Charmonium-like resonances in $D\overline{D}$ scattering

Sara Collins
University of Regensburg

In collaboration with: **D. Mohler**, **M. Padmanath**, **S. Piemonte**, **S. Prelovsek**.

See [M. Padamanath et al.,1811.04116], [Piemonte et al.,1905.03506] and [Prelovsek et al.,2011.02542] for more details.

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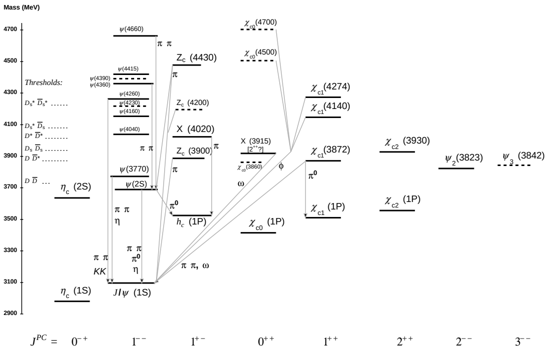
Overview

[PDG,(2019)]



★ Rich spectrum of states $J = 0 - 3$, **further states being discovered**: quark model and non-quark model states (exotics, $\bar{q}q\bar{c}c$, $\bar{c}cg$).

★ Theoretical interest in the internal structure of exotics: compact diquark-anti-diquark states, mesonic molecules, hadro-quarkonia, ...

★ [Piemonte et al.,1905.03506]: **Focus on conventional states $J^{PC} = 1^{--}$ near $D\overline{D}$** $\rightarrow$ demonstration of methods (3^{--} also considered).

★ [Prelovsek et al.,2011.02542]: **Focus on less well explored 0^{++} channel up to 4.1 GeV** including possible non-conventional states (2^{++} also considered).

Challenges and simplifications

Only one previous lattice calculations near/above threshold in $I = 0$ channel which takes into account strong decay [Lang, Leskovec, Mohler, Prelovsek,1503.05363] (vector and scalar channels studied).

(Investigation of $Z_c(3900)$ by [CLQCD,1907.03371] and [HALQCD,1602.03465,1706.07300] (using a potential approach).)

Challenges:

- ⋆ Dense spectrum of states: a number states with same/different J^{PC} in a narrow energy region.
- ⋆ Depending on the state of interest, multiple two-particle and three-particle decay channels are open.
- ⋆ Technical challenges: signal (need to determine ground state and high tower of excited states), spin identification (reduced symmetry on the lattice), ...
- ⋆ Lattice: relation of finite volume spectrum to infinite volume scattering information is not straightforward.

Simplifications:

- ⋆ Only "main" two-particle decay channels/nearby thresholds considered. Single lattice spacing, unphysical light quark masses, ...

Lattice details

Coordinated lattice simulations ensembles [CLS,1411.3982], $N_f = 2 + 1$, isospin limit $m_u = m_d = m_\ell$.

Single lattice spacing: $a = 0.086$ fm. Discretisation effects $O(a^2 m_c^2)$.

Flavour average quark mass $2m_\ell + m_s =$ constant as physical point approached $\rightarrow$ $m_\pi > m_\pi^{phys}$, $m_K < m_K^{phys}$.

$D\overline{D}$ and $D_s\overline{D}_s$ thresholds are closer together than in experiment.

	$\kappa_c = 0.12315$	$\kappa_c = 0.12522$	Expt. $\bar{D}^0 D^0 / D^+ D^-$
m_D [MeV]	1927(2)	1762(2)	$\bar{m}_D \simeq 1867$ MeV
m_{D_s} [MeV]	1981(1)	1818(1)	1968.34(7)
$M_{\rm av}$ [MeV]	3103(3)	2820(3)	3068.6(2)
m_π [MeV]	280	280	$\bar{m}_\pi \simeq 137$ MeV
m_K [MeV]	467	467	$\bar{m}_K \simeq 494$ MeV

$M_{\rm av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$, $am_c = \frac{1}{2}(\frac{1}{\kappa_c} - \frac{1}{\kappa_{crit}})$. Two values of $m_c \gtrsim m_c^{phys}$ (κ_c).

Two spatial volumes $L = 24a$, $L = 32a$. Only statistical uncertainty will be quantified.

Lattice details

Determine finite volume (FV) spectrum in energy region of interest.

Compute two-point correlation functions

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_n \frac{1}{2E_n} Z_n^i Z_n^{j*} e^{-E_n t}$$

Construct O_i with definite lattice symmetry $\Lambda \to$ tower of states with different J^{PC} (in the continuum limit). State at rest: e.g. $\Lambda = A_1 \to J = 0, 4, \ldots$, $P = \pm$.

Reliable isolation of E_n for many n requires single ($\bar{c}\Gamma c$) and multi-particle ($\bar{c}\Gamma_1 q \bar{q}\Gamma_2 c$, $\bar{c}\Gamma_1 c \bar{q}\Gamma_2 q$) O_i.

Use methods of [HSC,1004.4930,1107.1930] to ensure O_i has a good overlap with specific J^P.

Spin-identification [M. Padamanath et al.,1811.04116] is based on overlap factors $Z_n^i = \langle 0|O_i|n\rangle$.

Additional information provided by projecting to finite momentum. Symmetry further reduced, P no longer a good QN. State with $\vec{p} = \frac{2\pi}{L}(0,0,1)$, A_1 (Dic_4) $\to 0^+, 1^-, 2^+, 3^-, 4^\pm, \ldots$.

From FV energies to infinite volume scattering information

Quantisation condition see e.g. [Briceño,1401.3312] and [Hansen and Sharpe,1204.0826].

Scattering of spin-less particles: determinant equation for each E_{cm}.

$$\det\left[\tilde{K}^{-1}_{\ell,ij}(E_{cm})\delta_{\ell\ell'} - B^{\vec{P},\Lambda}_{\ell'\ell}(E_{cm})\delta_{ij}\right] = 0$$

Infinite volume: $\tilde{K}^{-1}_{\ell,ij}(E_{cm})$ related to the t-matrix

$$(t^{-1})_{ij} = \frac{2}{E_{cm}\ p_i^l p_j^l}\ (\tilde{K}^{-1})_{ij} - i\ \rho_i\ \delta_{ij}\ , \qquad \rho_i \quad \equiv 2p_i/E_{cm} = \sqrt{1-(2m_i)^2/E_{cm}^2}$$

Finite volume: known "box" functions $B^{\vec{P},\Lambda}_{\ell'\ell}(E_{cm})$, see [Morningstar et al.,1707.05817].
Reduced lattice symmetry (irrep Λ) $\rightarrow$ off-diagonal entries in partial wave indices ℓ, ℓ'.

Straightforward for one-channel scattering with single ℓ: $\tilde{K}^{-1}_{\ell}(E_{cm}) = B^{\vec{P},\Lambda}_{\ell\ell}(E_{cm})$.

For coupled channel scattering: e.g. single ℓ, $i,j = 1,2$, relation between $\tilde{K}^{-1}_{11}$, $\tilde{K}^{-1}_{12}$, $\tilde{K}^{-1}_{22}$ for each E_{cm} $\rightarrow$ under constrained problem.

Parameterise $\tilde{K}^{-1}_{\ell}(E_{cm})$, χ^2 minimisation, then search for poles in t-matrix.

Utilise multiple $\vec{P}$, L, Λ with TwoHadronsInBox package [Morningstar et al.,1707.05817].

Warm-up: 1^{--} and 3^{--} charmonia

Interested in $\psi(3770)$, $J^{PC} = 1^{--}$.

Only consider $D\overline{D}$ scattering: branching ratio $\text{Br}(\psi(3770) \to D\overline{D}) = 93 \pm 9\%$.

Quantisation condition, scattering of spinless particles:

$$\det\left[\tilde{K}_{\ell}^{-1}(E_{cm})\delta_{\ell\ell'} - B_{\ell'\ell}^{\vec{P},\Lambda}(E_{cm})\right] = 0$$

Scattering of particles with equal masses: partial waves $\ell = 1, 3, \ldots$

$J = \ell \to$ determine FV spectrum for $J^{PC} = 1^{--}$ and 3^{--}.

Interested in capturing energy region including $\psi(2S)$ and $\psi(3770) \Rightarrow$ double pole parameterisation for $\ell = 1$.

$$\tilde{K}_{\ell=1}^{-1}(E_{cm}) = \frac{p^3 \cot(\delta_1)}{\sqrt{s}} = \left(\frac{G_1^2}{m_1^2 - s} + \frac{G_2^2}{m_2^2 - s}\right)^{-1} \qquad s = E_{cm}^2$$

Single pole form for $\ell = 3$:

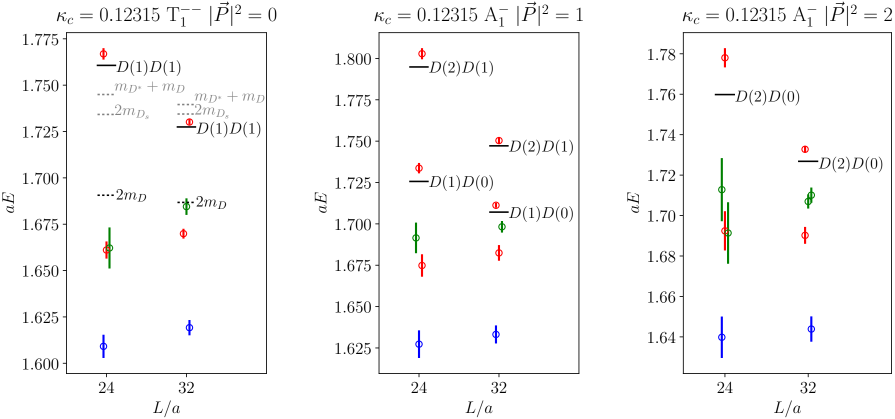
$$\tilde{K}_{\ell=3}^{-1}(E_{cm}) = \frac{p^7 \cot(\delta_3)}{\sqrt{s}} = \frac{m_3^2 - s}{g_3^2}$$

Other parameterisations also explored.

1^{--} and 3^{--} charmonia: finite volume spectrum

Energies given in lattice units. Two spatial volumes $L = 24a, 32a$, $|\vec{P}|^2 = 0, 1, 2$, $\vec{p} = \frac{2\pi}{L}\vec{P}$.

[Piemonte et al.,1905.03506]

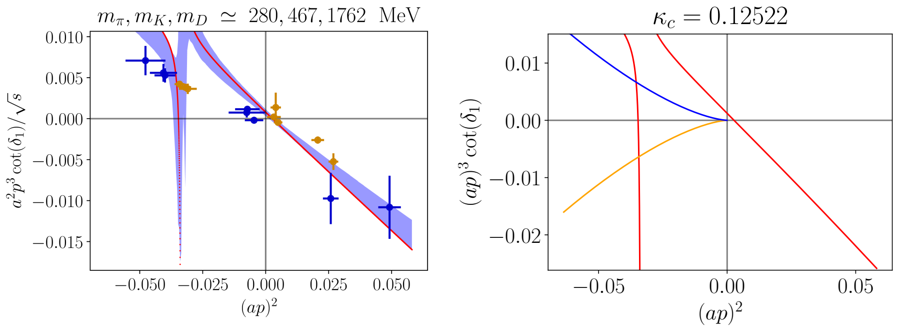


Data points are naive $\psi(2S)$ energy levels (blue), charmonia with $J^{PC} = 1^{--}$ and $D\overline{D}$ (red), levels identified as $J^{PC} = 3^{--}$ (green).

Analysis performed with and without 3^{--} states.

1^{--} and 3^{--} charmonia

[Piemonte et al.,1905.03506]



Shown: only considering $\ell = 1$, results for smaller m_c (larger κ_c).

Right: (red) central values of double pole fit.

(Blue/yellow) bound/virtual bound state condition: $p^3 \cot(\delta_1) = (-)(p^2)\sqrt{-p^2}$

Obtain: bound state and resonance.

Larger m_c (smaller κ_c): two bound states.

1^{--} and 3^{--} charmonia: final spectrum

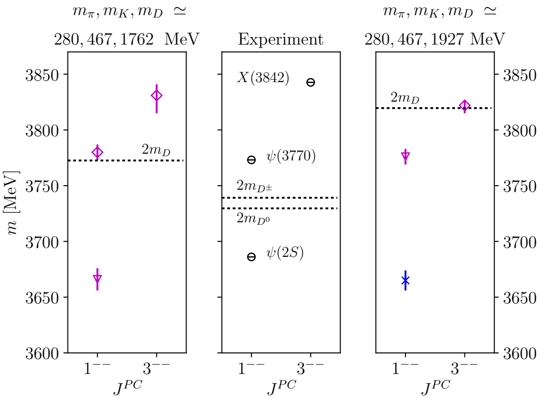
Consistent results obtained when $\ell = 3$ (3^{--} states) included in the analysis.
$\ell = 3$ has neglible influence on extraction of $\psi(3770)$.

[Piemonte et al.,1905.03506]

m relative to
$M_{\rm av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$:

$m = m^{latt} - M_{\rm av}^{\rm lat} + M_{\rm av}^{\rm exp}$.

Statistical errors only.



Left: Coupling of $\psi(3770)$: $g = 16(^{+2.1}_{-0.2})$ consistent with $g^{\rm exp} = 18.7$, $\Gamma = g^2 p^3/(6\pi s)$.
$m_{3^{--}}$ compatible with $X(3842)$ [LHCb,1903.12240].

0^{++} and 2^{++} charmonia

Interested in 0^{++} channel in $\sim 3.7 - 4.1$ GeV energy region.

Consider $D\overline{D}$ and $D_s\overline{D}_s$ thresholds ($J/\psi\omega$ and $\eta\eta_c$ omitted).

Quantisation condition, coupled channel scattering of spinless particles:

$$\det\left[\tilde{K}^{-1}_{\ell,ij}(E_{cm})\delta_{\ell\ell'} - B^{\vec{P},\Lambda}_{\ell'\ell,i}(E_{cm})\delta_{ij}\right] = 0 \qquad \ell = 0, 2, \ldots$$

$s = E_{cm}^2$

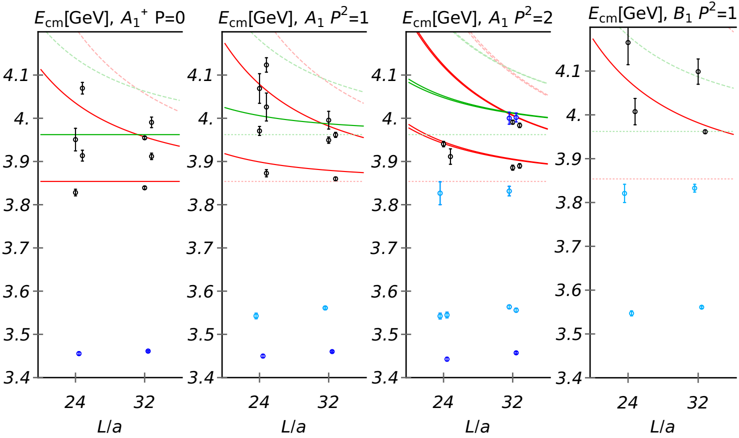
$$\left[\begin{array}{cc} D\overline{D} \to D\overline{D} & D\overline{D} \to D_s\overline{D}_s \\ D_s\overline{D}_s \to D\overline{D} & D_s\overline{D}_s \to D_s\overline{D}_s \end{array}\right] \qquad \frac{\tilde{K}^{-1}_{\ell,ij}(s)}{\sqrt{s}} = \left[\begin{array}{cc} a_{11} + b_{11}s & a_{12} \\ a_{12} & a_{22} + b_{22}s \end{array}\right]$$

Piecewise analysis of the energy region:

- 0^{++} channel close to $D\overline{D}$ threshold: $\ell = 0$, ignore coupling to $D_s\overline{D}_s$.
- 2^{++} channel around first resonance ($\chi_{c2}(3930)$): $\ell = 2$, ignore coupling to $D_s\overline{D}_s$.
- 0^{++} channel $E_{cm} \sim 3.93 - 4.13$ GeV: coupled channel analysis with $\ell = 2$ (fixed from single channel analysis).

0^{++} and 2^{++} charmonia: finite volume spectrum

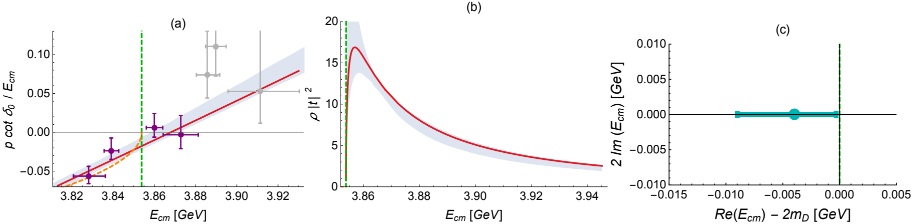
[Prelovsek et al.,2011.02542]: Larger m_c (smaller κ_c). $E_{cm,n} = \sqrt{E_n^2 - \vec{p}^2}$, $\vec{p} = \frac{2\pi}{L}\vec{P}$. $\vec{P}^2 = 0, 1, 2$, two spatial volumes $L = 24a, 32a$.



Black and dark blue data points: $J^{PC} = 0^{++}$.
Light blue data points: $J^{PC} = 2^{++}$ and 2^{-+}.
Bold red lines $D\overline{D}$ non-interactng energies and green lines for $D_s\overline{D}_s$.

0^{++} charmonia: (low) energy region close to $D\overline{D}$ threshold

[Prelovsek et al.,2011.02542]



Left: $D\overline{D}$ scattering in s-wave, $\frac{\tilde{K}^{-1}_{\ell=0,11}(s)}{\sqrt{s}} = a_{11} + b_{11}s$. Equivalent to effective range expansion.

$t^{-1} = \frac{2}{E_{cm}p^{2\ell}}\tilde{K}^{-1} - i\frac{2p}{E_{cm}}$. Right: pole in the t-matrix.

Bound state just below threshold, $\boldsymbol{m} - \boldsymbol{2m_D} = -4^{+3.7}_{-5.0}$ MeV. Statistical errors only.

Middle: number of $D\overline{D}$ events seen in expt. $\propto \rho|t|^2$, $\rho = 2p/E_{cm}$. Peak in number of events above threshold.

Shallow 0^{++} bound state

Not observed as yet in experiment.

Resampling of data to estimate statistical uncertainty, small number of samples give a virtual bound state.

Not clear if this state would also feature in a simulation with $m_\pi = m_\pi^{phys}$, $m_K = m_K^{phys}$.

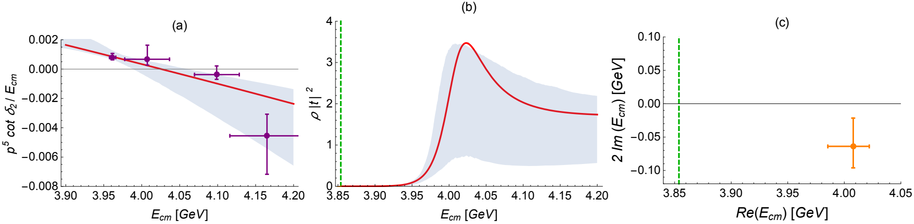
Phenomenological models: shallow bound state suggested in [Gammermann et al.,hep-ph/0612179].

Discussion of experimental evidence in [Gamermann and Oset,0712.1758].

In a molecular picture, using heavy quark symmetry arguments, a 0^{++} partner to the $X(3872)$ is expected [Hildago Duque et al.,1305.4487], [Baru et al.,1605.09649].

2^{++} charmonia: $D\overline{D}$ scattering with $I = 2$

[Prelovsek et al.,2011.02542]



Left: $D\overline{D}$ scattering in d-wave, $\frac{\tilde{K}^{-1}_{\ell=2,11}(s)}{\sqrt{s}} = a_{11} + b_{11}s = \frac{m^2 - s}{g^2}$. Breit-Wigner form.

$t^{-1} = \frac{2}{E_{cm}p^{2\ell}}\tilde{K}^{-1} - i\frac{2p}{E_{cm}}$. Right: pole in the t-matrix.

Resonance, $\boldsymbol{m} - \boldsymbol{M}_{\mathbf{av}} = 905^{+14}_{-22}$ MeV, $g = 4.5^{+0.7}_{-1.5}$ GeV^{-1} (statistical errors only).

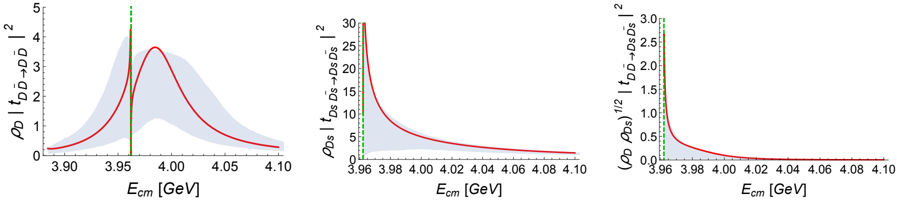
Expt. $\chi_{c2}(3930)$: $\boldsymbol{m} - \boldsymbol{M}_{\mathbf{av}} = 854 \pm 1$ MeV, $g = 2.65 \pm 0.12$ GeV^{-1}.

Middle: number of $D\overline{D}$ events seen in expt. $\propto \rho|t|^2$, $\rho = 2p/E_{cm}$.

0^{++} charmonia: coupled channel scattering around $D_s\overline{D}_s$ threshold

Two states found. a_{12} small.

[Prelovsek et al.,2011.02542]



Narrow resonance close to $D_s\overline{D}_s$ threshold (weakly coupled to $D\overline{D}$). (statistical errors only)

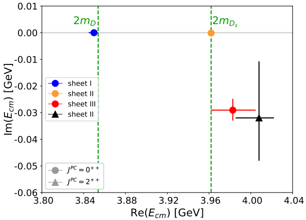
$\boldsymbol{m} - \boldsymbol{2m_{D_s}} = -0.2^{+0.16}_{-4.9}$ MeV, $g = 0.10^{+0.21}_{-0.03}$ GeV^{-1}.

Expt. $\chi_{c0}(3930)$:

$\boldsymbol{m} - \boldsymbol{2m_{D_s}} = -12.9 \pm 1.6$ MeV, $g = 0.67 \pm 0.10$ GeV^{-1}.

Expt. $X(3915)$:

$\mathbf{m} - \boldsymbol{2m_{D_s}} = -18.3 \pm 1.9$ MeV, $g = 0.72 \pm 0.10$ GeV^{-1}.

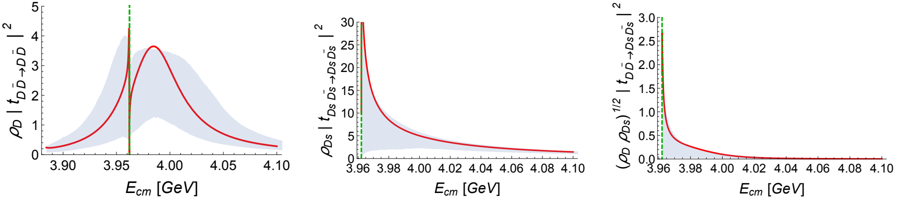


[Polosa and Lebed,1602.08421]: proposed $X(3915)$ to be a $c\bar{c}s\bar{s}$ state. See also e.g. [Liu et al.,2103.12425].

0^{++} charmonia: coupled channel scattering around $D_s\overline{D}_s$ threshold

Two states found.

[Prelovsek et al.,2011.02542]

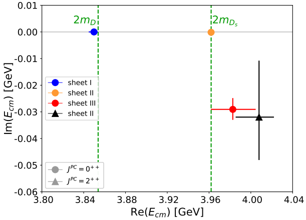


Broad resonance (coupling mostly to $D\overline{D}$). (statistical errors only)

$\boldsymbol{m} - \boldsymbol{M}_{\mathbf{av}} = 880^{+28}_{-20}$ MeV, $g = 1.35^{+0.04}_{-0.08}$ GeV^{-1}.

Expt. $X(3860)$:

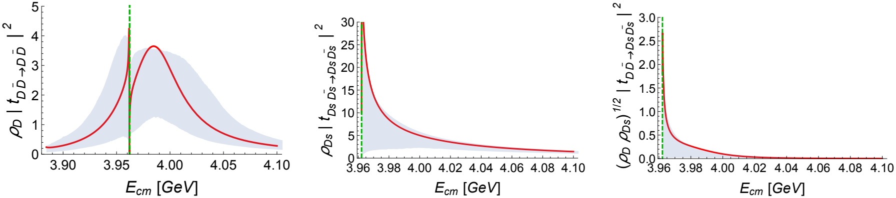
$\boldsymbol{m} - \boldsymbol{M}_{\mathbf{av}} = 793^{+48}_{-35}$ MeV, $g = 2.5^{+1.2}_{-0.9}$ GeV^{-1}.



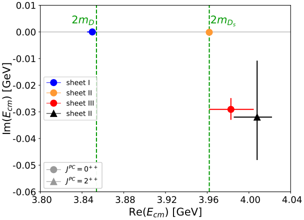
0^{++} charmonia: coupled channel scattering around $D_s\overline{D}_s$ threshold

Two states found.

[Prelovsek et al.,2011.02542]



Narrow resonance gives rise to dip in $D\overline{D} \to D\overline{D}$ events and sharp rise in $D_s\overline{D}_s \to D_s\overline{D}_s$ and $D\overline{D} \to D_s\overline{D}_s$ above $2m_{D_s}$.



Broad resonance gives peak in $D\overline{D} \to D\overline{D}$.

0^{++} and 2^{++} charmonia: final spectrum

[Prelovsek et al.,2011.02542]

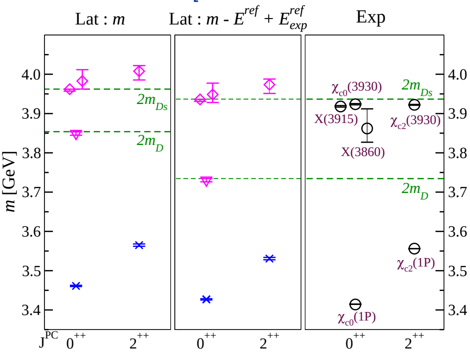
Unphysical quark masses used in the simulation.

Middle: $mass = m - E^{ref} + E^{ref}_{exp}$.

$E^{ref} = 2m_D$ for 0^{++} (pink) bound state.

$E^{ref} = 2m_{D_s}$ for 0^{++} narrow resonance.

$E^{ref} = M_{\rm av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$ for others (pink and blue).



Statistical errors only.

Blue crosses: $\chi_{c0}(1P)$ and $\chi_{c2}(1P)$ extracted from FV energies.

$J^{PC} = 0^{++}$, $X(3915)$ and $\chi_{c0}(3930)$ may be the same state.

Summary

Presented an investigation of vector and scalar charmonia.

- ★ A number of simplifications are made (the thresholds considered, unphysical quark masses, single lattice spacing).
- ★ These simplifications can be removed in future work.
- ★ Only statistical uncertainty quantified: qualitative comparison with experiment.
- ★ Vector channel around $D\overline{D}$ threshold: demonstration of methods.
  - Smaller m_c set: m and g for $\psi(3770)$ consistent with experiment (and $J^{PC} = 3^{--}$ with mass consistent with $X(3842)$).
- ★ Scalar channel around $D\overline{D}$ to above $D_s\overline{D}_s$:
  - State just below $D\overline{D}$ threshold, not yet observed in experiment.
  - Narrow resonance just below $D_s\overline{D}_s$ which may be related to $X(3915)/\chi_{c0}(3930)$.
  - Broad resonance which may be related to $X(3860)$.
  - ($J^{PC} = 2^{++}$ similar to $\chi_{c2}(3930)$)
- ★ Consider additional channels in the future.