The gap equation in QCD and the origin of the light hadron's masses

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FAPESP
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CNPq
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"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELBIS IS AN AUTHORITY, BUT NOT AN EXPERT."

Work in collaboration with

- Luis Albino, Instituto de Física Teórica, Unesp, Brazil
- Adnan Bashir, Universidad de Michoacán, Mexico
- José Roberto Lessa, Instituto Tecnológico de Aeronáutica, Brazil
- Orlando Oliveira, Universidade de Coimbra, Portugal
- Eduardo Rojas, Universidad de Nariño, Colombia
- Fernando Serna, Universidade Cruzeiro do Sul, Brazil
- Roberto Correa da Silveira, Universidade Cruzeiro do Sul, Brazil

Quantum ChromoDynamics

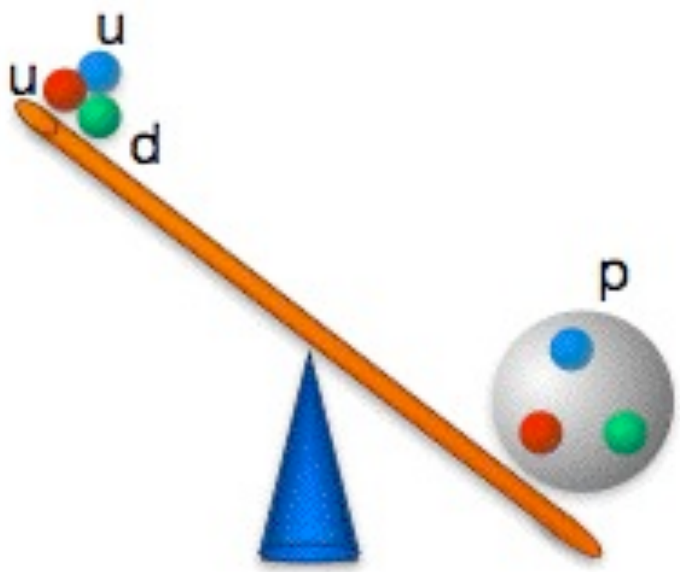
- We strive for a description of interactions between quarks and gluons which form hadrons as observed in *Nature*.

- The key issue is: while the Brout-Englert-Higgs mechanism has been established as the essential explicit source of elementary particle's masses, the same cannot be said of the atoms and their nuclei.

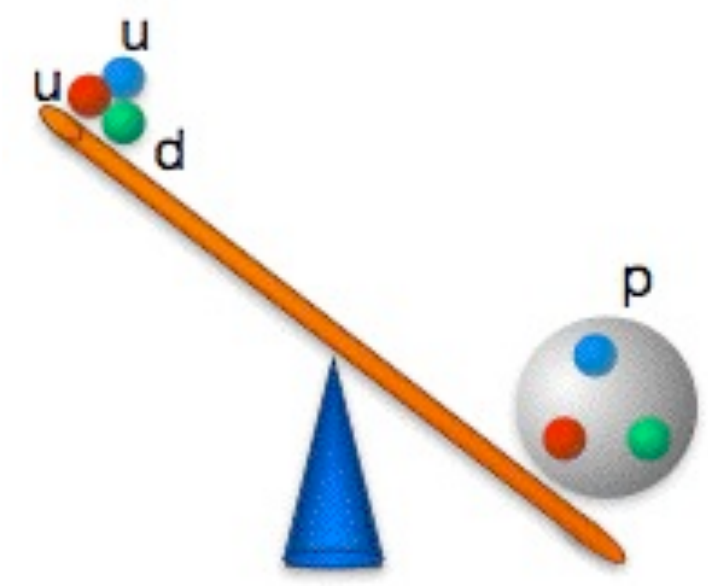
- The lightest Nambu-Goldstone mode of QCD, the pion, is more than an order of magnitude heavier than the sum of two light current quarks

- The formation of hadronic and nuclear bound states via its fundamental constituents is an inherently *nonperturbative* problem.

- It involves precise knowledge of the infrared (long distance) regime of QCD and the dynamical generation of a constituent quark mass.



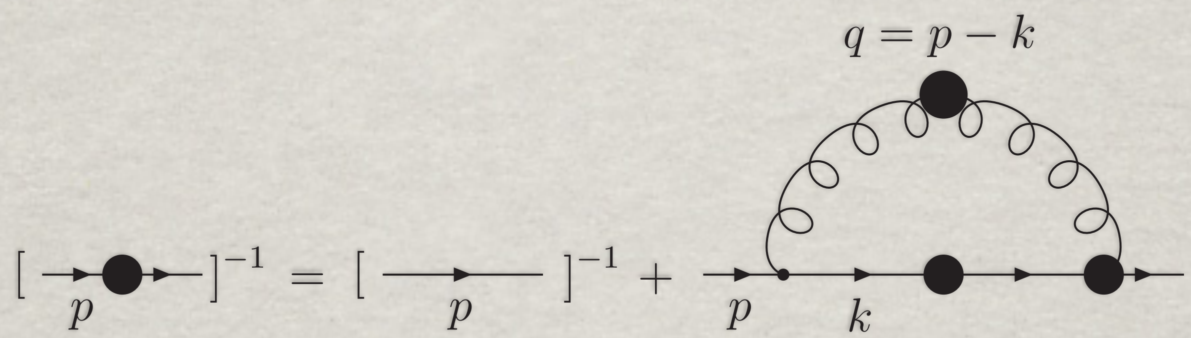
So where do the Hadron’s masses come from after all?! The Higgs boson isn’t doing the job alone!



Hint: the gluons interact with each other and have infinite ways to interact with the quark and “dress it”.

Dyson-Schwinger equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.



$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)
\end{aligned}
$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$	:	dressed-gluon propagator
$\Gamma^a_\nu(q,p)$	:	dressed quark-gluon vertex
Z_2	:	quark wave function renormalization constant
Z_1	:	quark-gluon vertex renormalization constant

Each satisfies it's own DSE !

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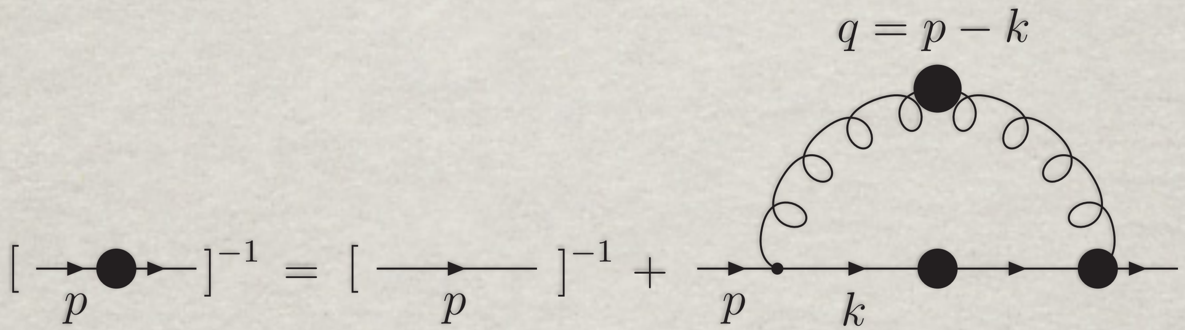

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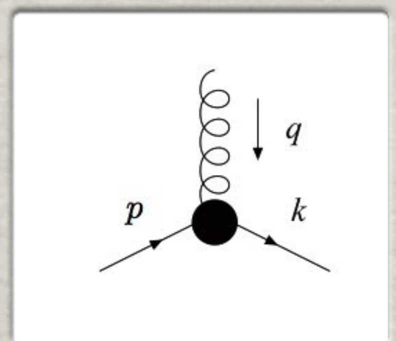

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Dyson-Schwinger equation in QCD

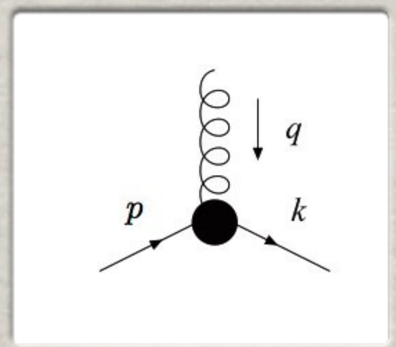
The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fe … coupled equations.

Running Quark Mass

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma \cdot p\, A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q)\, \Gamma^a_\nu(q,p)$$



with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

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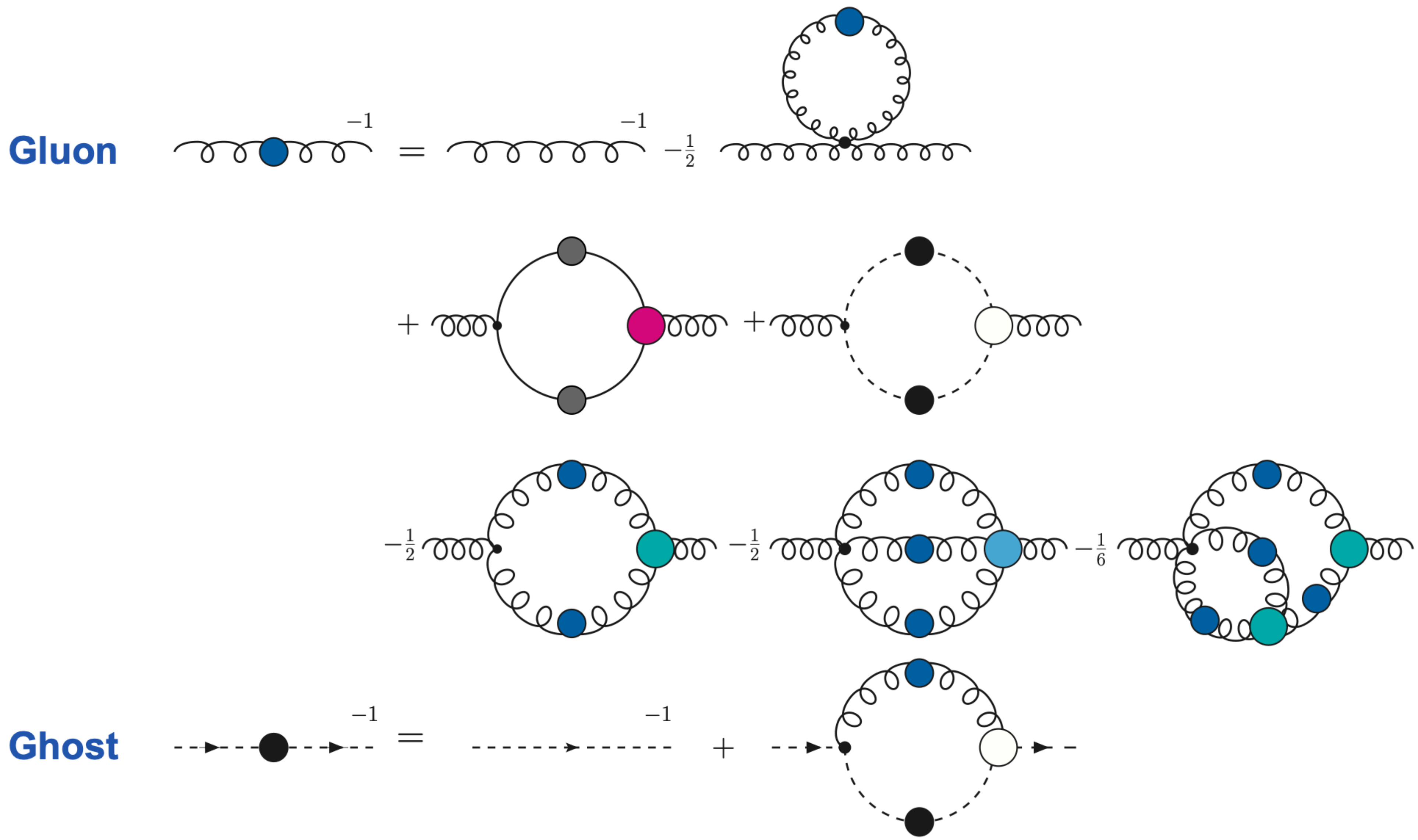
Each satisfies it's own DSE !

More DSE in QCD: Gluon and Ghost Propagators



However, in this work we employ dressed gluon and ghost propagators from lattice QCD.

Rainbow Truncation

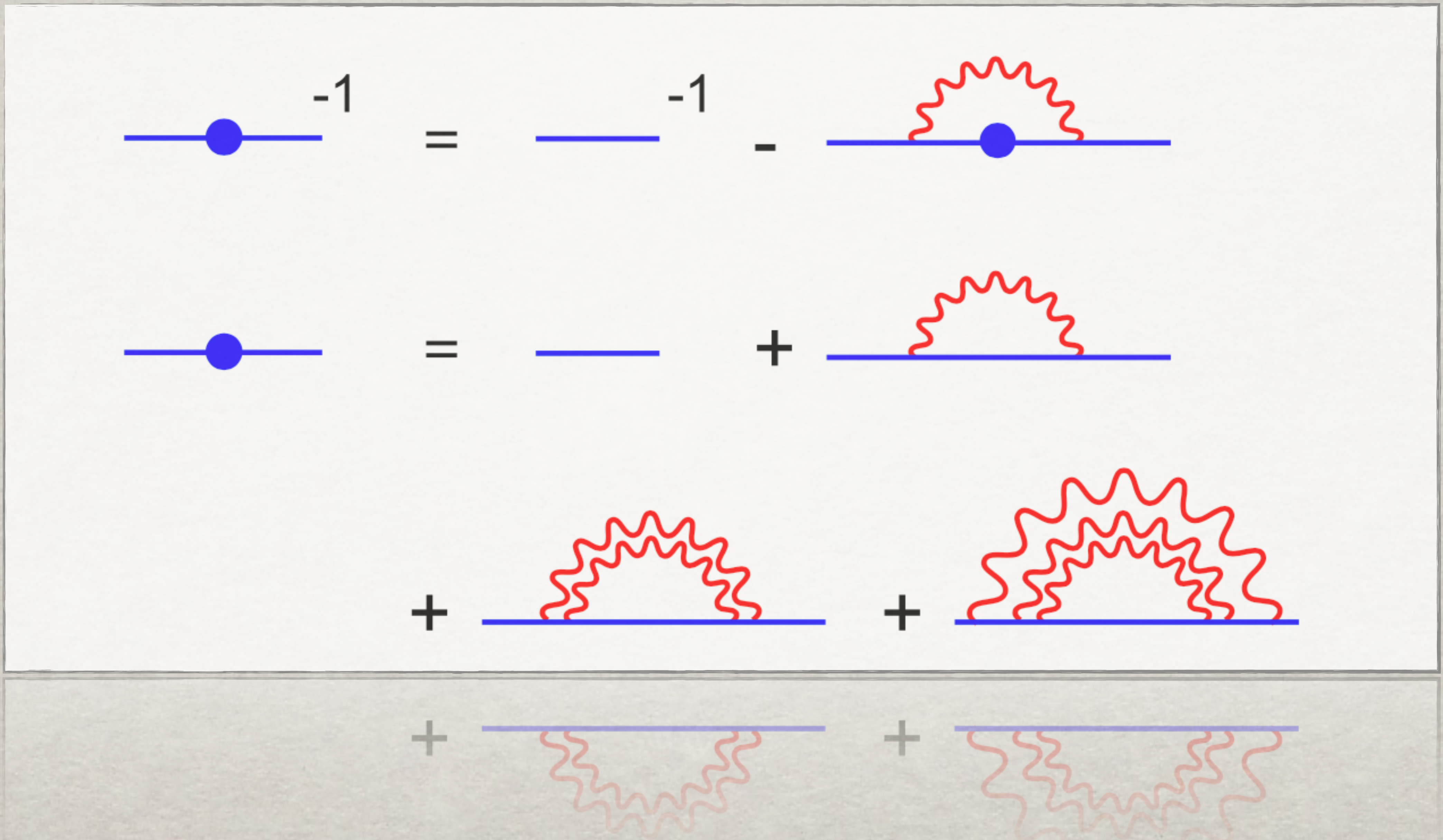
Since the Dyson-Schwinger Equation for QCD imply an infinite tower of non-linear integral equations, a symmetry preserving truncation scheme must be employed. The leading term in such a scheme is the Rainbow-Ladder (RL) truncation (Abelian approach).

$$\Gamma_\nu \rightarrow \gamma_\nu$$

RL truncation satisfies vector and flavor non-singlet axial-vector Ward-Takahashi identities but has bad gauge dependence.

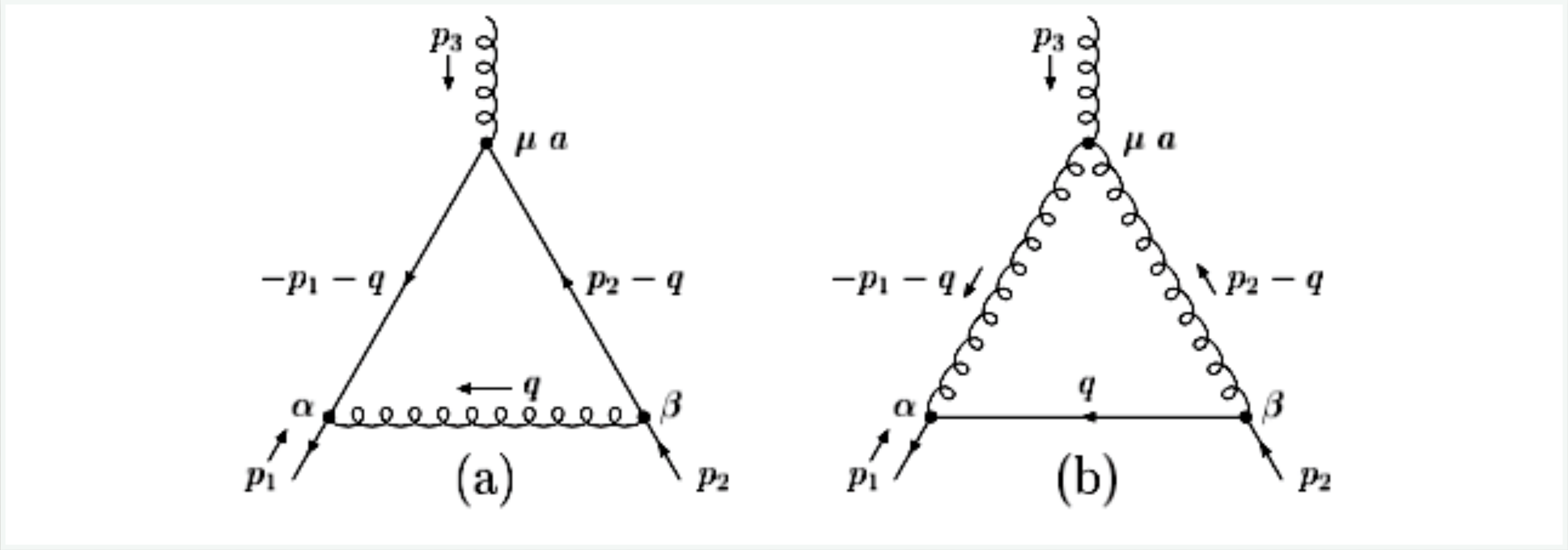
$\Rightarrow$ Landau gauge!

Rainbow Truncation



Here the bare gauge-boson propagator is used, can also be dressed.

The Quark-Gluon Vertex in QCD



(a) *Abelian* correction at one loop
(b) *Non-Abelian* correction at one loop

- The quark-gluon vertex in a tree-order is just $i\,\frac{\lambda_i}{2}\gamma_\mu$.
- However, already at one loop the Dirac-tensor structure is very complex.

Davydychev, *Osland and Saks* (2000)

Nonperturbative quark-gluon vertex: *restrictions*

- Must satisfy Slavnov-Taylor identities.
- $\Gamma_\mu(k,p)$ must be free of kinematic singularities for $k^2 \to p^2$.
- Must transform as bare vertex γ_μ under C, P and T transformations.
- Correct weak-coupling limit: must reduce to the perturbative limit when coupling is small.
- Vertex ansatz should lead to gauge independent physical observables, i.e. condensates, hadron masses, decay constants, form factors etc.

Nonperturbative quark-gluon vertex: tensor structure

The fermion-gauge-boson vertex can be decomposed into "longitudinal" and transverse components: $\Gamma_\mu(k,p) = \Gamma^L_\mu(k,p) + \Gamma^T_\mu(k,p)$

$$\Gamma^L_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k^2,p^2)\, L^i_\mu(k,p)$$

$$\Gamma^T_\mu(k,p) = \sum_{i=1}^{8} \tau_i(k^2,p^2)\, T^i_\mu(k,p)$$

$$\Gamma_\mu(k,p)\big|_{k^2=p^2=q^2=\mu^2} = \gamma_\mu$$

$$q \cdot \Gamma^T_\mu(k,p) = 0$$

Nonperturbative quark-gluon vertex: tensor structure

Which independent tensor structures to specify the longitudinal and transverse vertex?
Following Ball and Chiu (1980), one can write:

RL approximation

$$
\begin{aligned}
L^1_\mu(k,p) &= \gamma_\mu \\
L^2_\mu(k,p) &= \tfrac{1}{2}(k+p)_\mu\, \gamma\cdot(k+p) \\
L^3_\mu(k,p) &= -i\,(k+p)_\mu \\
L^4_\mu(k,p) &= -\sigma_{\mu\nu}\,(k+p)_\mu \\
\\
T^1_\mu(k,p) &= i\,[p_\mu(k\cdot q) - k_\mu(p\cdot q)] \\
T^2_\mu(k,p) &= [p_\mu(k\cdot q) - k_\mu(p\cdot q)]\,\gamma\cdot t \\
T^3_\mu(k,p) &= q^2\gamma_\mu - q_\mu\,\gamma\cdot q \\
T^4_\mu(k,p) &= -\,[p_\mu(k\cdot q) - k_\mu(p\cdot q)]\,p^\nu k^\rho \sigma_{\nu\,\rho} \\
T^5_\mu(k,p) &= \sigma^{\mu\,\nu} q_\nu \\
T^6_\mu(k,p) &= -\gamma_\mu\,(k^2-p^2) + t_\mu\,\gamma\cdot q \\
T^7_\mu(k,p) &= \tfrac{i}{2}(k^2-p^2)\,[\gamma_\mu\gamma\cdot t - t_\mu] + t_\mu\,p^\nu k^\rho\sigma_{\nu\rho} \\
T^8_\mu(k,p) &= -i\gamma_\mu\,p^\nu k^\rho \sigma_{\nu\,\rho} - p_\mu\,\gamma\cdot k + k_\mu\,\gamma\cdot p
\end{aligned}
$$

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy a gauge dependence? Clearly must be beyond rainbow truncation as bare vertex violates gauge variance:

$$iq^\mu\gamma_\mu \;\neq\; i\gamma\cdot k\, A(k^2) + B(k^2) - i\gamma\cdot p\, A(p^2) - B(p^2)$$

Best "prepared" with Landau gauge to minimize dependence.

First step is the proposal for the longitudinal vertex by Ball and Chiu:

$$\Gamma^L_\mu(k,p) = \sum_{i=1}^{4} \lambda_i(k^2,p^2)\, L^i_\mu(k,p)$$

$$\lambda_1(k^2,p^2) = \tfrac{1}{2}\left[A(k^2)+A(p^2)\right] \qquad \lambda_2(k^2,p^2) = \frac{A(k^2)-A(p^2)}{k^2-p^2}$$

$$\lambda_3(k^2,p^2) = \frac{B(k^2)-B(p^2)}{k^2-p^2} \qquad \lambda_4(k^2,p^2) = 0$$

Widely employed in phenomenology though transverse part remains undetermined.

What about gauge covariance, does it satisfy Landau-Khalatnikov-Fradkin transformations?

What about multiplicative renormalizability?

Non-Abelian Ward-Takahashi identities: *divergence and curl*

Slavnov-Taylor identity:

$$i\,q\cdot\Gamma^a(k,p) = G(q^2)\Big[S^{-1}(-k)\,H^a(k,p) - \bar{H}^a(p,k)S^{-1}(p)\Big]$$

Transverse Slavnov-Taylor identities: H.-X. He, PRD 80, 016004 (2009)

$$\begin{aligned} q_\mu\Gamma^a_\nu(k,p) - q_\nu\Gamma^a_\mu(k,p) &= G(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}\,H^a(k,p) + \bar{H}^a(p,k)\,\sigma_{\mu\nu}S^{-1}(k)\right] \\ &+ 2im\,\Gamma^a_{\mu\nu}(k,p) + t_\alpha\epsilon_{\alpha\mu\nu\beta}\,\Gamma^{5a}_\beta(k,p) + A^a_{\mu\nu}(k,p) \end{aligned}$$

$$\begin{aligned} q_\mu\Gamma^{5a}_\nu(k,p) - q_\nu\Gamma^{5a}_\mu(k,p) &= G(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu}\,H^a(k,p) - \bar{H}^a(p,k)\,\sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &+ t_\alpha\epsilon_{\alpha\mu\nu\beta}\,\Gamma^a_\beta(k,p) + V^a_{\mu\nu}(k,p) \end{aligned}$$

Vector and axialvector vertices must be decoupled !

Slavnov-Taylor Identity

One can relate the longitudinal form factors λ_i to the quark propagator's scalar and vector pieces, $B(p^2)$ and $A(p^2)$ via an ST

$$i\, q\cdot\Gamma^a(k,p) = G(q^2)\left[S^{-1}(-k)\, H^a(k,p) - \bar{H}^a(p,k) S^{-1}(p)\right]$$

Ghost dressing function   Quark-ghost scattering kernel

Decomposition of $H(k,p)$ and its conjugate in terms of Lorentz covariants:

$$H(p_1,p_2,p_3) = X_0\,\mathbb{I}_D + i\,X_1\,\gamma\cdot p_1 + i\,X_2\,\gamma\cdot p_2 + i\,X_3\,\sigma_{\alpha\beta}\,p_1^\alpha p_2^\beta$$

$$\overline{H}(p_2,p_1,p_3) = \overline{X}_0\,\mathbb{I}_D - i\,\overline{X}_2\,\gamma\cdot p_1 - i\,\overline{X}_1\,\gamma\cdot p_2 + i\,\overline{X}_3\,\sigma_{\alpha\beta}\,p_1^\alpha p_2^\beta$$

$X_i \equiv X_i(p_1,p_2,p_3)$ $X_i(p,k,q) = \bar{X}_i(k,p,q)$

Davydychev, Osland & Saks (2001)
A .C. Aguilar and J. Papavassiliou (2011)
A. C. Aguilar, J. C. Cardona, M. N. Ferreira and J.~Papavassiliou (2016, 2018)

Decoupling the transverse STIs

$$
\begin{aligned}
q_\mu \Gamma^a_\nu(k,p) - q_\nu \Gamma^a_\mu(k,p) &= G(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}\, H^a(k,p) + \bar{H}^a(p,k)\,\sigma_{\mu\nu} S^{-1}(k)\right] \\
&+ 2im\,\Gamma^a_{\mu\nu}(k,p) + t_\alpha \epsilon_{\alpha\mu\nu\beta}\,\Gamma^{5a}_\beta(k,p) + A^a_{\mu\nu}(k,p)
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$$

$$
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&+ t_\alpha \epsilon_{\alpha\mu\nu\beta}\,\Gamma^a_\beta(k,p) + V^a_{\mu\nu}(k,p)
\end{aligned}
$$

- The decoupling of the vector and axialvector vertices can be achieved by appropriate projections with two tensors which lead to two independent equations for each vertex ! S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

- Using the two identities for the vector vertex, we can use another set of projections to isolate the 8 tensor structures of the transverse vertex as functions of the *quark propagator, the ghost dressing function, the quark-ghost scattering form factors and an hitherto undetermined nonlocal tensor structure.*

The unknown ingredient ...

The cumbersome nonlocal tensor structure originates in the Fourier transform of a 4-point function with a vector-vertex insertion and a Wilson line.

$$V_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} 2k_\lambda \varepsilon^{\lambda\mu\nu\rho} \Gamma_{A\rho}(p_1, p_2; k)$$

H.-X. He, PRD 80, 016004 (2009)

$$\int d^4x d^4x' d^4x_1 d^4x_2 e^{i\left(p_1\cdot x_1 - p_2\cdot x_2 + (p_2-k)\cdot x - (p_1-k)\cdot x'\right)}$$
$$\times \left\langle 0 \left| T\bar{\psi}(x') \gamma_\rho\gamma_5 U_P(x', x)\psi(x)\psi(x_1)\bar{\psi}(x_2) \right| 0 \right\rangle$$
$$= (2\pi)^4 \delta^4(p_1 - p_2 - q)\, iS_F(p_1)\, \Gamma_{A\rho}(p_1, p_2; k) iS_F(p_2)$$

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$$\times \left\langle 0 \left| T\bar{\psi}(x')\,\gamma_\rho\gamma_5\, U_P(x', x)\psi(x)\psi(x_1)\,\bar{\psi}(x_2)\right| 0\right\rangle$$
$$= (2\pi)^4 \delta^4(p_1 - p_2 - q)\, iS_F(p_1)\,\Gamma_{A\rho}(p_1, p_2; k) iS_F(p_2)$$

The unfamiliar, complicated components in these identities can be decomposed:

$$i\,T^1_{\mu\nu} V_{\mu\nu} = Y_1(k,p)\mathbf{I}_D + Y_2(k,p)\gamma\cdot q + Y_3(k,p)\gamma\cdot t + Y_4(k,p)\left[\gamma\cdot q, \gamma\cdot t\right]$$

$$i\,T^2_{\mu\nu} V_{\mu\nu} = Y_5(k,p)\mathbf{I}_D + Y_6(k,p)\gamma\cdot q + Y_7(k,p)\gamma\cdot t + Y_8(k,p)\left[\gamma\cdot q, \gamma\cdot t\right]$$

The unknown ingredient …

We constrain the Y_i form factors with a known ansatz for transverse vertex based on *perturbation theory, symmetry considerations and multiplicative renormalizability* in a given limit $k^2 \gg p^2$.

Bashir, Bermudez, Chang & Roberts (2012)

$$
\begin{aligned}
\tau_1(k^2,p^2) &= \frac{a_1\,\Delta_B(k^2,p^2)}{(k^2+p^2)} \\
\tau_2(k^2,p^2) &= \frac{a_2\,\Delta_A(k^2,p^2)}{(k^2+p^2)} \\
\tau_3(k^2,p^2) &= a_3\,\Delta_A(k^2,p^2) \\
\tau_4(k^2,p^2) &= \frac{a_4\,\Delta_B(k^2,p^2)}{[k^2+M^2(k^2)[p^2+M^2(p^2)]} \\
\tau_5(k^2,p^2) &= a_5\,\Delta_B(k^2,p^2) \\
\tau_6(k^2,p^2) &= \frac{a_6(k^2+p^2)\,\Delta_A(k^2,p^2)}{[(k^2-p^2)^2+(M^2(k^2)+M^2(p^2))^2]} \\
\tau_7(k^2,p^2) &= \frac{a_7\,\Delta_B(k^2,p^2)}{(k^2+p^2)} \\
\tau_8(k^2,p^2) &= a_8\Delta_A(k^2,p^2)
\end{aligned}
$$

Gluon and ghost dressing functions

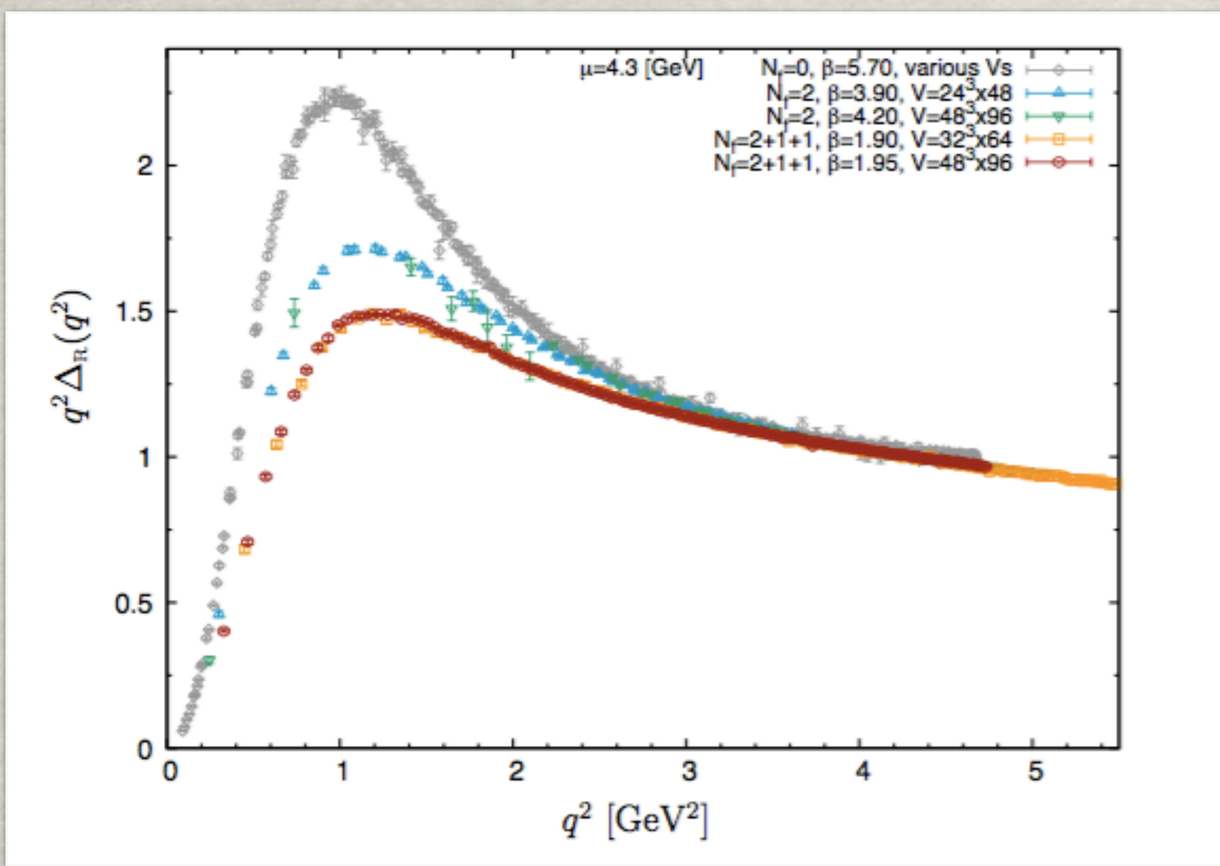
The gluon propagator in Landau gauge is:

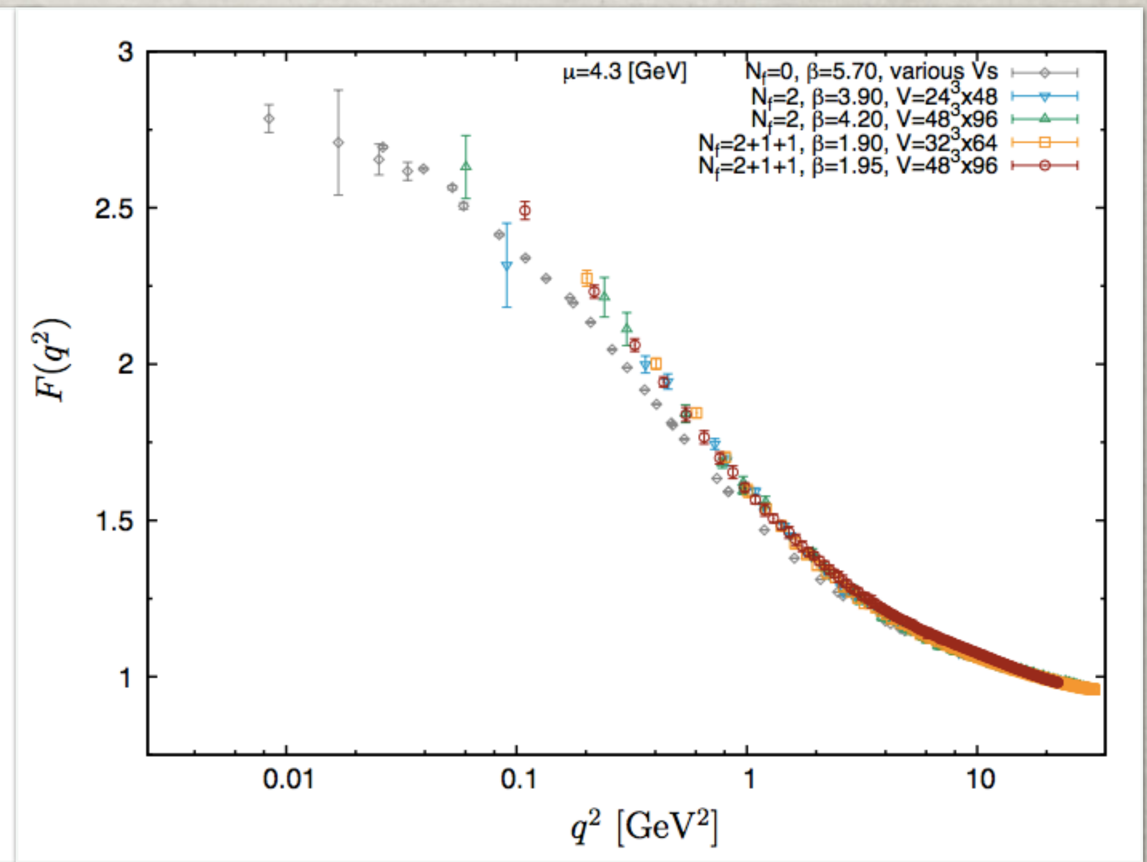
$$\Delta^{ab}_{\mu\nu}(q) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Delta(q^2) \qquad \Delta(q^2) \stackrel{q^2\to\infty}{\longrightarrow} \frac{1}{q^2}$$

The ghost propagator is:

$$D^{ab}(q^2) = -\,\delta^{ab}\,\frac{G(q^2)}{q^2} \qquad G(q^2) \stackrel{q^2\to\infty}{\longrightarrow} 1$$

Gluon and ghost dressing functions

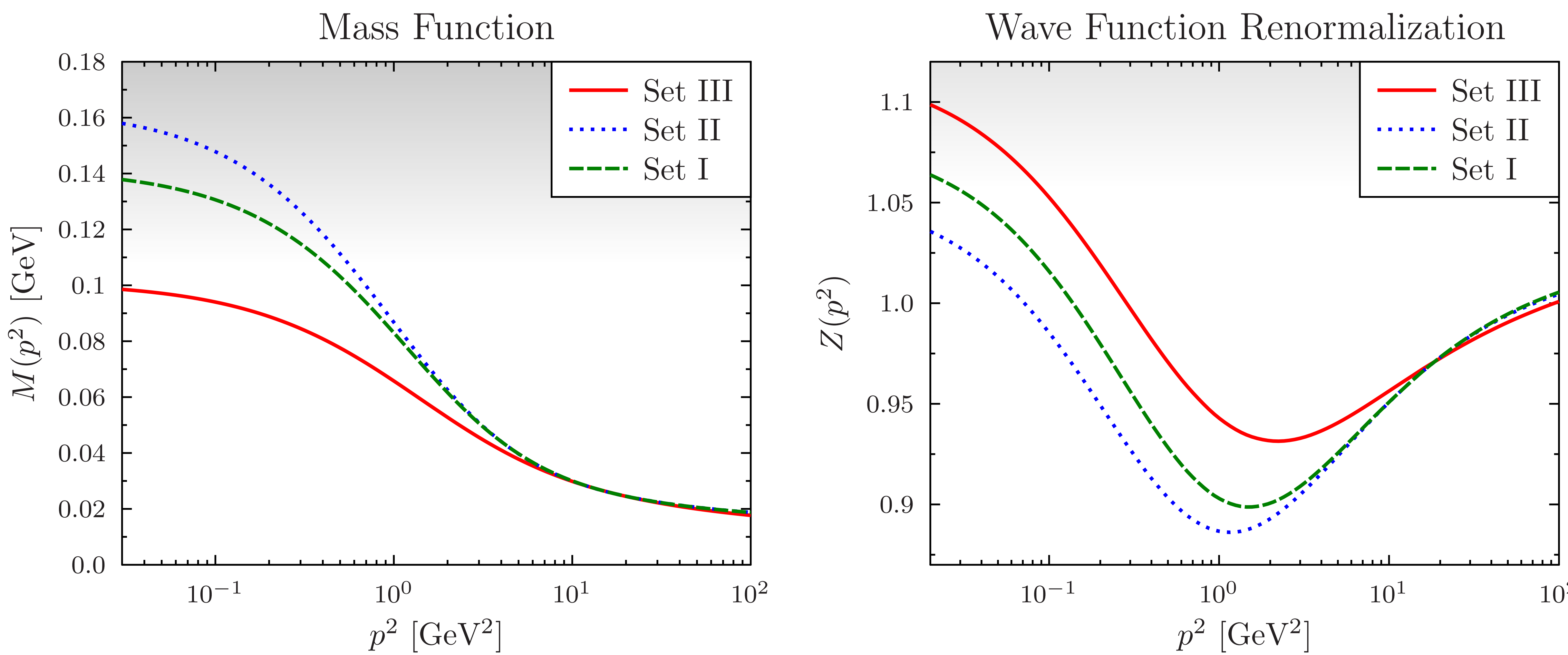




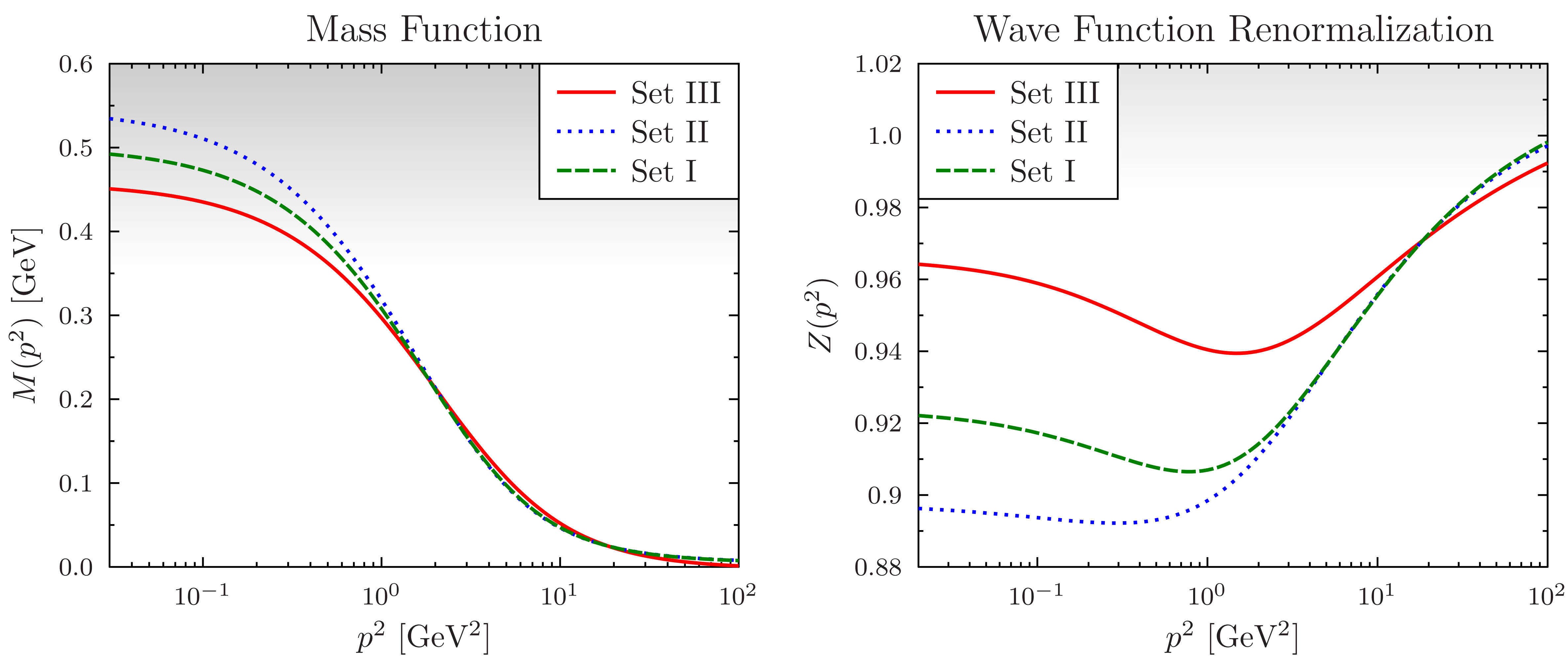
We study three sets of propagators from different collaborations:

- Set I: Bogolubsky *et al.*, Phys. Lett. B 676, 69 (2009)
- Set II: Dudal *et al.*, Annals Phys. 397, 351-364 (2018)
  Duarte *et al.*, Phys. Rev. D 94 (2016)
- Set III: A. Ayala et al., Phys. Rev. D 86, 074512 (2012)

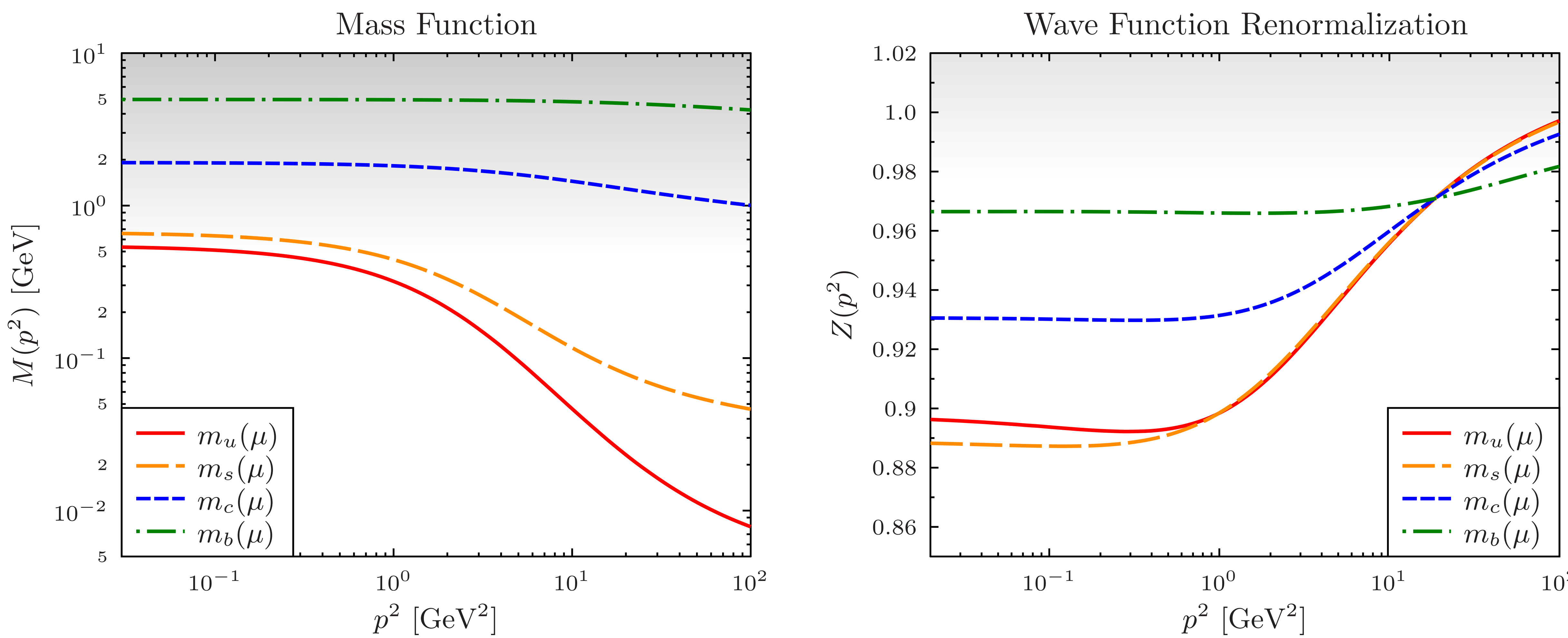
DSE Solutions with non-transverse vertex



DSE solutions with full quark-gluon vertex



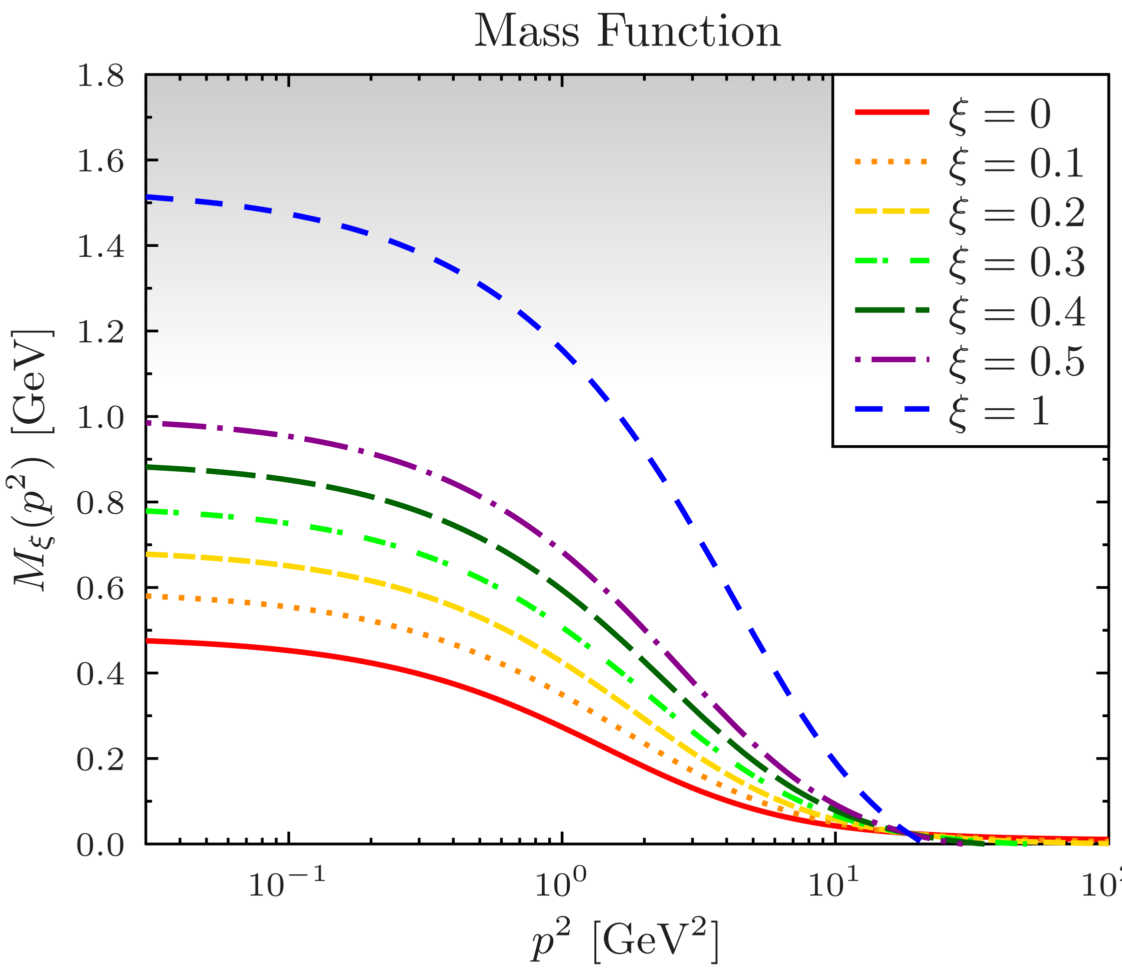
Flavor dependence of DSE solutions

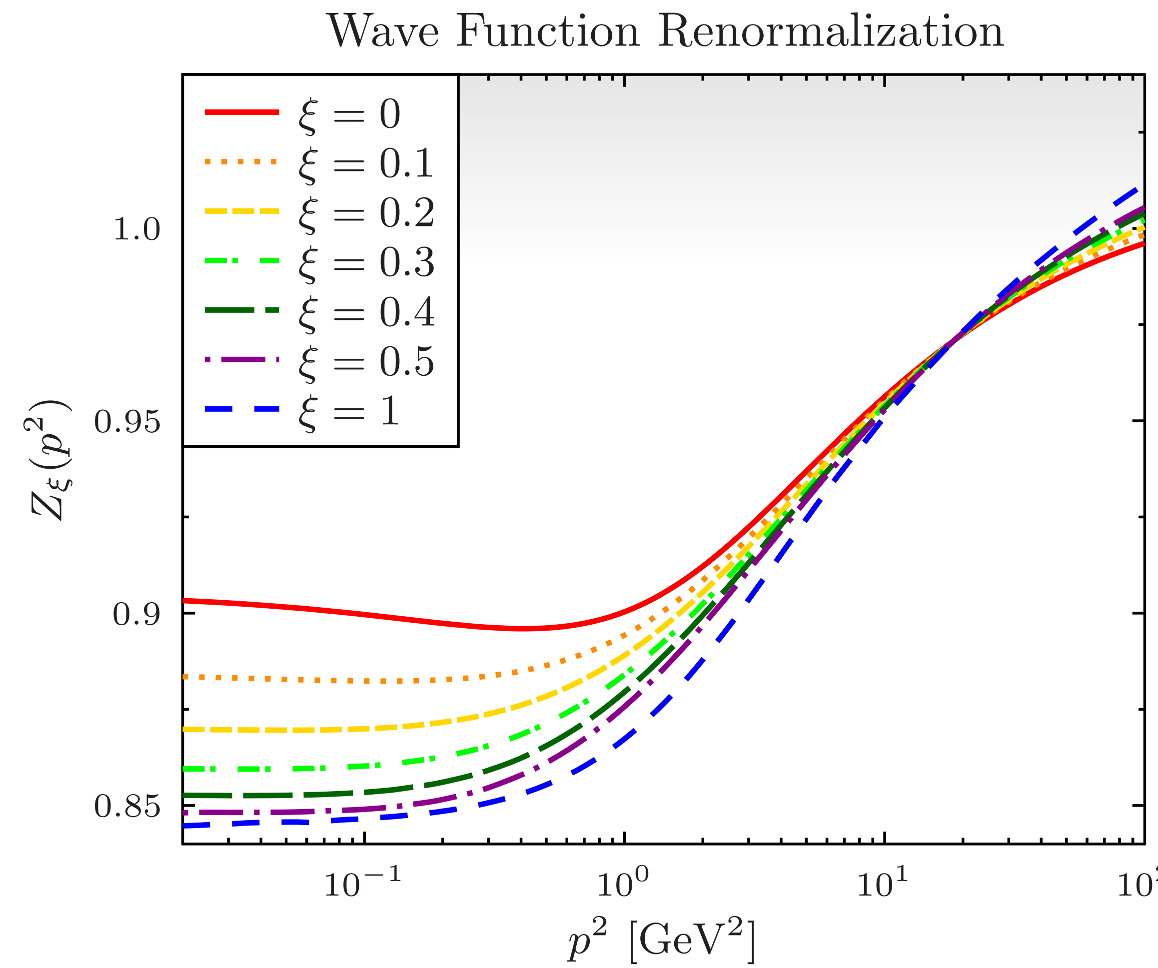


DSE with gluon propagators in R_ξ gauge

Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)

Extrapolation to Feynman Gauge using Padé parametrization of lattice gluon and ghost dressing function.



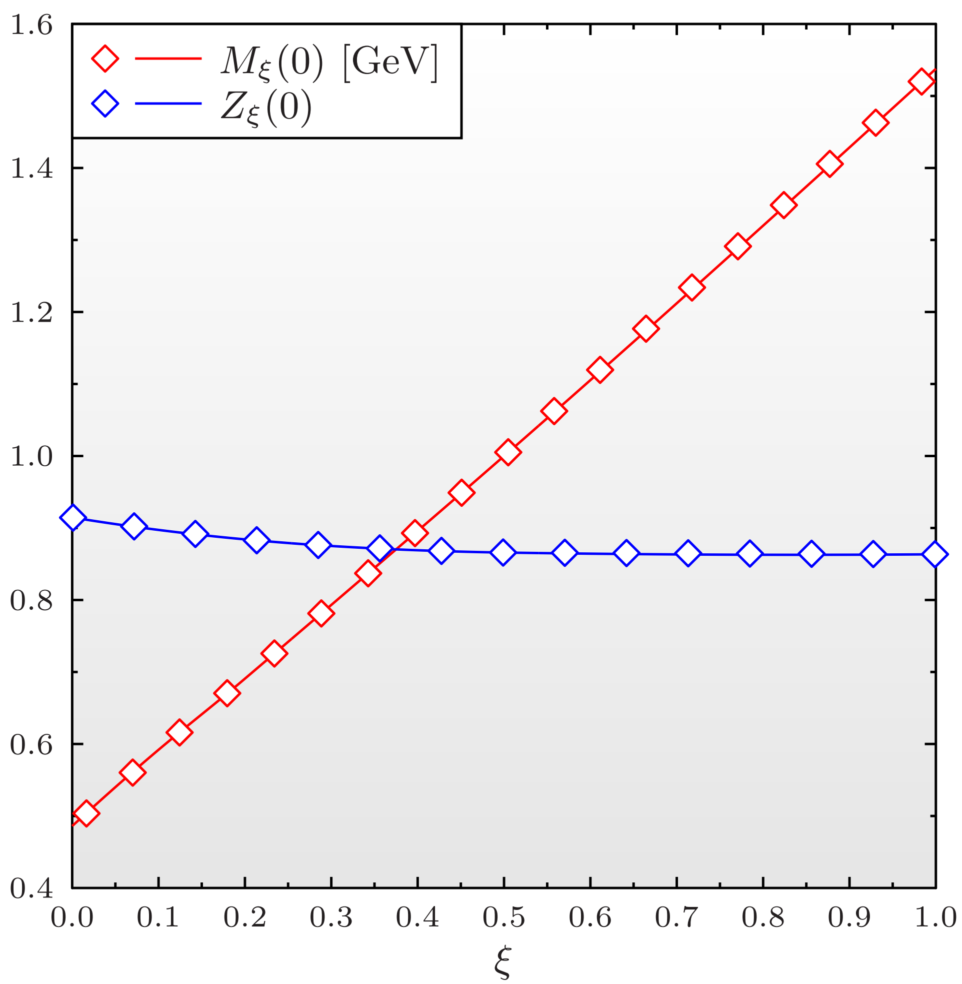


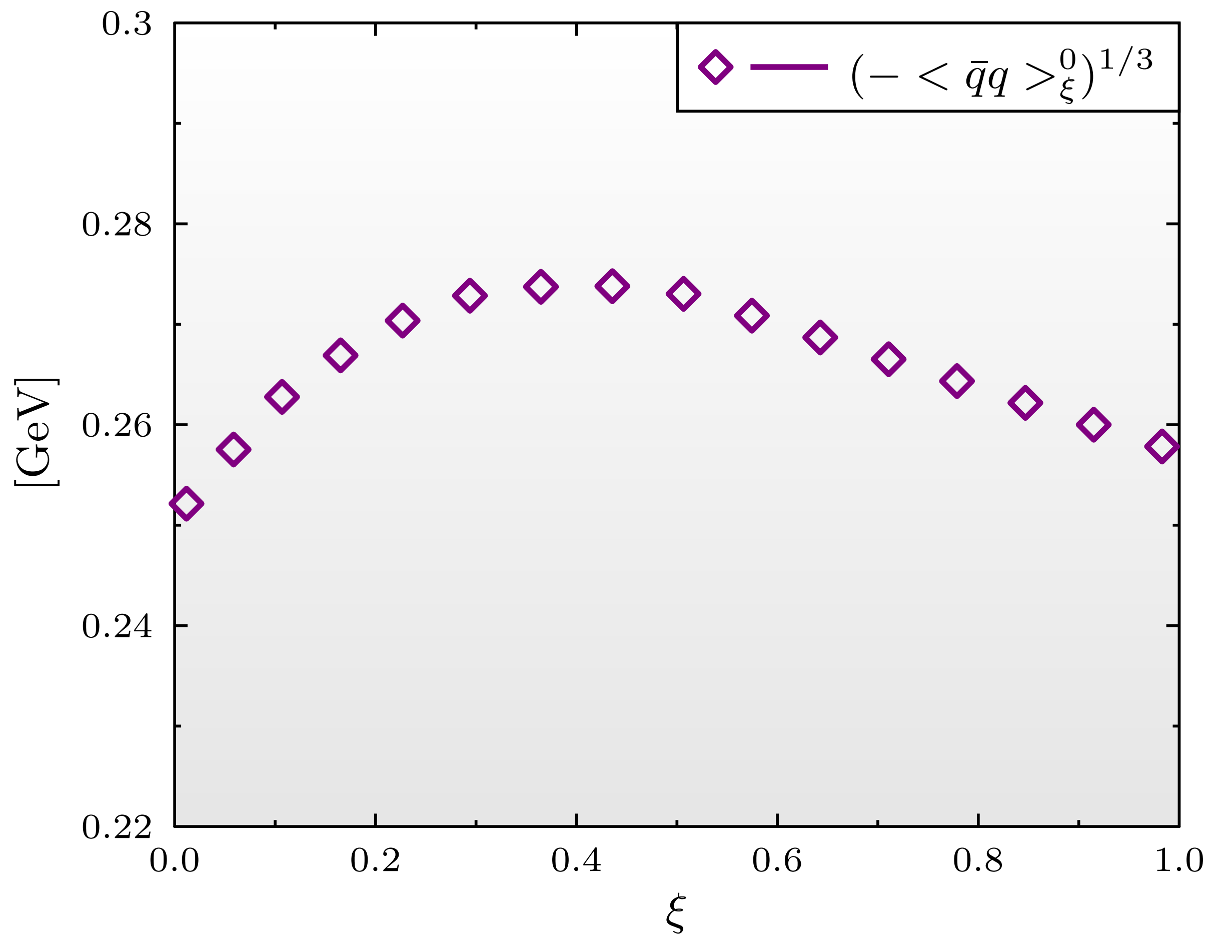
$$\alpha_s^\xi = 0.29 + 0.098\xi - 0.064\xi^2$$

A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D95, 034017 (2017)

DSE with gluon propagators in R_ξ gauge: constituent mass and quark condensate

Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)





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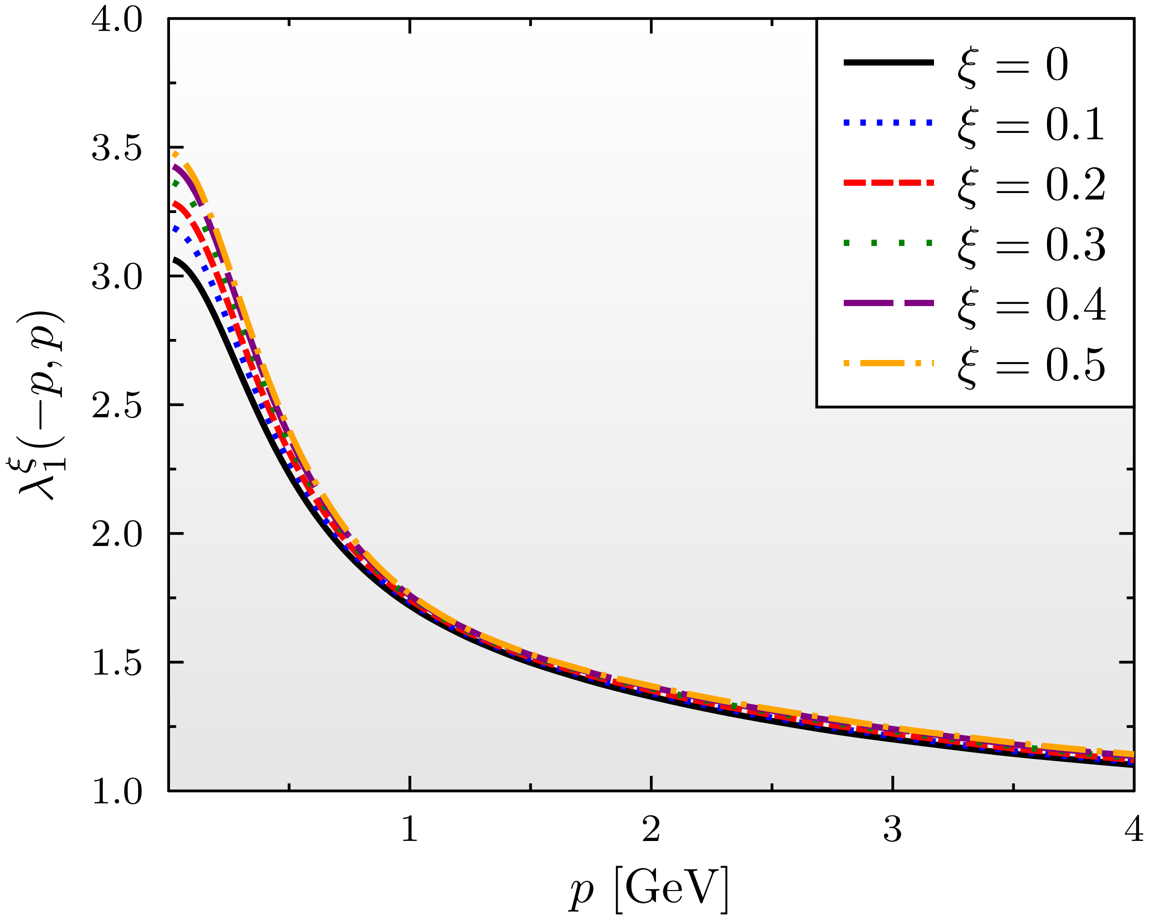
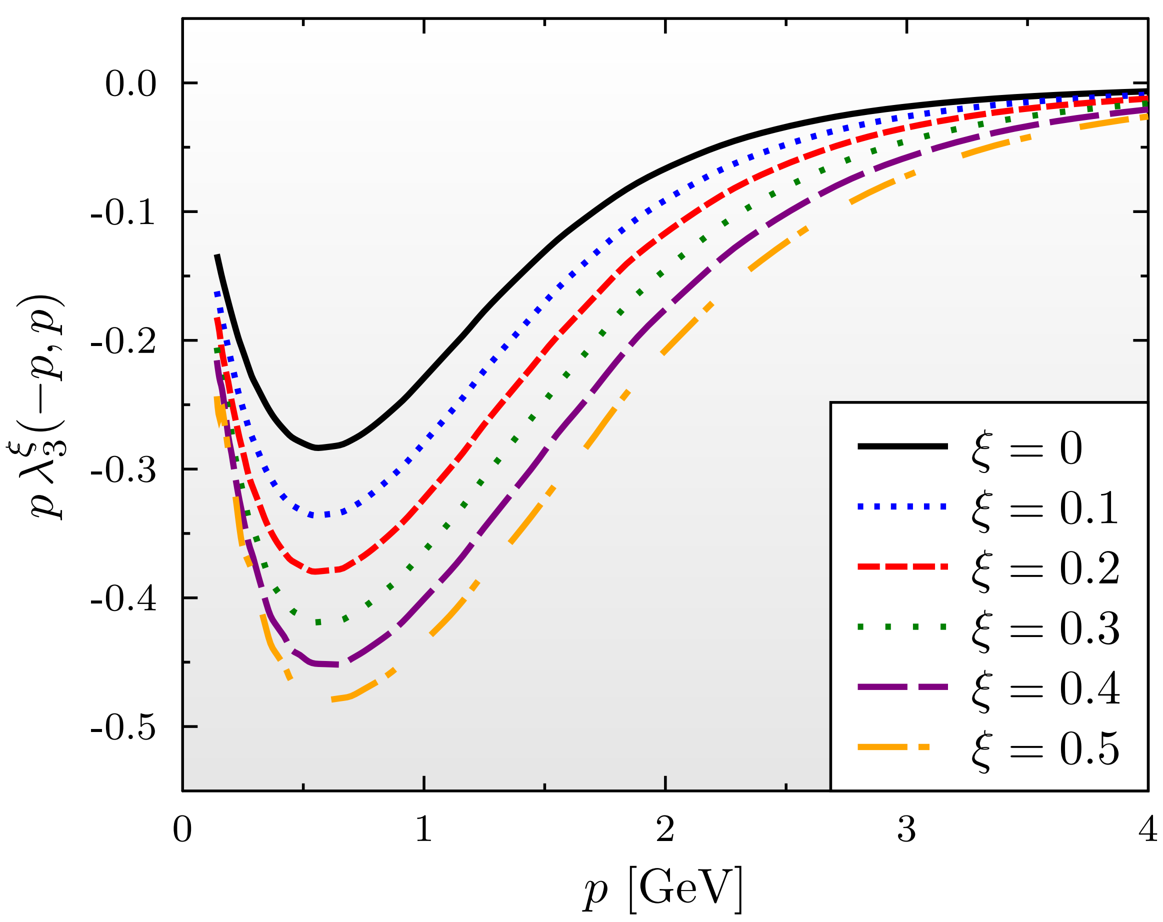
DSE with gluon propagators in R_ξ gauge: quark-gluon vertex

Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)





Conclusions & Progress

- We derived a quark-gluon vertex from symmetries (gauge + Lorentz), that is we don't solve the inhomogeneous BSE for the quark-gluon vertex.
- The self-consistent solutions employ as ingredients gluon and ghost propagators from lattice QCD.
- Current status of DCSB still unsatisfying when only known terms are kept and multiplicative renormalizability is not satisfied.
- Next step: use Y_i form factors from nonlocal tensors (line integral).
- Adding these terms yields a mass function and DCSB known from phenomenological model, but still has parameters.
- A full self-consistent with additional coupled integral equations is underway.