LONGITUDINAL DYNAMICS IN LIGHT-FRONT HOLOGRAPHIC QCD AND HADRON SPECTROSCOPY

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Overview

Light-Front Schrödinger Wave Equation

Light-Front Holography with Invariant Mass Ansatz

The 't Hooft Equation and Longitudinal Dynamics

Hadron Mass Spectra and Pion structure

Conclusions

Light-Front Schrödinger Wave Equation

In light-front QCD, the mass of a quark-antiquark meson

\[ M^2 = \int dx d^2 b \Psi^*(x, b) \left[ -\nabla_b^2 \frac{x}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right] \Psi(x, b) + \text{interactions} \]

- Light-front variable, \( \zeta = \sqrt{x(1-x)} b_\perp \)
- Assuming factorized form of wavefunction as

\[ \Psi(x, \zeta, \varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi \zeta}} e^{iL\varphi} X(x) \]

- \( X(x) = \sqrt{x(1-x)} \chi(x) \) and \( L = |L_{z_{\text{max}}}^z| \) is the light-front OAM

Light-front wave equation

\[ \left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} - \frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\text{eff}} \right) \chi(x) \phi(\zeta) = M^2 \chi(x) \phi(\zeta) \]

- For massless quarks: \( U_{\text{eff}} \rightarrow U(\zeta) \)

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In the chiral limit \( (m_q \to 0) \)

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)
\]

- No longitudinal dynamical equation
- Wave function: \( \Psi(x, \zeta) = \sqrt{x(1-x)} \phi(\zeta) \)

For massive quark \( m_q \neq 0 \)

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_\perp(\zeta) \right) \phi(\zeta) = M^2_\perp \phi(\zeta)
\]

\[
\left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel \right) \chi(x) = M^2_\parallel \chi(x)
\]

- Assumption: \( U_{\text{eff}} = U_\perp(\zeta) + U_\parallel(x) \)
- Masses: \( M^2 = M^2_\perp + M^2_\parallel \)
- Wave functions: \( \Psi(x, \zeta) = \sqrt{x(1-x)}\chi(x) \phi(\zeta) \)
- Normalization: \( \int dx |\chi(x)|^2 = \int d^2\zeta |\phi(\zeta)|^2 = 1 \)

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Light-Front Holography QCD

- String modes in anti-de-Sitter (AdS) space are mapped into the hadron states of the light-front Hamiltonian equations
- Mapping between $\zeta$ and fifth dimension $z$ in anti de-Sitter space
- Unique confining potential: $U^\text{LFH}_\perp(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)$
- Meson mass spectra:

$$M^2_\perp(n_\perp, J, L) = 4\kappa^2 \left( n_\perp + \frac{J + L}{2} \right) ; \quad J = L + S$$

- Massless pion (chiral limit): $M^2_\pi = M^2_\perp(0, 0, 0) = 0$
- Transverse part of the wavefunction:

$$\phi_{n_\perp L}(\zeta) \sim \zeta^{1/2 + L} \exp\left(\frac{-\kappa^2 \zeta^2}{2}\right) L^L_{n_\perp}(\kappa^2 \zeta^2)$$

- Longitudinal mode: $X(x) = \sqrt{x(1 - x)}$, fixed by mapping the spacelike EM / GFF form factor calculated in AdS and LF formalism

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**Invariant Mass Ansatz for Light-Front Holography**

\[
\frac{\vec{k}_\perp^2}{x(1-x)} \rightarrow \frac{\vec{k}_\perp^2}{x(1-x)} + \frac{m^2_q}{x} + \frac{m^2_{\bar{q}}}{1-x}
\]

- **Brodsky-de Téramond prescription** Mass correction due to \( m_q \neq 0 \)

\[
\Delta M^2_{\text{BdT}} = \int \frac{dx}{x(1-x)} X^2_{\text{BdT}}(x) \left( \frac{m^2_q}{x} + \frac{m^2_{\bar{q}}}{1-x} \right)
\]

with

\[
X_{\text{BdT}}(x) = \sqrt{x(1-x)} \exp \left[ -\frac{1}{2\kappa^2} \left( \frac{m^2_q}{x} + \frac{m^2_{\bar{q}}}{1-x} \right) \right]
\]

- Masses: \( M^2_\perp (n_\perp, J, L) + \Delta M^2_{\text{BdT}} \); (Pion mass: \( M_\pi = \Delta M^2_{\text{BdT}} \))

- Phenomenologically successful: describes mass spectroscopy, form factors and parton distributions, etc., \(^{1234}\)

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SHORTCOMING WITH INVARIANT MASS ANSATZ

• Not consistent with the Gell-Mann-Oakes-Renner (GMOR) relation $M^2_\pi \sim 2m_u/d$; from ansatz $\rightarrow M^2_\pi \sim 2m^2_u/d \ln(\kappa^2/m^2_u/d - \gamma E)$

• No of longitudinal excitations $\rightarrow$ the longitudinal wave functions, hence the distribution amplitudes, are the same for all mesons.

• Endpoint : strongly suppressed $\chi(x) \sim \exp(-a/x)$; expected $\sim x^a$

Shortcoming has been overcome by solving\(^1\)

$$\left( \frac{m^2_q}{x} + \frac{m^2_{\bar{q}}}{1-x} + U_{\parallel}\right) \chi(x) = M^2_{\parallel} \chi(x)$$

• ’t Hooft Equation \(^2\) : QCD\(_{1+1}\) in the $N_c \gg 1$ approximation.

• Li-Maris-Zhao-Vary phenomenological confinement potential \(^3\) : 
  $U_{\parallel}^{BLFQ} = -\sigma^2 \partial_x (x(1-x)\partial_x)$

• The chiral limit and the phenomenon of chiral symmetry breaking \(^4\)

---


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$$
\left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel \right) \chi(x) = M_\parallel^2 \chi(x)
$$

- 't Hooft Equation $^2$: QCD$_{1+1}$ in the $Nc \gg 1$ approximation.
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---

The 't Hooft Equation

Derived from the (1 + 1)-dim QCD Lagrangian in the large $N_c$ limit $^1$:

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + \frac{g^2}{\pi} \mathcal{P} \int dy \frac{|\chi(x) - \chi(y)|}{(x-y)^2} = M^2_\parallel \chi(x)$$

- $g = g_s \sqrt{N_c}$ is the (finite) 't Hooft coupling with mass dimensions $\to$ plays the role of $\Lambda_{\text{QCD}}$.
- Assuming $\chi(x) \approx x^{\beta_1} (1-x)^{\beta_2} \to$ end-point behavior characterized by:

$$\pi m_i^2 g^2 - 1 + \pi \beta_i \cot(\pi \beta_i) = 0$$

- In the chiral limit, $m_i \to 0$ $^2$,

$$\beta_i = \sqrt{\frac{2m_i^2}{\pi g^2}}$$

$$M_\pi^2 = g \sqrt{\frac{\pi}{3}} (m_u + m_d) + \mathcal{O}(m_u + m_d)^2$$

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$^2$ G. F. De Téramond and S. J. Brodsky, 2103.10950 (2021)
**Key Observations within 't Hooft Model**

- In chiral limit: $M_\pi^2 \propto m_{u/d}$ (GMOR relation)

- $\beta = 0 \rightarrow \chi(x) = 1$ and $X(x) = \sqrt{x(1-x)}$, i.e. the longitudinal mode of light-front holography

- In heavy quark limit $^{1,2}$: $f_M \propto m_Q^{-1/2}$ (HQET)

- The 't Hooft potential is consistent with the holographic potential in the non-relativistic limit and thus reflecting the restoration of manifest 3-dimensional rotational symmetry $^3$

- Following $^4$:

  $$U_{LF} = V_{IF}^2 + 4mV_{IF}$$

  One finds: $V_\perp = \frac{1}{4}\kappa^2 b_\perp$ and $V_\parallel = \frac{1}{4}g^2 b_\parallel$

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### Meson Mass Spectrum

\[ M^2(n_\parallel, n_\perp, J, L) = M_{\perp}^2(n_\perp, J, L) + M_{\parallel}^2(n_\parallel; m_q, m_{\bar{q}}, g) \]

- Parity and charge conjugation quantum numbers to the meson to be 
  \[ P = (-1)^{L+1} \] and \[ C = (-1)^{L+S+n_\parallel}. \]

- Important points to mention:
  - We use universal holographic mass scale \( \kappa = 0.523 \text{ GeV} \)
  - The universality of \( \kappa \) seems to be lost with BdT prescription for heavy mesons/baryons/tetraquarks: \( \kappa \propto \sqrt{m_Q}. \)
  - We expected \( \kappa \approx g \) for heavy-heavy mesons because of the restoration of the 3-dimensional rotational symmetry.
  - We find that \( n_\parallel \geq n_\perp + L \) (underlying link between the holographic Schrödinger Equation and the ‘t Hooft Equation).

- Parameters to compute masses:

<table>
<thead>
<tr>
<th>Mesons</th>
<th>( g )</th>
<th>( m_{u/d} )</th>
<th>( m_s )</th>
<th>( m_c )</th>
<th>( m_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light</td>
<td>0.128</td>
<td>0.046</td>
<td>0.357</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Heavy-light</td>
<td>0.680</td>
<td>0.046</td>
<td>0.357</td>
<td>1.370</td>
<td>4.640</td>
</tr>
<tr>
<td>Heavy-heavy</td>
<td>0.523</td>
<td>-</td>
<td>-</td>
<td>1.370</td>
<td>4.640</td>
</tr>
</tbody>
</table>

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**Light mesons**

\[
\begin{align*}
\pi(140) & \quad b_1(1235) & \quad \pi_2(1670) \\
\pi(1300) & \quad \pi_2(1880) & \quad \rho(770) & \quad a_2(1320) & \quad \rho(1450) & \quad a_2(1690) \\
\pi(1800) & \quad \rho(1700) & \quad a_4(1970) & \quad a_2(1320) & \quad \rho(1450) & \quad a_2(1690) \\
\end{align*}
\]

\[
\begin{align*}
\rho(1450) & \quad \rho(1700) & \quad a_4(1970) & \quad a_2(1690) & \quad a_2(1700) \\
\rho(1700) & \quad a_2(1690) & \quad a_4(1970) & \quad a_2(1700) & \quad a_2(1700) \\
\rho(1700) & \quad a_2(1700) & \quad a_4(1970) & \quad a_2(1700) & \quad a_2(1700) \\
\end{align*}
\]

\[
\begin{align*}
\pi(1300) & \quad \pi_2(1880) & \quad \rho(1700) & \quad a_2(1690) & \quad a_2(1700) \\
\pi(1800) & \quad \rho(1700) & \quad a_4(1970) & \quad a_2(1700) & \quad a_2(1700) \\
\pi(1800) & \quad \rho(1700) & \quad a_4(1970) & \quad a_2(1700) & \quad a_2(1700) \\
\end{align*}
\]

\[
\begin{align*}
\omega(782) & \quad f_2(1270) & \quad \omega(1420) & \quad f_2(1950) & \quad f_2(2300) \\
\omega(1420) & \quad f_2(1950) & \quad f_2(2300) & \quad \omega_3(1670) & \quad \omega_3(1670) \\
\omega(1650) & \quad f_2(1950) & \quad f_2(2300) & \quad \omega_3(1670) & \quad \omega_3(1670) \\
\end{align*}
\]

\[
\begin{align*}
\omega(1650) & \quad f_2(1950) & \quad f_2(2300) & \quad \omega_3(1670) & \quad \omega_3(1670) \\
\omega(1420) & \quad f_2(1950) & \quad f_2(2300) & \quad \omega_3(1670) & \quad \omega_3(1670) \\
\omega(1650) & \quad f_2(1950) & \quad f_2(2300) & \quad \omega_3(1670) & \quad \omega_3(1670) \\
\end{align*}
\]
Heavy mesons

$\eta_c(1S)$

$\psi(2S)$

$\eta_b(1S)$

$\chi_{b2}(1P)$

$M^2 [\text{GeV}^2]$
Heavy-light mesons

$M^2 [\text{GeV}^2]$ vs $L$ for different values of $n_T$.

1 Particle Data Group (2020)
Supersymmetric Formulation of LFH

- Provides a unified description of baryons and mesons/tetraquarks
- Each baryon possesses two superpartners: a meson and a tetraquark
- Supersymmetric holographic Schrödinger Equation: $H \phi = M^2_{\perp} \phi$ \(^{12}\)

\[
H = \begin{pmatrix}
  -\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + U_M(\zeta) & 0 \\
  0 & -\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + U_B(\zeta)
\end{pmatrix}
\]

with

\[
U_M(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L_M + S_M - 1)
\]
\[
U_B(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L_B + S_D)
\]

- Mesons with $L_M$ and $S_M$ are the superpartners of baryons with $L_B = L_M - 1$ and the diquark with $S_D = S_M$.
- Baryons with $L_B$ and diquark with $S_D$ are the superpartners with $L_T = L_B$ and $S_T = S_D$.

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HARDONIC SUPERPARTNERS

\[
M^2_M(n_\perp, n_\parallel, L_M, S_M) = 4\kappa^2 \left(n_\perp + L_M + \frac{S_M}{2}\right) + M^2_\parallel(n_\parallel; m_q, m_{\bar{q}}, g)
\]

\[
M^2_B(n_\perp, n_\parallel, L_B, S_D) = 4\kappa^2 \left(n_\perp + L_B + \frac{S_D}{2} + 1\right) + M^2_\parallel(n_\parallel; m_q, m_{[qq]}, g)
\]

\[
M^2_T(n_\perp, n_\parallel, L_T, S_T) = 4\kappa^2 \left(n_\perp + L_T + \frac{S_T}{2} + 1\right) + M^2_\parallel(n_\parallel; m_{[\bar{q}q]}, m_{[qq]}, g)
\]

- Parity: \( P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T} \)

- Charge conjugation: \( C = (-1)^{n_\parallel + L_M + S_M} = (-1)^{n_\parallel + L_T + S_T - 1} \)

- We find that \( n_\parallel \geq n_\perp + L \) (underlying link between the holographic Schrödinger Equation and the ‘t Hooft Equation)

\[2\text{ M. Neilson, S.J. Brodsky, Phys. Rev. D 97, 114001 (2018)}\]
Light baryons

\[ M^2 \text{ [GeV}^2] = n_{\perp} \]

- \( n_{\perp} = 0 \)
- \( n_{\perp} = 1 \)
- \( n_{\perp} = 2 \)
- \( n_{\perp} = 3 \)

\[ \begin{array}{c}
\Sigma(1190) \\
\Sigma(1915) \\
\Sigma(1660) \\
\Sigma(1880) \\
\Xi(1320) \\
\Omega(1672) \\
\Lambda(1115) \\
\Lambda(1690) \\
\Lambda(2350) \\
\end{array} \]

\[ \begin{array}{c}
N(940) \\
N(1440) \\
N(1520) \\
N(1680) \\
N(2100) \\
N(1520) \\
N(1680) \\
N(1440) \\
N(2220) \\
\end{array} \]

\[ \begin{array}{c}
\Delta(1232) \\
\Delta(1270) \\
\Delta(1600) \\
\Delta(1930) \\
\Delta(2420) \\
\Delta(1700) \\
\Delta(1950) \\
\Delta(1600) \\
\Delta(1950) \\
\end{array} \]

\[ \begin{array}{c}
\Lambda(1115) \\
\Lambda(1600) \\
\Lambda(1520) \\
\Lambda(1690) \\
\Lambda(2100) \\
\Lambda(1690) \\
\Lambda(2350) \\
\Lambda(1690) \\
\Lambda(2100) \\
\end{array} \]
HEAVY-LIGHT BARYONS

\[
\begin{align*}
M^2 \text{ [GeV}^2] & \quad n_{\perp}=0 \\
\Lambda_c(2290) & \quad \Lambda_c(2940) \\
\Sigma_c(2455) & \quad \Xi_c(2455) \\
\Lambda_b(5620) & \quad \Xi_b(5797) \\
\Omega_c(2625) & \quad \Omega_c(2880) \\
\Omega_c(2625) & \quad \Omega_c(2880) \\
\Omega_b(5920) & \quad \Omega_b(6046) \\
\end{align*}
\]

\[1\] Particle Data Group (2020)
Application of LFWFs : Pion PDFs

LF wavefunction ⇒ Initial PDFs ⇒ Scale evolution (0.240 ± 0.024 → 16 GeV$^2$)

- Pion valence PDF falls off as $(1 - x)^{1.9}$, favoring a slightly slower falloff compared to the $(1 - x)^2$ predicted by perturbative QCD and DSEs.

<table>
<thead>
<tr>
<th>Valence moments</th>
<th>$\langle x \rangle$</th>
<th>$\langle x \rangle$</th>
<th>$\langle x^2 \rangle$</th>
<th>$\langle x^2 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>0.271</td>
<td>0.245</td>
<td>0.121</td>
<td>0.103</td>
</tr>
<tr>
<td>JAM (2018)</td>
<td>0.268</td>
<td>0.245</td>
<td>0.127</td>
<td>0.108</td>
</tr>
<tr>
<td>BLFQ-NJL (2019)</td>
<td>0.271</td>
<td>0.245</td>
<td>0.124</td>
<td>0.106</td>
</tr>
<tr>
<td>BSE (2018)</td>
<td>0.268</td>
<td>0.240</td>
<td>0.125</td>
<td>—</td>
</tr>
<tr>
<td>ETM (2018) [Lattice]</td>
<td>—</td>
<td>0.207(11)</td>
<td>—</td>
<td>0.163(33)</td>
</tr>
</tbody>
</table>

- Gluons: $\{36.8, 39.8, 42.7\}$% of pion momentum at $\{1.69, 4, 10, 16\}$ GeV$^2$.

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Distribution Amplitude and Transition Form Factor

\[ \pi \rightarrow \gamma^* \gamma \text{ TFF}:^1 \]

\[ F_{\pi \gamma}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \! dx \, T_H(x, Q^2) \phi(x, \bar{x}Q) \]

- DA evolution: ERBL evol. (Gegenbauer basis) [Ruiz, et. al. PRD 66, (2002)]
- Our DA is close to Asymptotic DA
- Our prediction agrees well with data reported by Belle Collaboration.
- It deviates from the rapid growth of the large \( Q^2 \) data reported by BaBar Collaboration.

Decay constant \( f_\pi \):

LFH + 't Hooft Equation: 166.4 MeV
Experimental data: 130.2 ± 1.7 MeV

Decay constants for excited pions:

\[ f_{\pi'} = 3.5 \text{ MeV}; \quad f_{\pi''} = 2.6 \text{ MeV} \]

\(^1\) S. J. Brodsky, F. G. Cao and G. F. de Teramond, Phys. Rev. D 84, 033001 (2011)
Pion Electromagnetic Form Factor

Charge radius:

\[
\langle r_c^2 \rangle = -6 \lim_{Q^2 \to 0} \frac{d}{dQ^2} F_P(Q^2).
\]

Pion charge radius: 0.69 fm

Experimental data: 0.672 ± 0.008 fm

\(^1\) in preparation
Conclusions

• The ‘t Hooft Equation is complementary to the holographic Schrödinger Equation in predicting the full hadron spectrum.

• We find that the emerging hadronic scale $\kappa$ of LFH remains universal across the full spectrum. For heavy-heavy mesons, $\kappa$ coincides with the ’t Hooft coupling as expected for rotational symmetry in nonrelativistic limit.

• The baryons and mesons spectra are in good agreement with the data. For the tetraquark candidates, the agreement is less impressive.

• Wave functions lead to a good description of Pion FF, PDFs, PDA, and transition form factor ($\pi^0 \rightarrow \gamma^*\gamma$).

Thank You
Light super-partners

\[ \pi(140) \]

\[ h_1(1170) \]

\[ \eta_2(1645) \]

\[ \nabla \nabla \]

\[ N(940) \]

\[ N(1520) \]

\[ \Delta(1232) \]

\[ \Delta(1700) \]

\[ \Delta(1950) \]

\[ f_0(500) \]

\[ f_1(1260) \]

\[ f_2(1270) \]

\[ f_4(2050) \]

\[ a_1(1260) \]

\[ a_2(1320) \]

\[ a_4(1970) \]

\[ \rho(770) \]

\[ \rho_3(1690) \]

\[ \omega(782) \]

\[ \omega_3(1670) \]

\[ L \]

\[ L^2 \text{ [GeV}^2\text{]} \]
Light-Strange super-partners

\[ K(495) \]
\[ K_1(1270) \]
\[ K_2(1770) \]
\[ \Lambda(1115) \]
\[ \Lambda(1520) \]
\[ K_0^*(1430) \]
\[ \phi(1020) \]
\[ f_1(1420) \]
\[ f_0(980) \]
\[ a_0(1450) \]
\[ \eta(980) \]
\[ \eta(958) \]
\[ \phi(1850) \]
\[ \Omega(1520) \]
\[ \phi(1850) \]
\[ \eta(1415) \]
\[ \eta(1870) \]
\[ \Omega(1672) \]

\[ M^2 [\text{GeV}^2] \]

\[ L \]
Heavy super-partners

\[ M^2 \text{ [GeV}^2\text{]} \]

- \( \eta_c(2984) \)
- \( h_c(3525) \)
- \( \chi_{c0}(3415) \)
- \( \chi_{c1}(3510) \)
- \( \chi_{c2}(3556) \)
- \( \psi'(3686) \)
- \( \eta_b(9400) \)
- \( h_b(9900) \)
- \( \chi_{b0}(9860) \)
- \( \chi_{b1}(9893) \)
- \( \chi_{b2}(9910) \)
- \( \Upsilon(9460) \)
- \( \Upsilon(10020) \)

\( L \)
HEAVY-LIGHT SUPER-PARTNERS

\[
M^2 \text{ [GeV}^2\text{]} = \begin{cases} 
D(1870) & \Lambda_c(2290) \\
D_1(2420) & \Lambda_c(2625) \\
D_0^{*}(2300) & D_s(1968) \\
D_s(2010) & D_s(2460) \\
D_s^{*}(2536) & \Omega_c(2695) \\
D_s(2536) & D_s(2815) \\
D_s(2860) & \Omega(2770) \\
\end{cases}
\]