THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

PRISMA+

HADRON 2021

Meson-baryon scattering and $\Lambda(1405)$ in resummed baryon chiral perturbation theory

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In collaboration with:

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Eur. Phys. J. C80 (2020) 406; Eur. Phys. J. C81 (2021) 582

OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and outlook

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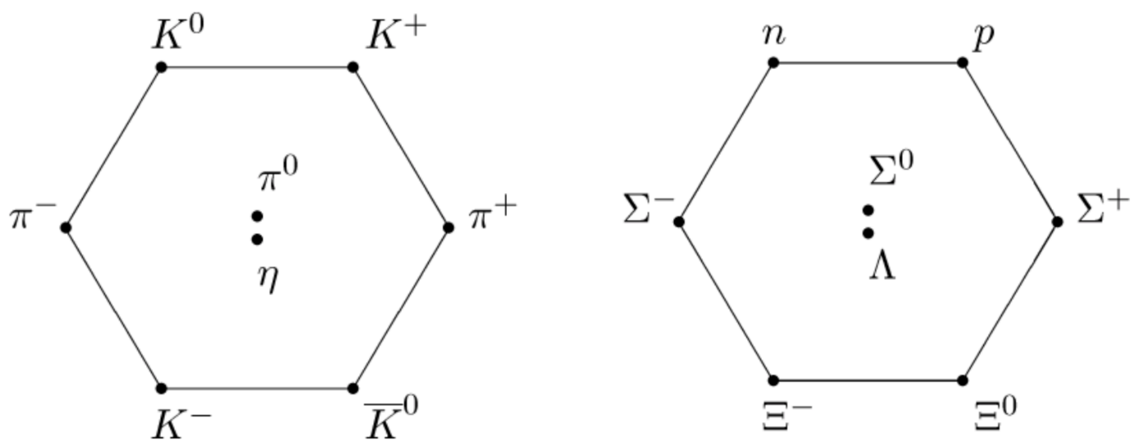
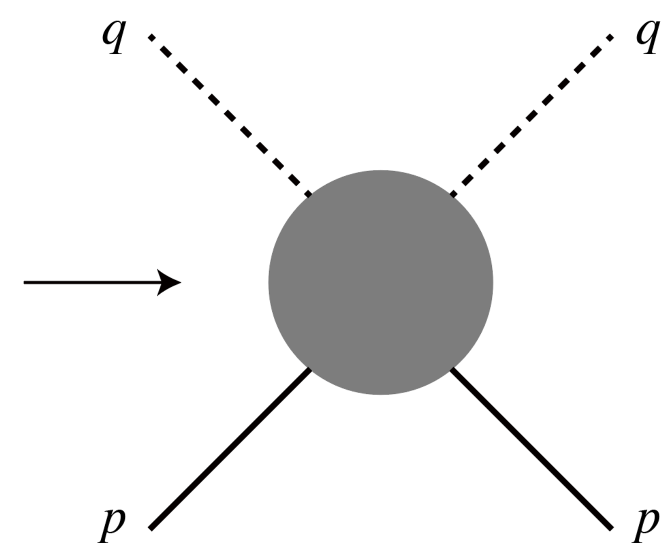
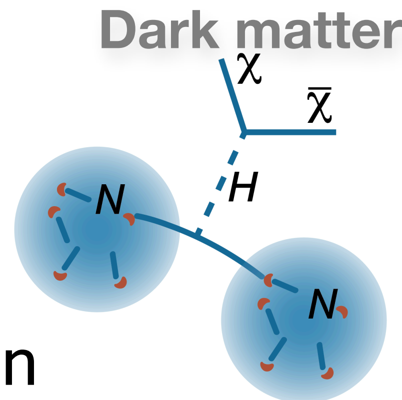
Meson−baryon scattering

- **Lowest lying** pseudoscalar meson-baryon scattering process





- Simple process but with interesting phenomena
  - $\pi N \to \pi N$ scattering: large amount experimental data
    - ✓ Extract sigma term $\sigma_{\pi N}$, key input of neutralino-nucleon cross section
      M. Hoferichter,et al.,PRL115,092301(2015) *J.R. Ellis, et al., PRD77(2008)065026*

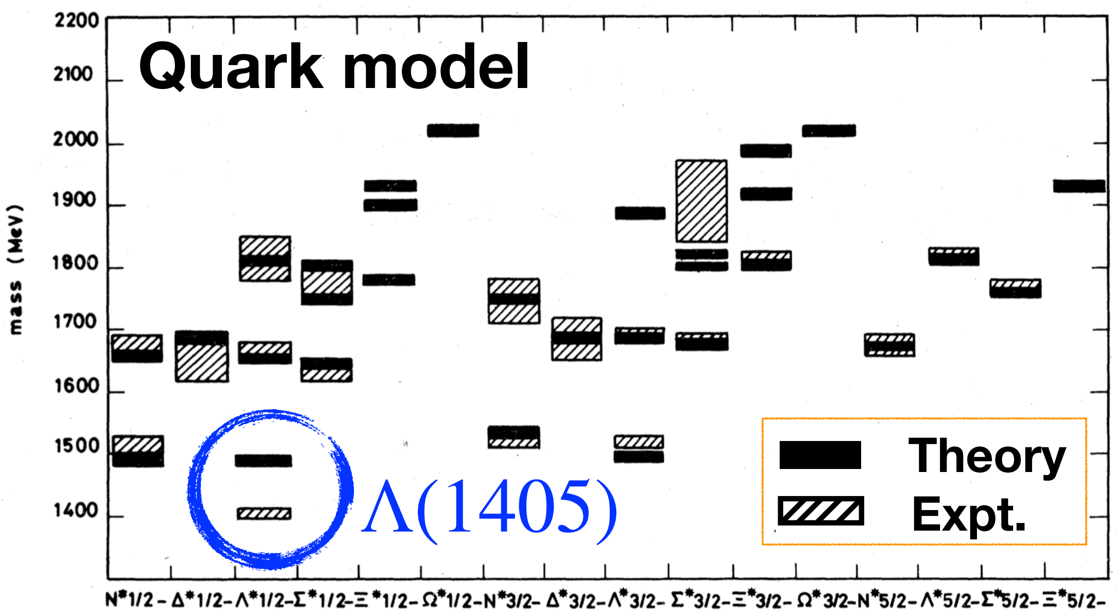


  - $\bar{K}N$ interaction is important in strangeness nuclear physics
    - ✓ Interaction is strongly attractive, generating $\Lambda(1405)$ resonance
    - ✓ $\bar{K}NN, \bar{K}NNN,$ multi-antikaonic nuclei **J-PARC, FINUDA@DAΦNE, etc**
    - ✓ *Kaon*-condensate could change Equation of State of neutron stars
      S.Pal et al., NPA674(2000)553
  - Deepen understanding of SU(3) dynamics in nonperturbative QCD

$\Lambda(1405)$ resonance

PDG2021: **** $I(J^P) = 0(1/2^-)$, $M = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma = 50.5 \pm 2.0$ MeV

- $\Lambda(1405)$ state is an **exotic candidate**



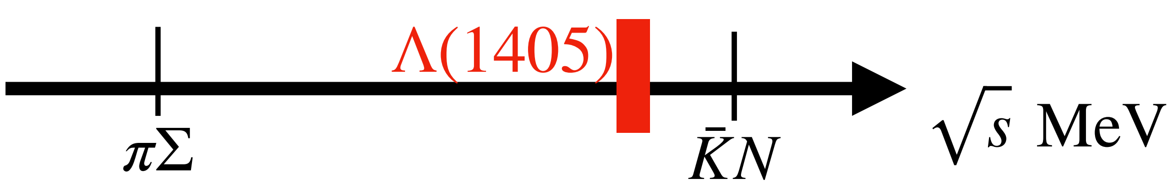
N. Isgur and G. Karl, Phys.Rev.D 18 (1978) 4187

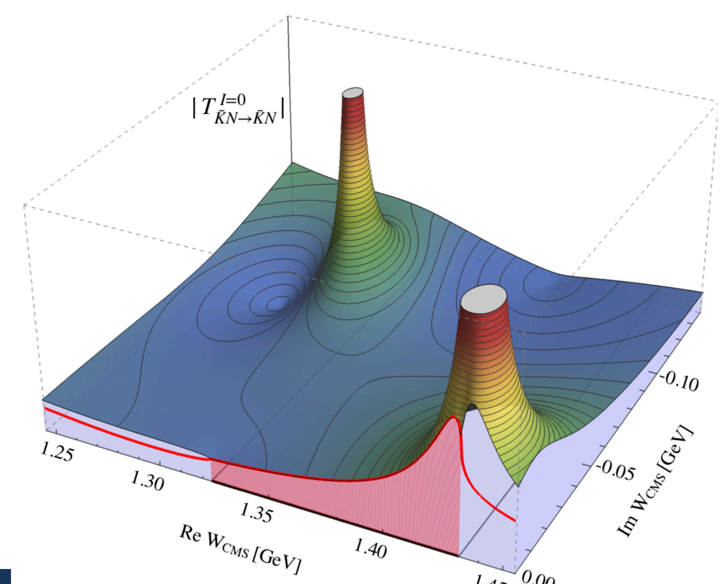
Variety of theoretical studies (e.g.):

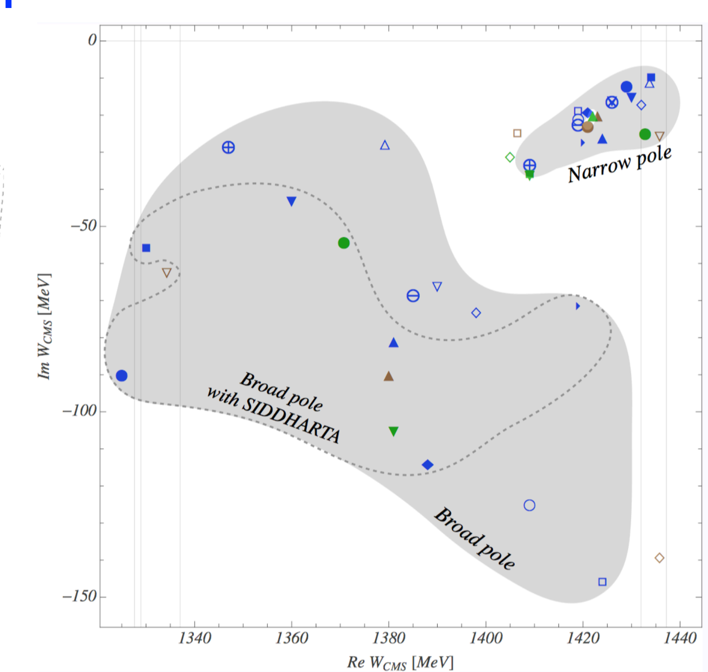
- QCD sum rules *L.S. Kisslinger,EPJA2011…*
- Phenomenological potential model *A. Cieplý, NPA2015*
- Skyrme model *T. Ezoe,PRD2020…*
- Hamiltonian effective field theory *Z.-W. Liu,PRD2017*
- Chiral unitary approach

N.Kaiser,NPA1995; E.Oset,NPA1998; J.A.Oller&U.-G.Meißner,PLB2001…

- **Double-pole structure of $\Lambda(1405)$** predicted by chiral unitary approaches
  - ✓ Two poles of the scattering amplitude in the complex energy plane between the $\bar{K}N$ and $\pi\Sigma$ thresholds.







M.Mai, EPJ.ST(2021)

$\Lambda(1405)$ resonance

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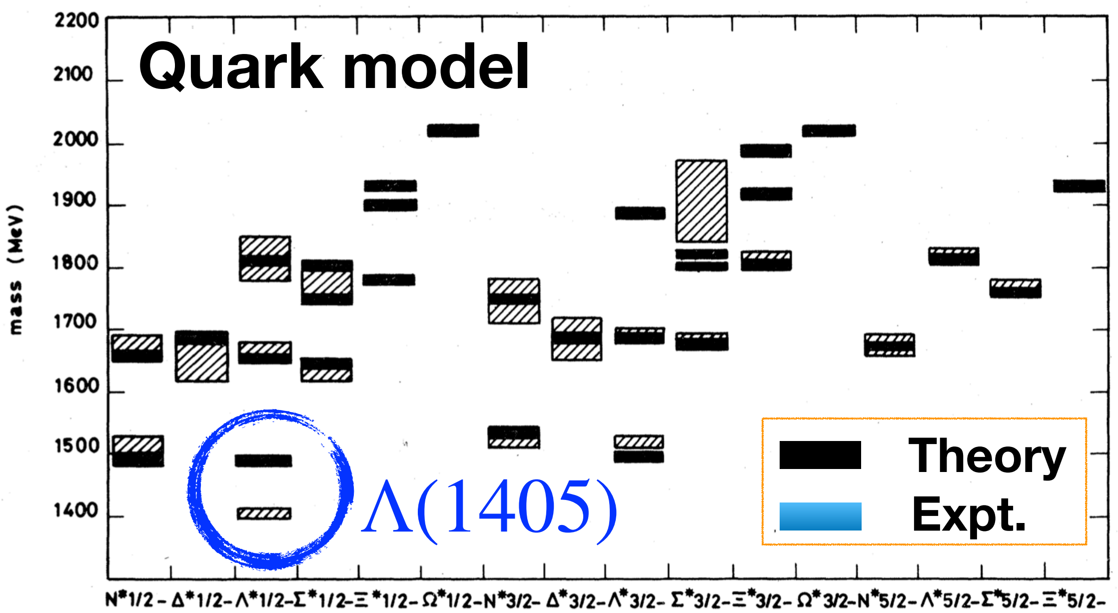

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- **Double-pole structure of $\Lambda(1405)$** predicted by chiral unitary approaches
  - ✓ Two poles of the scattering amplitude in the complex energy plane between the $\bar{K}N$ and $\pi\Sigma$ thresholds.
  - ✓ This fact is now part of the PDG book i.e.
    PDG review “Pole Structure of the Λ(1405) Region” by U.-G. Meißner and T. Hyodo

Article

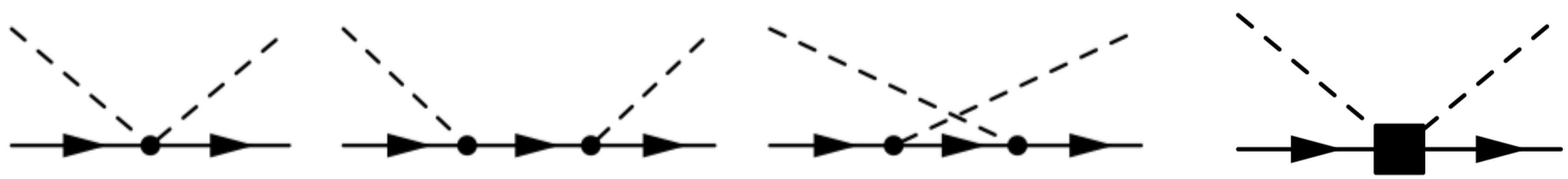
Two-Pole Structures in QCD: Facts, Not Fantasy!

Ulf-G. Meißner [1,2,3]

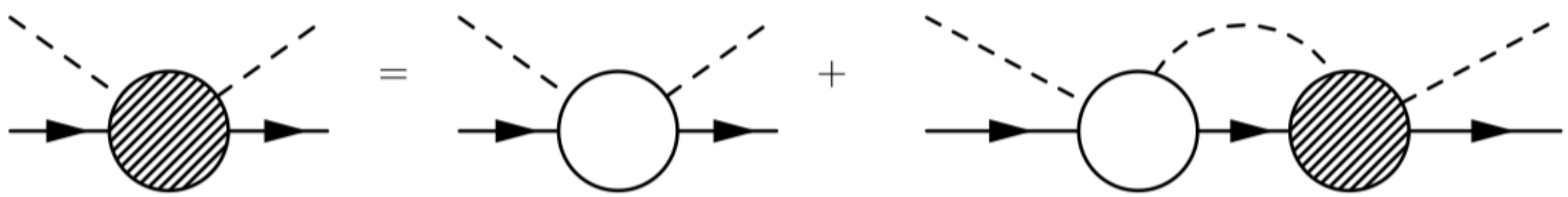
Symmetry **2020**, *12*(6), 981

Chiral Unitary approach

- Chiral symmetry of low-energy QCD is imposed to dynamical generate the resonances *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP67(2012)55-98 …*
  - Interaction kernel V: calculated in ChPT order by order
    - ✓ Leading, Next-to-leading order, …



  - Scattering T-matrix: by solving scattering equations



    - ✓ Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + \int \frac{d^3k}{(2\pi)^3} V(p',k)\, G(k)\, T(k,p)$$

    - ✓ On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(\int \frac{d^3k}{(2\pi)^3} G(k)\right) T(p',p)$

Neglecting off-shell effect

    - ✓ Introduce finite cutoff or subtraction constant to renormalize the loop integral

Cutoff/Model dependence

In this work

- **To avoid those approximations/obstacles,** we tentatively propose a renormalized framework for meson-baryon scattering using time-ordered perturbation theory with the covariant chiral Lagrangians
  - Obtain the potential and scattering equation **on an equal footing**
  - Include the **off-shell effects of potential** and utilize the **subtractive renormalization** to obtain the **renormalized T-matrix**
  - Apply to the pion-nucleon scattering at LO
  - Extend to $S = -1$ sector and investigate the pole structure of $\Lambda(1405)$ state

X.-L. Ren, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406; Eur. Phys. J. C81 (2021) 582

OUTLINE

- Introduction
- Theoretical framework
  - Briefly introduce time-ordered perturbation theory
  - Obtain the potential and scattering equation in TOPT
  - Use subtractive renormalization to obtain the renormalized T-matrix
- Results and discussion
- Summary and outlook

TOPT with covariant Lagrangian

Time-ordered perturbation theory (TOPT)

- Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

  - ✓ Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit. This form is called **TOPT or old-fashioned PT**
  - ✓ (In short) Instead the propagators for internal lines as the energy denominators for intermediate states

- Advantages
  - ✓ Explicitly show the unitarity
  - ✓ One-to-one relation between internal lines and intermediate states
  - ✓ Easily tell the contributions of a particular diagram

- Derive the rules for time-ordered diagrams
  - ✓ Perform Feynman integrations over the zeroth components of the loop momenta
  - ✓ Decompose Feynman diagram into sums of time-ordered diagrams
  - ✓ Match to the rules of time-ordered diagrams

Diagram rules in TOPT

V.Baru, E.Epelbaum, J. Gegelia, XLR, PLB 798 (2019) 134987

External lines

- Incoming (outgoing) baryon lines: $u(p)\ [\bar{u}(p')]$

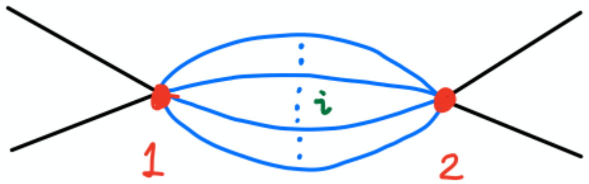
Internal lines

- Pseudo-scalar meson lines: $\dfrac{1}{2\omega(q_i, M_i)}$ $\qquad \omega(q, M) = \sqrt{q^2 + M^2}$
- Baryon lines: $\dfrac{m_i}{\omega(p_i, m)} \sum u(p_i)\bar{u}(p_i)$

Interaction vertices

- Follow the standard Feynman rules
- Take care of zeroth components of momenta p^0
  - ✓ Replaced as $\omega(p, m)$ for particle
  - ✓ Replaced as $-\omega(p, m)$ for antiparticle

Intermediate state: a set of lines between any two vertices

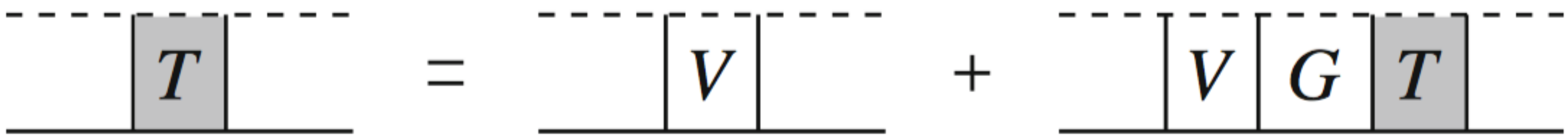


$$[E - \sum_i \omega(p_i, m_i) + i\epsilon]^{-1}$$

E is the total energy of the system

Meson-baryon scattering in TOPT

- Interaction kernel / potential V
  - **Define: sum up the one-meson and one-baryon irreducible time-ordered diagrams**
  - **Power counting: Q/Λ_χ systematic ordering of all graphs**
- Scattering equation (non-perturbative)



  - Coupled-channel integral equation for T-matrix

$$
\begin{aligned}
T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) &= V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) \\
&+ \sum_{MB}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)
\end{aligned}
$$

  - **Meson-baryon Green function in TOPT**:

$$
G_{MB}(E) = \frac{1}{2\omega(k,M)\,\omega(k,m)}\,\frac{m}{E-\omega(k,M)-\omega(k,m)+i\epsilon}
$$

Meson–baryon scattering in TOPT

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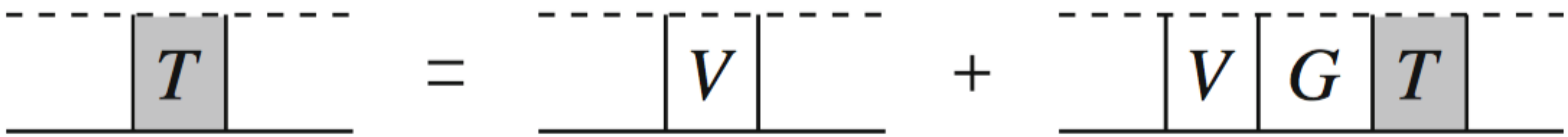

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$$\begin{aligned} T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) &= V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) \\ &+ \sum_{MB}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\, G_{MB}(E)\, T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E) \end{aligned}$$

  - **Meson-baryon Green function in TOPT**:

$$G_{MB}(E) = \frac{1}{2\omega(k,M)\,\omega(k,m)}\frac{m}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

Potential and scattering equation are obtained on an equal footing!

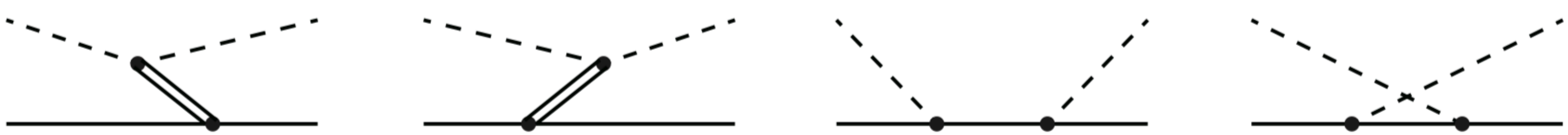
Leading order potential

Chiral effective Lagrangian

$$\mathcal{L}_{\rm LO} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle$$
$$-\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\langle\bar{B}\gamma_\mu[V^\mu, B]\rangle$$

- Vector mesons included as explicit degrees of freedom
  - ✓ One-vector meson exchange potential instead the Weinberg-Tomozawa term
  - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

Time ordered diagrams



(a) (b) (c) (d)

- Applying time-ordered rules, LO potential

$$V^{(a+b)}_{M_jB_j,M_iB_i} = -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right)$$
$$\times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)}+\frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]$$

$$V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.$$
$$V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.$$

  - ✓ Here, Dirac spinor is decomposed as $u_B(p,s) = u_0 + [u(p) - u_0] \equiv (0,1)^\dagger\chi_s$+high order

Subtractive renormalization

- Leading order potential as the sum of **the one-baryon reducible and irreducible parts**

$$V_{\rm LO} = V_I + V_R = V^{(a)} + V^{(b)} + V^{(d)} + V^{(c)}$$

- Leading order T-matrix

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G\, T_{\rm LO}\,. \Longrightarrow$$

$$T_{\rm LO} = T_I + (1 + T_I\, G)\, T_R\, (1 + G\, T_I)$$

$$T_I = V_I + V_I\, G\, T_I$$

$$T_R = V_R + V_R\, G\, (1 + T_I\, G)\, T_R$$

- Irreducible part: $T_I \xrightarrow{\Lambda \sim \infty}$ Finite
- Reducible part: $T_R \xrightarrow{\Lambda \sim \infty}$ Divergent
  - ✓ Potential can be rewritten as separable form $V_R(p', p; E) = \xi^T(p')\, C(E)\, \xi(p)$ **C(E): constant** $\xi^T(q) := (1, q)$
  - ✓ T_R can be rewritten as $T_R(p', p; E) = \xi^T(p')\, \chi(E)\, \xi(p)$ $\chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

  - ✓ Using **subtractive renormalization**, replacing Green function $G^{Rn} = G(E) - G(m_B)$

E. Epelbaum, et al., EPJA56(2020)152

Renormalized T-matrix

$$\boxed{T_{\rm LO}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)}$$

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C(E): constant

$\xi^T(q) := (1, q)$

We can take cutoff to infinity and avoid introducing the cutoff dependence or subtraction constants!

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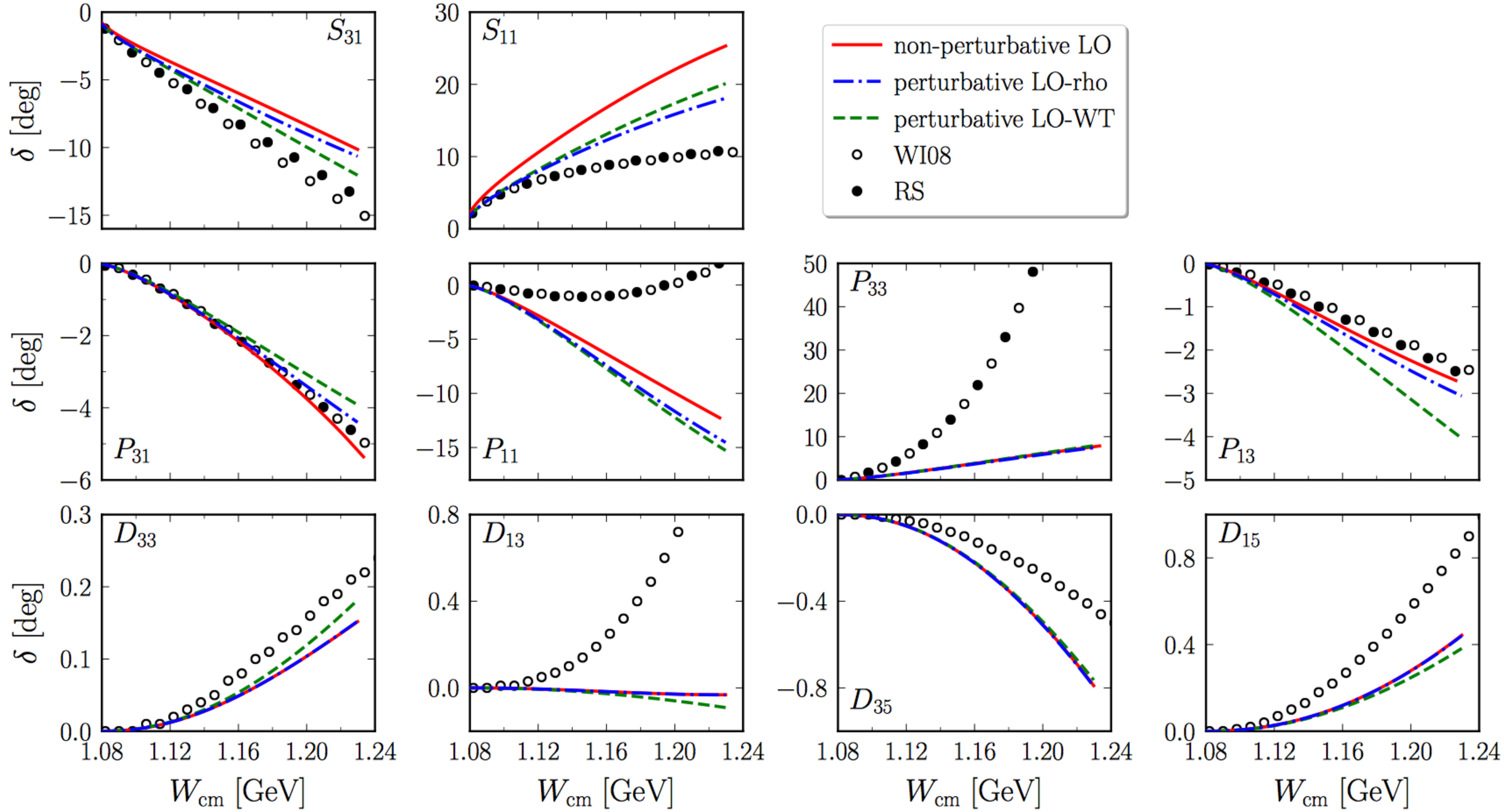
Pion–Nucleon scattering

- Description phase shifts of pion-nucleon scattering



- Rho-meson-exchange contribution is similar as WT term.
- Phase shifts from non-perturbative renormalized amplitude are only slightly different from the ones of the perturbative approach.

✓ Our non-perturbative treatment is valid, since ChPT has good convergence in SU(2) sector

X.-L. Ren, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

S=-1 meson-baryon scattering

Four coupled channel $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$

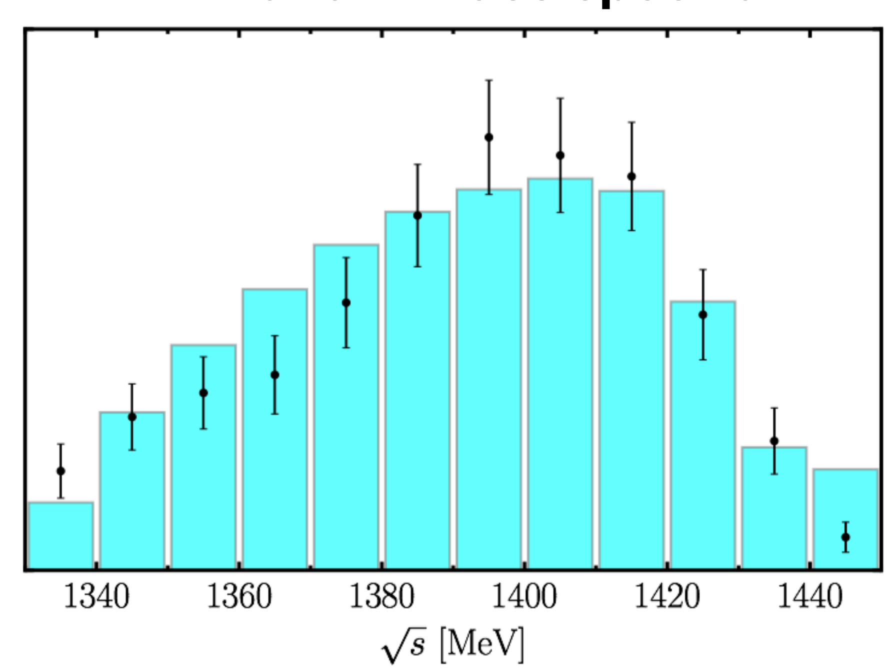
- Solving coupled-channel scattering equation in isospin basis
- Taking into account the off-shell effects of potential
- **Using subtractive reormalization and taking cutoff to infinity** to obtain the renormalized T-matrix

No free parameters needed to be fitted!

Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work (LO)	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



- Varying the meson-decay constant, the width of lower pole is increasing and the higher pole lies close and moves beyond the threshold of $\bar{K}N$ channel, and its width decreases
- Our LO results are consistent with M.Mai EPJA(2015), in particular for the lower pole.

Coupling strengths for Λ(1405)

- On-shell scattering T-matrix can be approximated by

$$T_{ij} \simeq 4\pi \frac{g_i \, g_j}{z - z_R}$$

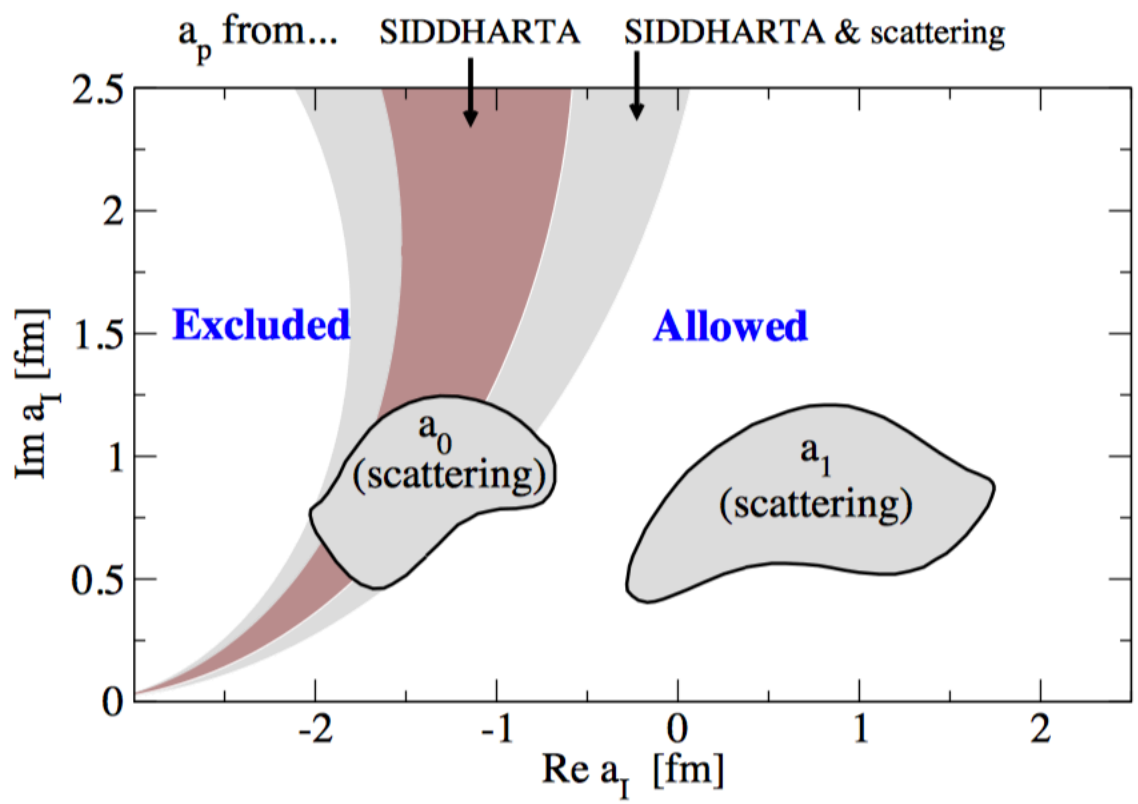
- g_i (g_j): coupling strength of the initial (final) transition channel

	lower pole		higher pole	
	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$
$\pi\Sigma$	$1.83 + i1.90$	2.64	$-0.38 + i0.84$	0.92
$\bar{K}N$	$-1.59 - i1.47$	2.17	$2.16 - i0.83$	2.31
$\eta\Lambda$	$-0.19 - i0.67$	0.69	$1.59 - i0.36$	1.63
$K\Xi$	$0.72 + i0.81$	1.08	$-0.10 + i0.34$	0.35

- Two poles of Lambda(1405) have different coupling nature
  - ✓ the lower pole couples predominantly to the $\pi\Sigma$ channel
  - ✓ the higher pole couples strongly to the $\bar{K}N$ channel

$\bar{K}N$ scattering observables

- Scattering length: constrained by scattering + SIDDHARTA kaonic deuterium data



M. Döring and U.-G. Meißner, Phys. Lett. B 704, 663 (2011).

Our LO prediction (isospin basis)

- **Isospin I=0**

$$a_0 = -2.50 + i\,1.37\,\text{fm}$$

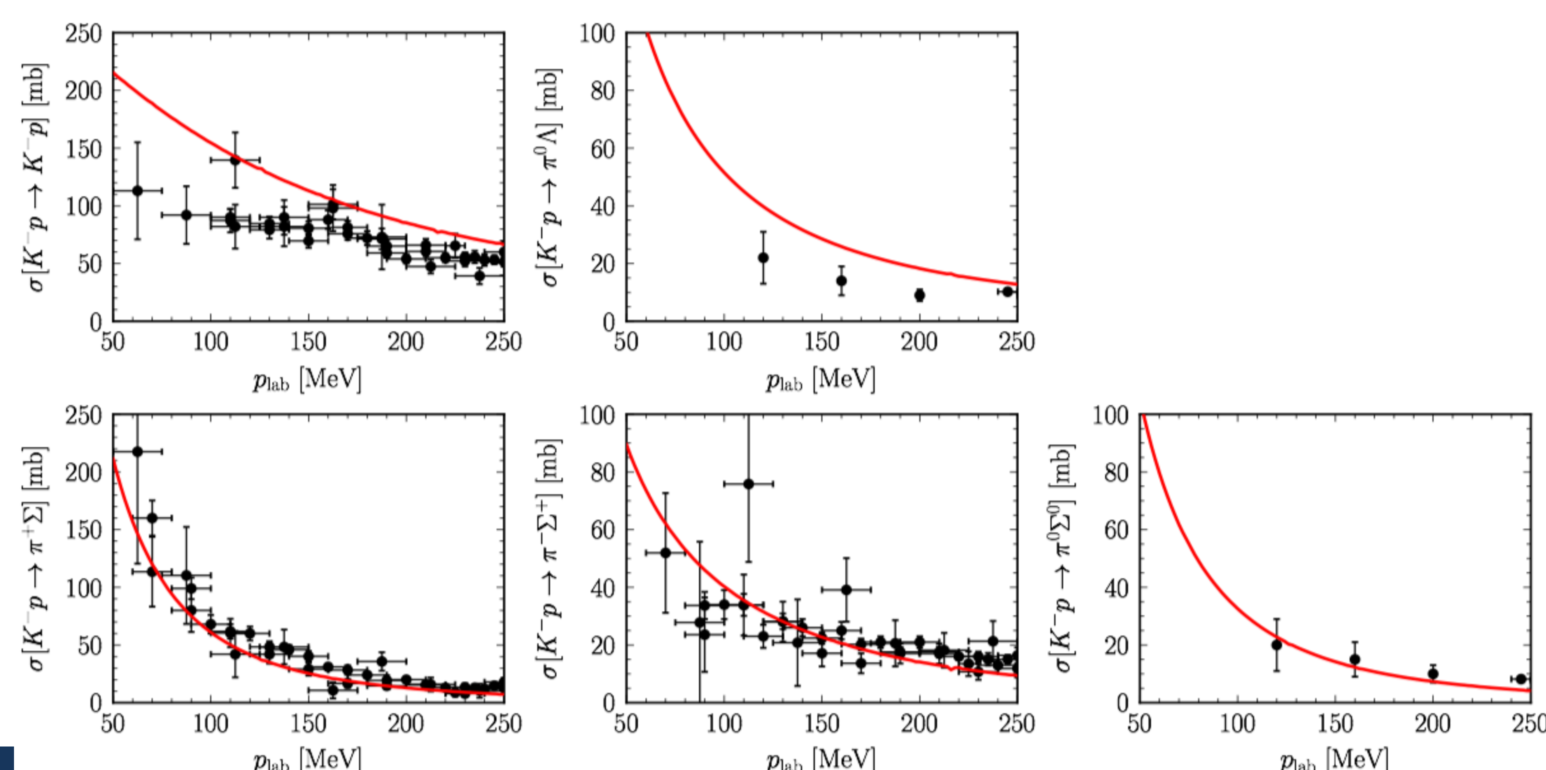
outside the allowed region

- **Isospin I=1**

$$a_1 = 0.33 + i\,0.72\,\text{fm}$$

within the allowed region

- Total cross section of $K^- p$
  - Our LO prediction covers well $K^- p \to \pi^{\pm,0}\Sigma^{\pm,0}$ cross section
  - slightly larger than the data of $K^- p \to K^- p, \pi^0\Lambda$



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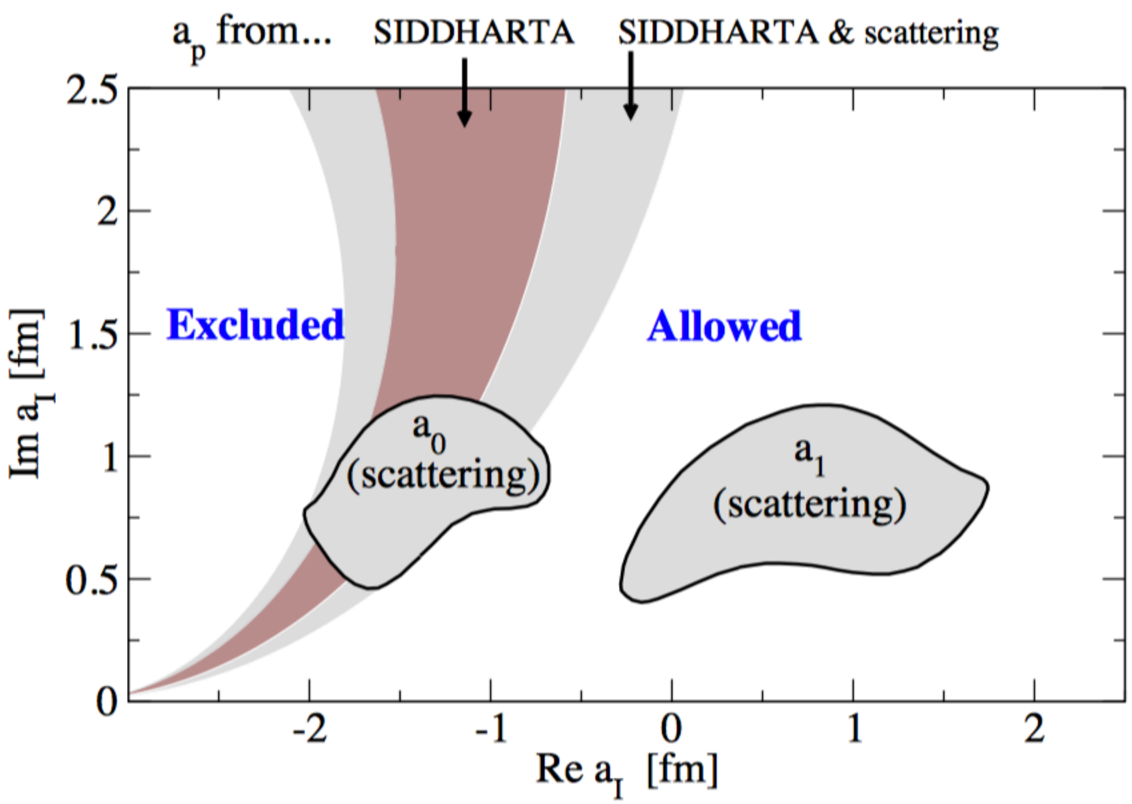

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Next-to-leading order calculation

- Total cross section of K^-p
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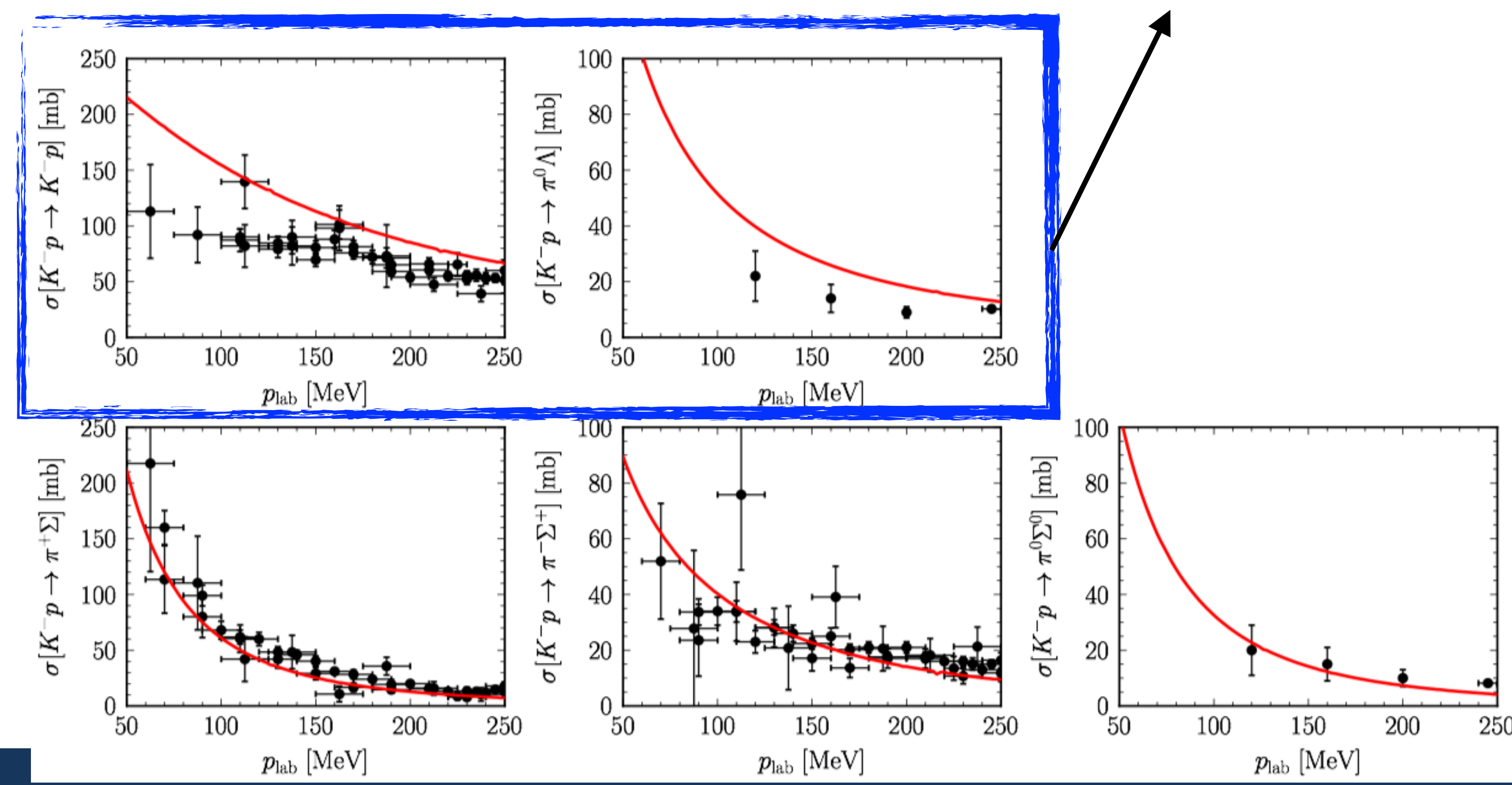
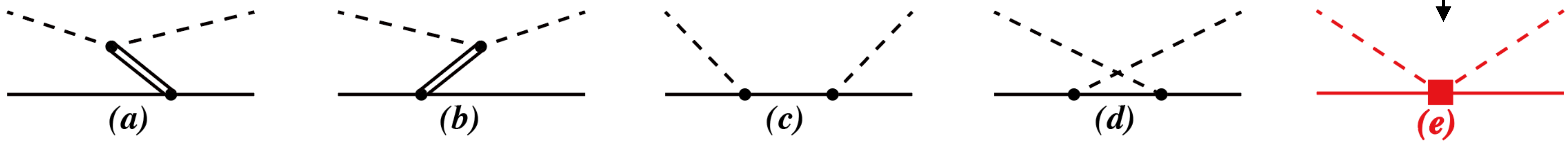


Next-to-leading order study

- NLO potential:

14 Free parameters



- T-matrix at NLO:

$$T = T_{\mathrm{LO}} + T_{\mathrm{NLO}} \qquad T_{\mathrm{LO}} = V_{\mathrm{LO}} + V_{\mathrm{LO}}\, G\, T_{\mathrm{LO}}$$

  - NLO correction is **perturbatively included** to maintain the scattering T-matrix renormalizable

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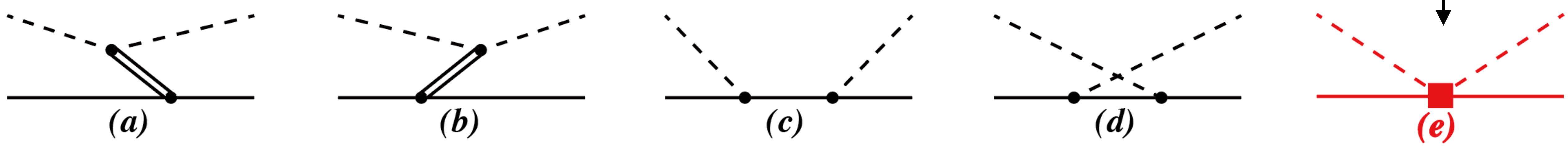
- Solving the T-matrix in the particle basis
  - S=-1 sector, 10 coupled channels:
    $K^-p, \bar{K}^0 n, \pi^0\Lambda, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+, \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$
  - Describe the scattering cross section, decay ratios, the energy shift and width of kaonic hydrogen from SIDDHARTA

Next-to-leading order study

- NLO potential:

14 Free parameters



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Stay Tuned !

Summary

- We tentatively propose a renormalized framework for meson-baryon scattering using time-ordered perturbation theory with the covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Use subtractive renormalization to obtain renormalized T-matrix
    - ✓ This avoids to introduce the cutoff dependence or subtraction parameters
  - Apply to πN scattering and extend to the S=-1 sector at LO
  - Obtain the two-pole structure of $\Lambda(1405)$

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Thank you for your attention!