A possible interpretation of Λ baryon spectrum with pentaquark components

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Introduction

- Our previous works have constructed the full wave functions of multiquark systems, including meson, baryon, tetraquark and pentaquarks in a group theory approach. Model parameters are fixed by fitting to the experimental data. (Phys. Rev. C93 (2016), 025201. Phys. Rev. C100 (2019), 065207, Phys. Rev. D101 (2020), and Phys. Rev. D103 (2021),116027)

- The masses of low-lying q^3 states and ground $q^4\bar{q}$, $q^3s\bar{s}$ states are evaluated.

- $N_{1/2+}(1440)$ is mainly a q^3 first radial excitation,
- $N_{3/2-}(1520)$ may contain a large ground-state $(l=0)$ non-strange pentaquark component,
- $N_{1/2-}(1535)$ may contain a large ground-state $(l=0)$ $uuds\bar{s}$ component.

Mass of excited low-lying q^3 states with the model parameters

Ground state baryons applied to fit the model parameters. The last column shows the deviation between the experimental and theoretical mean values, M^{exp} are taken from PDG.

Baryon	$M^{exp}(\mathrm{MeV})$	$M^{cal}(\mathrm{MeV})$	$D = 100 \cdot (M^{exp} - M^{cal})/M^{exp}$ (%)
N(939)	939	939	0
$\Delta(1232)$	1232	1232	0
$\Lambda(1116)$	1116	1112	0.34
$\Sigma(1193)$	1193	1146	3.96
$\Sigma^*(1385)$	1385	1372	0.97
$\Xi(1318)$	1318	1318	0
$\Xi^*(1530)$	1533	1510	1.49
$\Omega(1672)$	1672	1662	0.62
$\Lambda_C(2286)$	2286	2272	0.62
$\Sigma_C(2455)$	2454	2428	1.06
$\Sigma_C^*(2520)$	2518	2486	1.26
$\Xi_C(2470)$	2469	2489	-0.82
$\Xi_C^*(2645)$	2646	2633	0.47
$\Omega_C(2695)$	2695.	2751	-2.07
$\Omega_C^*(2770)$	2766	2789	-0.84
$\Lambda_B(5620)$	5620	5599	0.37
$\Sigma_B(5811)$	5811	5781	0.51
$\Sigma_B^*(5832)$	5832	5801	0.54
$\Xi_B(5792)$	5792	5819	-0.47
$\Xi_B^*(5945)$	5950	5953	-0.05
$\Omega_B(6046)$	6046	6097	-0.84

Nucleon resonances of positive parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{exp}(\mathrm{MeV})$	$M^{cal}(\mathrm{MeV})$
$N(56, {}^{2}8, 0, 0^{+})$	****	$\frac{1}{2}^{+}$	939	939
$N(56, {}^{2}8, 2, 0^{+})$	****	$\frac{1}{2}^{+}$	N(1440)	1499
$N(56, {}^{2}8, 2, 2^{+})$	****	$\frac{5}{2}^{+}$	N(1720)	1655
$N(56, {}^{2}8, 2, 2^{+})$	****	$\frac{3}{2}^{+}$	N(1680)	1655
$N(20, {}^{2}1, 2, 1^{+})$	***	$\frac{1}{2}^{+}$	N(1880)	1749
$N(20, {}^{4}1, 2, 1^{+})$	-	$\frac{3}{2}^{+}$	missing	1749
$N(70, {}^{2}10, 2, 0^{+})$	****	$\frac{1}{2}^{+}$	N(1710)	1631
$N(70, {}^{4}10, 2, 0^{+})$	****	$\frac{3}{2}^{+}$	N(1900)	1924
$N(70, {}^{2}10, 2, 2^{+})$	-	$\frac{3}{2}^{+}$	missing	1702
$N(70, {}^{2}10, 2, 2^{+})$	**	$\frac{5}{2}^{+}$	N(1860)	1702
$N(70, {}^{4}10, 2, 2^{+})$	***	$\frac{1}{2}^{+}$	N(2100)	1994
$N(70, {}^{4}10, 2, 2^{+})$	*	$\frac{3}{2}^{+}$	N(2040)	1994
$N(70, {}^{4}10, 2, 2^{+})$	**	$\frac{5}{2}^{+}$	N(2000)	1994
$N(70, {}^{4}10, 2, 2^{+})$	**	$\frac{7}{2}^{+}$	N(1990)	1994

The talk on Wednesday by Prof. Mokeev has implemented this missing baryon resonance as N(1720)$3/2^{+}$.

Resonances of negative-parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{exp}(\mathrm{MeV})$	$M^{cal}(\mathrm{MeV})$
$N(70, {}^{2}10, 1, 1^-)$	****	$\frac{3}{2}^-$	N(1520)	1380
$N(70, {}^{2}10, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1535)	1380
$N(70, {}^{4}10, 1, 1^-)$	****	$\frac{1}{2}^-$	N(1650)	1672
$N(70, {}^{4}10, 1, 1^-)$	****	$\frac{5}{2}^-$	N(1675)	1672
$N(70, {}^{4}10, 1, 1^-)$	***	$\frac{3}{2}^-$	N(1700)	1672
$\Delta(70, {}^{2}10, 1, 1^-)$	****	$\frac{1}{2}^-$	Δ(1620)	1380
$\Delta(70, {}^{2}10, 1, 1^-)$	****	$\frac{3}{2}^-$	Δ(1700)	1380

- Except for the two missing $\Delta(70, {}^{2}10, 2, 2^+)$ states and the two missing nucleon states $N(20, {}^{2}1, 2, 1^+)$ and $N(70, {}^{2}10, 2, 2^+)$, most positive-parity states are reasonably reproduced.
- We have much more negative parity resonance states than the model prediction.

Baryon resonances of negative parity below 2 GeV

Table: Masses of negative-parity resonances after including ground state pentaquark components.

Resonance	Status	J^P	M^{exp} (MeV)	M^{cal} (MeV)
$N(1520)$	****	$\frac{3}{2}^-$	1510-1520	1515
$N(1535)$	****	$\frac{1}{2}^-$	1515-1545	1530
$N(1685)$	*	$\frac{1}{2}^-?$	1665-1675	1683
$N(1875)$	***	$\frac{3}{2}^-$	1850-1920	1899/1914
$N(1895)$	****	$\frac{1}{2}^-$	1870-1920	1882
$\Delta(1620)$	****	$\frac{1}{2}^-$	1590-1630	1610
$\Delta(1700)$	****	$\frac{3}{2}^-$	1690-1730	1710
$\Delta(1900)$	***	$\frac{1}{2}^-$	1840-1920	1893
$\Delta(1940)$	**	$\frac{3}{2}^-$	1940-2060	2024

- We proposed a way to understand the long-standing ordering problem of $N(1440)$, $N(1520)$ and $N(1535)$.
- We have given possible theoretical interpretations for $N(1895)1/2^-$, $N(1875)3/2^-$, $\Delta(1900)1/2^-$, and $\Delta(1940)3/2^-$ states.

We got a question from APFB2020 conference: how about Λ spectrum?
Apply the Hamiltonian, wave functions and model parameters to Λ resonances and ground state $q^3s\bar{q}$ pentaquarks

Table: Λ resonances (Experimental parameters (exp) are from PDG).

Particle	Status	J^P	M_{BW}^{exp} MeV	Γ_{BW}^{exp} MeV
$\Lambda(1600)$	****	$\frac{1}{2}^+$	1570-1630	150-250
$\Lambda(1710)$	*	$\frac{1}{2}^+$	1700-1726	138-222
$\Lambda(1810)$	***	$\frac{1}{2}^+$	1740-1840	50-170
$\Lambda(1820)$	****	$\frac{5}{2}^+$	1815-1825	70-90
$\Lambda(1890)$	****	$\frac{3}{2}^+$	1870-1910	80-160
$\Lambda(2070)$	*	$\frac{3}{2}^+$	2046-2094	320-420
$\Lambda(2085)$	**	$\frac{7}{2}^+$	2000-2040	130-190
$\Lambda(2110)$	***	$\frac{5}{2}^+$	2050-2130	200-300
$\Lambda(1405)$	****	$\frac{1}{2}^-$	1404-1406	48.5-52.5
$\Lambda(1520)$	****	$\frac{3}{2}^-$	1518-1520	15-17
$\Lambda(1670)$	****	$\frac{1}{2}^-$	1670-1678	25-35
$\Lambda(1690)$	****	$\frac{3}{2}^-$	1685-1695	60-80
$\Lambda(1800)$	***	$\frac{1}{2}^-$	1750-1850	150-250
$\Lambda(1830)$	****	$\frac{5}{2}^-$	1820-1830	60-120
$\Lambda(2000)$	*	$\frac{1}{2}^-$	2000	100-150
$\Lambda(2050)$	*	$\frac{3}{2}^-$	2034-2078	432-554
$\Lambda(2080)$	*	$\frac{5}{2}^-$	2069-2095	152-210
$\Lambda(2100)$	****	$\frac{7}{2}^-$	2090-2110	100-250
$\Lambda(2325)$	*	$\frac{3}{2}^-$	2307-2347	120-200

OUTLINE

- Construction of the wave-function of $q^4\bar{q}$ pentaquark system

- Constituent quark model with a cornell-like potential and spin-orbit interaction

  q^2s baryon mass spectra in 3q pictures

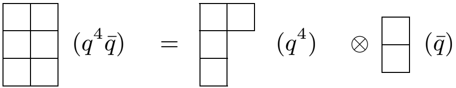
- $q^3s\bar{q}$ pentaquark states and possible mixtures of three-quark and pentaquark states

  Two-body mixture

  Three-body mixture

Pentaquark wave functions

- $q^4\bar{q}$ configuration young tabloid construction



- Pentaquark wave function in $q^4\bar{q}$ configuration:

$$\psi = \frac{1}{\sqrt{3}}\left(\psi^{c}_{[211]_\lambda}\psi^{osf}_{[31]_\rho} - \psi^{c}_{[211]_\rho}\psi^{osf}_{[31]_\lambda} + \psi^{c}_{[211]_\eta}\psi^{osf}_{[31]_\eta}\right) \tag{1}$$

- All possible spin-flavor [31] configurations of q^4 cluster

$[31]_{FS}$			
$[31]_{FS}[31]_F[22]_S$	$[31]_{FS}[31]_F[31]_S$	$[31]_{FS}[31]_F[4]_S$	$[31]_{FS}[211]_F[22]_S$
$[31]_{FS}[211]_F[31]_S$	$[31]_{FS}[22]_F[31]_S$	$[31]_{FS}[4]_F[31]_S$	

Constituent quark model with a Cornell-like potential and spin-orbit interaction

- The realistic Hamiltonian:

$$
\begin{aligned}
H &= H_0 + H^{OGE}_{hyp} + H^{SO}_{q^3}, \\
H_0 &= \sum_{k=1}^{N}(m_k + \frac{p_k^2}{2m_k}) + \sum_{i<j}^{N}(-\frac{3}{8}\lambda_i^C \cdot \lambda_j^C)(A_{ij}r_{ij} - \frac{B_{ij}}{r_{ij}}), \\
H^{OGE}_{hyp} &= -C_{OGE}\sum_{i<j}\frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j}\,\vec{\sigma}_i \cdot \vec{\sigma}_j, \\
H_{SO} &= \frac{3}{4m_q^2\rho}\frac{d(V - S/3)}{d\rho}\left[3(\vec{\rho} \times \vec{p}_\rho)(\vec{S_1} + \vec{S_2}) - \frac{1}{\sqrt{3}}(\vec{\rho} \times \vec{p}_\lambda)(\vec{S_1} - \vec{S_2})\right]
\end{aligned} \tag{2}
$$

- where A_{ij} and B_{ij} are mass dependent coupling parameters, taking the form,

$$
A_{ij} = a\sqrt{\frac{m_{ij}}{m_u}}, \quad B_{ij} = b\sqrt{\frac{m_u}{m_{ij}}} \tag{3}
$$

- with m_{ij} being thereduced mass of i^{th} and j^{th}quarks, defined as $m_{ij} = \frac{m_i m_j}{m_i + m_j}$. $C_{OGE} = C_m\, m_u^2$, with m_u being the constituent u quark mass and C_m a constant. λ_i^C in the above equations are the generators of color SU(3) group.

Model parameters

- The 3 model coupling constants and 4 constituent quark masses are fitted,

$$m_u = m_d = 327 \text{ MeV}\,, \quad m_s = 498 \text{ MeV}\,,$$
$$m_c = 1642 \text{ MeV}\,, \quad m_b = 4960 \text{ MeV}\,,$$
$$C_m = 18.3 \text{ MeV}, \quad a = 49500 \text{ MeV}^2, \quad b = 0.75 \tag{4}$$

- The spin-orbit parameters k_1, k_2, k_3, k_4 corresponding to the baryon supermultiplets: $[70, 1^-]$, $[20, 1^+]$, $[56, 2^+]$ and $[70, 2^+]$ are determined by the gaps between known Λ, Σ and Σ^* states which we believe in the pure q^3 pictures,

$$k_1 = 4 \text{ MeV}\,, \quad k_2 = 6 \text{ MeV}\,,$$
$$k_3 = -10 \text{ MeV}\,, \quad k_4 = -10 \text{ MeV}\,, \tag{5}$$

$$C(^{2S+1}D_J) = 2 < \vec{L}\cdot\vec{S} >= [J(J+1) - L(L+1) - S(S+1)] \tag{6}$$

The method is from N, Isgur and G, Karl, PhysRevD.18.4187(1978).

$(70\ 1^-)^{2S+1}D_J$	$^48_{5/2}$	$^48_{3/2}$	$^48_{1/2}$	$^28_{3/2}$	$^28_{1/2}$	$^210_{3/2}$	$^210_{1/2}$	$^21_{3/2}$	$^21_{1/2}$
$C(^{2S+1}D_J)$	3	-2	-5	1	-2	1/3	-2/3	2/3	-4/3
$k_1C(^{2S+1}D_J)$	12	-8	-20	4	-8	1	-3	3	-5

Λ resonances of positive parity applied to fit the model parameters.

Table: For L=0 states, no SL interaction should be included, so $M^{sl} = M^{cal}$.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{exp}(\mathrm{MeV})$	$M^{cal}(\mathrm{MeV})$	$M^{sl}(\mathrm{MeV})$
$\Lambda(56, {}^2 8, 0, 0^+)$	****	$\frac{1}{2}^+$	$\Lambda(1116)$	1112	-
$\Lambda(56, {}^2 8, 2, 0^+)$	****	$\frac{1}{2}^+$	$\Lambda(1600)$	1689	-
$\Lambda(56, {}^2 8, 2, 2^+)$	****	$\frac{5}{2}^+$	$\Lambda(1820)$	1845	1825
$\Lambda(56, {}^2 8, 2, 2^+)$	****	$\frac{3}{2}^+$	$\Lambda(1890)$	1845	1875
$\Lambda(20, {}^2 8, 2, 1^+)$	-	$\frac{1}{2}^+$	missing	1939	1927
$\Lambda(20, {}^2 8, 2, 1^+)$	-	$\frac{3}{2}^+$	missing	1939	1945
$\Lambda^*(20, {}^4 1, 2, 1^+)$	-	$\frac{1}{2}^+$	missing	2182	2162
$\Lambda^*(20, {}^4 1, 2, 1^+)$	-	$\frac{3}{2}^+$	missing	2182	2174
$\Lambda^*(20, {}^4 1, 2, 1^+)$	-	$\frac{5}{2}^+$	missing	2182	2194
$\Lambda(70, {}^2 8, 2, 0^+)$	***	$\frac{1}{2}^+$	$\Lambda(1810)$	1821	-
$\Lambda(70, {}^4 8, 2, 0^+)$	*	$\frac{3}{2}^+$	$\Lambda(2070)$	2081	-
$\Lambda^*(70, {}^2 1, 2, 0^+)$	-	$\frac{1}{2}^+$	missing	1838	-
$\Lambda(70, {}^2 8, 2, 2^+)$	****	$\frac{3}{2}^+$	$?\Lambda(1890)$	1892	1922
$\Lambda(70, {}^2 8, 2, 2^+)$	****	$\frac{5}{2}^+$	$?\Lambda(1820)$	1892	1872
$\Lambda(70, {}^4 8, 2, 2^+)$	-	$\frac{1}{2}^+$	missing	2152	2242
$\Lambda(70, {}^4 8, 2, 2^+)$	-	$\frac{3}{2}^+$	missing	2152	2212
$\Lambda(70, {}^4 8, 2, 2^+)$	***	$\frac{5}{2}^+$	$\Lambda(2110)$	2152	2142
$\Lambda(70, {}^4 8, 2, 2^+)$	**	$\frac{7}{2}^+$	$\Lambda(2085)$	2152	2092
$\Lambda^*(70, {}^2 1, 2, 2^+)$	-	$\frac{3}{2}^+$	missing	1909	1929
$\Lambda^*(70, {}^4 1, 2, 2^+)$	-	$\frac{5}{2}^+$	missing	1909	1896

Λ resonances of positive parity applied to fit the model parameters.

Table: For L=0 states, no SL interaction should be included, so $M^{sl} = M^{cal}$.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	M^{exp}(MeV)	M^{cal}(MeV)	M^{sl}(MeV)
$\Lambda(56, {}^2 8, 0, 0^+)$	****	$\frac{1}{2}^+$	$\Lambda(1116)$	1112	-
$\Lambda(56, {}^2 8, 2, 0^+)$	****	$\frac{1}{2}^+$	$\Lambda(1600)$	1689	-
$\Lambda(56, {}^2 8, 2, 2^+)$	****	$\frac{5}{2}^+$	$\Lambda(1820)$	1845	1825
$\Lambda(56, {}^2 8, 2, 2^+)$	****	$\frac{3}{2}^+$	$\Lambda(1890)$	1845	1875
$\Lambda(20, {}^2 8, 2, 1^+)$	-	$\frac{1}{2}^+$	missing	1939	1927
$\Lambda(20, {}^2 8, 2, 1^+)$	-	$\frac{3}{2}^+$	missing	1939	1945
$\Lambda(70, {}^2 8, 2, 0^+)$	***	$\frac{1}{2}^+$	$\Lambda(1810)$	1821	-
$\Lambda(70, {}^4 8, 2, 0^+)$	*	$\frac{3}{2}^+$	$\Lambda(2070)$	2081	-
$\Lambda(70, {}^2 8, 2, 2^+)$	****	$\frac{3}{2}^+$	$?\Lambda(1890)$	1892	1922
$\Lambda(70, {}^2 8, 2, 2^+)$	****	$\frac{5}{2}^+$	$?\Lambda(1820)$	1892	1872
$\Lambda(70, {}^4 8, 2, 2^+)$	-	$\frac{1}{2}^+$	missing	2152	2242
$\Lambda(70, {}^4 8, 2, 2^+)$	-	$\frac{3}{2}^+$	missing	2152	2212
$\Lambda(70, {}^4 8, 2, 2^+)$	***	$\frac{5}{2}^+$	$\Lambda(2110)$	2152	2142
$\Lambda(70, {}^4 8, 2, 2^+)$	**	$\frac{7}{2}^+$	$\Lambda(2085)$	2152	2092

Most positive-parity Λ resonances are well repeated.

Λ Resonances of negative-parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	M^{exp}(MeV)	M^{cal}(MeV)	M^{sl}(MeV)
$\Lambda(70, {}^2 8, 1, 1^-)$	****	$\frac{1}{2}^-$	Λ(1670)	1587	1579
$\Lambda(70, {}^2 8, 1, 1^-)$	****	$\frac{3}{2}^-$	Λ(1690)	1587	1591
$\Lambda(70, {}^4 8, 1, 1^-)$	****	$\frac{1}{2}^-$	Λ(1800)	1830	1810
$\Lambda(70, {}^4 8, 1, 1^-)$	****	$\frac{5}{2}^-$	Λ(1830)	1830	1842
$\Lambda(70, {}^4 8, 1, 1^-)$	-	$\frac{3}{2}^-$	missing	1830	1822
$\Lambda^*(70, {}^2 1, 1, 1^-)$	****	$\frac{1}{2}^-$	Λ(1405)	1587	1582
$\Lambda^*(70, {}^2 1, 1, 1^-)$	****	$\frac{3}{2}^-$	Λ(1520)	1587	1590

- We have much more negative parity resonance states than the model prediction.
- Λ(1405) and Λ(1520) are identified as the flavor-singlet baryons while Λ(1670) and Λ(1690) are identified as the flavor-octet baryons, but they are degenerate in our model.
- The expected $\frac{3}{2}^-$ spin-orbit coupling pair of $\Lambda(1800)\frac{1}{2}^-$ and $\Lambda(1830)\frac{5}{2}^-$ is missing, as shown by the new partial-wave amplitudes data of K^-p scattering [M. Matveev, et. al., Eur. Phys. J. A 55, 179 (2019)].

Ground state pentaquark mass spectra in $q^4\bar{q}$ system

- $q^3 s\bar{q}$ ground state pentaquark masses.

$q^4\bar{q}$ configurations	J^P	$M(q^3 s\bar{q})$ (MeV)
$\Psi^{csf}_{[211]_C[31]_{FS}[4]_F[31]_S}(q^3 s\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	2659, 2392
$\Psi^{csf}_{[211]_C[31]_{FS}[31]_F[4]_S}(q^3 s\bar{q})$	$\frac{3}{2}^-, \frac{5}{2}^-$	2170 (2216), 2408 (2408)
$\Psi^{csf}_{[211]_C[31]_{FS}[31]_F[31]_S}(q^3 s\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	2277 (2296), 2203 (2244)
$\Psi^{csf}_{[211]_C[31]_{FS}[31]_F[22]_S}(q^3 s\bar{q})$	$\frac{1}{2}^-$	2183 (2229)
$\Psi^{csf}_{[211]_C[31]_{FS}[211]_F[31]_S}(q^3 s\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	1794, 2116
$\Psi^{csf}_{[211]_C[31]_{FS}[211]_F[22]_S}(q^3 s\bar{q})$	$\frac{1}{2}^-$	2001
$\Psi^{csf}_{[211]_C[31]_{FS}[22]_F[31]_S}(q^3 s\bar{q})$	$\frac{1}{2}^-, \frac{3}{2}^-$	1894, 2229

- For $q^3 s$ $[31]$ configuration, there are two possible Weyl tabloids corresponding two isospin states $I = 1/2$ $\begin{array}{|c|c|c|}\hline u & u & s \\ \hline d \\ \cline{1-1}\end{array}$,and $I = 3/2$ $\begin{array}{|c|c|c|}\hline u & u & d \\ \hline s \\ \cline{1-1}\end{array}$
- Λ has isospin$= 0$, so only the wave functions from projection $\begin{array}{|c|c|c|}\hline u & u & s \\ \hline d \\ \cline{1-1}\end{array}$ with $I = 1/2$ are possible to mix up with negative parity Λ resonance states.

Mass of ground state pentaquark $q^3c\bar{c}$

- Ground hidden-charm pentaquark $q^3c\bar{c}$ mass spectrum, where the q^3 and $Q\bar{Q}$ components are in the color octet states.

J^P	$q^3s\bar{q}$ configurations	$(S^{q^3s}, S^{\bar{q}}, S)$	$M^{EV}(q^3s\bar{q})$
$\frac{5}{2}^-$	$\Psi^{sf}_{[31]_F[4]_S}(q^3s\bar{q})$	(2,1/2,5/2)	2408
$\frac{3}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^3s\bar{q})$	(1,1/2,3/2)	2392
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[4]_S}(q^3s\bar{q})\\ \Psi^{sf}_{[31]_F[31]_S}(q^3s\bar{q})\end{pmatrix}^{mix1}$	(2,1/2,3/2) (1,1/2,3/2)	$\begin{pmatrix}1966\\2407\end{pmatrix}$
	$\Psi^{sf}_{[211]_F[31]_S}(q^3s\bar{q})$	(1,1/2,3/2)	2116
	$\Psi^{sf}_{[22]_F[31]_S}(q^3s\bar{q})$	(1,1/2,3/2)	2229
$\frac{1}{2}^-$	$\Psi^{sf}_{[4]_F[31]_S}(q^3s\bar{q})$	(1,1/2,1/2)	2659
	$\begin{pmatrix}\Psi^{sf}_{[31]_F[31]_S}(q^3s\bar{q})\\ \Psi^{sf}_{[31]_F[22]_S}(q^3s\bar{q})\end{pmatrix}^{mix2}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix}2162\\2314\end{pmatrix}$
	$\begin{pmatrix}\Psi^{sf}_{[211]_F[31]_S}(q^3s\bar{q})\\ \Psi^{sf}_{[211]_F[22]_S}(q^3s\bar{q})\end{pmatrix}^{mix3}$	(1,1/2,1/2) (0,1/2,1/2)	$\begin{pmatrix}1742\\2052\end{pmatrix}$
	$\Psi^{sf}_{[22]_F[31]_S}(q^3s\bar{q})$	(1,1/2,1/2)	1894

- Only four $\frac{3}{2}^-$ and five $\frac{1}{2}^-$ $q^3s\bar{q}$ ground state pentaquarks with $I=0$ could mix with negative parity Λ resonance states.

Possible mixtures of three-quark and pentaquark states

- The mixing states of two-body and three-body systems are realized by unitary transformations,

$$U_1^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}; \; U_1 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$U = \begin{pmatrix} \cos(\theta)\cos(\psi)\cos(\phi)+\sin(\psi)\sin(\phi) & \cos(\psi)\sin(\phi)-\cos(\theta)\sin(\psi)\cos(\phi) & \sin(\theta)\cos(\phi) \\ \sin(\psi)\cos(\phi)-\cos(\theta)\cos(\psi)\sin(\phi) & \cos(\theta)\sin(\psi)\sin(\phi)+\cos(\psi)\cos(\phi) & -\sin(\theta)\sin(\phi) \\ -\sin(\theta)\cos(\psi) & \sin(\theta)\sin(\psi) & \cos(\theta) \end{pmatrix};$$

$$U^{-1} = \begin{pmatrix} \cos(\theta)\cos(\psi)\cos(\phi)+\sin(\psi)\sin(\phi) & \sin(\psi)\cos(\phi)-\cos(\theta)\cos(\psi)\sin(\phi) & -\sin(\theta)\cos(\psi) \\ \cos(\psi)\sin(\phi)-\cos(\theta)\sin(\psi)\cos(\phi) & \cos(\theta)\sin(\psi)\sin(\phi)+\cos(\psi)\cos(\phi) & \sin(\theta)\sin(\psi) \\ \sin(\theta)\cos(\phi) & -\sin(\theta)\sin(\phi) & \cos(\theta) \end{pmatrix};$$

- The Hamiltonian for the mixture of two three-quark states is,

$$H = \begin{pmatrix} M_{\Lambda_1} & \epsilon \\ \epsilon & M_{\Lambda_8} \end{pmatrix}, M_{\psi_1} + M_{\psi_2} = M_{\Lambda_1} + M_{\Lambda_8}$$

$$H = \begin{pmatrix} M_{q^3_{\Lambda_1}} & \epsilon & \Delta_1 \\ \epsilon & M_{q^3_{\Lambda_8}} & \Delta_2 \\ \Delta_1 & \Delta_2 & M_{q^4\bar{q}} \end{pmatrix}, M_{\psi_1} + M_{\psi_2} + M_{\psi_3} = M_{\Lambda_1} + M_{\Lambda_8} + M_{q^3 s\bar{q}}$$

Λ_1 and Λ_8 mixture

- The mixture of two three-quark states. Both the masses are fit to data perfectly with weighted mass squared distance

ψ_1 State	J^P	θ	ψ_2 State	Λ_1 State	Λ_8 State
1444($\Lambda(1405)$)	$\frac{1}{2}^-$	$45.32°$	1718 ($\Lambda(1670)$)	1582	1579
1506($\Lambda(1520)$)	$\frac{3}{2}^-$	$44.83°$	1675 ($\Lambda(1690)$)	1590	1591

- The mixture of two extremely close states are quite unphysical.

- The mixture for $\frac{3}{2}^-$ states are not bad, but two $\frac{1}{2}^-$ eigenvalues can't repeat the experimental data.

- It's naturally to include the three-body mixture including the single pentaquark state.

Three-body mixture

- The mixture of two three-quark states and one pentaquark state.

ψ_1	ψ_2	ψ_3	J^P	Λ_1/Λ_8 States	$q^3s\bar{q}$ config.	$q^3s\bar{q}$ Mass
1405	1670	2248	$\frac{1}{2}^-$	1582 /1579	$q^3s\bar{q}^{mix2}_{[31]_F[31]_S}$	2162
1405	1670	2400	$\frac{1}{2}^-$	1582 /1579	$q^3s\bar{q}^{mix2}_{[31]_F[22]_S}$	2314
1405	1670	1828	$\frac{1}{2}^-$	1582 /1579	$q^3s\bar{q}^{mix3}_{[211]_F[31]_S}$	1742
1405	1670	2138	$\frac{1}{2}^-$	1582/1579	$q^3s\bar{q}^{mix3}_{[211]_F[22]_S}$	2052
1405	1670	1980($\Lambda(2000)$)	$\frac{1}{2}^-$	1582/1579	$q^3s\bar{q}_{[22]_F[31]_S}$	1894

- The angles θ, ψ and ϕ are not shown here. The $q^3s\bar{q}_{[22]_F[31]_S}$ configuration produces the highest eigenvalue M_{ψ_3} closest to $\Lambda(2000)$.

ψ_1	ψ_2	ψ_3	J^P	Λ_1/Λ_8 States	$q^3s\bar{q}$ config.	$q^3s\bar{q}$ Mass
1520	1650($\Lambda(1690)$)	1977	$\frac{3}{2}^-$	1590/1591	$q^3s\bar{q}^{mix1}_{[31]_F[4]_S}$	1966
1520	1650 ($\Lambda(1690)$)	2418	$\frac{3}{2}^-$	1590/1591	$q^3s\bar{q}^{mix1}_{[31]_F[31]_S}$	2407
1520	1650 ($\Lambda(1690)$)	2127	$\frac{3}{2}^-$	1590/1591	$q^3s\bar{q}_{[211]_F[31]_S}$	2116
1520	1650 ($\Lambda(1690)$)	2240	$\frac{3}{2}^-$	1590/1591	$q^3s\bar{q}_{[22]_F[31]_S}$	2229

- No M_{ψ_3} mass is close to $\Lambda(2325)\frac{3}{2}^-$ state for three-body mixture.

Summary

- We have extended our model to $q^2 s$ Λ resonance states and the ground state $q^3 s\bar{q}$ pentaquark states for isospin $I = 0$.
- A possible interpretation was proposed: the $\Lambda(1520)3/2^-$ and $\Lambda(1690)3/2^-$ are more likely to be the two three-quark negative-parity Λ_1 and Λ_8 mixing states, and the $\Lambda(1405)1/2^-$, $\Lambda(1670)1/2^-$ and $\Lambda(2000)1/2^-$ could be the three-body mixtures.

Future work

- A more realistic Hamiltonian may be introduced to reproduce the mass spectrum.
- Reveal the inner structures of the baryon resonance states by studying the dynamical properties like (transition) form factors and helicity transition amplitudes in the electro- and photoproductions. (Parallel talk by Attaphon on this Wednesday.)

Thank You Very Much For Your Attentions!

Explicit q^2s color-orbital-spin-flavor wave functions for Isospin=0

	$SU(6)_{SF}$	l^P	$SU(6)_{SF}\times O(3)$ wave functions	
N	Representations	O(3)	$SU(3)_F$ singlet	$SU(3)_F$ octet
0	56	0^+		$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^0_{00s}(\Phi_\lambda\chi_\lambda+\Phi_\rho\chi_\rho)$
1	70	1^-	$J^P=\frac{1}{2}^-,\ \frac{3}{2}^-$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}(\chi_\rho\phi^1_{1m\lambda}-\chi_\lambda\phi^1_{1m\rho})\Phi_A$	$J^P=\frac{1}{2}^-,\ \frac{3}{2}^-$ $\frac{1}{2}\psi^c_{[111]}[\phi^1_{1m\rho}(\Phi_\lambda\chi_\rho+\Phi_\rho\chi_\lambda)+\phi^1_{1m\lambda}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)]$ $J^P=\frac{1}{2}^-,\ \frac{3}{2}^-,\ \frac{5}{2}^-$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^1_{1m\lambda}\Phi_\lambda+\phi^1_{1m\rho}\Phi_\rho)$
2	56	0^+		$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^2_{00s}(\Phi_\lambda\chi_\lambda+\Phi_\rho\chi_\rho)$
	70	0^+	$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}(\chi_\rho\phi^2_{00\lambda}-\chi_\lambda\phi^2_{00\rho})\Phi_A$	$J^P=\frac{1}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}[\phi^2_{00\rho}(\Phi_\lambda\chi_\rho+\Phi_\rho\chi_\lambda)+\phi^2_{00\lambda}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)]$ $J^P=\frac{3}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^2_{00\lambda}\Phi_\lambda+\phi^2_{00\rho}\Phi_\rho)$
2	20	1^+	$J^P=\frac{1}{2}^+,\ \frac{3}{2}^+,\ \frac{5}{2}^+$ $\psi^c_{[111]}\phi^2_{1mA}\chi_S\Phi_A$	$J^P=\frac{1}{2}^+,\ \frac{3}{2}^+$ $\psi^c_{[111]}\phi^2_{1mA}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)$
2	56	2^+		$J^P=\frac{3}{2}^+,\ \frac{5}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\phi^2_{2mS}(\Phi_\rho\chi_\rho+\Phi_\lambda\chi_\lambda)$
	70	2^+	$J^P=\frac{3}{2}^+,\ \frac{5}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}(\chi_\rho\phi^2_{2m\lambda}-\chi_\lambda\phi^2_{2m\rho})\Phi_A$	$J^P=\frac{3}{2}^+,\ \frac{5}{2}^+$ $\frac{1}{2}\psi^c_{[111]}[\phi^2_{2m\rho}(\Phi_\lambda\chi_\rho+\Phi_\rho\chi_\lambda)+\phi^2_{2m\lambda}(\Phi_\rho\chi_\rho-\Phi_\lambda\chi_\lambda)]$ $J^P=\frac{1}{2}^+,\ \frac{3}{2}^+,\ \frac{5}{2}^+,\ \frac{7}{2}^+$ $\frac{1}{\sqrt{2}}\psi^c_{[111]}\chi_S(\phi^2_{2m\lambda}\Phi_\lambda+\phi^2_{2m\rho}\Phi_\rho)$

The two Λ_1^* states take masses, 1582 MeV for $\frac{1}{2}^-$ and 1590 MeV for $\frac{3}{2}-$

ψ_1 State	J^P	θ	ψ_2 State	$q^3 s\bar{q}$ configuration	$q^3 s\bar{q}$ Mass
1405	$\frac{1}{2}^-$	25.8°	2239	$q^3 s\bar{q}_{[31]_F[31]_S}$	2162
1405	$\frac{1}{2}^-$	23.8°	2491	$q^3 s\bar{q}_{[31]_F[22]_S}$	2314
1405	$\frac{1}{2}^-$	35.9°	1919	$q^3 s\bar{q}_{[211]_F[31]_S}$	1742
1405	$\frac{1}{2}^-$	27.6°	2229	$q^3 s\bar{q}_{[211]_F[22]_S}$	2052
1405	$\frac{1}{2}^-$	31.0°	2071 ($\Lambda(2000)$)	$q^3 s\bar{q}_{[22]_F[31]_S}$	1894
1520	$\frac{3}{2}^-$	21.6°	2036 ($\Lambda(2050)$)	$q^3 s\bar{q}_{[31]_F[4]_S}$	1966
1520	$\frac{3}{2}^-$	15.7°	2477	$q^3 s\bar{q}_{[31]_F[31]_S}$	2407
1520	$\frac{3}{2}^-$	18.9°	2186	$q^3 s\bar{q}_{[211]_F[31]_S}$	2116
1520	$\frac{3}{2}^-$	17.4°	2299 ($\Lambda(2325)$)	$q^3 s\bar{q}_{[22]_F[31]_S}$	2229

Possible mixtures of $q^2 s$ and $q^3 s\,\bar{q}$ states

- The two Λ_8^* states take the same mass, 1579 MeV for $\frac{1}{2}^-$ and 1591 MeV for $\frac{3}{2}-$.

ψ_1 State	J^P	θ	ψ_2 State	$q^3 s\bar{q}$ configuration	$q^3 s\bar{q}$ Mass
1670	$\frac{1}{2}^-$	$i26.4^\circ$	2074	$q^3 s\bar{q}_{[31]_F[31]_S}$	2162
1670	$\frac{1}{2}^-$	$i22.6^\circ$	2226	$q^3 s\bar{q}_{[31]_F[22]_S}$	2314
1670	$\frac{1}{2}^-$	?	1654	$q^3 s\bar{q}_{[211]_F[31]_S}$	1742
1670	$\frac{1}{2}^-$	$i30.6^\circ$	1964	$q^3 s\bar{q}_{[211]_F[22]_S}$	2052
1670	$\frac{1}{2}^-$	$i43.2^\circ$	1806	$q^3 s\bar{q}_{[22]_F[31]_S}$	1894
1690	$\frac{3}{2}^-$	$i39.6^\circ$	1866	$q^3 s\bar{q}_{[31]_F[4]_S}$	1966
1690	$\frac{3}{2}^-$	$i22.4^\circ$	2307	$q^3 s\bar{q}_{[31]_F[31]_S}$	2407
1690	$\frac{3}{2}^-$	$i30.1^\circ$	2016	$q^3 s\bar{q}_{[211]_F[31]_S}$	2116
1690	$\frac{3}{2}^-$	$i26.2^\circ$	2129	$q^3 s\bar{q}_{[22]_F[31]_S}$	2229

- The mixture for higher negative parity Λ states is highly complex, the transformation may not be realized by an unitary transformation.

Mass of ground state pentaquark $q^3c\bar{c}$

- Ground hidden-charm pentaquark $q^3c\bar{c}$ mass spectrum, where the q^3 and $Q\bar{Q}$ components are in the color octet states.

$q^3Q\bar{Q}$ configurations	J^P	$S^{c\bar{c}}$	$M(q^3c\bar{c})$(MeV)
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[21]_S}(q^3c\bar{c})$	$\frac{1}{2}^-$	0	4483
	$\frac{1}{2}^-, \frac{3}{2}^-$	1	4452, 4495
$\Psi^{csf}_{[21]_C[21]_{FS}[3]_F[21]_S}(q^3c\bar{c})$	$\frac{1}{2}^-$	0	4702
	$\frac{1}{2}^-, \frac{3}{2}^-$	1	4701, 4701
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[3]_S}(q^3c\bar{c})$	$\frac{3}{2}^-$	0	4556
	$\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$	1	4481, 4525, 4598

- The mixture of configurations of the same flavor are considered.

Mass of ground state pentaquark $q^3c\bar{c}$ after mixing of the hyperfine interaction part

$q^3c\bar{c}$ pentaquark, with hyperfine mixture(the states without considering the mixture are shown in brackets.)

$q^3c\bar{c}$ configurations	$(S^{q^3}, S^{c\bar{c}}, S)$	J^P	$M^{EV}(q^3c\bar{c})$ (MeV)
$\Psi^{csf}_{[21]_C[21]_{FS}[3]_F[21]_S}(q^3c\bar{c})$	(1/2,1,1/2)	$\frac{1}{2}^-$	4702
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[21]_S}(q^3c\bar{c})$	(1/2,1,1/2)	$\frac{1}{2}^-$	4701
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[21]_S}(q^3c\bar{c})$	(1/2,0,1/2)		$4433(4483)$
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[21]_S}(q^3c\bar{c})$	(1/2,1,1/2)	$\frac{1}{2}^-$	$4456(4452)$
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[3]_S}(q^3c\bar{c})$	(3/2,1,1/2)		$4525(4481)$
$\Psi^{csf}_{[21]_C[21]_{FS}[3]_F[21]_S}(q^3c\bar{c})$	(1/2,1,3/2)	$\frac{3}{2}^-$	4701
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[21]_S}(q^3c\bar{c})$	(1/2,1,3/2)		$4473(4495)$
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[3]_S}(q^3c\bar{c})$	(3/2,0,3/2)	$\frac{3}{2}^-$	$4531(4556)$
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[3]_S}(q^3c\bar{c})$	(3/2,1,3/2)		$4570(4525)$
$\Psi^{csf}_{[21]_C[21]_{FS}[21]_F[3]_S}(q^3c\bar{c})$	(3/2,1,5/2)	$\frac{5}{2}^-$	4598

The hidden-charm pentaquark mass spectra in this work is slightly higher than the three narrow pentaquark-like states, $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$ measured by LHCb. But two $1/2^-$ mixing states could be $P_c(4440)^+$, and $P_c(4457)^+$.

N=3, L =1 Λ Resonances of negative-parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{cal}_{N=3}$	M^{exp}
$\Lambda(70, {}^{2}8, 3, 1^-)$	*	$\frac{1}{2}^-$	2019	Λ(2000)
$\Lambda(70, {}^{2}8, 3, 1^-)$	*	$\frac{3}{2}^-$	2019	Λ(2050)
$\Lambda(70, {}^{4}8, 3, 1^-)$	-	$\frac{1}{2}^-$	2262	missing
$\Lambda(70, {}^{4}8, 3, 1^-)$	*	$\frac{5}{2}^-$	2262	$?\Lambda$(2080)
$\Lambda(70, {}^{4}8, 3, 1^-)$	*	$\frac{3}{2}^-$	2262	$?\Lambda$(2325)
$\Lambda^*(70, {}^{2}1, 3, 1^-)$	*	$\frac{1}{2}^-$	2019	$?\Lambda$(2000)
$\Lambda^*(70, {}^{2}1, 3, 1^-)$	*	$\frac{3}{2}^-$	2019	$?\Lambda$(2050)

The L=1 radial excited Λ octet and singlet states can also be the Λ(2000) and Λ(2050) resonances.

N=3 L=3 only mixed symmetric type, the Λ Resonances of negative-parity applied to fit the model parameters.

$(\Gamma, {}^{2s+1}D, N, L^P)$	Status	J^P	$M^{cal}_{L=3}$	M^{exp}
$\Lambda(70, {}^2 8, 3, 3^-)$	*	$\frac{5}{2}^-$	2113	$\Lambda(2080)$
$\Lambda(70, {}^2 8, 3, 3^-)$	****	$\frac{7}{2}^-$	2113	$\Lambda(2100)$
$\Lambda(70, {}^4 8, 3, 3^-)$	*	$\frac{3}{2}^-$	2356	$\Lambda(2325)$
$\Lambda(70, {}^4 8, 3, 3^-)$	-	$\frac{5}{2}^-$	2356	missing
$\Lambda(70, {}^4 8, 3, 3^-)$	-	$\frac{7}{2}^-$	2356	missing
$\Lambda(70, {}^4 8, 3, 3^-)$	-	$\frac{9}{2}^-$	2356	missing
$\Lambda^*(70, {}^2 1, 3, 3^-)$	*	$\frac{5}{2}^-$	2113	$?\Lambda(2080)$
$\Lambda^*(70, {}^2 1, 3, 3^-)$	****	$\frac{7}{2}^-$	2113	$?\Lambda(2100)$

The Λ resonances can also have some proper assignments, but the $\Lambda(1405)$, $\Lambda(1520)$, $\Lambda(1670)$ and $\Lambda(1690)$ are not able to be described at all.