Isospin symmetry breaking effects in mesonic masses from a Poincaré Covariant Bethe-Salpeter approach.

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Introduction

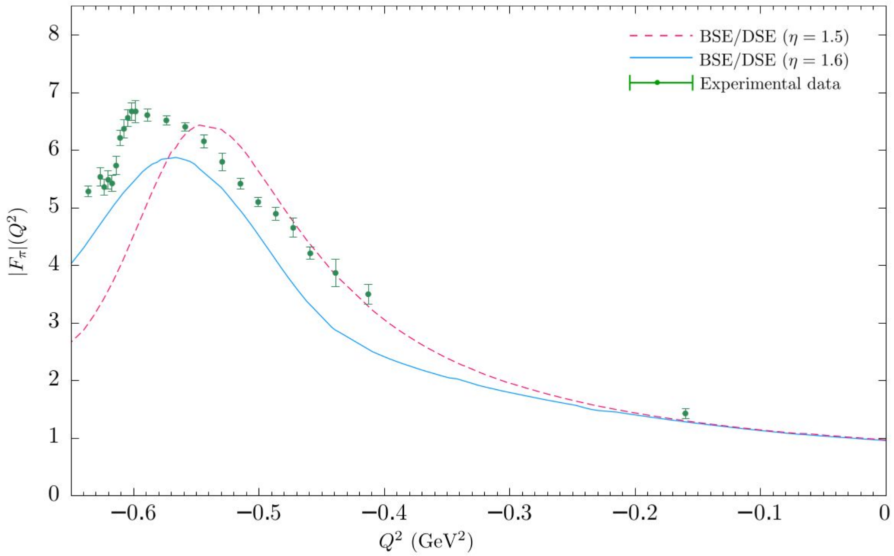
- Isospin symmetry is broken at the level of QED due to the difference in the charges between the up and down quarks
- Also broken in QCD through the small mass difference between up and down quarks in the QCD lagrangian.
- Almost an exact symmetry, mass difference much smaller than QCD scale.
- For this reason, theoretical predictions of different physical quantities assume isospin symmetry taking the mass of the quarks up and down to be the same and neglecting electromagnetic effects.

Introduction

- With the improvement of theoretical calculations and experimental determinations is becoming phenomenological relevant.
- Important for hadron spectroscopy, where the mass of hadrons is determined by QED and QCD dynamics.
- For QCD interactions, since the coupling is too large at low energy, non-perturbative methods are required.
- In the past, Isospin breaking effects have been investigated using Chiral perturbation theory.
- The splitting of flavour singlet and non-singlet mesons have been investigated by calculating contributions of disconnected quark diagrams in Lattice QCD.
- The tool of our choice: the combination of **Dyson-Schwinger** and **Bethe Salpeter-equations**.

Motivation

- As a consequence of isospin breaking, the rho and omega are not eigenstates of isospin, and then, there will be a matrix element mixed between the omega-rho mesons.
- Experimentally, the omega-rho mass splitting is small (7MeV).
- Necessary its implementation for the correct description of the time-like electromagnetic pion form factor.
- Main goal of this investigation: study strong isospin breaking and the splitting of the omega-rho mesons
- Warm up, study the splitting of the pion system.

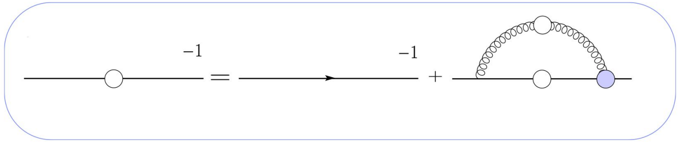


A. S. Miramontes, H. Sanchis Alepuz, and R. Alkofer, Phys. Rev. D 103, 116006 (2021)

Formalism

Dyson-Schwinger equations

- They are the equations of motion of a quantum field theory.
- Infinite set of coupled integral equations.

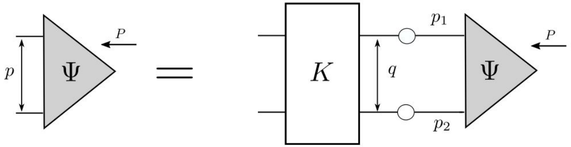


Bethe-Salpeter equations

The solutions of the Bethe-Salpeter equations encode all information about the hadron.

- Mass
- Decay constants
- Decay width

We have access to form factors when we couple to an external field



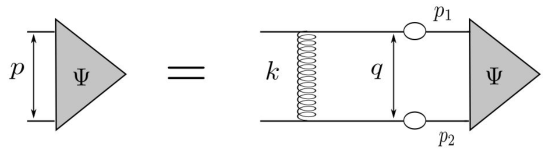
G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer and C. S. Fischer, Prog. Part. Nucl. Phys.91(2016) doi:10.1016/j.ppnp.2016.07.001 [arXiv:1606.09602 [hep-ph]]

Formalism

- They conform a non perturbative method to study QCD.
- Infinite set of integral equations, truncation will be necessary.
- The rainbow-ladder is the simplest truncation which preserves chiral symmetry.

$$\alpha(k^2) = \boxed{\pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 \exp^{-\eta^2 \frac{k^2}{\Lambda^2}}} + \boxed{\alpha_{UV}}$$

- Enough strength for dynamical chiral symmetry breaking to take place.



- Ultraviolet part reproduces the one-loop QCD behaviour of the propagator at large momenta.
- Two free parameters to be fitted to reproduce pion mass and decay constant.

P. Maris and P. C. Tandy, Phys. Rev. C 60 (1999) 055214
doi:10.1103/PhysRevC.60.055214

Formalism

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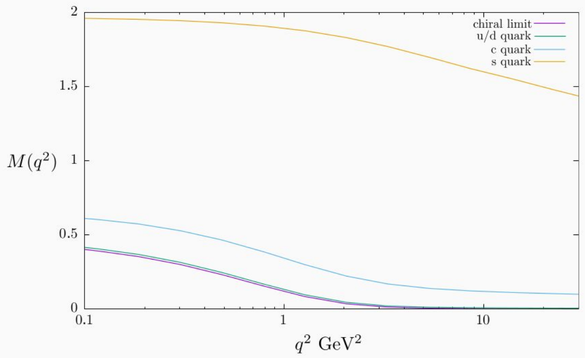
$$S(p) = \frac{1}{A(p^2)}\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)} = -i\not{p}\,\sigma_v(p^2) + \sigma_s(p^2);$$



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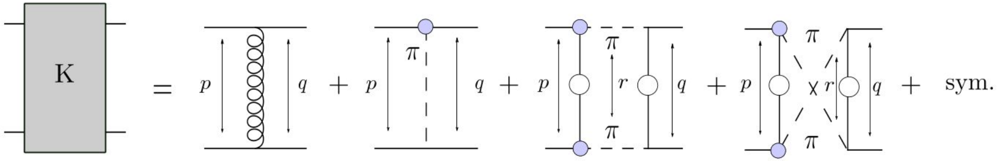
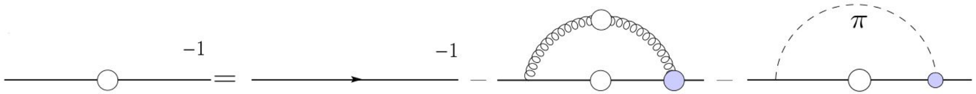
Beyond the rainbow

- Pionic effects are considered by including explicit pions as degrees of freedom.
- Besides the gluon contribution, a pion loop and pion exchange will appear on the DSE/BSE system.

$$\Gamma_\pi^i(p,P) = \tau^i\ \gamma_5\{E_\pi(p,P) - i\not{P}F_\pi(p,P) - i\not{p}(p\cdot P)G_\pi(p,P) - [\not{P},\not{p}]\,H_\pi(p,P)\}$$





C. S. Fischer, D. Nickel and R. Williams, Eur. Phys. J. C 60 (2009) 47 doi:10.1140/epjc/s10052-008-0821-1

A. S. Miramontes and H. Sanchis-Alepuz, Eur. Phys. J. A 55 (2019) no.10, 170 doi:10.1140/epja/i2019-12847-6

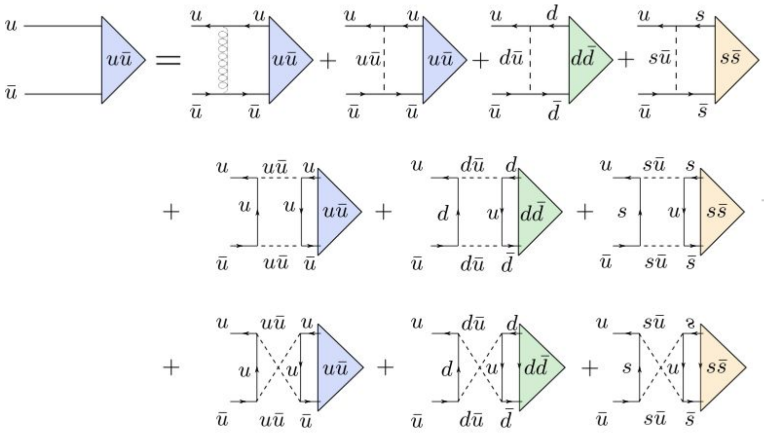
Towards mass splitting

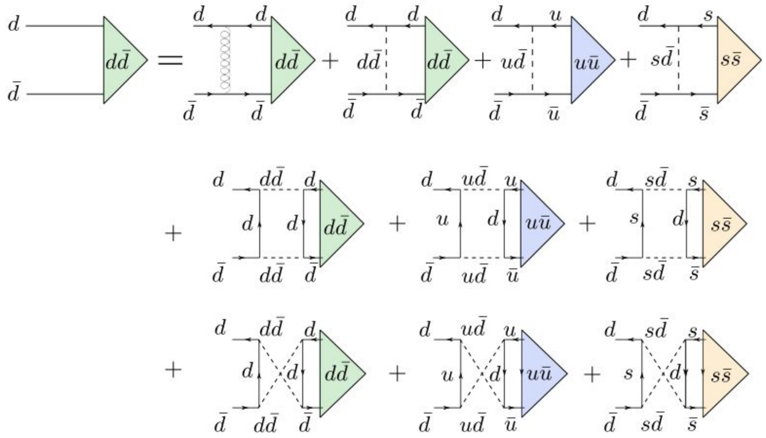
- As a first step towards the omega-rho mixing, we study the splitting for pions using the formalism of Dyson-Schwinger and Bethe-Salpeter
- The pions, as dictated in the quark model, are given by:

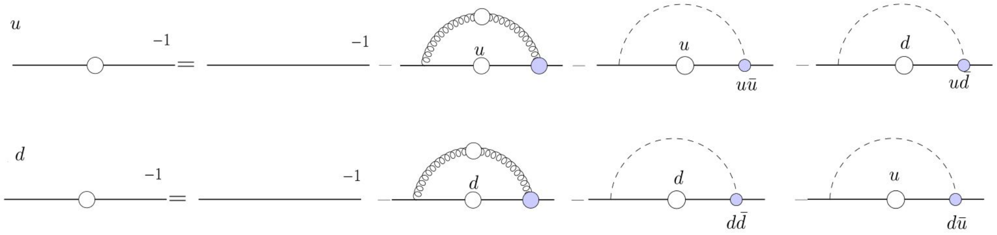
$$
\begin{aligned}
|\pi^+\rangle &= |u\bar{d}\rangle \\
|\pi^-\rangle &= -|d\bar{u}\rangle \\
|\pi^0\rangle &= \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |\bar{d}d\rangle)
\end{aligned}
$$

DSE/BSE setup

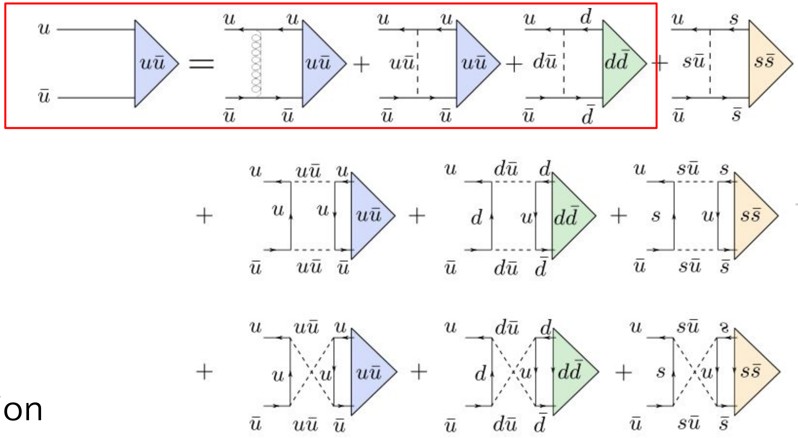
- We need to solve a coupled system of DSE, using different masses for up and down quarks.
- Then, we use the computed propagators into a coupled system of BSE



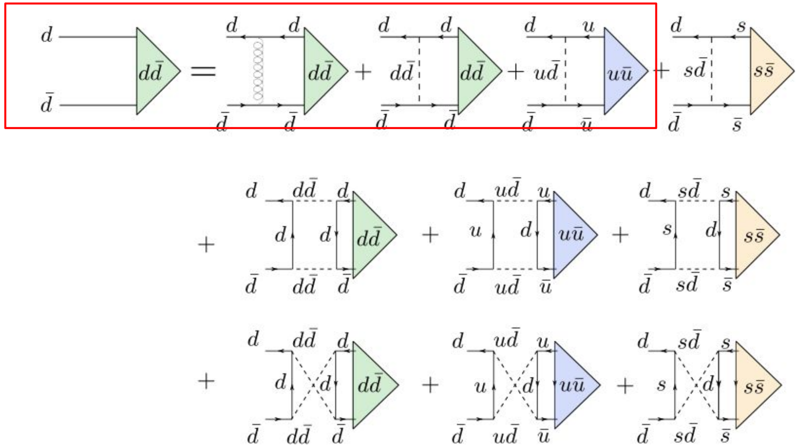


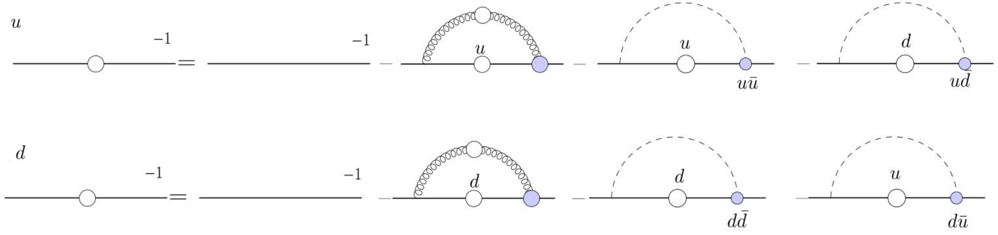


DSE/BSE setup



We start solving the system including only RL + t-channel pion exchange and neglecting strange quarks.



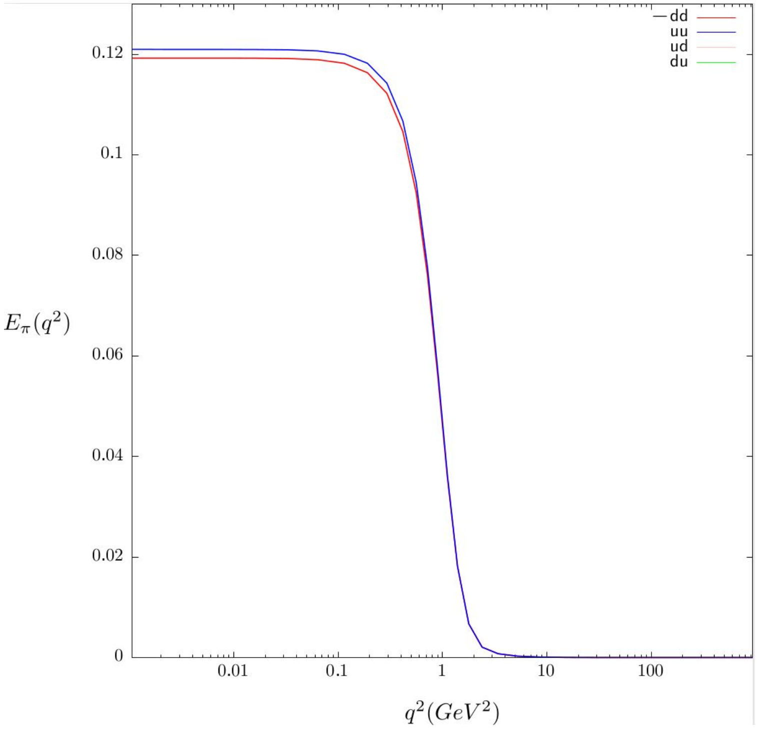


Results: Neutral pion

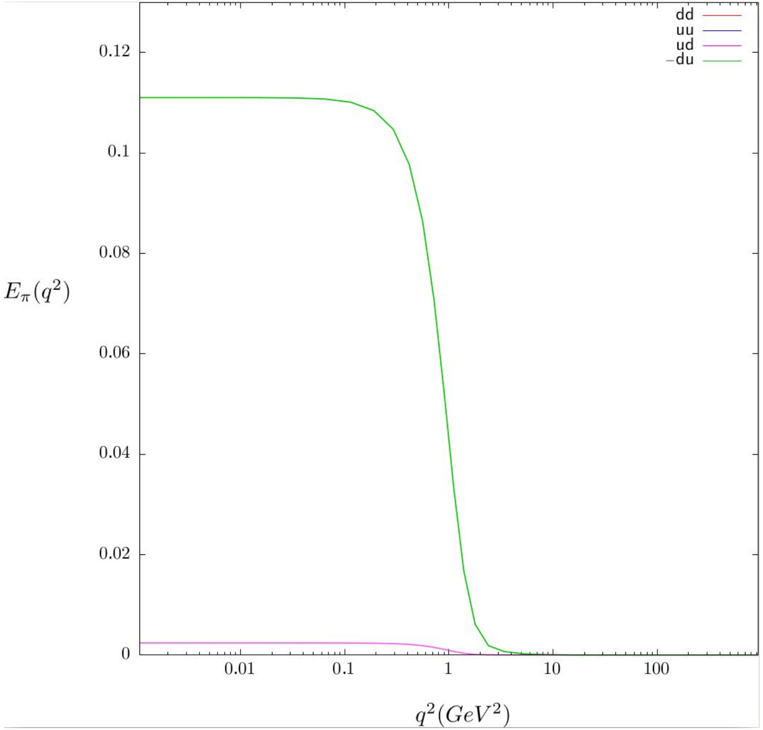
- The corresponding results are two degenerate solutions with purely ud and purely du, respectively and one solution with mixed content uu, dd

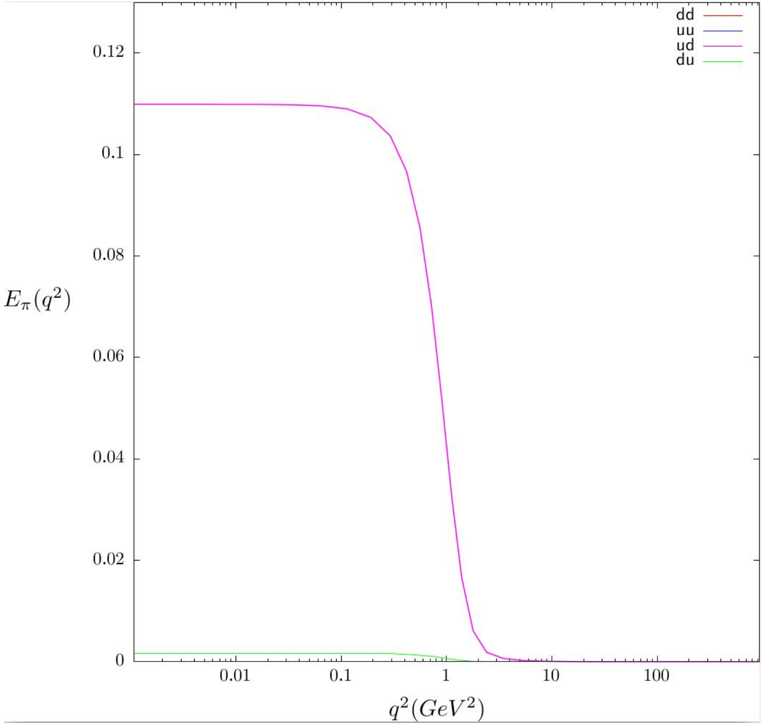
m_d	m_u	m_{π^0}	$m_{\pi^\pm}$	mass difference
7.0	5.0	133.0	136.0	3.0
6.8	5.2	134.2	135.7	1.5
6.6	5.4	134.5	135.6	1.1

Quark and pion masses using Λ = 0.75, η=1.42 in the Maris-Tandy model

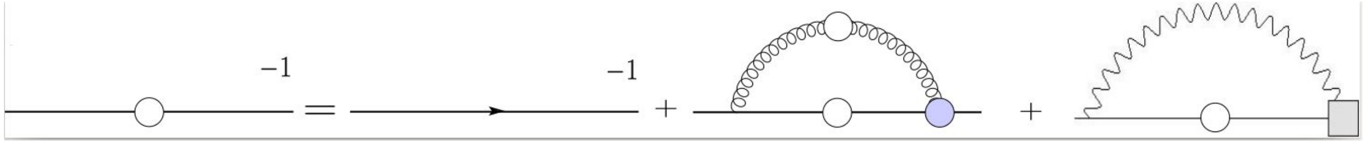


Results: Charged pions





QED effects



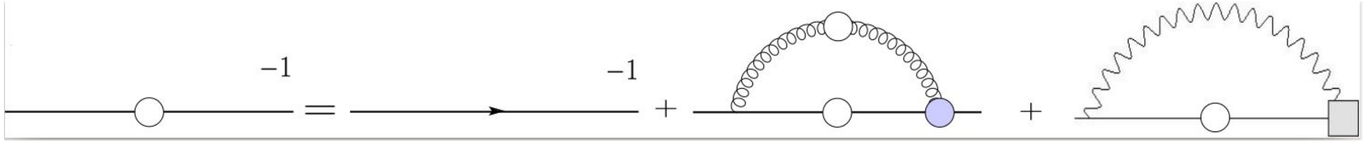
$$\alpha_{eff}^{QED} = \alpha_0 Q_q^2 \tilde{Z}_{QED}\left(q^2\right)$$

$$\tilde{Z}_{QED}\left(q^2\right) = f_m \frac{A_1\left(q^2\right) + A_2\left(q^2\right)}{2}$$

- In addition to the gluon loop, a QED contribution was included.
- Tune the free parameter to get the correct splitting.

Jan bonnet, PhD thesis (2017), Uni Giessen

QED effects



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- In addition to the gluon loop, a QED contribution was included.
- Tune fm to get the correct splitting.

We want to include this QED contribution to our calculation

Summary/Outlook

- We have introduced the first steps to study strong isospin breaking and omega-rho mass splitting.
- As a warm up, we have computed the pion mass splitting solving a coupled system of DSE/BSE, reproducing the states given by the quark model.
- Next step:
- Include QED interaction to study the two different sources of isospin breaking.
- Include strangeness content to study the kaon.