Electromagnetic Pion Form Factor in a deformed background

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Motivation

From holographic Grounds

When we consider Δ as the scaling dimension of the operators creating hadrons, can we calculated a consistent pion form factor?

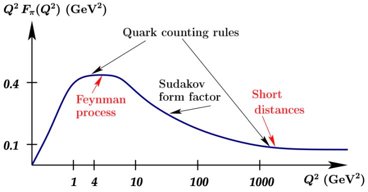
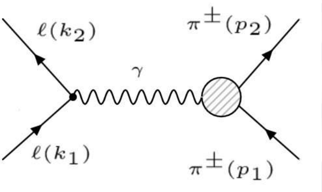


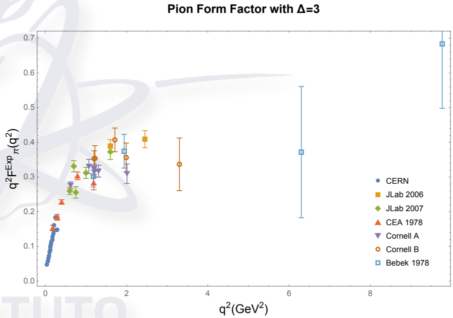
Figure: Taken from Efremov 2009

From phenomenological grounds

- The behavior of the πFF at moderate q^2 (5-20 GeV^2) is expected to exhibit a Sudakov suppression up to the asymptotic region where counting rules apply. See Efremov 2009.
- Lattice-QCD analysis for the transitions from soft to hard πFF shows a suppressed behavior at intermediate q^2 regions. See Choi 2006 and Gao et al. 2021.

Pion form factor





- It is one of the most valuable QCD quantities related to the transition from the non-perturbative to the perturbative regime.
- It is defined by the matrix element

$$\langle \pi^{\pm}(p_2)\,|J_{\pi}^{\mu}(0)|\,\pi^{\pm}(p_1)\rangle = g_{\pi^{\pm}}\,(p_1+p_2)^{\mu}\,F_{\pi}(q^2).$$

Deformed background AdS/QCD model

In a general bottom-up model

Confinement is realized *via* the Schrödinger-like holographic potential

$$V(z) = \frac{1}{4}B'(z)^2 - \frac{1}{2}B''(z) + M_5^2\, R^2\, e^{2\,A(z)} \text{ for integer spin hadrons}$$

where $B(z) = \Phi(z) + \beta\, A(z)$.

- $\Phi(z)$ is the KKSS dilaton (static or dynamic generated).
- $h(z)$ is a metric deformation.

Both proposals induce confinement. However, the latter works better with fermions than the former. See Eduardo's talk about proton DIS.

General Idea:

Induce confinement geometrically with a quadratic deformation and $\Phi(z) = 0$ (See Capossolli et al. 2020):

$$dS^2 = \frac{R^2}{z^2} e^{\frac{k_H\, z^2}{2}} \left[dz^2 + \eta_{\mu\,\nu}\, dx^\mu\, dx^\nu\right]$$

where k_H sets confinement for a given hadron.

Pion in the context of Deformed Background Model

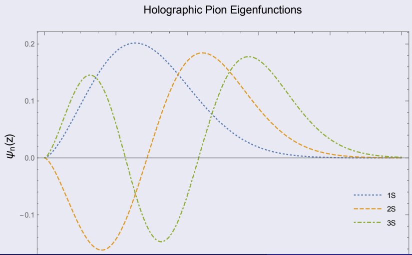
Pion dual to a bulk massive scalar field living on its background defined by k_π:

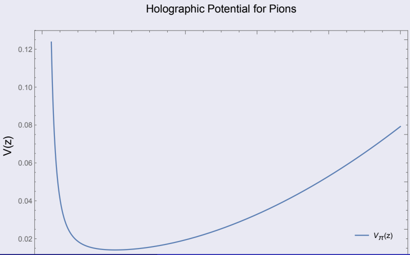
$$S_\pi = \int d^5x \sqrt{-g_\pi}\, [g_\pi^{mn}\partial_m X \partial^n X + M_5^2 X^2]$$

$$M_5^2\, R^2 = (\Delta - S)(\Delta + S - 4)$$

- $\Delta = 3 \rightarrow M_5^2\, R^2 = -3$
- $\beta = -3$ and $k_\pi = -0.0425^2$ GeV^2.

$$V_\pi(z) = \frac{15}{4\,z^2} - \frac{3}{z^2}\, e^{-\frac{k_\pi\, z^2}{2}} + \frac{k_\pi^2\, z^2}{4} + k_\pi$$





Virtual photon in the background model

Since the photon γ is virtual, the associated deformation slope k_γ does not set confinement. This k_γ is associated with the kinematics of the elastic process.

Photon dual to a massless bulk vector field:

$$S_\gamma = -\frac{1}{c_\gamma^2}\int d^5x\sqrt{-g_\gamma}\,\frac{1}{4}F^{mn}F_{mn}\,,$$

Photon Bulk to boundary propagator dual to the EM current in the pion vertex at the boundary:

$$\begin{aligned}\phi_\mu(z,q) &= -\frac{\eta_\mu e^{iq\cdot y}}{2}\,k_\gamma\,z^2\,\Gamma\left[1-\frac{q^2}{2k_\gamma}\right]\mathcal{U}\left(1-\frac{q^2}{2k_\gamma};\,2;\,-\frac{k_\gamma\,z^2}{2}\right)\\ &\equiv -\frac{\eta_\mu e^{iq\cdot y}}{2}\,\mathcal{B}(z,q)\,,\end{aligned}$$

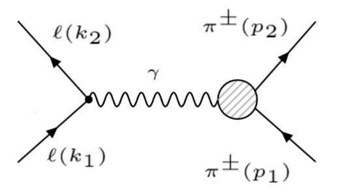
Interaction Picture: Pion Form Factor

General Action

$$S = S_\pi + S_\gamma + S_{\text{Int}}$$

where

$$\begin{aligned} S_{\text{Int}} &= g_{\text{eff}} \int d^5x \sqrt{-g_\pi}\, g_\pi^{mn}\, \phi_m \left[X_{p_1}\, \partial_m X^*_{p_2} - X^*_{p_2}\, \partial_m X_{p_1} \right], \\ &= i\,(2\,\pi)^4\, \delta^4(q - (p_2 - p_1))\, \eta^{\mu\nu} \eta_\mu\, (p_1 + p_2)_\nu\, g_{\text{eff}} \int dz\, e^{3\,A_\pi(z)}\, \psi_0^2\, \mathcal{B}, \end{aligned}$$

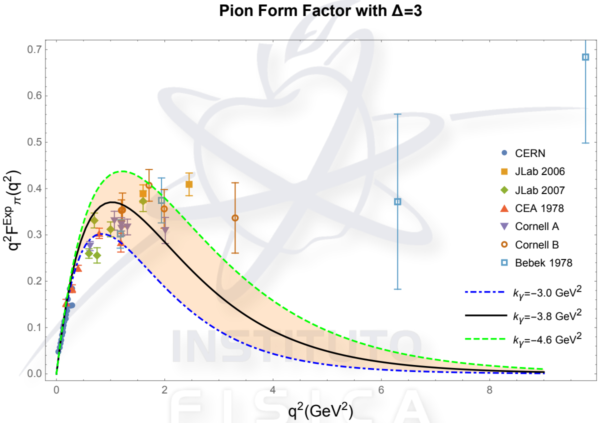


We define the pion form factor as

$$F_\pi(q^2) = \int dz\, e^{3\,A_\pi(z)}\, v(z)\, \mathcal{B}(z, q^2)\, v(z).$$

We evaluate πFF in the pion geometry!!

Results πFF try one: Comparison with Exp. Data



- In agreement with low q^2 limit.
- Not good for intermediate q^2
- Highly Sudakov suppressed.
- Not in agreement with Brodsky-Lepage counting rule.

Results πFF try one: Comparison with Exp. Data

No Brodsky-Lepage counting rule (C.R.)

Let us take the large q^2 limit for the πFF:

$$F_\pi(q^2)\Big|_{q^2\to\infty} = \frac{1}{8\,\Delta^2\,(\Delta-1)}\left(\frac{1}{q^2}\right)^{\Delta-1}\left[1+\gamma_e\,(\Delta-1)\,\Delta-\Delta^2\,(\log 4-3)+\ldots\right]$$

LF πFF (Brodsky 2008)

$$F_\pi(q^2) = \frac{4\,\kappa^2}{4\,\kappa^2+q^2} \to \left(\frac{1}{q^2}\right)^{\Delta-1}\Bigg|_{q\to\infty}$$

- $\Delta = 2 \to$ constituent number.
- FF independent of hadronic spin.

Other bottom-up models (non-LF)

- $\Delta = 3 \to$ scaling dimension.
- Scalar mesons does not have the right C.R.(Ops!)!
- Only in the vector meson case, the C.R. is fulfilled.

Our Solution: $k_\gamma \to k_\gamma(q)$

Ansatz

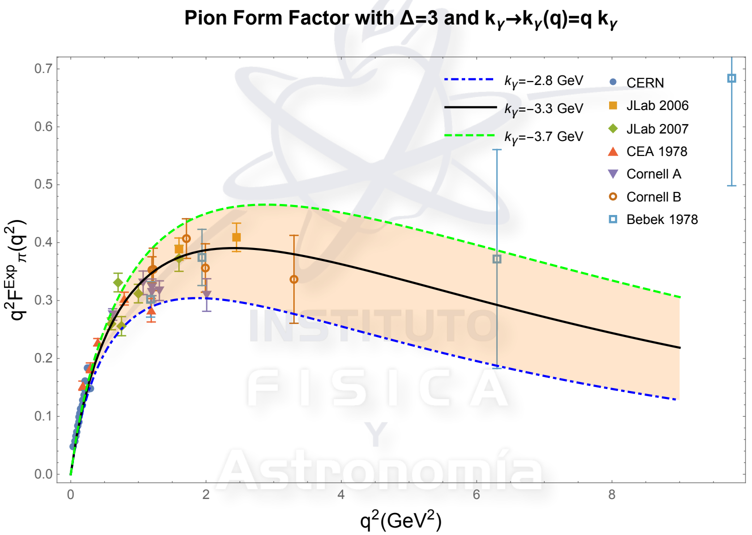
Since the geometric slope k_γ is related with the kinematic of the scattering process, we propose

$$k_\gamma(q) = q\,\kappa_\gamma$$

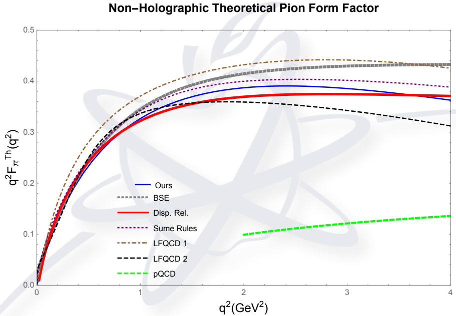
with κ_γ defined as a constant given in energy units.

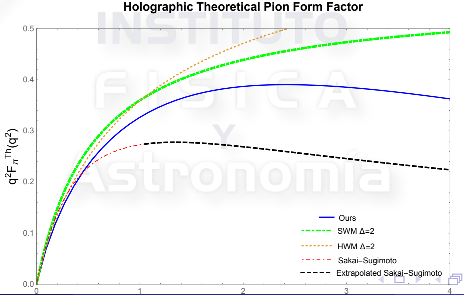
This choice guarantees that the Brodsky-Lepage C.R. be satisfied.

Results πFF try two: Comparison with Exp. Data



Results πFF try two: Comparison with other models





Results πFF try two: low q^2, the pion charge radius

Pion charge radius is extracted from the low q^2 behavior of the πFF as:

$$\langle r_\pi^2 \rangle = -6 \left. \frac{dF_\pi(q^2)}{dq^2} \right|_{q^2=0} .$$

In our case, with $\kappa_\gamma = -3.3$ GeV,we obtain

Exp. PDG (fm)	**Theo. (fm)**	**R. E.(%)**
0.659 ± 0.004	0.671	2.0

Conclusions

- We have calculated the πFF in the AdS deformed background model, with $\Delta = 3$.
- The calculated pion form factor exhibits a Sudakov-like suppression for the intermediate q^2 region.
- In order to have the proper C.R. behavior, we impose that $k_\gamma = k_\gamma(q)$, since it is not related to confinement.
- The pion charge radius in our proposal has 2.0% of R.E in comparison with experimental data (PDG).
- This holographic procedure is an alternative to the L.F. holography calculation, where Δ is interpreted in terms of the constituent number.

Thank you!