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Direct and Indirect Methods of Vortex Identification in continuum theory

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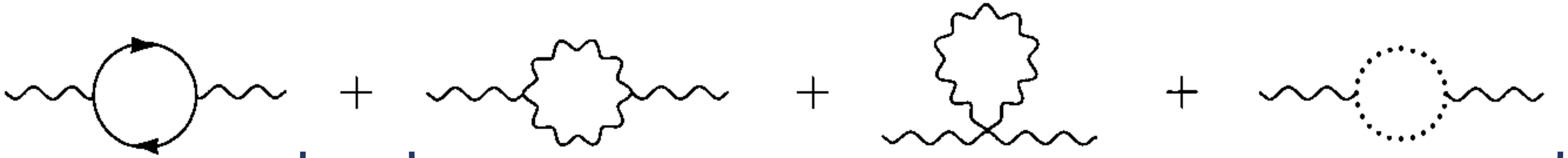
Outline

- Confinement
- Lattice QCD and continuum theory
- The direct method of identifying vortices in SU(2)- vortices
- The indirect method of identifying vortices in SU(2)- chain

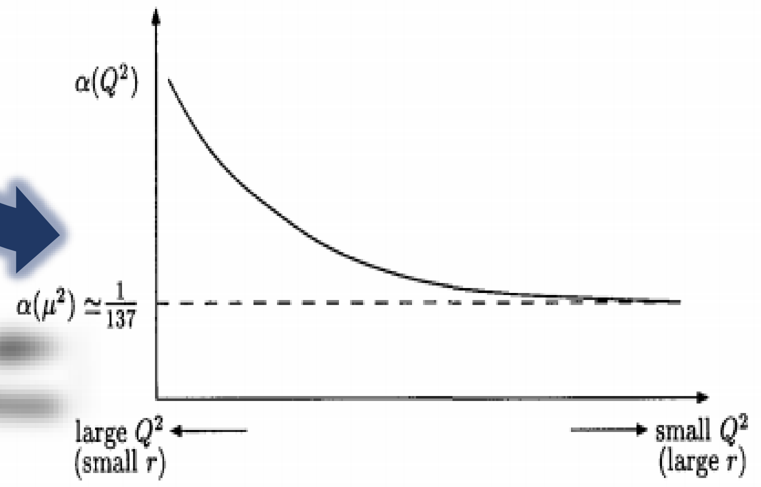
1 Confinement

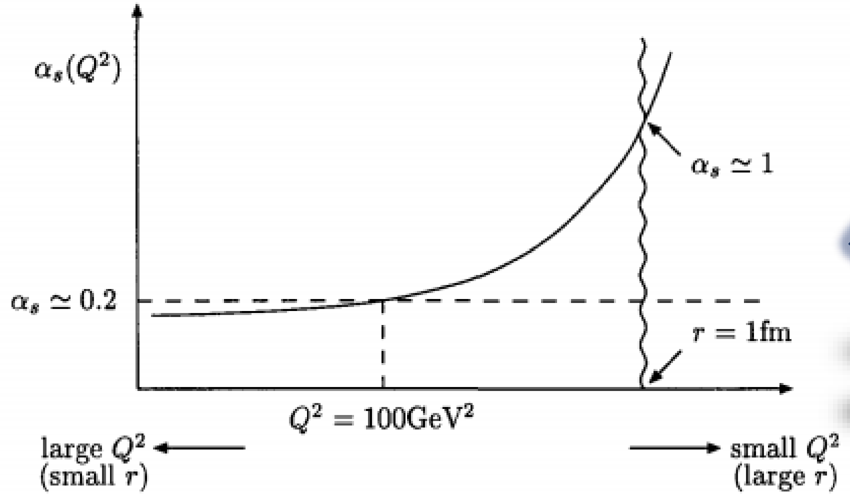
Non-Abelian theories: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu$



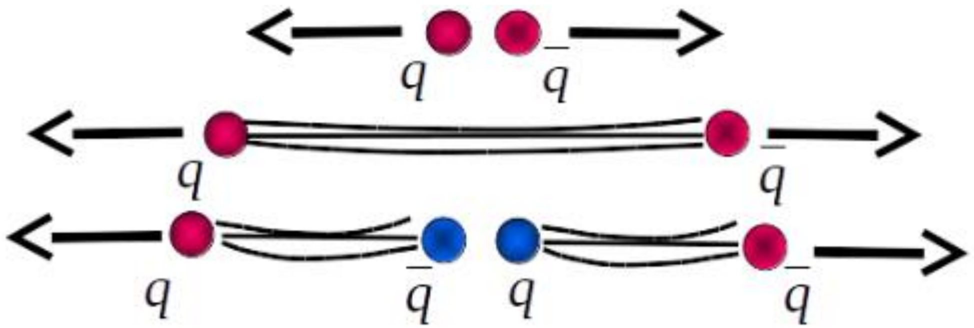
$$\alpha(Q^2) = \frac{\alpha(m^2)}{1 + \frac{\alpha(m^2)}{3\pi}\ln(Q^2/m^2)}$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_F)\ln(Q^2/\mu^2)}$$

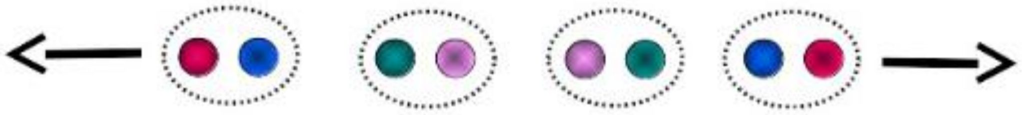


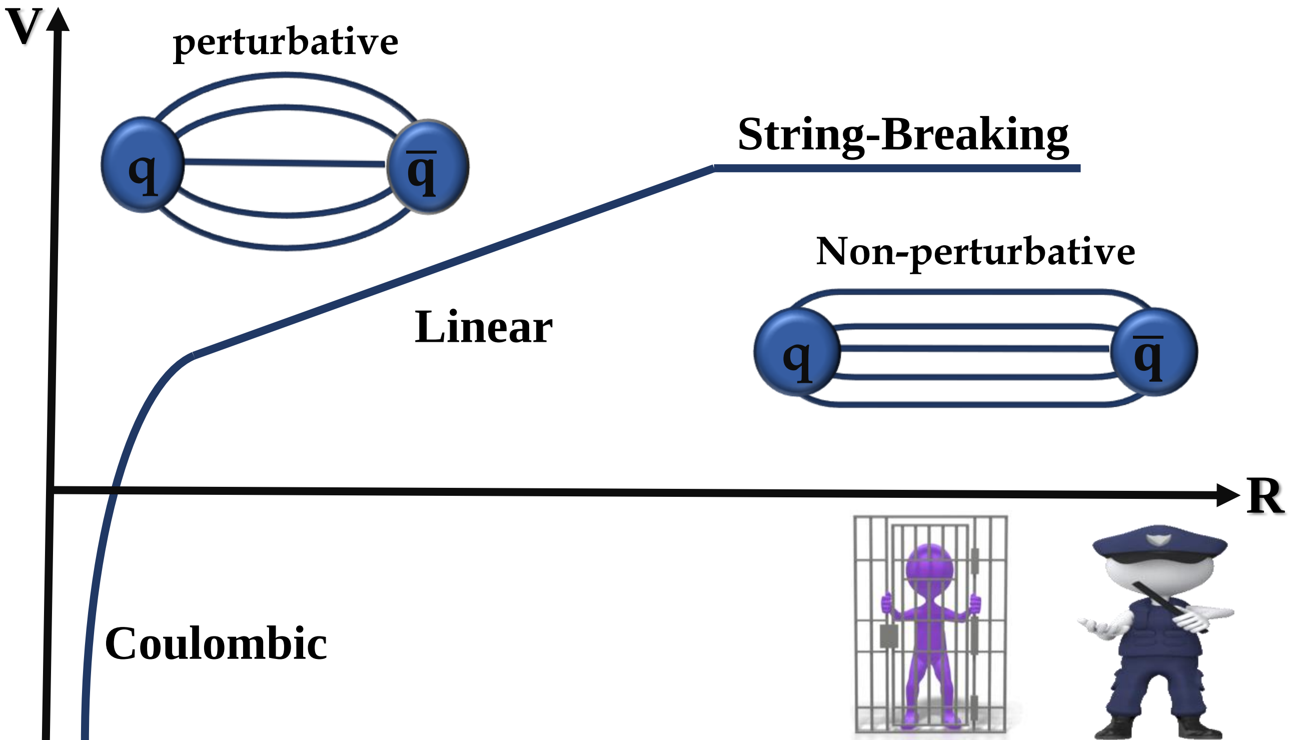


2 Confinement



As energy decreases, hadrons (mainly mesons) freeze out





Non-perturbative methods

- **Lattice QCD** $\xrightarrow{a \to 0}$ **Continuum theory**
- **Phenomenological models**

QCD vacuum

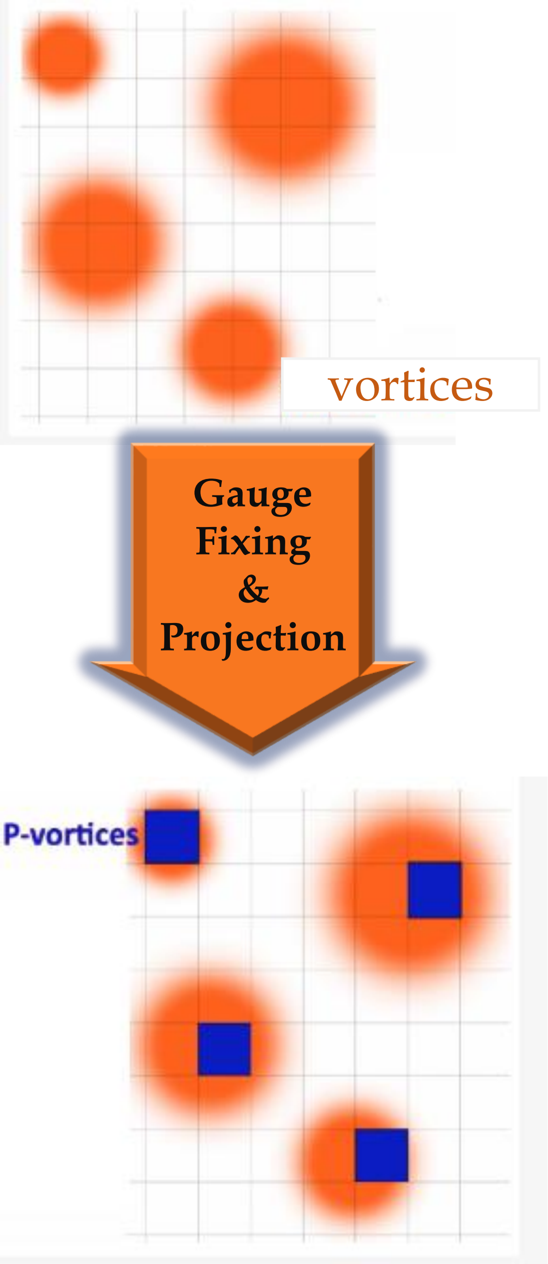
Monopoles

Dual Superconductor

Vortices

Center vortex model

Lattice QCD



$$\text{Center (SU(N))} = Z_N = e^{i2\pi n/N} \times \boldsymbol{I} \qquad n = 1,2,\dots,N-1$$

Direct Maximal Center Gauge (DMCG)

$$U_\mu^G \rightarrow Z(2) = sign\left[TrU_\mu^G\right] \times \boldsymbol{I} = \{-1,1\} \times \boldsymbol{I}$$

1 2

L. Del Debbio et al, Phys. Rev. D58 (1998) 094501.

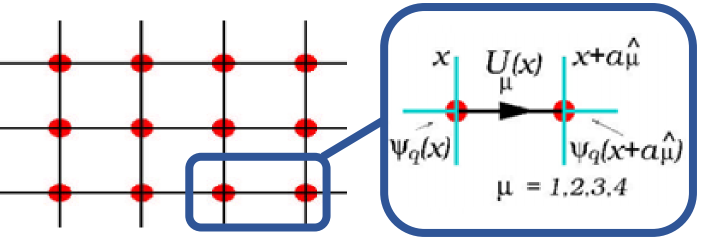
Indirect Maximal Center Gauge (IMCG)

1. Abelian gauge fixing + Abelian projection $\mathrm{SU(N)} \rightarrow [U(1)]^{N-1}$
2. center gauge fixing + center projection $[U(1)]^{N-1} \rightarrow Z_N$

$$U_\mu^G \rightarrow Z(2) = sign[cos\theta(x,\mu)] \times \boldsymbol{I}$$

L. Del Debbio, M. Faber, J. Greensit e and S. Olejnik, Phys. Rev. D55 (1997) 2298.

Lattice & continuum theory



$$U_\mu(x) = e^{iagA_\mu(x)} \in SU(N_c) \qquad U_\mu(x) \xrightarrow{G(x)} U_\mu^G(x)$$

$$U_\mu^G(x) = 1 + iag\left[G(x)A_\mu(x)G_\mu^\dagger(x) - \frac{i}{g}G(x)\partial_\mu G^\dagger(x)\right] + O(a^2) \equiv e^{iagA_\mu^G(x)}$$

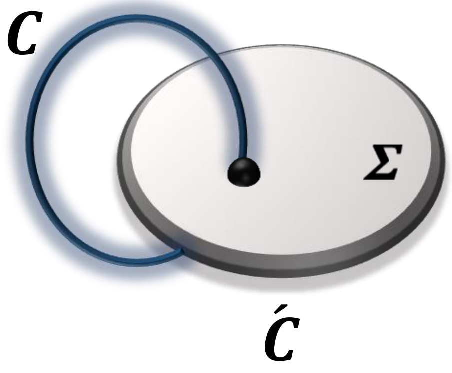
In limit $a \to 0$; $\quad A_\mu^G(x) = G(x)A_\mu(x)G^\dagger(x) - \frac{i}{g}G(x)\partial_\mu G^\dagger(x)$

1. $\boldsymbol{G(x) \equiv M(x) \in SU(N_c)}$ ***is an Abelian gauge***;

$$A_\mu^M(x) = M(x)A_\mu(x)M^\dagger(x) - \frac{i}{g}M(x)\partial_\mu M^\dagger(x)$$

2. $\boldsymbol{G(x) \equiv N(x) \in SU(N_c)}$ ***is a center gauge***;

$$A_\mu^N(x) = N(x)A_\mu(x)N^\dagger(x) - \frac{i}{g}N(x)\partial_\mu N^\dagger(x)$$



$$W(C) \to W^N(C) = N(x)W(C)N^\dagger(x + a\hat{\mu})$$

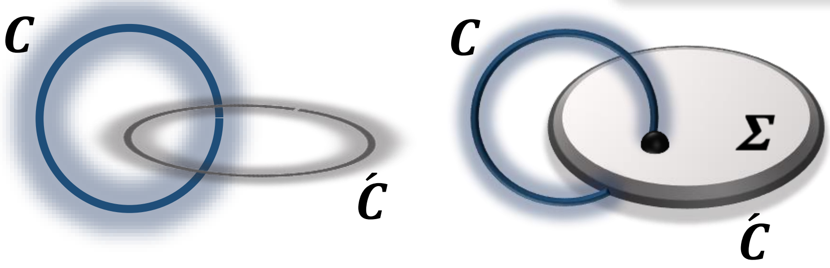
$$w(c) = 1 + O(\epsilon)$$

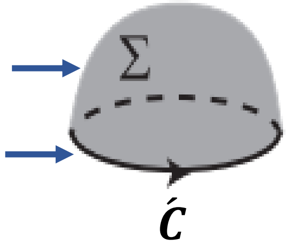
$$W^N(C) = N(x)N^\dagger(x + a\hat{\mu}) = Z(k)$$

M. Engelhardt, H. Reinhardt, Nucl. Phys. B 567 (2000) 249.

Lattice & continuum theory

K-I. Kondo, S. Kato, A. Shibata, T. Shinohara, Quark confinement. arXive: 1409.1599v3 [hep-th]





$$Thin\ vortex = \frac{i}{g} N(x)\partial_\mu N^\dagger(x) + ideal\ vortex$$

$$A_\mu^N(x) = N(x)A_\mu(x)N^\dagger(x) - \frac{i}{g} N(x)\partial_\mu N^\dagger(x) \longrightarrow A_\mu'^N(x) = N(x)A_\mu(x)N^\dagger(x) - Thin\ vortex + ideal\ vortex - ideal\ vortex$$

$$A_\mu'^N(x) = N(x)A_\mu(x)N^\dagger(x) - Thin\ vortex$$

for $x \notin \Sigma$ $\quad Thin\ vortex = \frac{i}{g} N(x)\partial_\mu N^\dagger(x)$

3. ***If $M(x)$ is an Abelian gauge & $N(x)$ a Center gauge*:** $\quad U_\mu(x) \xrightarrow{M(x)} U_\mu^M \xrightarrow{N(x)} U_\mu^{NM}$

$$U_\mu^{NM} = N(x)M(x)e^{iagA_\mu}M^\dagger(x+a\hat{\mu})N^\dagger(x+a\hat{\mu}) = 1 + iag\left(N(x)\left[M(x)A_\mu(x)M^\dagger(x) - \frac{i}{g}M(x)\partial_\mu M^\dagger(x)\right]N^\dagger(x) - \frac{i}{g}N(x)\partial_\mu N^\dagger(x)\right) + O(a^2) = e^{iagA_\mu^{NM}}$$

In limit $a \to 0$: $\quad A_\mu^{NM}(x) = N(x)\left[M(x)A_\mu(x)M^\dagger(x) - \frac{i}{g}M(x)\partial_\mu M^\dagger(x)\right]N^\dagger(x) - \frac{i}{g}N(x)\partial_\mu N^\dagger(x)$

$$A_\mu'^{NM}(x) = N(x)\left[M(x)A_\mu(x)M^\dagger(x) - \frac{i}{g}M(x)\partial_\mu M^\dagger(x)\right]N^\dagger(x) - Thin\ vortex$$

M. Engelhardt, H. Reinhardt, Nucl. Phys. B 567 (2000) 249.

Lattice & continuum theory

$\mathcal{L} = -\frac{1}{2} Tr(\vec{F}_{\mu\nu}.\vec{F}^{\mu\nu})$ With local $SU(N_c)$ symmetry $\vec{F}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + ig[\vec{A}_\mu, \vec{A}_\nu] \in SU(N_c)$

Regular system: $\vec{F}_{\mu\nu} = \frac{1}{ig}[\hat{D}_\mu, \hat{D}_\nu]$ Where, $\hat{D}_\mu = \hat{\partial}_\mu + ig\vec{A}_\mu$

$$\frac{1}{ig}[\hat{D}_\mu, \hat{D}_\nu] = \frac{1}{ig}[\hat{\partial}_\mu, \hat{\partial}_\nu] + \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + ig[\vec{A}_\mu, \vec{A}_\nu]$$

Singular system: $\vec{F}_{\mu\nu} = \frac{1}{ig}[\hat{D}_\mu, \hat{D}_\nu] - \frac{1}{ig}[\hat{\partial}_\mu, \hat{\partial}_\nu]$

→ Topological defects

Gauge Transformation $\xrightarrow{G(x) \in SU(N_c)}$ $\vec{F}_{\mu\nu} \to \vec{F}^G_{\mu\nu} = G(x)(\vec{F}_{\mu\nu})G^\dagger(x)$

$$\vec{F}^G_{\mu\nu} = (\partial_\mu \vec{A}^G_\nu - \partial_\nu \vec{A}^G_\mu) + ig[\vec{A}^G_\mu, \vec{A}^G_\nu] + \frac{i}{g} G(x)[\hat{\partial}_\mu, \hat{\partial}_\nu]G^\dagger(x) \in SU(N_c)$$

- ✓ Abelian Gauge $G(x)=M(x)$
- ✓ Center Gauge $G(x)=N(x)$

$$\vec{F}^{NM}_{\mu\nu} = (\partial_\mu \vec{A}^{NM}_\nu - \partial_\nu \vec{A}^{NM}_\mu) + ig[\vec{A}^{NM}_\mu, \vec{A}^{NM}_\nu] + \frac{i}{g} N(x)M(x)[\hat{\partial}_\mu, \hat{\partial}_\nu]M^\dagger(x)N^\dagger(x) \in SU(N_c)$$

H. Ichie, H. Suganuma, Nucl. Phys. B 574 (2000) 70-106.

The direct method of identifying vortices in SU(2)

Step 1: Center gauge fixing

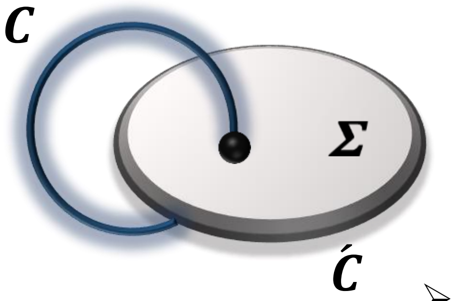
$$G(x) = \begin{pmatrix} e^{\frac{i}{2}[\gamma(x)+\alpha(x)]}\cos\frac{\beta(x)}{2} & e^{\frac{i}{2}[\gamma(x)-\alpha(x)]}\sin\frac{\beta(x)}{2} \\ -e^{-\frac{i}{2}[\gamma(x)-\alpha(x)]}\sin\frac{\beta(x)}{2} & e^{-\frac{i}{2}[\gamma(x)+\alpha(x)]}\cos\frac{\beta(x)}{2} \end{pmatrix} \in SU(2)$$

$\alpha(x) \in [0,2\pi)$

$\beta(x) \in [0,\pi]$

$\gamma(x) \in [0,2\pi)$

$$if \quad G(x) \equiv N(x) \in SU(2)\ is\ a\ Center\ gauge\ transformation$$



$$N(x_\perp, t=\epsilon)N^\dagger(x_\perp, t=-\epsilon) = Z(2) \xrightarrow[\beta(x)=0]{\alpha(x)=\gamma(x)=\frac{\varphi}{2}} N = \begin{pmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-i\frac{\varphi}{2}} \end{pmatrix} \quad with\ \varphi \in [0,2\pi)$$

$t = 0\ is\ on\ \Sigma$

- $N(\varphi=\epsilon)N^\dagger(\varphi=2\pi-\epsilon) = -\boldsymbol{I} \in Non-trivial\ center\ element\ Z(2) \xrightarrow{\varphi=0} ideal\ vortex\ contribution$
- $Thin\ vortex \equiv B_\mu = \frac{i}{g}N(x)\partial_\mu N^\dagger(x) = \frac{1}{g}\partial_\mu\varphi\, T^3 = \frac{1}{g\rho}T^3$ Away from Σ

$$\vec{A}'^N_\mu.\vec{T} = N(x)(\vec{A}_\mu.\vec{T})N^\dagger(x) - Thin\ vortex = A^1_\mu(cos\varphi\, T^1 - sin\varphi T^2) + A^2_\mu(sin\varphi\, T^1 + cos\varphi T^2) + \left(A^3_\mu - \frac{1}{g}\partial_\mu\varphi\right)T^3$$

$$Magnetic\ flux \quad \Phi^{flux} = \int d\vec{X}.\vec{A}^{singular}_\mu = -\frac{1}{2g}\int \rho d\varphi\hat{\phi}.\begin{pmatrix} \partial_\mu\varphi & 0 \\ 0 & -\partial_\mu\varphi \end{pmatrix} = -\frac{2\pi}{g}T^3$$

$$\hat{n}_a \equiv R^{-1}(N)\hat{e}_a = \begin{pmatrix} cos\varphi & sin\varphi & 0 \\ -sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}\hat{e}_a,\ R^{-1} \in SO(3) \qquad \vec{A}'^N_\mu = A^1_\mu\hat{n}_1 + A^2_\mu\hat{n}_2 + \left(A^3_\mu - \frac{1}{g}\partial_\mu\varphi\right)\hat{k}$$

The direct method of identifying vortices in SU(2)

Step 1: Center gauge fixing

$$\vec{F}^{N}_{\mu\nu} = \left(\partial_\mu \vec{A}'^{N}_\nu - \partial_\nu \vec{A}'^{N}_\mu\right) + ig\left[\vec{A}'^{N}_\mu, \vec{A}'^{N}_\nu\right] + \frac{i}{g} N(x)\left[\hat{\partial}_\mu, \hat{\partial}_\nu\right] N^\dagger(x)$$

$$\vec{F}^{linear}_{\mu\nu} \equiv \left(\partial_\mu \vec{A}'^{N}_\nu - \partial_\nu \vec{A}'^{N}_\mu\right) = \left(\partial_\mu A^1_\nu - \partial_\nu A^1_\mu\right)\hat{n}_1 + \left(\partial_\mu A^2_\nu - \partial_\nu A^2_\mu\right)\hat{n}_2 + \left(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu\right)\hat{k}$$

$$-g\left(A^1_\nu \frac{1}{g}\partial_\mu\varphi - A^1_\mu \frac{1}{g}\partial_\nu\varphi\right)\hat{n}_1 + g\left(A^2_\nu \frac{1}{g}\partial_\mu\varphi - A^2_\mu \frac{1}{g}\partial_\nu\varphi\right)\hat{n}_2 - \left(\frac{1}{g}\left[\partial_\mu, \partial_\nu\right]\varphi\right)\hat{k}$$

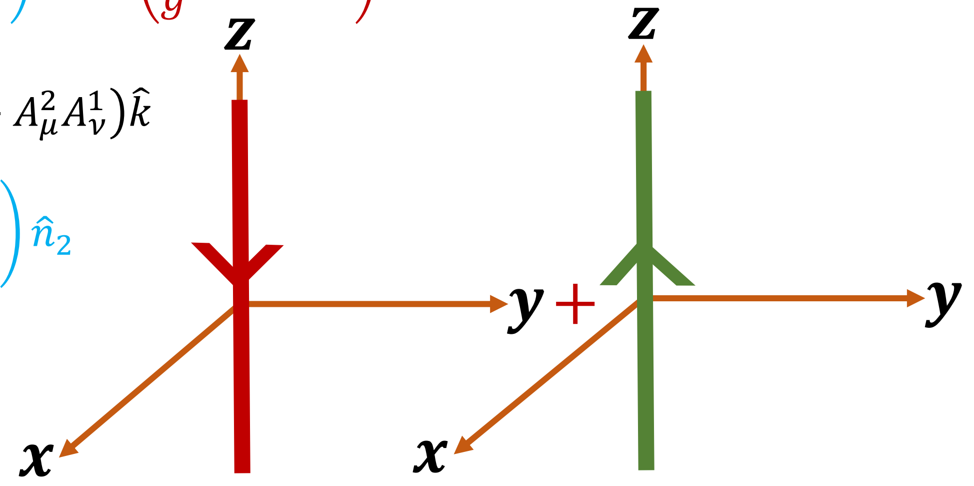
vortex on z − axis with $\Phi^{flux} = -\frac{2\pi}{g} T^3$

$$\vec{F}^{bilinear}_{\mu\nu} \equiv ig\left[\vec{A}'^{N}_\mu, \vec{A}'^{N}_\nu\right] = -g\left(A^2_\mu A^3_\nu - A^3_\mu A^2_\nu\right)\hat{n}_1 - g\left(A^3_\mu A^1_\nu - A^1_\mu A^3_\nu\right)\hat{n}_2 - g\left(A^1_\mu A^2_\nu - A^2_\mu A^1_\nu\right)\hat{k}$$

$$+g\left(A^1_\nu \frac{1}{g}\partial_\mu\varphi - A^1_\mu \frac{1}{g}\partial_\nu\varphi\right)\hat{n}_1 - g\left(A^2_\nu \frac{1}{g}\partial_\mu\varphi - A^2_\mu \frac{1}{g}\partial_\nu\varphi\right)\hat{n}_2$$

$$\vec{F}^{singular}_{\mu\nu} \equiv \frac{i}{g} N(x)\left[\hat{\partial}_\mu, \hat{\partial}_\nu\right] N^\dagger(x) = +\left(\frac{1}{g}\left[\partial_\mu, \partial_\nu\right]\varphi\right)\hat{k}$$

anti − vortex on z − axis with $\Phi^{flux} = +\frac{2\pi}{g} T^3$



$$\vec{F}^{N}_{\mu\nu} = \underbrace{\vec{F}^{linear}_{\mu\nu} + \vec{F}^{bilinear}_{\mu\nu}}_{\text{Center projection}} + \vec{F}^{singular}_{\mu\nu}$$

The direct method of identifying vortices in SU(2)

Step 2: Center projection

$$\vec{F}^{CP}_{\mu\nu} \equiv \vec{F}^{linear}_{\mu\nu} + \vec{F}^{bilinear}_{\mu\nu} = \left(\partial_\mu A^1_\nu - \partial_\nu A^1_\mu\right)\hat{n}_1 - g\left(A^2_\mu A^3_\nu - A^3_\mu A^2_\nu\right)\hat{n}_1$$

$$+\left(\partial_\mu A^2_\nu - \partial_\nu A^2_\mu\right)\hat{n}_2 - g\left(A^3_\mu A^1_\nu - A^1_\mu A^3_\nu\right)\hat{n}_2$$

$$+\left(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu\right)\hat{k} - g\left(A^1_\mu A^2_\nu - A^2_\mu A^1_\nu\right)\hat{k}$$

$$+\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)\hat{k}$$

$$\mathrm{SU(2)} \xrightarrow{\mathrm{CP}} \mathrm{SU(2)}/Z_2 \cong \mathrm{SO(3)}$$

$$\downarrow$$

$$\Pi_1(\mathrm{SO(3)}) = Z_2$$

$$\mathcal{L}_{CP} = -\frac{1}{4}\vec{F}^{CP}_{\mu\nu}.\vec{F}^{CP}_{\mu\nu}$$

$$= \mathcal{L}_{QCD} - \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2$$

$$-\frac{g}{2}\left(A^1_\mu A^2_\nu - A^2_\mu A^1_\nu\right)\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right) - \frac{1}{2}\left(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu\right)\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)$$

The indirect method of identifying vortices in SU(2)

Step 1: Abelian gauge fixing

$$G(x) = \begin{pmatrix} e^{\frac{i}{2}[\gamma(x)+\alpha(x)]}\cos\frac{\beta(x)}{2} & e^{\frac{i}{2}[\gamma(x)-\alpha(x)]}\sin\frac{\beta(x)}{2} \\ -e^{-\frac{i}{2}[\gamma(x)-\alpha(x)]}\sin\frac{\beta(x)}{2} & e^{-\frac{i}{2}[\gamma(x)+\alpha(x)]}\cos\frac{\beta(x)}{2} \end{pmatrix} \in SU(2)$$

$$\alpha(x) \in [0,2\pi) \quad \beta(x) \in [0,\pi] \quad \gamma(x) \in [0,2\pi)$$

$$if \quad G(x) \equiv M(x) \in SU(2) \; is \; an \; Abelian \; gauge \; transformation$$

$$\Phi^M = M(x)\Phi M^\dagger(x) = \frac{r}{2}M(x)\begin{pmatrix} cos\theta & sin\theta e^{-i\varphi} \\ sin\theta e^{i\varphi} & -cos\theta \end{pmatrix}M^\dagger(x) \xrightarrow[\gamma(x)=-\varphi]{\alpha(x)=\varphi\,,\beta(x)=\theta} \Phi^M = \frac{r}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad thus \quad M = \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2} \\ -e^{i\varphi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$\vec{A}^M_\mu(x) = M(x)\vec{A}_\mu(x)M^\dagger(x) - \frac{i}{g}M(x)\partial_\mu M^\dagger(x)$$

Regular Term ($M(x)\vec{A}_\mu(x)M^\dagger(x)$) **Singular Term** ($-\frac{i}{g}M(x)\partial_\mu M^\dagger(x)$)

$$\vec{A}^{singular}_\mu = -\frac{i}{g}M(x)\partial_\mu M^\dagger(x) = \frac{1}{2g}\begin{pmatrix} [1-cos\theta]\partial_\mu\varphi & [i\partial_\mu\theta + sin\theta\partial_\mu\varphi]e^{-i\varphi} \\ [-i\partial_\mu\theta + sin\theta\partial_\mu\varphi]e^{i\varphi} & -[1-cos\theta]\partial_\mu\varphi \end{pmatrix}$$

$$[1-cos\theta]\partial_\mu\varphi = \frac{1-cos\theta}{rsin\theta} \qquad sin\theta\partial_\mu\varphi = \frac{sin\theta}{rsin\theta} \qquad \partial_\mu\theta = \frac{1}{r}$$

$$r = 0 \to Monopole \qquad \theta = \pi \to Dirac - string$$

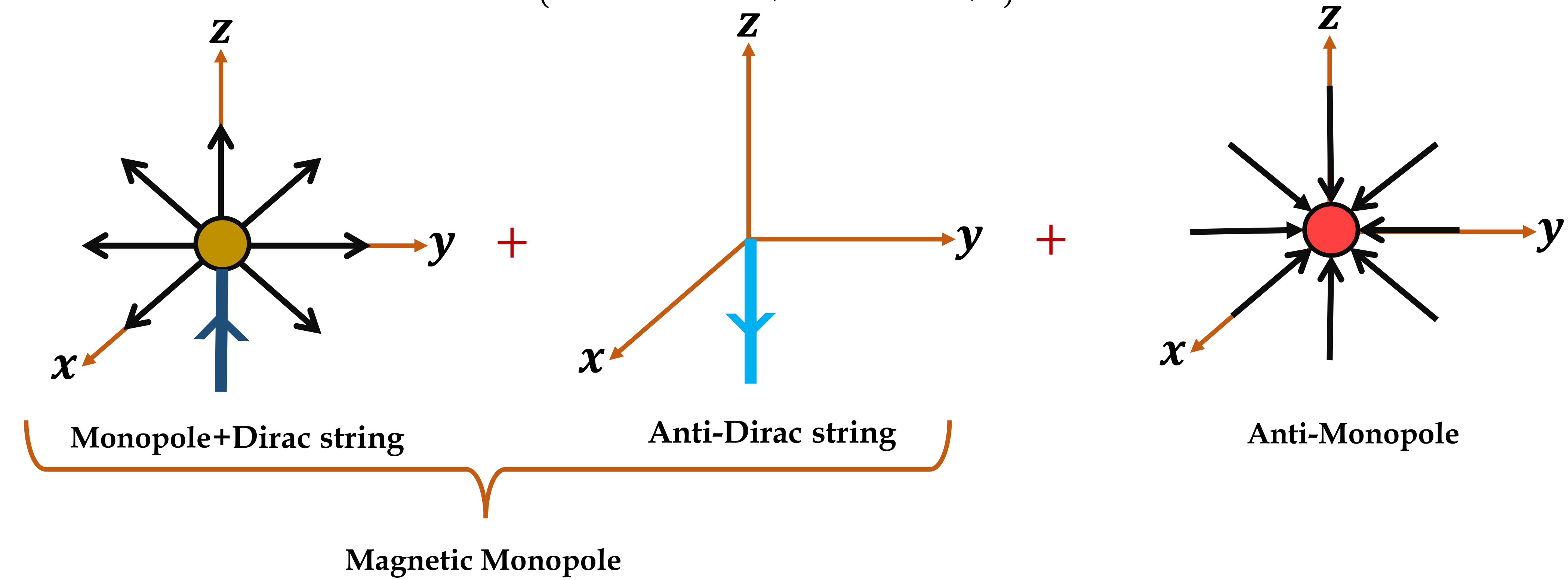
$$\Phi^{flux} = \int_c d\vec{X}.\vec{A}^{singular}_\mu = \frac{2\pi}{2g}\begin{pmatrix} 1-cos\theta & 0 \\ 0 & -(1-cos\theta) \end{pmatrix} = \frac{2\pi}{g}(1-cos\theta)T^3 \xrightarrow{\theta=\pi} \Phi^{flux} = \frac{4\pi}{g}T^3$$

The indirect method of identifying vortices in SU(2)

Step 2: Abelian projection

$$\vec{F}^M_{\mu\nu}.\vec{T} = \left(\partial_\mu \vec{A}^M_\nu - \partial_\nu \vec{A}^M_\mu\right) + ig\left[\vec{A}^M_\mu, \vec{A}^M_\nu\right] + \frac{i}{g} M\left[\hat{\partial}_\mu, \hat{\partial}_\nu\right] M^\dagger \in SU(2)$$

$$\left\{\left(F^{linear}_{\mu\nu}\right)^3 \equiv \partial_\mu \left(\vec{A}^M_\nu\right)^3 - \partial_\nu \left(\vec{A}^M_\mu\right)^3\right\} + \left\{\left(F^{singular}_{\mu\nu}\right)^3 \equiv \left(\frac{i}{g} M\left[\hat{\partial}_\mu, \hat{\partial}_\nu\right] M^\dagger\right)^3\right\} + \left\{\left(F^{bilinear}_{\mu\nu}\right)^3 \equiv ig\left[\left(\vec{A}^M_\mu\right)^1, \left(\vec{A}^M_\nu\right)^2\right]\right\}$$



$$\vec{A}^M_\mu \rightarrow \mathcal{A}_\mu = \left(\vec{A}^M_\mu\right)^3 T^3$$

The indirect method of identifying vortices in SU(2)

Step 3: center gauge fixing

$$\vec{A}'^{NM}_\mu.\vec{T} = N(x)\left[M(x)\vec{A}_\mu M^\dagger(x) - \frac{i}{g}M(x)\partial_\mu M^\dagger(x)\right]N^\dagger(x) - Thin\ vortex$$

$$\vec{A}^M_\mu \xrightarrow{\textbf{AP}} \mathcal{A}_\mu = \left(\vec{A}^M_\mu\right)^3 T^3$$

$$\vec{A}'^{NM}_\mu.\vec{T} = N(x)\mathcal{A}_\mu N^\dagger(x) - Thin\ vortex \qquad ,N = \begin{pmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-i\frac{\varphi}{2}} \end{pmatrix} \qquad Thin\ vortex = \frac{1}{g}\partial_\mu\varphi\, T^3$$

$$\vec{A}'^{NM}_\mu.\vec{T} = \left[A^1_\mu sin\theta cos\varphi + A^2_\mu sin\theta sin\varphi + A^3_\mu cos\theta - \frac{1}{g}cos\theta\partial_\mu\varphi\right]T^3$$

$$-\frac{1}{g}cos\theta\frac{1}{rsin\theta}\hat{\varphi} = -\frac{1}{g}cos\theta\partial_\mu\varphi = \frac{1}{g}(1-cos\theta)\partial_\mu\varphi - \frac{1}{g}\partial_\mu\varphi$$

$r = 0 \rightarrow Monopole$

$\theta = 0,\pi \rightarrow line\ vortex$

$$\Phi^{flux} = \int_c d\vec{X}.\vec{A}^{singular}_\mu = \frac{2\pi}{2g}\begin{pmatrix} 1-cos\theta & 0 \\ 0 & -(1-cos\theta) \end{pmatrix} - \frac{2\pi}{2g}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \left\{\frac{2\pi}{g}(1-cos\theta) - \frac{2\pi}{g}\right\}T^3$$

Monopole + Dirac string — vortex

$\theta = 0 \qquad \Phi^{flux} = -\frac{2\pi}{g}T^3$

$\theta = \pi \qquad \Phi^{flux} = \frac{2\pi}{g}T^3$

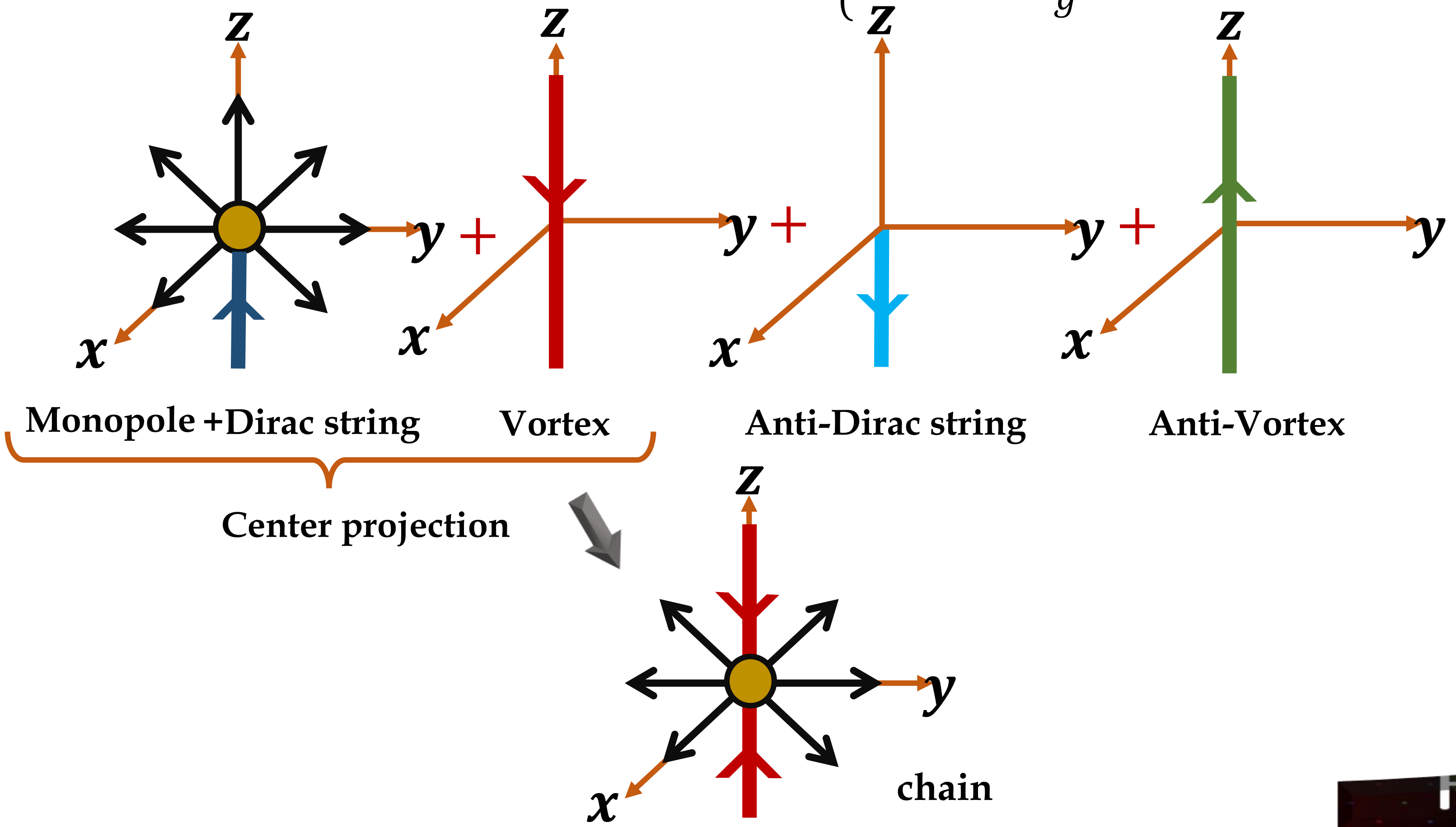
The indirect method of identifying vortices in SU(2)

Step 4: center projection

$$\vec{F}^{NM}_{\mu\nu} = \left(\partial_\mu \vec{A}^{NM}_\nu - \partial_\nu \vec{A}^{NM}_\mu\right) + \underbrace{ig\left[\vec{A}^{NM}_\mu, \vec{A}^{NM}_\nu\right]}_{\textbf{Zero}} + \frac{i}{g} N(x)M(x)\left[\hat{\partial}_\mu, \hat{\partial}_\nu\right]M^\dagger(x)N^\dagger(x)$$

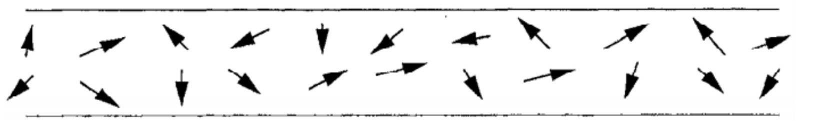
$$\vec{F}^{bilinear}_{\mu\nu}.\vec{T} = ig\left\{\left(\vec{A}^{NM}_\mu\right)^2\left(\vec{A}^{NM}_\nu\right)^3 - \left(\vec{A}^{NM}_\mu\right)^3\left(\vec{A}^{NM}_\nu\right)^2\right\}T^1 + ig\left\{\left(\vec{A}^{NM}_\mu\right)^3\left(\vec{A}^{NM}_\nu\right)^1 - \left(\vec{A}^{NM}_\mu\right)^1\left(\vec{A}^{NM}_\nu\right)^3\right\}T^2 + ig\left\{\left(\vec{A}^{NM}_\mu\right)^1\left(\vec{A}^{NM}_\nu\right)^2 - \left(\vec{A}^{NM}_\mu\right)^2\left(\vec{A}^{NM}_\nu\right)^1\right\}T^3$$

$$\vec{F}^{NM}_{\mu\nu} = \left\{ \vec{F}^{linear}_{\mu\nu} \equiv \left(\partial_\mu \vec{A}^{NM}_\nu - \partial_\nu \vec{A}^{NM}_\mu\right)\right\} + \left\{\vec{F}^{singular}_{\mu\nu} \equiv \frac{i}{g} N(x)M(x)\left[\hat{\partial}_\mu, \hat{\partial}_\nu\right]M^\dagger(x)N^\dagger(x)\right\}$$

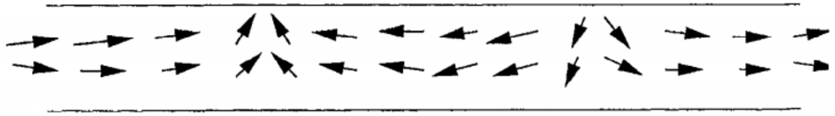


$$\mathcal{L}_{CP} = -\frac{1}{4}\vec{F}^{CP}_{\mu\nu}.\vec{F}^{CP}_{\mu\nu}$$
$$= \mathcal{L}_{QCD} - \frac{1}{4}\left(\partial_\mu E_\nu - \partial_\nu E_\mu\right)^2$$
$$- \frac{1}{2}\left(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu\right)\left(\partial_\mu E_\nu - \partial_\nu E_\mu\right) + \cdots$$

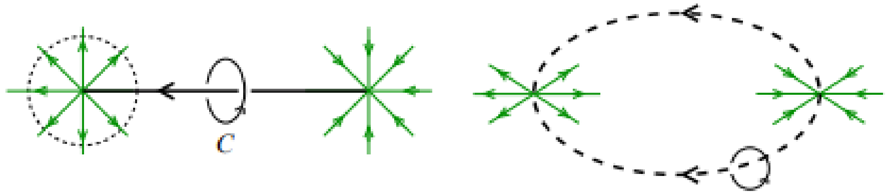
L. Del Debbio, M. Faber, J. Greensit e and S. Olejnik, Phys. Rev. D **55**, 2298 (1997)



Fixing to maximal abelian gauge, the field strength tends to line up mainly (but not entirely) in the diagonal ($\pm\sigma^3$) color direction



Upon abelian projection, the regions interpolating between $+\sigma^3$ and $-\sigma^3$ emerge as "monopoles." Their location is gauge (and Gribov copy) dependent.

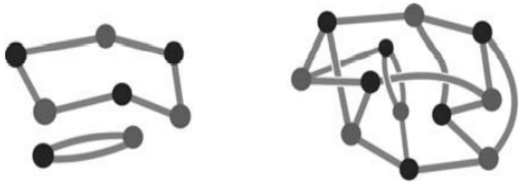


(a) (b)

Figure 3: Illustration of the connection between Dirac string (a) and center vortex (b) (see text)

which is the background gauge fixing with the thin vortex $a(\partial\Sigma)$ figuring as background field.

M. Engelhardt, H. Reinhardt, Nucl. Phys. B **567**, 249 (2017)



SU(2) chains *SU*(3) nets

Fig. 2. The monopole–vortex chains and the monopole-vortex nets in $SU(2)$ and $SU(3)$ gauge theories, respectively.

naïve Coulomb-like distribution of the magnetic fields around the monopole [22]. Each monopole is a source of two vortex fluxes which must be connected to other

M. N. Chernodub, V. I. Zakharov, Phys. Atom. Nucl 72 (2009)

Conclusions

- Inspired by DMCG and IMCG methods that identified vortices in lattice calculations and using connection formalism, we show that in both methods under a singular center gauge fixing, vortices appear in QCD vacuum in the continuum theory.

- In the direct method, we show that under the singular center gauge fixing, vortex and anti-vortex appear in the gauge theory. Then by removing the term that represents the anti-vortex, namely defining the center projection, we show that the SU(2) gauge theory is reduced to a theory involving the vortex.

- In the indirect method, in addition to the center gauge fixing and center projection, an intermediate step called Abelian gauge fixing and then Abelian projection is used. In fact, in the indirect method, we will not have single vortices but a chain that includes monopoles and vortices.