all possible thresholds in the quark loop diagram.

[3] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. K¨orner and V. E.

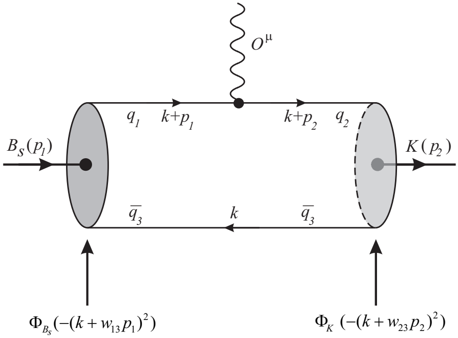
Model parameters

Quark masses m_{qi}, the infrared cutoff parameter λ and the size parameters Λ_{H_i} (all in GeV)

m_s	m_b	m_u	λ
0.428	5.05	0.24	0.181

Λ_{B_s}	Λ_K	Λ_{K^*}
2.05	1.01	0.80

Semileptonic B_s-Meson Decays



- The invariant matrix element of semi leptonic decays of $B_s \to K\ell^+\nu_\ell$

$$M(B_s \to K\ell^+\nu_\ell) = \frac{G_F}{\sqrt{2}} V_{ub}\langle K|\bar{b}O^\mu u|B_s\rangle \ell^+ O^\mu \nu_\ell$$

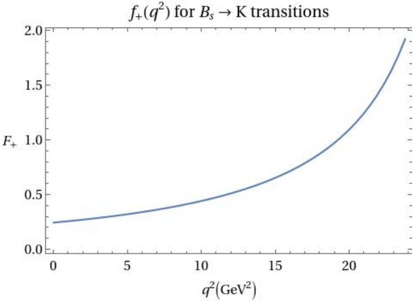
Form Factors of Semi Leptonic B_s-Meson Decays

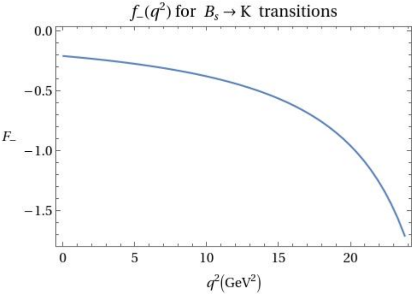
The matrix element of semileptonic $B_s \to K$ transitions

$$\langle K(p_2)|\bar{b}O^{\mu}u|B_s(p_1)\rangle = F_+(q^2)P^{\mu} + F_-(q^2)q^{\mu}$$

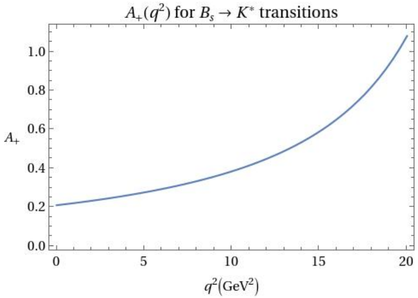
$$\begin{aligned}\langle K(p_2,\epsilon_\nu)|\bar{b}O^{\mu}u|B_s(p_1)\rangle = \frac{\epsilon_\nu^\dagger}{m_1+m_2}\Big[&-g^{\mu\nu}P\cdot qA_0(q^2)\\ &+P^\mu P^\nu A_+(q^2)+q^\mu P^\nu A_-(q^2)\\ &+i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta V(q^2)\Big]\end{aligned}$$

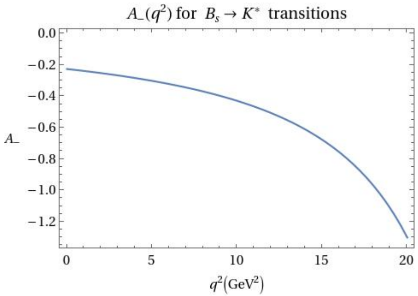
Form Factors



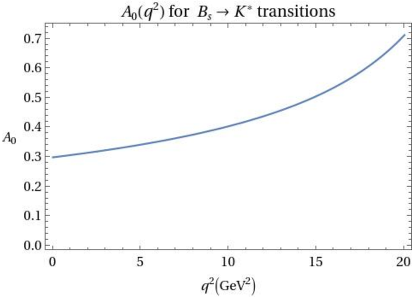


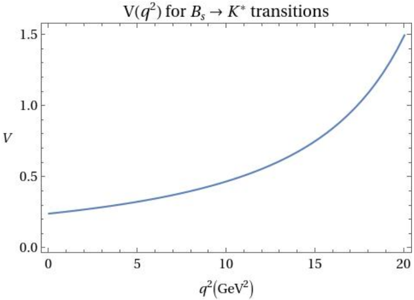
Form Factors





Form Factors





Form Factors

Dipole interpolation

$$F(q^2) = \frac{F(0)}{1 - as + bs^2}, \quad s = \frac{q^2}{m_{B_s}^2}$$

The paramaters of dipole interpolation:

	$\boldsymbol{F_+}$	$\boldsymbol{F_-}$	$\boldsymbol{A_0}$	A_+	A_-	V
$F(0)$	0.25	−0.20	0.30	0.21	−0.23	0.24
a	1.44	1.47	0.65	0.1.46	1.54	1.58
b	0.46	0.50	−0.26	0.43	0.50	0.54

Results

Table: F_+ for $B_s \to K$ at maximum recoil

Present	Data	Reference
0.25	$0.30^{+0.04}_{-0.03}$	LCSR[4]
	0.336 ± 0.023	LCSR[5]
	0.284	RQM[6]
	0.31	RDA[7]
	$0.24^{+0.05}_{-0.04}$	PQCD[8]
	0.290	LCQM with SCET[9]
	0.23	CLFM[10]

[4] G.Duplancic and B.Melic, Phys.Rev.D**78**,054015(2008).

[5] A.Khodjamirian and A.V.Rusov, JHEP**08**,112(2017).

[6] R.N.Faustov and V.O.Galkin, Phys.Rev.D**87**,094028(2013).

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[9] C.D.Lu, W.Wang and Z.T.Wei, Phys.Rev.D**76**,014013(2007).

[10] R.C.Verma, J.Phys.G:Nucl.Part.Phys**39**,025005(2012).

Conclusion

We have computed

- The transition form factors are computed in the entire range of momentum transfer.
- Our preliminary results shows good agreement with other theoritical models
- Further detailed study on these channels are under way.
- We have employed CCQM for different channels
  - $D^{+(0)} \to (\bar{K}^{0(*)}, \pi)\ell^+\nu_\ell$ (Phys. Rev. D **96**, 016017 (2017))
  - $D \to (\omega, \rho, \eta^{(\prime)})\ell^+\nu_\ell$, $D_s^+ \to (\phi, K^{0(*)}, \eta^{(\prime)})\ell^+\nu_\ell$ (Phys. Rev. D **98**, 114031 (2018))
  - $D_{(s)}^+ \to f_0(980)\ell^+\nu_\ell$ and $D \to a_0(980)\ell^+\nu_\ell$ (Phys. Rev. D **102**, 016013 (2020))

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Thank You