Ghost dynamics from Schwinger-Dyson equations

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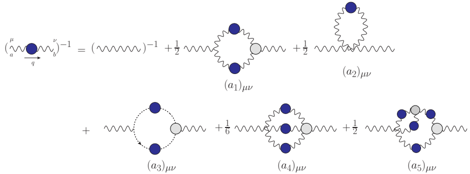
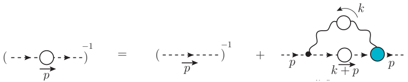
Based on:
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou and J. Rodríguez-Quintero, arXiv:2107.00768.

Motivation

- Ghost sector Green's functions, in particular the ghost propagator and ghost-gluon vertex, appear as ingredients in several Schwinger-Dyson equations (SDEs), *e.g.* the gluon gap equation





- Studied using a variety of nonperturbative approaches, including Lattice QCD, SDEs, Functional Renormalization Group.

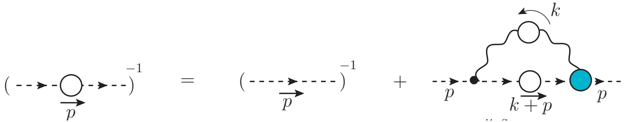
I. L. Bogolubsky, E. M. Ilgenfritz , M. Muller-Preussker and A. Sternbeck, PoS **LATTICE2007**, 290 (2007).
P. Boucaud, J. P. Leroy, A. L. Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, JHEP **06**, 012 (2008).
M. Q. Huber and L. von Smekal, JHEP **04**, 149 (2013).
A. C. Aguilar, D. Ibáñez and J. Papavassiliou, Phys. Rev. D **87**, no.11, 114020 (2013).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski and N. Strodthoff, Phys. Rev. D **94**, no.5, 054005 (2016).

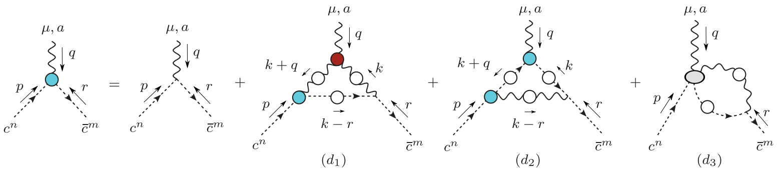
★ Ghost sector SDEs are the simplest: Clean environment for testing agreement with lattice and probe the effect of other Green's functions.

This work: Coupled dynamics of the ghost propagator and ghost-gluon vertex, consistency between SDE and lattice and impact of the three-gluon vertex.

Coupled SDEs

We solve the system of equations for the ghost propagator and the ghost-gluon vertex





Coupled SDEs

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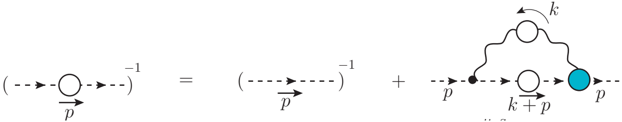

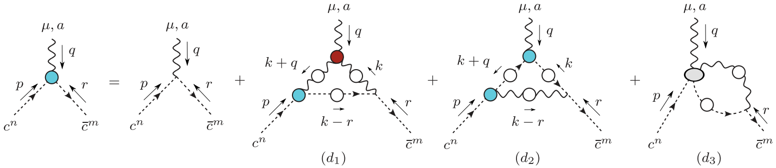

To close the system we need to provide

- Gluon propagator [all loop diagrams];

Coupled SDEs

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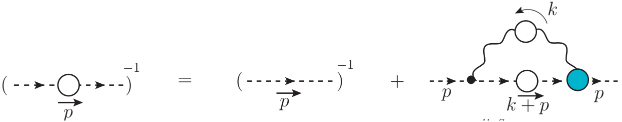

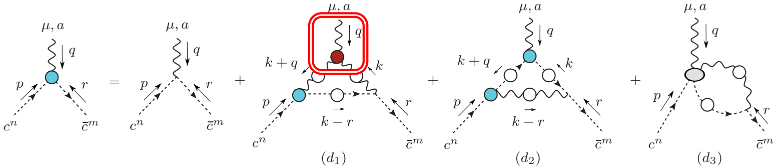

To close the system we need to provide

- Gluon propagator [all loop diagrams];
- three-gluon vertex [diagram (d_1)];

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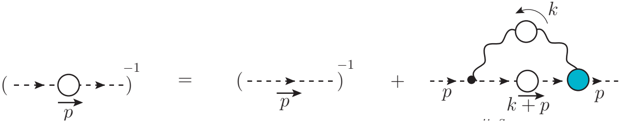
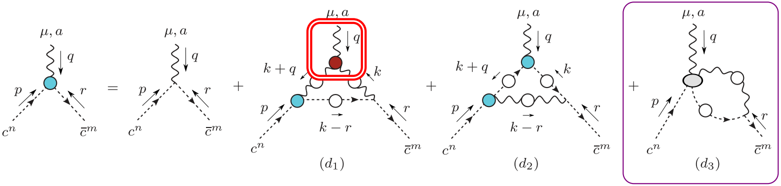




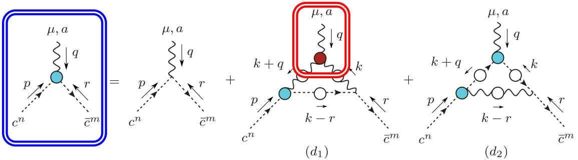
≈ 2%

To close the system we need to provide

Huber, M. Q. Eur. Phys. J. C77, 733 (2017).

- Gluon propagator [all loop diagrams];
- three-gluon vertex [diagram (d_1)];
- ghost-ghost-gluon-gluon vertex [diagram (d_3)].
  This we will omit ⇒ "**one-loop dressed**" truncation.

- Main remaining uncertainty is in the **three-gluon** vertex,

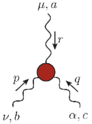


Sensitive to the three-gluon vertex.

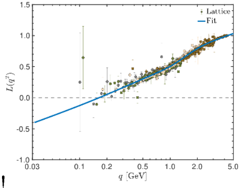
- Because this SDE has little contamination of more higher order functions

Clean probe for the three-gluon vertex.

- In the soft gluon limit, the **three-gluon vertex** is **completely determined by the lattice:**



Soft gluon

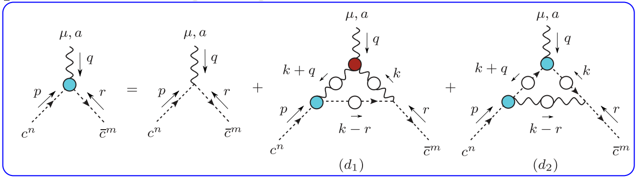


- Provides a nontrivial consistency check!

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).

Coupled SDEs: ghost-gluon vertex equation

Special properties of the ghost-gluon vertex simplify the analysis further:



- The most general structure of the ghost-gluon vertex is

$$\Gamma_\mu(r,p,q) = \boxed{B_1(r,p,q)}\, r_\mu + B_2(r,p,q) q_\mu \,.$$

- At tree level, $B_1^{(0)} = 1$ and $B_2^{(0)} = 0$.
- Now, in Landau gauge, by virtue of

$$\boxed{\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)\,, \qquad P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2\,,}$$

the form factor B_2 decouples from B_1,

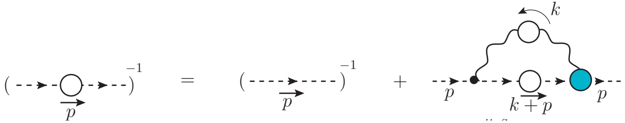
$$B_1(r,p,q) = Z_1 + \int_\ell \mathcal{K}_1[B_1] + \int_\ell \mathcal{K}_2[B_1]\,.$$

- Also, in Landau gauge this vertex is **finite**: $Z_1 = 1$ (Taylor scheme).
  $\Rightarrow$ Further simplifies analysis.

Taylor, J. C. Nucl. Phys. B33, 436–444 (1971).

Coupled SDEs: ghost propagator equation

We define the ghost dressing function, $F(p^2)$, as usual $D(p^2) = F(p^2)/p^2$.
The SDE for the ghost propagator also simplifies in Landau gauge,



$$F^{-1}(p^2) = Z_c + \Sigma(p^2)\,,$$

with

$$\Sigma(p^2) = ig^2 C_A Z_1 \int_k \left[1 - \frac{(k\cdot p)^2}{k^2p^2}\right] B_1(-p, k+p, -k)\Delta(k)D(k+p)\,.$$

- Only depends on the form factor B_1 (Landau gauge).
- Renormalization in the Taylor scheme, where $F(\mu^2) = 1$ and $Z_1 = 1$ entails,

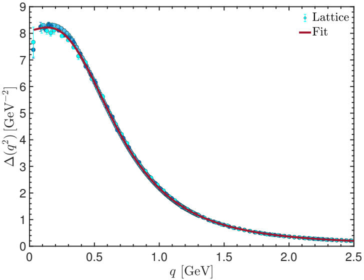
$$\boxed{F^{-1}(p^2) = 1 + \Sigma(p^2) - \Sigma(\mu^2)\,.}$$

Inputs: gluon propagator

For the gluon propagator, we can use a fit to lattice data for $\Delta(q^2)$.

P. Boucaud, F. De Soto, K. Raya, J. Rodríguez-Quintero and S. Zafeiropoulos, Phys. Rev. D **98**, no.11, 114515 (2018).

I. L. Bogolubsky, E. M. Ilgenfritz , M. Muller-Preussker and A. Sternbeck, PoS **LATTICE2007**, 290 (2007).



- Most notable feature: IR saturation.
- Associated with the dynamical generation of a gluon mass scale.
  A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
  A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).

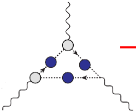
Inputs: Three-gluon vertex

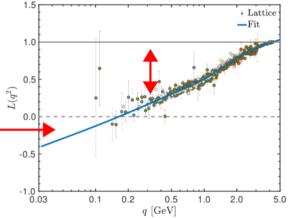
The most complicated ingredient is the general kinematics three-gluon vertex appearing in the ghost-gluon SDE.

- Complicated tensor structure with 14 independent tensors.
- Each accompanied with a scalar form factor that is a function of three variables.

- Rich nonperturbative structure:
  - **IR suppression.**
  - **Zero-crossing**
  - IR divergences due to massless ghost loops.

log divergence





A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).
A. Athenodorou, D. Binosi, Ph. Boucaud, et al. Phys.Lett.B 761 (2016) 444-449.
A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **89**, no.8, 085008 (2014).

In this work we resort to the Gauge Technique construction for the three-gluon vertex.

A. C. Aguilar, M.N.F., C. T. Figueiredo and J. Papavassiliou, Phys.Rev.D 99 (2019) 3, 034026.

Gauge Technique three-gluon vertex

The three-gluon vertex satisfies a Slavnov-Taylor identity

$$q \cdot \Gamma^{\mathrm{STI}} = F[\Delta^{-1}H - \Delta^{-1}H]\,, \qquad q \cdot \Gamma^{\mathrm{Tr}} = 0\,, \qquad \Gamma = \Gamma^{\mathrm{STI}} + \Gamma^{\mathrm{Tr}}\,,$$

with H the ghost-gluon kernel $[q^{\mu} H_{\nu\mu}(q,p,r) = \Gamma_{\mu}(q,p,r)]$.

A. C. Aguilar, M.N.F., C. T. Figueiredo and J. Papavassiliou, Phys.Rev.D 99 (2019) 9, 094010.

- The **transverse part** cannot be determined by the STI.

★ **But we can determine Γ^{STI}.**

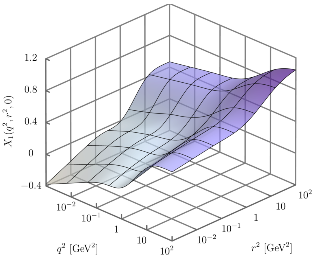
For simplicity, we retain only the "classical" tensor structure,

$$\Gamma^{\mathrm{STI}}_{\alpha\mu\nu}(q,r,p) \approx (q-r)_{\nu} g_{\alpha\mu} X_1(q,r,p) + (r-p)_{\alpha} g_{\mu\nu} X_1(r,p,q) + (p-q)_{\mu} g_{\nu\alpha} X_1(p,q,r)\,,$$

which reduces to the tree-level vertex with $X_1^{(0)} = 1$.

The form factor $X_1(q,r,p)$ captures the main features of the three-gluon vertex:

- IR suppression;
- logarithmic IR divergences;
- perturbative anomalous dimension.



Results: Ghost dressing function

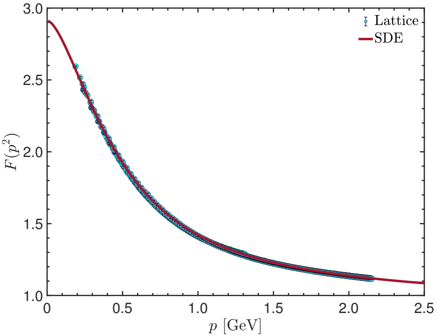
We first compare the resulting ghost dressing function to lattice data of P. Boucaud, F. De Soto, K. Raya, J. Rodríguez-Quintero and S. Zafeiropoulos, Phys. Rev. D **98**, no.11, 114515 (2018).

- Value of $\alpha_s = 0.244$ adjusted for best agreement.

★ **Excellent agreement!**

★ Careful treatment of the continuum limit of the lattice is key to this agreement.

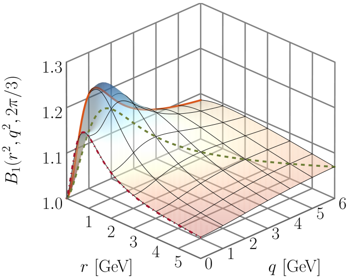
- In previous works, with lattice not cured from discretization the agreement was not so perfect, *e.g.*
  A. C. Aguilar, M. N. F., C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **99**, no.3, 034026 (2019).
  A. C. Aguilar, D. Ibáñez and J. Papavassiliou, Phys. Rev. D **87**, no.11, 114020 (2013)



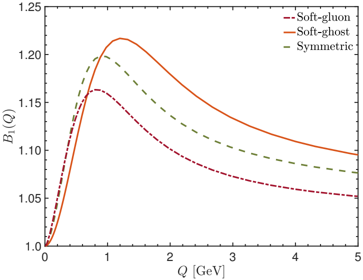
Results: ghost-gluon vertex

We can parametrize $B_1(r,p,q)$ in terms of the gluon and antighost momenta, q and r, respectively, and the angle θ between them.

A representative case: $\theta = 2\pi/3$



Some special limits:



- For all kinematics we find moderate deviations only from the tree level value, $B_1^{(0)} = 1$.
- Qualitative agreement with previous works.

Ilgenfritz, E.-M., Muller-Preussker, M., Sternbeck, et al. Braz.J. Phys. 37, 193–200 (2007).
Cucchieri, A., Maas, A. and Mendes, T. Phys. Rev. D77, 094510 (2008).
A. C. Aguilar, D. Ibáñez and J. Papavassiliou, Phys. Rev. D **87**, no.11, 114020 (2013).
Huber, M. Q. and von Smekal, L. JHEP 04, 149 (2013).
Cyrol, A. K., Fister, L., Mitter, M., et al. Phys. Rev. D94, 054005 (2016).
Boucaud, P., Dudal, D., Leroy, J., et al. JHEP 12, 018 (2011).
Barrios, N., Pelaez, M., Reinosa, U., et al. Phys.Rev.D 102, 11401 (2020).

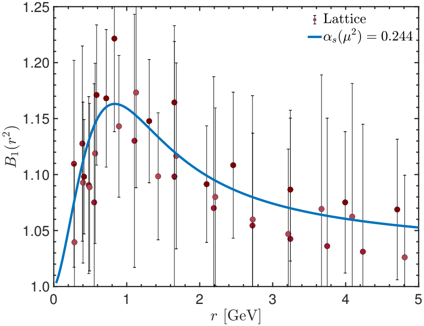
Results: ghost-gluon vertex

In the soft gluon limit there is lattice data available for the ghost-gluon vertex.

A. Sternbeck, (2006), arXiv:hep-lat/0609016 [hep-lat].

E.-M. Ilgenfritz, et al., Braz. J. Phys. 37, 193 (2007).

Again in agreement with the lattice:



Soft gluon limit: A nontrivial consistency check

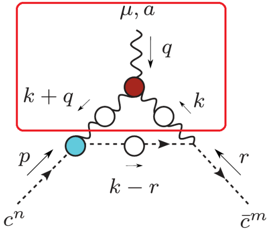
In the soft gluon limit, $q = 0$, the three-gluon vertex contribution to the ghost-gluon SDE simplifies dramatically.

It can be shown that

$$P_\mu^{\mu'}(r)P_\nu^{\nu'}(-r)\Gamma_{\mu'\nu'\alpha}(r,-r,0) = L(r^2)\lambda_{\mu'\nu'\alpha}(r)\,, \qquad \lambda_{\mu'\nu'\alpha}(r) := 2r_\alpha P_{\mu\nu}(r)\,,$$

recalling that $P_{\mu\nu}(r) = g_{\mu\nu} - r_\mu r_\nu/r^2$ is the transverse projector.

- **Only one tensor structure.**
- In the ghost-gluon SDE, for $q = 0$, the above is all we need:



Since in Landau gauge

$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)\,,$$

the combination to the left becomes

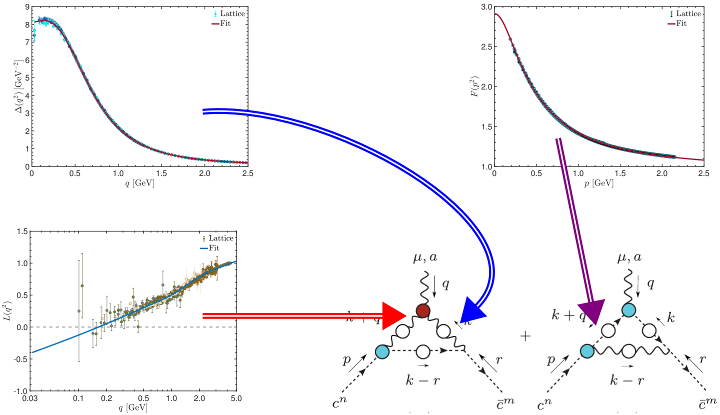
$$-\Delta^2(k^2)L(k^2)\lambda_{\alpha\beta\mu}(k)\,.$$

★ **Projection $L(q^2)$ accurately determined on the lattice.**

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).

In this limit all external ingredients are known from lattice simulations:

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **818**, 136352 (2021).
A. Athenodorou, D. Binosi, Ph. Boucaud, et al. Phys.Lett.B 761 (2016) 444-449.
P. Boucaud, F. De Soto, K. Raya, J. Rodríguez-Quintero and S. Zafeiropoulos, Phys. Rev. D **98**, no.11, 114515 (2018).

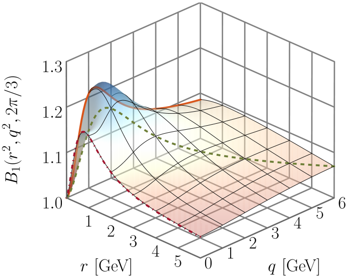


- All from the same lattice setups;
- and renormalized at the same point, $\mu = 4.3$ GeV.

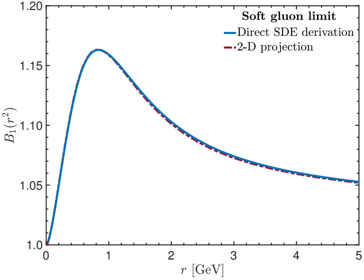
Comparison to general kinematics

Comparing the soft gluon limit of the coupled system with the result of the soft-gluon SDE with lattice inputs only provides a consistency check:

Coupled system:



Soft gluon SDE:



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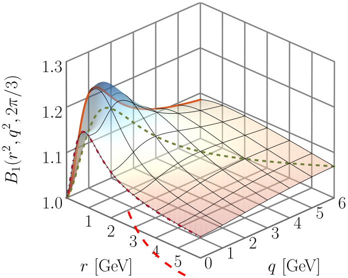

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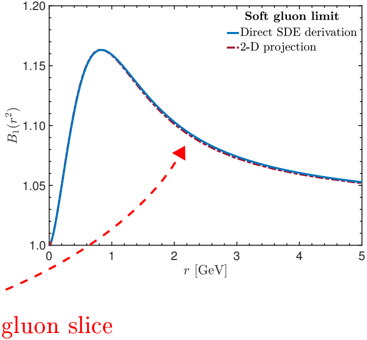


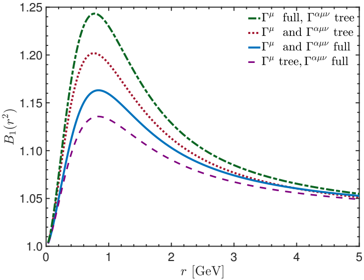
★ **Results agree at the 2% level in quantum correction ($B_1 - 1$).**

Impact of the vertex dressings

We now consider the effect of the the vertex dressings in the result for B_1.

Results exhibit the expected hierarchy:

- Significant suppression due to the three-guon vertex;
- and enhancement due to the ghost-gluon vertex itself.
- **Effects are most prominent around ~ 1 GeV.**



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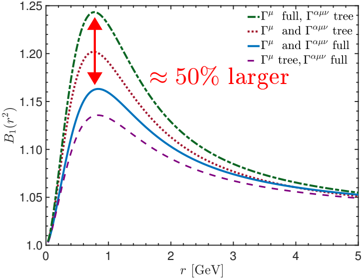

★ Most importantly, neglecting the three-gluon vertex yields 50% too big quantum correction for B_1.

Conclusions

- We solved the coupled SDEs for the ghost propagator and general kinematics ghost-gluon vertex with most up-to-date lattice and gauge technique inputs.
- Results exhibit **excellent agreement with lattice** data, after careful treatment of discretization artifacts.
- In the soft gluon limit, a consistency check is possible using lattice inputs for all external ingredients.
  - ★ Consistency of **Lattice**, SDE and Gauge Technique.
- The characteristic **IR suppression of the three-gluon vertex** has an important quantitative effect on the ghost-gluon vertex.