Minkowskian three-body model of the proton and Ioffe-time imaging

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Introduction

- In hadron physics, one of the most important remaining challenges is to describe the dynamics and structure of the proton in terms of its basic constituents (quarks and gluons).
- The proton light-front wave function, defined one the null plane $x^+ = t + z = 0$, gives through the parton probability densities access to various observables.
- For example:
  - Electromagnetic form factors
  - The parton distribution function
  - Generalized parton distribution functions
- Additionally, the double parton scattering cross section depends on the double parton distribution function (DPDF):

$$D(x_1, x_2, \vec{q}_\perp) = \sum_{n=3}^{\infty} D_n(x_1, x_2, \vec{q}_\perp) = \sum_{n=3}^{\infty} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \left\{ \prod_{i \neq 1,2} \int \frac{d^2k_i}{(2\pi)^2} \int_0^1 dx_i \right\}$$
$$\times \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta\left(\sum_{i=1}^{n} \vec{k}_{i\perp}\right) \Psi_n^\dagger(x_1, \vec{k}_{1\perp} + \vec{q}_\perp, x_2, \vec{k}_{2\perp} - \vec{q}_\perp, ...) \Psi_n(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, ...)\,, \quad (1)$$

- The DPDF has recently been calculated within lattice QCD.

- In the present work the proton is studied in an simple valence LF model based on the zero-range interaction.
- The main dynamical characteristic is the diquark, either as a bound state are as a virtual one.
- The proton structure will be explored through the LF wave function and its Ioffe-time representation. Results for the momentum distributions will also be presented.

Three-body Faddeev-Bethe-Salpeter equation with zero interaction

- Our starting point is the Faddeev-Bethe-Salpeter (FBS) equation with zero interaction with zero interaction [1]:

$$v(q,p) = 2\mathrm{i}F(M_{12}^2) \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon} \frac{\mathrm{i}}{(p-q-k)^2 - m^2 + i\epsilon} v(k,p) \tag{2}$$

- Equal-mass case, bare propagators and spinless quarks.
- $v(q,p)$ is one of the Faddeev components of the total vertex function.
- $F(M_{12}^2)$: two-body scattering amplitude characterized by scattering length a and $M_{12}^2 = (p-q)^2$ given by

$$\mathcal{F}(M_{12}^2) = \frac{\Theta(-M_{12}^2)}{\frac{1}{16\pi^2 y}\log\frac{1+y}{1-y} - \frac{1}{16\pi m a}} + \frac{\Theta(M_{12}^2)\ \Theta(4m^2 - M_{12}^2)}{\frac{1}{8\pi^2 y'}\arctan y' - \frac{1}{16\pi m a}}\,, \tag{3}$$

- 1) $a < 0$: Borromean system, virtual diquark state, 2) $a > 0$: bound diquark state.
- The FBS equation was recently solved including the infinite number of Fock components in Euclidean [2] and Minkowski [3] space.

[1] T. Frederico, PLB 282 (1992) 409
[2] E. Ydrefors et al, PLB 770 (2017) 131
[3] E. Ydrefors et al, PLB 791 (2019) 276

Valence LF equation

- After the LF projection, i.e. introducing $k_\pm = k_0 \pm k_z$ and integrating over k_-, one obtains the three-body LF equation [1, 2]:

$$\Gamma(k_\perp, x) = \frac{F(M_{12}^2)}{(2\pi)^3} \int_0^{1-x} \frac{dx'}{x'(1-x-x')} \int_0^\infty \frac{d^2k'_\perp}{M_0^2 - M_N^2} \Gamma(k'_\perp, x') \tag{4}$$

with the squared free three-body mass

$$M_0^2 = (k'^2_\perp + m^2)/x' + (k_\perp^2 + m^2)/x + ((k'_\perp + k_\perp)^2 + m^2)/(1-x-x') \tag{5}$$

- The three-body valence LF wave function is given by

$$\Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}) = \frac{\Gamma(x_1, \vec{k}_{1\perp}) + \Gamma(x_2, \vec{k}_{2\perp}) + \Gamma(x_3, \vec{k}_{3\perp})}{\sqrt{x_1 x_2 x_3}(M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2\vec{k}_{2\perp}, x_3\vec{k}_{3\perp}))}, \tag{6}$$

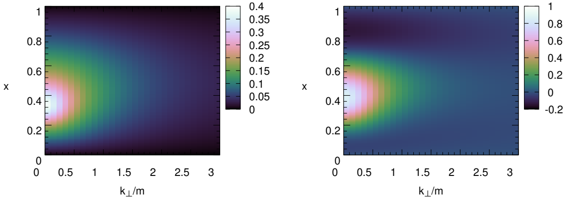
where $x_3 = 1 - x_2 - x_3$ and $\vec{k}_{3\perp} = -\vec{k}_{1\perp} - \vec{k}_{2\perp}$.

[1] J. Carbonell and V.A. Karmanov, PRC 67 (2003) 037001

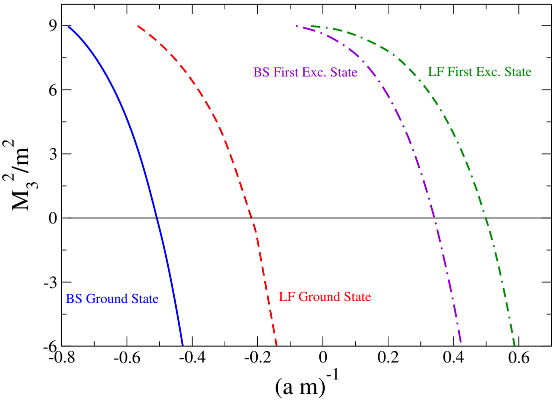
[2] T. Frederico, PLB 282 (1992) 409

Results for the vertex function

Model	m [MeV]	a $[m^{-1}]$	M_2 [MeV]	M_N/m	r_{F_1} [fm]
I	317	-1.84	-	2.97	0.97
II	362	3.60	681	2.60	0.72

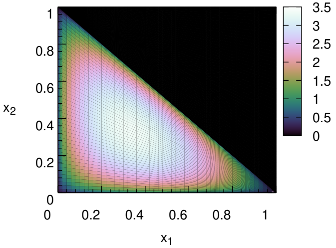


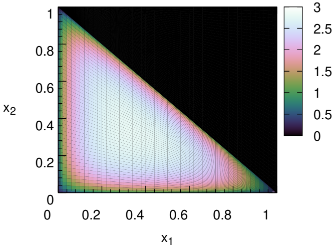
- Two different values of a considered, with negative and positive a, fitted to reproduce the experimental Dirac form factor. For the model with a bound diquark the obtained value of the di-quark mass same as a recent Lattice QCD calculation.
- The proton structure contained in the vertex function $\Gamma(k_\perp, x)$. As seen for the bound diquark case it has a node at roughly $x = 0.8$.



- As studied in PLB 770 (2017) 131, it exists lower-lying unphysical solution with $M_N^2 < 0$. This is the relativistic analog of the well-known Thomas collapse. But, contrary to the non-relativistic case the unphysical state has a finite energy.

Distribution amplitude





- The distribution amplitude is defined as

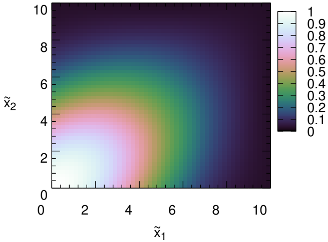
$$\phi(x_1,x_2) = \int d^2k_{1\perp} d^2k_{2\perp} \Psi_3(x_1,\vec{k}_{1\perp},x_2,\vec{k}_{2\perp},x_3,\vec{k}_{3\perp}). \tag{7}$$

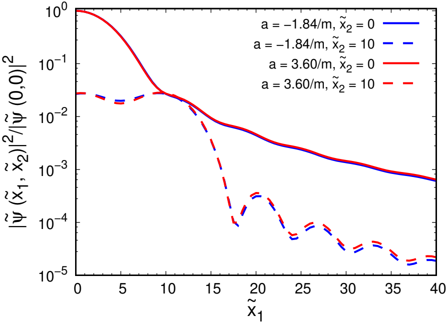
- It shows the dependence of the wave function on the momentum fractions for the case when the quarks share the same position.
- For the two considered cases similar results.

Ioffe-time image of the valence state

- The proton can also be studied in the configuration space associated with the null-plane, where the coordinates of each particle are the transverse position ($\vec{b}_{i\perp}$) and the Ioffe-time $\tilde{x}_i = b_i^- p^+$. This obtained through the Fourier transform of the proton LF wave function.
- For simplicity, we consider here the case $\vec{b}_{1\perp} = \vec{b}_{2\perp} = \vec{0}_\perp$, and then one has

$$\Phi(\tilde{x}_1, \tilde{x}_2) \equiv \tilde{\Psi}_3(\tilde{x}_1, \vec{0}_\perp, \tilde{x}_2, \vec{0}_\perp) = \int_0^1 dx_1 \, \mathrm{e}^{i\tilde{x}_1 x_1} \int_0^{1-x_1} dx_2 \, \mathrm{e}^{i\tilde{x}_2 x_2} \, \phi(x_1, x_2) \,, \qquad (8)$$





- For $\tilde{x}_2 = 0$ the two parameter sets give almost identical results.
- For $\tilde{x}_2 = 10$ and $\tilde{x}_1 >= 10$ a rather dramatic decrease of the amplitude is seen. Similar behavior for the two parameter sets.

- The valence contribution to the Dirac form factor is given by

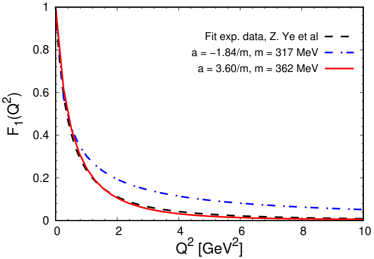
$$F_1(Q^2) = \left\{ \prod_{i=1}^{3} \int \frac{d^2 k_{i\perp}}{(2\pi)^2} \int_0^1 dx_i \right\} \delta\left(1 - \sum_{i=1}^{3} x_i\right) \delta\left(\sum_{i=1}^{3} \vec{k}^{\mathrm{f}}_{i\perp}\right) \times \Psi_3^\dagger(x_1, \vec{k}^{\mathrm{f}}_{1\perp}, \ldots) \Psi_3(x_1, \vec{k}^{\mathrm{i}}_{1\perp}, \ldots), \tag{9}$$

where $Q^2 = \vec{q}_\perp \cdot \vec{q}_\perp$ and the magnitudes of the momenta read

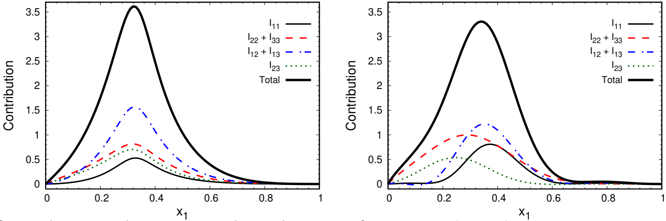
$$\left|\vec{k}^{\mathrm{f(i)}}_{i\perp}\right|^2 = \left|\vec{k}_{i\perp} \pm \frac{\vec{q}_\perp}{2} x_i\right|^2 = \vec{k}^2_{i\perp} + \frac{Q^2}{4} x_i^2 \pm \vec{k}_{i\perp} \cdot \vec{q}_\perp x_i \quad (i = 1, 2), \tag{10}$$

and

$$\left|\vec{k}^{\mathrm{f(i)}}_{3\perp}\right|^2 = \left|\pm \frac{\vec{q}_\perp}{2}(x_3 - 1) - \vec{k}_{1\perp} - \vec{k}_{2\perp}\right|^2 = (1 - x_3)^2 \frac{Q^2}{4} \pm (1 - x_3) \vec{q}_\perp \cdot (\vec{k}_{1\perp} + \vec{k}_{2\perp}) + (\vec{k}_{1\perp} + \vec{k}_{2\perp})^2. \tag{11}$$



- Both parameters give a fair reproduction of experimental data for low Q^2, i.e $Q^2 < 1\mathrm{GeV}^2$, where the model should be applicable.
- The diquark case give also quite good agreement for moderate Q^2. But, this should be viewed with caution since the scaling laws of the QCD are not built-in.



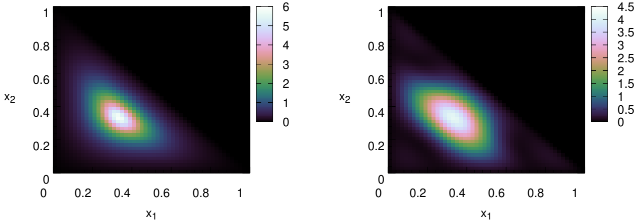
- We define the single parton distribution function (PDF) as

$$f_1(x_1) = \frac{1}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} |\Psi_3(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp})|^2 = I_{11} + I_{22} + I_{33} + I_{12} + I_{13} + I_{23}. \quad (12)$$

with the Faddeev contributions

$$I_{ii} = \frac{1}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma^2(x_i, \vec{k}_{i\perp})}{x_1 x_2 x_3 (M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))^2}$$
$$I_{ij} = \frac{2}{(2\pi)^6} \int_0^{1-x_1} dx_2 \int d^2k_{1\perp} d^2k_{2\perp} \frac{\Gamma(x_i, \vec{k}_{i\perp})\Gamma(x_j, \vec{k}_{j\perp})}{x_1 x_2 x_3 (M_N^2 - M_0^2(x_1, \vec{k}_{1\perp}, x_2, \vec{k}_{2\perp}, x_3, \vec{k}_{3\perp}))^2}; \quad i \neq j. \quad (13)$$

- Evolution of the PDF will be performed in the near future.



- The valence double parton distribution function (DPDF) is given by

$$D_3(x_1, x_2; \vec{q}_\perp) = \frac{1}{(2\pi)^6} \int d^2k_{1\perp} d^2k_{2\perp} \\ \times \Psi_3^\dagger(x_1, \vec{k}_{1\perp} + \vec{q}_\perp; x_2, \vec{k}_{2\perp} - \vec{q}_\perp; x_3, \vec{k}_{3\perp}) \Psi_3(x_1, \vec{k}_{1\perp}; x_2, \vec{k}_{2\perp}; x_3, \vec{k}_{3\perp}). \tag{14}$$

- Fourier transform of $D_3(x_1, x_2, \vec{q}_\perp)$ in $\vec{q}_\perp$ gives the probability of finding the quarks 1 and 2 with momentum fractions x_1 and x_2 at a relative distance $\vec{y}_\perp$ within the proton.
- In the figure is shown results for $Q^2 = 0$. For the case of virtual diquark (left panel) a rather narrow distribution is obtained due to the small binding energy.

Conclusions

- We have, in this work, studied the proton in a simple but fully dynamical valence LF model based on a zero-range interaction.
- The model is based on the concept on a strongly interacting diquark, either virtual or bound.
- We have studied the structure of the proton by computing the LF wave function in its Ioffe-time representation and also longitudinal momentum distributions.
- However, the model is rather crude since e.g. the spin degree of freedom hasn't been included yet. But is a first step towards studying the proton directly in Minkowski space.
- Future plans:
  - Generalization to the infinite set of Fock components (The Faddeev-Bethe-Salpeter equation solved in PLB 791 (2019) 276)
  - Implementation of a more realistic interaction (gluon exchange)
  - Inclusion of spin degree of freedom