$\Lambda(1405)$ *mediated triangle singularity in the* $K^- d \longrightarrow p\Sigma^-$ *reaction.*

arXiv:2105.09654 [nucl-th]

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Motivation: $\overline{K}N$ interaction background

Aim:

Study of the $K^- d \longrightarrow p\Sigma^-$ ($p\Sigma^- \longrightarrow K^- d$) reactions close to threshold for the first time.

- **Process driven by a triangle singularity (TS).**
- **This reaction have access to $\overline{K}N$ *subthreshold amplitudes***

$\overline{K}N$ Interaction:

Perturbative QCD is inappropriate to treat low energy hadron interactions.

Chiral Perturbation Theory (ChPT) is an effective theory with hadrons as degrees of freedom which respects the symmetries of QCD.

- limited to a moderate range of energies above threshold
- not applicable close to a resonance (singularity in the amplitude)

But it is not so straight forward …

Motivation: $\overline{K}N$ interaction background

$\overline{K}N$ interaction is dominated by the presence of the $\Lambda(1405)$ resonance, located only 27 MeV below the $\overline{K}N$ threshold.

- In 1995 Kaiser, Siegel and Weise reformulated the problem in terms of a **Unitary extension of ChPT (UChPT)** in coupled channels.

The pioneering work -- *Kaiser, Siegel, Weise,* NP A594 (1995) 325

E. Oset, A. Ramos, Nucl. Phys. A 636, 99 (1998).

J. A. Oller, U. -G. Meissner, Phys. Lett. B 500, 263 (2001).
M. F. M. Lutz, E. Kolomeitsev, Nucl. Phys. A 700, 193 (2002).
B. Borasoy, E. Marco, S. Wetzel, Phys. Rev. C 66, 055208 (2002).
C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003).
D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A 725, 181 (2003).
B. Borasoy, R. Nissler, W. Wiese, Eur. Phys. J. A 25, 79 (2005).
V.K. Magas, E. Oset, A. Ramos, Phys. Rev. Lett. 95, 052301 (2005).
B. Borasoy, U. -G. Meissner and R. Nissler, Phys. Rev. C 74, 055201 (2006).

All of them obtaining in general similar features:

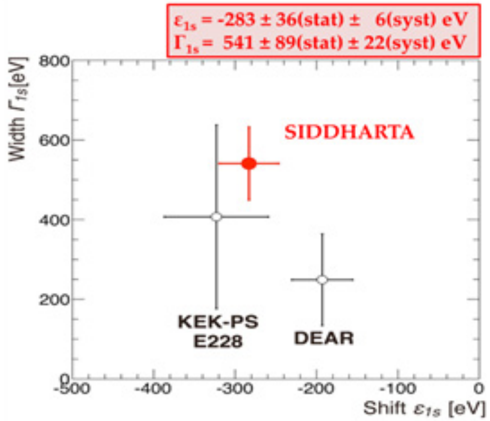
- $\overline{K}N$ scattering data reproduced very satisfactorily
- Two-pole structure of $\Lambda(1405)$

Motivation: $\overline{K}N$ interaction background

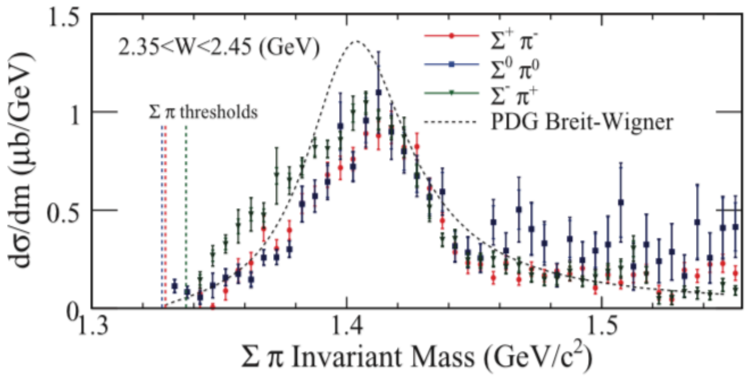
This topic has experienced a renewed interest after recent experimental advances:

The energy shift and width of the 1s state in kaonic hydrogen measured by SIDDHARTA@DAΦNE fixes the K^-p scattering length with a 20% precision!!!

M. Bazzi et al.,
Phys. Lett. B 704, 113 (2011).



Photoproduction $\gamma p \longrightarrow K^+\pi\Sigma$ data by the CLAS@Jlab provided detailed line shape results of the $\Lambda(1405)$

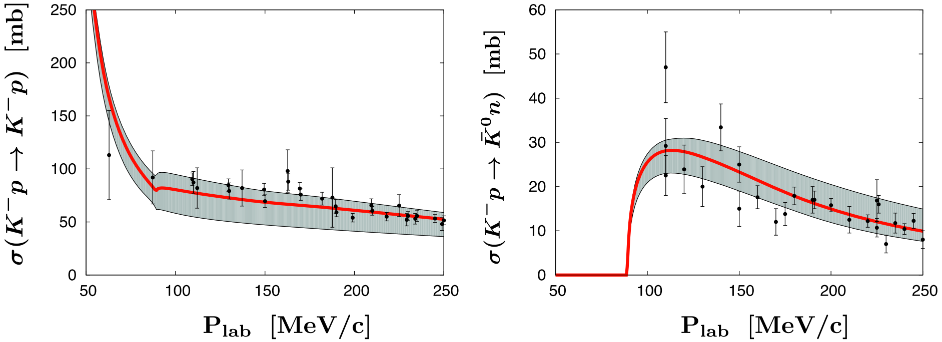


K. Moriya *et al.,* Phys. Rev. C**87**, 035206(2013).

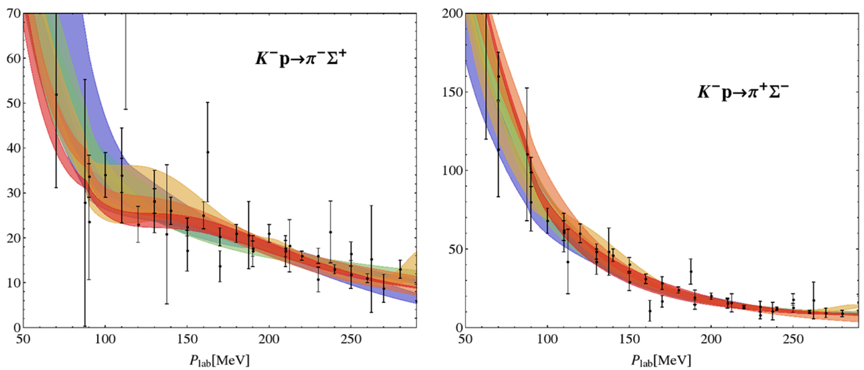
Y. Ikeda, T. Hyodo, W. Wiese, Nucl. Phys. A 881, 98 (2012).
A. Cieply and J. Smejkal, Nucl. Phys. A 881, 115 (2012).
Zhi-Hui Guo, J. A. Oller, Phys. Rev. C 87, 035202 (2013).
T. Mizutani, C. Fayard, B. Saghai and K. Tsushima, Phys. Rev. C 87, 035201 (2013).
L. Roca and E. Oset: Phys. Rev. C 87, 055201 (2013), Phys. Rev. C 88, 055206 (2013).
M. Mai and U. G. Meissner, Eur. Phys. J. A 51, 30 (2015).
Feijoo, V. Magas, A. Ramos, Phys. Rev. C 92, 015206 (2015); Nucl. Phys. A 954, 58 (2016); Phys. Rev. C 99 (2019) 035211.

Motivation: $\overline{K}N$ interaction background

$K^- p \longrightarrow MB\ (S=-1)$ *total cross sections from different groups:*

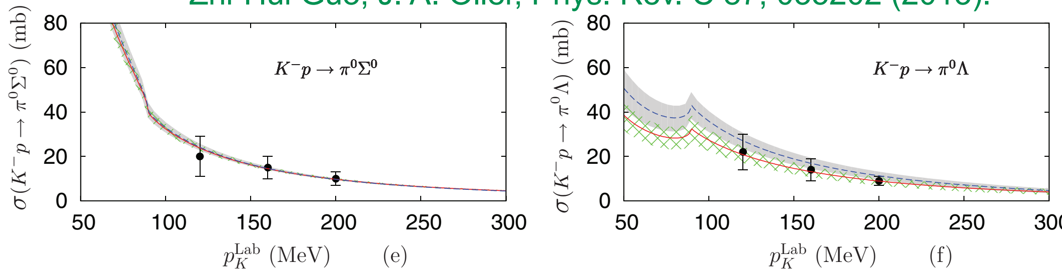


Y. Ikeda, T. Hyodo, W. Wiese, Nucl. Phys. A 881, 98 (2012).

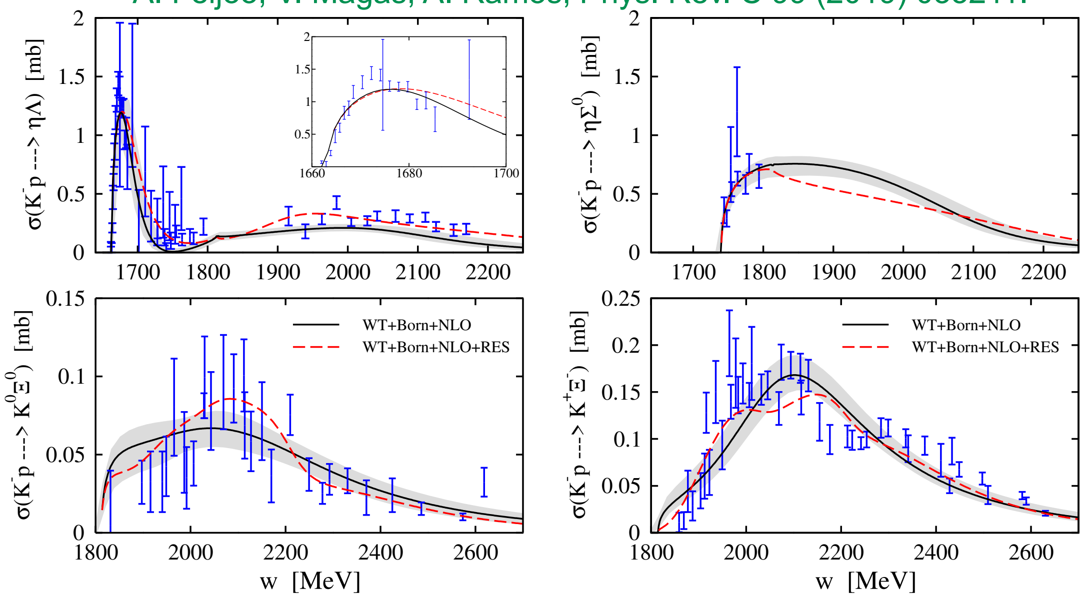


M. Mai and U. G. Meissner, Eur. Phys. J. A 51, 30 (2015).

Zhi-Hui Guo, J. A. Oller, Phys. Rev. C 87, 035202 (2013).



A. Feijoo, V. Magas, A. Ramos, Phys. Rev. C 99 (2019) 035211.



Motivation: $\overline{K}N$ interaction background

Threshold observables obtained from recent studies:

	γ	R_n	R_c	$a_p(K^-p \to K^-p)$	ΔE_{1s}	Γ_{1s}
Ikeda-Hyodo-Weise (NLO) [23]	2.37	0.19	0.66	$-0.70 + \mathrm{i}\,0.89$	306	591
Guo-Oller (fit I + II) [25]	$2.36^{+0.24}_{-0.23}$	$0.188^{+0.028}_{-0.029}$	$0.661^{+0.012}_{-0.011}$	$(-0.69 \pm 0.16) + \mathrm{i}\,(0.94 \pm 0.11)$	308 ± 56	619 ± 73
Mizutani et al (Model s) [26]	2.40	0.189	0.645	$-0.69 + \mathrm{i}\,0.89$	304	591
Mai-Meissner (fit 4) [29]	$2.38^{+0.09}_{-0.10}$	$0.191^{+0.013}_{-0.017}$	$0.667^{+0.006}_{-0.005}$		288^{+34}_{-32}	572^{+39}_{-38}
Cieply-Smejkal (NLO) [76]	2.37	0.191	0.660	$-0.73 + \mathrm{i}\,0.85$	310	607
Shevchenko (two-pole Model) [77]	2.36			$-0.74 + \mathrm{i}\,0.90$	308	602
WT+Born+NLO	$2.36^{+0.03}_{-0.03}$	$0.188^{+0.010}_{-0.011}$	$0.659^{+0.005}_{-0.002}$	$-0.65^{+0.02}_{-0.08} + \mathrm{i}\,0.88^{+0.02}_{-0.05}$	288^{+23}_{-8}	588^{+9}_{-40}
WT+NLO+Born+RES	2.36	0.189	0.661	$-0.64 + \mathrm{i}\,0.87$	283	587
Exp.	2.36 ± 0.04	0.189 ± 0.015	0.664 ± 0.011	$(-0.66 \pm 0.07) + \mathrm{i}\,(0.81 \pm 0.15)$	283 ± 36	541 ± 92

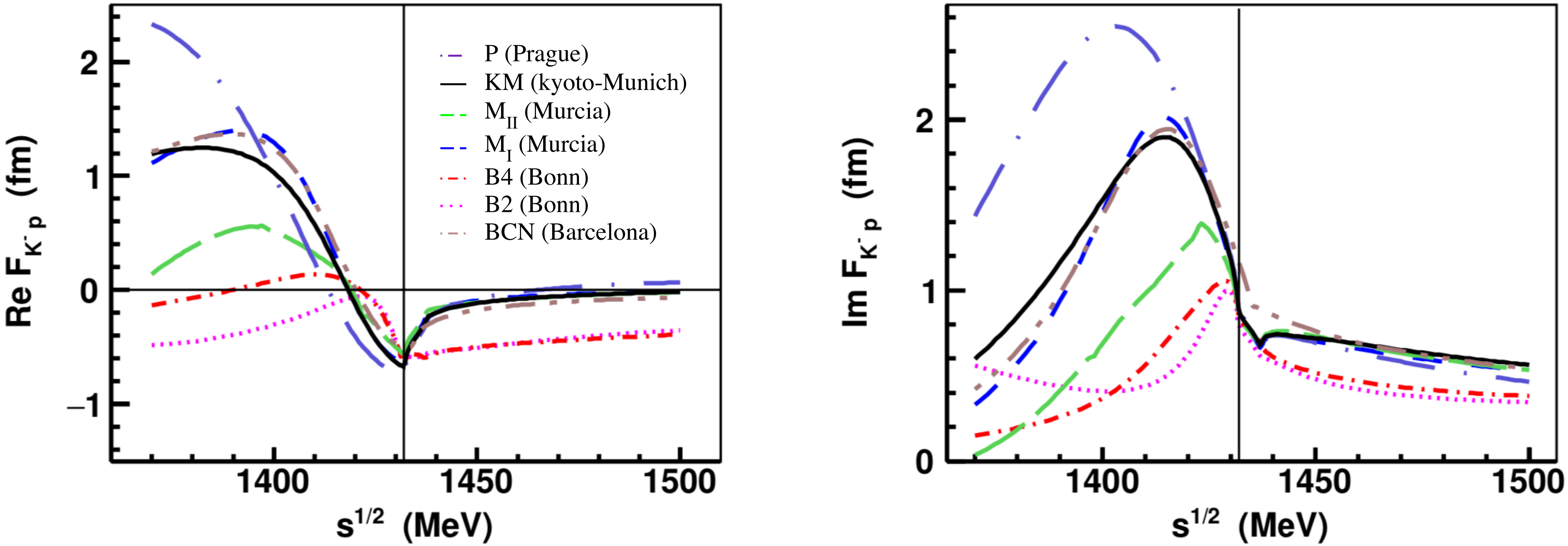
A. F., V. Magas, A. Ramos, Phys. Rev. C 99 (2019) 035211.

$$\gamma = \frac{\Gamma(K^-p \to \pi^+\Sigma^-)}{\Gamma(K^-p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

$$R_n = \frac{\Gamma(K^-p \to \pi^0\Lambda)}{\Gamma(K^-p \to \text{neutral states})} = 0.664 \pm 0.011$$

$$R_c = \frac{\Gamma(K^-p \to \pi^+\Sigma^-, \pi^-\Sigma^+)}{\Gamma(K^-p \to \text{inelastic channels})} = 0.189 \pm 0.015$$

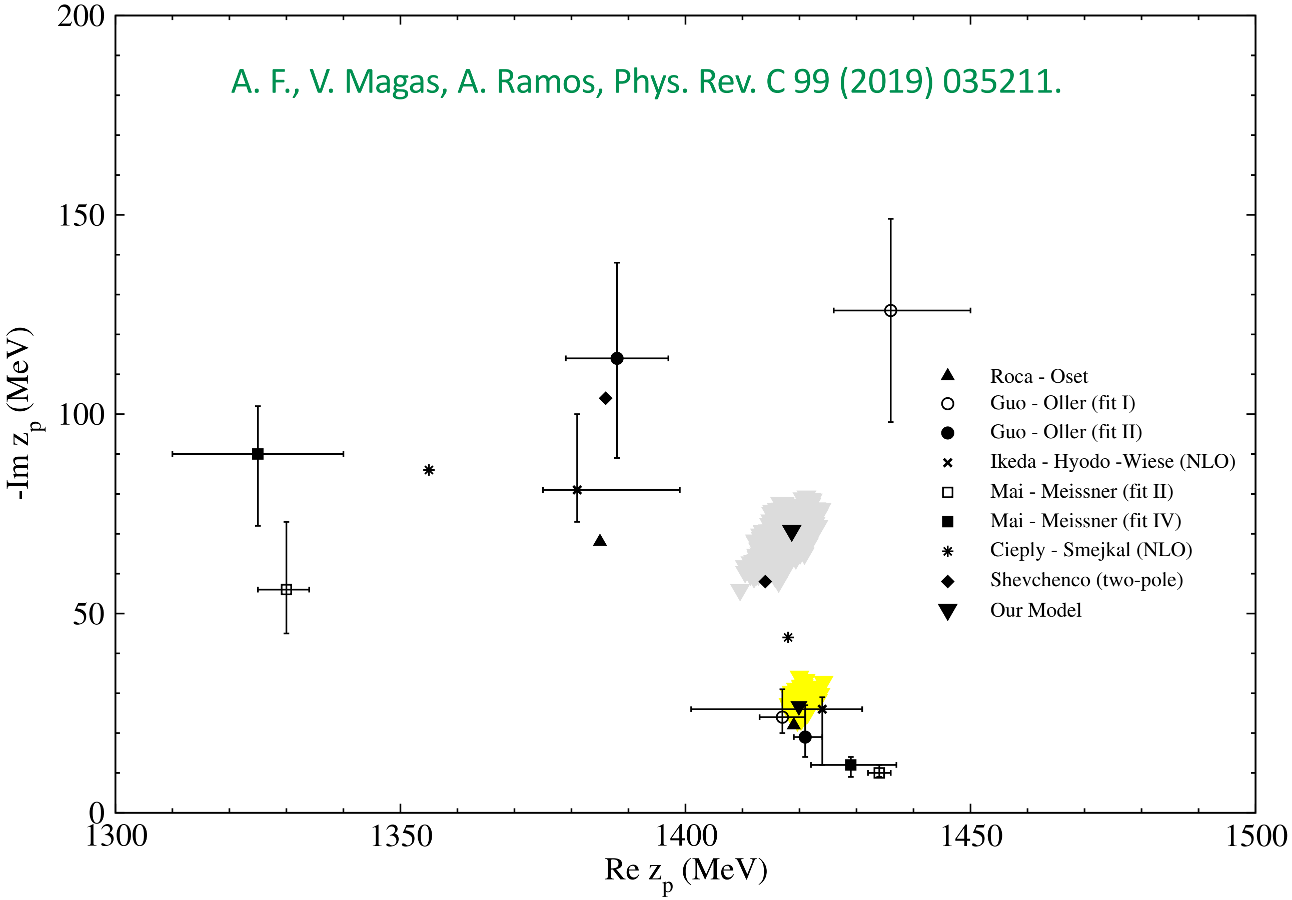
$K^-p \longrightarrow K^-p$ scattering amplitudes generated by recent chirally motivated approaches:



A. Cieply, J. Hrtánková, J. Mareš, E. Friedman, A. Gal and A. Ramos, AIP Conf. Proc. 2249, no.1, 030014 (2020).

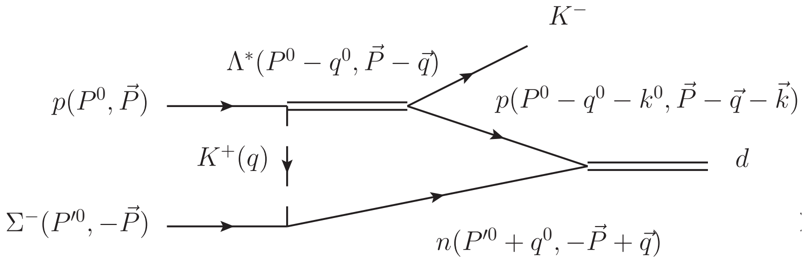
Motivation: $\overline{K}N$ interaction background

Pole positions of the $\Lambda(1405)$ for some state-of-the-art models:

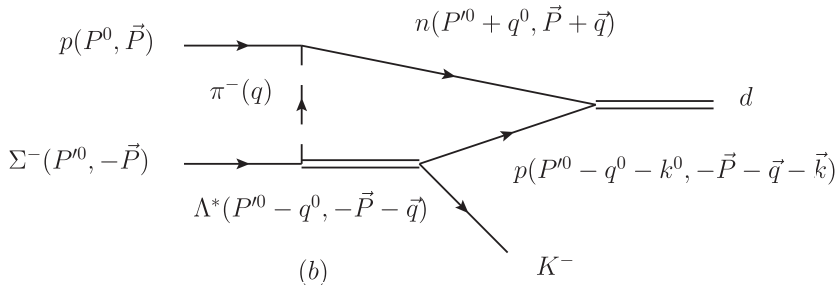


Formalism I: Mechanisms + Amplitudes

$p\Sigma^- \longrightarrow K^- d$ reaction proceeds via these 2 mechanisms:



(a)



(b)

$$-it^{(a)} = (-i)g_{\Lambda^*,K^-p}(-i)g_{\Lambda^*,K^-p}(-i)g_d\frac{D-F}{2f}\int\frac{d^4q}{(2\pi)^4}\vec{\sigma}_2\vec{q}\frac{i}{q^2-m_K^2+i\epsilon}\frac{M_{\Lambda^*}}{E^*_\Lambda}\frac{i}{P^0-q^0-E^*_\Lambda(\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}}{2}}$$
$$\times\frac{M_N}{E_N}\frac{i}{P^0-q^0-k^0-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon}\frac{M_N}{E'_N}\frac{i}{P'^0+q^0-E'_N(-\vec{P}+\vec{q})+i\epsilon}\theta(q_{\rm max}-|\vec{P}-\vec{q}-\frac{\vec{k}}{2}|),$$
$$-it^{(b)} = -g_{\Lambda^*,K^-p}g_{\Lambda^*,\pi^+\Sigma^-}g_d\frac{f_{\pi NN}}{m_\pi}i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_\pi^2+i\epsilon}\frac{M_{\Lambda^*}}{E_{\Lambda^*}}\frac{1}{P'^0-q^0-E_{\Lambda^*}(-\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}}{2}}$$
$$\times\vec{\sigma}_1\cdot\vec{q}\frac{M_N}{E_N}\frac{1}{P'^0-q^0-k^0-E_N(-\vec{P}-\vec{q}-\vec{k})+i\epsilon}\frac{M_N}{E'_N}\frac{1}{P^0+q^0-E'_N(\vec{P}+\vec{q})+i\epsilon}\theta(q_{\rm max}-|-\vec{P}-\vec{q}-\frac{\vec{k}}{2}|),$$

Formalism I: Mechanisms + Amplitudes

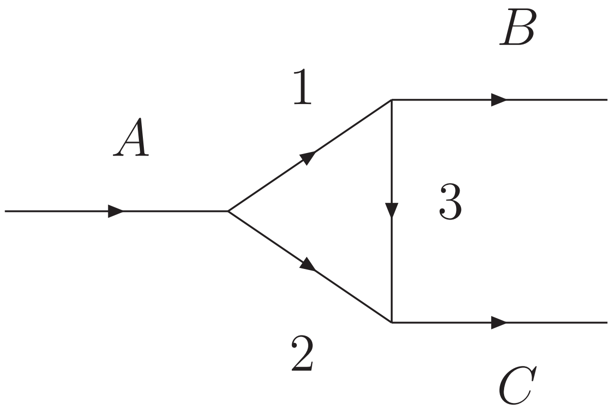
$$-it_{ij}^{(a)} = g_{\Lambda^*,K^-p}\, g_{\Lambda^*,K^-p}\, g_d \frac{D-F}{2f} \int \frac{d^3q}{(2\pi)^3} V_{ij}(q) F(P^0, P'^0, \vec{q}, \omega_K(\vec{q}), \vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_K)$$

$$-it_{ij}^{(b)} = -g_{\Lambda^*,K^-p}\, g_{\Lambda^*,\pi^+\Sigma^-}\, g_d \frac{f_{\pi NN}}{m_\pi} \int \frac{d^3q}{(2\pi)^3} W_{ij}(q) F(P'^0, P^0, \vec{q}, \omega_\pi(\vec{q}), -\vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_\pi)$$

$$\begin{aligned}
F(P^0, P'^0, \vec{q}, \omega, \vec{P}, \vec{k}) &= \frac{1}{2\omega(\vec{q})} \frac{M_N}{E_N(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{M_{\Lambda^*}}{E_{\Lambda^*}(\vec{P}-\vec{q})} \\
&\times \frac{\theta(q_{\max} - |\vec{P}-\vec{q}-\frac{\vec{k}}{2}|)}{\sqrt{s} - k^0 - E_N(-\vec{P}+\vec{q}) - E_N(\vec{P}-\vec{q}-\vec{k}) + i\epsilon} \\
&\times \left\{ \frac{1}{P^0 - \omega(\vec{q}) - E_{\Lambda^*}(\vec{P}-\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \frac{1}{P^0 - \omega(\vec{q}) - k^0 - E_N(\vec{P}-\vec{q}-\vec{k}) + i\epsilon} \right. \\
&+ \frac{1}{P^0 - E_{\Lambda^*}(\vec{P}-\vec{q}) - \omega(\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \frac{1}{\sqrt{s} - E_{\Lambda^*}(\vec{P}-\vec{q}) - E_N(-\vec{P}+\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \\
&+ \left. \frac{1}{P'^0 - E_N(-\vec{P}+\vec{q}) - \omega(\vec{q}) + i\epsilon} \frac{1}{\sqrt{s} - E_{\Lambda^*}(\vec{P}-\vec{q}) - E_N(-\vec{P}+\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \right\}
\end{aligned}$$

Formalism I: Triangle singularity

TS can be developed when the 3 intermediate particles $\Lambda(1405)$ (1), n (2), p (3):



+ ***1, 2, 3 particles are simultaneously placed on Shell and they are colinear fulfilling Norton-Coleman theorem***

S. Coleman and R. E. Norton, Nuovo Cim. 38, 438 (1965).

These conditions are encoded in the following equation:

*Momentum of the **n** in the $p\Sigma^-$ rest frame* → $$q_{\rm on} = q_{a^-}$$ ← *Solution for the **n** momentum in the decay of the **d** for the moving **d** in the $p\Sigma^-$ rest frame*

M. Bayar, F. Aceti, F.-K. Guo, and E. Oset, Phys. Rev. D 94, 074039 (2016)

→ For this study, **TS** should appear at $\sqrt{s} \approx 2380 MeV$

Formalism I:

Differential cross section for the $K^- d \longrightarrow p\Sigma^-$ reaction.

$$\frac{d\sigma}{d\cos\theta_p} = \frac{1}{4\pi}\frac{1}{s} M_p M_{\Sigma^-} M_d \frac{p}{k} \bar{\sum}\sum |t|^2 \qquad \bar{\sum}\sum |t|^2 = \frac{1}{3}\sum_{i,j} |t_{ij}^{(a)} + t_{ij}^{(b)}|^2$$

$d\ (S=1)$ polarizations

$$j = \uparrow\uparrow, \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \downarrow\downarrow$$

$p\Sigma^-$ spin configurations

$$i = \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow$$

Pole couplings and coordinates needed to compute the cross section:

State	$g_{\Lambda^*,\bar{K}N}$	$g_{\Lambda^*,\pi\Sigma}$	(Mass, $\frac{\Gamma}{2}$)
$\Lambda(1390)$	$1.2 + i\,1.7$	$-2.5 - i\,1.5$	$(1390, 66)$
$\Lambda(1426)$	$-2.5 + i\,0.94$	$0.42 - i\,1.4$	$(1426, 16)$

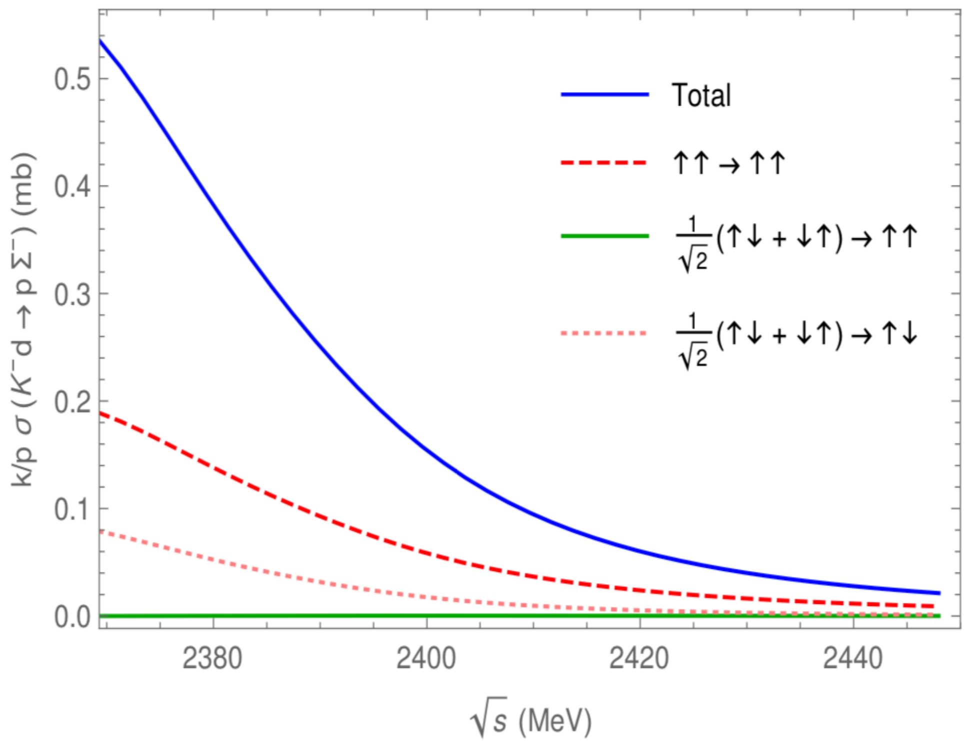
$$g_{\Lambda^*,K^-p} = \frac{1}{\sqrt{2}} g_{\Lambda^*,\bar{K}N}$$

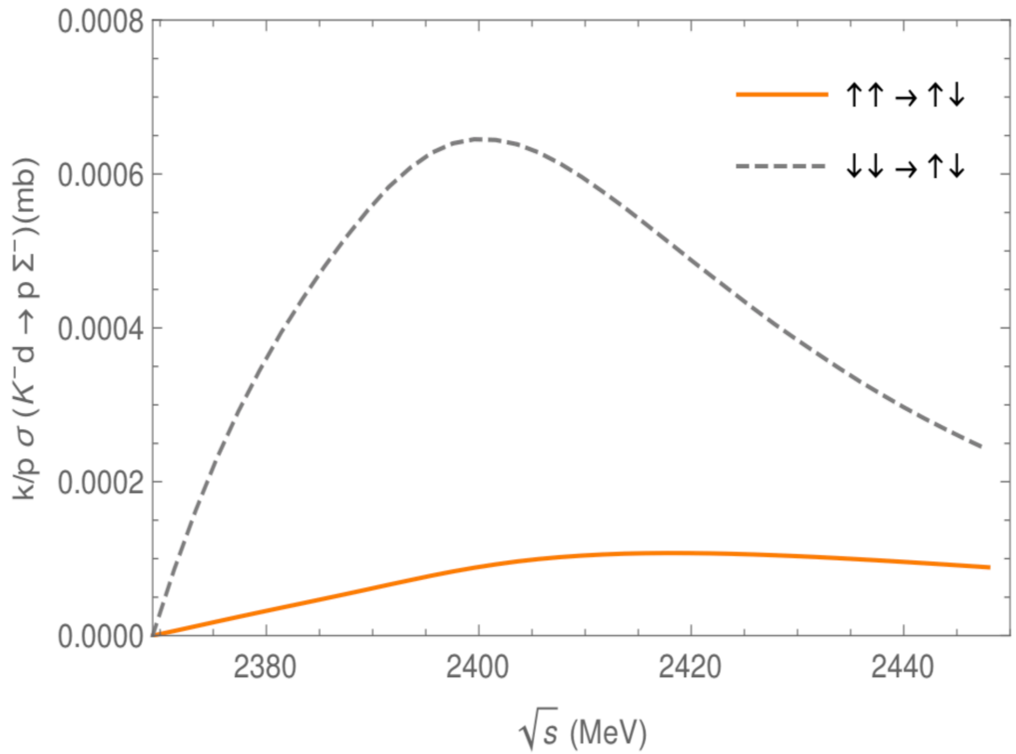
$$g_{\Lambda^*,\pi^+\Sigma^-} = -\frac{1}{\sqrt{3}} g_{\Lambda^*,\pi\Sigma}$$

E. Oset, A. Ramos, Nucl. Phys. A 636, 99 (1998).

Results I: spin transitions

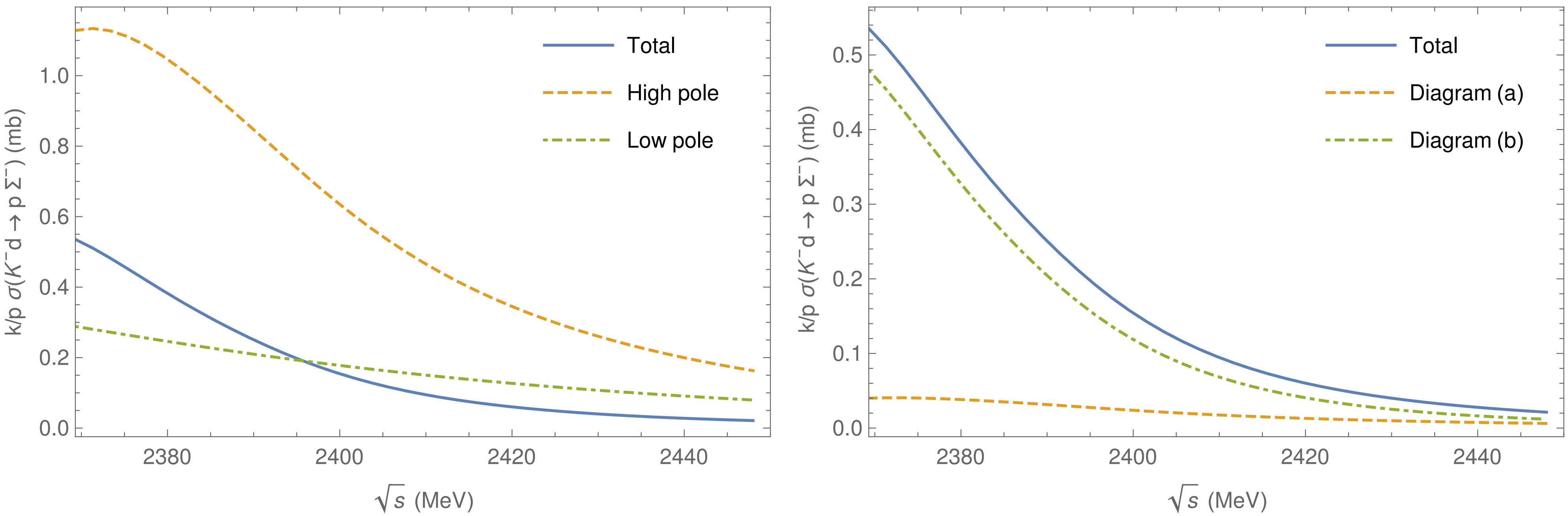
Contribution of several spin transitions to the $K^- d \longrightarrow p\Sigma^-$ cross section.





Results I: role of the mechanisms and poles in the total cross section

Contribution of the high and low mass poles and the mechanisms (a) and (b) to $\mathrm{K}/p \cdot \sigma(K^- d \longrightarrow p\Sigma^-)$.



Formalism II: imcorporation of the explicit $\overline{K}N$ amplitudes and ψ Bonn deuteron wave function

Deuteron wave function replacement:

$$g_d \frac{M_N}{E(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{\theta(q_{max}-|\vec{P}-\vec{q}-\frac{\vec{k}}{2}|)}{\sqrt{s}-k^0-E_N(-\vec{P}+\vec{q})-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon} \longrightarrow -(2\pi)^{3/2}\psi(\vec{P}-\vec{q}-\frac{\vec{k}}{2})$$

R. Machleidt, Phys. Rev. C 63, 024001 (2001)

Formal equivalence between Breit-Wigner amplitudes and theoretical amplitudes:

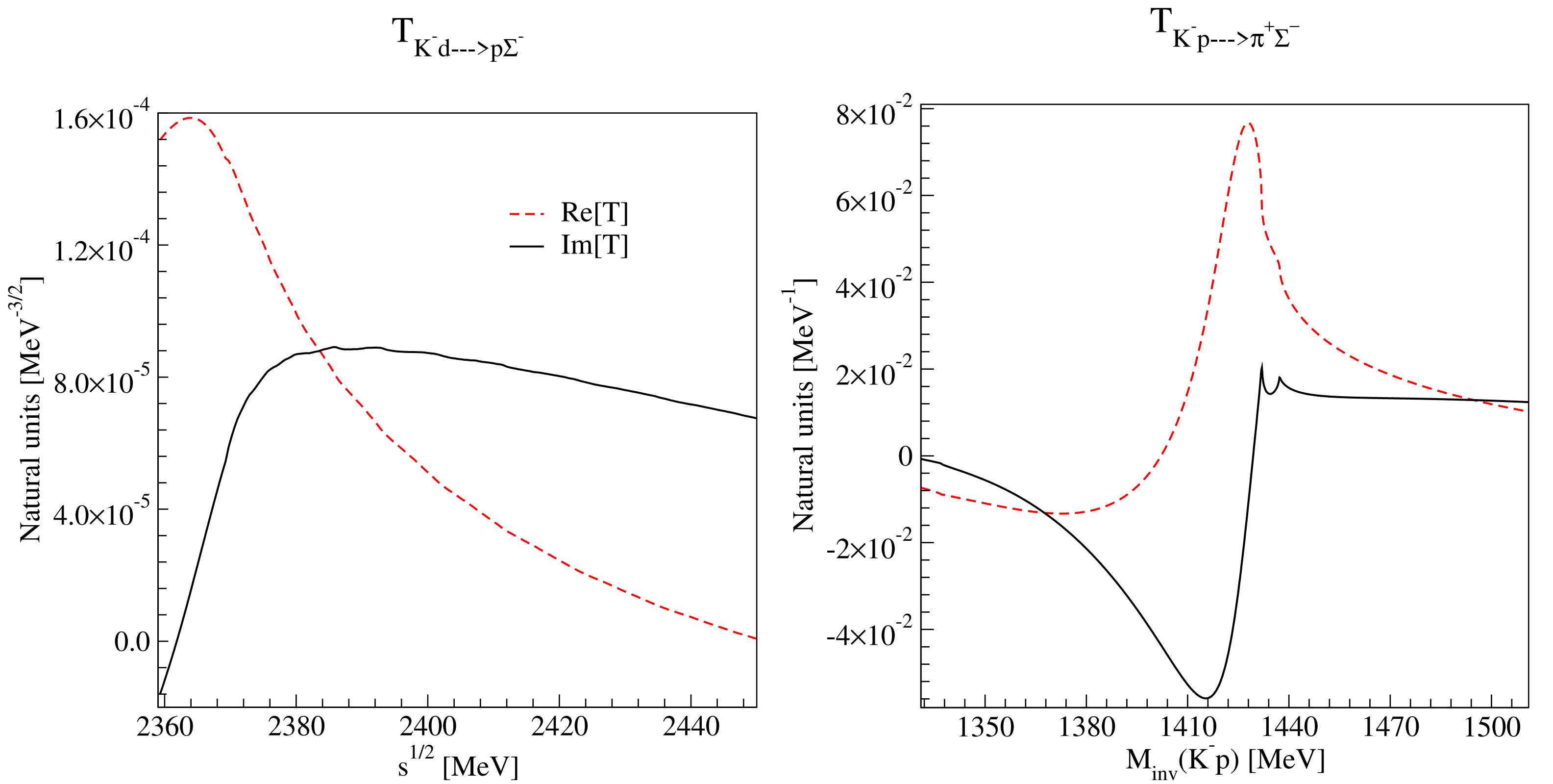
$$\sum_{i=1}^{2} \frac{M^{(i)}_{\Lambda^*}}{E^{(i)}_{\Lambda^*}(\vec{P}-\vec{q})} \frac{g^{(i)}_{\Lambda^*,K^-p} g^{(i)}_{\Lambda^*,K^-p}}{\sqrt{s}-E_N(-\vec{p}+\vec{q})-E^{(i)}_{\Lambda^*}(\vec{P}-\vec{q})+i\frac{\Gamma^{(i)}_{\Lambda^*}}{2}} \equiv t_{K^-p,K^-p}(M_{inv})$$

$$\sum_{i=1}^{2} \frac{M^{(i)}_{\Lambda^*}}{E^{(i)}_{\Lambda^*}(\vec{P}-\vec{q})} \frac{g^{(i)}_{\Lambda^*,K^-p} g^{(i)}_{\Lambda^*,\pi^+\Sigma^-}}{\sqrt{s}-E_N(\vec{p}+\vec{q})-E^{(i)}_{\Lambda^*}(-\vec{P}-\vec{q})+i\frac{\Gamma^{(i)}_{\Lambda^*}}{2}} \equiv t_{K^-p,\pi^+\Sigma^-}(M'_{inv})$$

$$M^2_{\text{inv}} = s + M_N^2 - 2\sqrt{s}E_N(-\vec{P}+\vec{q}) \qquad M'^2_{\text{inv}} = s + M_N^2 - 2\sqrt{s}E_N(\vec{P}+\vec{q})$$

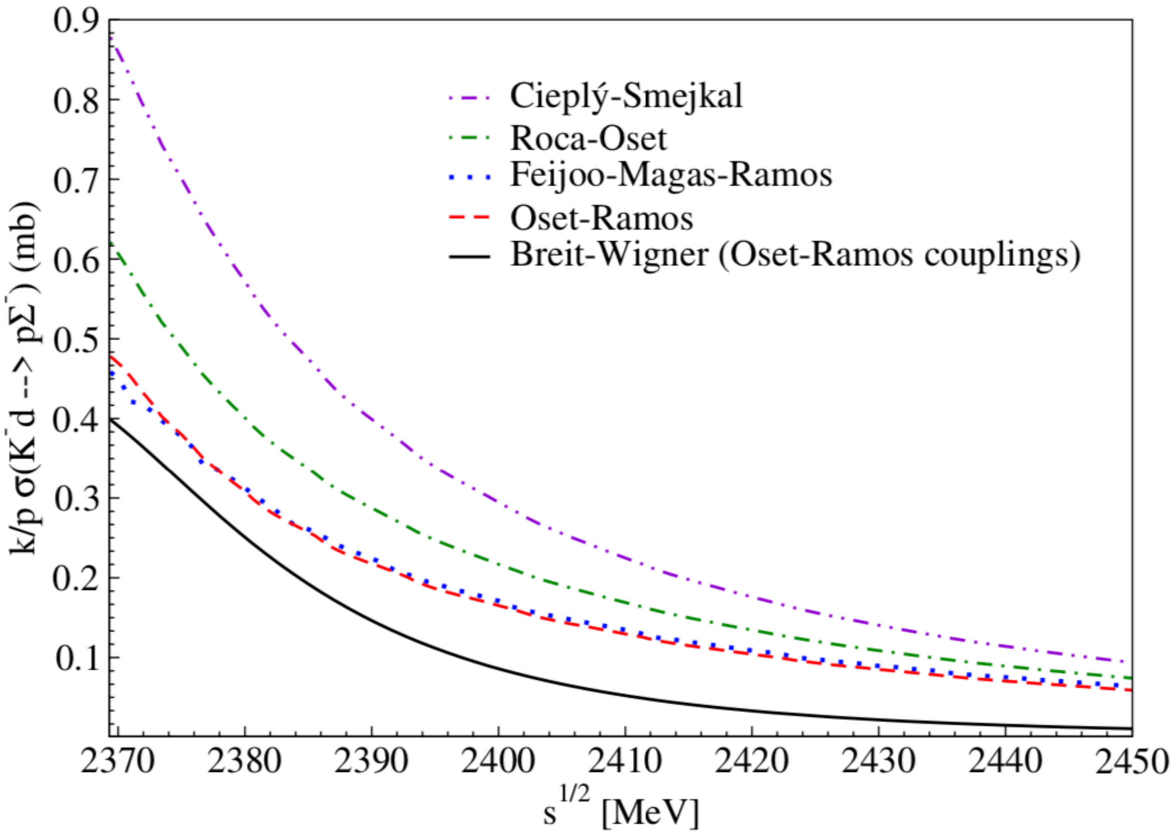
Results II: *using explicit* $\overline{K}N$ *amplitudes* + ψ *Bonn deuteron wave function*

Energy dependence of the real and the imaginary parts of the $K^-d \longrightarrow p\Sigma^-$ and $K^-p \longrightarrow \pi^+\Sigma^-$ amplitudes.



Results II: using explicit $\overline{K}N$ amplitudes + ψ Bonn deuteron wave function

$K^- d \longrightarrow p\Sigma^-$ cross sections for different considered models.



CONCLUSIONS

We have studied the $K^- d \longrightarrow p\Sigma^-$ ($p\Sigma^- \longrightarrow K^- d$) reaction via a triangular topology (with two possible mechanisms) that embeds a TS.

- The peak associated to the TS shows up few MeV above $K^- d$ threshold, being clearly visible in the case of the narrow (high mass) $\Lambda(1405)$ state.
- The mechanism involving the pion exchange has shown to be the dominant one.
- We have seen that the particular dependence of the $K^- d \longrightarrow p\Sigma^-$ transition on the $\bar{K}N$ amplitudes below threshold weighted by the structures tied to TS makes this process very sensitive to the different models.

The measurement of this reaction will provide valuable information for $\bar{K}$ bound states in nuclei as well as it will help to narrow the uncertainty around the location of the lower mass pole of the $\Lambda(1405)$.

Backup slides

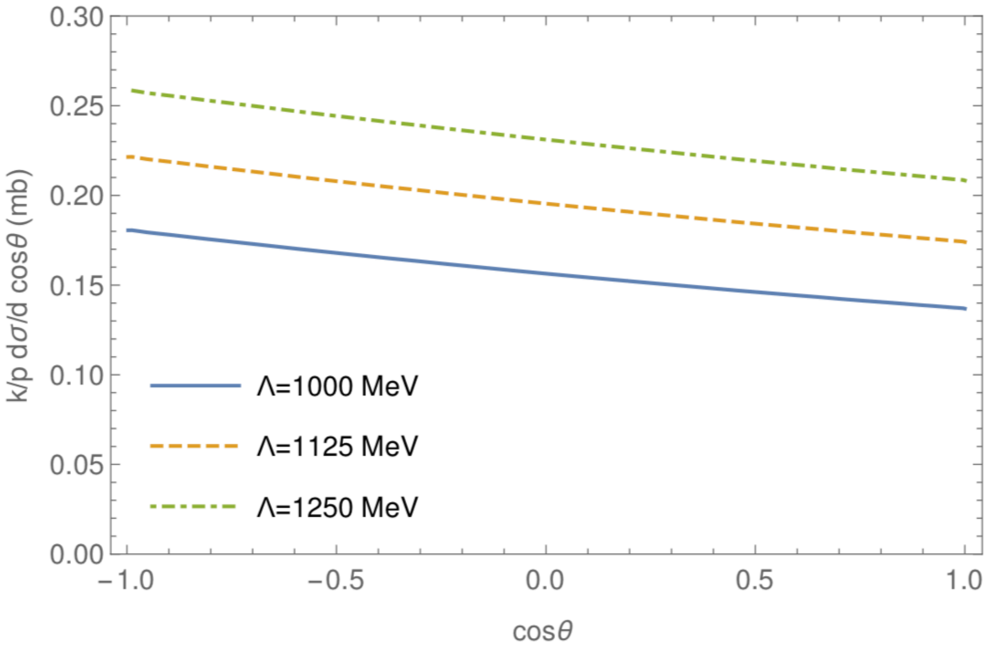
$$-it_{ij}^{(a)} = g_d \frac{D-F}{2f} \int \frac{d^3q}{(2\pi)^3} V_{ij}(q) F'(P^0, P'^0, \vec{q}, \omega_K(\vec{q}), \vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_K)$$

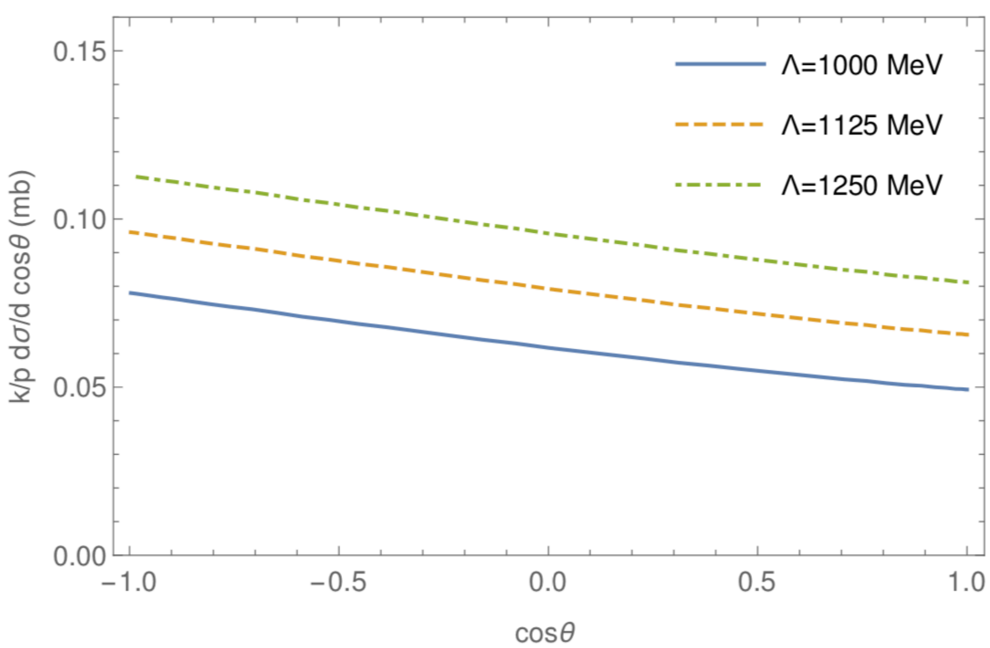
$$-it_{ij}^{(b)} = -g_d \frac{D+F}{2f} \int \frac{d^3q}{(2\pi)^3} W_{ij}(q) G'(P^0, P'^0, \vec{q}, \omega_\pi(\vec{q}), \vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_\pi)$$

$$F'(P^0, P'^0, \vec{q}, \omega_K, \vec{P}, \vec{k}) = \frac{1}{2\omega(\vec{q})} \frac{M_N}{E_N(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{1}{\sqrt{s}-k^0-E_N(-\vec{P}+\vec{q})-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon}$$
$$\times \Bigg\{ \sum_{i=1,2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})} \frac{(g_{\Lambda^*,K^-p}^{(i)})^2}{P^0-\omega_K(\vec{q})-E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \frac{1}{P^0-\omega_K(\vec{q})-k^0-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon}$$
$$+ \left(\frac{1}{P^0-E_{\Lambda^*}(\vec{P}-\vec{q})-\omega_K(\vec{q})+i\frac{\Gamma_{\Lambda^*}}{2}} + \frac{1}{P'^0-E_N(-\vec{P}+\vec{q})-\omega_K(\vec{q})+i\epsilon} \right) t_{K^-p,K^-p}(M_{\text{inv}}) \Bigg\}$$

$$G'(P^0, P'^0, \vec{q}, \omega_K, \vec{P}, \vec{k}) = \frac{1}{2\omega(\vec{q})} \frac{M_N}{E_N(-\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(\vec{P}+\vec{q})} \frac{1}{\sqrt{s}-k^0-E_N(\vec{P}+\vec{q})-E_N(-\vec{P}-\vec{q}-\vec{k})+i\epsilon}$$
$$\times \Bigg\{ \sum_{i=1,2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(-\vec{P}-\vec{q})} \frac{g_{\Lambda^*,K^-p}^{(i)} g_{\Lambda^*,\pi^+\Sigma^-}^{(i)}}{P^0-\omega_\pi(\vec{q})-E_{\Lambda^*}^{(i)}(-\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \frac{1}{P'^0-\omega_\pi(\vec{q})-k^0-E_N(-\vec{P}-\vec{q}-\vec{k})+i\epsilon}$$
$$+ \left(\frac{1}{P'^0-E_{\Lambda^*}(-\vec{P}-\vec{q})-\omega_\pi(\vec{q})+i\frac{\Gamma_{\Lambda^*}}{2}} + \frac{1}{P^0-E_N(\vec{P}+\vec{q})-\omega_\pi(\vec{q})+i\epsilon} \right) t_{K^-p,\pi^+\Sigma^-}(M'_{\text{inv}}) \Bigg\}$$

Backup slides





Form Factor $$\mathcal{F}(\Lambda, m_i) = \frac{\Lambda^2 - m_i^2}{\Lambda^2 + \vec{q}^2}$$

Effective Chiral Lagrangian:

$$\mathcal{L}^{eff}(B,U) = \mathcal{L}^{(1)}_{MB}(B,U) + \mathcal{L}^{(2)}_{MB}(B,U)$$

→ derive an interaction kernel $\mathbf{V_{ij}}$

- **Leading order (LO)**

$$\mathcal{L}^{(1)}_{MB} = \langle \bar{B}(i\gamma_\mu D^\mu - M_0)B\rangle + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle$$

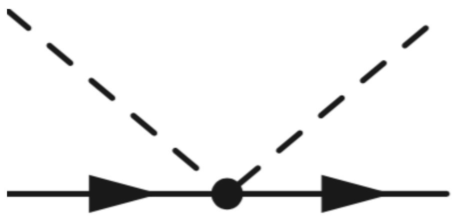
Motivation: $\overline{K}N$ interaction background

Effective Chiral Lagrangian:

$$\mathcal{L}^{eff}(B,U) = \mathcal{L}^{(1)}_{MB}(B,U) + \mathcal{L}^{(2)}_{MB}(B,U)$$ → derive an interaction kernel $\mathbf{V}_{ij}$

- **Leading order (LO)**

$$\mathcal{L}^{(1)}_{MB} = \underbrace{\boxed{\langle \bar{B}(i\gamma_\mu D^\mu - M_0)B\rangle}}_{\text{Tomozawa-Weinberg term (WT)}} + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle$$



1. Dominant contribution.
2. Interaction mediated, basically, by the constant f of the leptonic decay of the pseudoscalar meson

$$V_{ij}^{WT} = -\frac{N_i N_j}{4f^2}C_{ij}\left\{(2\sqrt{s} - M_i - M_j)\chi_f^{\dagger s'}\chi_0^s + \frac{2\sqrt{s}+M_i+M_j}{(E_i+M_i)(E_j+M_j)}\chi_f^{\dagger s'}\left[\vec{q}_j\cdot\vec{q}_i + i(\vec{q}_j\times\vec{q}_i)\cdot\vec{\sigma}\right]\chi_0^s\right\}$$

Lagrangian:

$$\mathcal{L}^{eff}(B,U) = \mathcal{L}^{(1)}_{MB}(B,U) + \mathcal{L}^{(2)}_{MB}(B,U)$$

→ derive an interaction kernel $\mathbf{V_{ij}}$

- **Leading order (LO)**

$$\mathcal{L}^{(1)}_{MB} = \langle \bar{B}(i\gamma_\mu D^\mu - M_0)B\rangle + \boxed{\frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle}$$

Born terms

1. Direct diagram (s-channel Born term)

$$V^D_{ij} = V^D_{ij}(D,F)$$

2. Cross diagram (u-channel Born term)

$$V^C_{ij} = V^C_{ij}(D,F)$$

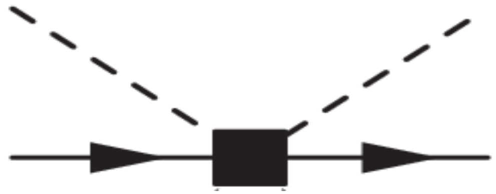
1. Direct diagram (s-channel Born term)

$$
\begin{aligned}
V_{ij}^{D} \quad = \quad & \frac{N_i N_j}{12 f^2} \sum_k \frac{C_{\bar{i}i,k}^{(\mathrm{Born})} C_{\bar{j}j,k}^{(\mathrm{Born})}}{s - M_k^2} \Big\{ (\sqrt{s} - M_k)(s + M_i M_j - \sqrt{s}(M_i + M_j)) \chi_j^{\dagger s'} \chi_i^{s} \\
& + \frac{(s + \sqrt{s}(M_i + M_j) + M_i M_j)(\sqrt{s} + M_k)}{(E_i + M_i)(E_j + M_j)} \chi_j^{\dagger s'} \left[\vec{q}_j \cdot \vec{q}_i + i(\vec{q}_j \times \vec{q}_i) \cdot \vec{\sigma} \right] \chi_i^{s} \Big\}
\end{aligned}
$$

2. Cross diagram (u-channel Born term)

$$
\begin{aligned}
V_{ij}^{C} \quad = \quad & -\frac{N_i N_j}{12 f^2} \sum_k \frac{C_{\bar{j}k,i}^{(\mathrm{Born})} C_{\bar{i}k,j}^{(\mathrm{Born})}}{u - M_k^2} \Big\{ \left[u(\sqrt{s} + M_k) + \sqrt{s}(M_j(M_i + M_k) + M_i M_k) \right. \\
& \left. - M_j(M_i + M_k)(M_i + M_j) - M_i^2 M_k \right] \chi_j^{\dagger s'} \chi_i^{s} + \left[u(\sqrt{s} - M_k) + \sqrt{s}(M_j(M_i + M_k) + M_i M_k) \right. \\
& \left. + M_j(M_i + M_k)(M_i + M_j) + M_i^2 M_k \right] \chi_j^{\dagger s'} \frac{\vec{q}_j \cdot \vec{q}_i + i(\vec{q}_j \times \vec{q}_i) \cdot \vec{\sigma}}{(E_i + M_i)(E_j + M_j)} \chi_i^{s} \Big\}
\end{aligned}
$$

Motivation: $\overline{K}N$ interaction background



- **Next to leading order (NLO),** just considering the **contact term**

$$
\begin{aligned}
\mathcal{L}_{\phi B}^{(2)} \quad = \quad & b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + d_1\langle\bar{B}\{u_\mu,[u^\mu,B]\}\rangle \\
& +d_2\langle\bar{B}[u_\mu,[u^\mu,B]]\rangle + d_3\langle\bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle\bar{B}B\rangle\langle u^\mu u_\mu\rangle \\
& -\frac{g_1}{8M_N^2}\langle\bar{B}\{u_\mu,[u_\nu,\{D^\mu,D^\nu\}B]\}\rangle - \frac{g_2}{8M_N^2}\langle\bar{B}[u_\mu,[u_\nu,\{D^\mu,D^\nu\}B]]\rangle \\
& -\frac{g_3}{8M_N^2}\langle\bar{B}u_\mu\rangle\langle[u_\nu,\{D^\mu,D^\nu\}B]\rangle - \frac{g_4}{8M_N^2}\langle\bar{B}\{D^\mu,D^\nu\}B\rangle\langle u_\mu u_\nu\rangle \\
& -\frac{h_1}{4}\langle\bar{B}[\gamma^\mu,\gamma^\nu]Bu_\mu u_\nu\rangle - \frac{h_2}{4}\langle\bar{B}[\gamma^\mu,\gamma^\nu]u_\mu[u_\nu,B]\rangle - \frac{h_3}{4}\langle\bar{B}[\gamma^\mu,\gamma^\nu]u_\mu\{u_\nu,B\}\rangle \\
& -\frac{h_4}{4}\langle\bar{B}[\gamma^\mu,\gamma^\nu]u_\mu\rangle\langle u_\nu,B\rangle + h.c.
\end{aligned}
$$

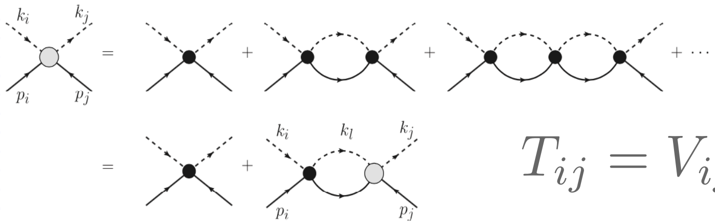
terms taken into account at higher energies

- Contributions with g_3 get cancelled
- $b_0, b_D, b_F, d_1, d_2, d_3, d_4, g_1, g_2, g_4, h_1, h_2, h_3, h_4$ are not well established, so they should be treated as parameters of the model!

- **Next to leading order (NLO),** just considering the **contact term**

$$
\begin{aligned}
V_{ij}^{NLO} \quad = \quad & \frac{N_i N_j}{f^2}\left[D_{ij} - 2L_{ij}q_j{}^{\mu}q_{i\,\mu} + \frac{1}{2M_N^2}g_{ij}(p_i{}^{\mu}q_{j\,\mu}p_i{}^{\nu}q_{i\,\nu} + p_j{}^{\mu}q_{j\,\mu}p_j{}^{\nu}q_{i\,\nu})\right]\Bigg(\chi_j^{\dagger s'}\chi_i^s \\
& -\chi_j^{\dagger s'}\frac{\vec{q}_j\cdot\vec{q}_i + i(\vec{q}_j\times\vec{q}_i)\cdot\vec{\sigma}}{(E_i+M_i)(E_j+M_j)}\chi_i^s\Bigg) + \frac{N_i N_j}{f^2}h_{ij}\left[-\left(\frac{q_{j\,0}q_i{}^2}{E_i+M_i} + \frac{q_{i\,0}q_j{}^2}{E_j+M_j}\right.\right. \\
& \left. + \frac{q_j{}^2 q_i{}^2}{(E_i+M_i)(E_j+M_j)} + \frac{(\vec{q}_j\cdot\vec{q}_i)^2}{(E_i+M_i)(E_j+M_j)}\right)\chi_j^{\dagger s'}\chi_i^s \\
& + \left(\frac{q_{i\,0}}{E_i+M_i} + \frac{q_{j\,0}}{E_j+M_j}\right)\chi_j^{\dagger s'}\vec{q}_j\cdot\vec{q}_i\chi_i^s + \left(\frac{q_{i\,0}}{E_i+M_i} + \frac{q_{j\,0}}{E_j+M_j} + \right. \\
& \left.\left. \frac{\vec{q}_j\cdot\vec{q}_i}{(E_i+M_i)(E_j+M_j)} - 1\right) i\chi_j^{\dagger s'}(\vec{q}_j\times\vec{q}_i)\cdot\vec{\sigma}\chi_i^s\right]
\end{aligned}
$$

Unitarization via the Bethe-Salpeter equation which it is solved by factorizing ***V*** and ***T*** matrices on-shell out the internal integrals



$$T_{ij} = V_{ij} + V_{il} G_l V_{lj} + V_{il} G_l V_{lk} G_k V_{kj} + \ldots$$

$$T_{ij} = V_{ij} + V_{il} G_l T_{lj} \longrightarrow \boxed{T_{ij} = (1 - V_{il} G_l)^{-1} V_{lj}}$$

Pure algebraic equation

$$G_l = \frac{2M_l}{(4\pi)^2}\left\{a_l(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} + \frac{q_{\rm cm}}{\sqrt{s}}\ln\left[\frac{(s+2\sqrt{s}q_{\rm cm})^2 - (M_l^2 - m_l^2)^2}{(s-2\sqrt{s}q_{\rm cm})^2 - (M_l^2 - m_l^2)^2}\right]\right\}$$

subtraction constants for the dimensional regularization scale μ in all the k channels.

$$V_{ij} = V_{ij}^{WT} + V_{ij}^{D} + V_{ij}^{C} + V_{ij}^{NLO} \Longrightarrow T = (1 - VG)^{-1} V \Longrightarrow T_{ij}$$

Fitting parameters:

- Decay constant f either partially constrained $1.12 f_\pi^{exp} \leq f \leq 1.26\, f_\pi^{exp}$, f_π^{exp}=93 *MeV*
  or taking fixed values depending on the process f_π , f_K , f_η
- Axial vector couplings *D, F* varying around the experimental values yet imposing $g_A = D + F = 1.26$

→ Most of the models fix them at the exp. values $D = 0.80, F = 0.46$

- 7 (14) coefficients of the NLO lagrangian terms
  $b_0, b_D, b_F, d_1, d_2, d_3, d_4$ $(+\, g_1, g_2, g_4, h_1, h_2, h_3, h_4)$
- Parameters from the regularization method

→ vast majority employs dim. reg. with 6 subtracting constants (isospin symmetry):

$$a_{K^-p} = a_{\bar{K}^0 n} = a_{\bar{K}N}$$

$$a_{\pi\Lambda}$$

$$a_{\pi^+\Sigma^-} = a_{\pi^-\Sigma^+} = a_{\pi^0\Sigma^0} = a_{\pi\Sigma}$$

$$a_{\eta\Lambda}$$

$$a_{\eta\Sigma}$$

$$a_{K^+\Xi^-} = a_{K^0\Xi^0} = a_{K\Xi}$$

Motivation: $\overline{K}N$ interaction background

Available experimental data used (or not) to constrain the parameters present in the chirally motivated models:

Observable	Points	Observable	Points
$\sigma_{K^-p\to K^-p}$	23	$\sigma_{K^-p\to \bar{K}^0 n}$	9
$\sigma_{K^-p\to \pi^0\Lambda}$	3	$\sigma_{K^-p\to \pi^0\Sigma^0}$	3
$\sigma_{K^-p\to \pi^-\Sigma^+}$	20	$\sigma_{K^-p\to \pi^+\Sigma^-}$	28
$\sigma_{K^-p\to \eta\Sigma^0}$	9	$\sigma_{K^-p\to \eta\Lambda}$	49
$\sigma_{K^-p\to K^+\Xi^-}$	46	$\sigma_{K^-p\to K^0\Xi^0}$	29
γ	1	ΔE_{1s}	1
R_n	1	Γ_{1s}	1
R_c	1		

K^-p scattering data at energies close to the production threshold (obviously above $\overline{K}N$ threshold)

Observables at $\bar{K}N$ threshold

Data commonly used in the fitting procedures

$$\gamma = \frac{\Gamma(K^-p\to\pi^+\Sigma^-)}{\Gamma(K^-p\to\pi^-\Sigma^+)} = 2.36\pm0.04$$

$$R_n = \frac{\Gamma(K^-p\to\pi^0\Lambda)}{\Gamma(K^-p\to \text{neutral states})} = 0.664\pm0.011$$

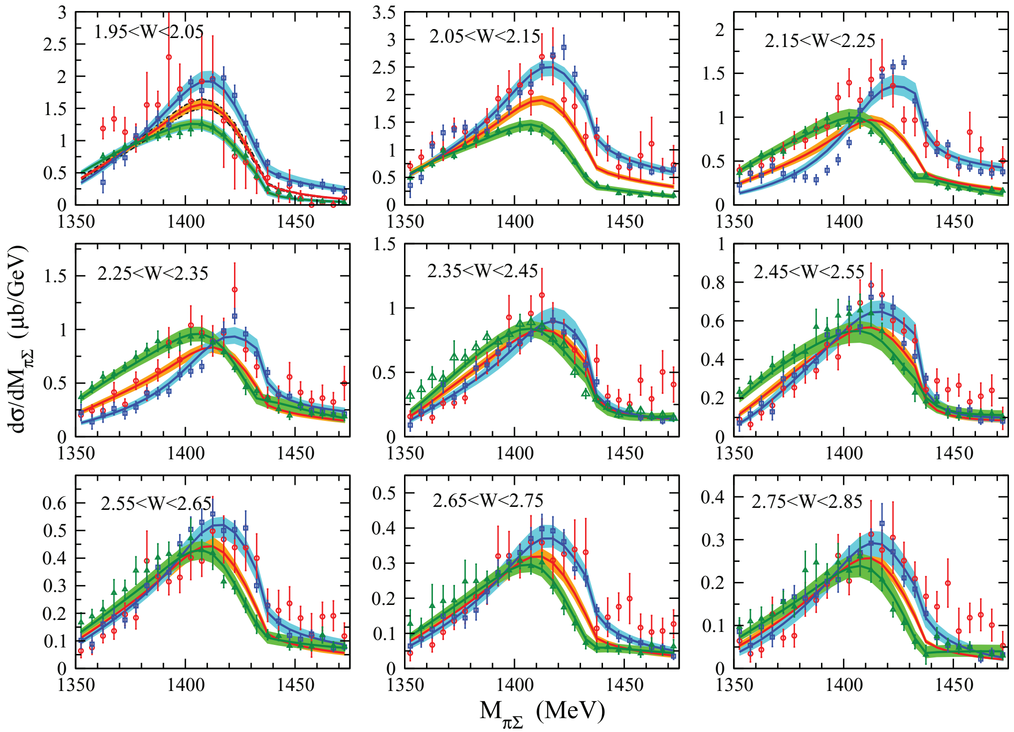
$$R_c = \frac{\Gamma(K^-p\to\pi^+\Sigma^-,\pi^-\Sigma^+)}{\Gamma(K^-p\to \text{inelastic channels})} = 0.189\pm0.015$$

CLAS Photoproduction processes provide subthreshold information (barely used!!!)

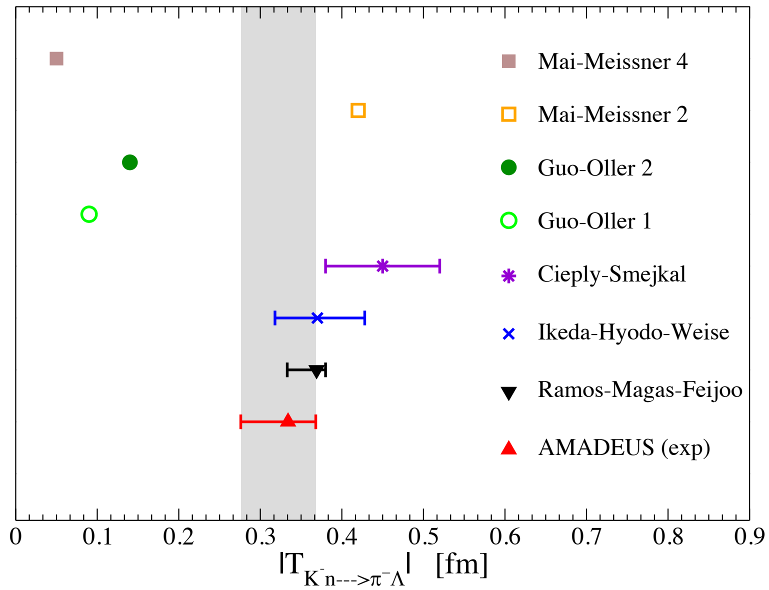
$$\frac{d\sigma(\gamma p\to\pi\Sigma K^+)}{dM_{\pi\Sigma}}$$

Motivation: $\overline{K}N$ interaction background

Many efforts have been made in order to extract information about subthreshold amplitudes…



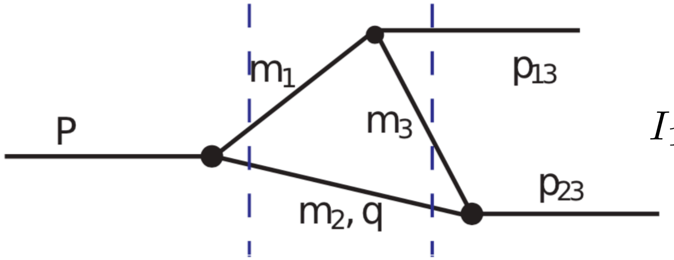
L. Roca and E. Oset, Phys. Rev. C 88, 055206 (2013).
Fit to photoproduction data from CLAS
K. Moriya et al. (CLAS Collaboration), Phys. Rev. C 87, 035206 (2013).



$K^- n \longrightarrow \pi^- \Lambda$ amplitude (pure $I = 1$ process)

K. Piscicchia et al.., Phys.Lett. B782 (2018) 339-345.
AMADEUS collaboration, KLOE detector at DAFNE

Formalism I: Triangle singularity



Explicit iintegral of the intermediate loop containing the 3 propagators:

$$I_1 = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_2^2 + i\epsilon)[(P-q)^2 - m_1^2 + i\epsilon][(P-q-p_{13})^2 - m_3^2 + i\epsilon]}$$

Integrating over q^0 … and taking only the part of the integral containing the singularity structure

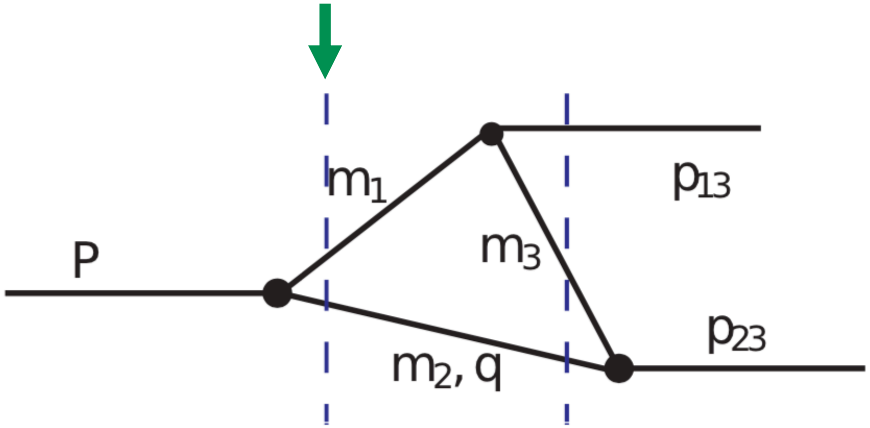
$$I(m_{23}) = \int \frac{d^3q}{(P^0 - \omega_1(\vec{q}\,) - \omega_2(\vec{q}\,) + i\,\epsilon)\left(E_{23} - \omega_2(\vec{q}\,) - \omega_3(\vec{k} + \vec{q}\,) + i\,\epsilon\right)}$$

$$= 2\,\pi \int_0^\infty dq\, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)\,, \quad f(q) = \int_{-1}^{1} dz\, \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 + 2\,q\,k\,z} + i\,\epsilon}$$

$$\omega_{1,2}(q) = \sqrt{m_{1,2}^2 + q^2},\ \omega_3(\vec{q} + \vec{k}) = \sqrt{m_3^2 + (\vec{q} + \vec{k})^2},\ E_{23} = P^0 - k^0$$

$$q = \vec{q},\ k = |\vec{k}| = \sqrt{\lambda(M^2, m_{13}^2, m_{23}^2)},\ M = \sqrt{P^2},\ m_{13,23} = \sqrt{p_{13,23}^2}$$

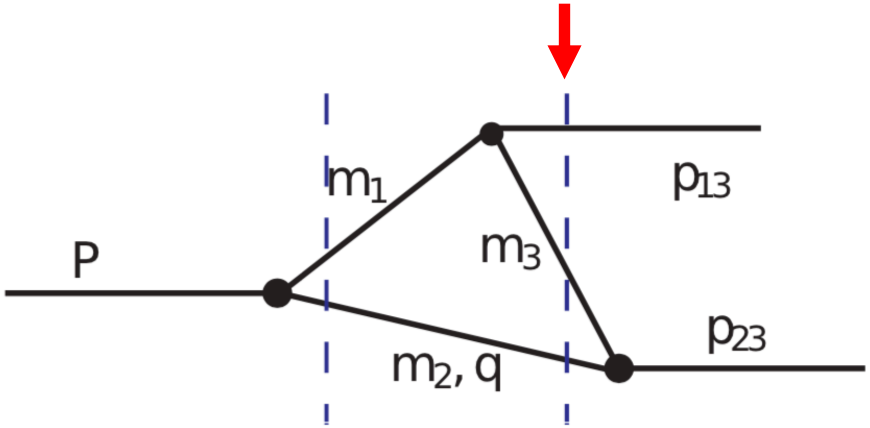
Formalism I: Triangle singularity



$$P^0 - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\epsilon = 0$$

$$q_{on+} = q_{on} + i\epsilon,\; q_{on} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_1^2)}$$

Formalism I: Triangle singularity



$$P^0 - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\epsilon = 0$$

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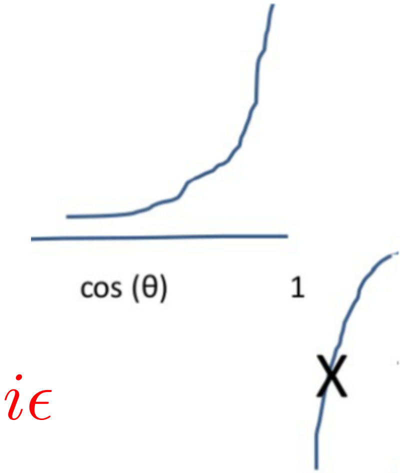


f(q) contains end-point singularities (logarithmic branch points) for $z = \pm 1$

$$E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 \pm 2qk} + i\epsilon = 0$$

$z = -1$

$$q_{a+} = \gamma(vE_2^* + p_2^*) + i\epsilon$$

$$q_{a-} = \gamma(vE_2^* - p_2^*) - i\epsilon$$

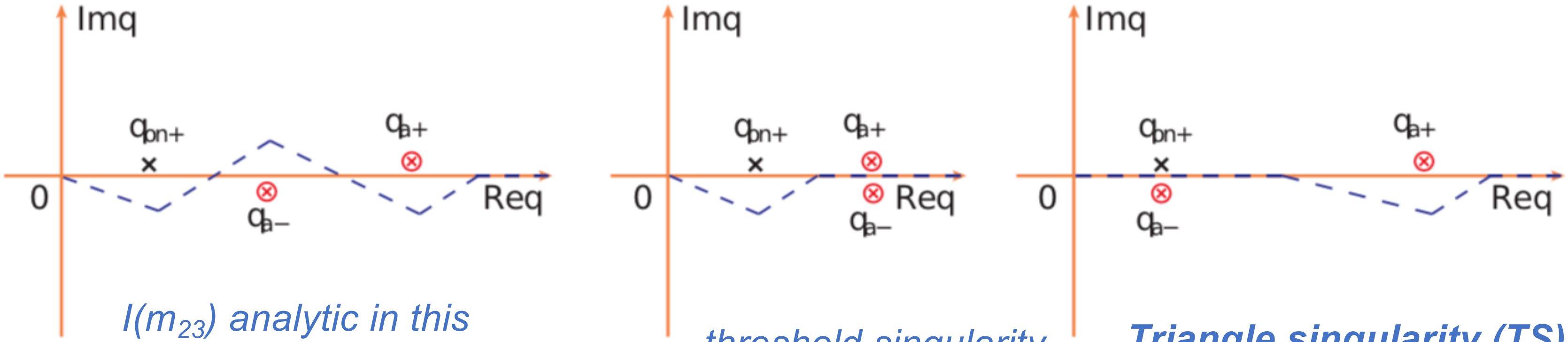
$z = 1$

$$q_{b+} = \gamma(-vE_2^* + p_2^*) + i\epsilon$$

$$q_{b-} = \cancel{-\gamma(vE_2^* + p_2^*)} - i\epsilon$$

Formalism I: Triangle singularity

q_{b+} and q_{a-} are mutually exclusive as solutions that are simultaneously in the q (positive) integration range.
The interesting casuistry for TS is given by $\boldsymbol{q_{a-}, q_{a+}, q_{on+}}$:



$I(m_{23})$ analytic in this Kinematic region

threshold singularity

Triangle singularity (TS)

$$\lim_{\epsilon\to 0}(q_{on+} - q_{a-}) = 0$$

This is only fulfilled when all three intermediate particles are placed on shell and when:

$z = -1$ *Momentum of part. 2 is anti-parallel to that of (2,3) system from the decaying particle rest system*

$$\omega_1(q_{on}) - p_{13}^0 - \sqrt{m_3^2 + (q_{on} - k)^2} = 0$$

→ For this study, **TS** should appear at $\sqrt{s} \approx 2380 MeV$