On the nature of $X(6900)$ and other structures in the LHCb di-J/ψ spectrum

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Based on [ZRL, Xiao-Yi Wu, De-Liang Yao, arXiv: 2104.08589.]

Outline

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  - Status of theoretical studies
- Theoretical Framework
  - Effective Lagrangian
  - Helicity amplitudes and partial-wave projection
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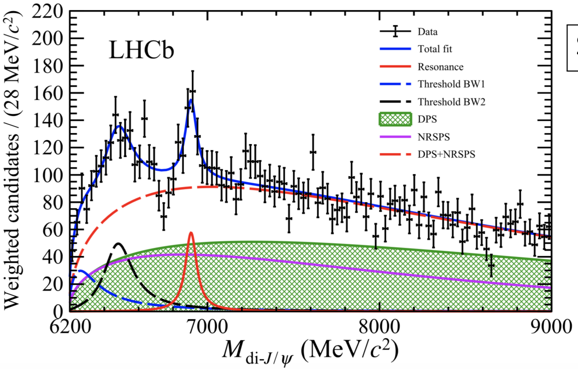
  - Description of the LHCb data with model with three-coupled channels
  - Analyses with four-coupled channels and hints of four possible states
  - Further consideration: a combined fit
- Summary and outlook

Background



Sci. Bull. 65 (2020), 1983-1993

- LHCb reported a narrow structure around 6.9 GeV in the di-J/ψ invariant mass spectrum: $X(6900)$.
- A possible board structure at range $[6.2\ \mathrm{GeV}, 6.8\ \mathrm{GeV}]$.
- A hint of another structure around $7.2\ \mathrm{GeV}$.

Status of theoretical studies

Many works have been accumulated:

- Quark model:
  - R. N. Faustov, V. O. Galkin and E. M. Savchenko, Universe **7** (2021) no.4, 94.
  - X. Jin, X. Liu, Y. Xue, H. Huang and J. Ping, arXiv:2011.12230.
  - Q. F. Lü, D. Y. Chen and Y. B. Dong, Eur. Phys. J. C **80** (2020), 871.
- QCD sum rules:
  - B. C. Yang, L. Tang and C. F. Qiao, Eur. Phys. J. C **81**, 324 (2021).
  - B. D. Wan and C. F. Qiao, Phys. Lett. B **817**, 136339 (2021).
  - Z. G. Wang, Chin. Phys. C **44** (2020), 113106.
- NRQCD factorization:
  - F. Feng, Y. Huang, Y. Jia, W. L. Sang and J. Y. Zhang, Phys. Lett. B **818**, 136368 (2021).
  - Y. Q. Ma and H. F. Zhang, arXiv:2009.08376.
  - F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y. Zhang, arXiv:2009.08450.

Status of theoretical studies

- Phenomenological model:
  - Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat and Y. Yan, Phys. Rev. D **103** (2021), 116027.
  - C. Gong, M. C. Du, B. Zhou, Q. Zhao and X. H. Zhong, arXiv:2011.11374.
  - Q. F. Cao, H. Chen, H. R. Qi and H. Q. Zheng, Chin. Phys. C **45**, 093113 (2021).
  - X. K. Dong, V. Baru, F. K. Guo, C. Hanhart and A. Nefediev, Phys. Rev. Lett. **126**, 132001 (2021).
  - Z. H. Guo and J. A. Oller, Phys. Rev. D **103** (2021), 034024.
- ...
- **Our work** (Effective Lagrangian & Unitarization)
  Reveal possible states and explore their J^{PC} quantum numbers
  - derive potentials from effective Lagrangians
  - take coupled-channel effects into account
  - employ helicity amplitude formalism and perform partial-wave analysis

Effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff.}} &= h_1(J/\psi\cdot J/\psi)^2 \\
&+ h_2(J/\psi\cdot J/\psi)(J/\psi\cdot\psi(2S)) \\
&+ h_3(J/\psi\cdot J/\psi)(J/\psi\cdot\psi(3770)) \\
&+ h_4(J/\psi\cdot\psi(2S))^2 \\
&+ h_4'(J/\psi\cdot J/\psi)(\psi(2S)\cdot\psi(2S)) \\
&+ h_5(J/\psi\cdot\psi(2S))(J/\psi\cdot\psi(3770)) \\
&+ h_5'(J/\psi\cdot J/\psi)(\psi(2S)\cdot\psi(3770)) \\
&+ h_6(J/\psi\cdot\psi(3770))^2 \\
&+ h_6'(J/\psi\cdot J/\psi)(\psi(3770)\cdot\psi(3770)) \\
&+ h_7(J/\psi\cdot\psi(2S))(\psi(2S)\cdot\psi(2S)) \\
&+ h_8(J/\psi\cdot\psi(3770))(\psi(2S)\cdot\psi(2S)) \\
&+ h_8'(J/\psi\cdot\psi(2S))(\psi(3770)\cdot\psi(2S)) \\
&+ h_9(\psi(2S)\cdot\psi(2S))^2
\end{aligned}
$$

- Four channels: $\{J/\psi J/\psi,\ J/\psi\psi(2S),\ J/\psi\psi(3770),\ \psi(2S)\psi(2S)\}$
- The Lagrangian satisfies the basic symmetries, such as Lorentz invariance, P and T parity symmetries and so on.
- The unknown couplings $h_{1,2,..9}$ and $h'_{4,5,6,8}$ need to be determined by experimental data.

Potentials for the scattering processes

- The generic form of the potentials for $V_1(p_1, \epsilon_1) + V_2(p_2, \epsilon_2) \to V_3(p_3, \epsilon_3) + V_4(p_4, \epsilon_4)$

$$V_{ij} = \mathcal{C}_1 \epsilon_1 \cdot \epsilon_2 \epsilon_3^\dagger \cdot \epsilon_4^\dagger + \mathcal{C}_2 \epsilon_1 \cdot \epsilon_3^\dagger \epsilon_2 \cdot \epsilon_4^\dagger + \mathcal{C}_3 \epsilon_1 \cdot \epsilon_4^\dagger \epsilon_2 \cdot \epsilon_3^\dagger$$

- Coefficients for different processes

V_{ij}	Channels	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
11	$J/\psi J/\psi \to J/\psi J/\psi$	$8h_1$	$8h_1$	$8h_1$
12	$J/\psi J/\psi \to \psi(2S) J/\psi$	$2h_2$	$2h_2$	$2h_2$
13	$J/\psi J/\psi \to \psi(3770) J/\psi$	$2h_3$	$2h_3$	$2h_3$
14	$J/\psi J/\psi \to \psi(2S)\psi(2S)$	$4h_4'$	$2h_4$	$2h_4$
22	$\psi(2S) J/\psi \to \psi(2S) J/\psi$	$2h_4$	$4h_4'$	$2h_4$
23	$\psi(2S) J/\psi \to \psi(3770) J/\psi$	h_5	$2h_5'$	h_5
24	$\psi(2S) J/\psi \to \psi(2S)\psi(2S)$	$2h_7$	$2h_7$	$2h_7$
33	$\psi(3770) J/\psi \to \psi(3770) J/\psi$	$2h_6$	$4h_6'$	$2h_6$
34	$\psi(3770) J/\psi \to \psi(2S)\psi(2S)$	$2h_8$	h_8'	h_8'
44	$\psi(2S)\psi(2S) \to \psi(2S)\psi(2S)$	$8h_9$	$8h_9$	$8h_9$

Helicity amplitude

- Helicity amplitudes (81 in total), $\lambda_i = \pm 1, 0$

$$V_{\lambda_1\lambda_2\lambda_3\lambda_4} = \epsilon_3^{\rho\dagger}(p_3, \lambda_3)\epsilon_4^{\sigma\dagger}(p_4, \lambda_4) V_{\mu\nu\rho\sigma}\epsilon_1^{\mu}(p_1, \lambda_1)\epsilon_2^{\nu}(p_2, \lambda_2)$$

$$V_{\mu\nu\rho\sigma} = \mathcal{C}_1 g_{\mu\nu} g_{\rho\sigma} + \mathcal{C}_2 g_{\mu\rho} g_{\nu\sigma} + \mathcal{C}_3 g_{\mu\sigma} g_{\nu\rho}$$

According to P and T parity symmetries ($81 \to 25$)
e.g.

$$V_{++++} = V_{----}$$
$$V_{+++0} = -V_{---0} = V_{-0--} = -V_{+0++}$$

- Explicit expressions for helicity amplitudes
e.g.

$$V_{++++} = V_{----} = \mathcal{C}_1 + \frac{1}{4}(\mathcal{C}_3(-1+z_s)^2 + \mathcal{C}_2(1+z_s)^2)$$

with $z_s = \cos\theta$, θ being scattering angle .

Partial-Wave Projection

- Partial wave amplitudes

$$V^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s) = \frac{1}{2}\int_{-1}^{+1} dz_s\, V_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, t(s, z_s)) d^J_{\lambda,\lambda'}(z_s)$$

  where $\lambda = \lambda_1 - \lambda_2$, $\lambda' = \lambda_3 - \lambda_4$.
  Relations due to P, T symmetries and properties of $d^J_{\lambda\lambda'}$ functions:
  e.g.
  - $V^J_{++++} = V^J_{----}$
  - $V^J_{+++0} = V^J_{---0} = V^J_{-0--} = V^J_{+0++}$
- Helicity basis $\rightarrow$ JLS basis (definite J^{PC} quantum number)

$$\mathcal{V}^J = \sum_{\substack{\lambda_1\lambda_2 \\ \lambda_3\lambda_4}} U^J_{\lambda_3\lambda_4} \mathcal{V}^J_{\lambda_1\lambda_2\lambda_3\lambda_4} [U^J_{\lambda_1\lambda_2}]^\dagger$$

  - The transformation matrix

$$U^J_{\lambda_1\lambda_2} = \frac{1}{\sqrt{2S+1}} \langle S_1\lambda_1 S_2 - \lambda_2 | S\lambda\rangle \quad => \boxed{\text{C-G coefficient}}$$

S-Wave Amplitude

S-wave: $L = 0$, $J = L + S = S \Rightarrow$ $\boxed{\text{For identical particles: } L + S = \mathrm{even}}$

- $J^{PC} = 0^{++}$

$$\mathcal{V}(0^{++}) = \frac{2}{3} V^{J=0}_{++++} + \frac{2}{3} V^{J=0}_{++--} + \frac{1}{3} V^{J=0}_{0000} - \frac{4}{3} V^{J=0}_{++00}$$

- $J^{PC} = 2^{++}$

$$\begin{aligned}
\mathcal{V}(2^{++}) &= \frac{2}{15} V^{J=2}_{0000} + \frac{4}{15} V^{J=2}_{00++} + \frac{4\sqrt{6}}{15} V^{J=2}_{00+-} + \frac{2\sqrt{6}}{15} (V^{J=2}_{+-++} + V^{J=2}_{-+++}) \\
&+ \frac{4\sqrt{3}}{15} (V^{J=2}_{00+0} + V^{J=2}_{000+}) + \frac{2\sqrt{3}}{15} (V^{J=2}_{+0++} + V^{J=2}_{0+++} + V^{J=2}_{0-++} + V^{J=2}_{-0++}) \\
&+ \frac{1}{5} (V^{J=2}_{+0+0} + V^{J=2}_{-0+0} + V^{J=2}_{0+0+} + V^{J=2}_{0-0+}) \\
&+ \frac{2\sqrt{2}}{5} (V^{J=2}_{+-+0} + V^{J=2}_{0++-} + V^{J=2}_{-++0} + V^{J=2}_{0-+-}) \\
&+ \frac{2}{5} (V^{J=2}_{0++0} + V^{J=2}_{0-+0} + V^{J=2}_{+-+-} + V^{J=2}_{-++-}) + \frac{1}{15} (V^{J=2}_{++++} + V^{J=2}_{--++})
\end{aligned}$$

Unitarization

- Requirement of unitarity
  $\longrightarrow$ Bethe-Salpeter equation method is employed to restore unitarity.
- The unitarized amplitude under on-shell approximation

$$\mathcal{T}_J(s) = \mathcal{V}^J(s) \cdot \left[1 - \mathcal{G}(s) \cdot \mathcal{V}^J(s)\right]^{-1}$$

Graphic representation

- For coupled-channel:

$$\mathcal{V}^J(s) = \begin{pmatrix} V^J_{11}(s) & V^J_{12}(s) & V^J_{13}(s) & V^J_{14}(s) \\ V^J_{12}(s) & V^J_{22}(s) & V^J_{23}(s) & V^J_{24}(s) \\ V^J_{13}(s) & V^J_{23}(s) & V^J_{33}(s) & V^J_{34}(s) \\ V^J_{14}(s) & V^J_{24}(s) & V^J_{34}(s) & V^J_{44}(s) \end{pmatrix}$$

- two-point functions

$$\mathcal{G}(s) = \begin{pmatrix} g_1(s) & & & \\ & g_2(s) & & \\ & & g_3(s) & \\ & & & g_4(s) \end{pmatrix}$$

Unitarization

- The explicit form of two-point loop function

$$
\begin{aligned}
g_i(s) = & \frac{1}{16\pi^2}\Bigg\{ a(\mu) + \ln \frac{M_{V_1}^2}{\mu^2} + \frac{s - M_{V_1}^2 + M_{V_2}^2}{2s} \ln \frac{M_{V_2}^2}{M_{V_1}^2} \\
& + \frac{\sigma(s)}{2s}\Big[\ln(s - M_{V_2}^2 + M_{V_1}^2 + \sigma(s)) - \ln(-s + M_{V_2}^2 - M_{V_1}^2 + \sigma(s)) \\
& + \ln(s + M_{V_2}^2 - M_{V_1}^2 + \sigma(s)) - \ln(-s - M_{V_2}^2 + M_{V_1}^2 + \sigma(s))\Big] \Bigg\}
\end{aligned}
$$

where

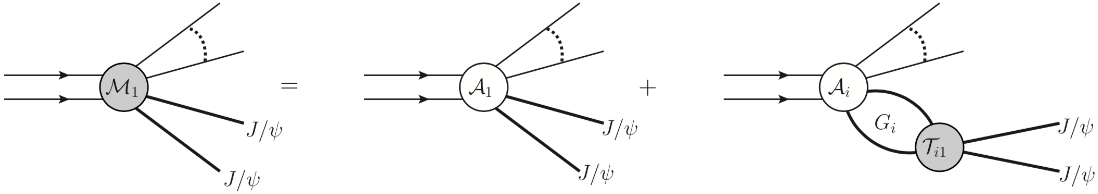
- $\sigma(s) = \{[s - (M_{V_2} + M_{V_1})^2][s - (M_{V_2} - M_{V_1})^2]\}^{1/2}$.
- The subtraction scale $a(\mu) = -3.0$ at renormalization scale $\mu = 1\ \mathrm{GeV}$.
- Definition of Riemann Sheet

$$
g_i(s) \to g_i(s) + i\frac{p_i(s)}{4\pi\sqrt{s}},
$$

with $p_i(s)$ being the CM momentum in the i-th channel.

Production Amplitudes

- Production of di-J/ψ via pp collision



- The production amplitude

$$\mathcal{M}_1(s) = \mathcal{A}_1 + \sum \mathcal{A}_i \mathcal{G}_i(s) \mathcal{T}_{i1}(s) = \mathcal{A}_1 \left[1 + \sum r_i \mathcal{G}_i(s) \mathcal{T}_{i1}(s)\right]$$

- $\gamma_i = \mathcal{A}_i / \mathcal{A}_1$
- $\mathcal{A}_1$ represent direct production of J/ψ pairs.

The invariant mass formula

- The invariant mass

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\sqrt{s}} \propto \rho(s)|\mathcal{M}_1(s)|^2 = \rho(s)|\mathcal{A}_1(s)|^2\left|\gamma + \sum_{i=1}^{3}\mathcal{G}_i(s)\mathcal{T}_{i1}(s)\right|^2$$

with γ coherent background.

- Phase factor

$$\rho(s) = \frac{p_1(s)}{8\pi\sqrt{s}} = \frac{\lambda^{1/2}(s, m_{J/\psi}^2, m_{J/\psi}^2)}{16\pi s}$$

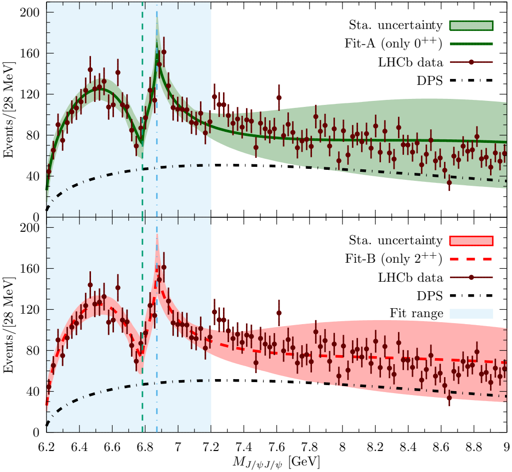
- The direct production amplitude is parameterized as

$$|\mathcal{A}_1(s)|^2 = \alpha^2 e^{-2\beta s}$$

  - Normalization factor α
  - The slope parameter β is fixed ($\beta = 0.0123$)
    [X. K. Dong, V. Baru, F. K. Guo, C. Hanhart and A. Nefediev, Phys. Rev. Lett. **126**, 132001 (2021).]

Fits with three coupled-channels

- Description of LHCb data:



- consider three coupled-channels: $J/\psi J/\psi$, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$;
- perform two different kinds of fits 0^{++} and 2^{++} within range [6.2 GeV, 7.2 GeV] ;
- Both fits well describe the di-J/ψ spectrum within 1-σ uncertainty.

Fits with three coupled-channels

- Pole positions and residues:
  - 0^{++}

RS	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$
I	$6173.9^{+19.6}_{-41.5}$	$16.8^{+5.8}_{-10.3}$	$22.6^{+4.1}_{-11.3}$	$5.1^{+3.7}_{-2.8}$
II	$6191.4^{+2.4}_{-4.8}$	$4.2^{+1.3}_{-1.0}$	$5.7^{+1.2}_{-1.0}$	$1.3^{+0.9}_{-0.7}$
II	$6976.2^{+70.6}_{-75.5} - i153.0^{+182.6}_{-123.3}$	$27.6^{+8.1}_{-7.1}$	$36.2^{+6.8}_{-6.7}$	$19.5^{+10.9}_{-6.3}$

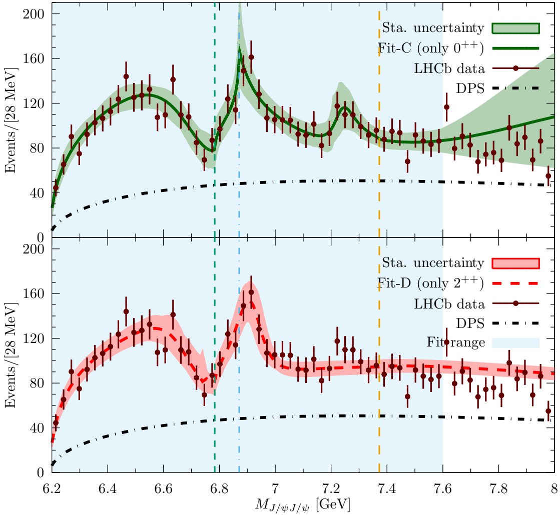
  - 2^{++}

RS	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$
I	$6169.3^{+23.9}_{-44.2}$	$17.8^{+5.5}_{-10.4}$	$22.8^{+3.6}_{-10.3}$	$5.3^{+3.1}_{-3.0}$
II	$6190.9^{+2.8}_{-5.1}$	$4.4^{+1.2}_{-1.2}$	$5.7^{+1.1}_{-1.0}$	$1.3^{+0.8}_{-0.7}$
II	$6991.7^{+121.6}_{-87.8} - i176.1^{+291.6}_{-171.1}$	$29.3^{+9.8}_{-8.8}$	$38.5^{+8.7}_{-9.6}$	$20.4^{+17.9}_{-9.2}$

    - A near-threshold bound state is found in these two cases, referred to as $X(6200)$.
    - A peak located around 6.9 GeV appears due to the $J/\psi\psi(3770)$ threshold effects.

Fits with four coupled-channels

- Description of LHCb data:



- consider four coupled-channels $J/\psi J/\psi$, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$, $\psi(2S)\psi(2S)$.
- perform two different kinds of fits 0^{++} and 2^{++} and extend the energy range to [6.2 GeV,7.6 GeV] ;
- Fit-C (0^{++}) and Fit-D (2^{++}) behave differently. A peak around 7.2 GeV is observed in 0^{++} case.

Fits with four coupled-channels

- Pole positions and residues:

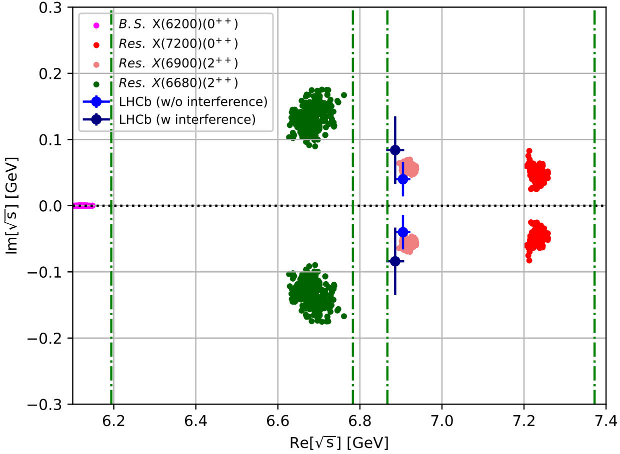
RS	Position $\sqrt{s_{\text{pole}}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$	$\psi(2S)\psi(2S)$
I (0^{++})	$6124.8^{+23.9}_{-121.8}$	$24.6^{+7.5}_{-2.3}$	$21.0^{+21.2}_{-6.7}$	$1.1^{+1.1}_{-1.1}$	$2.7^{+18.2}_{-2.6}$
VIII (0^{++})	$7234.3^{+24.2}_{-28.5} - i45.1^{+37.8}_{-20.0}$	$5.6^{+2.1}_{-1.8}$	$6.2^{+1.1}_{-1.1}$	$0.9^{+0.3}_{-0.3}$	$\boxed{37.0^{+2.6}_{-2.3}}$
II (2^{++})	$6680.3^{+80.9}_{-53.0} - i136.1^{+39.3}_{-46.4}$	$14.9^{+2.1}_{-2.4}$	$26.5^{+1.8}_{-2.8}$	$7.8^{+5.9}_{-1.7}$	$39.0^{+4.9}_{-5.2}$
VIII (2^{++})	$6919.8^{+17.2}_{-23.7} - i58.8^{+10.9}_{-12.2}$	$5.7^{+1.6}_{-1.3}$	$9.9^{+1.1}_{-1.8}$	$2.9^{+1.2}_{-0.5}$	$\boxed{52.7^{+1.7}_{-1.1}}$

- Fit-C:
  (1) A narrow resonance $X(7200)$;
  (2) A bound state $X(6200)$;
  (3) Their J^{PC} numbers are 0^{++}.

- Fit-D:
  (1) A narrow resonance $X(6900)$;
  (2) A broad resonance $X(6680)$;
  (3) Their J^{PC} numbers are 2^{++}.

Fits with four coupled-channels

- The locations of poles:



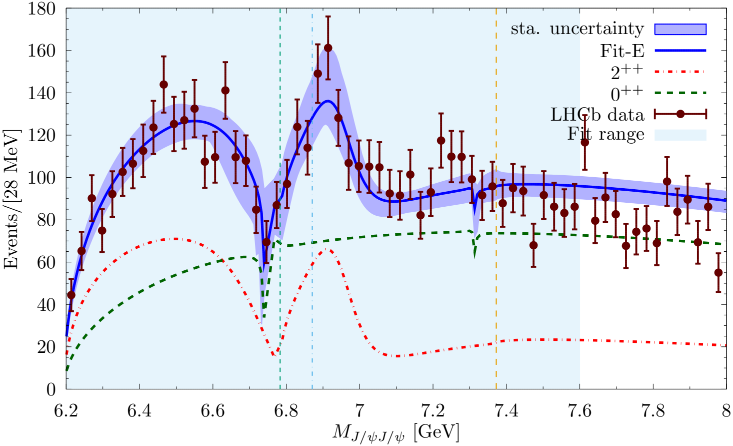
- Green $\rightarrow$ the 2^{++} $X(6680)$ resonance
- Pink $\rightarrow$ the 2^{++} $X(6900)$ resonance
- Red $\rightarrow$ the 0^{++} $X(7200)$ resonance
- Magenta $\rightarrow$ the 0^{++} $X(6200)$ bound state

LHCb results (the blue error bar):

- $\mathrm{M} = 6905 \pm 11 \pm 7$ MeV
  $\Gamma = 80 \pm 19 \pm 33$ MeV
- $\mathrm{M} = 6886 \pm 11 \pm 11$ MeV
  $\Gamma = 168 \pm 33 \pm 69$ MeV

Combined fit

- Description of LHCb data:



- three coupled-channels $J/\psi J/\psi$, $J/\psi\psi(2S)$, $\psi(2S)\psi(2S)$
- To judge which partial-wave is dominant $\rightarrow$ both are sizeable

Combined fit

- Pole positions and residues

RS (J^{PC})	Position $\sqrt{s_{\text{pole}}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$\psi(2S)\psi(2S)$
I (0^{++})	$5979.6^{+3.6}_{-9.1}$	$36.4^{+0.6}_{-0.2}$	$6.1^{+0.6}_{-1.7}$	$2.9^{+0.3}_{-0.8}$
II (2^{++})	$6769.7^{+163.1}_{-140.2} - i204.5^{+46.5}_{-62.3}$	$17.5^{+0.7}_{-1.5}$	$22.4^{+6.7}_{-6.9}$	$44.8^{+4.1}_{-4.7}$
IV (2^{++})	$6951.1^{+36.1}_{-48.2} - i89.1^{+15.4}_{-17.8}$	$7.2^{+1.7}_{-1.5}$	$11.2^{+1.8}_{-2.0}$	$\boxed{50.8^{+3.0}_{-1.8}}$

- The $X(6200)$, $X(6680)$ and $X(6900)$ still exist.
- Their pole locations are shifted, due to the absence of the $J/\psi\psi(3770)$ channel.
- The $X(7200)$ disappears, however, there still exist an enhancement around 7.2 GeV.
  $\rightarrow$ The $J/\psi\psi(3770)$ channel is important for the existence of the $X(7200)$ state.

Summary and Outlook

- Exploring all possible states in the range $[6.2\ \mathrm{GeV}, 7.6\ \mathrm{GeV}]$ by means of PWA.
- Four states are found

  - $X(6200)$: a bound state with $J^{PC} = 0^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6124.8^{+23.9}_{-121.8}\ \mathrm{MeV}$
  - $X(6680)$: a broad resonant state with $J^{PC} = 2^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6680.3^{+80.9}_{-53.0} - i136.1^{+39.3}_{-46.4}\ \mathrm{MeV}$
  - $X(6900)$: a narrow resonant state with $J^{PC} = 2^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6919.8^{+17.2}_{-23.7} - i58.8^{+10.9}_{-12.2}\ \mathrm{MeV}$
  - $X(7200)$: a narrow resonant state with $J^{PC} = 0^{++}$, $\sqrt{s_{\mathrm{pole}}} = 7234.3^{+24.2}_{-28.5} - i45.1^{+37.8}_{-20.0}\ \mathrm{MeV}$

  - The above results need to be confirmed by more precise experimental data;
  - Study the structures of these states in future.

Many thanks for your patience!