The light-quark mass dependence of the nucleon axial charge

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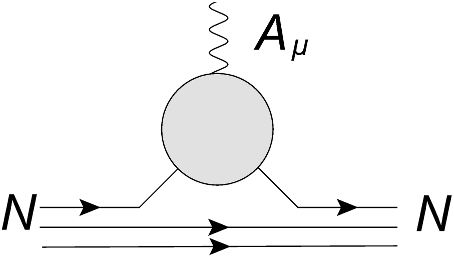
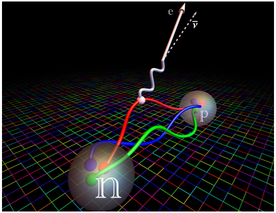
Nucleon Axial Form Factor

- Nucleon Axial Form Factor, $F_A(q^2)$
  - fundamental property of the nucleon
  - information on the spins distribution
  - $A^i_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5\frac{\tau^i}{2}q(x)$
  - $\langle N(p')|A^i_\mu|N(p)\rangle =$
    $\bar{u}\left\{\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N}G_P(q^2)\right\}\gamma_5\frac{\tau^i}{2}u(p)$
- $$\boxed{F_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]}$$
- g_A and F_A dependence in q^2 are necessary in ν oscillations experiments
- μ capture, β-decay



Axial charge, g_A

- Lattice QCD
  - β decay: $g_A = 1.2723(23)$ (PDG) $\Longrightarrow$ benchmark for lattice QCD
  - Recent lattice progress
    - reduction of the systematics: excited state contamination
- Chiral Perturbation Theory (χPT)
  - EFT for QCD at low energy
  - QCD based parametrization $\Longrightarrow$ extrapolate lattice results to the phys. point
  - account for finite volume, lattice spacing and excited states



$\Longrightarrow$ Calculation of $g_A(M_\pi)$ in χPT

- LQCD useful to determine χPT LECs
  - $\Rightarrow g_A = \mathring{g}_A + 4d_{16}M_\pi^2 + g_{A\text{loop}}$
  - we extract d_{16} from g_A
  - d_{16}: key source of uncertainty in m_q dependence of nuclear properties: ground-state and binding energies [1,2,3]
  - previously extracted from $\pi N \to \pi\pi N$
    - considerable uncertainty

Motivations:

1. test the LECs extracted from πN scattering
2. determine d_{16} from fit to lattice
3. study more deeply the convergence of g_A in χPT

[1] Beane et al. Nucl. Phys. A 717 (2003), [2] Berengut et al. PRD 87 (2013), [3] Epelbaum et al Eur. Phys. J. A 49 (2013)

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d_{16}:

$$\mathcal{L}_{\pi N}^{(3)} = \bar{\Psi}\frac{d_{16}}{2}\gamma^\mu\gamma_5\langle\chi_+\rangle u_\mu\Psi + \ldots$$

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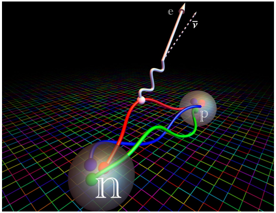

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g_A in Baryon χPT

- χPT: the EFT for QCD at low energy
  - $\mathscr{L}_{SM}$ ~invariance under chiral symmetry: $G \equiv SU(n_f)_L \otimes SU(n_f)_R$
  - quarks, gluons $\longrightarrow$ hadronic degrees of freedom
  - successful description of meson properties
- Baryon χPT
  - problem: $m_B|_{\chi \text{limit}} \nrightarrow 0 \Rightarrow$ Power counting breaking (PCB)
  - $\Longrightarrow$ additional finite renormalisation: extended on mass-shell (EOMS) renormalisation
    - PCB terms absorbed by LECs
    - covariance and analytic properties of loops preserved

g_A Calculation

- NNLO $\mathcal{O}(p^4)$ in relativistic Baryon χPT
- Explicit $\Delta(1232)$
  - SSE: $\delta = m_\Delta - m_N \sim \mathcal{O}(p)$
- EOMS renormalisation
- Correct power counting and analytical properties $\Rightarrow$ appropriate for chiral extrapolations
- $\mathscr{L}$:
  - $\mathscr{L}^{(1)}_{\pi N} \Longrightarrow \mathring{g}_A, \quad \mathscr{L}^{(3)}_{\pi N} \Longrightarrow d_{16},$
    $\mathscr{L}^{(2)}_{\pi N} \Longrightarrow c_1, c_2, c_3, c_4$
  - $\mathscr{L}^{(1)}_{\pi N\Delta} \Longrightarrow h_A, g_1,$
    $\mathscr{L}^{(2)}_{\pi\Delta} \Longrightarrow a_1, \quad \mathscr{L}^{(2)}_{\pi N\Delta} \Longrightarrow b_4, b_5$

$$g_A = \mathring{g}_A + 4d_{16}M_\pi^2 + g^{(3)\not\Delta}_{\text{Aloop}}(\mathring{g}_A;M_\pi) + g^{(3)\Delta}_{\text{Aloop}}(\mathring{g}_A,h_A,g_1;M_\pi)$$
$$+ g^{(4)\not\Delta}_{\text{Aloop}}(\mathring{g}_A,c_1,\tilde{c}_4;M_\pi) + g^{(4)\Delta}_{\text{Aloop}}(\mathring{g}_A,h_A,g_1,c_1,a_1,\tilde{b}_4;M_\pi),$$

$$\tilde{c}_4 = c_4 - c_3/2, \quad \tilde{b}_4 = b_4 + 12/13 b_5$$

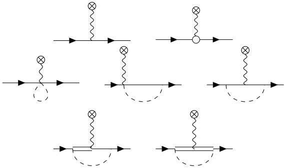


Figure: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

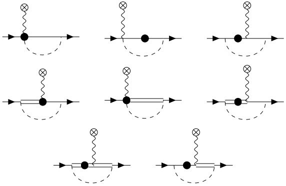
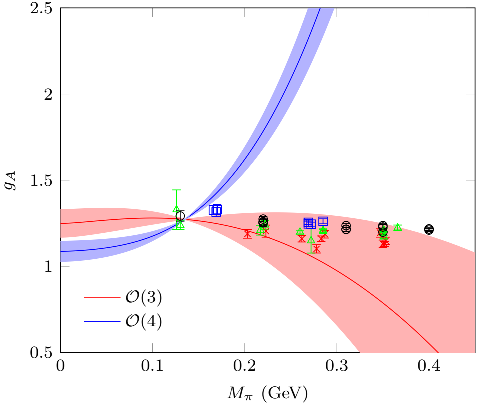


Figure: $\mathcal{O}(p^4)$

M_π dependence of g_A from pheno

- $\left.\begin{array}{l}\pi N \to \pi\pi N \Rightarrow d_{16}, c_i \\ \pi N \to \pi N \Rightarrow c_i\end{array}\right\} \to$ combined fit Ref. [4,5]
- $g_A^{\text{phys}} = g_A(M_\pi^{\text{phys}}) \Rightarrow \mathring{g}_A$

$\Rightarrow$ **Δ-less $g_A(M_\pi)$ prediction**



- $\mathcal{O}(p^3)$: partial agreement with LQCD
- $\mathcal{O}(p^4)$: steep rise [6]

Table: d_{16} (GeV^{-2}), c_i (GeV^{-1}) LECs from*

	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$
d_{16}	-2.2 ± 1.1	-1.86 ± 0.80
c_1	-	-0.89 ± 0.06
c_2	-	3.38 ± 0.15
c_3	-	-4.59 ± 0.09
c_4	-	3.31 ± 0.13

[4] Siemens et al. PRC 94 (2016)
[5] Siemens et al. PRC 96 (2017)
(value converted to standard EOMS)
[6] Bernard et al PLD 639 (2006)

Combined Fit to LQCD

- Objectives:
  - more precise determination of d_{16}
  - study the $g_A(M_\pi)$ convergence
- Combined fit to LQCD data:
  - "Mainz"[7] + CalLat[8] + RQCD[9] + PNDME[10]
  - data without q^2, FV, a or M_π extrapolation
  - large vol. only, $M_\pi L \geq 3.5$
- We correct lattice spacing a
  - $g_A(M_\pi, a) = g_A(M_\pi) + \sum_i x_i a^{n_i}$
- Chiral series truncation error
  - $\Delta X^{(n)} = \max\left\{Q^{n+1}|X^{(0)}|, Q^n|X^{(1)}|, \ldots, Q|X^{(n)}|\right\} \sim Q^{n+1}$ (from [11])
  - $g_A^{(4)} \sim (M_\pi/\Lambda)^3 \; (Q = M_\pi/\Lambda)$
    $\Delta g_{A\chi}^{(4)} = \max\left\{\left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda}|g_A^{(4)}|\right\}$
    $\Delta g_{A\chi}^{(4)} \sim (M_\pi/\Lambda)^4$

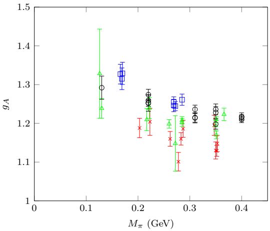
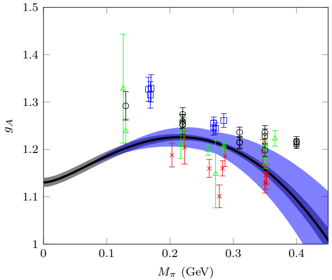
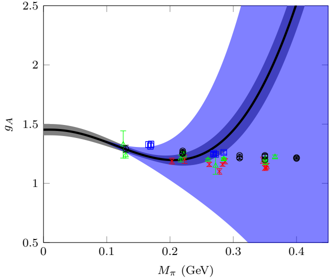


Figure: Red crosses="Mainz"[7]; black circles=CalLat[8]; green triangles=RQCD[9]; blue squares=PNDME[10].

[7] Harris et al. PRD 100 (2019)
[8] Chang et al. Nature 558 (2018)
[9] Bali et al. JHEP 05 (2020)
[10] Park et al. 2103.05599
[11] Epelbaum et al. EPJA 53 (2015)

Combined Fit to LQCD: Δ-less





- χ^2:
  - $\chi^2 = \chi^2_{\text{free}} + \chi^2_{\text{prior}}$

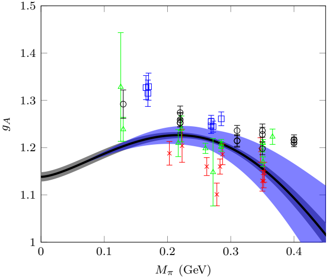
    $\chi^2_{\text{free}} = \sum_{M_{\pi\text{latt}}} \frac{(g_A(M_{\pi\text{latt}},a) - g_{A\text{latt}})^2}{\Delta g^2_{A\text{latt}} + \Delta g^2_{A\chi}}$

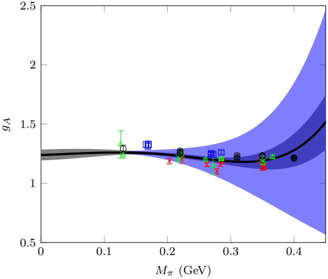
    $\chi^2_{\text{prior}} = \left(\frac{d_{16}}{5}\right)^2 + \left(\frac{\tilde{b}_4}{5}\right)^2$
- χ^2 plateau $\Rightarrow M_\pi^{\text{cut}} \simeq 400$ MeV
- Δ-less Fits
  - $\mathcal{O}(p^3)$:

    $\Rightarrow g_A = \mathring{g}_A + 4 d_{16} M_\pi^2 + g^{(3)\not{\Delta}}_{A\text{loop}}(\mathring{g}_A; M_\pi)$
    - $d_{16} = -0.925 \pm 0.055$ GeV^{-2}
    - underestimates truncation error
  - $\mathcal{O}(p^4)$:

    $\Rightarrow g_A = \mathring{g}_A + 4 d_{16} M_\pi^2 + g^{(3)\not{\Delta}}_{A\text{loop}}(\mathring{g}_A; M_\pi) + g^{(4)\not{\Delta}}_{A\text{loop}}(\mathring{g}_A, c_1, \tilde{c}_4; M_\pi)$
    - c_i fixed to $\pi N \to \pi N$ from Ref. [4]
    - does not describe data $\Longrightarrow$ include Δ

Combined Fit to LQCD: explicit Δ





- $\mathcal{O}(p^3)$ with explicit Δ
  - $\Rightarrow g_A = \mathring{g}_A + 4d_{16}M_\pi^2 + g^{(3)\not\Delta}_{A\text{loop}}(\mathring{g}_A; M_\pi) + g^{(3)\Delta}_{A\text{loop}}(\mathring{g}_A, h_A, g_1; M_\pi)$
  - h_A, g_1 $\rightarrow$ large-N_c
  - underestimates truncation error

- $\mathcal{O}(p^4)$ with explicit Δ
  - reproduces LQCD

Combined Fit to LQCD: explicit Δ $\mathcal{O}(p^4)$

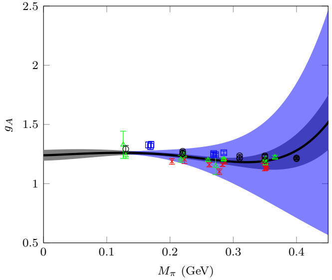


Figure: g_A $\mathcal{O}(p^4)$ with explicit Δ fit.

- $\mathcal{O}(p^4)$ with explicit Δ

$$\Rightarrow g_A = \mathring{g}_A + 4d_{16}M_\pi^2 + g_{A\text{loop}}^{(3)\not\Delta}(\mathring{g}_A;M_\pi) + g_{A\text{loop}}^{(3)\Delta}(\mathring{g}_A,h_A,g_1;M_\pi)$$
$$+ g_{A\text{loop}}^{(4)\not\Delta}(\mathring{g}_A,c_1,\tilde{c}_4;M_\pi) + g_{A\text{loop}}^{(4)\Delta}(\mathring{g}_A,h_A,g_1,c_1,a_1,\tilde{b}_4;M_\pi)$$

- large $\mathcal{O}(p^4)$ contribution
  $\Rightarrow$ sizeable truncation error at high M_π
  $\Leftrightarrow$ slow g_A convergence
- $\Rightarrow d_{16} = -0.88 \pm 0.88$ GeV^{-2}
  - (without truncation error $\to \Delta d_{16} = \pm 0.17$ GeV^{-2})
- $\pi N \to \pi\pi N$ from Ref. [5] $\Rightarrow d_{16} = -1.0 \pm 1.0$ GeV^{-2}
- $\Rightarrow$ good agreement between $\pi N \to \pi\pi N$ and LQCD

	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free par.)	1.240 ± 0.046
d_{16} (free par.)	-0.88 ± 0.88
h_A	$h_A^{Nc} = 1.35$
g_1	$-\lvert g_1^{Nc}\rvert = -2.29$
c_1	-1.15 ± 0.05
c_2	1.57 ± 0.10
c_3	-2.54 ± 0.05
c_4	2.61 ± 0.10
a_1	0.90
$\tilde{b}_4$ (free par.)	-12.3 ± 1.0
$\mathring{m}$	0.855
$\mathring{m}_\Delta$	1.166
χ^2/dof	$11.14/(43-7) = 0.31$

Combined Fit to LQCD: explicit Δ $\mathcal{O}(p^4)$

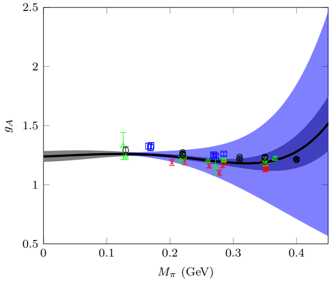


Figure: g_A $\mathcal{O}(p^4)$ with explicit Δ fit.

- $\mathcal{O}(p^4)$ with explicit Δ
  - reproduces LQCD
  - slow g_A convergence
  - $\Rightarrow d_{16} = -0.88 \pm 0.88$ GeV^{-2}
- Work in progress: $F_A(q^2, M_\pi)$ fit
  - $\Rightarrow$ more robust description of LQCD
  - $\Rightarrow$ reduce LECs' errors
  - Roper or two loop $\mathcal{O}(p^5)$ may be necessary to reach the full chiral convergence of g_A

Work in progress: $F_A(q^2)$

- $F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
- fundamental for ν oscillation experiments
  - QE: $n\nu \to p\mu$
- LQCD analysis recently improved
- BχPT fit to LQCD
  $\Rightarrow$ extrapolate in q^2 without ad hoc parametrisation
  $\Rightarrow$ extract reliable $\langle r_A^2\rangle$
- we are improving previous $\mathcal{O}(p^3)$ extraction of $\langle r_A^2\rangle$ up to $\mathcal{O}(p^4)$
  - $\mathcal{L}_{\pi N}^{(3)} \Longrightarrow d_{22}, \quad \mathcal{L}_{\pi N\Delta}^{(2)} \Longrightarrow b_1, b_2$
  - d_{22} and g_1 are correlated

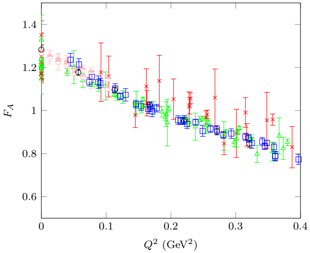


Figure: $F_A(Q^2)$ from the lattice. Green triangles=RQCD[9]; blue squares=PNDME[10]; red crosses="Mainz"[12]; pink half-filled circeles=PACS[13]; black circles="Cyprus"[14].

[12] Meyer et al. Modern Phys. A 34 (2019)
[13] Shintani et al. PRD 102 (2020)
[14] Alexandrou et al. PRD 103 (2021)

Conclusion and Outlook

- Explicit Δ is necessary to describe the M_π dependence of g_A in χPT (and to better agree with $\pi N \to \pi\pi N$)
  - slow convergence
- We extract $d_{16} = -0.88 \pm 0.88\ \mathrm{GeV}^{-2}$ from the lattice in $\mathcal{O}(p^4)$ model with explicit Δ
  - good agreement with $\pi N \to \pi\pi N$ [5]
  - error dominated by chiral truncation uncertainty
- Work in progress: $\mathcal{O}(p^4)$ analysis of F_A with explicit Δ
  - may shed more light on d_{16} and also d_{22}
  - important quantity in ν oscillation and other processes

Thanks for watching!
Any questions?