Quarkonium Suppression in the Open Quantum Systems Approach

Xiaojun Yao
MIT

Review: XY, 2102.01736

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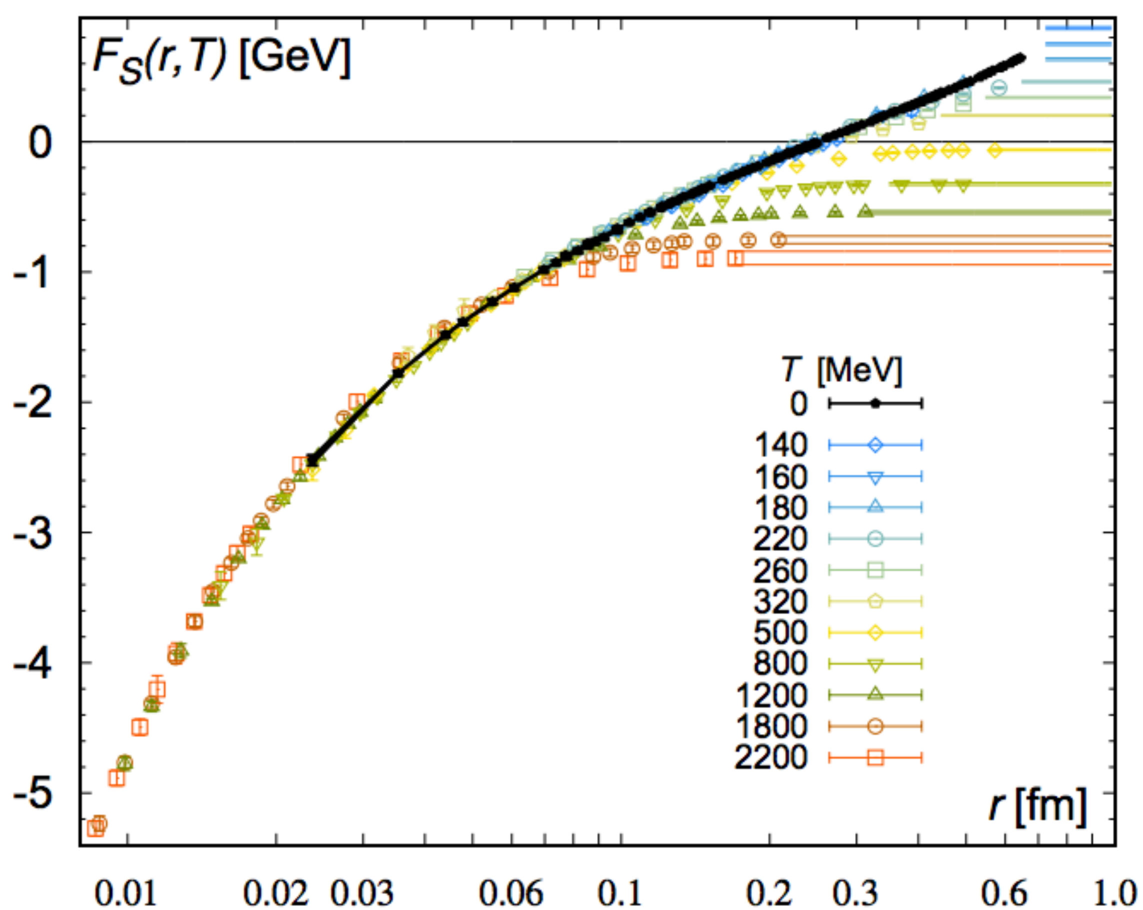
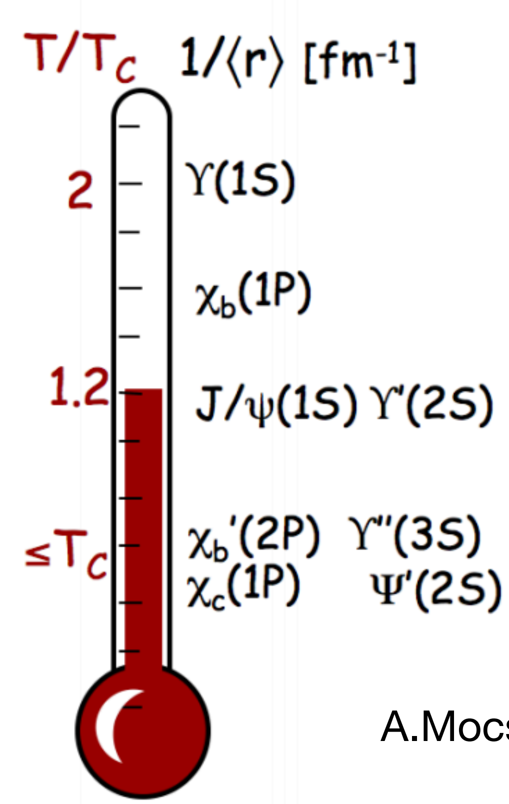
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



1804.10600, A.Bazavov, N.Brambilla P.Petreczky, A.Vairo, J.H.Weber



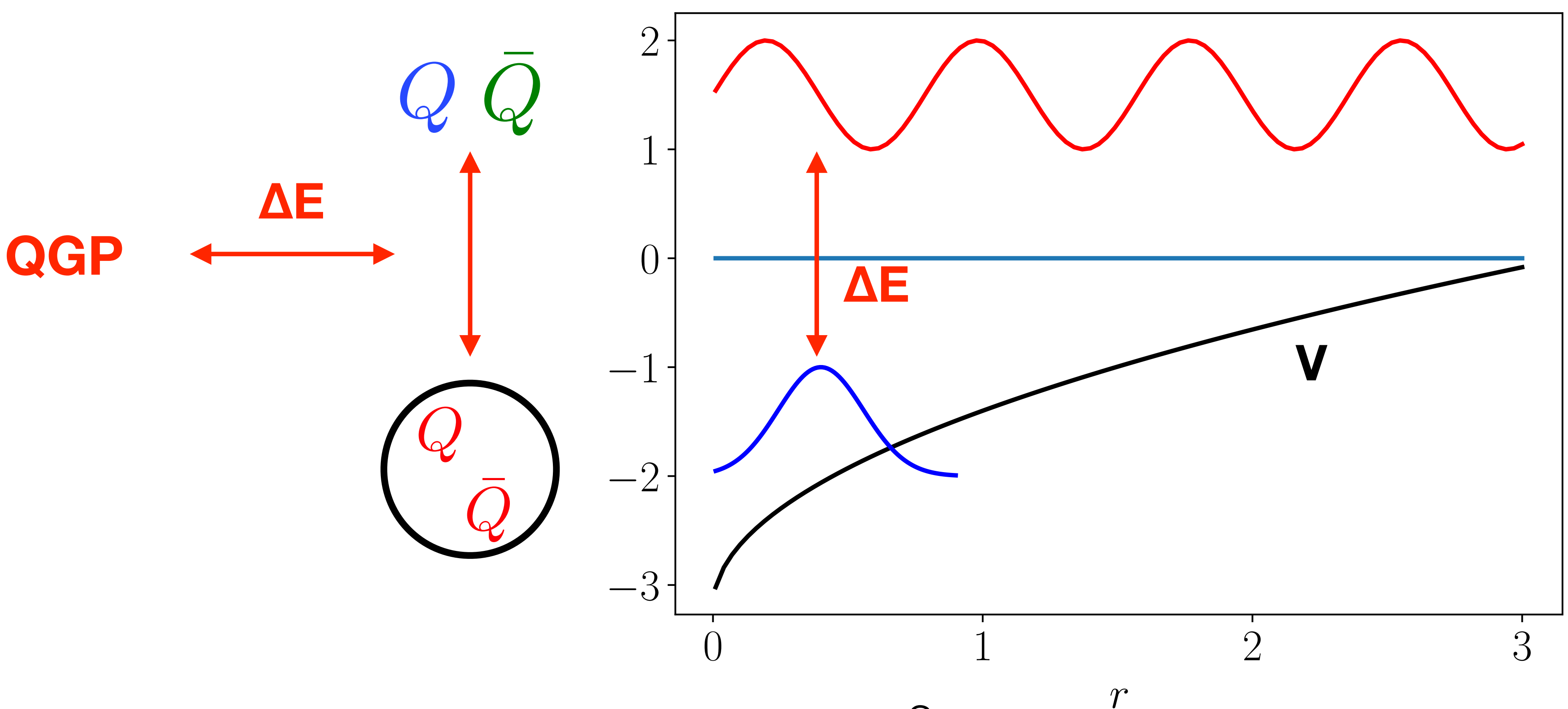
A.Mocsy, 0811.0337

Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: related to imaginary potential, **dissociation** induced by dynamical process, lead to suppression even when T(QGP) < melting T

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, crucial for phenomenology and theory consistency



Quarkonium as Probe of Quark-Gluon Plasma

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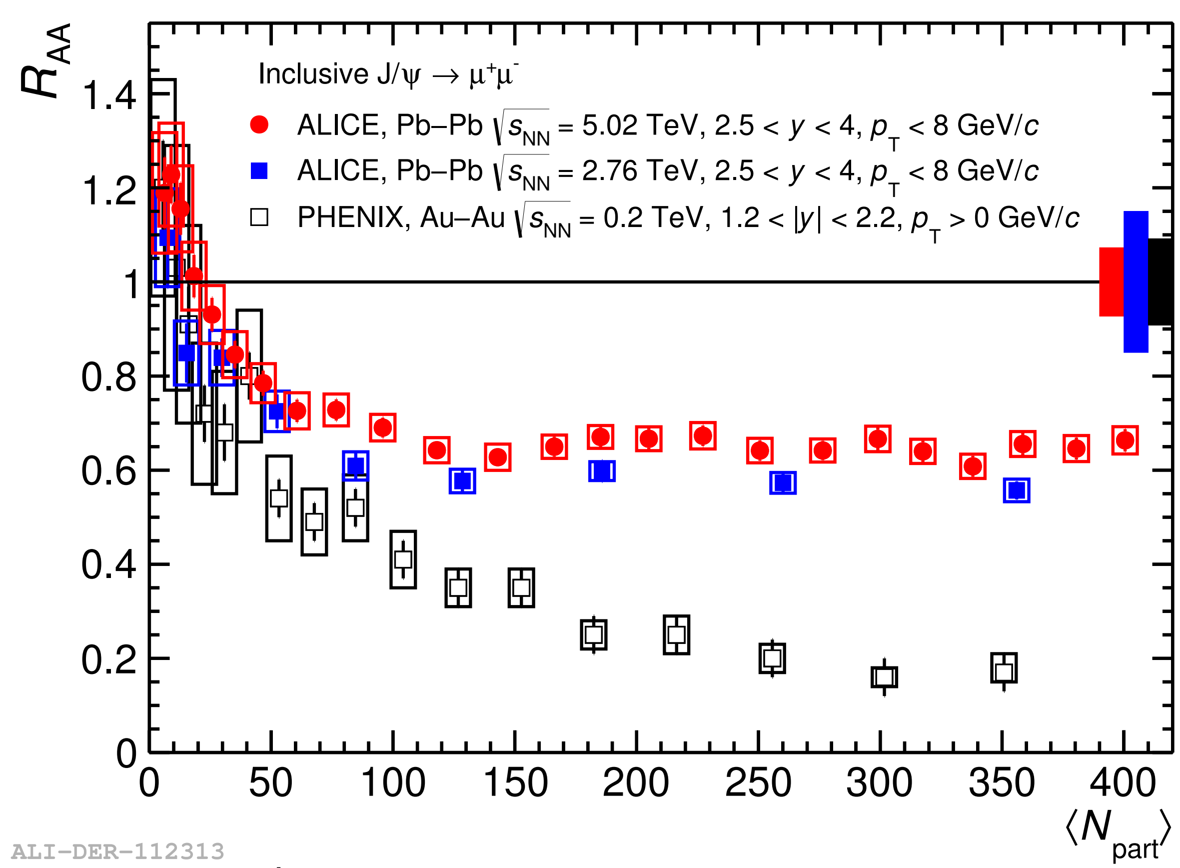
$$R_{\mathrm{AA}} = \frac{\sigma_{\mathrm{AA}}}{N_{\mathrm{coll}}\sigma_{pp}}$$



Simple Thermometer Picture Breaks Down

What QGP properties are we probing by measuring quarkonium?

This talk:

- **In certain limit, we are probing chromoelectric correlators of QGP/nuclear medium**
- **Gauge invariant object, all-order (in coupling) construction**
- **Tools: open quantum systems + effective field theory (EFT)**

Contents

- Introduction: **open quantum system**
- General procedure: derive semiclassical transport from open quantum system, with **effective field theory**
- Two temperature regimes:
    - **High temperature:** quantum Brownian motion, **Langevin** equations
    - **Low temperature:** quantum optical limit, **Boltzmann** equations
- Momentum-dependent & independent chromoelectric correlators of QGP

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

Subsystem & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

Subsystem & environment

Trace out (integrate out) environment

$$\rho_S(t=0)$$

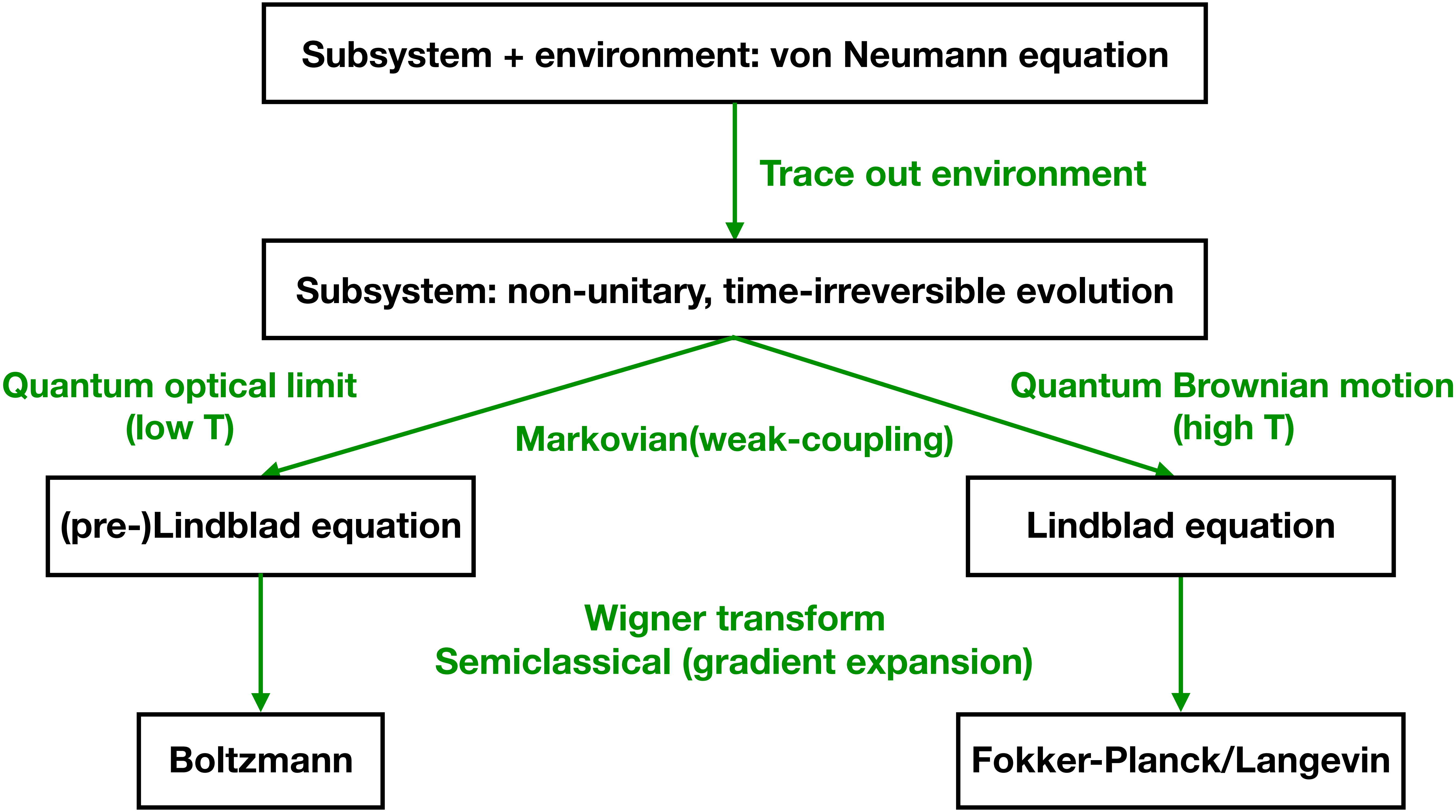
Subsystem

Non-unitary

Time irreversible

$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

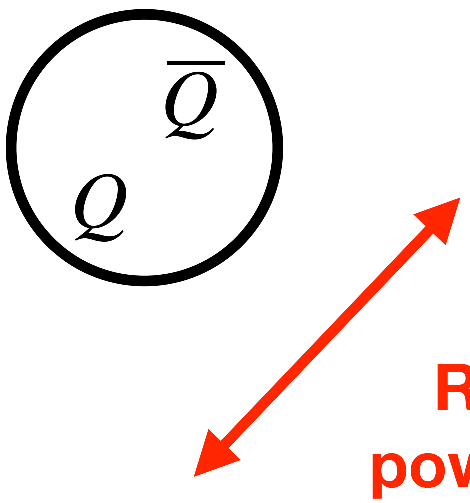
From Open Quantum System to Semiclassical Transport



Wigner transform $$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1 \middle| \rho_S(t) \middle| \boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1 \right\rangle$$

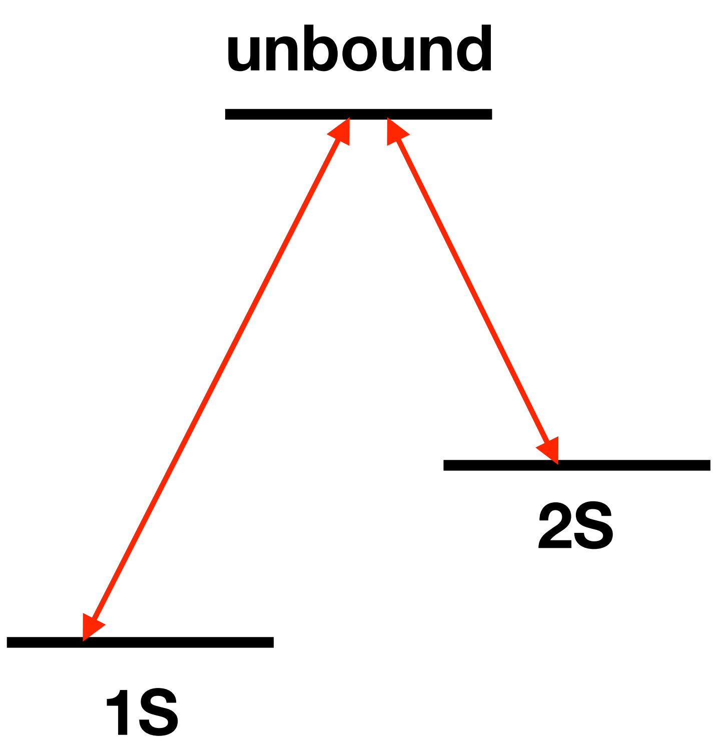
Physical Pictures of Two Limits

- **Quantum optical limit (low T)**

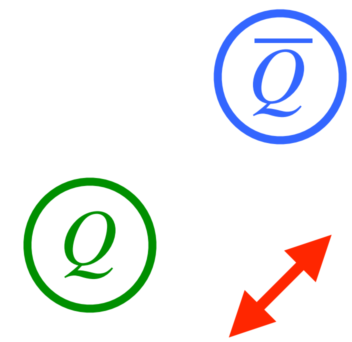


Resolving power of QGP

Transitions between levels

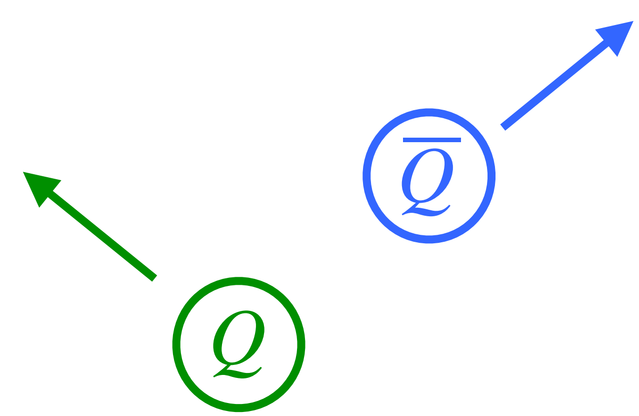


- **Quantum Brownian motion (high T)**



Resolving power of QGP

Diffusion of heavy Q pair



Wavefunction decoherence
—> dissociation

Two Limits and Hierarchy of Time Scales

- **Quantum optical limit (low T)**

$$\tau_R \gg \tau_E,\ \tau_R \gg \tau_S$$

- **Quantum Brownian motion (high T)**

$$\tau_R \gg \tau_E,\ \tau_S \gg \tau_E$$

- τ_E: **environment correlation time**, $\tau_E \sim \dfrac{1}{T}$ for QGP at equilibrium

- τ_S: **subsystem intrinsic time scale**, $\tau_S \sim \dfrac{1}{E_b}$, inverse of quarkonium binding energy

- τ_R: **subsystem relaxation time**, depends on coupling strength between subsystem and environment

- $\tau_R \gg \tau_E$: **Markovian dynamics**, environment correlation lost during subsystem evolution, **generally true in weak coupling limit** (between subsystem and environment)

Separation of Scales and NREFT

Separation of scales

$$M \gg Mv \gg Mv^2,\ \Lambda_{QCD}$$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium

QCD

Hard M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Soft Mv — perturbative / non-perturbative matching

potential NRQCD

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Ultrasoft

$Mv^2 \sim 500\ \mathrm{MeV}$

Different descriptions depending on where T fits into the hierarchy

High Temperature 1: NRQCD $T \gg Mv^2$

Lindblad equation in limit of quantum Brownian motion

$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S, \rho_S(t)\right] + \frac{1}{N_c^2-1}\int \frac{d^3q}{(2\pi)^3} D^{>}(q_0=0,\boldsymbol{q}) \times \left(\widetilde{O}^a(\boldsymbol{q})\rho_S(t)\widetilde{O}^{a\dagger}(\boldsymbol{q}) - \frac{1}{2}\left\{\widetilde{O}^{a\dagger}(\boldsymbol{q})\widetilde{O}^a(\boldsymbol{q}), \rho_S(t)\right\}\right)$$

Environment correlator $D^{>ab}(x_1,x_2) = g^2\mathrm{Tr}_E\left(\rho_E A_0^a(t_1,\boldsymbol{x}_1)A_0^b(t_2,\boldsymbol{x}_2)\right)$

$$\widetilde{O}^a(\boldsymbol{q}) = e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}\left(1-\frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_Q}{4MT}\right)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_Q}T_F^a - e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}\left(1-\frac{\boldsymbol{q}\cdot\hat{\boldsymbol{p}}_{\bar{Q}}}{4MT}\right)e^{\frac{i}{2}\boldsymbol{q}\cdot\hat{\boldsymbol{x}}_{\bar{Q}}}T_F^{*a}$$

Dissipation effect, important for thermalization

Approximations:

Stochastic Schrödinger equation with dissipation R.Katz, P.B.Gossiaux, 1504.08087; T.Miura, Y.Akamatsu, M.Asakawa, A.Rothkopf, 1908.06293

Semiclassical limit Langevin equations J.-P. Blaizot, M.A.Escobedo, 1711.10812

High Temperature 2: pNRQCD $Mv \gg T \gg Mv^2$

Lindblad equation in limit of quantum Brownian motion

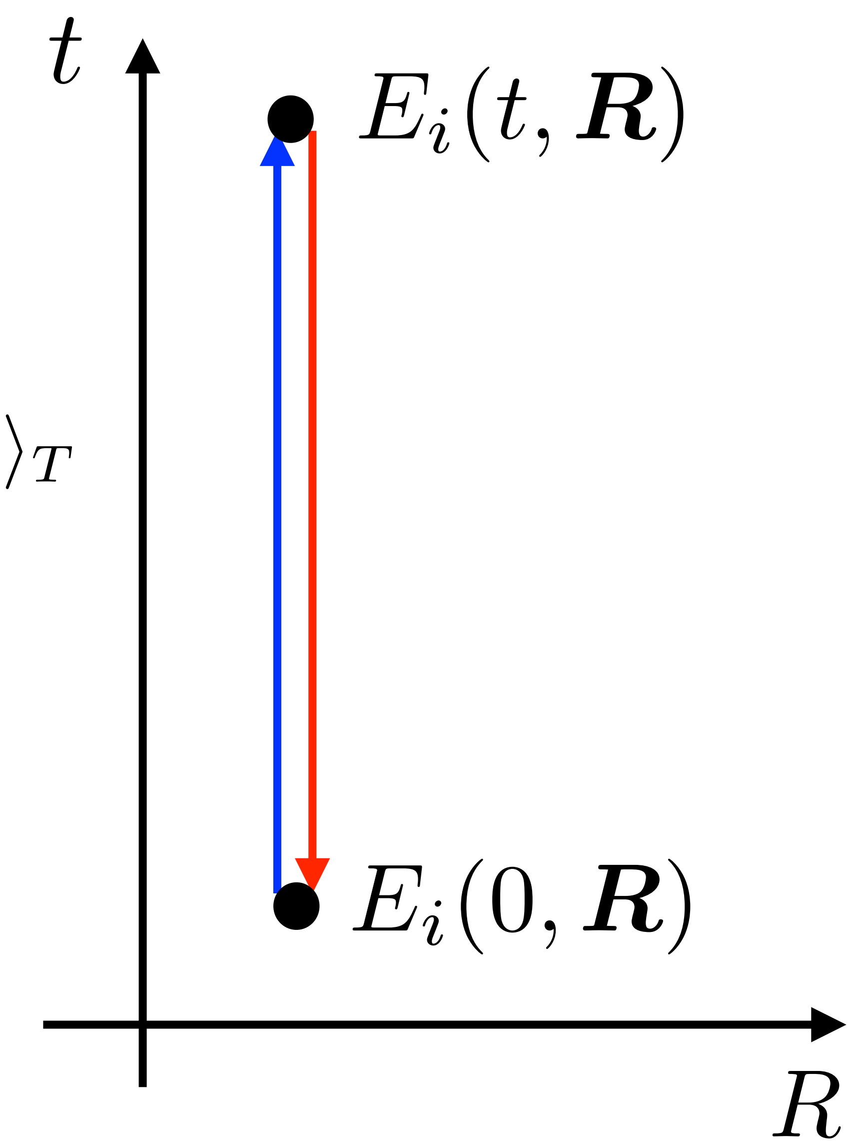
$$\frac{d\rho_S(t)}{dt} = -i\left[H_S + \Delta H_S,\, \rho_S(t)\right] + \frac{D(\omega=0,\boldsymbol{R}=0)}{N_c^2-1}\left(L_{\alpha i}\rho_S(t)L_{\alpha i}^{\dagger} - \frac{1}{2}\left\{L_{\alpha i}^{\dagger}L_{\alpha i},\, \rho_S(t)\right\}\right)$$

N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber, 2012.01240, 2107.06222

Evolution determined by transport coefficients

$$D(\omega=0,\boldsymbol{R}=0) = g^2 \int dt\, \langle E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$

$$\Sigma(\omega=0,\boldsymbol{R}=0) = g^2 \mathrm{Im} \int dt\, \langle \mathcal{T} E_i(t,\boldsymbol{R})\mathcal{W}_{[t,0]}E_i(0,\boldsymbol{R})\rangle_T$$



D is just the heavy quark diffusion coefficient

Why HQ diffusion coefficient affects quarkonium?

$T \gg Mv^2$ **binding energy effect is subleading**

Low Temperature: pNRQCD $Mv \gg Mv^2 \gtrsim T$

Quantum optical and semiclassical limits: Boltzmann equation

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) + \frac{\boldsymbol{k}}{2M} \cdot \nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{+}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{-}_{nl}(\boldsymbol{x}, \boldsymbol{k}, t)$$

T.Mehen, XY: 1811.07027, 2009.02408

Dissociation term

$$\mathcal{C}^{-}_{nl} = \frac{T_F}{N_c} \sum_{i_1, i_2} \int \frac{d^3 p_{\rm cm}}{(2\pi)^3} \frac{d^3 p_{\rm rel}}{(2\pi)^3} \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(E_{nl} - E_p + q^0)$$
$$\times \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle \langle \Psi_{\boldsymbol{p}_{\rm rel}} | r_{i_2} | \psi_{nl} \rangle D_{i_1 i_2}(q^0, \boldsymbol{q}) f_{nl}(\boldsymbol{x}, \boldsymbol{k})$$

Chromoelectric correlator of QGP (gauge invariant, scale independent)

$$D_{i_1 i_2}(q^0, \boldsymbol{q}) = g^2 \int dt\, d^3 R\, e^{iq^0(t_1 - t_2) - i\boldsymbol{q}\cdot(\boldsymbol{R}_1 - \boldsymbol{R}_2)} \langle E_{i_1}(t_1, \boldsymbol{R}_1) \mathcal{W} E_{i_2}(t_2, \boldsymbol{R}_2) \rangle_T$$

More general than the previous case:

Binding energy effect matters here: different quarkonium states respond differently

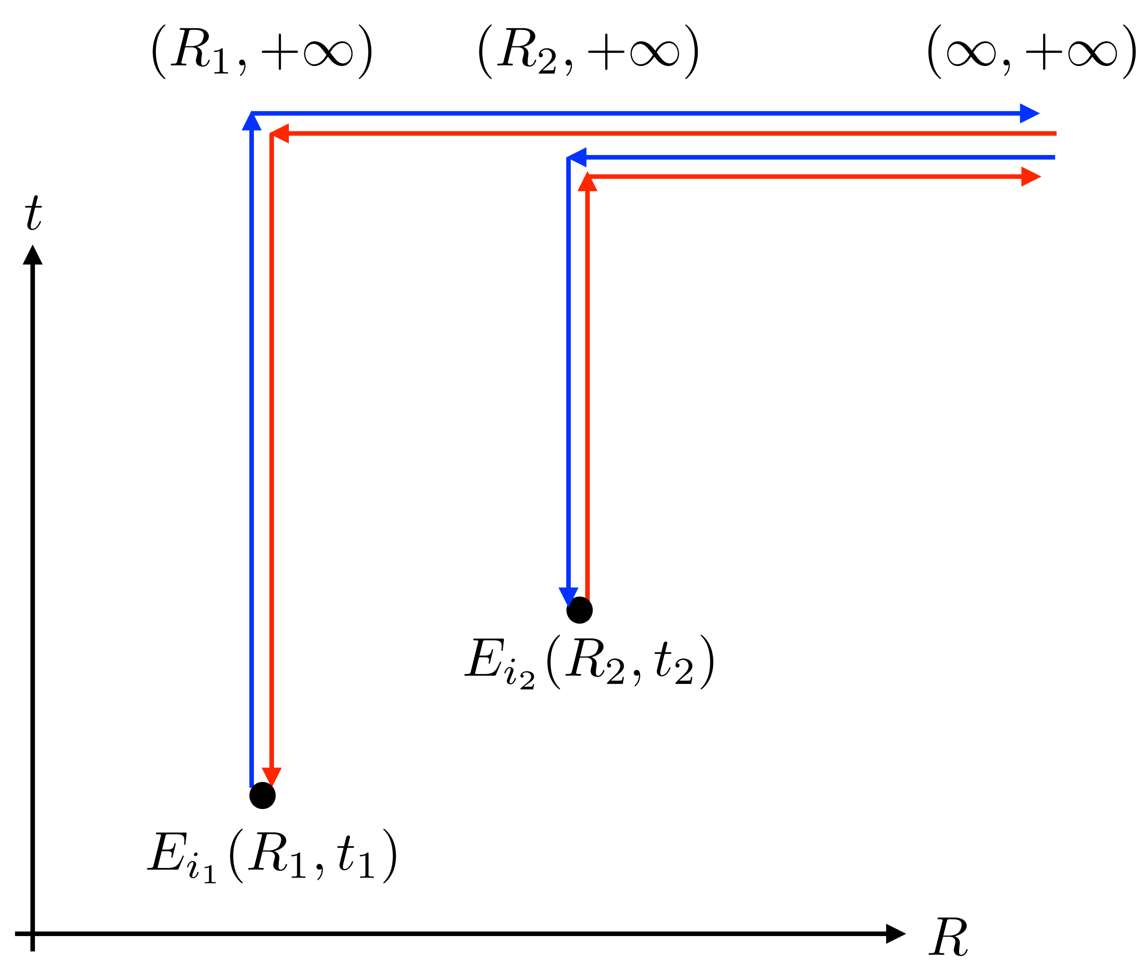
Finite momentum transfer, momentum dependence

Chromoelectric Correlator of QGP

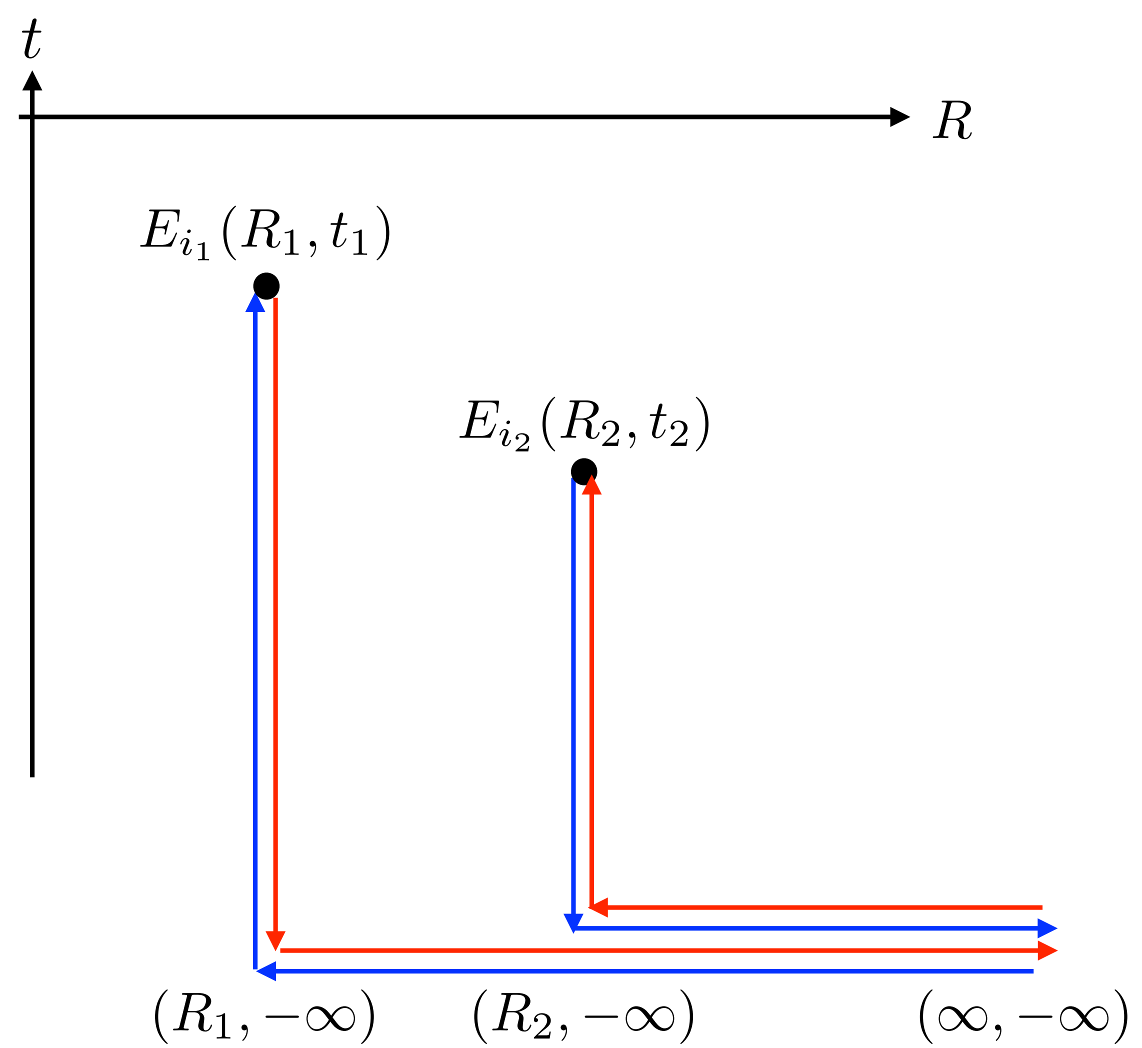
Staple shaped Wilson lines

$$D_{i_1 i_2}(q^0, \boldsymbol{q}) = g^2 \int dt\, d^3R\, e^{iq^0(t_1-t_2)-i\boldsymbol{q}\cdot(\boldsymbol{R}_1-\boldsymbol{R}_2)} \langle E_{i_1}(t_1, \boldsymbol{R}_1)\mathcal{W}E_{i_2}(t_2, \boldsymbol{R}_2)\rangle_T$$

For dissociation: final-state interaction



For recombination: initial-state interaction



Inclusive v.s. Differential Reaction Rates

Take dissociation rate as example

$$R^-_{nl} = \sum_{i_1,i_2} \int \frac{d^3p_{\rm cm}}{(2\pi)^3} \frac{d^3p_{\rm rel}}{(2\pi)^3} \frac{d^4q}{(2\pi)^4} (2\pi)^4 \delta^3(\boldsymbol{k} - \boldsymbol{p}_{\rm cm} + \boldsymbol{q}) \delta(E_{nl} - E_p + q^0) d^{nl}_{i_1 i_2}(\boldsymbol{p}_{\rm rel}) D_{i_1 i_2}(q^0, \boldsymbol{q})$$

$$d^{nl}_{i_1 i_2}(\boldsymbol{p}_{\rm rel}) = \frac{T_F}{N_c} \langle \psi_{nl} | r_{i_1} | \Psi_{\boldsymbol{p}_{\rm rel}} \rangle \langle \Psi_{\boldsymbol{p}_{\rm rel}} | r_{i_2} | \psi_{nl} \rangle$$

Inclusive rate

$$R^-_{nl} = \int \frac{d^3p_{\rm rel}}{(2\pi)^3} \bar{d}^{nl}(\boldsymbol{p}_{\rm rel}) D\Big(\frac{p^2_{\rm rel}}{M} - E_{nl}, \boldsymbol{R} = 0\Big)$$

$$D(q^0, \boldsymbol{R} = 0) = g^2 \int dt\, e^{iq^0 t} \langle E_i(t, \boldsymbol{R}) \mathcal{W}_{[t,0]} E_i(0, \boldsymbol{R}) \rangle_T$$

Momentum independent distribution

Zero frequency limit = HQ diffusion coefficient, appear in quantum Brownian motion

Differential rate

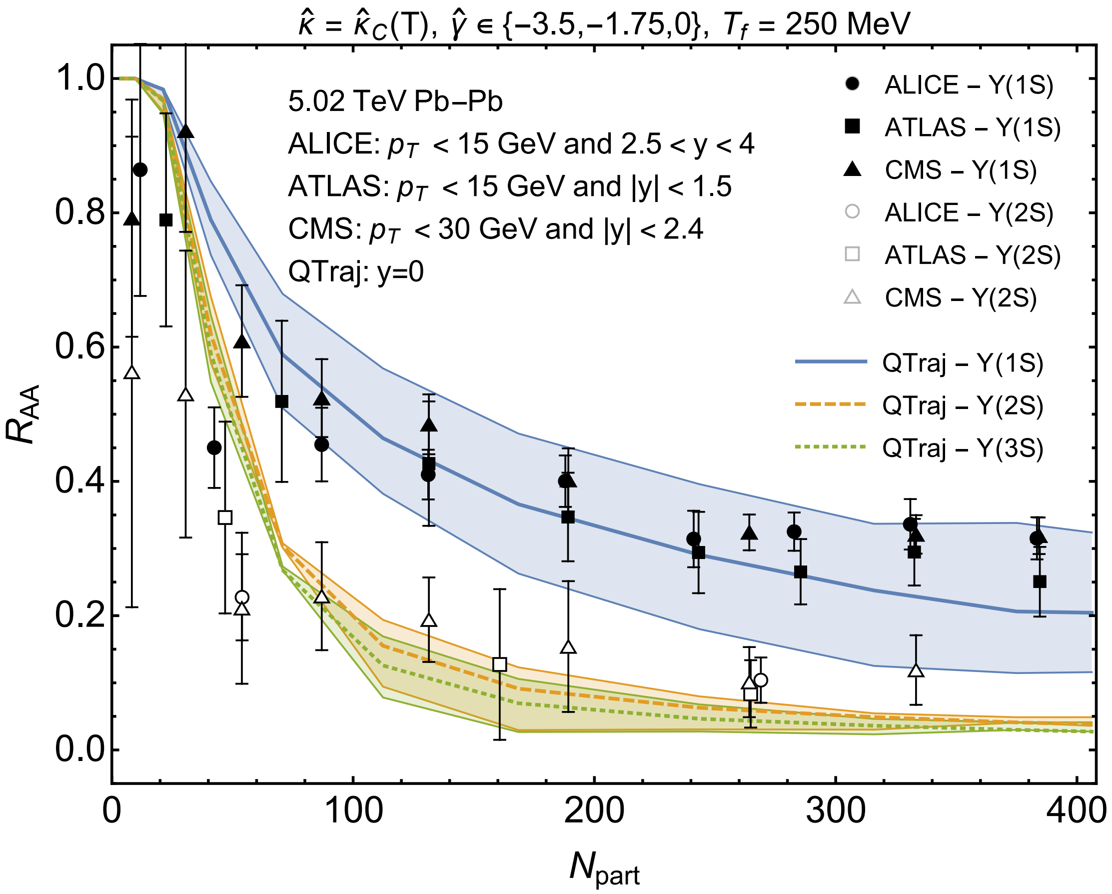
$$(2\pi)^3 \frac{dR^-_{nl}}{d^3p_{\rm cm}} = \int \frac{d^3p_{\rm rel}}{(2\pi)^3} \bar{d}^{nl}(\boldsymbol{p}_{\rm rel}) D\Big(\frac{p^2_{\rm rel}}{M} - E_{nl}, \boldsymbol{p}_{\rm cm} - \boldsymbol{k}\Big)$$

Momentum dependent distribution

Similar to PDF v.s. TMDPDF, though different in time axis

Phenomenological Results for Bottomonia

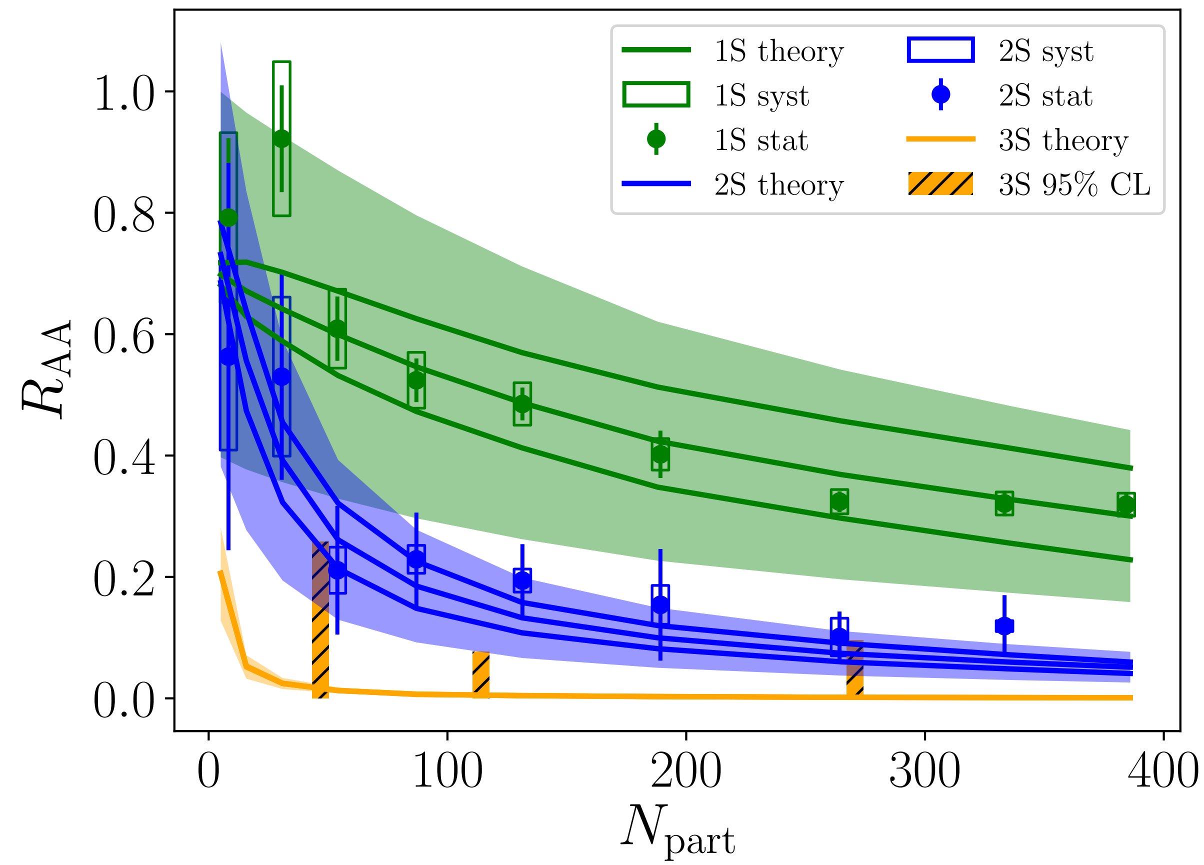
Lindblad equation for quantum Brownian motion



N.Brambilla, M.A.Escobedo, M.Strickland, A.Vairo, P.V.Griend, J.H.Weber arXiv:2012.01240

Uncertainty of transport coefficients

Coupled Boltzmann equation for quantum optical limit



XY, W.Ke, Y.Xu, S.A.Bass, B.Müller, 2004.06746

Uncertainty of nPDF dominates

Summary

- What are we probing by measuring quarkonium in heavy ion collisions? **Chromoelectric correlator of QGP**
- **Open quantum + EFT**: derive quantum and semiclassical transport equations
  - High temperature: Langevin equations, dynamics governed by heavy quark diffusion coefficient & another transport coefficient
  - Low temperature: Boltzmann equations, dynamics governed by energy and momentum dependent chromoelectric correlator