MESON FORM FACTORS IN A UNIFIED SCHEME

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HADRON 2021

MOTIVATION

- Hadron physics experiments probe the asymptotic predictions of Quantum Chromodynamics and its non-perturbative emergent phenomena of dynamical chiral symmetry breaking and confinement.
- The theoretical study of the dynamics of Hadrons represents a challenge since the extrapolation from fundamental particles to bound state systems is a hard task.
- The use of the SDE and BSE to study static and dynamic properties of Hadrons with the minimum number of input parameters is a long term goal.

- At leading-order in a symmetry preserving truncation of the SDEs, internal properties of pseudoscalar, vector, scalar and axial-vector mesons such as pion, rho, sigma and a_1 can be studied in a consistent manner.

- In this work we are interested on the determination of Elastic Meson Form Factors (EMFF).

- The calculation of all elastic meson form factors requires the computation of the quark propagator, the BSA of mesons, their masses as well as the knowledge of the quark-photon interaction at different probing momenta.

GAP EQUATION

- Dressed masses of quarks are obtained by use of the GAP equation,

$$S(p)^{-1} = \imath\gamma \cdot p + m_f + \Sigma(p)$$

- Where

$$\Sigma(p) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q)\gamma_\mu S(q)\Gamma_\nu(q,p)$$

- And m_f is the current quark mass and Γ_ν is the quark-vector boson vertex

C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33, 477 (1994).
C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45, S1 (2000).

- By means of the Contact Interaction (CI),

$$g^2 D_{\mu\nu}(q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\rm IR}}{m_G^2}$$

$$\Gamma_\nu(q,p) = \gamma_\nu$$

- it is possible to find dressed masses of mesons M_f by,

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \mathcal{C}(M_f^2)$$

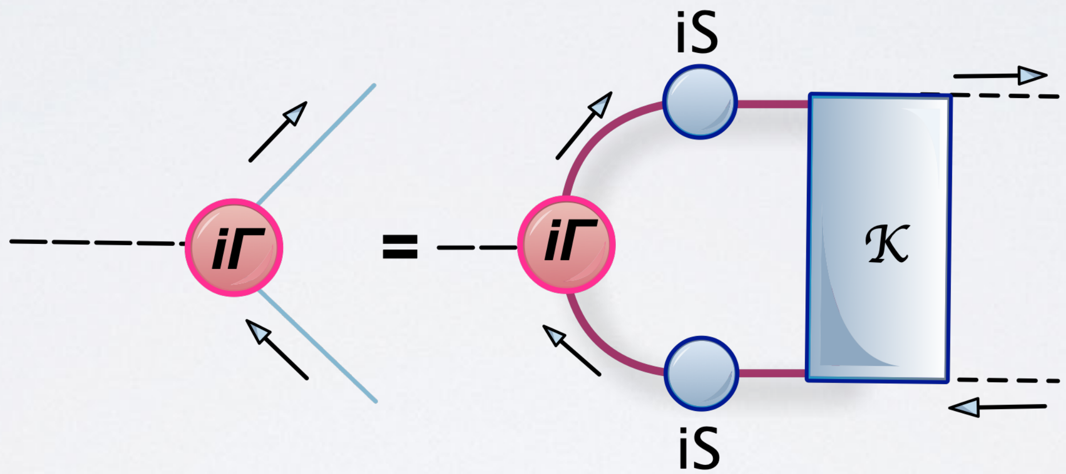
- Where,

$$\mathcal{C}(z)/z = \Gamma(-1, z\,\tau_{\rm UV}^2) - \Gamma(-1, z\,\tau_{\rm IR}^2)$$

L. X. Gutierrez-Guerrero, A. Bashir, I. C. Cloet, and C. D. Roberts, Phys. Rev. C81, 065202 (2010), 1002.1968.

BETHE-SALPETER EQUATION

- The bound-state problem for Hadrons characterized by two valence-fermions may be studied using the homogeneous BS equation,



- Where Γ the is the Bethe-Salpeter Amplitude (BSA).

- Since this equation is rather general for all kind of mesons, we present the analysis for Scalar, Pseudoscalar, Vector and Axial-Vector mesons.

E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951)

- A general decomposition of the BSA in the CI for Scalar (S), Pseudoscalar (PS), Vector (V) and Axial-Vector (AV) mesons is given by,

$$\Gamma_{\rm S}(P) = i\, E_{\rm S}\,(P)\, I$$

$$\Gamma_{\rm PS}(P) = i\,\gamma_5\, E_{\rm PS}\,(P) + \frac{1}{2M_R}\gamma_5\,\gamma\cdot P\, F_{\rm PS}(P)$$

$$\Gamma_{{\rm V},\mu}(P) = \gamma^T_\mu\, E_{\rm V}\,(P)$$

$$\Gamma_{{\rm AV},\mu}(P) = \gamma_5\,\gamma^T_\mu\, E_{\rm AV}\,(P)$$

- Such as $P_\mu \gamma^T_\mu = 0$ and, the reduced mass with dressed quark masses is

$$M_R = M_{f_1} M_{f_2}/(M_{f_1} + M_{f_2})$$

C. H. Llewellyn-Smith, Annals Phys. 53, 521 (1969).

QUARK-PHOTON VERTEX

- Since the Elastic Form Factors of mesons shall be extracted from the process $\gamma M \to M$, the dressed quark-photon vertex is needed.

- Within the framework described in the CI, the quark-photon vertex is given by,

$$\Gamma_\mu(Q^2) = \gamma_\mu^{\rm L} P_{\rm L}(Q^2) + \gamma_\mu^{\rm T} P_{\rm T}(Q^2)$$

- Where, $P_{\rm L}(Q^2) = 1$ and $P_{\rm T}(Q^2) = (1 + K_\gamma(Q^2))^{-1}$, with

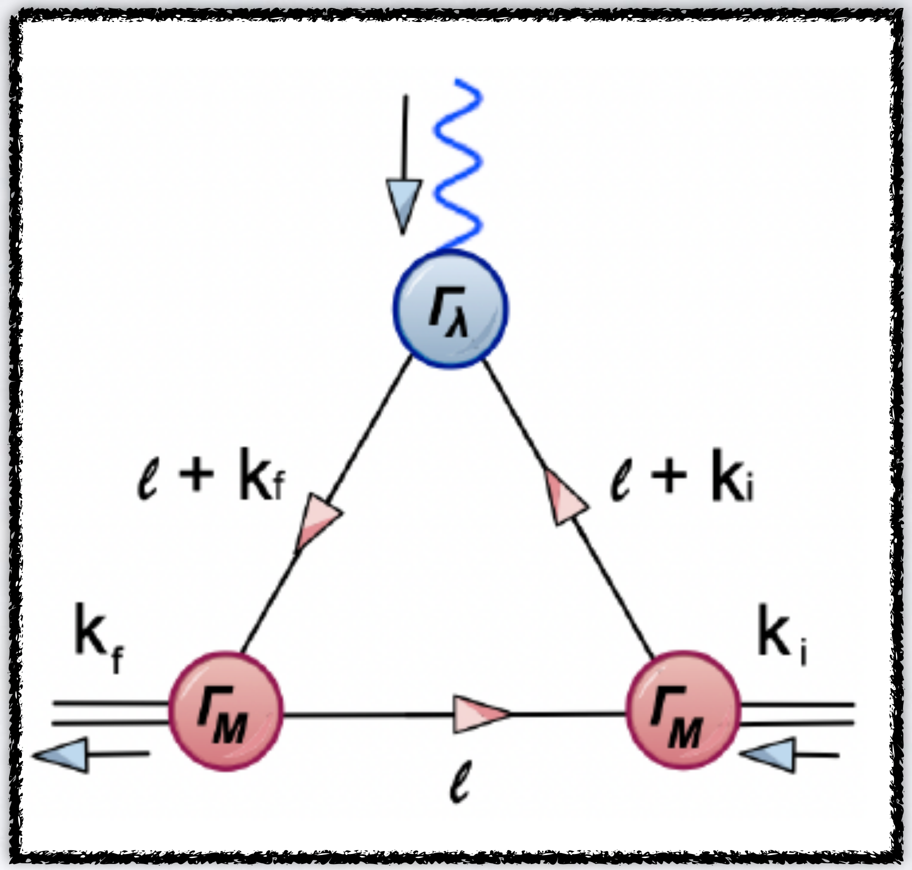
$$K_\gamma(Q^2) = \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \int_0^1 d\alpha \alpha(1-\alpha) Q^2 \overline{\mathcal{C}}_1(\omega)$$

- And $\overline{\mathcal{C}}_1(z) = \Gamma(0, z\tau_{\rm UV}^2) - \Gamma(0, z\tau_{\rm IR}^2)$, $\omega(x, y, z) = x + y(1-y)z$.

H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts, D.J. Wilson, *Phys.Rev.C* 83 (2011) 065206

ELASTIC FORM FACTORS

- The kinematics of the process is given by the Feynman diagram



- And the standard momentum parametrization,

$$k_i = K - Q/2$$

$$k_f = K + Q/2$$

- Using Feynman rules, we find that for a general meson, this process can be written as,

$$\Lambda^{M,f_1} = N_c \int \frac{d^4\ell}{(2\pi)^4} \mathrm{Tr}\, \mathcal{G}^{M,f_1}$$

- where we have omitted the Dirac indices, and,

$$\begin{aligned}\mathcal{G}^{M,f_1} &= -\imath\, \Gamma_M(k_f)\, S(\ell + k_i, M_{f_1})\, \Gamma_\lambda(Q) \\ &\times S(\ell + k_f, M_{f_1})\, \overline{\Gamma}_M(-k_i)\, S(\ell, M_{f_2})\end{aligned}$$

- The function Λ^{M,f_1} labels all M kind of mesons. Besides, the photon interacts with the quark with flavor f_1, and the fermion f_2 is a spectator.

- Λ^{M,f_1} can be written as a function of the Elastic Form Factors as,

Scalar	$\Lambda^{S,f_1} = -2K_\lambda F^{S,f_1}$
Pseudoscalar	$\Lambda^{PS,f_1} = -2K_\lambda F^{PS,f_1}$
Vector	$\Lambda^{V,f_1} = \sum_{i=1}^{3} T^{(i)}_{\lambda\mu\nu} F_i^{V,f_1}$
Axial-Vector	$\Lambda^{AV,f_1} = \sum_{i=1}^{3} T^{(i)}_{\lambda\mu\nu} F_i^{AV,f_1}$

- Where the tensor structure is given by

$$T^{(1)}_{\lambda\mu\nu} = 2K_\lambda \, \mathcal{P}^{\mathrm{T}}_{\mu\alpha}(p_i) \, \mathcal{P}^{\mathrm{T}}_{\alpha\nu}(p_f) \, ,$$

$$T^{(2)}_{\lambda\mu\nu} = \left(Q_\mu - p_{i,\mu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \mathcal{P}^{\mathrm{T}}_{\lambda\nu}(p_f) - \left(Q_\nu - p_{f,\nu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \mathcal{P}^{\mathrm{T}}_{\lambda\mu}(p_i) \, ,$$

$$T^{(3)}_{\lambda\mu\nu} = \frac{K_\lambda}{m^2_{\langle q\bar{q}\,'\rangle}} \left(Q_\mu - p_{i,\mu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \left(Q_\nu - p_{f,\nu} \frac{Q^2}{2m^2_{\langle q\bar{q}\,'\rangle}} \right) \, ,$$

- And $\mathcal{P}^{\mathrm{T}}_{\mu\nu}(p) = \delta_{\mu\nu} - p_\mu p_\nu / p^2$.

- In order to consider the total elastic form factor of mesons, we use,

$$F_M = e_{f_1} F^{M,f_1} + e_{\bar{f}_2} F^{M,\bar{f}_2}$$

- Furthermore, in the case of vector and axial-vector mesons, we present the results of the electric, magnetic and quadrupole form factors, defined as,

$$G_E = F_1 + \frac{2}{3}\eta G_Q$$
$$G_M = -F_2$$
$$G_Q = F_1 + F_2 + (1+\eta)F_3$$

- Where $\eta = Q^2/(4m^2_{\langle q\bar{q}'\rangle})$ and we label $\langle q\bar{q}'\rangle$ to the meson state.

ANALYSIS OF EFF

- The generation of dressed quark masses are,

$m_u = 0.007$	$m_d = 0.007$	$m_s = 0.17$	$m_c = 1.08$	$m_b = 3.92$
$M_u = 0.367$	$M_d = 0.367$	$M_s = 0.53$	$M_c = 1.52$	$M_b = 4.68$

- In the unified scheme for the generation of masses of mesons, it depends on the following parameters,

Quarks	Z_H	$\Lambda_{\rm UV}/\rm GeV$
u, d, s	1	0.905
c, d, s	3.034	1.322
c	13.122	2.305
b, u, s	16.473	2.522
b, c	59.056	4.131
b	165.848	6.559

$$\tau = 1/\Lambda$$

$$\Lambda_{\rm IR} = 0.24\,\rm GeV$$

Shown earlier in this conference by G. Paredes

- With these parameters, masses of mesons and BSA are,

		Scalar		Pseudoscalar			Vector		Axial-Vector	
$\langle q\bar{q}'\rangle$	Charge	Mass	E_S	Mass	E_{PS}	F_{PS}	Mass	E_V	Mass	E_{AV}
$u\bar{d}$	1	1.22	0.66	0.14	3.60	0.47	0.93	1.53	1.37	0.32
$u\bar{s}$	1	1.33	0.65	0.49	3.81	0.59	1.03	1.62	1.48	0.32
$s\bar{s}$	0	1.45	0.64	0.69	4.04	0.74	1.12	1.73	1.58	0.32
$c\bar{u}$	0	2.32	0.39	1.87	3.03	0.37	2.06	1.23	2.41	0.20
$c\bar{s}$	1	2.43	0.37	1.96	3.24	0.51	2.14	1.32	2.51	0.19
$u\bar{b}$	1	5.50	0.21	5.28	1.50	0.09	5.33	0.65	5.55	0.11
$s\bar{b}$	0	5.59	0.20	5.37	1.59	0.13	5.41	0.67	5.64	0.10
$c\bar{b}$	1	6.45	0.08	6.29	0.73	0.11	6.32	0.27	6.48	0.04
$c\bar{c}$	0	3.35	0.16	2.98	2.16	0.41	3.15	0.61	3.40	0.08
$b\bar{b}$	0	9.50	0.04	9.40	0.48	0.10	9.42	0.15	9.52	0.02

- By suitable projector operators, we find the charge radius defined as,

- $$r^2_{\langle q\bar{q}'\rangle} = \left\| \left(6 \frac{d}{dQ^2} F_M \Big|_{Q^2\to 0} \right) \right\|$$

L. X. Gutierrez-Guerrero, A. Bashir, M. A. Bedolla and E. Santopinto, *Phys.Rev.D* 100 (2019) 11, 114032

- For scalar and pseudoscalar mesons, we find the following charge radii,

	Scalar	Pseudoscalar
$\langle q\bar{q}\rangle$	$r^{S}_{\langle q\bar{q}\rangle}/\text{fm}$	$r^{PS}_{\langle q\bar{q}\rangle}/\text{fm}$
$u\bar{d}$	0.546	0.450
$u\bar{s}$	0.539	0.424
$s\bar{s}$	0.501	0.359
$c\bar{u}$	0.463	0.361
$c\bar{s}$	0.420	0.257
$u\bar{b}$	0.210	0.528
$s\bar{b}$	0.408	0.341
$c\bar{b}$	0.531	0.422
$c\bar{c}$	0.434	0.202
$b\bar{b}$	0.426	0.329

[In preparation, A. Bashir, M. A. Bedolla, R. J. H-P, L.X. Gutierrez-Guerrero]

- For vector mesons, we find the following charge radii,

$\langle q\bar{q}\rangle$	$r^E_{\langle q\bar{q}\rangle}$/fm	$r^M_{\langle q\bar{q}\rangle}$/fm	$r^Q_{\langle q\bar{q}\rangle}$/fm	$\mu_{\langle q\bar{q}\rangle}$	$\mathcal{Q}_{\langle q\bar{q}\rangle}$
$u\bar{d}$	0.560	0.745	0.472	2.11	-0.85
$u\bar{s}$	0.535	0.733	0.471	2.18	-0.90
$s\bar{s}$	0.473	0.632	0.397	2.09	-0.83
$c\bar{u}$	0.425	0.820	0.572	-1.51	1.05
$c\bar{s}$	0.338	0.543	0.355	2.10	-0.77
$u\bar{b}$	0.537	1.507	1.031	5.99	-3.02
$s\bar{b}$	0.354	0.955	0.672	-2.06	1.32
$c\bar{b}$	0.451	0.874	0.580	3.07	-1.38
$c\bar{c}$	0.339	0.477	0.268	2.03	-0.70
$b\bar{b}$	0.361	0.510	0.280	2.00	-0.67

- And

- $G_M^{\langle q\bar{q}'\rangle}(Q^2=0)=\mu_{\langle q\bar{q}'\rangle}$ $\qquad$ $G_Q^{\langle q\bar{q}'\rangle}(Q^2=0)=\mathcal{Q}_{\langle q\bar{q}'\rangle}$

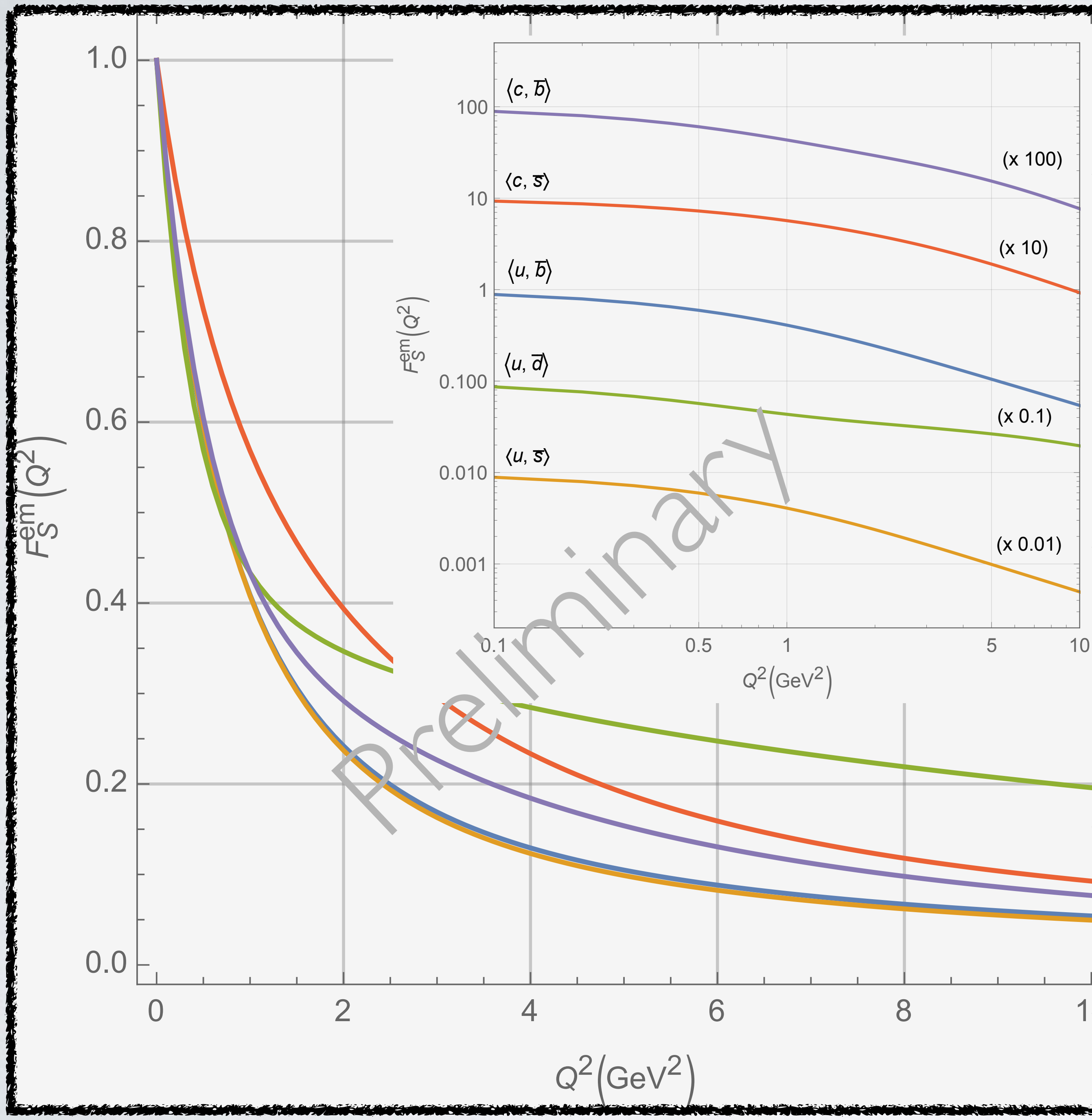
[In preparation, A. Bashir, M. A. Bedolla, R. J. H-P, L.X. Gutierrez-Guerrero]

- A similar analysis can be done for axial-vector mesons. Then, we find the following charge radii predictions,

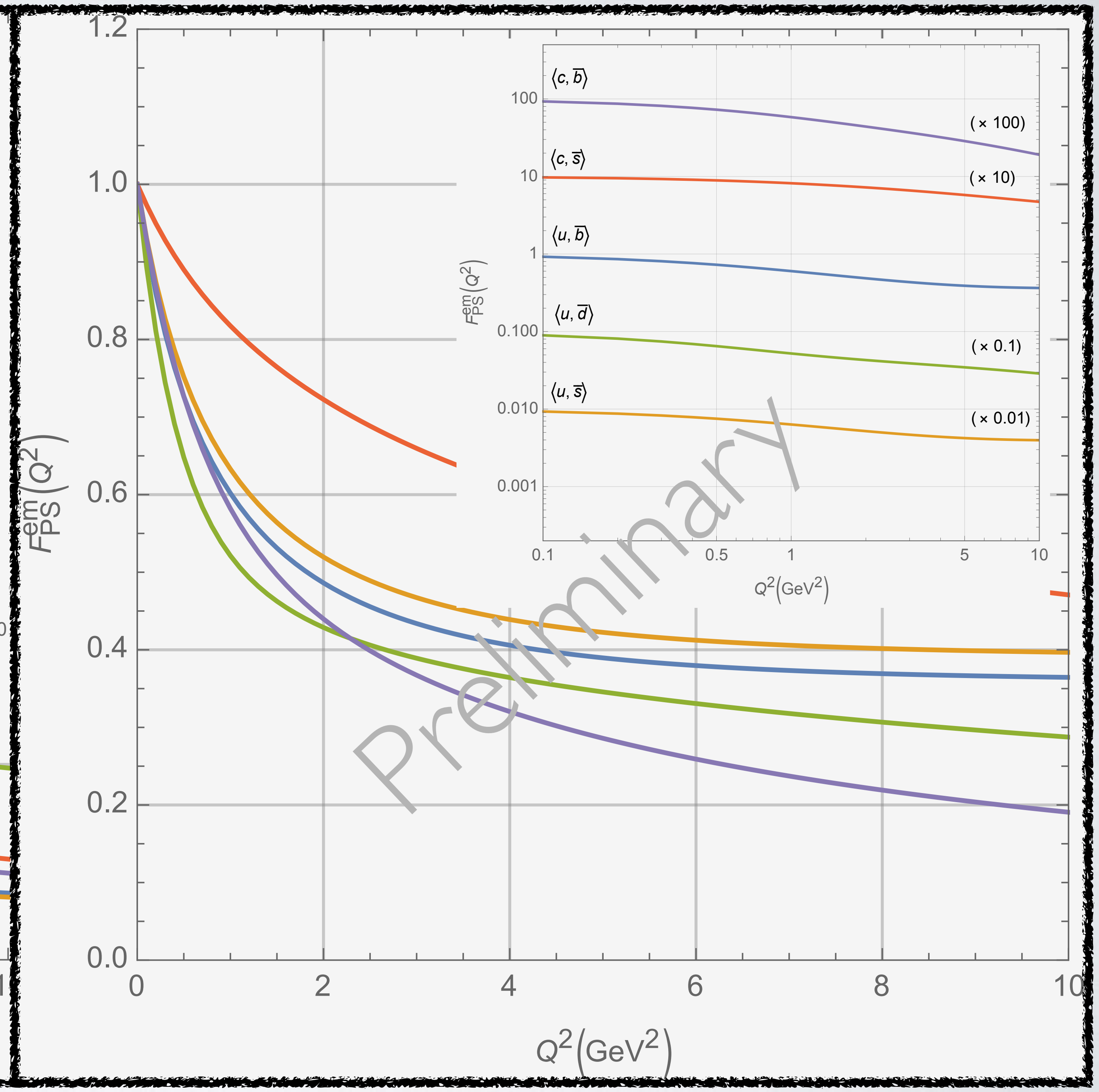
$\langle q\bar{q}\rangle$	$r^E_{\langle q\bar{q}\rangle}$/fm	$r^M_{\langle q\bar{q}\rangle}$/fm	$r^Q_{\langle q\bar{q}\rangle}$/fm	$\mu_{\langle q\bar{q}\rangle}$	$\mathcal{Q}_{\langle q\bar{q}\rangle}$
$u\bar{d}$	0.564	0.789	0.365	2.15	-0.52
$u\bar{s}$	0.556	0.785	0.323	2.19	-0.45
$s\bar{s}$	0.518	0.715	0.221	2.11	-0.30
$c\bar{u}$	0.460	0.862	0.117	-1.36	0.14
$c\bar{s}$	0.429	0.657	0.274	2.12	0.12
$u\bar{b}$	0.597	1.588	0.752	5.93	0.30
$s\bar{b}$	0.406	1.056	0.782	-2.15	-0.81
$c\bar{b}$	0.530	1.029	1.710	3.47	7.23
$c\bar{c}$	0.437	0.606	0.544	2.10	1.38
$b\bar{b}$	0.425	0.597	1.606	2.11	13.55

[In preparation, A. Bashir, M. A. Bedolla, R. J. H-P, L.X. Gutierrez-Guerrero]

- The behavior of the Electromagnetic Form Factors is presented in the next plots for charged mesons formed of different flavor quarks.

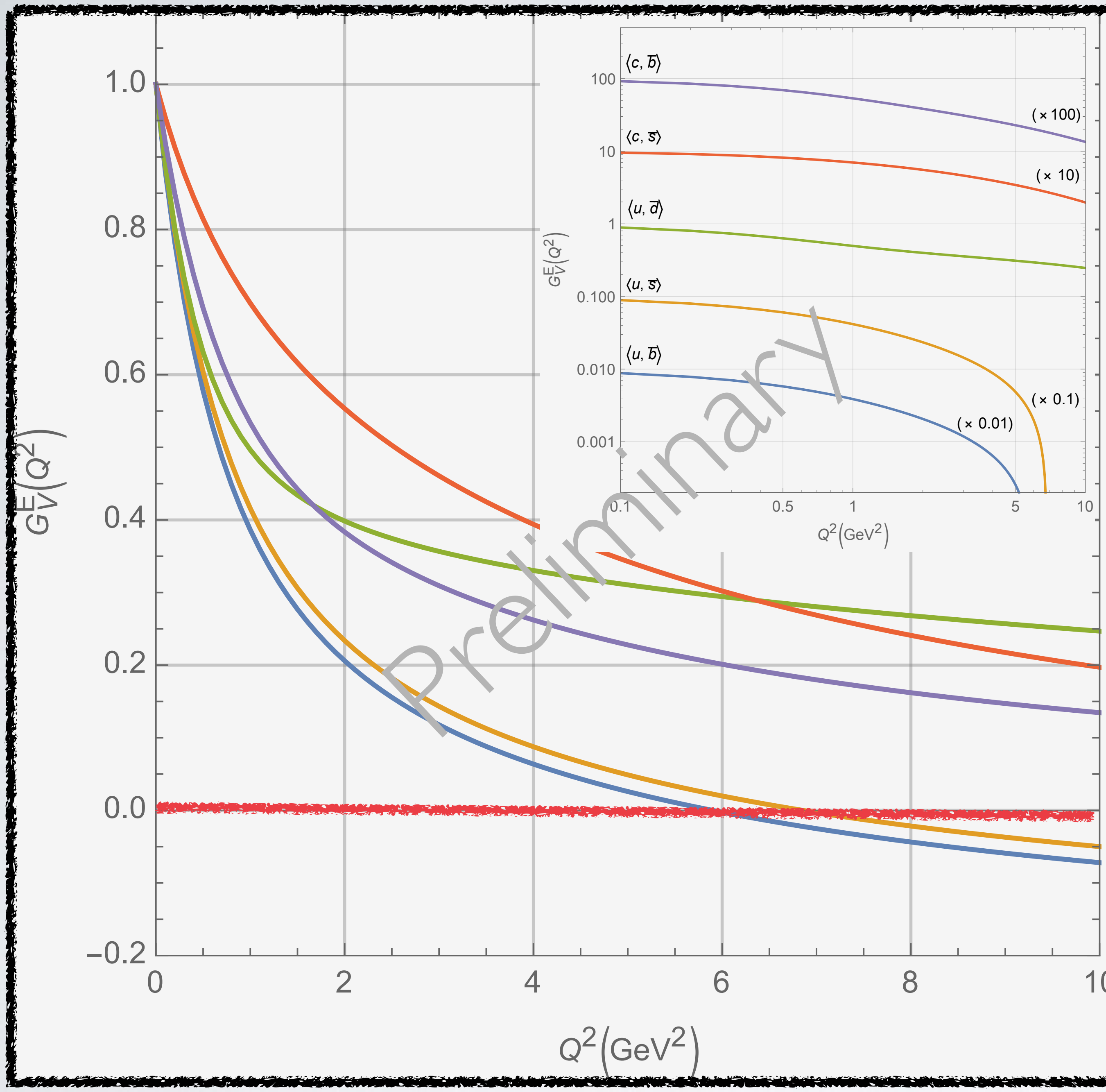


Scalar mesons

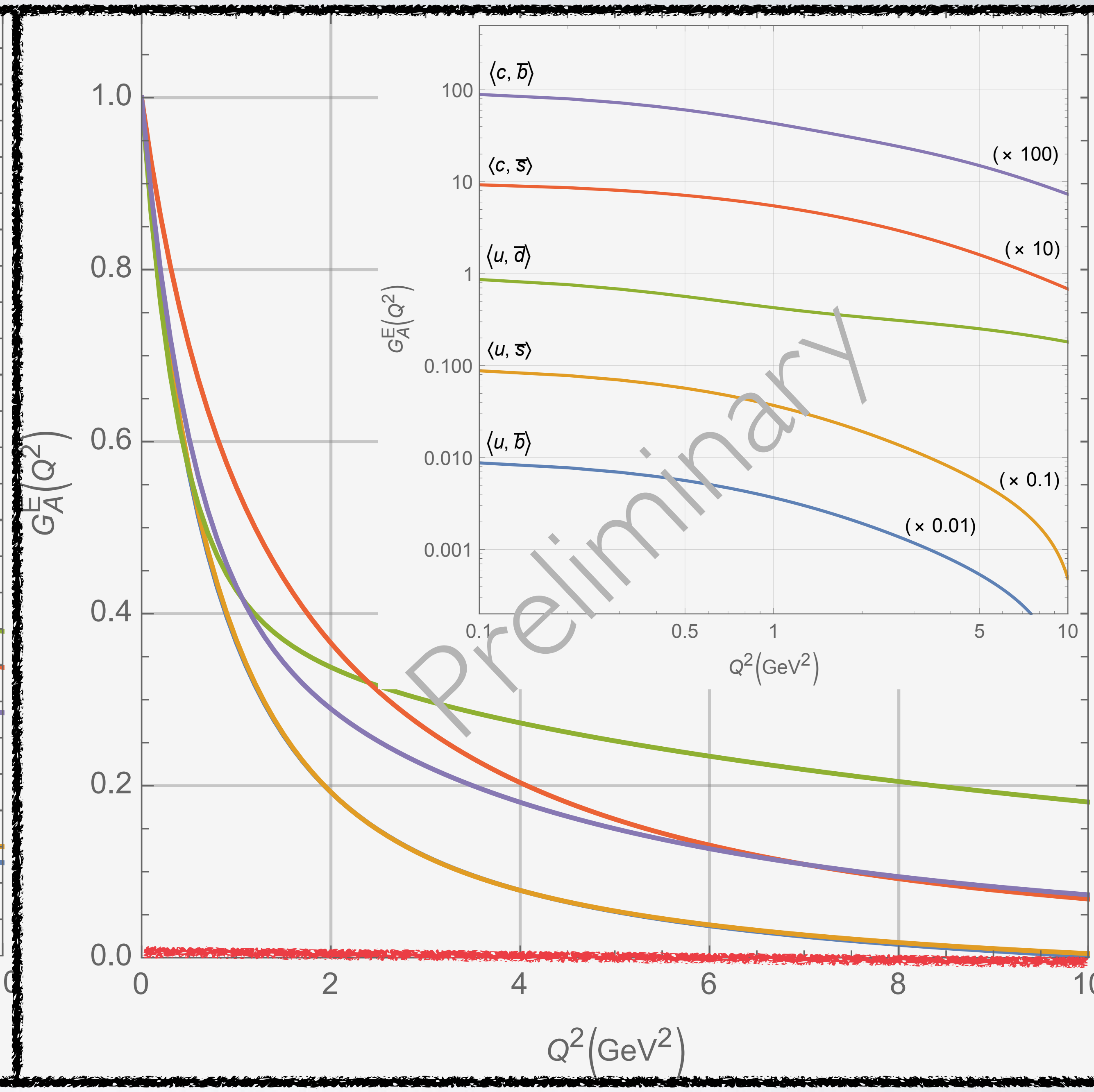


Pseudoscalar mesons

- Theoretical predictions for the Electric Form Factors are presented for charged vector and axial-vector mesons.



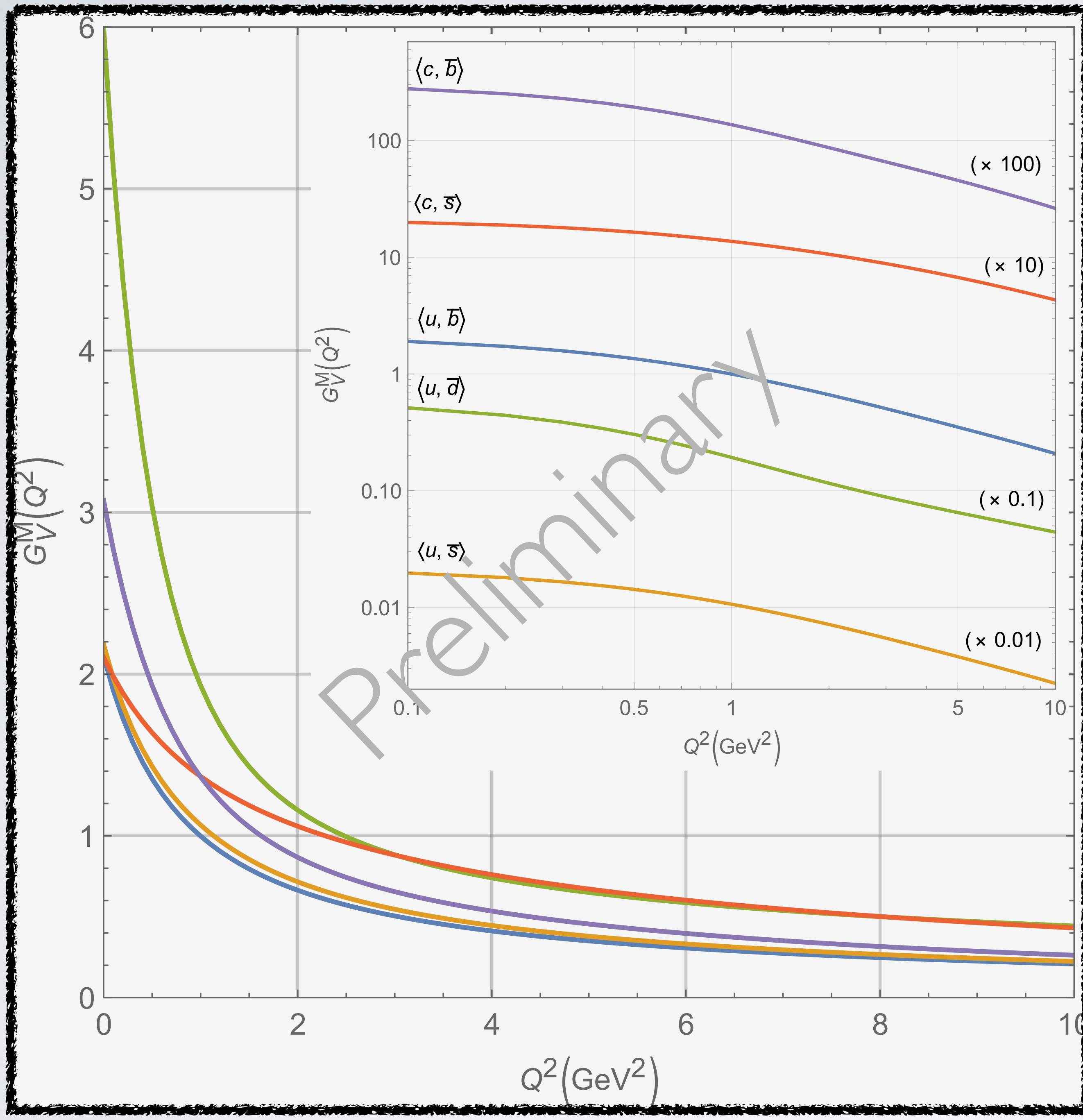
Vector mesons

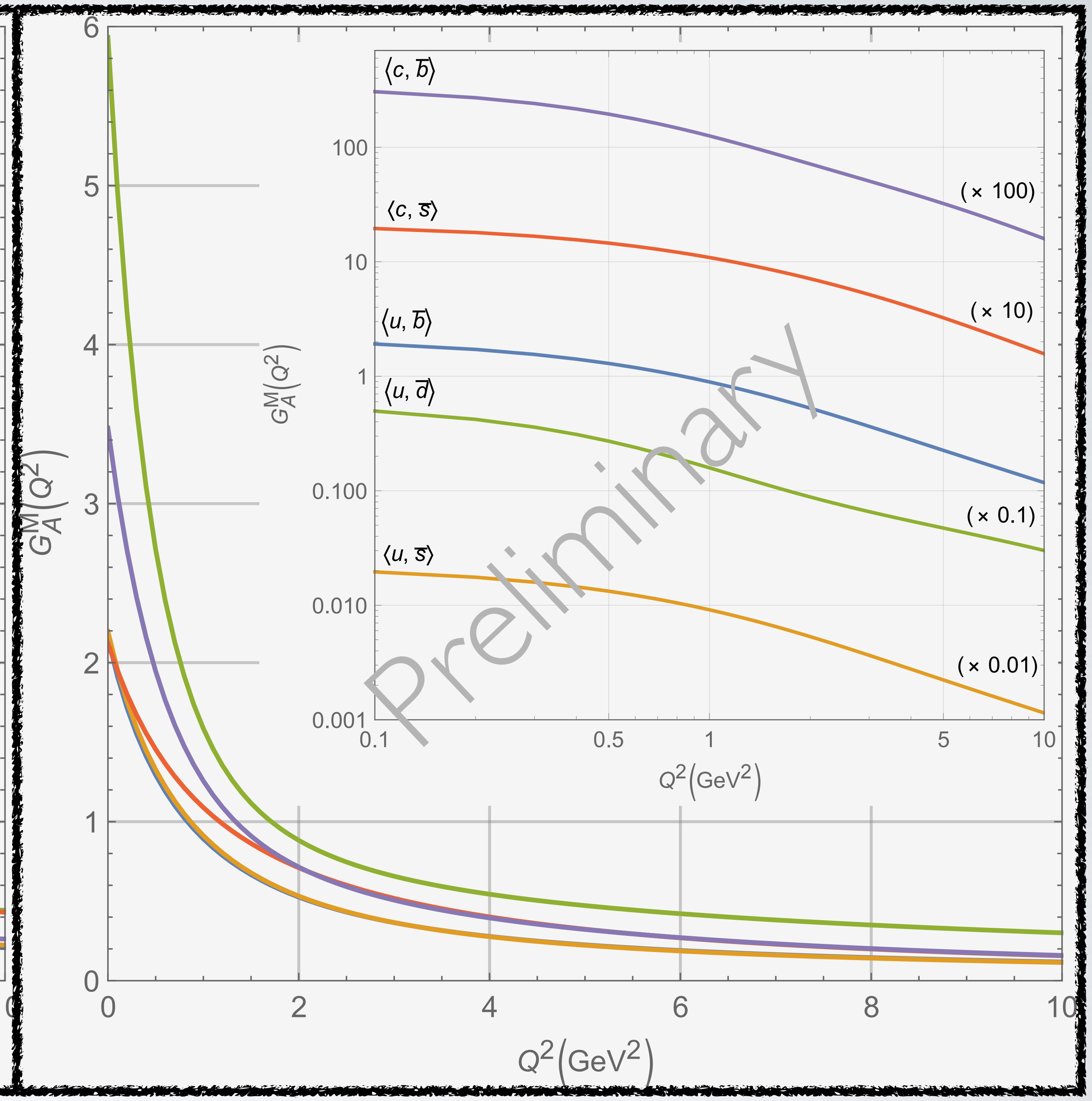


Axial-vector mesons

- Electric Form Factors diminish asymptotically with Q^2.

- Theoretical predictions for the Magnetic Form Factors are presented for charged vector and axial-vector mesons.



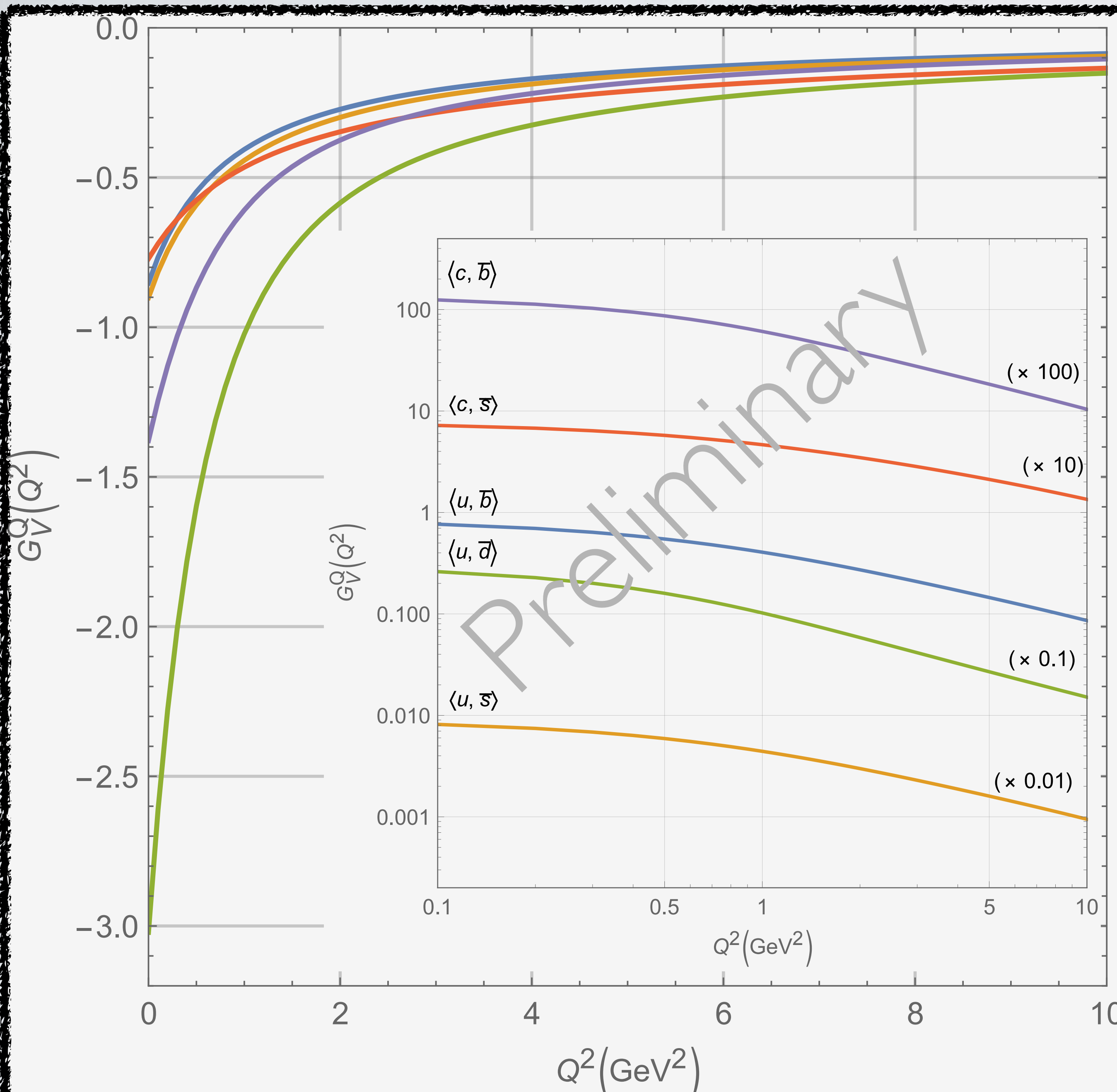


Vector mesons

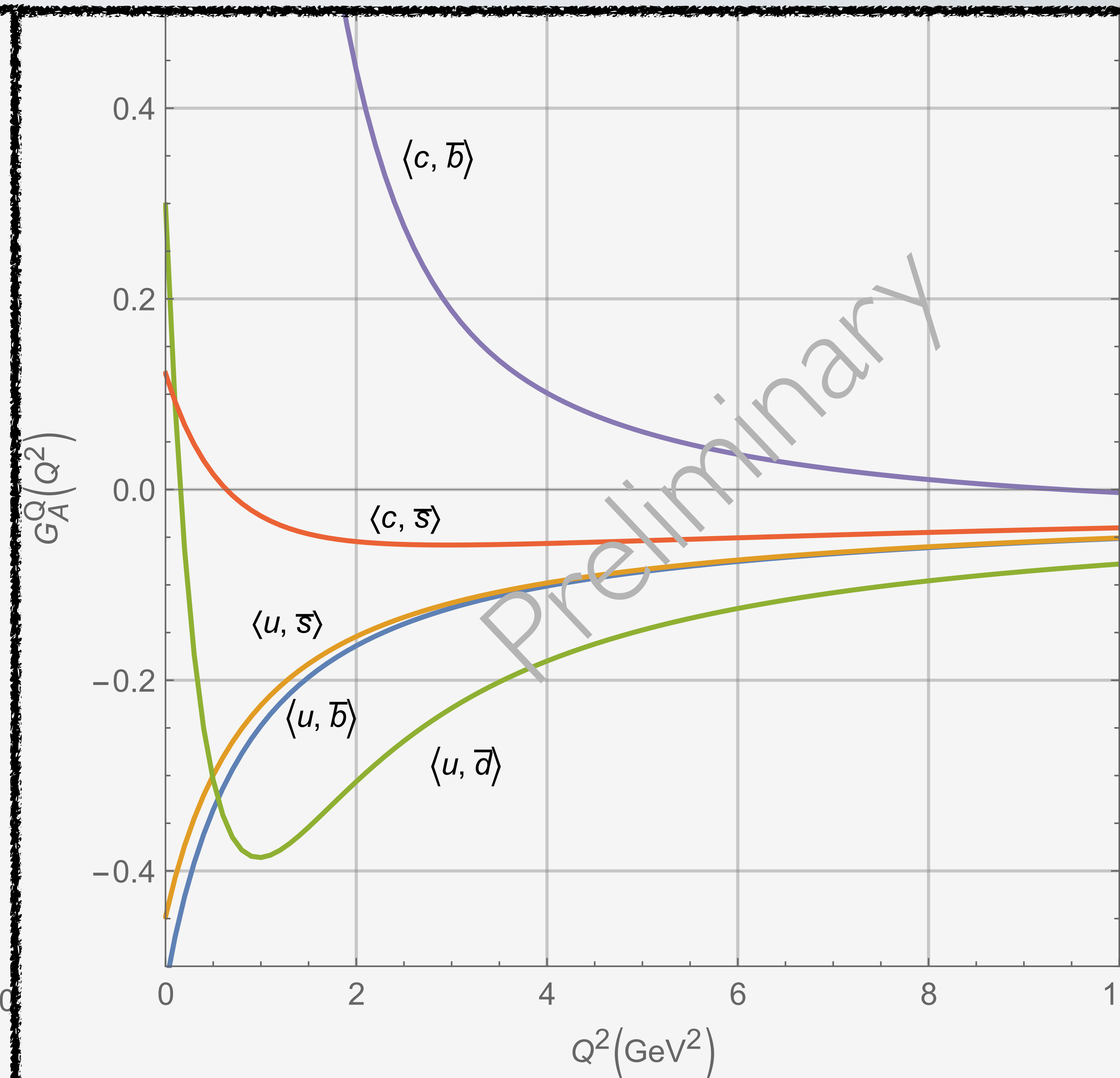
Axial-vector mesons

- Small fluctuations are found in these cases.

- Theoretical predictions for the Quadrupolar Form Factors are presented for charged vector and axial-vector mesons.



Vector mesons



Axial-vector mesons

- The impact of the γ_5 matrix is in the quadrupole component.

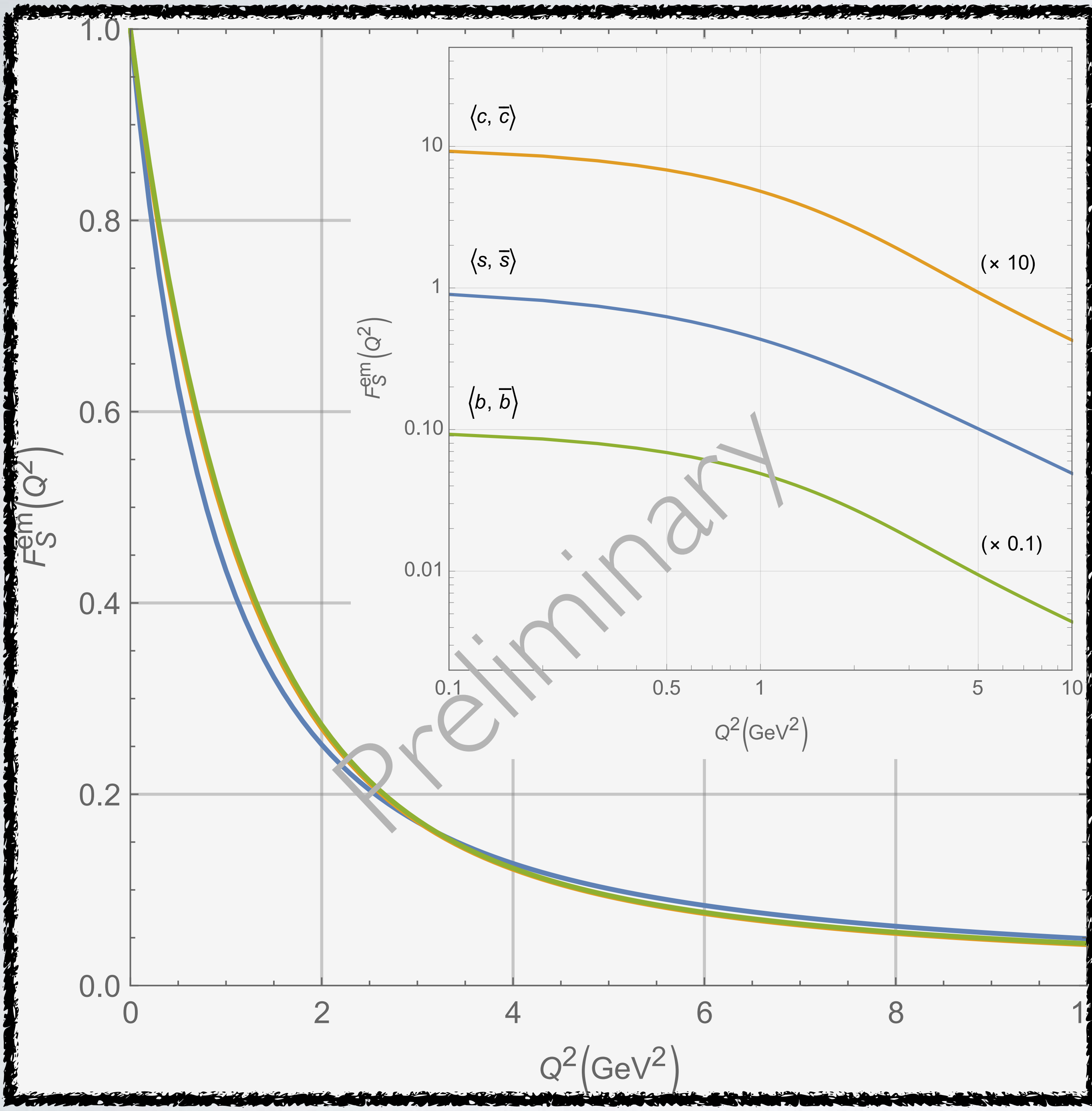
CONCLUSIONS

- The determination of the internal structure of mesons can be understood from Form Factors.
- In this work, we present the study of Form Factors of the elastic process $\gamma M \rightarrow M$, for scalar, pseudo scalar, vector and axial-vector mesons.
- By means of the Contact Interaction, we find meson masses, BSA, charge radii for all kinds of mesons in a Unified scheme.
- In the case of scalar and pseudoscalar mesons, we find a similar behavior of the Form Factors.
- However, for vector and axial-vector mesons, the quadrupole component receives important contributions which shall be interesting to study at experiments.

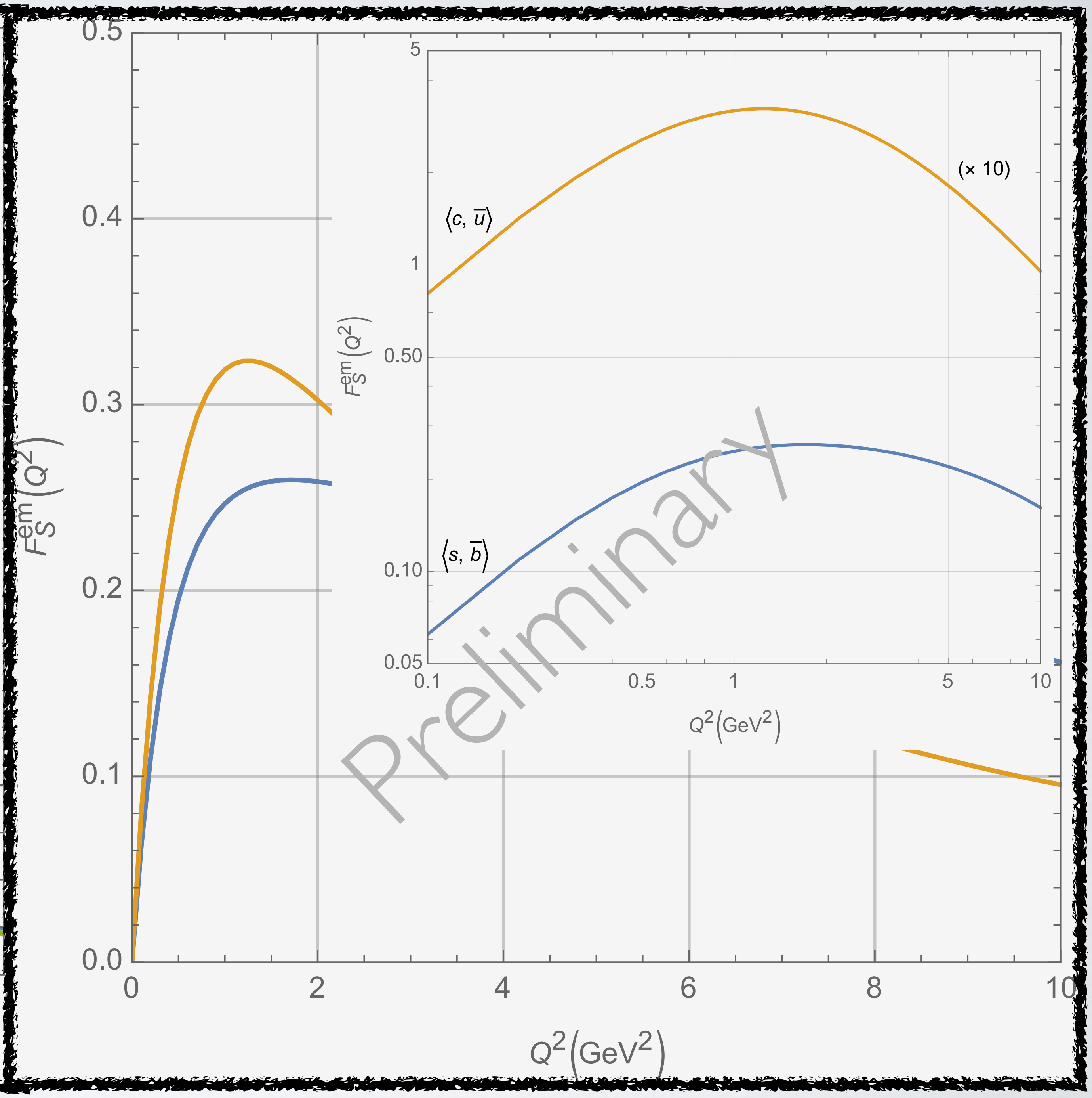
THANK YOU !

BACKUP

- Theoretical predictions for the electromagnetic Form Factors for neutral scalar mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.

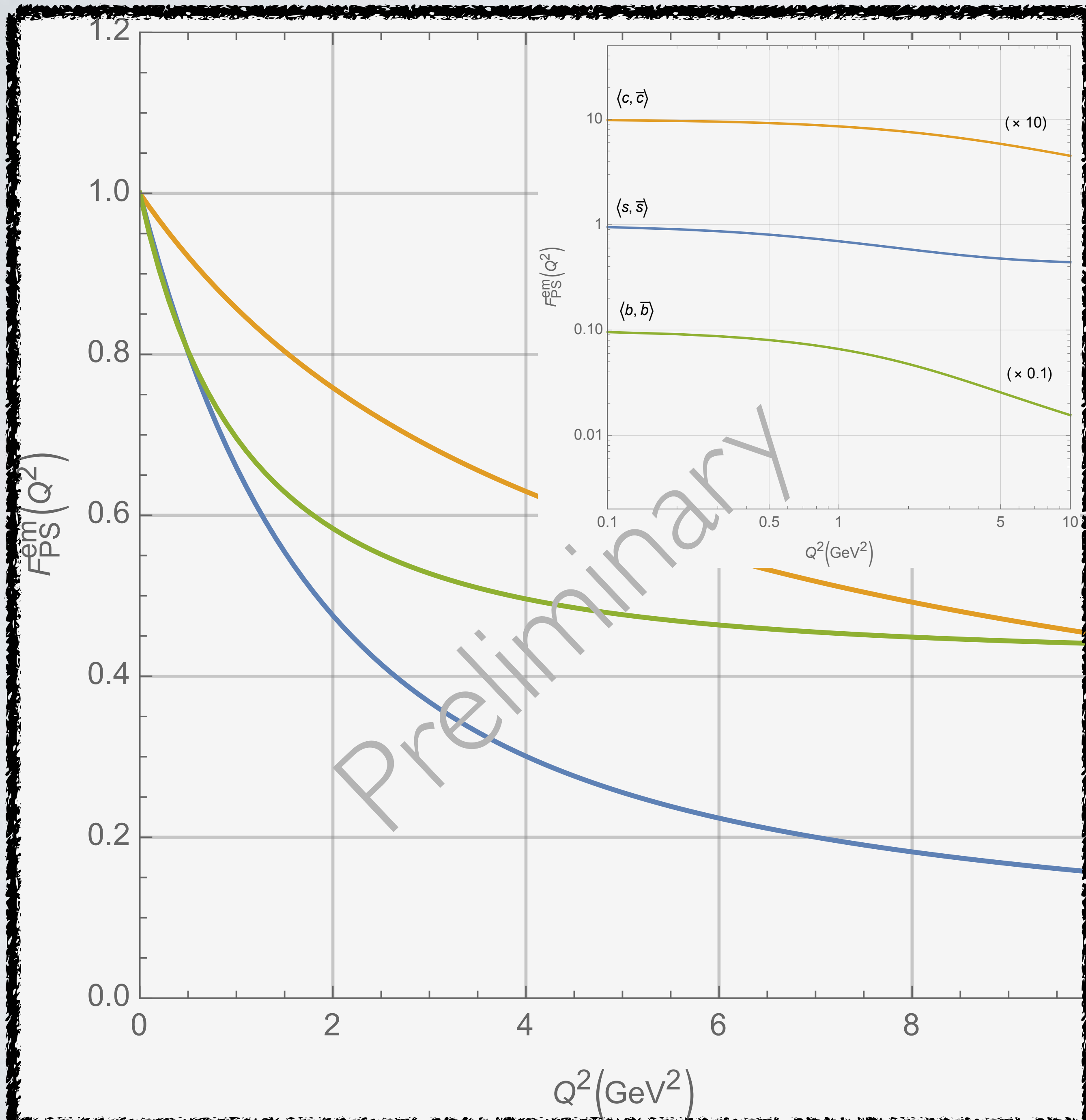


Same flavor



Different flavor

- Theoretical predictions for the electromagnetic Form Factors for neutral pseudoscalar mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.



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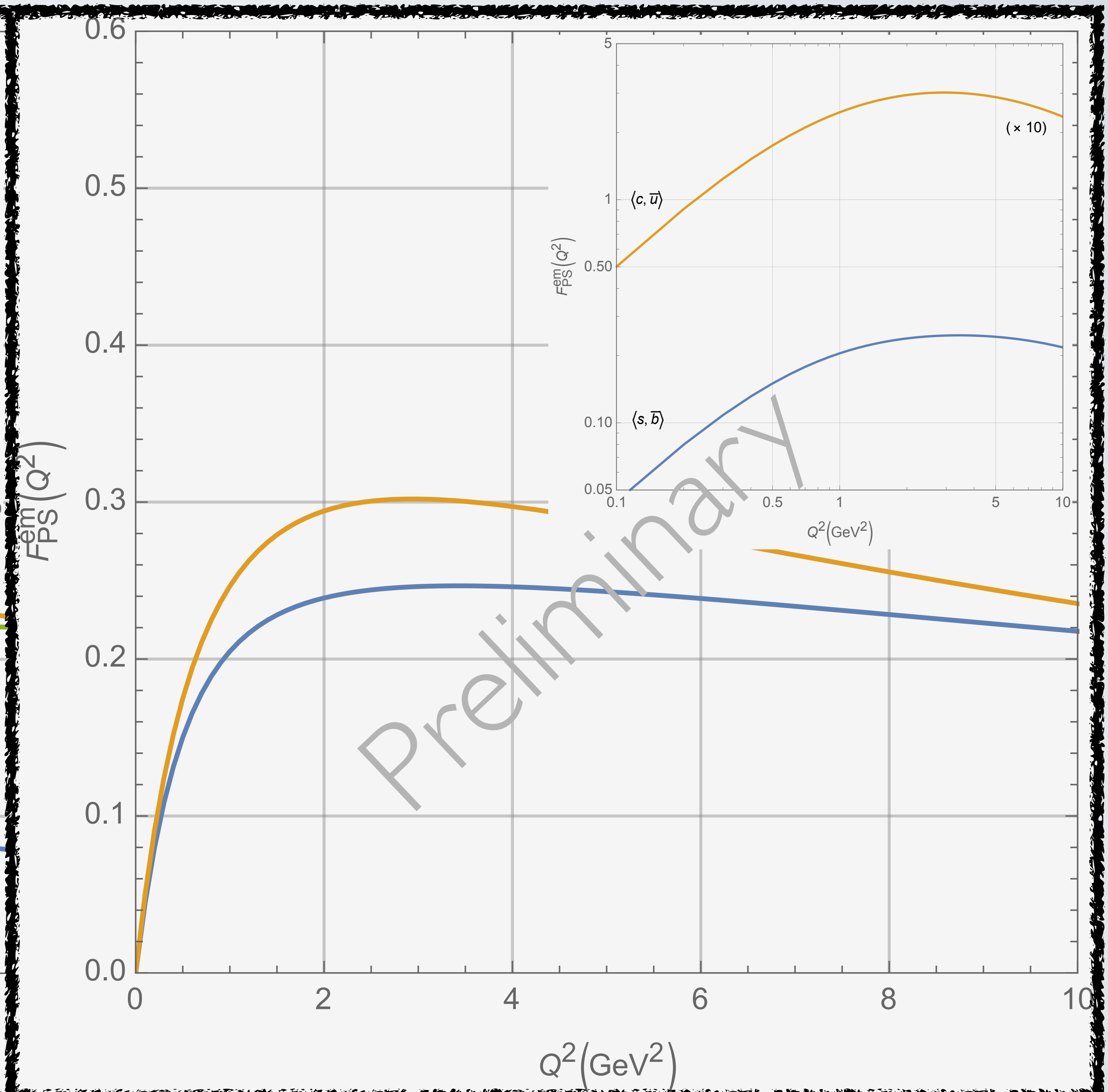
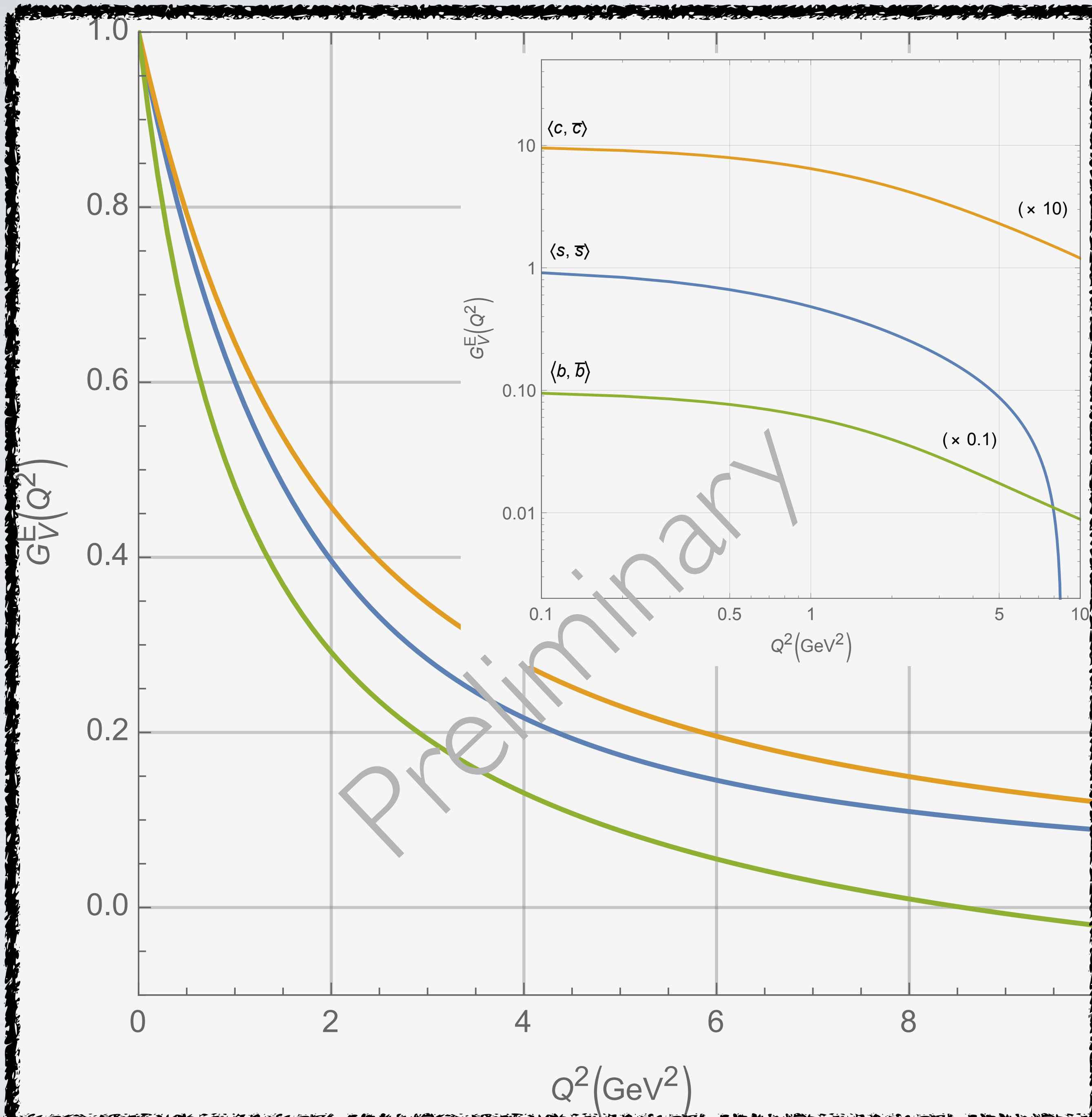


Different flavor

- Theoretical predictions for the Electric Form Factors for neutral vector mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.



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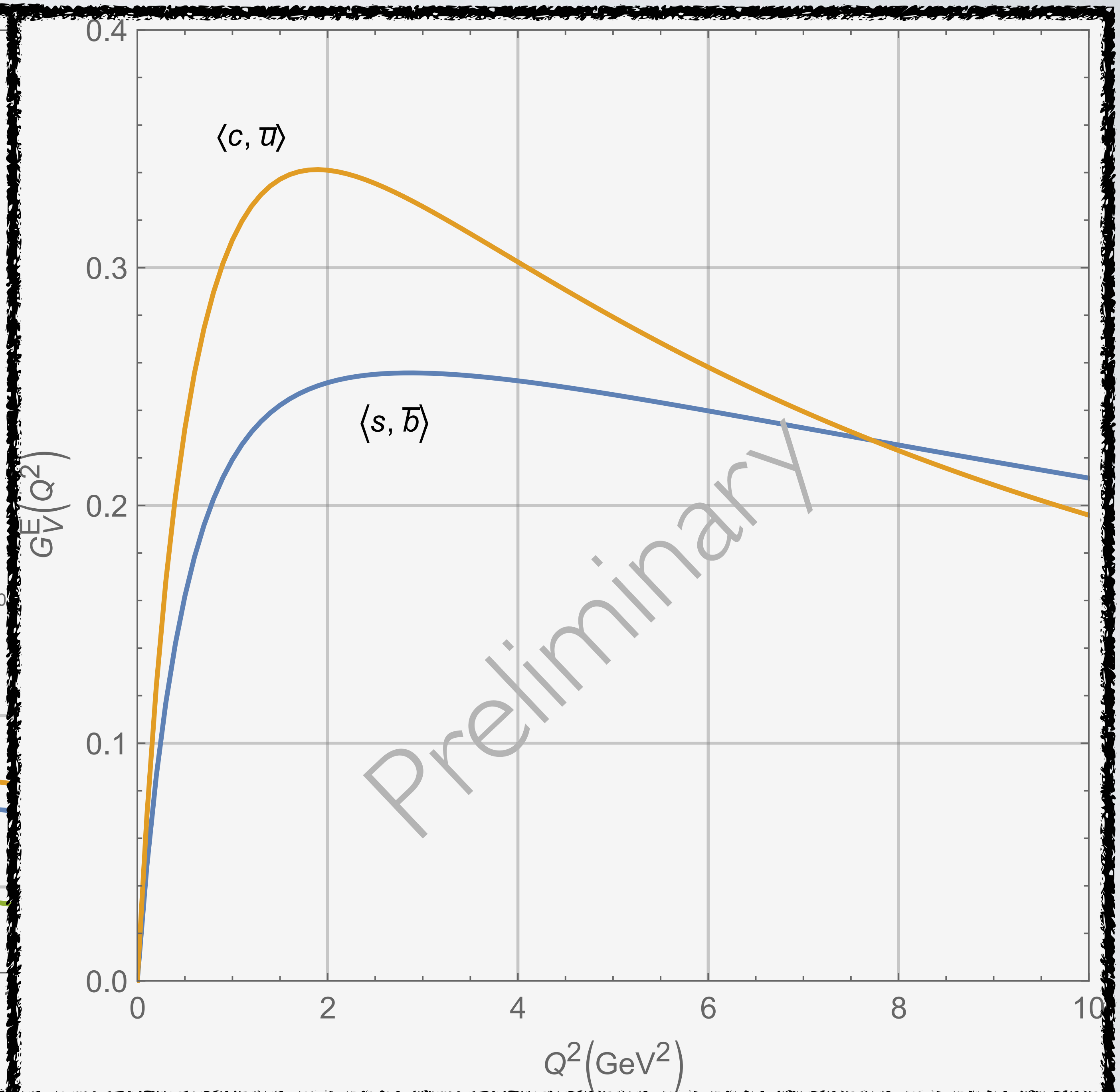
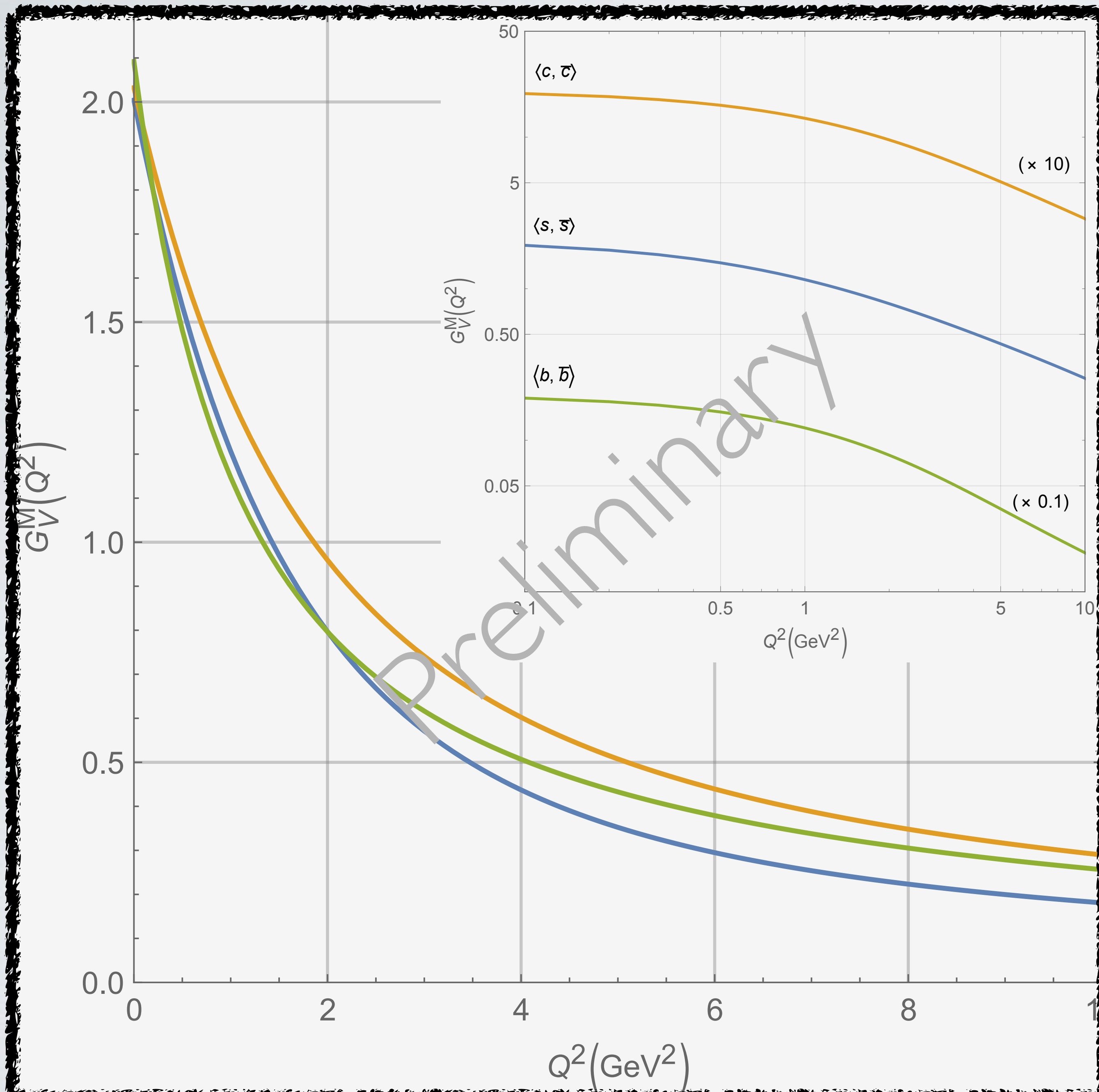


Different flavor

- Theoretical predictions for the Magnetic Form Factors for neutral vector mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.



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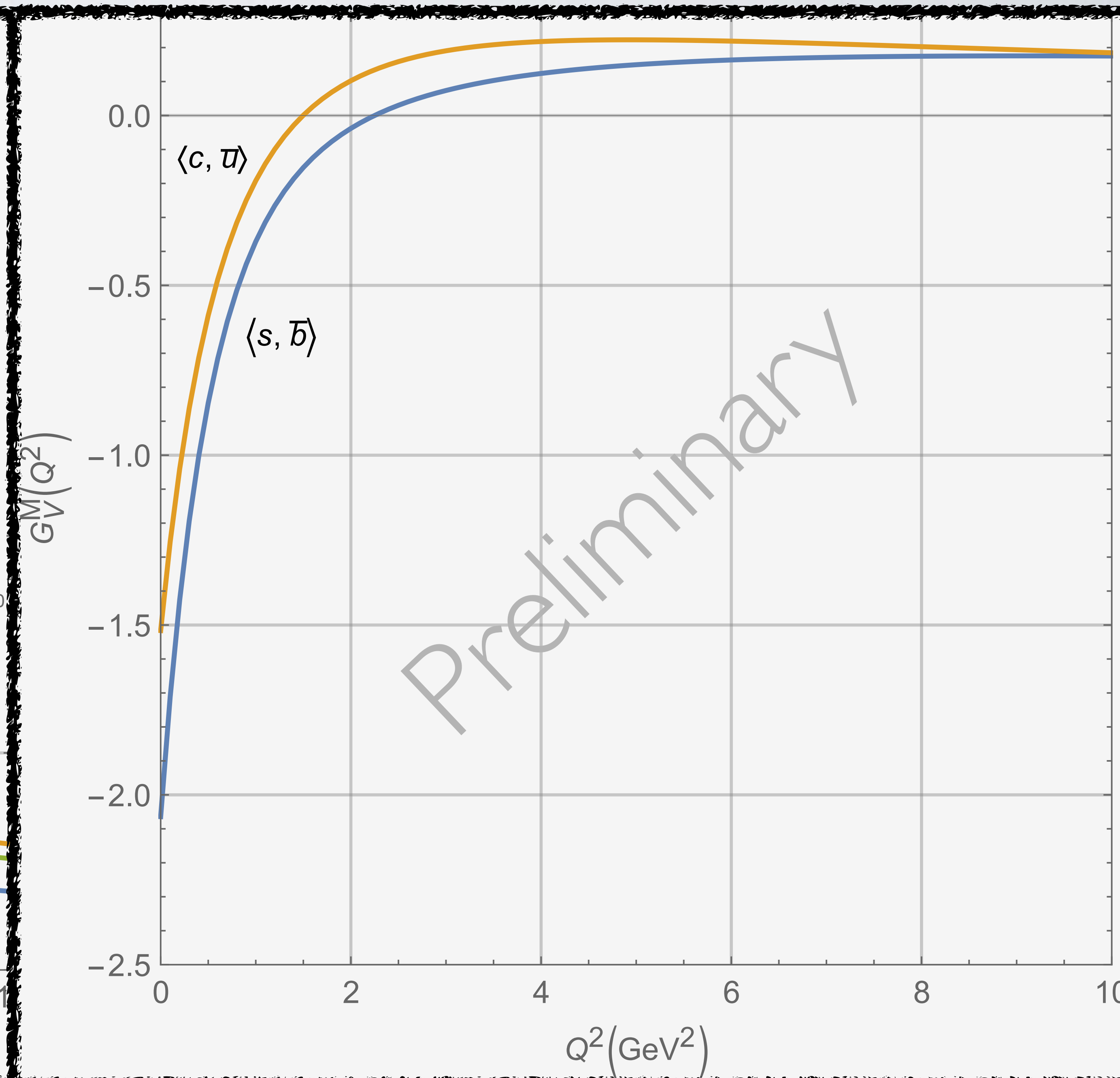
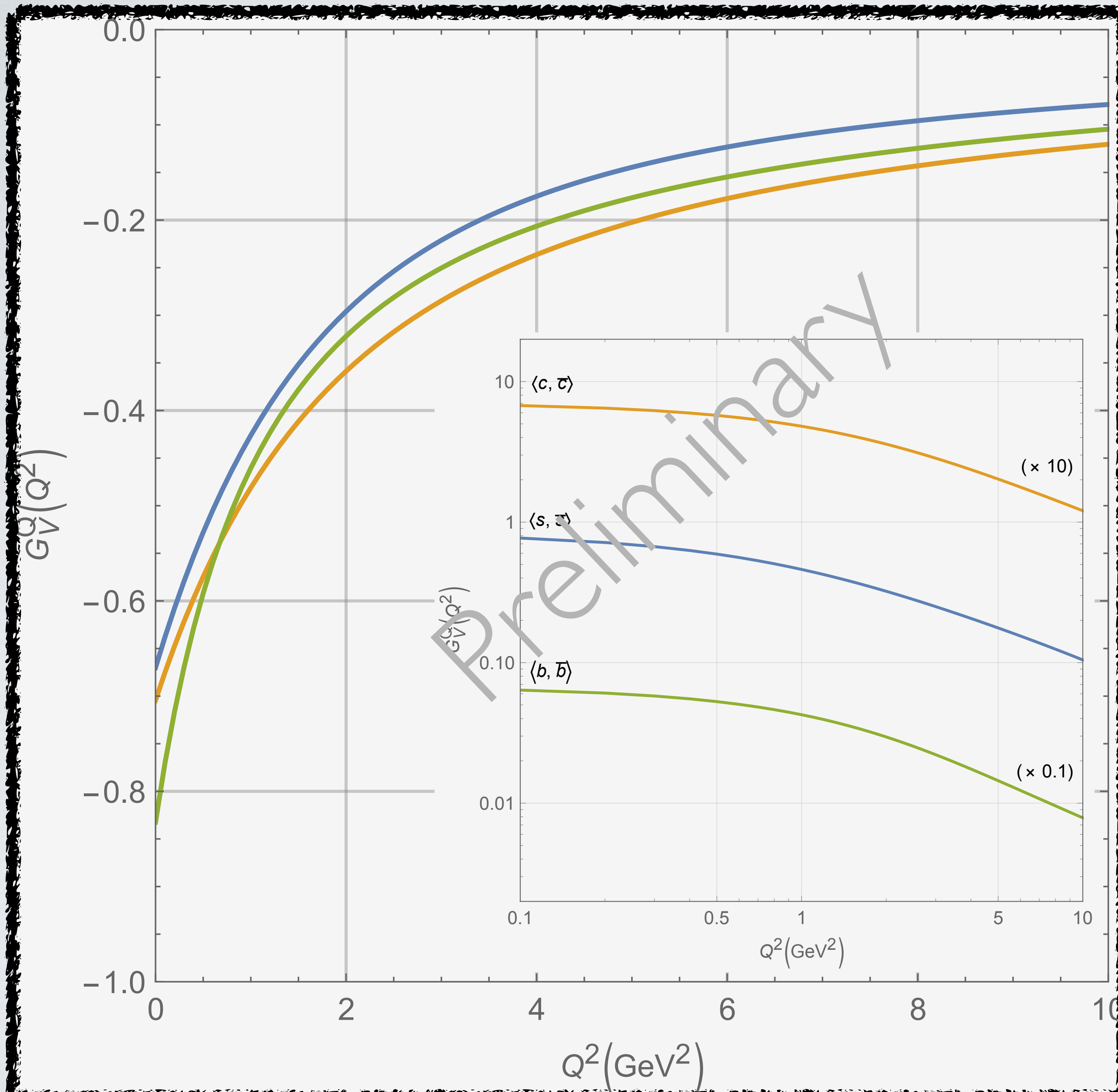
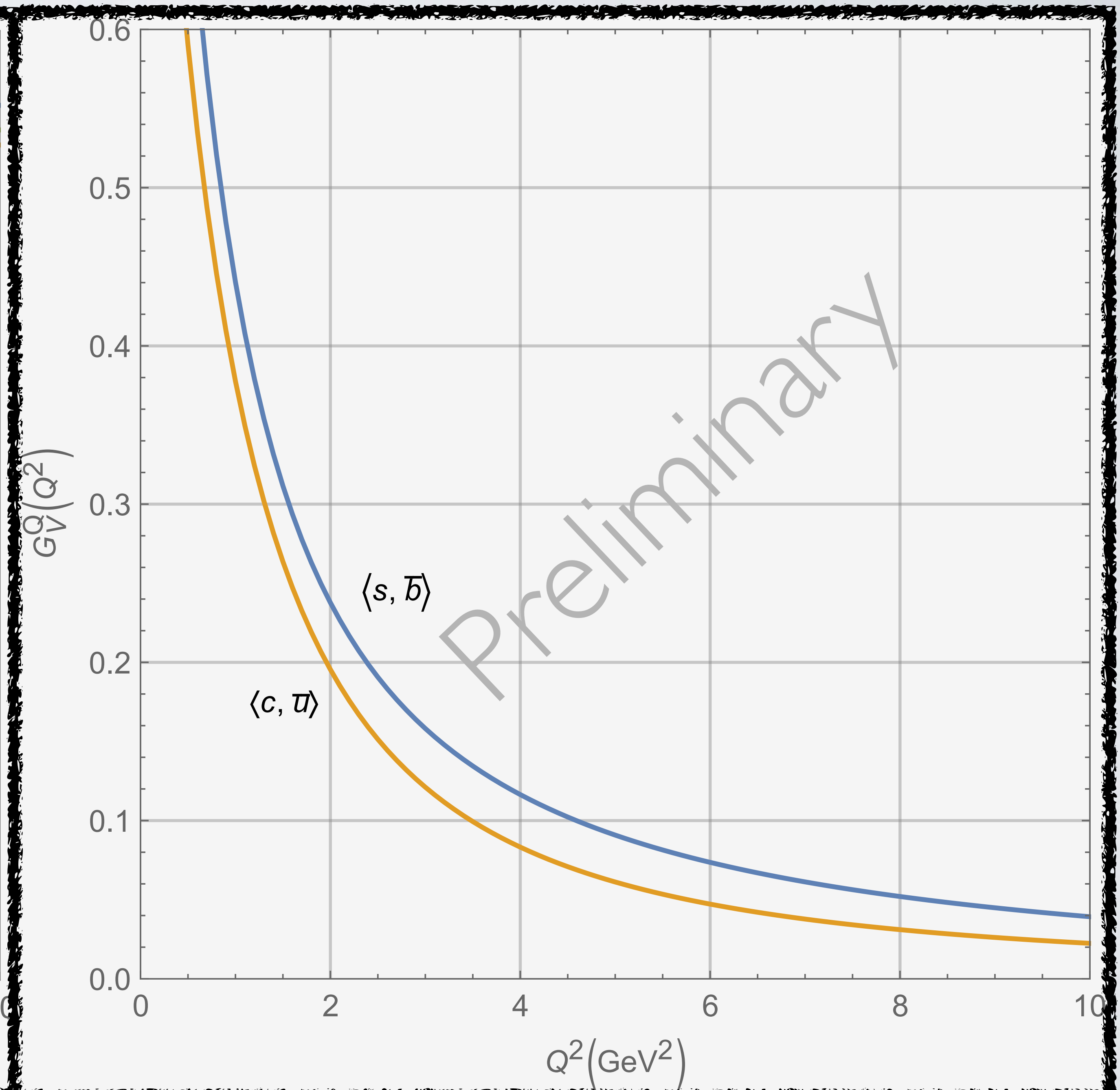


Different flavor

- Theoretical predictions for the Quadrupolar Form Factors for neutral vector mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.

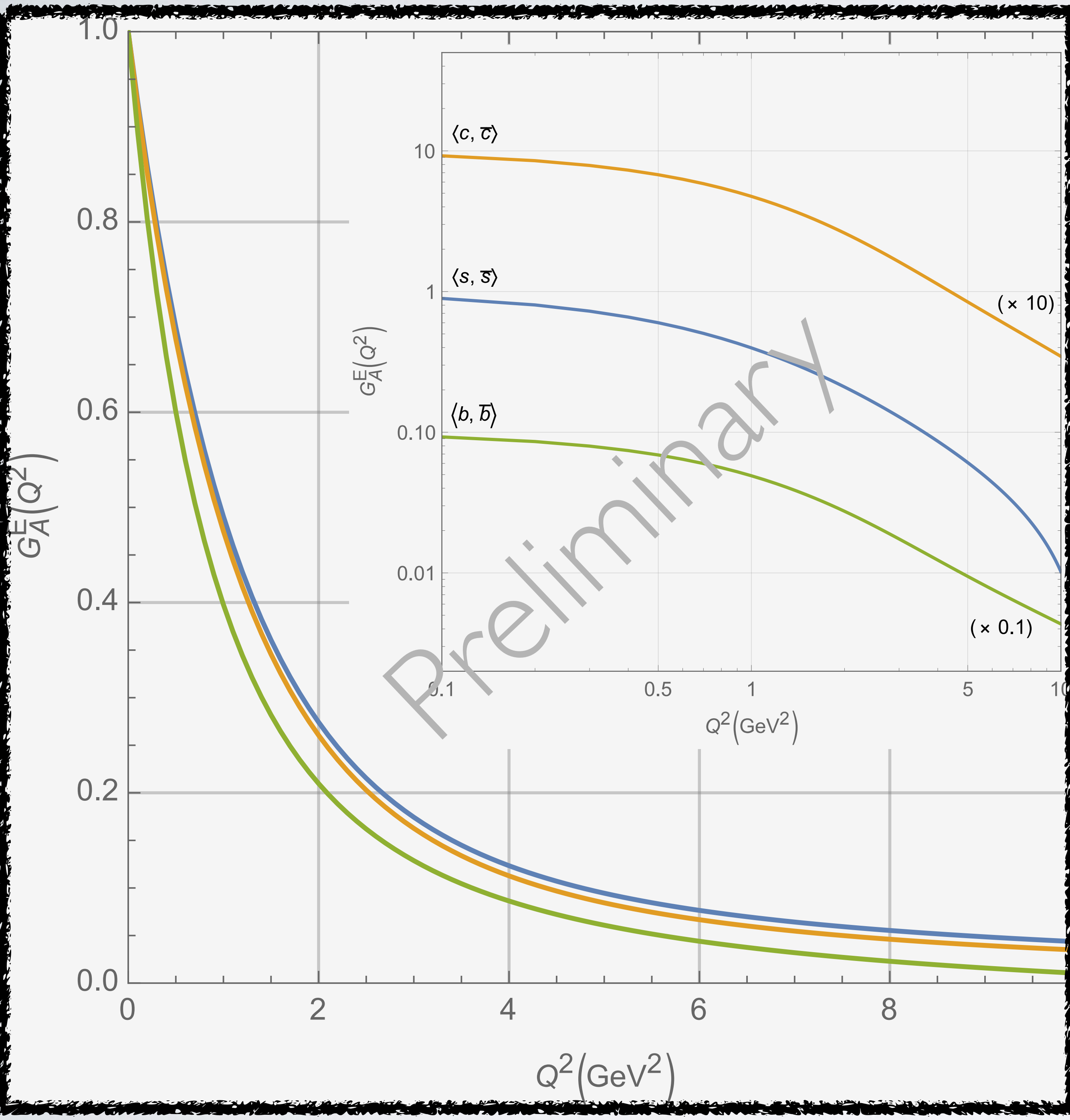


Same flavor



Different flavor

- Theoretical predictions for the Electric Form Factors for neutral axial-vector mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.



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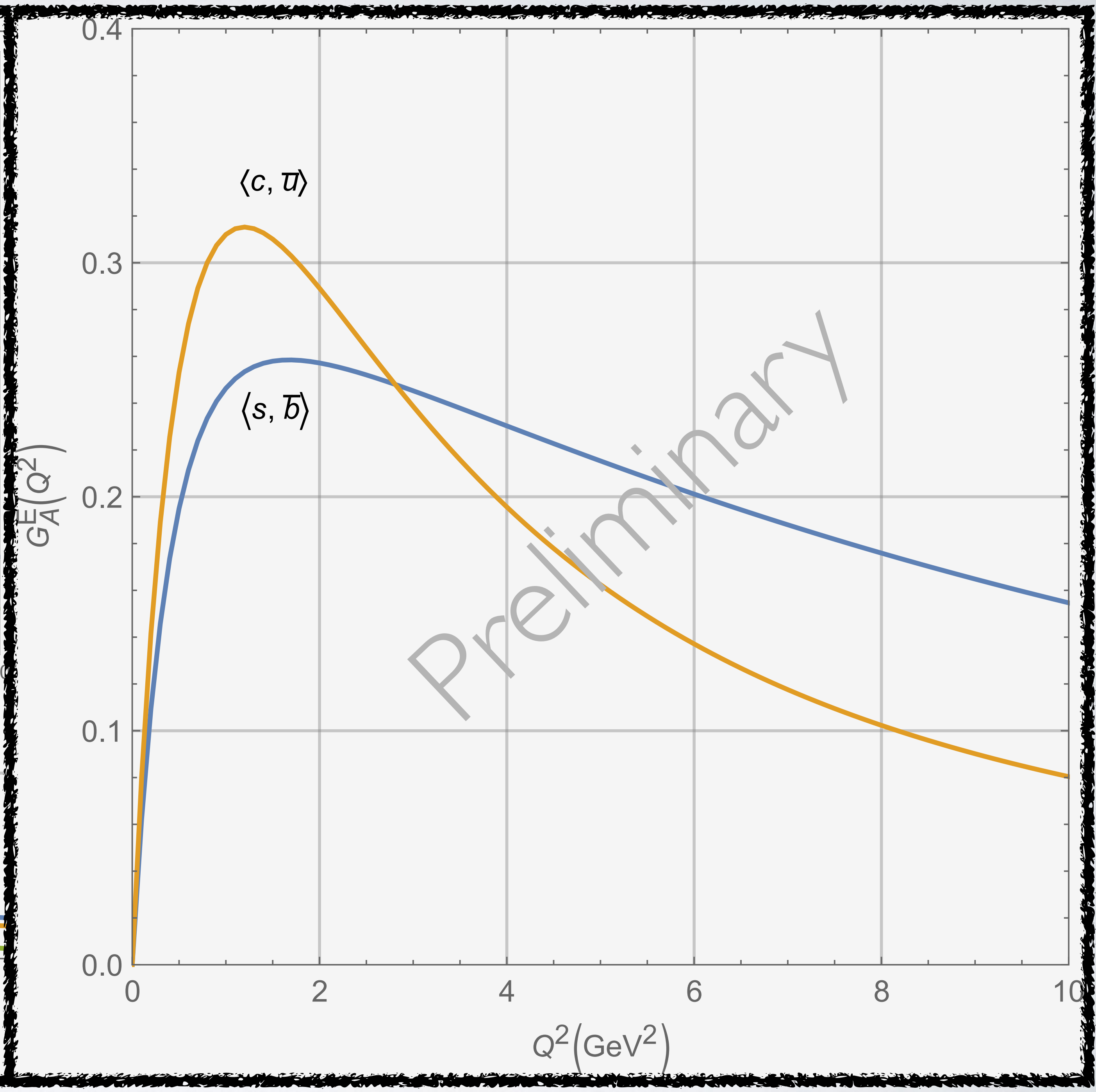

Different flavor

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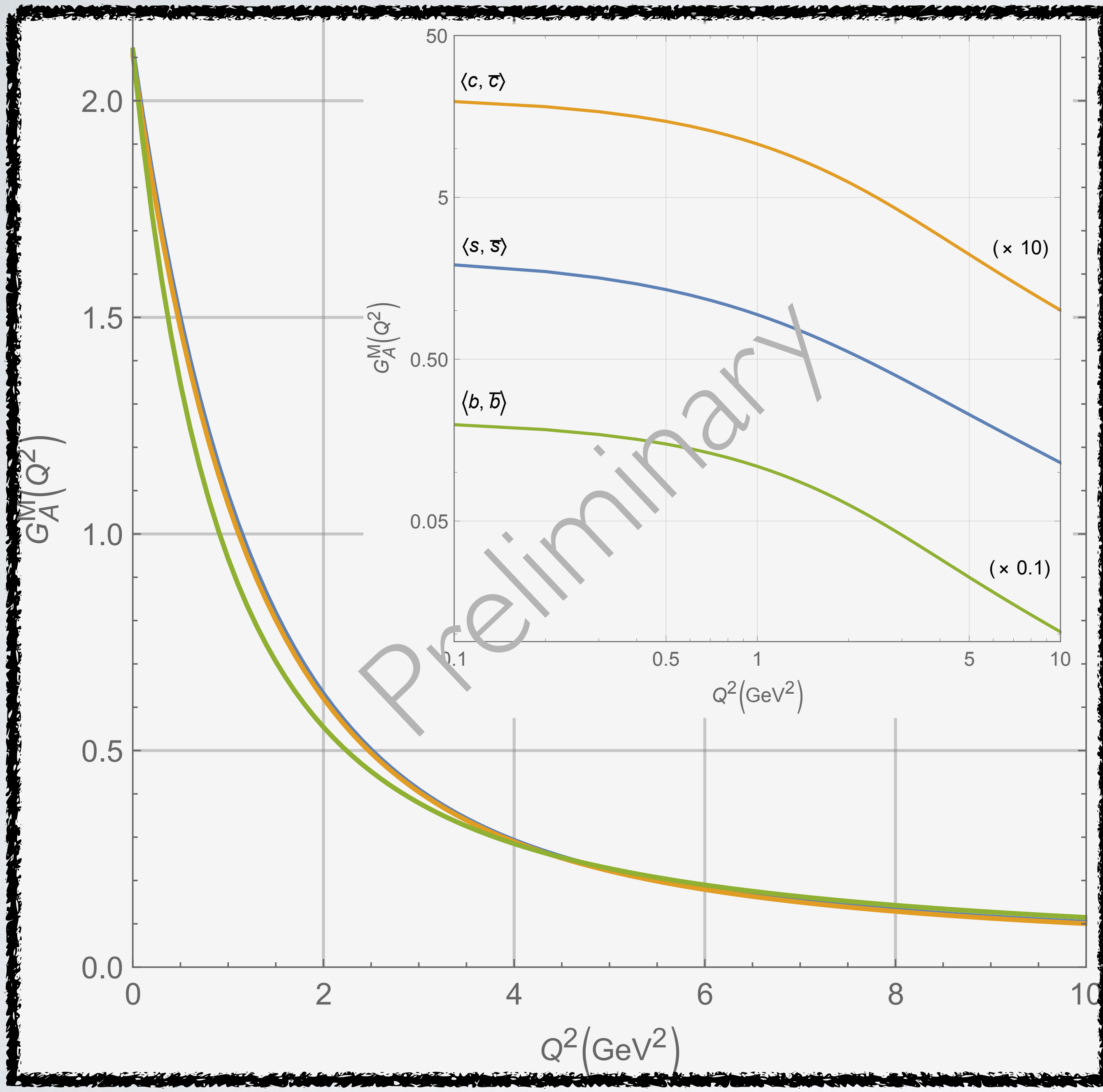

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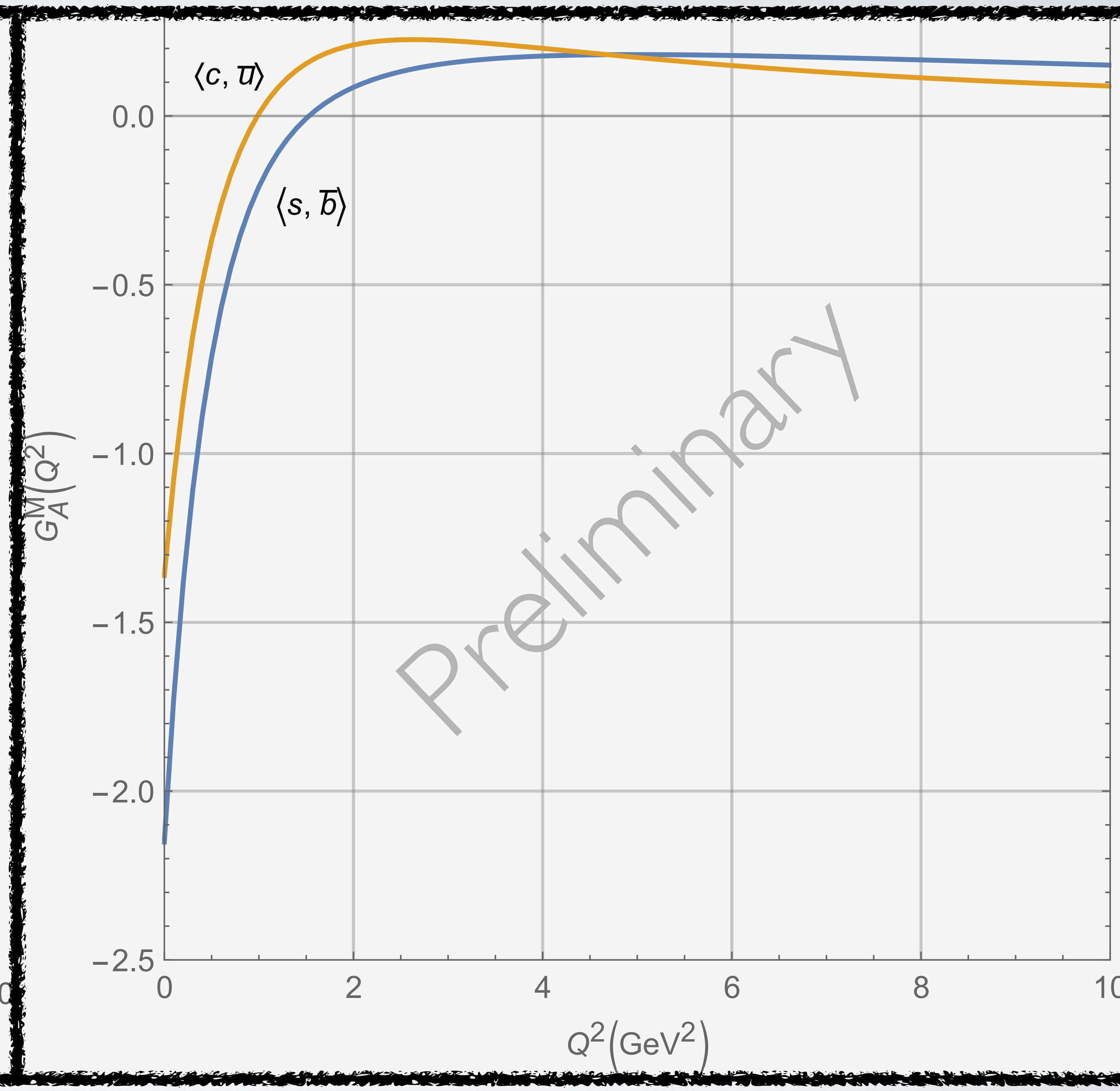
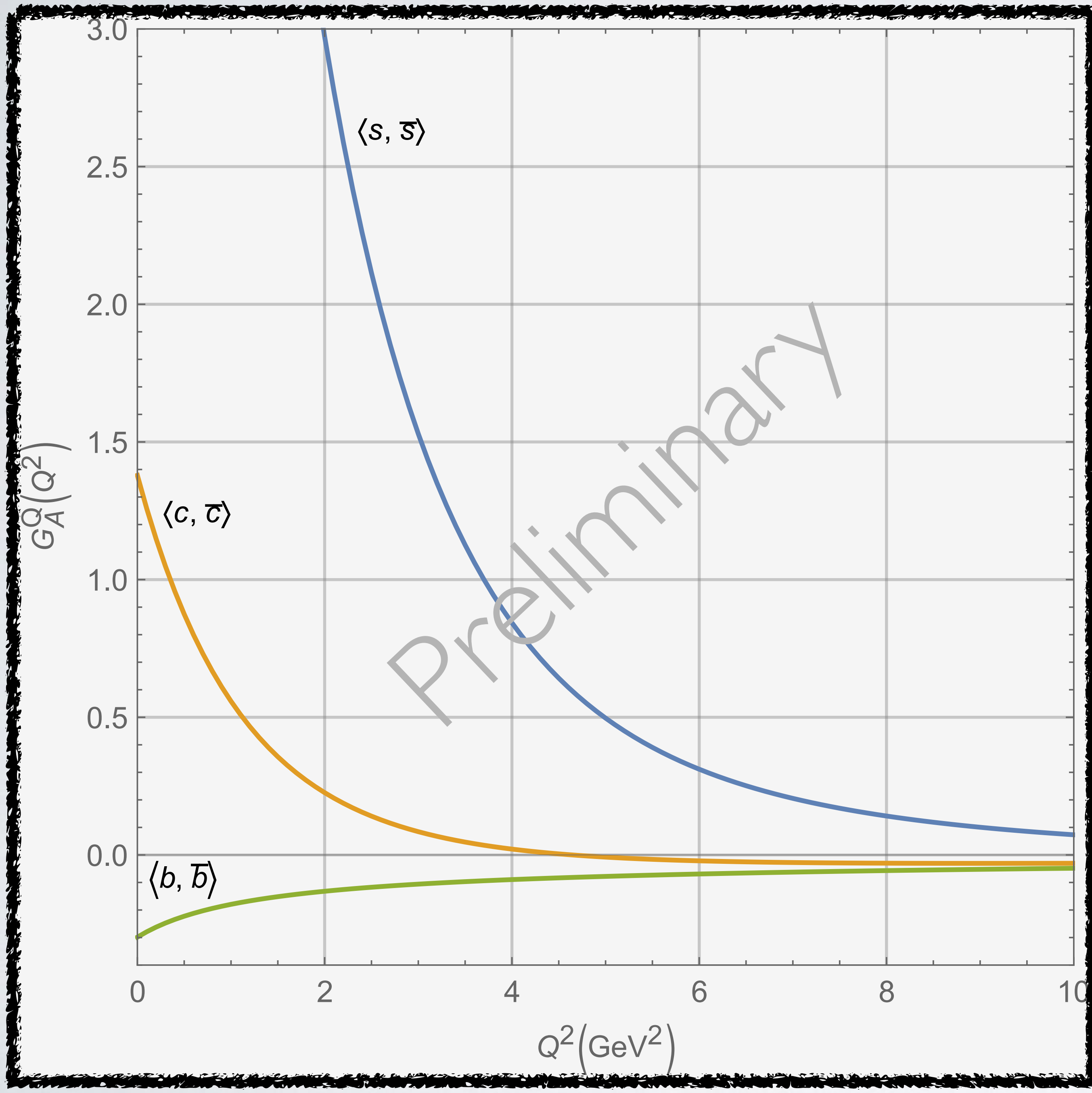
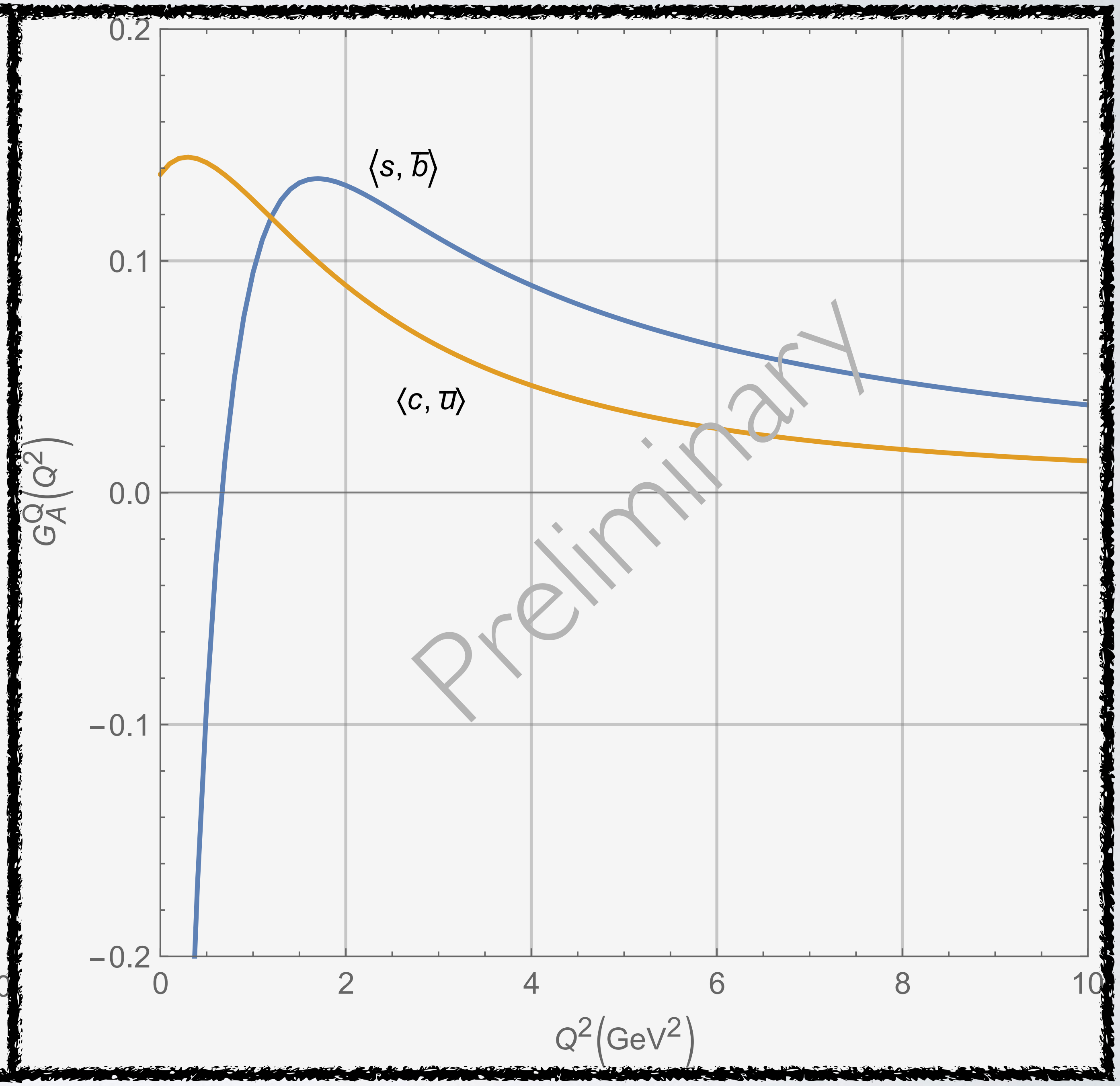


Different flavor

- Theoretical predictions for the Quadrupolar Form Factors for neutral axial-vector mesons. In the l.h.s., we present mesons formed with same flavored quarks, while in the r.h.s, the meson is formed with different flavored quarks.



Same flavor



Different flavor