Study of Tetraquark Spectroscopy in Group Theory and Quark Model

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Outline

- Introduction
- Estimation of the low-lying tetraquark mass spectrum
  - Construct tetraquark wave functions
  - Fix parameters
  - Calculate tetraquark mass spectrum
- Summary

These charged charmoniumlike states go beyond conventional $c\bar{c}$ meson picture and could be tetraquark systems $u\bar{d}c\bar{c}$ due to carrying one charge.

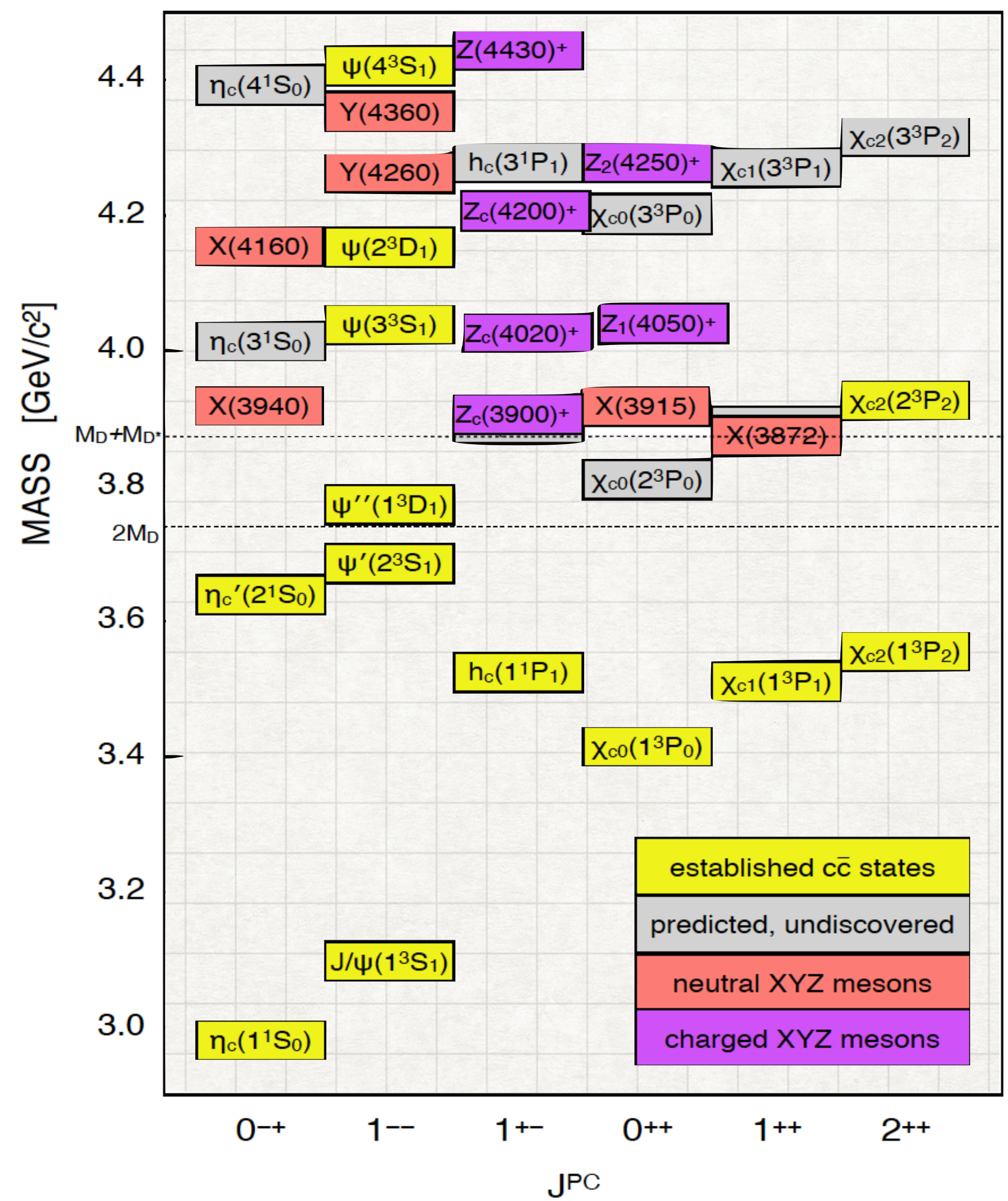


FIG 1: Charmonium meson spectrums include some charmonium-like XYZ states.[1]

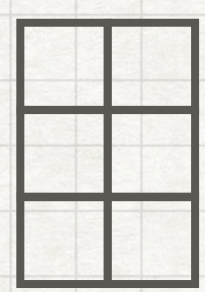
[1]Olsen, S. L. (2015). XYZ meson spectroscopy. In Proceedings, 53rd International Winter Meeting on Nuclear Physics (Bormio 2015): Bormio, Italy, January 26-30, 2015.

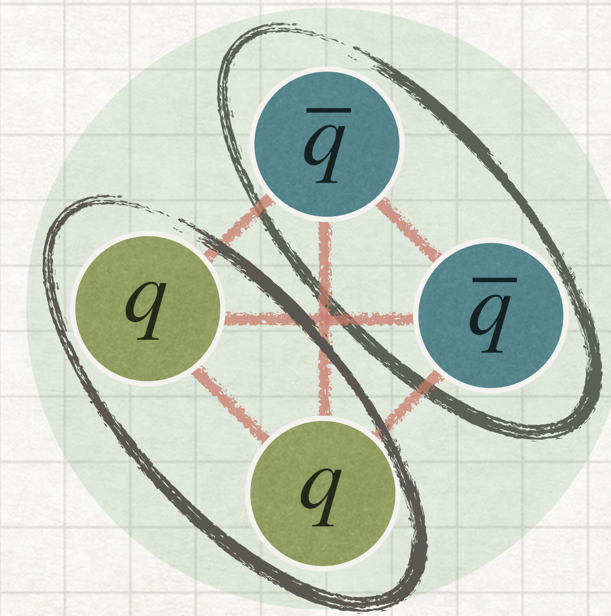
- Tetraquark are states of two quarks and two antiquarks. (eg. $qc\bar{q}\bar{c}$, $cc\bar{c}\bar{c}$ and $qq\bar{q}\bar{q}$)

- The construction of tetraquark wave function is guided by:
- The tetraquark wave function should be a color singlet. (for $qc\bar{q}\bar{c}$, $cc\bar{c}\bar{c}$ and $qq\bar{q}\bar{q}$)
- The tetraquark wave function should be antisymmetric under any permutation between identical quarks. (for $cc\bar{c}\bar{c}$ and $qq\bar{q}\bar{q}$)

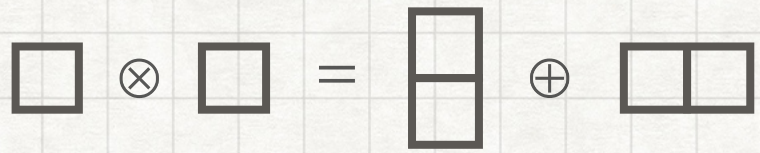
- It demands that the color part of tetra-quark wave function must be [222] singlet.

$$\psi^{c}_{[222]} =$$



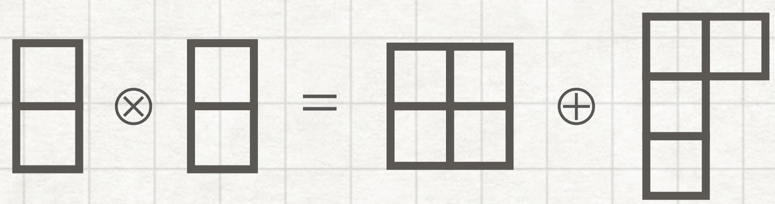


- The color part for two quarks in tetraquark states is



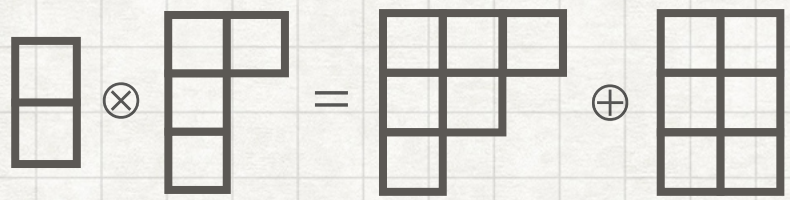
$$3 \otimes 3 = \bar{3} \oplus 6$$

- The color part for two anti-quarks in tetraquark states is

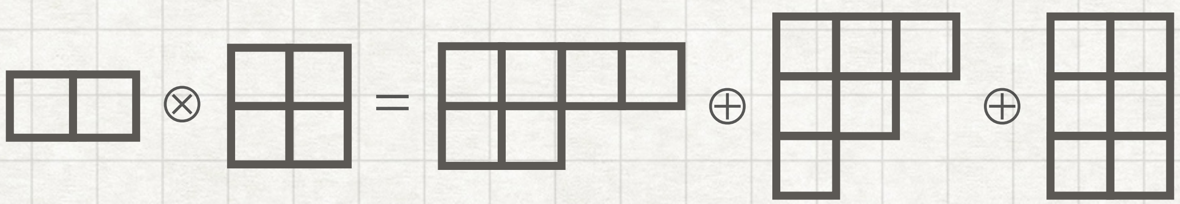


$$3 \otimes 3 = \bar{6} \oplus 3$$

- The color wave function should be a color singlet.



$$\bar{3} \otimes 3 = 8 \oplus 1$$



$$6 \otimes \bar{6} = 27 \oplus 8 \oplus 1$$

Color wave function

antitriplet-triplet state: $\psi^{c}_{\bar{3}-3} = \frac{1}{3}[\frac{1}{2}(rg - gr)(\bar{r}\bar{g} - \bar{g}\bar{r}) + \frac{1}{2}(br - rb)(\bar{b}\bar{r} - \bar{r}\bar{b}) + \frac{1}{2}(gb - bg)(\bar{g}\bar{b} - \bar{b}\bar{g})]$

sextet-antisextet state: $\psi^{c}_{6-\bar{6}} = \frac{1}{\sqrt{6}}[rr\bar{r}\bar{r} + gg\bar{g}\bar{g} + bb\bar{b}\bar{b} + \frac{1}{2}(rg + gr)(\bar{r}\bar{g} + \bar{g}\bar{r}) + \frac{1}{2}(rb + br)(\bar{r}\bar{b} + \bar{b}\bar{r})$

$+\frac{1}{2}(gb + bg)(\bar{g}\bar{b} + \bar{b}\bar{g})]$

- The total wave function should be antisymmetric for qq or cc cluster.
- The total wave function for qq or cc can be represented as

$$\psi_{total} = \psi_{spatial}\psi_{spin}\psi_{flavor}\psi_{color}$$

- Listed in tables below are all the possible configurations of spatial-spin-flavor part of the qq and cc cluster.

$\psi^c_{[2]}(qq) = \Box\Box \Rightarrow$	$\psi^c_{[2]}\psi^{osf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^{sf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^s_{[11]}\psi^f_{[2]}$
			$\psi^c_{[2]}\psi^o_{[2]}\psi^s_{[2]}\psi^f_{[11]}$
$\psi^c_{[11]}(qq) = \begin{smallmatrix}\Box\\\Box\end{smallmatrix} \Rightarrow$	$\psi^c_{[11]}\psi^{osf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^{sf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^s_{[11]}\psi^f_{[11]}$
			$\psi^c_{[11]}\psi^o_{[2]}\psi^s_{[2]}\psi^f_{[2]}$

$\psi^c_{[2]}(cc) = \Box\Box \Rightarrow$	$\psi^c_{[2]}\psi^{osf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^{sf}_{[11]}$	$\psi^c_{[2]}\psi^o_{[2]}\psi^s_{[11]}\psi^f_{[2]}$
$\psi^c_{[11]}(cc) = \begin{smallmatrix}\Box\\\Box\end{smallmatrix} \Rightarrow$	$\psi^c_{[11]}\psi^{osf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^{sf}_{[2]}$	$\psi^c_{[11]}\psi^o_{[2]}\psi^s_{[2]}\psi^f_{[2]}$

Jacobi coordinate for tetraquark

$$\sigma_1 = \frac{1}{\sqrt{2}}(r_1 - r_3) \qquad \sigma_2 = \frac{1}{\sqrt{2}}(r_2 - r_4) \qquad \lambda = \frac{m_1 r_1 + m_3 r_3}{m_1 + m_3} - \frac{m_2 r_2 + m_4 r_4}{m_2 + m_4}$$

Reduced mass:

$$m_{\sigma_1} = \frac{2 m_1 m_3}{m_1 + m_3} \qquad m_{\sigma_2} = \frac{2 m_2 m_4}{m_2 + m_4} \qquad m_\lambda = \frac{(m_1 + m_3)(m_2 + m_4)}{m_1 + m_2 + m_3 + m_4}$$

for $qc\bar{q}\bar{c}$: $\quad m_1 = m_3 = m_u \qquad m_2 = m_4 = m_c$

$$\sigma_1 = \frac{1}{\sqrt{2}}(r_1 - r_3) \qquad \sigma_2 = \frac{1}{\sqrt{2}}(r_2 - r_4) \qquad \lambda = \frac{1}{2}(r_1 + r_3 - r_2 - r_4)$$

$$m_{\sigma_1} = m_u \qquad m_{\sigma_2} = m_c \qquad m_\lambda = \frac{2 m_u m_c}{m_u + m_c}$$

[2]E. Santopinto and G. Galatà. Spectroscopy of tetraquark states. Phys. Rev. C75, 045206(2007).

Spatial wave function

- We construct the complete bases by using the harmonic oscillator wave function.
- The total spatial wave function of tetraquark, coupling among the σ_1, σ_2 and λ harmonic oscillator wave functions, may take the general form,

$$\psi_{NLM} = \sum_{\{n_i,l_i\}} A(n_{\sigma_1}, n_{\sigma_2}, n_\lambda, l_{\sigma_1}, l_{\sigma_2}, l_\lambda) \times \psi_{n_{\sigma_1} l_{\sigma_1}}(\vec{\sigma}_1) \otimes \psi_{n_{\sigma_2} l_{\sigma_2}}(\vec{\sigma}_2) \otimes \psi_{n_\lambda l_\lambda}(\vec{\lambda})$$

$$= \sum_{\{n_i,l_i,m_i\}} C_{n_{\sigma_1},l_{\sigma_1},m_{\sigma_1},n_{\sigma_2},l_{\sigma_2},m_{\sigma_2},n_\lambda,l_\lambda,m_\lambda} \times \psi_{n_{\sigma_1} l_{\sigma_1} m_{\sigma_1}}(\vec{\sigma}_1) \psi_{n_{\sigma_2} l_{\sigma_2} m_{\sigma_2}}(\vec{\sigma}_2) \psi_{n_\lambda l_\lambda m_\lambda}(\vec{\lambda})$$

- The complete bases of the tetraquarks are listed in table

$NLM = 000$		$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	
$NLM = 200$	$\Psi_{100}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{100}(\lambda)$
$NLM = 400$	$\Psi_{200}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{200}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{200}(\lambda)$
	$\Psi_{100}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{000}(\lambda)$	$\Psi_{100}(\sigma_1)\Psi_{000}(\sigma_2)\Psi_{100}(\lambda)$	$\Psi_{000}(\sigma_1)\Psi_{100}(\sigma_2)\Psi_{100}(\lambda)$

The non-relativistic Hamiltonian we study multiquark system reads:

$$H = \sum_{k=1}^{N} (\frac{1}{2}M_k^{ave} + \frac{p_k^2}{2m_k}) + \sum_{i<j}^{N} (-\frac{3}{16}\lambda_i^C \cdot \lambda_j^C)(A_{ij}r_{ij} - \frac{B_{ij}}{r_{ij}}) + H_{hyp}$$

The hyperfine interaction term takes the form:

$$H_{hyp} = -C_{ij} \sum_{i<j} \lambda_i^c \cdot \lambda_j^c \overrightarrow{\sigma_i} \cdot \overrightarrow{\sigma_j}$$

Solving the Schrödinger equation: $H|\psi_{total}> = E|\psi_{total}>$

3 mass-dependent coupling parameters are proposed:

$$A_{ij} = a + bm_{ij} \qquad B_{ij} = B_0\sqrt{\frac{1}{m_{ij}}} \qquad C_{ij} = C_0\sqrt{\frac{1}{m_{ij}}}$$

$$m_k = m_{ij} = 2\frac{m_i m_j}{m_i + m_j}$$

The non-relativistic Hamiltonian we study the meson system reads:

$$H = M_{ave} + \frac{P^2}{2m_r} + (-\frac{3}{16})\lambda_1\lambda_2(Ar - \frac{B}{r}) + H_{hyp} \qquad H_{hyp} = -C_{ij}\lambda_i\lambda_j\sigma_i\sigma_j$$

$$A_{ij} = a + bm_{ij} \qquad B_{ij} = B_0\sqrt{\frac{1}{m_{ij}}} \qquad C_{ij} = C_0\sqrt{\frac{1}{m_{ij}}} \qquad m_{ij} = 2\frac{m_i m_j}{m_i + m_j}$$

4 model coupling constants and 4 constituent quark masses are fitted:

$$a = 67413(\mathrm{MeV}^2) \qquad b = 35(\mathrm{MeV}) \qquad B_0 = 31.6635(\mathrm{MeV}^{1/2}) \qquad C_0 = -188.765(\mathrm{MeV}^{3/2})$$

$$m_u = m_d = 420\mathrm{MeV} \qquad m_s = 550\mathrm{MeV} \qquad m_c = 1270\mathrm{MeV} \qquad m_b = 4180\mathrm{MeV}$$

Meson (MeV)	Mave	Mij	Mv(1S)−Mps(1S)		pseudo meson						vector meson								
					n=0, s=0			n=1, s=0			n=0, s=1			n=1, s=1			n=2, s=1		
			Exp.	Ours	Exp.	Ours	%	Exp.	Ours	%	Exp.	Ours	%	Exp.	Ours	%	Exp.	Ours	%
$b\bar{b}$	9445	4180	61	62	9399	9408	-0.1	9999	10008	-0.1	9460	9470	-0.1	10023	10070	-0.5	–	–	–
$c\bar{c}$	**3069**	**1270**	**113**	**113**	**2984**	**2981**	**0.1**	**3638**	**3565**	**2**	**3097**	**3094**	**0.1**	**3686**	**3678**	**0.2**	**4040**	**4053**	**-0.3**
$B_s(s\bar{b})$	5403	972	48	129	5367	5310	1.1	–	–	–	5415	5439	-0.4	–	–	–	–	–	–
$B(u\bar{b})$	5314	763	46	146	5279	5221	1.1	–	–	–	5325	5367	-0.8	–	–	–	–	–	–
$D_s(c\bar{s})$	2076	767	144	145	1968	1981	-0.7	–	–	–	2112	2127	-0.7	2708	2733	-0.9	–	–	–
$D(c\bar{u})$	**1973**	**631**	**142**	**160**	**1870**	**1878**	**-0.4**	–	–	–	**2010**	**2038**	**-1.4**	–	–	–	–	–	–
$s\bar{s}$	952	550	–	–	–	–	–	–	–	–	1020	1029	-0.9	1680	1660	1.2	–	–	–
$q\bar{q}$	**675**	**420**	–	–	–	–	–	–	–	–	**770**	**779**	**-1.2**	**1450**	**1436**	**1**	–	–	–

[3]P. Zyla. et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

$qc\bar{q}\bar{c}$ configurations	$\psi^{S=0}_{(0_s\otimes 0_s)}$ $0^+0^{++}/1^-0^{++}$		$\psi^{S=0}_{(1_s\otimes 1_s)}$ $0^+0^{++}/1^-0^{++}$		$\psi^{S=1}_{(1_s\otimes 0_s)}$ $0^-1^{+-}/1^+1^{+-}$		$\psi^{S=1}_{(1_s\otimes 1_s)}$ $0^-1^{+-}/1^+1^{+-}$		$\psi^{S=2}_{(1_s\otimes 1_s)}$ $0^+2^{++}/1^-2^{++}$	
	Ours	**Data**	**Ours**	**Data**	**Ours**	**Data**	**Ours**	**Data**	**Ours**	**Data**
$\psi^c_{6-\bar{6}}(1S)$	**4202**	**Z(4250)**	**3925**	**X(3915)**	**4162**	**Z(4200)**	**4024**	**Z(4020)** **Z(4055)**	**4221**	
$\psi^c_{6-\bar{6}}(2S)$	**4566**		**4289**	**X(4350)**	**4526**	**Z(4430)**	**4388**		**4584**	
$\psi^c_{\bar{3}-3}(1S)$	**4033**	**Z(4050)**	**4114**	**Z(4100)**	**4113**	**X(4160)**	**4154**	**X(4160)**	**4233**	
$\psi^c_{\bar{3}-3}(2S)$	**4434**		**4516**		**4514**		**4555**		**4634**	

Table I. Masses, widths, J^{PC}, and processes of X and Z states in the $c\bar{c}$ region.

States	name in PDG	M(MeV)	Γ	J^{PC}	Process	Experment
$X(3860)$	$\chi_{c0}(3860)$	3862^{+26+40}_{-32-13}	$201^{+154+88}_{-67-82}$	0^{++}	$e^+e^- \to J/\psi(D\bar{D})$	Belle
$X(3915)$	$X(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{++}$	$B \to K(J/\psi\omega)$	Belle
$X(3940)$	$X(3940)$	$3942^{+7}_{-6} \pm 6$	$37^{+26}_{-15} \pm 18$	$?^{??}$	$e^+e^- \to J/\psi(D\bar{D}^*)$	Belle
$X(4160)$	$X(4160)$	$4156^{+25}_{-20} \pm 15$	$139^{+111}_{-61} \pm 21$	$?^{??}$	$e^+e^- \to J/\psi(D^*\bar{D}^*)$	Belle
$X(4350)$	$X(4350)$	$4350.6^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	$?^{?+}$	$\gamma\gamma \to \phi J/\psi$	Belle
$X(4500)$	$\chi_{c0}(4500)$	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	0^{++}	$B^+ \to (J/\psi\phi)K^+$	LHCb
$X(4700)$	$\chi_{c0}(4700)$	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	0^{++}	$B^+ \to (J/\psi\phi)K^+$	LHCb
$Z_c(3900)$	$Z_c(3900)$	3888.4 ± 2.5	28.3 ± 2.5	1^{+-}	$e^+e^- \to (D\bar{D}^*)^+\pi^-$	BESIII
$Z_c(4020)$	$X(4020)^\pm$	4024.1 ± 1.9	13 ± 5	$?^{?-}$	$e^+e^- \to \pi^-(\pi^+h_c)$	BESIII
$Z_c(4050)$	$X(4050)^\pm$	4051^{+24}_{-40}	82^{+50}_{-28}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle
$Z_c(4055)$	$X(4055)^\pm$	4054 ± 3.2	45 ± 13	$?^{?-}$	$e^+e^- \to \pi^-(\pi^+\psi(2S))$	Belle
$Z_c(4100)$	$X(4100)^\pm$	4096 ± 28	152^{+80}_{-70}	$0^{++}/1^{-+}$	$B^0 \to K^+(\pi^-\eta_c)$	LHCb
$Z_c(4200)$	$Z_c(4200)$	4196^{+35}_{-32}	370^{+100}_{-150}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+J/\psi)$	Belle
$Z_c(4250)$	$X(4250)^\pm$	4248^{+190}_{-50}	177^{+320}_{-70}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle
$Z_c(4430)$	$Z_c(4430)$	4478^{+15}_{-18}	181 ± 31	1^{+-}	$\bar{B}^0 \to K^-(\pi^+J/\psi)$	Belle

[3]P. Zyla. et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

$qq\bar{q}\bar{q}$ configurations	$\psi^{S=0}_{(0_s\otimes 0_s)}$ 0^{++}		$\psi^{S=0}_{(1_s\otimes 1_s)}$ 0^{++}		$\psi^{S=1}_{(1_s\otimes 0_s)}$ 0^{++}		$\psi^{S=1}_{(1_s\otimes 1_s)}$ 0^{++}		$\psi^{S=2}_{(1_s\otimes 1_s)}$ 0^{++}	
	Ours	**Data**	**Ours**	**Data**	**Ours**	**Data**	**Ours**	**Data**	**Ours**	**Data**
$\psi^c_{6-\bar{6}}(1S)$	**1890**	–	**1546**	**f0(1500)**	**1841**	**b1(1960)**	**1669**	**h1(1595)**	**1914**	**X2(1980)**
$\psi^c_{6-\bar{6}}(2S)$	**2283**	**f0(2200)**	**1939**	**f0(2020)**	**2234**	**b1(2240)**	**2062**	**h1(1965)**	**2308**	**f2(2300)**
$\psi^c_{\bar{3}-3}(1S)$	**1709**	**f0(1710)**	**1807**	**f0(1710)**	**1807**	**b1(1960)**	**1856**	**b1(1960)**	**1954**	**X2(1930)**
$\psi^c_{\bar{3}-3}(2S)$	**2189**	**f0(2100)**	**2287**	**f0(2200)**	**2287**	**b1(2240)**	**2336**	**b1(2240)**	**2434**	**f2(2340)**

Table I. Masses, widths, J^{PC} and processes of the light tetraquark candidates in the light-unflavored meson region.

States	M(MeV)	Γ	J^{PC}	Process	Experiment
$f_0(1500)$	1473 ± 5	108 ± 9	0^{++}	$p\bar{p} \to (\eta\eta)\pi$	E835[14]
$f_0(2020)$	2037 ± 8	296 ± 17	0^{++}	$p\bar{p} \to (\eta\eta)\pi$	E835[14]
$f_0(1710)$	$1759 \pm 6^{+14}_{-25}$	$172 \pm 10^{+32}_{-16}$	0^{++}	$J/\psi \to \gamma(\eta\eta)$	BESIII[15]
$f_0(2100)$	$2081 \pm 13^{+24}_{-36}$	273^{+27+70}_{-24-23}	0^{++}	$J/\psi \to \gamma(\eta\eta)$	BESIII[15]
$f_0(1710)$	$1760 \pm 15^{+15}_{-10}$	$125 \pm 25^{+10}_{-15}$	0^{++}	$\psi(2s) \to \gamma\pi^+\pi^-(K^+K^-)$	BES[16]
$f_0(2200)$	$2170 \pm 20^{+10}_{-15}$	$220 \pm 60^{+40}_{-45}$	0^{++}	$\psi(2s) \to \gamma\pi^+\pi^-(K^+K^-)$	BES[16]
$h_1(1595)$	$1594 \pm 15^{+10}_{-60}$	$384 \pm 60^{+70}_{-100}$	1^{+-}	$\pi^-p \to (\omega\eta)n$	BNL-E852[17]
$h_1(1965)$	1965 ± 45	345 ± 75	1^{+-}	$p\bar{p} \to \omega\eta, \omega\pi^0\pi^0$	SPEC[18]
$b_1(1960)$	1960 ± 35	230 ± 50	1^{+-}	$p\bar{p} \to \omega\pi^0, \omega\eta\pi^0, \pi^+\pi^-$	SPEC[19]
$b_1(2240)$	2240 ± 35	320 ± 85	1^{+-}	$p\bar{p} \to \omega\pi^0, \omega\eta\pi^0, \pi^+\pi^-$	SPEC[19]
$X_2(1930)$	1930 ± 25	450 ± 50	2^{++}	$\pi^-p \to (\eta\eta)n$	GAMS[20]
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	2^{++}	$J/\psi \to \gamma(\eta\eta)$	BESIII[15]
$X_2(1980)$	$1980 \pm 2 \pm 14$	$297 \pm 12 \pm 6$	2^{++}	$\gamma\gamma \to (K^+K^-)$	BELL[21]
$f_2(2300)$	$2327 \pm 9 \pm 6$	$275 \pm 36 \pm 20$	2^{++}	$\gamma\gamma \to (K^+K^-)$	BELL[21]

[3]P. Zyla. et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

$cc\bar{c}\bar{c}$ configurations	$\psi^{S=0}_{(0_s\otimes 0_s)}$ $0^+0^{++}/1^-0^{++}$			$\psi^{S=0}_{(1_s\otimes 1_s)}$ $0^+0^{++}/1^-0^{++}$			$\psi^{S=1}_{(1_s\otimes 1_s)}$ $0^-1^{+-}/1^+1^{+-}$			$\psi^{S=2}_{(1_s\otimes 1_s)}$ $0^+2^{++}/1^-2^{++}$		
	M(1S)	M(2S)	M(3S)	M(1S)	M(2S)	M(3S)	M(1S)	M(2S)	M(3S)	M(1S)	M(2S)	M(3S)
$\psi^c_{6-\bar{6}}$	6514	6840	7098									
$\psi^c_{\bar{3}-3}$				6466	6883	7225	6494	6911	7253	6551	6968	7310

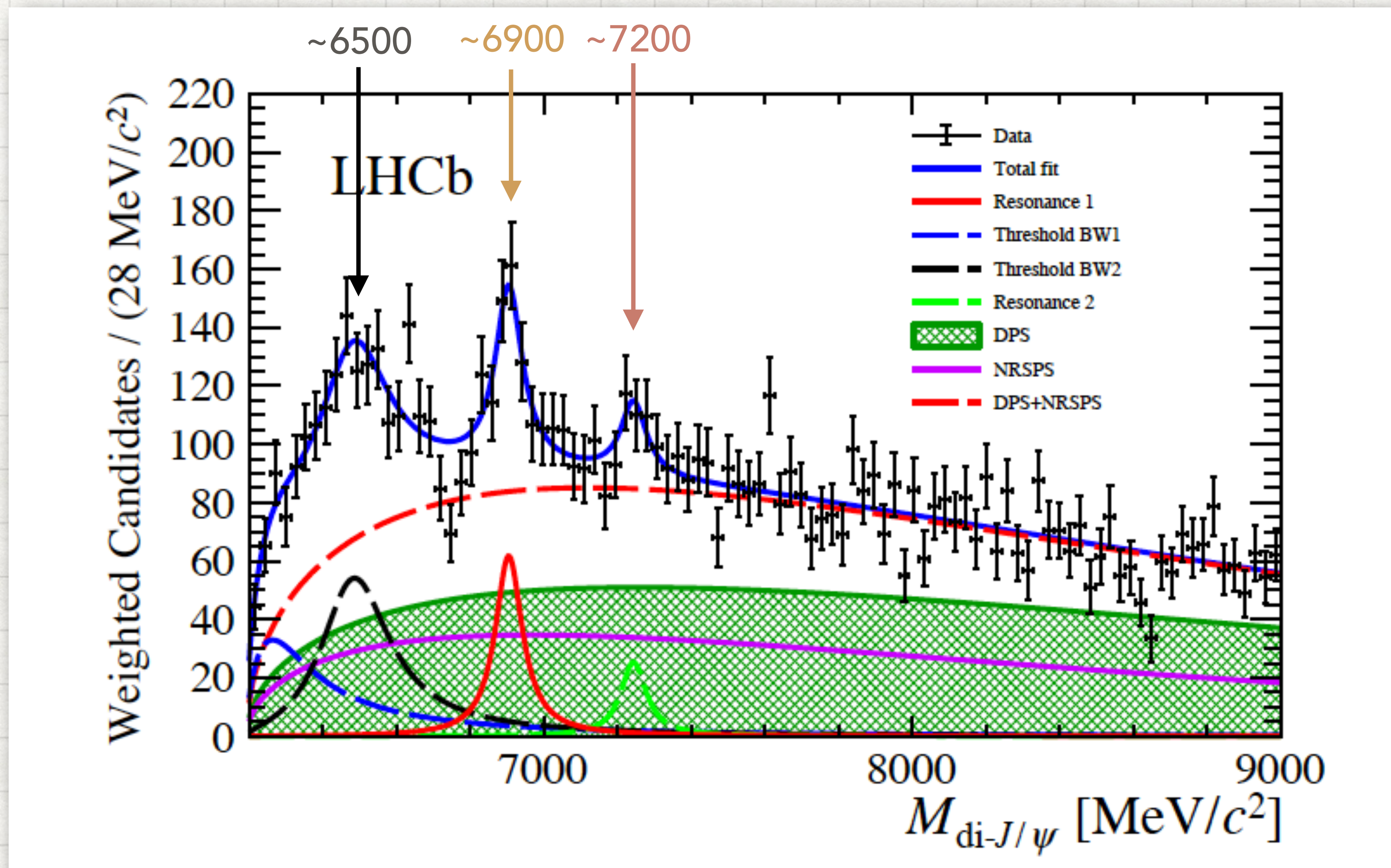


FIG. 2. Invariant mass spectra of weighted di-J/ψ candidates. Adapted from figure 7 in [4].

[4] R. Aaij and et al. (LHCb Collaboration), Sci. Bull. 65, 1983 (2020), arXiv:2006.16957 [hep-ex].

- All model parameters were predetermined by comparing the theoretical and experimental masses of light, charmed and bottom mesons.

- The tetraquark wave functions are constructed. We derived color wave function in the Yamanouchi basis framework with permutation group.

- We have evaluated the masses of ground and first radial excited $qc\bar{q}\bar{c}$ and $qq\bar{q}\bar{q}$ tetraquark states and of ground and first and second radial excited states of the $cc\bar{c}\bar{c}$ tetraquark states.

- We have made 2 tentative matchings between the predicted ground and first radial excited $qc\bar{q}\bar{c}$, and $qq\bar{q}\bar{q}$ tetraquark states and the experimental data.

- The work suggests that the $X(6900)$ observed by LHCb is likely the first radial excited $cc\bar{c}\bar{c}$ tetraquark state, with $J^{PC} = 1^{+-}$, in the $\bar{3}_c \otimes 3_c$ configuration.

Thank you for your attention.

- A charged state was observed by BESIII in 2017 with a mass around 4030 MeV (Phys. Rev. D 96, 032004 (2017)), which has similar mass with $Z_c(4020)^+$. But this charged state was observed in the same process of $Z_c(4055)^+$ which was observed by Belle in 2015 (Phys. Rev. D 91, 112007 (2015)).

- $Z_c(4100)^-$ was observed as a charged resonant state by LHCb in 2018. (Eur. Phys. J. C 78, 1019 (2018))

- Last year, the LHCb Collaboration presented evidence for the observation of at least one resonance in the J/ψ-pair spectrum at about 6900 MeV. (Sci. Bull. 2020, 65 (2020))

Spin wave function

- For $qc\bar{q}\bar{c}$ and $qq\bar{q}\bar{q}$, the possible spin combinations are $\left[\psi^{qc}_{[s=1]} \otimes \psi^{\bar{q}\bar{c}}_{[s=1]}\right]_{S=0,1,2}$, $\psi^{qc}_{[s=1]} \otimes \psi^{\bar{q}\bar{c}}_{[s=0]}$ and $\psi^{qc}_{[s=0]} \otimes \psi^{\bar{q}\bar{c}}_{[s=0]}$. The explicit spin wave functions $\psi^{S(qc\bar{q}\bar{c})}_{(S(qc)\otimes S(\bar{q}\bar{c}))}$ of $qc\bar{q}\bar{c}$ and $qq\bar{q}\bar{q}$ are listed:

$$\psi^{S=2}_{(1\otimes1)} = \uparrow\uparrow\bar{\uparrow}\bar{\uparrow}, \quad \psi^{S=1}_{(1\otimes1)} = \frac{1}{2}(\uparrow\uparrow\bar{\uparrow}\bar{\downarrow} + \uparrow\uparrow\bar{\downarrow}\bar{\uparrow} - \uparrow\downarrow\bar{\uparrow}\bar{\uparrow} - \downarrow\uparrow\bar{\uparrow}\bar{\uparrow}), \quad \psi^{S=1}_{(1\otimes0)} = \frac{1}{\sqrt{2}}(\uparrow\uparrow\bar{\uparrow}\bar{\downarrow} - \uparrow\uparrow\bar{\downarrow}\bar{\uparrow}),$$

$$\psi^{S=0}_{(1\otimes1)} = \frac{1}{\sqrt{3}}[\uparrow\uparrow\bar{\downarrow}\bar{\downarrow} - \frac{1}{2}(\uparrow\downarrow\bar{\uparrow}\bar{\downarrow} + \uparrow\downarrow\bar{\downarrow}\bar{\uparrow} + \downarrow\uparrow\bar{\uparrow}\bar{\downarrow} + \downarrow\uparrow\bar{\downarrow}\bar{\uparrow}) + \downarrow\downarrow\bar{\uparrow}\bar{\uparrow}],$$

$$\psi^{S=0}_{(0\otimes0)} = \frac{1}{2}(\uparrow\downarrow\bar{\uparrow}\bar{\downarrow} - \uparrow\downarrow\bar{\downarrow}\bar{\uparrow} - \downarrow\uparrow\bar{\uparrow}\bar{\downarrow} + \downarrow\uparrow\bar{\downarrow}\bar{\uparrow})$$

- For $cc\bar{c}\bar{c}$, the explicit spin wave functions $\psi^{S(qc\bar{q}\bar{c})}_{(S(qc)\otimes S(\bar{q}\bar{c}))}$ of $[2](c_1c_2) \otimes [22](\bar{c}_3\bar{c}_4)$ configuration is listed

$$\psi^{S=0}_{(0\otimes0)} = \frac{1}{2}(\uparrow\downarrow\bar{\uparrow}\bar{\downarrow} - \uparrow\downarrow\bar{\downarrow}\bar{\uparrow} - \downarrow\uparrow\bar{\uparrow}\bar{\downarrow} + \downarrow\uparrow\bar{\downarrow}\bar{\uparrow})$$

- For $cc\bar{c}\bar{c}$, the explicit spin wave functions $\psi^{S(qc\bar{q}\bar{c})}_{(S(qc)\otimes S(\bar{q}\bar{c}))}$ of $[11](c_1c_2) \otimes [211](\bar{c}_3\bar{c}_4)$ configuration is listed

$$\psi^{S=2}_{(1\otimes1)} = \uparrow\uparrow\bar{\uparrow}\bar{\uparrow}, \quad \psi^{S=1}_{(1\otimes1)} = \frac{1}{2}(\uparrow\uparrow\bar{\uparrow}\bar{\downarrow} + \uparrow\uparrow\bar{\downarrow}\bar{\uparrow} - \uparrow\downarrow\bar{\uparrow}\bar{\uparrow} - \downarrow\uparrow\bar{\uparrow}\bar{\uparrow}),$$

$$\psi^{S=0}_{(1\otimes1)} = \frac{1}{\sqrt{3}}[\uparrow\uparrow\bar{\downarrow}\bar{\downarrow} - \frac{1}{2}(\uparrow\downarrow\bar{\uparrow}\bar{\downarrow} + \uparrow\downarrow\bar{\downarrow}\bar{\uparrow} + \downarrow\uparrow\bar{\uparrow}\bar{\downarrow} + \downarrow\uparrow\bar{\downarrow}\bar{\uparrow}) + \downarrow\downarrow\bar{\uparrow}\bar{\uparrow}],$$

Color matrix element

Color	$\lambda_1\lambda_2$	$\lambda_1\lambda_3$	$\lambda_1\lambda_4$	$\lambda_2\lambda_3$	$\lambda_2\lambda_4$	$\lambda_3\lambda_4$	$\sum \lambda_i\lambda_j$
$<\psi^c_{\bar{3}-3}\vert\hat{O}\vert\psi^c_{\bar{3}-3}>$	–8/3	–4/3	–4/3	–4/3	–4/3	–8/3	–32/3
$<\psi^c_{6-\bar{6}}\vert\hat{O}\vert\psi^c_{6-\bar{6}}>$	4/3	–10/3	–10/3	–10/3	–10/3	4/3	–32/3

Spin matrix element of $qc\bar{q}\bar{c}$ and $qq\bar{q}\bar{q}$

Spin	$\sigma_1\sigma_2$	$\sigma_1\sigma_3$	$\sigma_1\sigma_4$	$\sigma_2\sigma_3$	$\sigma_2\sigma_4$	$\sigma_3\sigma_4$	$\sum \sigma_i\sigma_j$
$<\psi^{S=0}_{0\otimes0}\vert\hat{O}\vert\psi^{S=0}_{0\otimes0}>$	–3	0	0	0	0	–3	–6
$<\psi^{S=0}_{1\otimes1}\vert\hat{O}\vert\psi^{S=0}_{1\otimes1}>$	1	–2	–2	–2	–2	1	–6
$<\psi^{S=1}_{1\otimes0}\vert\hat{O}\vert\psi^{S=1}_{1\otimes0}>$	1	0	0	0	0	–3	–2
$<\psi^{S=1}_{1\otimes1}\vert\hat{O}\vert\psi^{S=1}_{1\otimes1}>$	1	–1	–1	–1	–1	1	–2
$<\psi^{S=2}_{1\otimes1}\vert\hat{O}\vert\psi^{S=2}_{1\otimes1}>$	1	1	1	1	1	1	6

Spin matrix element of $cc\bar{c}\bar{c}$

Spin	$\sigma_1\sigma_2$	$\sigma_1\sigma_3$	$\sigma_1\sigma_4$	$\sigma_2\sigma_3$	$\sigma_2\sigma_4$	$\sigma_3\sigma_4$	$\sum \sigma_i\sigma_j$
$<\psi^{C,S=0}_{(6\otimes\bar{6})(0\otimes0)}\vert\hat{O}\vert\psi^{C,S=0}_{(6\otimes\bar{6})(0\otimes0)}>$	–3	0	0	0	0	–3	–6
$<\psi^{C,S=0}_{(\bar{3}\otimes3)(1\otimes1)}\vert\hat{O}\vert\psi^{C,S=0}_{(\bar{3}\otimes3)(1\otimes1)}>$	1	–2	–2	–2	–2	1	–6
$<\psi^{C,S=1}_{(\bar{3}\otimes3)(1\otimes1)}\vert\hat{O}\vert\psi^{C,S=1}_{(\bar{3}\otimes3)(1\otimes1)}>$	1	–1	–1	–1	–1	1	–2
$<\psi^{C,S=2}_{(\bar{3}\otimes3)(1\otimes1)}\vert\hat{O}\vert\psi^{C,S=2}_{(\bar{3}\otimes3)(1\otimes1)}>$	1	1	1	1	1	1	6