Internal structure of pseudoscalar mesons: An algebraic model and its implications

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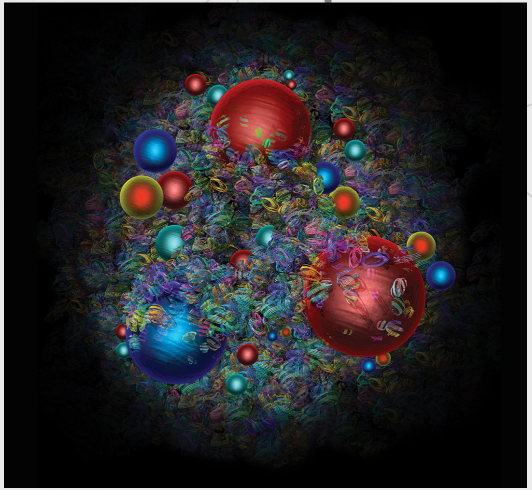
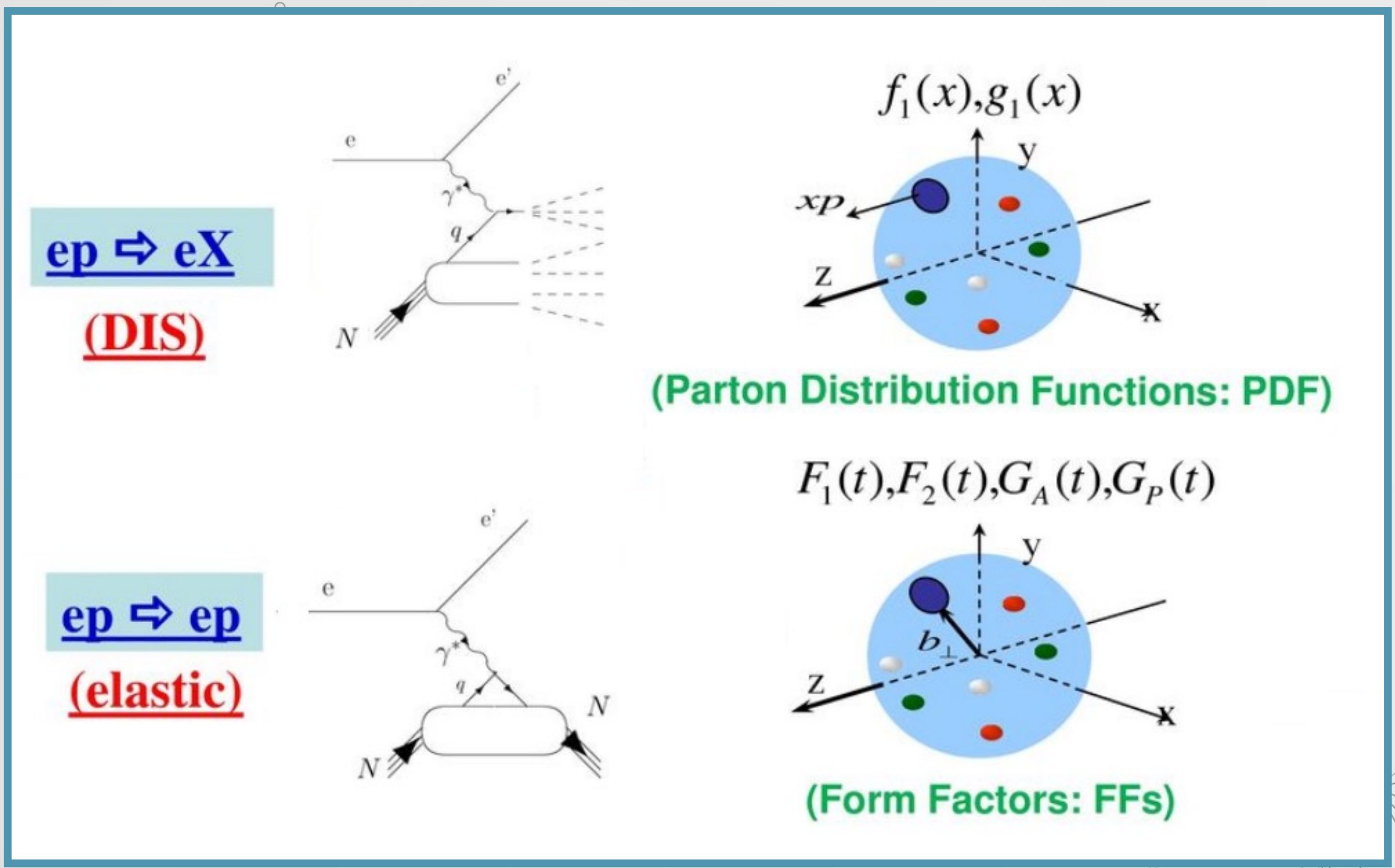
Content:

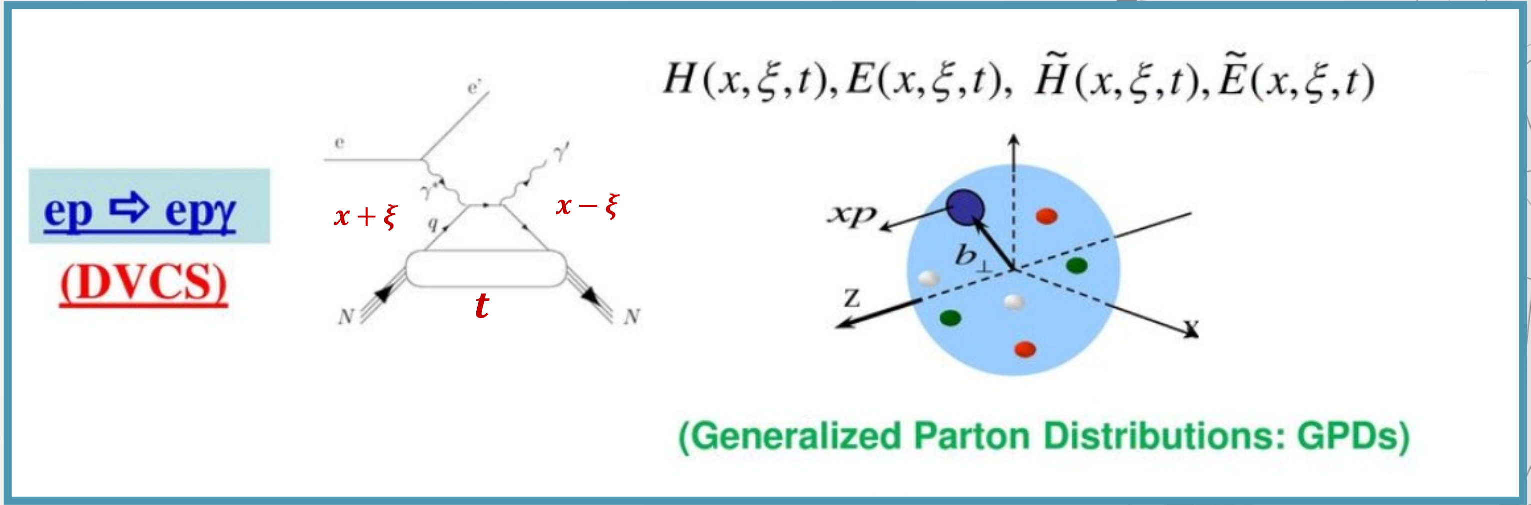
1. Introduction to the distribution functions: GPDs, PDFs and FFs.
2. Shwinger-Dyson equations.
3. Algebraic model and its implications.
4. Distribution functions.
5. Impact parameter space GPD.
6. Conclusions.



Introduction: PDFs y FFs



Introduction: GPDs



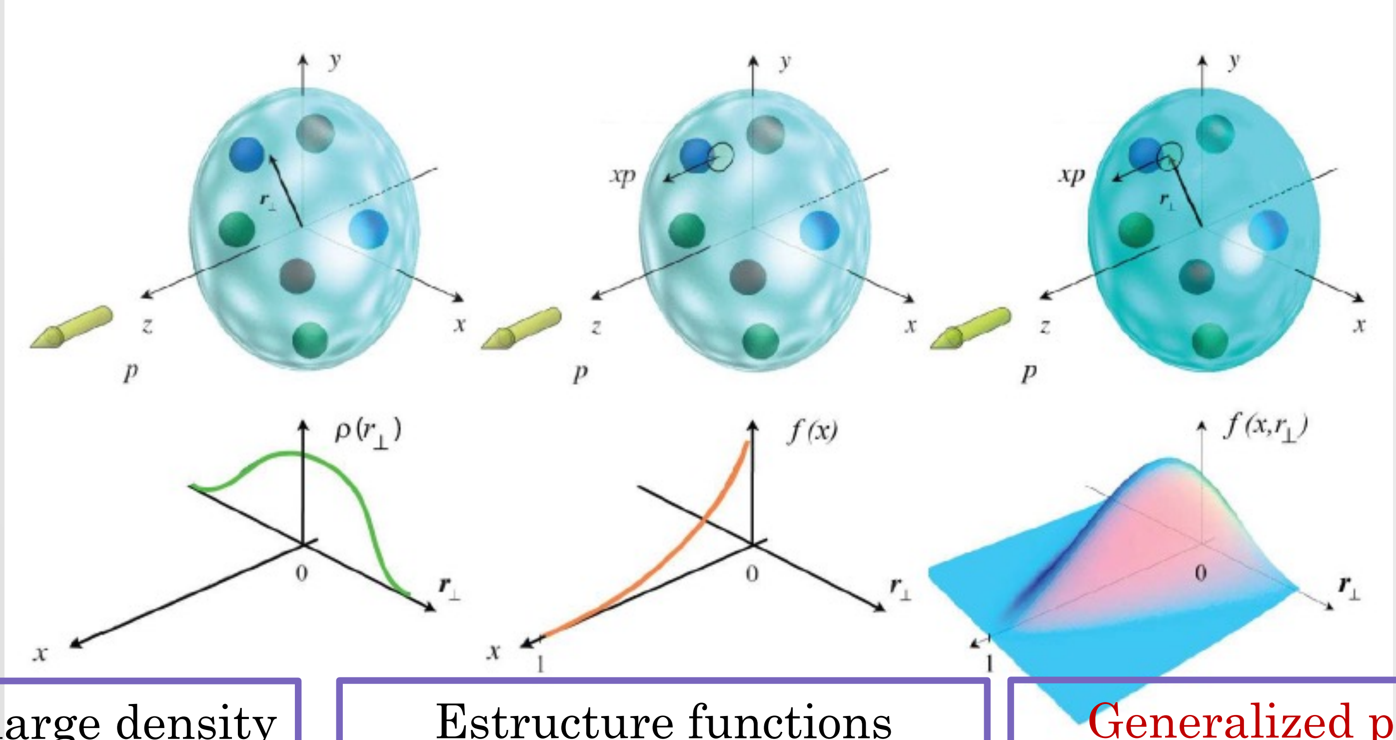
where

$x+\xi$ Is the relative longitudinal moment of the initial quark.

$x-\xi$ Is the relative longitudinal moment of the final quark.

$t=-\Delta^2$ Total square moment transferred to the hadron.

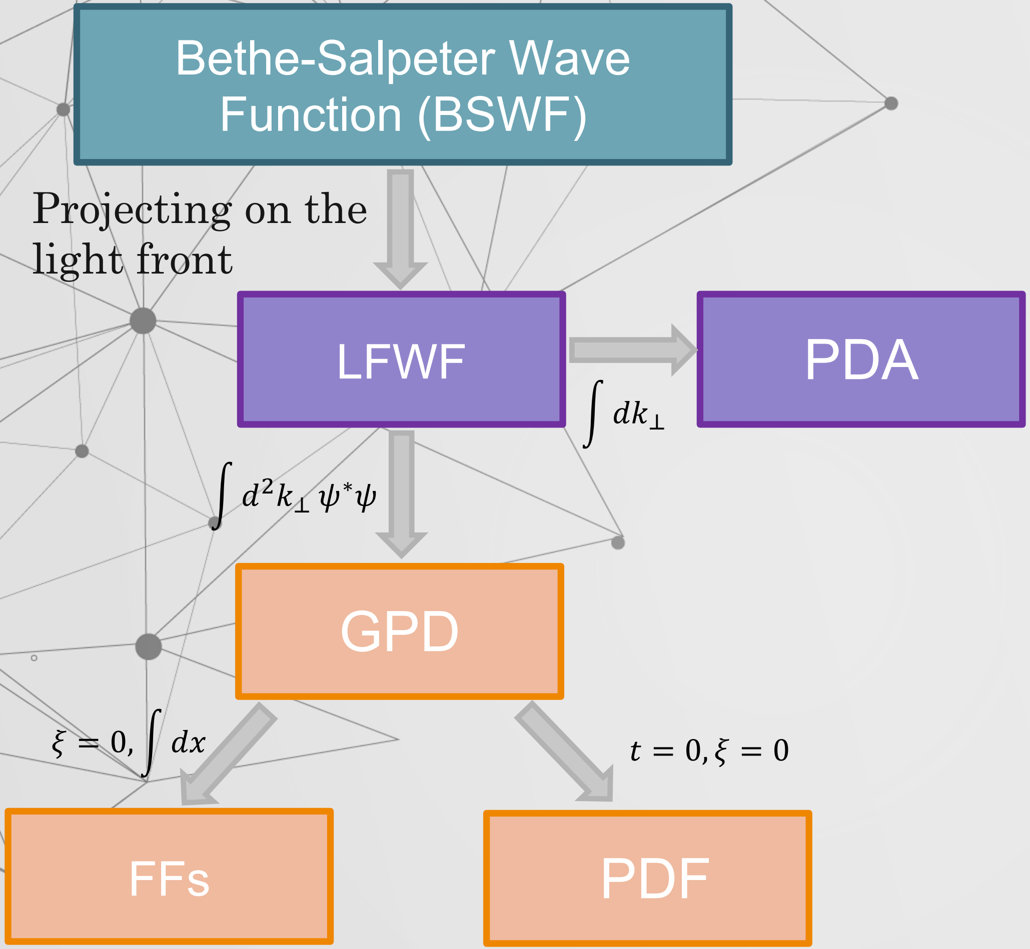
Introduction: FFs, PDFs, GPDs



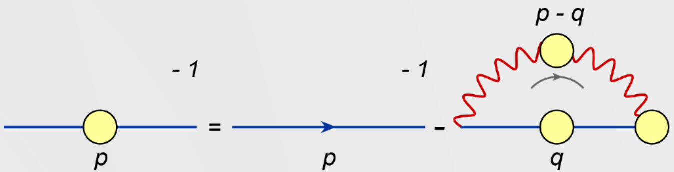
Charge density
Form Factors

Estructure functions
Parton distribution functions

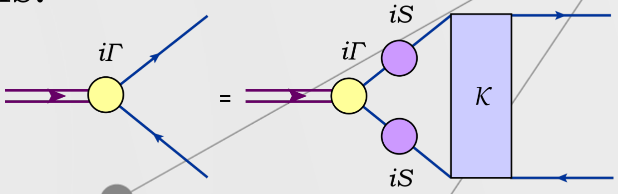
Generalized parton distribution functions



The SDE for the quark propagator is:



The meson Bethe-Salpeter equation (BSE) is:



The Bethe-Salpeter wave function (BSWF) is defined as:

$$\chi_{\mathrm{M}}\left(k_{-}, P\right)=S_q(k) \Gamma_{\mathrm{M}}\left(k_{-}, P\right) S_{\bar{h}}(k-P)$$

- Alternative

Algebraic Model

Algebraic Model

- The quark propagator and the BSA can be written as follows:

$$S_{q(\bar{h})}(k) = \left[-i\gamma \cdot k + M_{q(\bar{h})}\right] \Delta\left(k^2, M^2_{q(\bar{h})}\right)$$ and $$n_M \Gamma_M(k,P) = i\gamma_5 \int_{-1}^{1} dw\, \rho_M(w) \left[\hat{\Delta}\left(k_w^2, \Lambda_w^2\right)\right]^{\nu}$$

where, $\Delta(s,t) = \frac{1}{s+t}$, $\hat{\Delta}(s,t) = t\Delta(s,t)$, $k_\omega = k + \frac{\omega}{2}P$ and $P^2 = -m_M^2$.

$\rho(\omega)$ is the spectral density. Its shape determines the specific behavior of the associated BSA.

For our algebraic model:

$$\Lambda^2(w) = M_q^2 - \frac{1}{4}\left(1-w^2\right) m_M^2 + \frac{1}{2}(1-w)\left(M_{\bar{h}}^2 - M_q^2\right)$$

The **BSWF** is rewritten as:

$$n_M \chi_M(k_-, P) = \mathcal{M}_{q,\bar{h}}(k,P) \int_0^1 d\alpha \mathcal{F}_M(\alpha, \sigma^{\nu+2})$$

where

$$\mathcal{F}_M(\alpha, \sigma^{\nu+2}) = \nu(\nu+1)\left[\int_{-1}^{1-2\alpha} dw \int_{\frac{2\alpha}{w-1}+1}^{1} d\beta + \int_{1-2\alpha}^{1} dw \int_{\frac{2\alpha+(w-1)}{w+1}}^{1} d\beta\right] \frac{(1-\beta)^{\nu-1}\tilde{\rho}_M^{\nu}(w)}{\sigma^{\nu+2}}$$

$\sigma = [k - \alpha P]^2 + \Lambda^2_{1-2\alpha}$ and $\tilde{\rho}_M^{\nu}(w) \equiv \rho_M(w)\Lambda_w^{2\nu}$

Algebraic Model: LFWF and PDA

The LFWF for pseudoscalar mesons can be obtained in the light cone formalism by projecting the meson BSE:

$$\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right) = \operatorname{tr} \int_{dk_{\parallel}} \delta_{n}^{x}(k_{\mathrm{M}}) \gamma_5 \gamma \cdot n \, \chi_{\mathrm{M}}(k_{-}, P)$$

where $\delta_{n}^{x}(k_{\mathrm{M}}) = \delta(n \cdot k - x\, n \cdot P)$.

Then, from the Mellin moments of the LFWF $\langle x^m \rangle_{\psi_{\mathrm{M}}^{q}} = \int_0^1 dx\, x^m\, \psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right)$

we obtain:

$$\langle x^m \rangle_{\psi_{\mathrm{M}}}^{q} = \int_0^1 d\alpha \alpha^m \left[\frac{12}{n_{\mathrm{M}}} \frac{\mathcal{Y}_{\mathrm{M}}(\alpha, \sigma_{\perp}^{\nu+1})}{\nu+1}\right],$$
$$\mathcal{Y}_{\mathrm{M}}(\alpha, \sigma_{\perp}^{\nu+1}) = \mathcal{F}_{\mathrm{M}}(\alpha, \sigma_{\perp}^{\nu+1})(\alpha M_{\bar{h}} + (1-\alpha) M_q)$$

$\Rightarrow$

$$\psi_{\mathrm{M}}^{q}(x, k_{\perp}^{2}) = \left[\frac{12}{n_{\mathrm{M}}} \frac{\mathcal{Y}_{\mathrm{M}}(x, \sigma_{\perp}^{\nu+1})}{\nu+1}\right].$$

On the other hand, in the light-cone formalism the PDA can be expressed as:

$$f_{\mathrm{M}} \phi_{\mathrm{M}}^{q}(x) = \frac{1}{16\pi^3} \int d^2 k_{\perp} \psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right)$$

Algebraic Model: LFWF and PDA

Z.-F. Cuia, M. Dingb, F. Gaoc, K. Raya, **Eur. Phys. J. C Preprint No. NJU-INP 020/20**

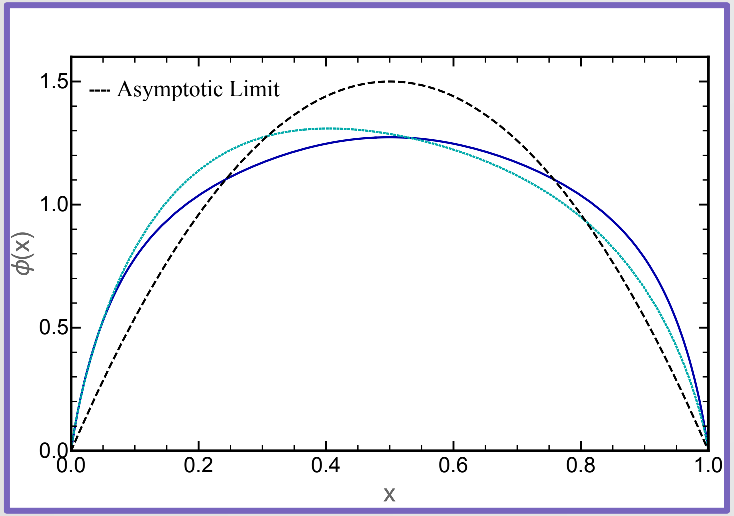
In the light-cone formalism the PDA can be expressed as:

$$f_{\mathrm{M}}\phi_{\mathrm{M}}^{q}(x) = \frac{1}{16\pi^3}\int d^2k_{\perp}\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^2\right)$$

Integrating over $k_{\perp}$ we obtain a new relationship between the LFWF and the PDA:

$$\psi_{\mathrm{M}}^{q}(x, k_{\perp}^2) = 16\pi^2 f_{\mathrm{M}}\frac{\nu\Lambda_{1-2x}^{2\nu}}{(k_{\perp}^2+\Lambda_{1-2x}^2)^{\nu+1}}\phi_{\mathrm{M}}^{q}(x)$$

From



[Blue]: Pion.
[Cyan]: Kaon.
[Black]:Asymptotic limit: $6x(1-x)$.

$$\phi_{\pi}^{\mathrm{DB}}(x;\zeta_H) = 20.227\,x(1-x) \times [1 - 2.5088\sqrt{x(1-x)} + 2.0250\,x(1-x)]$$

$$\phi_{K}^{u}(x;\zeta_H) = n_{\varphi_K}\,x(1-x) \times \left[1 + \rho x^{\frac{\alpha}{2}}(1-x)^{\frac{\beta}{2}} + \gamma x^{\alpha}(1-x)^{\beta}\right]$$

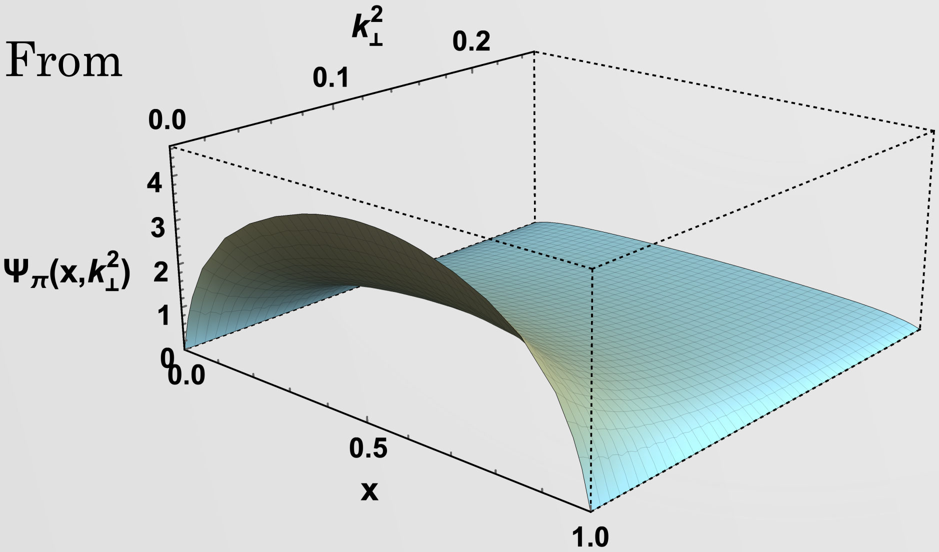
Algebraic Model: LFWF and PDA

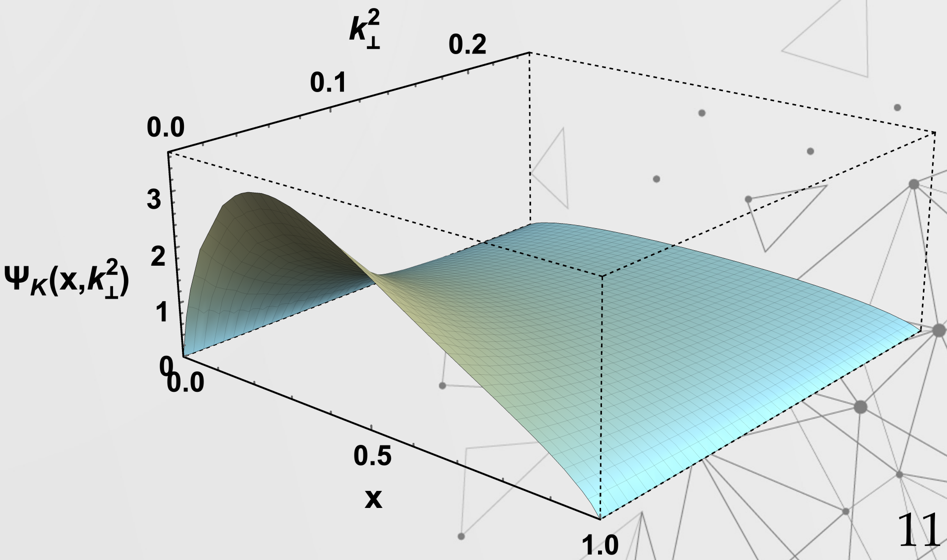
$$\psi_{\rm M}^{q}(x,k_\perp^2) = 16\pi^2 f_{\rm M}\frac{\nu\Lambda_{1-2x}^{2\nu}}{(k_\perp^2+\Lambda_{1-2x}^2)^{\nu+1}}\phi_{\rm M}^{q}(x)$$

Pion

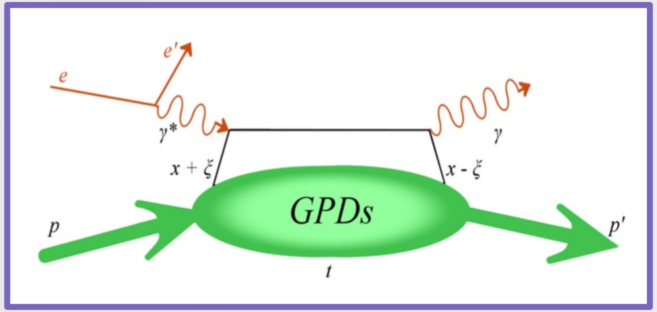
Kaon

From





Distribution Functions: GPDs

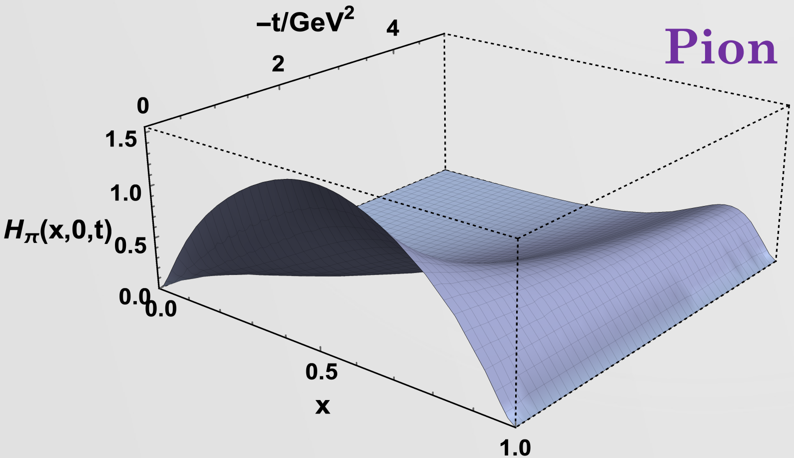


The GPD corresponding to the valence quarks can be obtained at a hadronic scale by means of the superposition configuration of the LFWF :

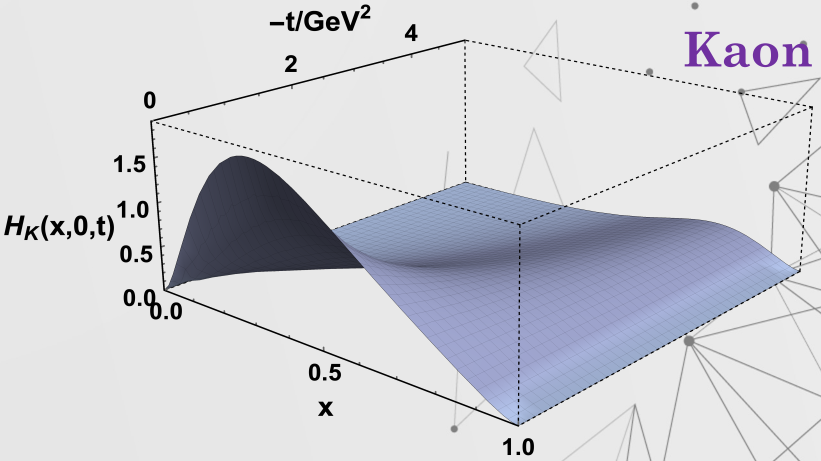
$$H_{\mathrm{M}}^{q}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3} \psi_{\mathrm{M}}^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_{\mathrm{M}}^{q}\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$

with $x^\pm = \frac{x \pm \xi}{1 \pm \xi} \quad , \quad \mathbf{k}_\perp^\pm = k_\perp \mp \frac{\Delta}{2}\frac{1-x}{1 \pm \xi}$.

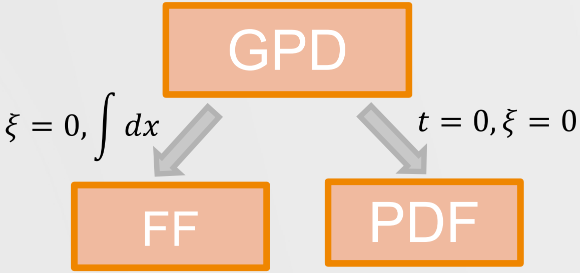
Pion



Kaon



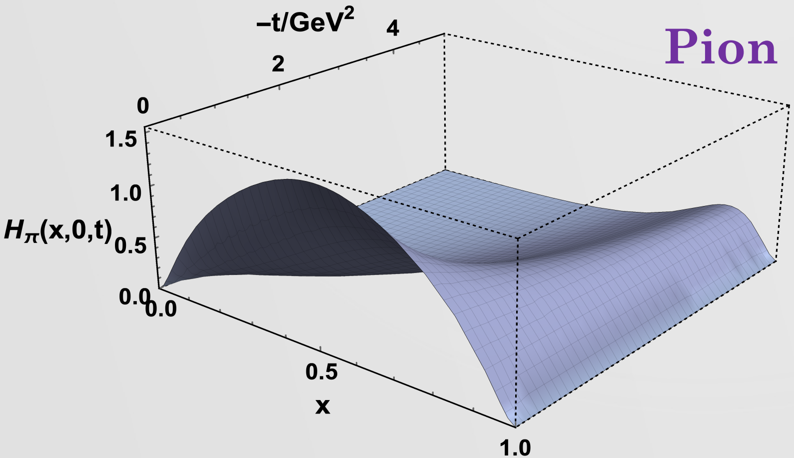
Distribution Functions: GPDs



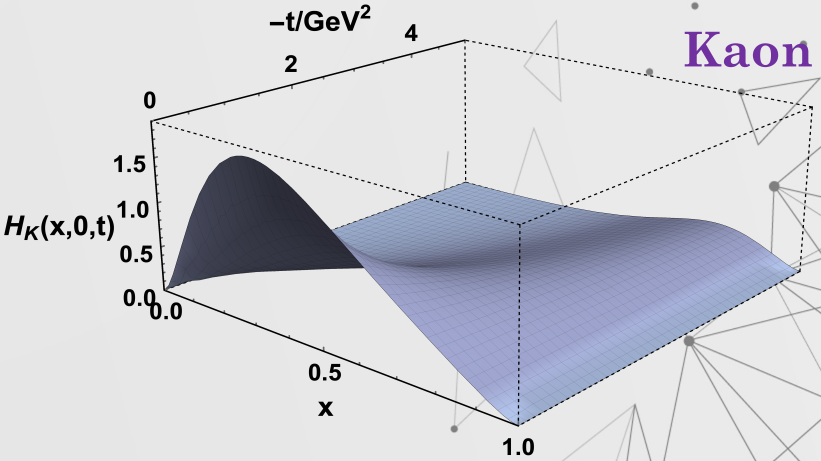
Taking advantage of the LFWF relation in terms of the PDA, solving the integration on $k_\perp$ by using Feynman parametrization, taking a suitable change of variable and $\xi = 0$

$$H^q_{\mathrm{M}}(x,0,t) = \frac{\Gamma(2\nu+2)}{\Gamma^2(\nu+1)} \mathcal{N} \phi^{q2}_{\mathrm{M}}(x) \Lambda^{4\nu}_{1-2x} \int_0^1 du \frac{u^\nu (1-u)^\nu}{[\mathbb{M}^2(u)]^{2\nu+1}}$$

with $\mathbb{M}^2(u) = t(1-x)^2 u(1-u)^2 + \Lambda^2_{1-2x-}$ ..

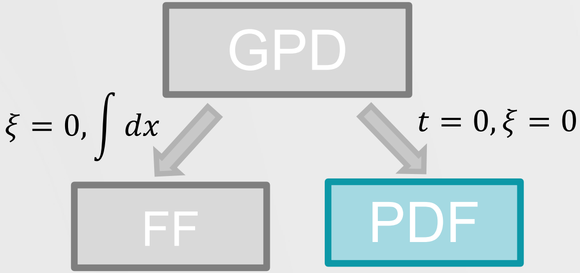


Pion



Kaon

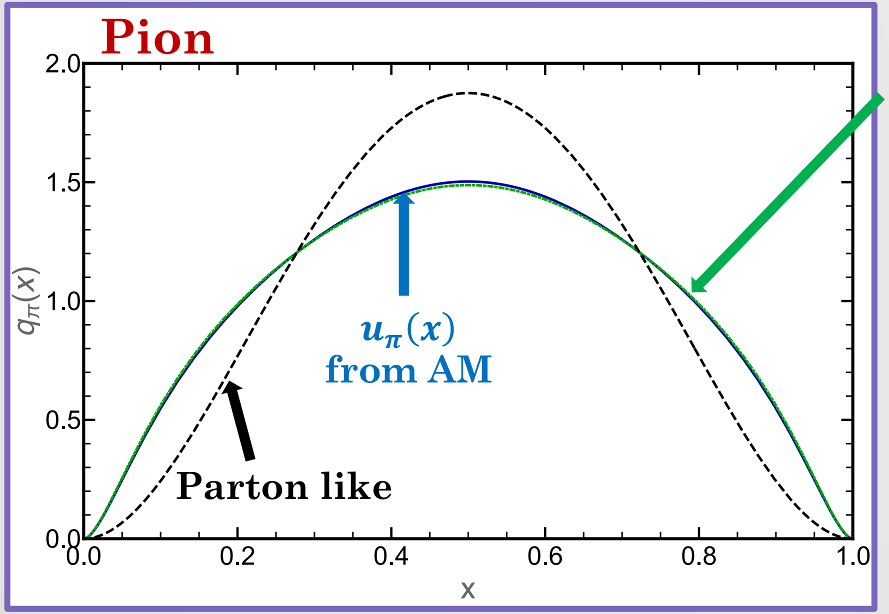
Distribution Functions: PDFs



The PDF is defined in terms of the GPDs as follows :

$$q_{\mathrm{M}}(x) \equiv H_{\mathrm{M}}^{q}(x,0,0) = \mathcal{N}\frac{\phi_{\mathrm{M}}^{2}(x)}{\Lambda_{1-2x}^{2}}$$

Where the normalization factor is: $\mathcal{N} = \left[\int_0^1 dx\, \frac{\phi_{\mathrm{M}}^2(x)}{\Lambda_{1-2x}^2}\right]^{-1}$



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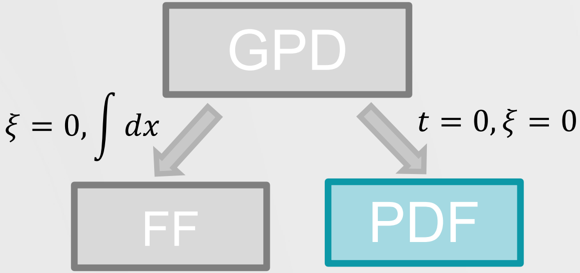
Where is considered

$$q_M(x;\zeta_H) \propto |\phi_M^q(x;\zeta_H)|^2$$

Taking into account the isospin symmetry for the pion in the AM, we recover this form.

Parton like: $30x^2(1-x)^2$.

Distribution Functions: PDFs



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$$q_{\rm M}(x) \equiv H^q_{\rm M}(x,0,0) = \mathcal{N}\frac{\phi^2_{\rm M}(x)}{\Lambda^2_{1-2x}}$$

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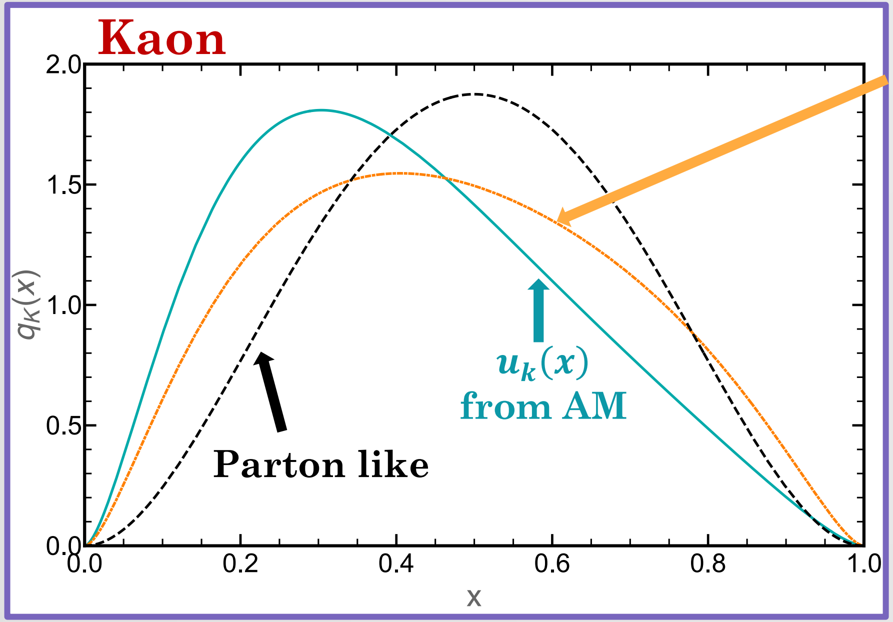

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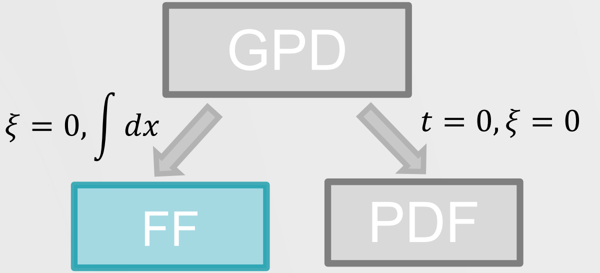
Where, again is considere

$$q_M(x;\zeta_H) \propto |\phi^q_M(x;\zeta_H)|^2$$

Parton like: $30x^2(1-x)^2$.

Distribution Functions: FFs



The contribution of the quark "q" to the elastic form factor is obtained by:

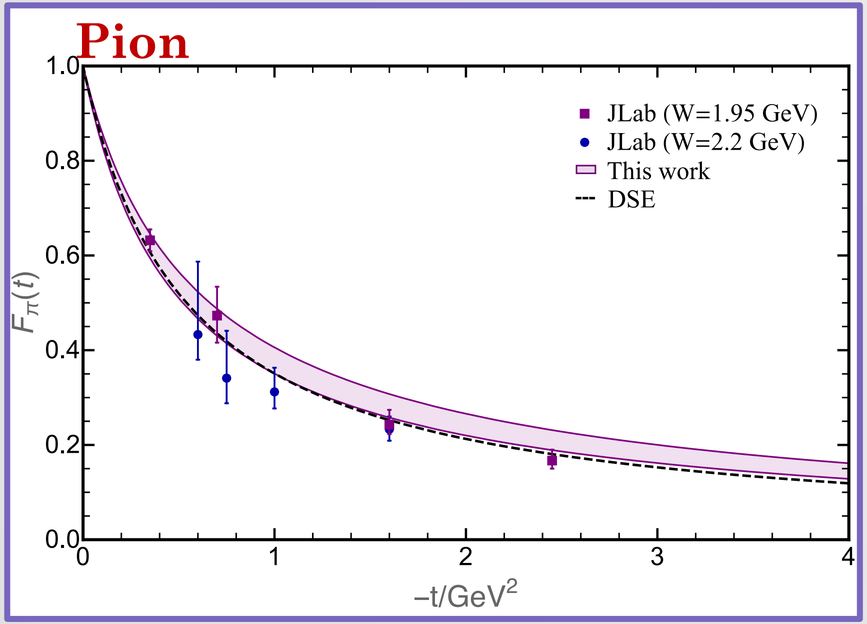
$$F_{\mathrm{M}}^{q}(t) = \int_0^1 dx\, H_{\mathrm{M}}^{q}(x,0,t)$$

The full FF of the meson is then:

$$F_{\mathrm{M}}(t) = e_q F_{\mathrm{M}}^{q}(t) + e_{\bar{h}} F_{\mathrm{M}}^{\bar{h}}(t)$$

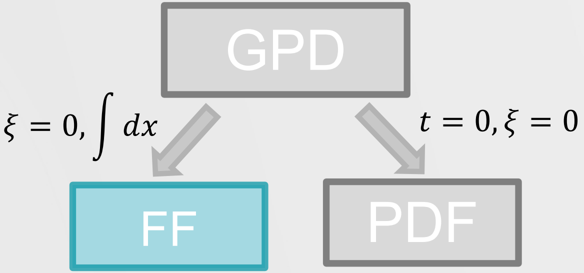
$$r_{\mathrm{M}}^2 = -6 \left.\frac{dF(t)}{dt}\right|_{t=0}$$

$$r_M^2 = 6 c_\nu^{(1)} \int_0^1 dx \left[e_q \frac{q_M(x)}{\Lambda_{1-2x}^2} + e_{\bar{h}} \frac{q_M(1-x)}{\Lambda_{2x-1}^2} \right] (1-x)^2$$



- Purple band- Pion FF from the AM for $M_l = 316$ MeV and $\upsilon = 1$ with a 5% variation on the value $r_\pi = 0.659$ fm.
- Dashed Gray line- DSE results **Physics Letters B Volume 797, 134855 (2019).**
- Circles y squares - **G. Huber et al., Phys. Rev. C 78, 045203 (2008).**

Distribution Functions: FFs



The contribution of the quark "q" to the elastic form factor is obtained by:

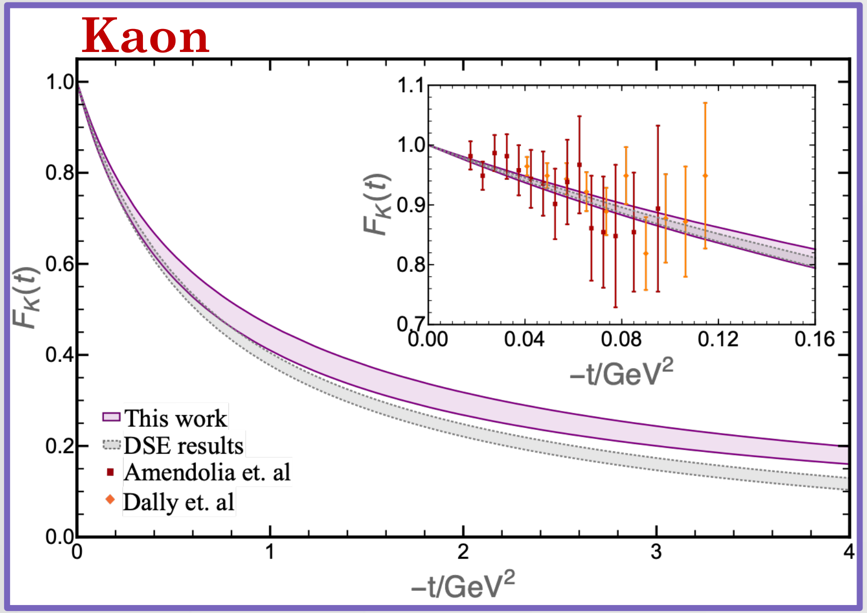
$$F_{\rm M}^{q}(t) = \int_0^1 dx\; H_{\rm M}^{q}(x,0,t)$$

The full FF of the meson is then:

$$F_{\rm M}(t) = e_q F_{\rm M}^{q}(t) + e_{\bar h} F_{\rm M}^{\bar h}(t)$$

$$r_{\rm M}^2 = -6 \frac{dF(t)}{dt}\bigg|_{t=0}$$

$$r_M^2 = 6 c_\nu^{(1)} \int_0^1 dx \left[e_q \frac{q_M(x)}{\Lambda_{1-2x}^2} + e_{\bar h} \frac{q_M(1-x)}{\Lambda_{2x-1}^2} \right] (1-x)^2$$



- Purple band – Kaon FF from AM for M_l = 316 MeV, Ms/Mu = 1.813 and $\upsilon = 1$ with a 5% variation on the value $r_K = 0.6$ fm.
- Gray band – DSE result **Phys. Rev. D 101, 054015 (2020).**
- Squars - **S. R. Amendolia et al., Phys. Lett. B178, 435 (1986).**
- Diamonds - **E. B. Dally et al., Phys. Rev. Lett. 45, 232 (1980).**

Distribution Functions: GPDs

At first orden in the Taylor expansion, the GPD can be aproximated as an exponential function

$$H_{\mathrm{M}}^{q}(x,0,t) \overset{t\to 0}{\approx} \mathcal{N}\frac{\phi_{\mathrm{M}}^{2}}{\Lambda_{1-2x}^{2}}\left[1-c_{\nu}^{(1)}(1-x)^{2}\left(\frac{-t}{\Lambda_{1-2x}^{2}}\right)+\ldots\right]$$

with $$c_{\nu}^{(1)} = \frac{(1+\nu)(1+2\nu)}{2(3+2\nu)}\,.$$

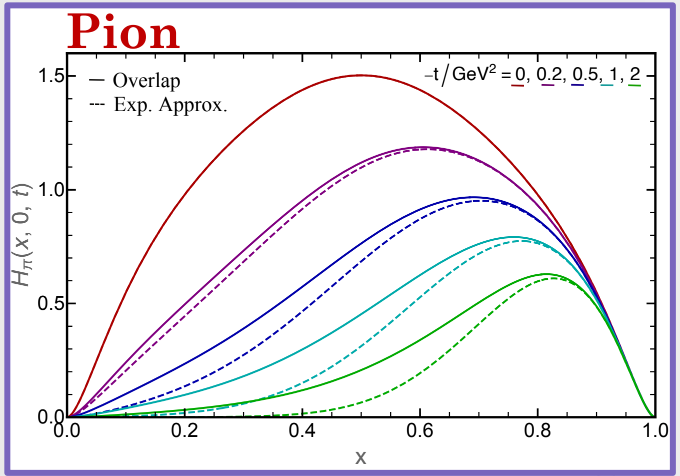
In the light-front holographic QCD aproach, the zero-skewness valence quark is expressed as

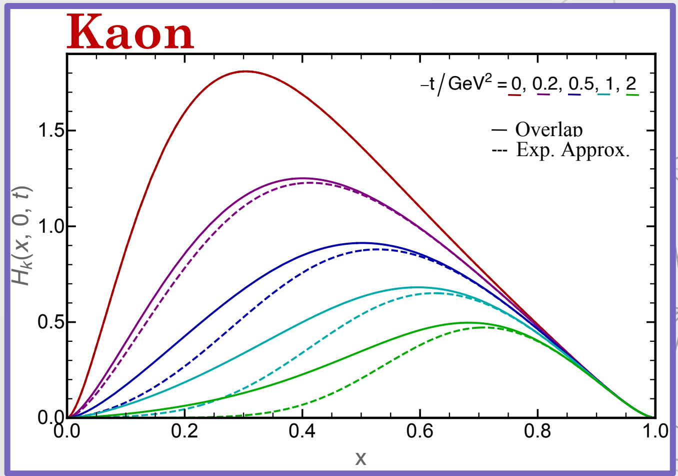
$$H_{\mathrm{M}}^{q}(x,0,t) = q_{\mathrm{M}}(x)\exp[tf(x)]$$

Where $f(x)$ is some profile function.

A subsequent matching enable us to identify

$$f(x) = \frac{c_{\nu}^{(1)}(1-x)^{2}}{\Lambda_{1-2x}^{2}}$$





Impact Parameter Space GPD

The impact parameter space is given by

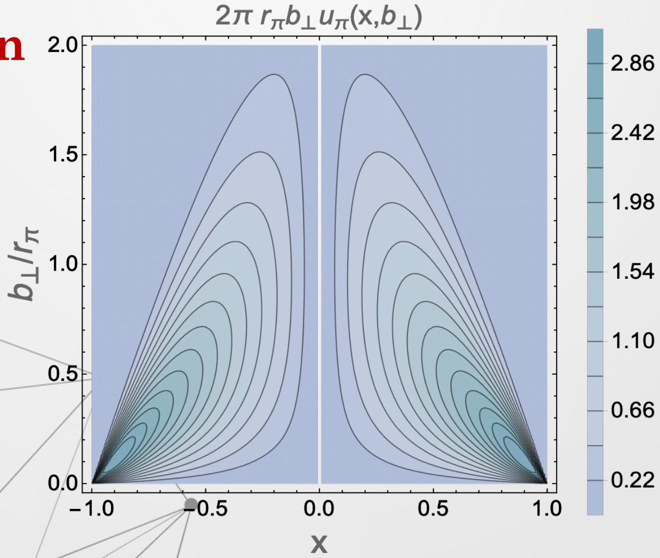
$$u_M(x, b_\perp^2, \zeta_H) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta) H_M(x, 0, t)$$

where J_0 is a cylindrical Bessel function.

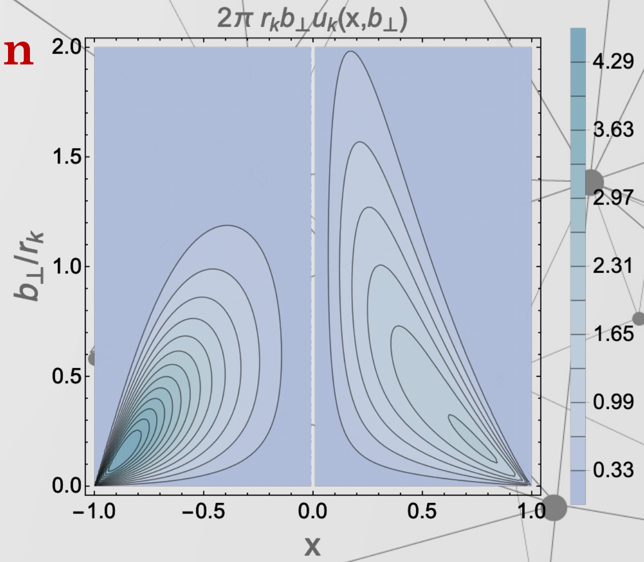
The AM allows analytical integration, leading to:

$$u_M(x, b_\perp^2, \zeta_H) = \frac{q_M(x)}{4\pi f(x)} e^{-\frac{b_\perp^2}{4f(x)}}$$

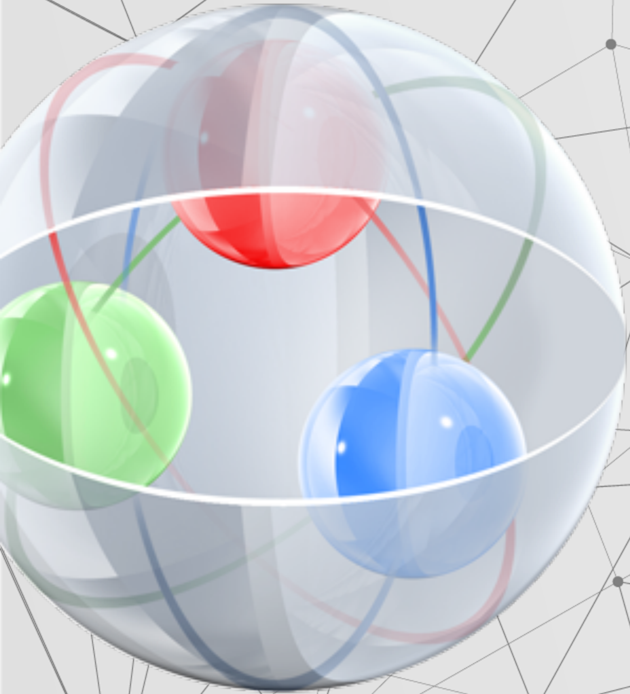
Pion



Kaon



Conclusions



- We observe that our porposal for lambda $(\Lambda \to \Lambda(\omega))$ makes possible to calculate analytically the distribution functions.

- This method allows us to find an analytical relationship between all the functions of interest and the PDAs.

- The model shows that the FFs obtained are in good agreement with both experimental results and SDE results, which allows making new predictions for pseudoscalar mesons like the GPDs.

- Currently, experimental data from proton GPDs are being extracted in several laboratories around the world such as: Jefferson Lab, DESY, CERN, among others, so we seek to make predictions for these experiments.

Thanks!

Algebraic Model: Elastic FF

Chi square

$$\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\delta E_i^2},$$

Wheree T_i is the theoretical estimation, E_i is the observed experimental value and δE_i is the asociated error to this medition.

For the Pion FF:

Experiments	No. Data	χ^2
G. Huber et al. (Jlab)	8	4.43382
T. Horn et al.	1	0.416452

For the Kaon FF:

Experiments	No. Data	χ^2
Dally et al.	10	4.13643
Amendolia et al.	15	4.02524

AM: Spectral Density

A relationship was found to find the spectral density in terms of parameterized PDA.

$$\rho(y) = -\frac{F_N}{2M}\left\{(1-y^2)\frac{d^2\phi(y)}{dy^2} + \frac{2y}{\Lambda_y^2}\left[M^2(\nu-1)+\frac{1}{4}\left(1-y^2\right)m_M^2\right]\frac{d\phi(y)}{dy} - \frac{\nu}{\Lambda_y^4}\left[M^4(\nu-1) - \frac{1}{2}m_M^2 M^2\left(\nu+(\nu-2)y^2\right) + \frac{1}{16}m_M^4(1-y^2)^2(\nu+1)\right]\phi(y)\right\}.$$

where $\Lambda_y^2 = M^2 - \frac{1}{4}(1-y^2)m_M^2$ for $M_q = M_{\bar{q}}$

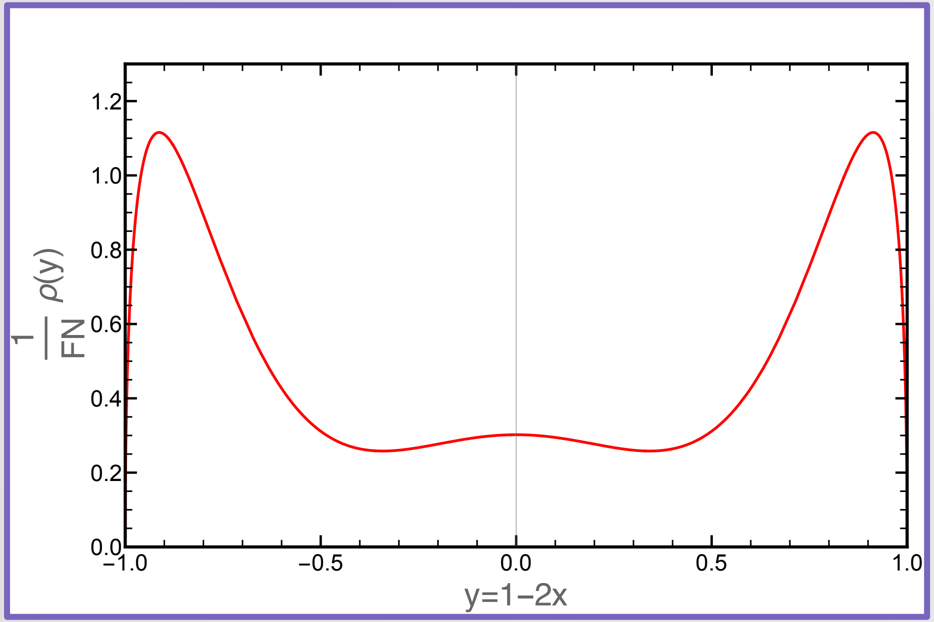
AM: Spectral Density

$M_l = 316\ \mathrm{MeV}$

$M_s = 1.8\ M_l$

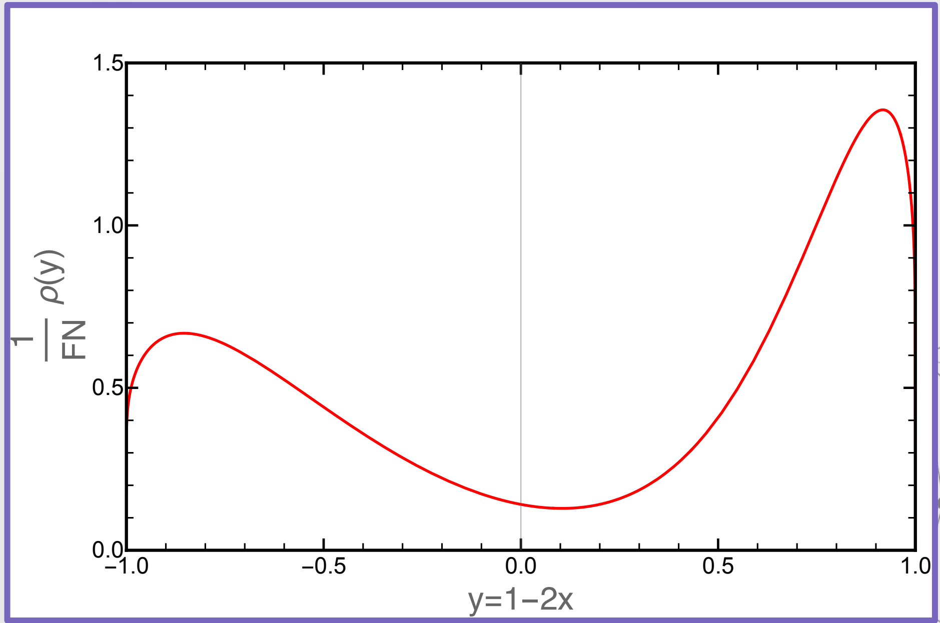
Spectral Density ρ

Pion



$r_\pi = 0.659\ \mathrm{fm}\ and\ \ v = 1.$

Kaon



$r_K = 0.6\ \mathrm{fm}\ and\ \ v = 1.$