# Glueball-glueball scattering and the glueballonium*

*article in preparation, soon to appear on arXiv

HADRON 2021, 30 July 2021

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- The Standard Model of particle interactions
- Characteristics of QCD
- Classification of mesons
- Glueballs
- Scattering of two scalar glueballs
- Bound state of two glueballs: glueballonium
- The effect of other glueballs on the scattering
- Conclusions

- Non-Abelian gauge field theory based on the symmetry group $SU(3) \otimes SU(2) \otimes U(1)$ with 8+3+1=12 generators [gluons + $W^{\pm}$, $Z^{o}$ + $\gamma$]

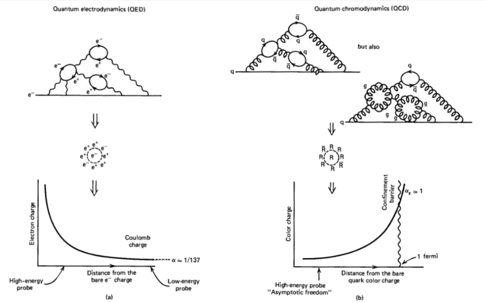
## Quantum chromodynamics (QCD)

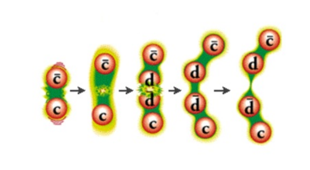
- Non-Abelian gauge theory [gauge group SU(3)]
- Self interaction among gluons $\rightarrow$idea of bound state (not yet observed)
- Theory for strong interactions between gluons and quarks

$$\mathscr{L}_{QCD} = -\frac{1}{4}\sum_{a=1}^{8} F^{a\mu\nu}F^{a}_{\mu\nu} + \sum_{j=1}^{n_f} \bar{q}_j(i\gamma^{\mu}D_{\mu} - m_j)q_j$$

$$D_{\mu} = \partial_{\mu} - igA_{\mu}$$
$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu};\ a, b, c = 1, \ldots, N_C^2 - 1$$

- The green part is called Yang-Mills (YM) Lagrangian



A.F.: Interactions between particles become asymptotically weaker as the energy scale increases
C.: Color charged particles cannot be isolated

*Hadrons* { *Baryons* $[qqq]$ / *Mesons* $[\bar{q}q]$ } *color singlets*

| Meson | $n^{2S+1}L_J$ | $J^{PC}$ | $S$ | $L$ | Hermitian quark current operators |
|---|---|---|---|---|---|
| pseudoscalar | $1^1S_0$ | $0^{-+}$ | 0 | 0 | $P_{ij} = \bar{q}_j \, i\gamma^5 \, q_i$ |
| vector | $1^3S_1$ | $1^{--}$ | 1 | | $V^\mu_{ij} = \bar{q}_j \, \gamma^\mu \, q_i$ |
| pseudovector | $1^1P_1$ | $1^{+-}$ | 0 | 1 | $P^\mu_{ij} = \bar{q}_j \, \gamma^5 \overleftrightarrow{\partial}^\mu \, q_i$ |
| scalar | $1^3P_0$ | $0^{++}$ | 1 | | $S_{ij} = \bar{q}_j \, q_i$ |
| axial vector | $1^3P_1$ | $1^{++}$ | 1 | | $A^\mu_{ij} = \bar{q}_j \, \gamma^5\gamma^\mu \, q_i$ |
| tensor | $1^3P_2$ | $2^{++}$ | 1 | | $X^{\mu\nu}_{ij} = \bar{q}_j \, i \left[\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3} \tilde{G}^{\mu\nu} \overleftrightarrow{\not\partial}\right] q_i$ |

Koenigstein A. and Giacosa F., Phenomenology of pseudotensor mesons and the pseudotensor glueball [10.1140/epja/i2016-16356-x, Eur. Phys. J. A, 2016]

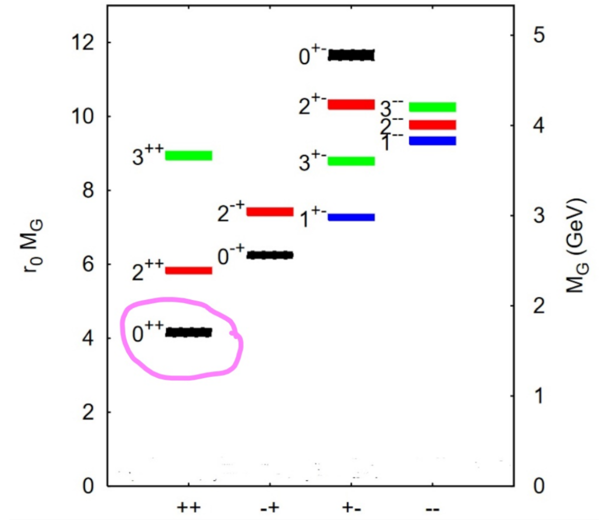
- QCD predicts also exotic mesons as tetraquarks ($\bar{q}\bar{q}qq$) and glueballs

- Not yet observed
- Gluons carry color charge and interact strongly with each other
- Expectation confirmed by numerous simulations of lattice QCD [arXiv:nucl-th/0309068], a technique by K. Wilson (1974) [Phys. Rev. D 10, 2445]
- Lattice QCD: QCD is reformulated on a discrete space time, a hypercubic lattice of sites (in the simplest realizations) with spacing $a$ and 4-volume $L^4$
- Possible state that is predominantly a glueball state is the resonance $f_0(1710)$ [arXiv:1408.4921v1 [hep-ph], Janowski et al., 2014]

$$\sigma_N \approx (\bar{u}u + \bar{d}d)/\sqrt{2}$$
$$\sigma_S \approx (\bar{s}s)$$

$$\begin{aligned} f_0(1370):&\quad 83\%\,\sigma_N, \quad 6\%\,\sigma_S, \quad 11\%\,G\,, \\ f_0(1500):&\quad 9\%\,\sigma_N, \quad 88\%\,\sigma_S, \quad 3\%\,G\,, \\ f_0(1710):&\quad 8\%\,\sigma_N, \quad 6\%\,\sigma_S, \quad 86\%\,G\,. \end{aligned}$$

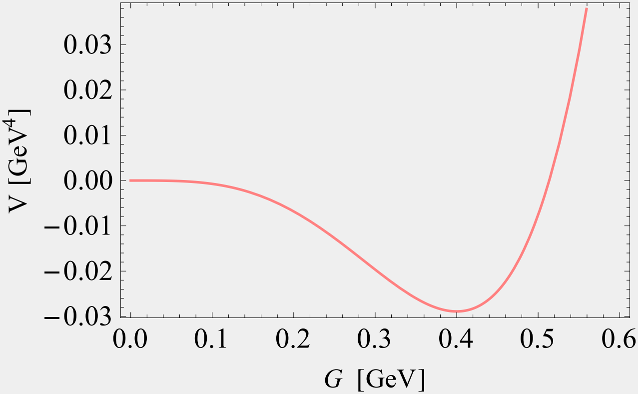
We worked with a single scalar glueball with $m_G = 1.7$ GeV.
Below: plot from Lattice QCD: a glueball spectrum is found.




Y. Chen et al, Phys. Rev. D73, 014516 (2006)

Now only the YM sector of QCD will be considered.

$$\mathscr{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right)$$

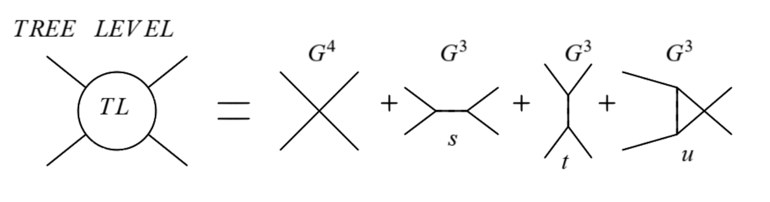



A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

Taylor expansion around min $V(G = \Lambda_G \approx 0.4 \text{ GeV})$

$$V(G) = V(\Lambda_G) + \frac{1}{2}m_G^2 G^2 + \frac{1}{3!}\left(5\frac{m_G^2}{\Lambda_G}\right)G^3 + \frac{1}{4!}\left(11\frac{m_G^2}{\Lambda_G^2}\right)G^4 + \frac{1}{5!}\left(6\frac{m_G^2}{\Lambda_G^3}\right)G^5 + \dots$$

$$A(s,t,u) = -11\frac{m_G^2}{\Lambda_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2\frac{1}{s-m_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2\frac{1}{t-m_G^2} - \left(5\frac{m_G^2}{\Lambda_G}\right)^2\frac{1}{u-m_G^2}$$



As $k^{2l+1}a_l(s) = \frac{1}{2}\frac{k}{8\pi\sqrt{s}}A_l(s)$ and

$$A_l(s) = \frac{1}{2}\int_{-1}^{+1} d\xi A(s,\theta)P_l(cos\theta),$$

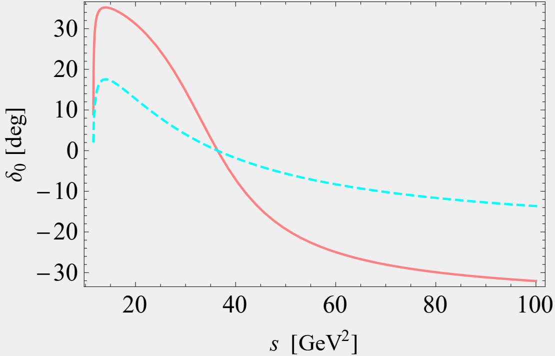
$s_{th} \equiv 4m_G^2$

$$a_0(s_{th}) = \frac{23m_G}{24\pi\Lambda_G^2}$$

$$\left( a_2(s_{th}) = \frac{5}{6\pi m_G^3\Lambda_G^2} \text{ and } a_4(s_{th}) = \frac{40}{63\pi m_G^7\Lambda_G^2} \right)$$

# *phase shift* (TL)

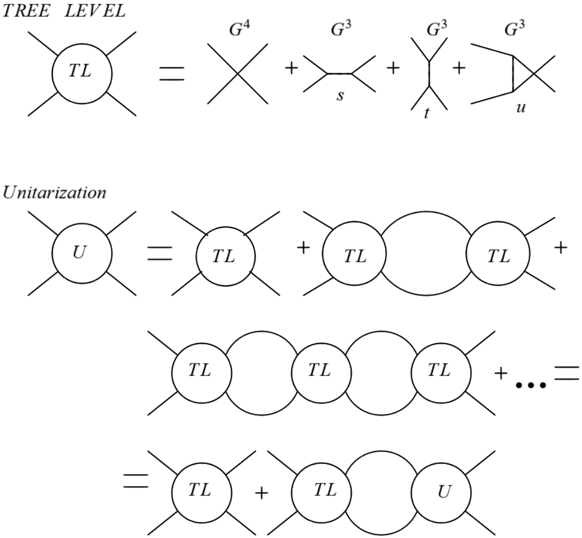
$$\delta_l(s) = \frac{1}{2} arg\left[1 + 2i\frac{\sqrt{\frac{s}{4} - m_G^2}}{16\pi\sqrt{s}}A_l(s)\right]$$




$$\left[\Lambda_G = 0.4\ GeV \text{ (pink) and } 0.8\ GeV \text{ (blue)}\right]$$

$$\Delta\delta_0 := \delta_0(s \to \infty) - \delta_0(4m_G^2) \neq n\pi \to \text{unitarization}$$

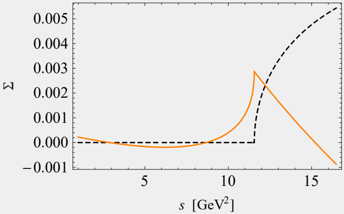
# SCHEMATIC REPRESENTATION OF THE UNITARIZATION THROUGH LOOPS




$$U = TL + (TL)\sum U$$

Loop function $\Sigma(s)$ in such a way to:
-preserve the pole corresponding to $s = m_G^2$
-preserve the logarithmic divergence at $s = 3m_G^2$ due to the $t$- and $u$-channel single-glueball exchange.
It follows that:

$$\Sigma(s) = \frac{(s - m_G^2)(s - 3m_G^2)}{\pi} \int_{4m_G^2}^{\infty} \frac{\frac{\frac{s'}{4} - m_G^2}{16\pi\sqrt{s'}}}{(s' - s)(s' - 3m_G^2)(s' - m_G^2)} ds'$$




$Re\Sigma$ (orange) vs. $Im\Sigma$ (black)

The unitarized amplitude is so given by:

$$A_l^{unit}(s) = [A_l^{-1}(s) - \Sigma(s)]^{-1}$$

Now:

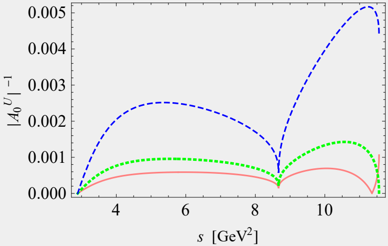
$$\delta_l^{unit}(s) = \frac{1}{2} arg\left[1 + 2i\frac{\sqrt{\frac{s}{4} - m_G^2}}{16\pi\sqrt{s}} A_l(s)^{unit}\right]$$

and

$$a_0^U(s_{th})\left(= \frac{1}{32\pi m_G} A_0^U(s)\right)_{s=s_{th}} = \frac{1}{32\pi m_G}\frac{1}{\frac{3\Lambda_G^2}{92m_G^2} - 0.0028715}.$$

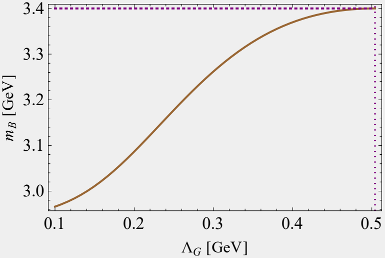
With our value of $m_G$:

$$a_0(s_{th}) = \infty \leftrightarrow \Lambda_G = \Lambda_{G,crit} \approx 0.504\ GeV$$




$|A_0^U|^{-1}(s)$ for:
▷ $\Lambda_G = 0.4$ *GeV* (pink)
▷ $\Lambda_G = \Lambda_{G,crit} \approx 0.504$ *GeV* (green)
▷ $\Lambda_G = 0.8$ *GeV* (cyan).

$$\Lambda_{G,crit} : a_0^U(s = s_{th})|_{\Lambda_G=\Lambda_{G,crit}} = \infty$$

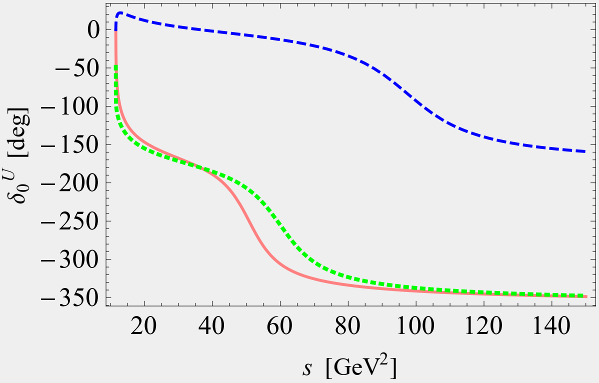



Mass of the glueballonium as function of $\Lambda_G$

For $m_B = 2m_G = 3.4$ GeV one has the critical value $\Lambda_{G,crit} \sim 0.5$ GeV.

For $\Lambda_G > \Lambda_{G,crit}$, $\nexists$ bound state.

# Phase shift (UNI)

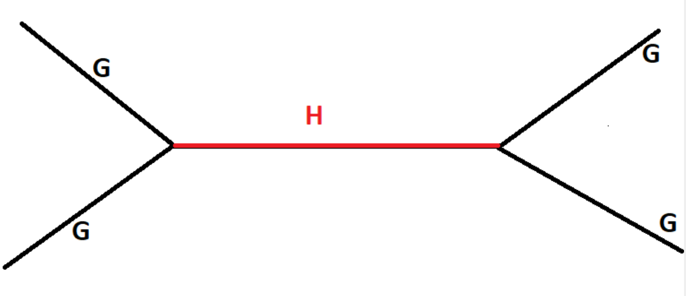



▷ $\Lambda_G = 0.4$ *GeV* (pink)
▷ $\Lambda_G = \Lambda_{G,crit} \approx 0.504$ *GeV* (green)
▷ $\Lambda_G = 0.8$ *GeV* (cyan).

$\Delta\delta_0^U \to -2\pi \to$ Levinson theorem fulfilled

H: other glueball entering the s-channel (non-scalar)

$$Amplitude \propto \frac{1}{4m_G^2 - m_H^2}$$




No expected contribution to TL for: quantum numbers of the glueballs, dilatation invariance...

- TL is not enough.
- Unitarized amplitudes for the l-th waves (l=0,2,4).
- $m_B(\Lambda_G)$. $\exists$ bound state if $\Lambda_G < \Lambda_{G,crit} \simeq 0.504$ GeV if $m_G = 1.7$ GeV
- Emergence of a 2 scalar glueball bound state, that we named "glueballonium", could be found on the lattice and/or in experiments.
- Other glueballs are not expect to sizably affect the results.
- The evaluated scattering lengths and phase shifts could be simulated on the lattice, hence a comparison is possible.
- FOR FUTURE: scattering of other glueballs.

Thank you for your attention