Resonance studies in the Bethe-Salpeter framework

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Hadron 2021
Mexico City
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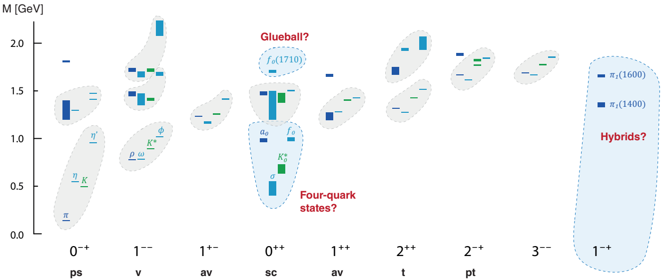
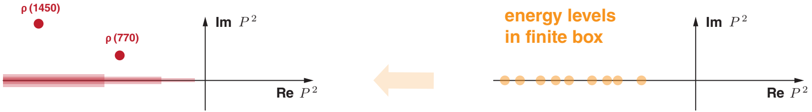
Resonances



Light mesons
(PDG 2020)

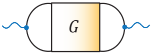
$$\langle\, j^\mu(x)\, j^\nu(0)\rangle \;= \int \mathcal{D}[\psi,\bar{\psi},A]\, e^{-S}\, j^\mu(x)\, j^\nu(0)$$

Lattice:



Resonances

$$\langle \underbrace{[\bar\psi\,\gamma^\mu\psi](x)}_{j^\mu(x)}\,\underbrace{[\bar\psi\,\gamma^\nu\psi](0)}_{j^\nu(0)}\rangle = \gamma^\mu_{\alpha\beta}\,\gamma^\nu_{\rho\sigma}\,\langle\,\bar\psi_\alpha(x)\,\psi_\beta(x)\,\bar\psi_\rho(0)\,\psi_\sigma(0)\,\rangle$$

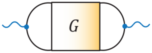
$=$ 

$q\bar{q}$ four-point function **G** contains **all meson poles:**

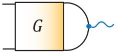
Pole in momentum space
⇒ exponential decay in Euclidean time

Bethe-Salpeter wave function
= overlap with state

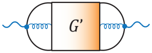
Same poles in all n-point function
that carry meson quantum numbers (but overlap may be small)



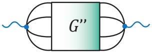
hadronic vacuum polarization



quark-photon vertex



hybrid operators



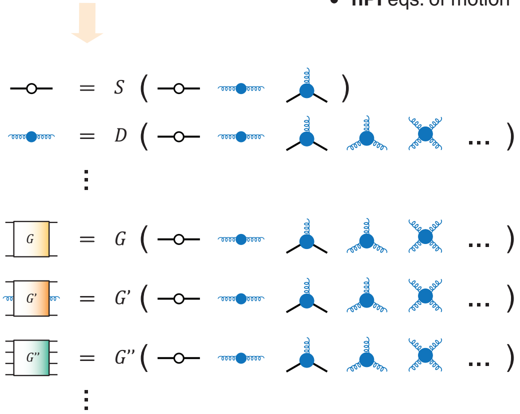
four-quark operators

Functional methods

Derive exact relations for n-point functions from path integral:

$$Z = \int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S} = e^{-\Gamma}$$

- Dyson-Schwinger equations **(DSEs)**
- Functional renormalization group **(FRG)** eqs.
- **nPI** eqs. of motion



much progress, approaching quantitative precision:

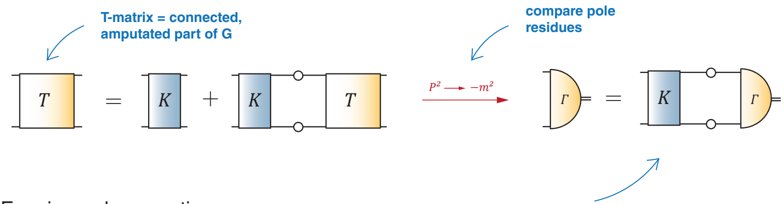
see Monday B1 & Friday A7:
J. Papavassiliou, J. Rodriguez-Quintero,
M. Huber, F. Gao, B. El-Bennich, ...

- quark mass generation
- gluon mass gap
- three-gluon vertex
- glueballs

compliated structure & eqs. for higher n-point functions, more efficient: solve **Bethe-Salpeter equations**

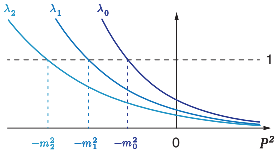
Bethe-Salpeter equations

Solve **homogeneous BSE**:



BSE = eigenvalue equation,
pole in G $\Leftrightarrow$ **eigenvalue = 1**

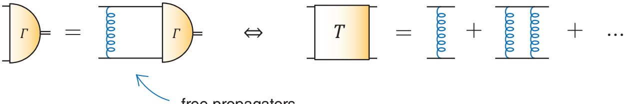
$$K G_0 \, \Gamma_i = \lambda_i \, \Gamma_i$$



- $q\bar{q}$ irreducible kernel
- chiral symmetry constraints (V + AV WTI)
- can be systematically derived from effective action, depends on QCD's n-point functions

Ladder

Simplest attempt:

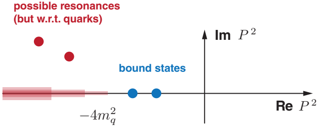


free propagators

$$\frac{-i\not{p} + m}{p^2 + m^2}$$

- breaks chiral symmetry: free propagators ⇔ NJL model ↯
- generates bound-state poles in G and T, possibly also resonances
- but also quark thresholds & cuts: “hadrons” decay into quarks, no confinement ↯

Analytic structure of G, T, etc. would look like this:

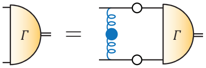


would be ok if elementary d.o.f. were not quarks but **hadrons** (→ EFTs)

Rainbow-ladder

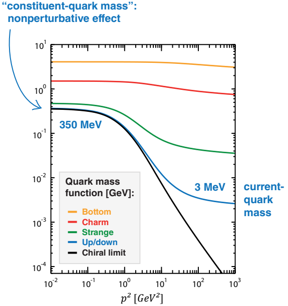
Better: **rainbow-ladder truncation**

Maris, Roberts, PRC 56 (1997), Maris, Tandy, PRC 60 (1999), ...

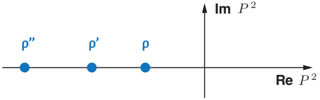


gluon = effective interaction,
dressed propagators
from quark DSE

$$\frac{1}{A(p^2)}\,\frac{-i\not{p} + M(p^2)}{p^2 + M(p^2)^2}$$



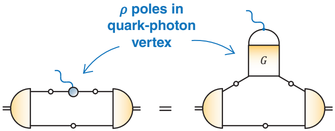
Analytic structure of G, T, etc.
would look like this:

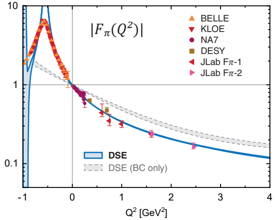


- chiral symmetry ✓
- dynamical propagators do not have real poles ⇒ no quark thresholds ✓
- but no resonances, only **bound states** ϟ

Pion form factor

- **Pion electromagnetic form factor** has ρ pole:
  Maris, Tandy, PRC 61 (2000), . . .

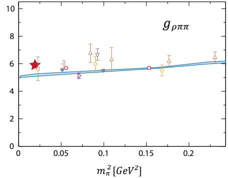




Absence of width has no visible effect on spacelike behavior

GE, Fischer, Weil, Williams, PLB 797 (2019)

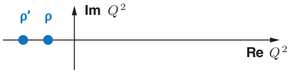
- Residue at pole = $g_{\rho\pi\pi}$



RL

Mader, GE, Blank, Krassnigg, PRD 84 (2011)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

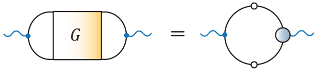


$\Rightarrow$ Main part of dynamics preserved even though ρ is not yet resonance

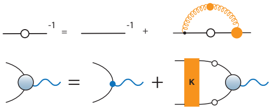
Hadronic vacuum polarization

Vector current correlator = **HVP** (→ muon g-2 problem)

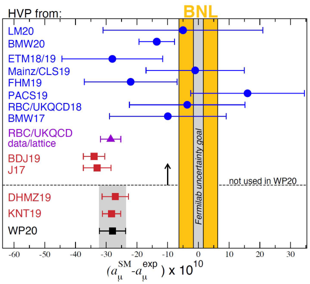
$\langle\, j^\mu(x)\, j^\nu(0)\,\rangle$ =



- Depends only on quark propagator and quark-photon vertex

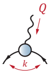


C. Lehner, CERN Seminar 2021



Goecke, Fischer, Williams, PLB 704 (2011)

- Quark-photon vertex has 12 tensors:



$$\Gamma^\mu(k,Q) = \Big[i\gamma^\mu\,\Sigma_A + 2k^\mu (i\not{k}\,\Delta_A + \Delta_B) \Big] + \Big[i \sum_{j=1}^{8} f_j\, \tau_j^\mu(k,Q) \Big]$$

Ball-Chiu vertex, determined by WTI, depends only on quark propagator

Ball, Chiu, PRD 22 (1980)

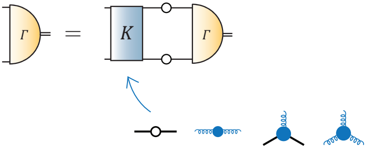
Transverse part, contains dynamics (VM poles, cuts, ...), 8 dressing functions

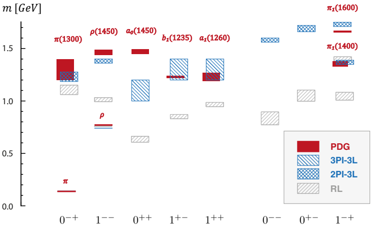
contributes 80% to g-2, resonance dynamics important

Beyond rainbow-ladder

Kernel from 3PI effective action

Williams, Fischer, Heupel, PRD 93 (2016)



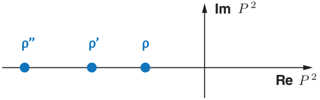


- also scalar and axialvector mesons move into right ballpark
- but still bound states

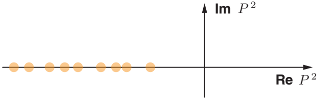
Much work also done for **baryons** (mostly RL)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

Barabanov et al., PPNP 116 (2021)

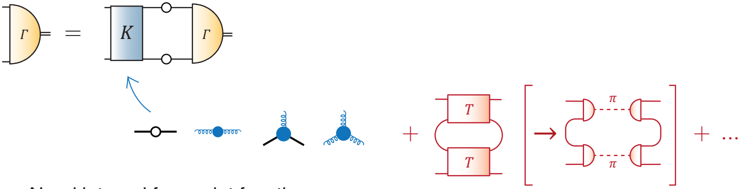


$\neq$



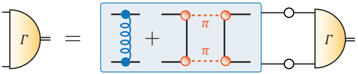
Resonances?

Resonance **mechanism** depends on truncations: need internal $\pi\pi$ dynamics



- Need internal four-point functions, must come from **higher-order truncations**

- Implement $\pi\pi$ dynamics explicitly
  Williams PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019), Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)



see talk by **A. Miramontes** right after

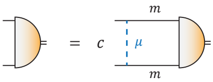
- Generates $\pi\pi$ cut, ρ meson becomes resonance

- How to **extract** resonance information on 2nd sheet?

Extracting resonances

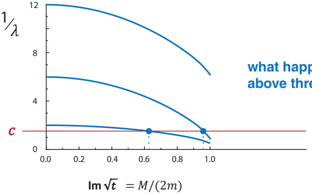
Simpler system: **scalar BSE** $\Gamma = KG_0\,\Gamma$

Wick 1954, Cutkosky 1954, Nakanishi 1969, ...

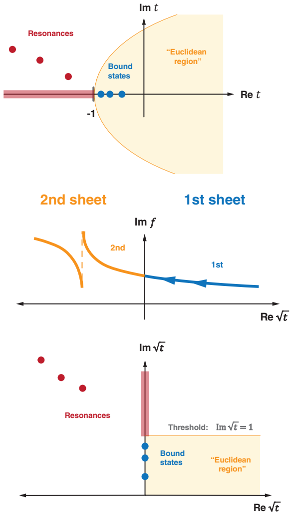


Define
$t = P^2/(4m^2)$

- **BSE** → eigenvalue spectrum of KG_0
  Condition for pole: $1/\lambda = c$
  ⇒ poles move with coupling parameter c



what happens above threshold?

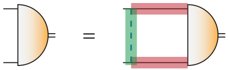


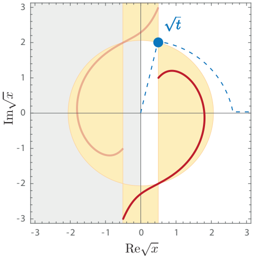
Extracting resonances

- **Contour deformations**
  Maris, PRD 52 (1995), Strauss, Fischer, Kellermann PRL 109 (2012), Windisch, Huber, Alkofer PRD 87 (2013), . . .

- Poles in propagators and kernel produce **cuts** in outermost integration variable x

$$\Gamma(X,Z,t) = \int_0^\infty dx \int_{-1}^{1} dz \; K(X,x,Z,z) \; G_0(x,z,t) \; \Gamma(x,z,t)$$

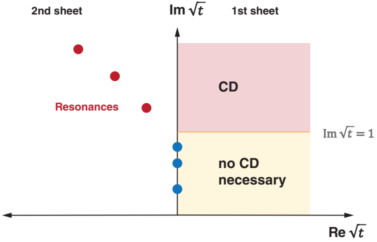




All possible cuts lie inside yellow area

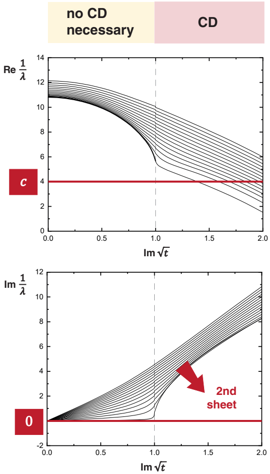
With contour deformations, can cover **entire complex** t **plane**

GE, Duarte, Pena, Stadler, PRD 100 (2019)



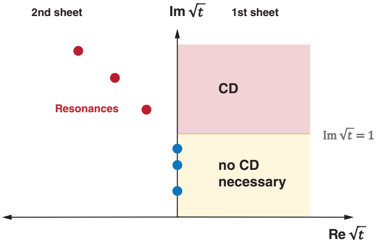
Extracting resonances

- Eigenvalues in complex t plane:



To extract resonances from homogeneous BSE, search for poles on **2nd sheet** defined by

$$\frac{1}{\lambda(t)} \overset{!}{=} c + 0 \cdot i$$

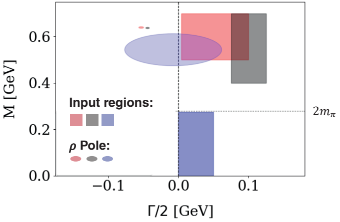


Extracting resonances

- To access **2nd sheet,** use Schlessinger method / Continued fraction:
  Schlessinger, Phys. Rev. 167 (1968),
  Tripolt, Haritan, Wambach, Moiseyev, PLB 774 (2017)

$$f(z) = \cfrac{c_1}{1 + \cfrac{c_2\,(z-z_1)}{1 + \cfrac{c_3\,(z-z_2)}{1 + \cfrac{c_4\,(z-z_3)}{\cdots}}}}$$

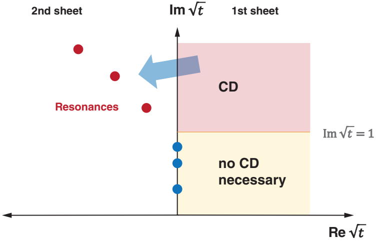
- Works well for ρ meson with clear resonance pole
  Williams PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019),
  Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)



Rainbow-ladder + $\pi\pi$, scale set by f_π:

$M_\rho = 638(2)$ MeV, $\Gamma_\rho = 108(4)$ MeV

→ **A. Miramontes**



Extracting resonances

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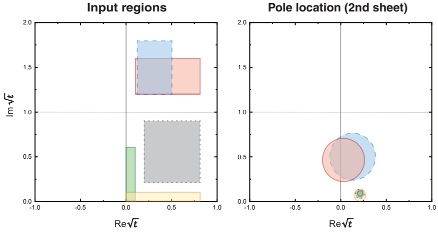
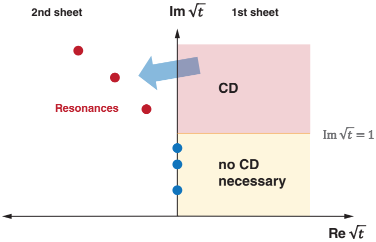
- For scalar model less clear: virtual state?
  GE, Duarte, Pena, Stadler, PRD 100 (2019)



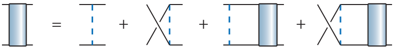


... is there **a more precise** way?

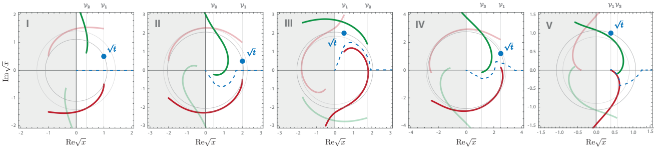
Scattering amplitude

Solve **scattering equation** $T = K + K G_0 T$

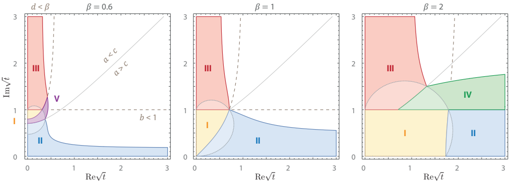
GE, Duarte, Pena, Stadler, PRD 100 (2019)



- Contour deformations become more complicated: two cuts, can overlap



- Can still cover **parts** of complex t plane:



- Advantage: two-body unitarity is automatic, can directly compute amplitude **on 2nd sheet**

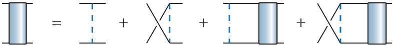
Partial-wave decomposition:

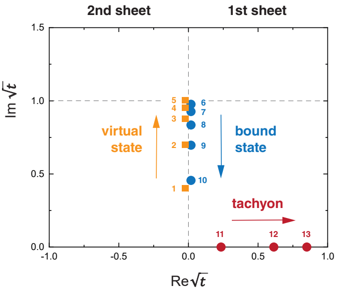
$$f_l(t)_{II} = \frac{f_l(t)_I}{1 - 2i\,\tau(t)\, f_l(t)_I}$$

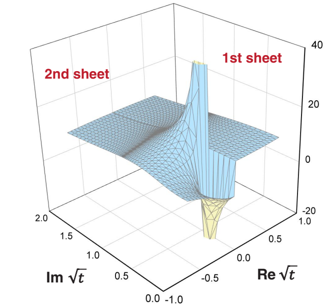
Scattering amplitude

Solve **scattering equation** $T = K + KG_0\,T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





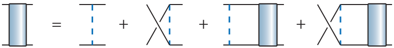


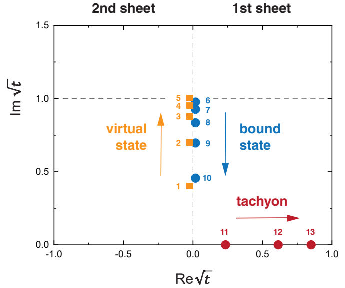
$c = 1$

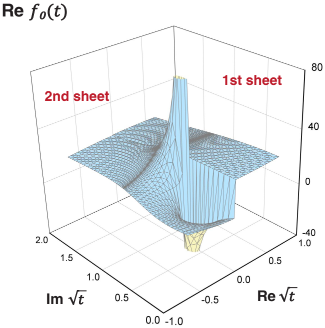
Scattering amplitude

Solve **scattering equation** $T = K + KG_0 T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





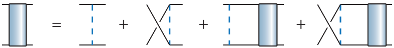


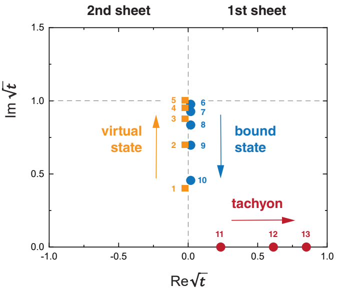
$c = 2$

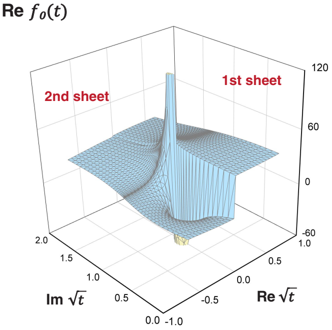
Scattering amplitude

Solve **scattering equation** $T = K + KG_0\, T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





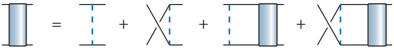


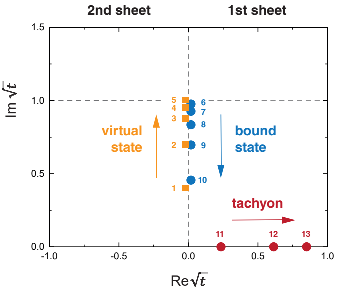
$c = 3$

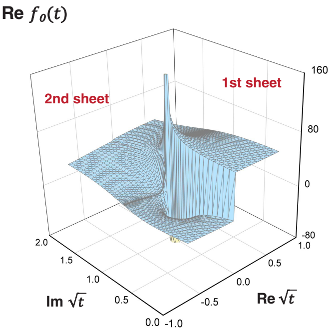
Scattering amplitude

Solve **scattering equation** $T = K + KG_0\,T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





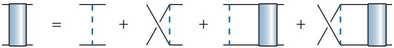


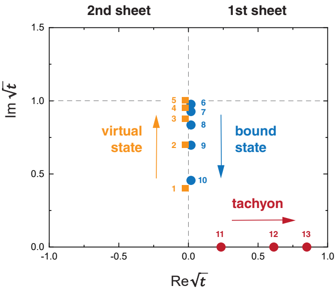
$c = 4$

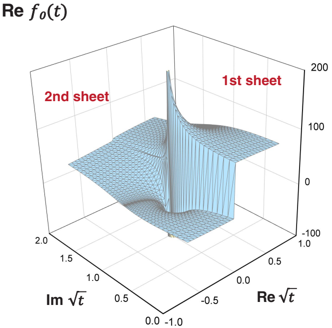
Scattering amplitude

Solve **scattering equation** $T = K + KG_0\,T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





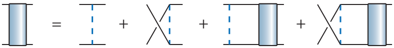


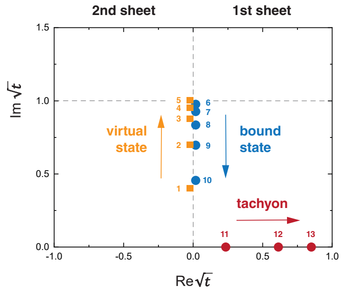
$c = 5$

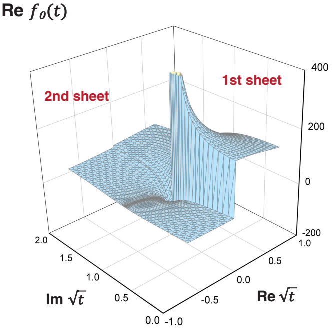
Scattering amplitude

Solve **scattering equation** $T = K + KG_0\,T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





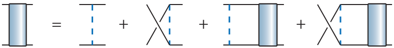


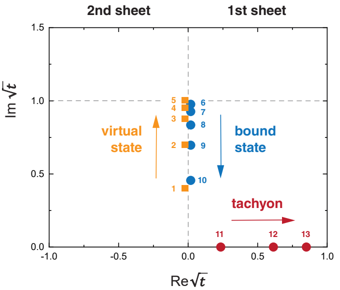
$c = 7$

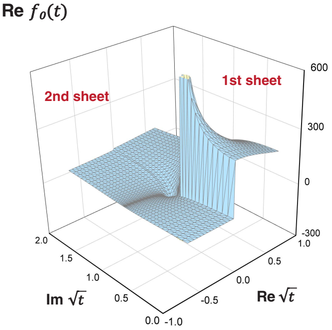
Scattering amplitude

Solve **scattering equation** $T = K + KG_0\,T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





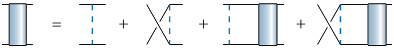


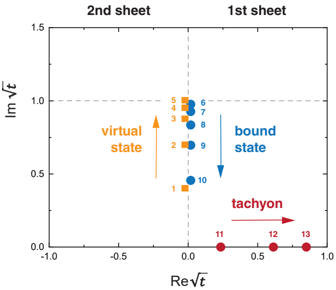
$c = 8$

Scattering amplitude

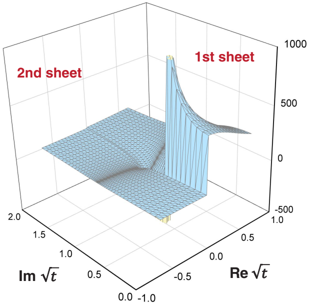
Solve **scattering equation** $T = K + KG_0\,T$

GE, Duarte, Pena, Stadler, PRD 100 (2019)





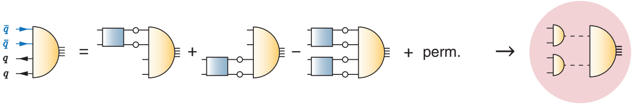
Re $f_0(t)$



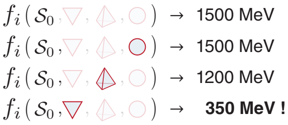
$c = 9$

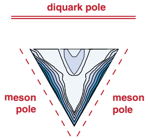
Four-quark states

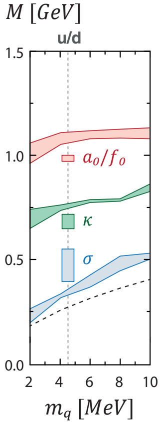
- Four-body system forms two-body clusters, **resonance dynamics automatic** GE, Fischer, Heupel, PLB 753 (2016)



- BSE dynamically generates meson poles in BS amplitude, light scalar mesons look like **meson molecules**







- Similar for heavy-light states: X(3872), ...
  Wallbott, GE, Fischer, PRD 100 (2019), PRD 102 (2020)
  Review: GE, Fischer, Heupel, Santowsky, Wallbott, FBS 61 (2020)

- $q\bar{q}$ admixture for σ meson is small
  Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)

Summary

- **Functional methods:** resonance dynamics for $q\bar{q}$ & qqq states depends on truncations (higher n-point functions)

- Recent progress & technical advances using **contour deformations**

  Williams PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019),
  GE, Duarte, Pena, Stadler, PRD 100 (2019),
  Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020),
  Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)

- **Four-quark states** form internal two-body clusters, resonance dynamics automatic

  GE, Fischer, Heupel, Santowsky, Wallbott, FBS 61 (2020)

Thank you!