Non-valence contributions to weak transition form factors of heavy-light mesons

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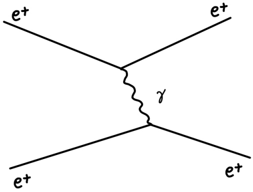
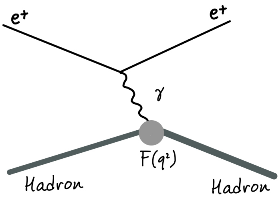
Motivation

- Form factors: source of information on the fundamental structure of hadrons
- Relativistic description of hadrons needed





- Weak decays: form factors provide CKM matrix elements
- Appropriate theoretical framework is required

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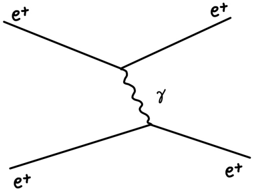
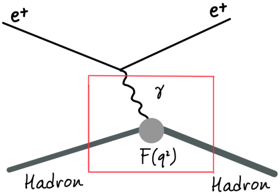


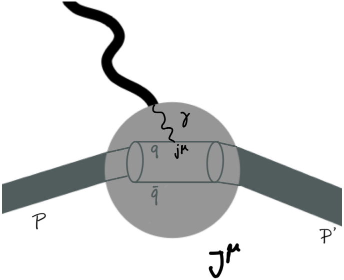


- Weak decays: form factors provide CKM matrix elements
- Appropriate theoretical framework is required

Motivation

Relativistic description of current in terms of constituent's currents not trivial

$$F(q^2) \sim \langle P', \sigma' | J^\mu | P, \sigma \rangle$$

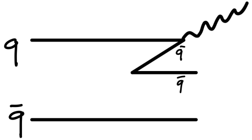


- Change of reference frame
- Covariance needed
- Cluster separability property required
- Heavy-light systems: heavy-quark symmetry in the limit

Motivation

Non-valence $q\bar{q}$-pair contributions: Z-graphs

- Suppressed in light-front dynamics in the $q^+ = 0$ frame
- Suppressed in instant form (and point form) dynamics in the Infinite Momentum frame (IF)



$\Rightarrow$ We will consider two different frames:

- **Infinite momentum frame (IF):** Mandelstam $s \to \infty$
- **Breit frame (BF):** Momentum is transferred in the direction of motion (as it happens in weak decay dynamics)

Point-form approach

Point-form version of the Bakamjian-Thomas construction

$$\hat{P}^\mu = \hat{\mathcal{M}}\,\hat{V}^\mu_{\text{free}} = \left(\hat{\mathcal{M}}_{\text{free}} + \hat{\mathcal{M}}_{\text{int}}\right)\hat{V}^\mu_{\text{free}}$$

Dynamics of the system encoded in $\hat{M}$.

The point form uses *velocity states:*

$$|\vec{k}_i, \mu_i\rangle \equiv |\vec{k}_1, \mu_1; \vec{k}_2, \mu_2; ...; \vec{k}_n, \mu_n\rangle \qquad \sum_{i=1}^{n} \vec{k}_i = 0.$$

They are boosted with velocity V^μ, $V^\mu V_\mu$

$$|V; \vec{k}_1, \mu_1; \vec{k}_2, \mu_2; ...; \vec{k}_n, \mu_n\rangle \;=\; \hat{U}_{B_c(V)}|\vec{k}_1, \mu_1; \vec{k}_2, \mu_2; ...; \vec{k}_n, \mu_n\rangle$$

The point form is very convenient for the description of *heavy-light systems* [PRD 90 (2014) 076003]

Point-form approach

Mass operator

Coupled channel

$$\hat{M} = \begin{pmatrix} \hat{M}^{\text{conf}}_{\bar{u}b\nu_e} & \hat{K}_{\bar{u}b\nu_e \to \bar{u}bWe} & 0 & \hat{K}_{\bar{u}b\nu_e \to \bar{u}cW\nu_e} \\ \hat{K}^{\dagger}_{\bar{u}b\nu_e \to \bar{u}bWe} & \hat{M}^{\text{conf}}_{\bar{u}bWe} & \hat{K}^{\dagger}_{\bar{u}ce \to \bar{u}bWe} & 0 \\ 0 & \hat{K}_{\bar{u}ce \to \bar{u}bWe} & \hat{M}^{\text{conf}}_{\bar{u}ce} & \hat{K}_{\bar{u}ce \to \bar{u}cW\nu_e} \\ \hat{K}^{\dagger}_{\bar{u}b\nu_e \to \bar{u}cW\nu_e} & 0 & \hat{K}^{\dagger}_{\bar{u}ce \to \bar{u}cW\nu_e} & \hat{M}^{\text{conf}}_{\bar{u}cW\nu_e} \end{pmatrix}$$

$$\hat{M} \begin{pmatrix} |\Psi_{\bar{u}b\nu_e}\rangle \\ |\Psi_{\bar{u}bWe}\rangle \\ |\Psi_{\bar{u}ce}\rangle \\ |\Psi_{\bar{u}cW\nu_e}\rangle \end{pmatrix} = m \begin{pmatrix} |\Psi_{\bar{u}b\nu_e}\rangle \\ |\Psi_{\bar{u}bWe}\rangle \\ |\Psi_{\bar{u}ce}\rangle \\ |\Psi_{\bar{u}cW\nu_e}\rangle \end{pmatrix}$$

Vertex operators

Creation/annihilation of particles

$$\langle V'; \vec{k}'_i, \mu'_i | \hat{K} | V; \vec{k}_i, \mu_i \rangle = N V^0 \delta^3(\vec{V} - \vec{V}') \langle \vec{k}'_i, \mu'_i | \hat{\mathcal{L}}_{\text{int}} | \vec{k}_i, \mu_i \rangle$$

Point-form approach

Some relevant results in this approach

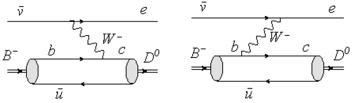
- Electromagnetic form factors of pseudoscalar mesons
  [Biernat *et al.* PRC 79 (2009) 055203]
- Electromagnetic form factors of vector mesons
  [Biernat *et al.* PRC 89 (2014) 5, 055205]
- Heavy quark symmetry in heavy light systems
  [Gomez-Rocha, Schweiger, PRD 86 (2021) 053010]
- Semileptonic weak form factors
  [Gomez-Rocha, PRD 90 (2014) 076003]
- ...

W-exchange – spacelike $q^2 < 0$

Optical potential contains the hadronic current

$$\langle V'; \mathbf{k}_D; \mathbf{k}_e, \mu_e | \hat{V}_{\text{opt}}^{\bar{u}b\nu_e \to \bar{u}ce}(m) | V; \mathbf{k}_B; \mathbf{k}_{\nu_e}, \mu_{\nu_e} \rangle_{\text{o.s}} \propto J^{\mu}_{B\to D}(\vec{k}_D; \vec{k}_B) \propto F(q^2)$$

$$\hat{V}_{\text{opt}}^{\bar{u}b\nu_e \to \bar{u}ce}(m) = \hat{K}_{\bar{u}ce\to\bar{u}bWe}(m - \hat{M}^{\text{conf}}_{\bar{u}bWe})^{-1} \hat{K}^{\dagger}_{\bar{u}b\nu_e\to\bar{u}bWe} + \hat{K}_{\bar{u}ce\to\bar{u}cW\nu_e}(m - \hat{M}^{\text{conf}}_{\bar{u}cW\nu_e})^{-1} \hat{K}^{\dagger}_{\bar{u}b\nu_e\to\bar{u}cW\nu_e}$$



Form factors are extracted from the hadronic current

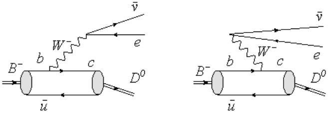
$$\tilde{J}^{\nu}_{M\to M'}(p_{M'}; p_M) = \left((p_M + p_{M'})^{\nu} - \frac{m_M^2 - m_{M'}^2}{q^2} q^{\nu}\right) F_1(q^2, s) + \frac{m_M^2 - m_{M'}^2}{q^2} q^{\nu} F_0(q^2, s)$$

Weak decay – timelike momentum transfer $q^2 > 0$

Optical potential contains the hadronic current

$$\langle V'; \mathbf{k}_D; \mathbf{k}_e, \mu_e | \hat{V}_{\text{opt}}^{\bar{u}b\nu_e \to \bar{u}ce}(m) | V; \mathbf{k}_B; \mathbf{k}_{\nu_e}, \mu_{\nu_e} \rangle_{\text{o.s}} \; \propto \; J^{\mu}_{B\to D}(\vec{k}_D; \vec{k}_B) \; \propto \; F(q^2)$$

$$\begin{aligned} \hat{V}_{\text{opt}}^{b\bar{d}\to c\bar{d}e\bar{\nu}_e}(m) \quad &= \quad \hat{K}_{c\bar{d}W\to c\bar{d}e\bar{\nu}_e}(m - M^{\text{conf}}_{c\bar{d}W})^{-1} \hat{K}^{\dagger}_{c\bar{d}W\to b\bar{d}} \\ &\quad + \hat{K}_{b\bar{d}We\bar{\nu}_e \to c\bar{d}e\bar{\nu}_e}(m - \hat{M}^{\text{conf}}_{b\bar{d}We\bar{\nu}_e})^{-1} \hat{K}^{\dagger}_{b\bar{d}We\bar{\nu}_e \to b\bar{d}} \,. \end{aligned}$$



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$$\begin{aligned} \tilde{J}^{\nu}_{M\to M'}(p_{M'}; p_M) \quad &= \quad \left((p_M + p_{M'})^{\nu} - \frac{m_M^2 - m_{M'}^2}{q^2} q^{\nu} \right) F_1(q^2) \\ &\quad + \frac{m_M^2 - m_{M'}^2}{q^2} q^{\nu} F_0(q^2) \end{aligned}$$

Numerical Studies

Model wave function

$$\psi_\alpha(\kappa) = \frac{2}{\pi^{\frac{4}{2}} a_\alpha^{\frac{3}{2}}} e^{-\frac{\kappa^2}{2a_\alpha^2}} \; ,$$

Model parameters

[Cheng *et al.*PRD 55 (1997)1559]

$M_B = 5.2795$ GeV	$m_b = 4.8$ GeV	$a_B = 0.55$
$M_D = 1.869$ GeV	$m_c = 1.6$ GeV	$a_D = 0.46$
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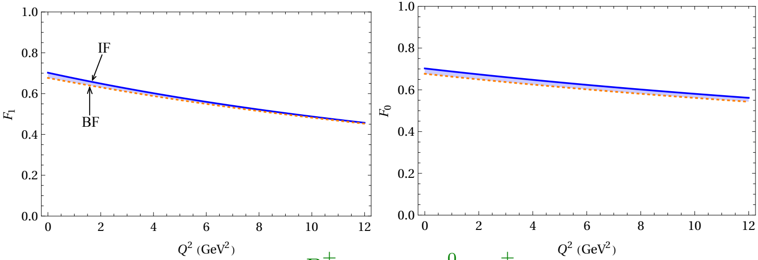
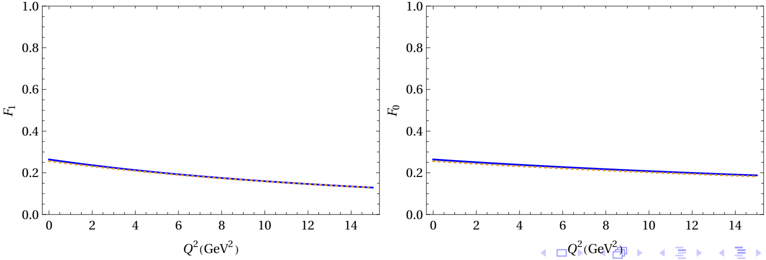
Breit Frame vs. Infinite Momentum Frame

Spacelike form factors $q^2 = q^\mu q_\mu = -Q^2$

$$B^\pm + \mu_e \to D^0 + e^\pm$$



$$B^\pm + \mu_e \to \pi^0 + e^\pm$$



Breit Frame vs. Infinite Momentum Frame

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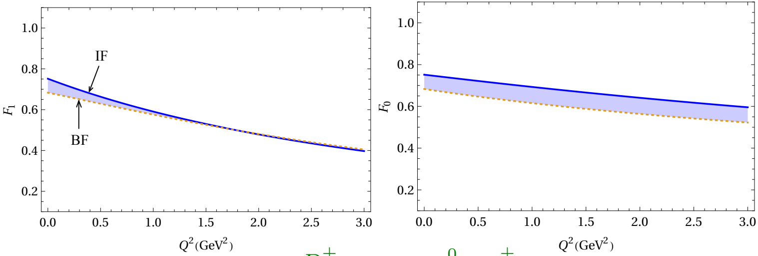
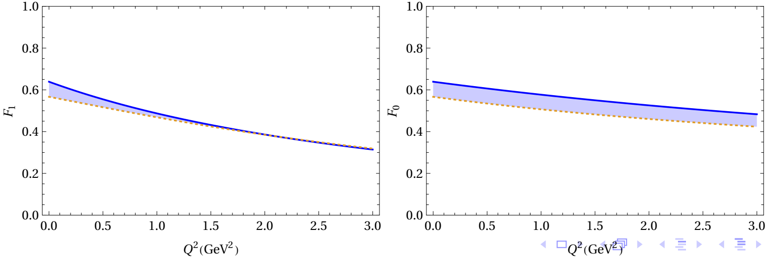
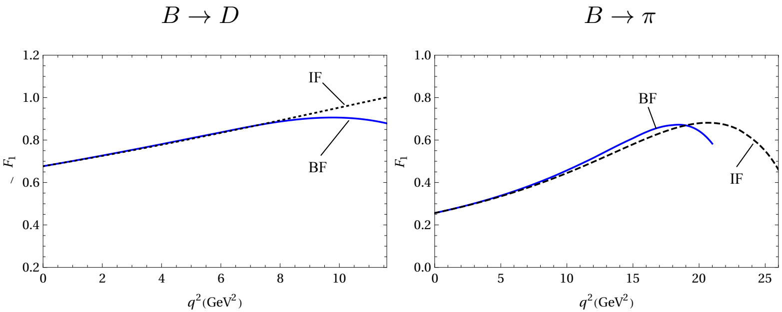


$$D^\pm + \mu_e \to \pi^0 + e^\pm$$



Analytic Continuation $Q \to iQ$ to timelike region

Analytic continuation of form factors calculated in the Breit frame (BF) and the Infinite Momentum (IF) frame for $0 \leq q^2 \leq (M_B - M_{D(\pi)})^2$



Remarks:

- Agreement at $q^2 = 0$ and up to $q^2 \approx 8$ GeV^2
- The difference increase near zero recoil point $q^2 = (M_B - M_{D(\pi)})^2$
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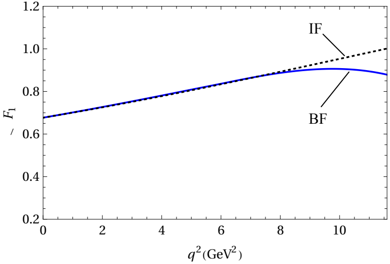
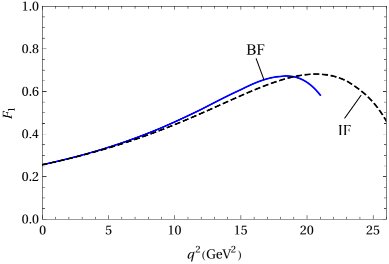


$B \to \pi$



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- Such difference may be due to cluster separability violation (s-dependence of form factors) or to Z-graphs

$\Rightarrow$ **We choose the Infinite Momentum frame**

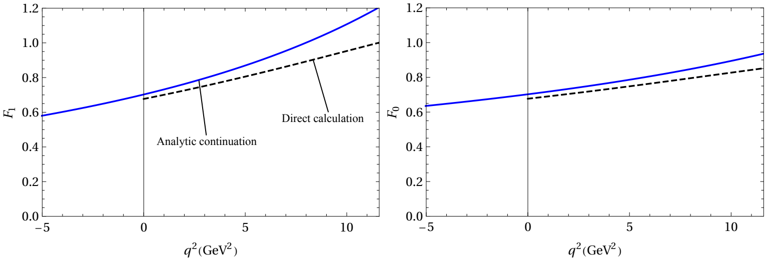
Estimation of Z-graph contributions

We estimate the importance of Z-graphs by comparing form factors

- Analytic continuation in IF $\Rightarrow$ Z-graphs suppressed
- Direct calculation $\Rightarrow$ Z-graphs not included

Difference of both: an estimate of the importance of Z-graphs

$B \to D$



Deviation is significant near zero recoil $\Rightarrow$ Z-graphs become relevant

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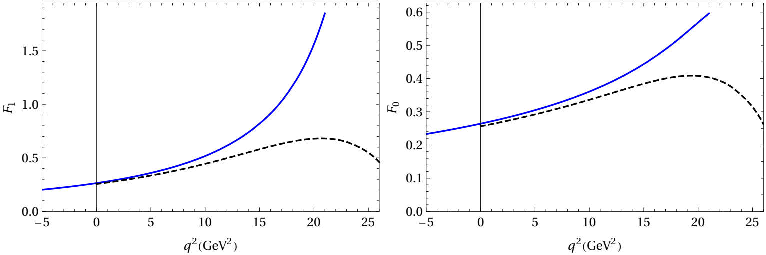
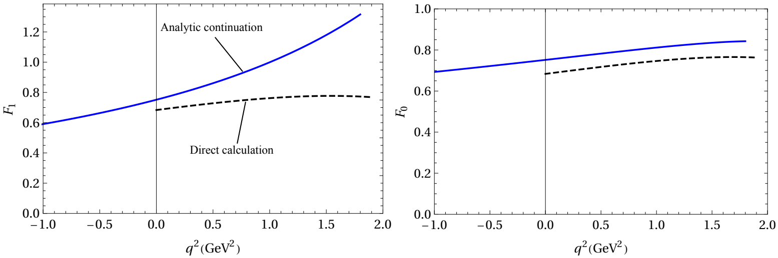


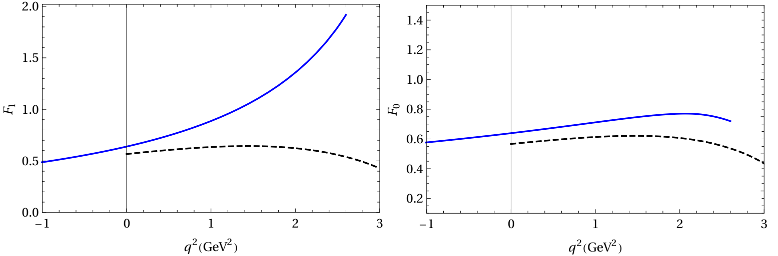
Deviation is significant near zero recoil $\Rightarrow$ Z-graphs become relevant

Estimation of Z-graph contributions

$D \to K$



$D \to \pi$



Estimation of Z-graph contributions

Experimental data near zero recoil not available $\Rightarrow$ Compare with lattice results
[Abada *et al.*, Nucl. Phys. B 619(2001) 565]

$B \to \pi$

q^2	Lattice	IF (this work)
13.6	$F_1 = 0.70(9)^{+.10}_{-.03}$	$F_1 = 0.71$
	$F_0 = 0.46(7)^{+.05}_{-.08}$	$F_0 = 0.42$
15.0	$F_1 = 0.79(10)^{+.10}_{-.04}$	$F_1 = 0.82$
	$F_0 = 0.49(7)^{+.06}_{-.08}$	$F_0 = 0.44$
17.9	$F_1 = 1.05(11)^{+.10}_{-.06}$	$F_1 = 1.15$
	$F_0 = 0.59(6)^{+.04}_{-.10}$	$F_0 = 0.51$
20.7	$F_1 = 1.53(17)^{+.08}_{-.11}$	$F_1 = 1.75$
	$F_0 = 0.71(6)^{+.03}_{-.10}$	$F_0 = 0.59$

$D \to \pi$

q^2	Lattice	IF (this work)
0.47	$F_1 = 0.67(6^{+.01}_{-.00}$	$F_1 = 0.74$
	$F_0 = 0.62(6)^{+.02}_{-.00}$	$F_0 = 0.67$
0.97	$F_1 = 0.81(7)^{+.02}_{-.00}$	$F_1 = 0.88$
	$F_0 = 0.70(6)^{+.01}_{-.00}$	$F_0 = 0.71$
1.48	$F_1 = 1.03(9)^{+.01}_{-.00}$	$F_1 = 1.07$
	$F_0 = 0.80(6)^{+.01}_{-.00}$	$F_0 = 0.75$

$D \to K$

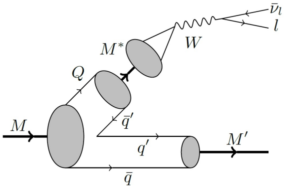
q^2	Lattice	IF (this work)
0.19	$F_1 = 0.70(5)(0)$	$F_1 = 0.79$
	$F_0 = 0.68(4)(0)$	$F_0 = 0.76$
0.69	$F_1 = 0.84(5)(0)$	$F_1 = 0.91$
	$F_0 = 0.76(4)(0)$	$F_0 = 0.79$
1.7	$F_1 = 1.29(7)(0)$	$F_1 = 1.27$
	$F_0 = 0.96(4)(0)$	$F_0 = 0.84$

$\Rightarrow$ This seems to confirm the statement that the analytic continuation in the IF provides the correct result (it includes non valence contr. implicitly).

Z-graph meson pole

Behavior near zero recoil

1. Z-graphs must be included in the decay calculation
2. Non-valence degrees of freedom may recombine with valence $Q\bar{q}$ via intermediate meson:

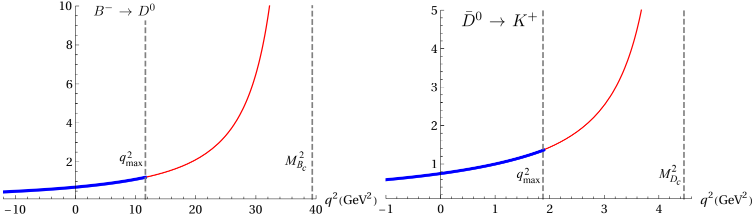


The form factor is traditionally parametrized with and explicit pole

$$F_1^{\text{pole}}(q^2) = \frac{F_1(0)}{\left(1 - \frac{q^2}{M_{\text{pole}}^2}\right)^{\alpha}}$$

Z-graph meson pole

- Setting $\alpha_{B\to D} = 1.55$ and $\alpha_{D\to K} = 1.09$:
- Our analytic continuation result follow $F_1^{\text{pole}}(q^2) = \dfrac{F_1(0)}{\left(1 - \frac{q^2}{M^2_{\text{pole}}}\right)^{\alpha}}$

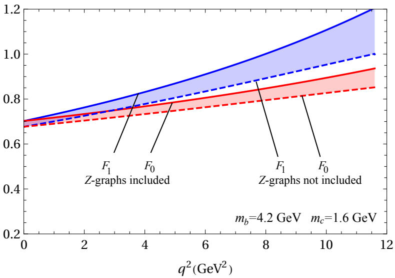


- ⋆ The procedure provides the expected pole-like behavior!!
- ⋆ IF includes implicit Z-contributions
- ⋆ There is a frame dependence in the results if Z-graphs not included

Z-graphs and the heavy-quark limit

Z-graphs are expected to be suppressed in the heavy-quark limit ($m_Q \approx m_M$, $m_q/m_Q \to 0$)

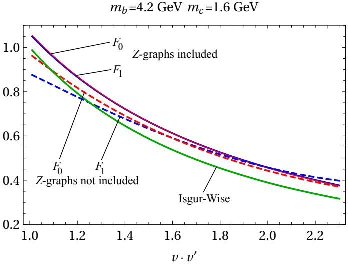
$B \to D$

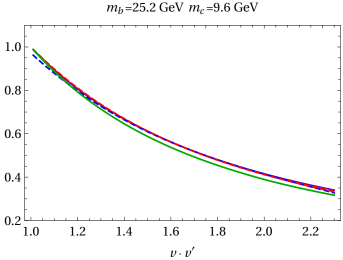


As the heavy quark mass increases, difference between direct calculation and analytic continuation vanish

Isgur-Wise function

Heavy quark symmetry predicts a universal form factor independent of mass and spin of the heavy quark: the Isgur-Wise function





Average deviation from the IW function

$$\sigma_{F_1}^{\text{val+Z-graph}} = 10.0\% , \qquad \sigma_{F_1}^{\text{val}} = 5.3\% ,$$
$$\sigma_{F_0}^{\text{val+Z-graph}} = 10.7\% , \qquad \sigma_{F_0}^{\text{val}} = 5\% .$$

NOTE: The comparison requires to multiply the form factors by appropriate kinematical factors $F_1 \to RF_1$, $\quad F_0 \to \dfrac{R}{1-\frac{q^2}{(M_B+M_D)^2}} F_0$, $\quad R = \dfrac{2\sqrt{M_B M_D}}{M_B + M_D}$

Comparison with experiments

We consider the slope of F_1 as a function of $v \cdot v'$ at zero recoil

$$\rho_D^2 = -\frac{F_1'(vv' = 1)}{F_1(vv' = 1)}$$

Experimental value: $\rho_D^2 = 1.122 \pm 0.023$

Direct calculation (val.): $\rho_D^2 = 0.55$

Analytic continuaton (val. + Z-graphs): $\rho_D^2 = 1.07$

Conclusions

- Analytic continuation in IF provide the best results comparable with lattice and experiments
- The IF in point form seems to be equivalent to the $q^+ = 0$ in front form and includes implicitly Z-graphs contributions
- Results seem to account for the pole structure in weak decays but a detailed explanation on the mechanism is still needed
- The formalism does not spoil heavy-quark symmetry or other properties of the current
- Frame dependence due to cluster separability still present

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Thank you!

Appendix

Masses and resonances

Model parameters

$M_B = 5.2795$ GeV	$m_b = 4.8$ GeV	$a_B = 0.55$
$M_D = 1.869$ GeV	$m_c = 1.6$ GeV	$a_D = 0.46$
$M_\pi = 0.1396$ GeV	$m_{u,d} = 0.25$ GeV	$a_\pi = 0.33$
$M_K = 0.4937$ GeV	$m_s = 0.4$ GeV	$a_K = 0.38$

Model wave function

$$\psi_\alpha(\kappa) = \frac{2}{\pi^{\frac{4}{2}} a_\alpha^{\frac{3}{2}}} e^{-\frac{\kappa^2}{2a_\alpha^2}} ,$$

Meson masses and resonances

Transition	Initial Meson	Final Meson	Resonance
$B^- \to D^0$	$M_{B^-} = 5.2795$ GeV	$M_{D^0} = 1.869$ GeV	$M_{B_c^*} > M_{B_c} = 6.274$ GeV
$\bar{B}^0 \to \pi^+$	$M_{B^0} = 5.2795$ GeV	$M_{\pi^+} = 1.869$ GeV	$M_{B^*} = 5.325$ GeV
$\bar{B}_S^0 \to K^+$	$M_{B_S^0} = 5.3667$ GeV	$M_{K^+} = 0.4937$ GeV	$M_{B^*} = 5.325$ GeV
$D^0 \to K^+$	$M_{D^0} = 1.864$ GeV	$M_{K^+} = 0.4937$ GeV	$M_{D_s^{*-}} = 2.112$ GeV
$D^- \to \pi^0$	$M_{D^-} = 1.869$ GeV	$M_{\pi^0} = 0.135$ GeV	$M_{D^{*-}} = 2.010$ GeV

Expression for the current

Spacelike:

$$\tilde{J}^{\nu}_{B\to D}(\mathbf{k}_D;\mathbf{k}_B) = \frac{\sqrt{k_D^0 k_B^0}}{4\pi}\int \frac{d^3\tilde{k}'_{\bar{u}}}{2k_b^0}\sqrt{\frac{k_{\bar{u}}^0+k_b^0}{k_{\bar{u}}'^0+k_c'^0}}\sqrt{\frac{\tilde{k}_{\bar{u}}'^0+\tilde{k}_c'^0}{\tilde{k}_{\bar{u}}^0+\tilde{k}_b^0}}\sqrt{\frac{\tilde{k}_{\bar{u}}^0\tilde{k}_b^0}{\tilde{k}_{\bar{u}}'^0\tilde{k}_c'^0}}$$
$$\times\sum_{\mu_b\mu'_c}\bar{u}_{\mu'_c}(\mathbf{k}'_c)\gamma^{\nu}(1-\gamma^5)u_{\mu_b}(\mathbf{k}_b)\,\psi^*_{\mu_{\bar{u}}\mu'_c}(|\tilde{\mathbf{k}}'_{\bar{u}}|)\,\psi_{\mu_{\bar{u}}\mu_b}(|\tilde{\mathbf{k}}_{\bar{u}}|)$$
$$\times D^{\frac{1}{2}}_{\mu_b\mu'_c}\left[R_W(\tilde{v}_b,B_c(v_{\bar{u}b}))\;R_W^{-1}(\tilde{v}_{\bar{u}},B_c(v_{\bar{u}b}))R_W(\tilde{v}'_{\bar{u}},B_c(v'_{\bar{u}c}))R_W^{-1}(\tilde{v}'_c,B_c(v'_{\bar{u}c}))\right].$$

Timelike:

$$J^{\nu}_{B\to D}(\mathbf{k}'_D;\mathbf{k}_B=\mathbf{0}) = \frac{\sqrt{\omega_{k_B}\omega_{k'_D}}}{4\pi}\int\frac{d^3\tilde{k}'_{\bar{q}}}{2\omega_{k_b}}\sqrt{\frac{\omega_{\tilde{k}'_c}+\omega_{\tilde{k}'_{\bar{q}}}}{\omega_{k'_c}+\omega_{k'_{\bar{q}}}}}\sqrt{\frac{\omega_{\tilde{k}_b}\omega_{\tilde{k}_{\bar{q}}}}{\omega_{\tilde{k}'_c}\omega_{\tilde{k}'_{\bar{q}}}}}\left\{\sum_{\mu_b,\mu'_c=\pm\frac{1}{2}}\bar{q}_{\mu'_c}(\mathbf{k}'_c)\,\gamma^{\nu}\,(1-\gamma^5)\,u_{\mu_b}(\mathbf{k}_b)\right.$$
$$\left.\times\, D^{1/2}_{\mu_b\mu'_c}\left[R_{\mathrm{W}}\left(\frac{\tilde{k}'_{\bar{q}}}{m_{\bar{q}}},B_c(v'_{c\bar{q}})\right)R_{\mathrm{W}}^{-1}\left(\frac{\tilde{k}'_c}{m_c},B_c(v'_{c\bar{q}})\right)\right]\right\}\psi^*_D\left(|\tilde{\mathbf{k}}'_{\bar{q}}|\right)\psi_B\left(|\tilde{\mathbf{k}}_{\bar{q}}|\right).$$