Longitudinal Dynamics in Light-Front Holographic QCD and Hadron Spectroscory

Chandan Mondal

Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China

Mexico City, July 29, 2021

Overview

Based on → M. Ahmady, H. Dahiya, S. Kaur, CM, R. Sandapen and N. Sharma, [arXiv:2105.01018 [hep-ph]] & recent progress.

Light-Front Schrödinger Wave Equation

In light-front QCD, the mass of a quark-antiquark meson

$$M^2 = \int \mathrm{d}x\mathrm{d}^2\mathbf{b}\Psi^*(x,\mathbf{b})\left[-\frac{\nabla_b^2}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right]\Psi(x,\mathbf{b}) + \text{interactions}$$

- Light-front variable, $\zeta = \sqrt{x(1-x)}\mathbf{b}_\perp$
- Assuming factorized form of wavefunction as

$$\Psi(x,\zeta,\varphi) = \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\varphi} X(x)$$

- $X(x) = \sqrt{x(1-x)}\chi(x)$ and $L = |L_z^{\max}|$ is the light-front OAM

Light-front wave equation

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} - \frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_{\text{eff}}\right)\chi(x)\,\phi(\zeta) = M^2\chi(x)\,\phi(\zeta)$$

- For massless quarks: $U_{\text{eff}} \to U(\zeta)$

[1] S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Phys. Rept. 584, 1 (2015)

In the chiral limit $(m_q \to 0)$

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- No longitudinal dynamical equation
- Wave function : $\Psi(x,\zeta) = \sqrt{x(1-x)}\,\phi(\zeta)$

For massive quark $m_q \neq 0$[1]

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2\phi(\zeta)$$

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel\right)\chi(x) = M_\parallel^2\chi(x)$$

- Assumption: $U_{\text{eff}} = U_\perp(\zeta) + U_\parallel(x)$
- Masses : $M^2 = M_\perp^2 + M_\parallel^2$
- Wave functions : $\Psi(x,\zeta) = \sqrt{x(1-x)}\chi(x)\,\phi(\zeta)$
- Normalization : $\int \mathrm{d}x|\chi(x)|^2 = \int \mathrm{d}^2\zeta|\phi(\zeta)|^2 = 1$

[1] S.S. Chabysheva and J.R. Hiller, Annals of Physics 337, 143 (2013); G.F. de Téramond and S.J. Brodsky, (2021), arXiv:2103.10950 [hep-ph]; Y. Li and J.P. Vary, (2021), arXiv:2103.09993 [hep-ph]

Light-Front Holography QCD

- String modes in anti-de-Sitter (AdS) space are mapped into the hadron states of the light-front Hamiltonian equations [1]
- Mapping between ζ and fifth dimension z in anti de-Sitter space
- Unique confining potential: $U_\perp^{\rm LFH}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- Meson mass spectra:

$$M_\perp^2(n_\perp, J, L) = 4\kappa^2\left(n_\perp + \frac{J+L}{2}\right) \quad ; \quad J = L + S$$

- Massless pion (chiral limit): $M_\pi^2 = M_\perp^2(0,0,0) = 0$
- Transverse part of the wavefunction:

$$\phi_{n_\perp L}(\zeta) \sim \zeta^{1/2+L}\exp\left(\frac{-\kappa^2\zeta^2}{2}\right) L_{n_\perp}^L(\kappa^2\zeta^2)$$

- Longitudinal mode: $X(x) = \sqrt{x(1-x)}$, fixed by mapping the spacelike EM / GFF form factor calculated in AdS and LF formalism

[1] S.J. Brodsky, G.F. de Téramond, H.G. Dosh, J. Erlich, Physics Reports 584, 1 (2015)

Invariant Mass Ansatz for Light-Front Holography

$$\frac{\vec{k}_\perp^2}{x(1-x)} \to \frac{\vec{k}_\perp^2}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}$$

- *Brodsky-de Téramond prescription* Mass correction due to $m_q \neq 0$ [1]

$$\Delta M_{\mathrm{BdT}}^2 = \int \frac{dx}{x(1-x)} X_{\mathrm{BdT}}^2(x) \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right)$$

with

$$X_{\mathrm{BdT}}(x) = \sqrt{x(1-x)} \exp\left[-\frac{1}{2\kappa^2} \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \right]$$

- Masses: $M_\perp^2(n_\perp, J, L) + \Delta M_{\mathrm{BdT}}^2$; (Pion mass: $M_\pi = \Delta M_{\mathrm{BdT}}$)

- Phenomenologically successful: describes mass spectroscopy, form factors and parton distributions, etc.,[1234]

[1] S.J. Brodsky, G.F. de Téramond, H.G. Dosh, J. Erlich, Physics Reports 584, 1 (2015)
[2] J. R. Forshaw, R. Sandapen, Phys. Rev. Lett. 109, 081601 (2012)
[3] M. Ahmady, R. Sandapen, N. Sharma, Phys. Rev. D94, 074018 (2016)
[4] M. Ahmady, C. Mondal, R. Sandapen, Phys. Rev. D98, 034010 (2018) & many more

Shortcoming with Invariant Mass Ansatz

- Not consistent with the Gell-Mann-Oakes-Renner (GMOR) relation $M_\pi^2 \sim 2m_{u/d}$; *from ansatz* $\rightarrow$ $M_\pi^2 \sim 2m_{u/d}^2 \ln(\kappa^2/m_{u/d}^2 - \gamma_E)$
- No of longitudinal excitations $\rightarrow$ the longitudinal wave functions, hence the distribution amplitudes, are the same for all mesons.
- Endpoint : strongly suppressed $\chi(x) \sim \exp(-a/x)$; expected $\sim x^a$

Shortcoming has been overcome by solving[1]

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel\right)\chi(x) = M_\parallel^2 \chi(x)$$

- *'t Hooft Equation* [2] : QCD_{1+1} in the $Nc \gg 1$ approximation.
- *Li-Maris-Zhao-Vary* phenomenological confinement potential [3] : $U_\parallel^{\mathrm{BLFQ}} = -\sigma^2 \partial_x (x(1-x)\partial_x)$
- The chiral limit and the phenomenon of chiral symmetry breaking [4]

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[1] S.S. Chabysheva and J.R. Hiller, Annals of Physics 337, 143 (2013); G.F. de Téramond and S.J. Brodsky, (2021), arXiv:2103.10950 [hep-ph]; Y. Li and J.P. Vary, (2021), arXiv:2103.09993 [hep-ph]

[2] G. 't Hooft, Nucl. Phys. B 75, 461 (1974)

[3] Y. Li, P. Maris, X. Zhao, and J. P. Vary, Phys. Lett. B 758, 118 (2016)

[4] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, A. Vega, Phys. Rev. D 87, 056001 (2013)

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The 't Hooft Equation

Derived from the $(1+1)$-dim QCD Lagrangian in the large N_c limit [1] :

$$\left(\frac{m_q^2}{x}+\frac{m_{\bar{q}}^2}{1-x}\right)\chi(x)+\frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y\,\frac{|\chi(x)-\chi(y)|}{(x-y)^2}=M_\parallel^2\chi(x)$$

- $g = g_s\sqrt{N_c}$ is the (finite) 't Hooft coupling with mass dimensions $\rightarrow$ plays the role of Λ_{QCD}.
- Assuming $\chi(x)\approx x^{\beta_1}(1-x)^{\beta_2}$ $\rightarrow$ end-point behavior characterized by :

$$\frac{\pi m_i^2}{g^2}-1+\pi\beta_i\cot(\pi\beta_i)=0$$

- In the chiral limit, $m_i \rightarrow 0$ [2],

$$\beta_i=\sqrt{2m_i^2/\pi g^2}$$

$$M_\pi^2=g\sqrt{\frac{\pi}{3}}(m_u+m_d)+\mathcal{O}(m_u+m_d)^2$$

[1] G. 't Hooft, Nucl. Phys. B 75, 461 (1974)

[2] G. F. De Téramond and S. J. Brodsky, 2103.10950 (2021)

Key observations within 't Hooft model

- In chiral limit : $M_\pi^2 \propto m_{u/d}$ (GMOR relation)
- $\beta = 0 \to \chi(x) = 1$ and $X(x) = \sqrt{x(1-x)}$, i.e. the longitudinal mode of light-front holography
- In heavy quark limit [1] [2] : $f_M \propto m_Q^{-1/2}$ (HQET)
- The 't Hooft potential is consistent with the holographic potential in the non-relativistic limit and thus reflecting the restoration of manifest 3-dimensional rotational symmetry [3]
- Following [4] :

$$U_{\rm LF} = V_{\rm IF}^2 + 4mV_{\rm IF}$$

One finds: $V_\perp = \frac{1}{4}\kappa^2 b_\perp$ and $V_\parallel = \frac{1}{4}g^2 b_\parallel$

[1] B. Grinstein, R. F. Lebed, Phys. Rev. D 57, 1366 (1998)
[2] B. Grinstein and P.F. Mende, Phys. Rev. Lett. 69, 1018 (1992)
[3] M. Ahmady, H. Dahiya, S. Kaur, C. Mondal, R. Sandapen and N. Sharma, [arXiv:2105.01018]
[4] Trawiński, lazek, Brodsky, de Téramond, and Dosch, Phys. Rev. D 90, 074017 (2014)

Meson Mass Spectrum

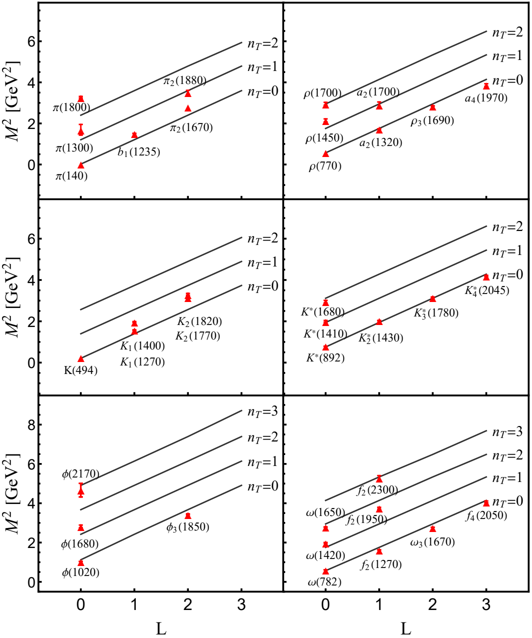
$$M^2(n_\parallel, n_\perp, J, L) = M_\perp^2(n_\perp, J, L) + M_\parallel^2(n_\parallel; m_q, m_{\bar{q}}, g)$$

- Parity and charge conjugation quantum numbers to the meson to be $P = (-1)^{L+1}$ and $C = (-1)^{L+S+n_\parallel}$.
- Important points to mention :
  - We use universal holographic mass scale $\kappa = 0.523$ GeV
  - The universality of κ seems to be lost with BdT prescription for heavy mesons/baryons/tetraquarks [1]: $\kappa \propto \sqrt{m_Q}$.
  - We expected $\kappa \approx g$ for heavy-heavy mesons because of the restoration of the 3-dimensional rotational symmetry [2]
  - We find that $n_\parallel \geq n_\perp + L$ (underlying link between the holographic Schrödinger Equation and the 't Hooft Equation)
- Parameters to compute masses:

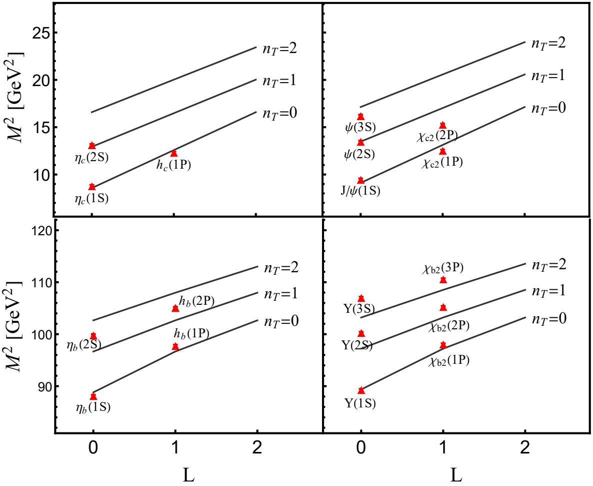
Mesons	g	$m_{u/d}$	m_s	m_c	m_b
Light	0.128	0.046	0.357	-	-
Heavy-light	0.680	0.046	0.357	1.370	4.640
Heavy-heavy	0.523	-	-	1.370	4.640

[1] H. G. Dosch, G. F. de Teramond and S. J. Brodsky, Phys. Rev. D **95**, no.3, 034016 (2017)
[2] M. Ahmady, H. Dahiya, S. Kaur, C. Mondal, R. Sandapen and N. Sharma, [arXiv:2105.01018]

Light mesons



Heavy mesons



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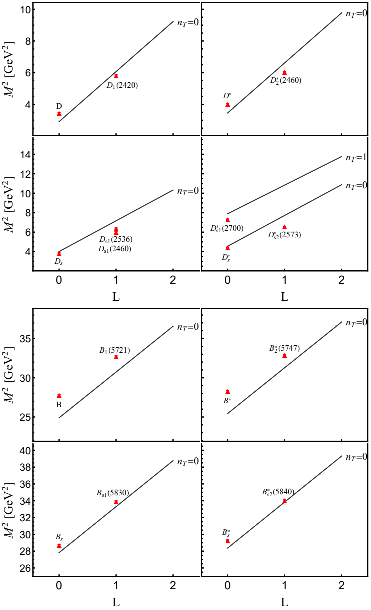
[1] Particle Data Group (2020)

Heavy-light mesons



[1] Particle Data Group (2020)

Supersymmetric Formulation of LFH

- Provides a unified description of baryons and mesons/tetraquarks
- Each baryon possesses two superpartners: a meson and a tetraquark
- Supersymmetric holographic Schrödinger Equation: $H|\phi\rangle = M_\perp^2 |\phi\rangle$ [1,2]

$$H = \begin{pmatrix} -\frac{d^2}{d\zeta^2} + \frac{4L_M^2-1}{4\zeta^2} + U_M(\zeta) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} + \frac{4L_B^2-1}{4\zeta^2} + U_B(\zeta) \end{pmatrix}$$

with

$$U_M(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L_M + S_M - 1)$$
$$U_B(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L_B + S_D)$$

- Mesons with L_M and S_M are the superpartners of baryons with $L_B = L_M - 1$ and the diquark with $S_D = S_M$.
- Baryons with L_B and diquark with S_D are the superpartners with $L_T = L_B$ and $S_T = S_D$.

[1] S.J. Brodsky, G.F. de Téramond, H.G. Dosch, C. Lorcé, Phys. Lett. B 759, 171 (2016)
[2] M. Neilson, S.J. Brodsky, Phys. Rev. D 97, 114001 (2018)

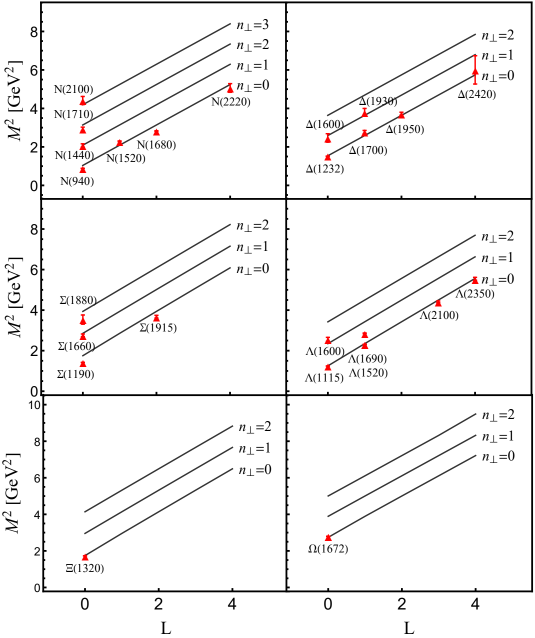
Hardonic Superpartners

$$M_M^2(n_\perp, n_\parallel, L_M, S_M) = 4\kappa^2\left(n_\perp + L_M + \frac{S_M}{2}\right) + M_\parallel^2(n_\parallel; m_q, m_{\bar{q}}, g)$$
$$M_B^2(n_\perp, n_\parallel, L_B, S_D) = 4\kappa^2\left(n_\perp + L_B + \frac{S_D}{2} + 1\right) + M_\parallel^2(n_\parallel; m_q, m_{[qq]}, g)$$
$$M_T^2(n_\perp, n_\parallel, L_T, S_T) = 4\kappa^2\left(n_\perp + L_T + \frac{S_T}{2} + 1\right) + M_\parallel^2(n_\parallel; m_{[\bar{q}q]}, m_{[qq]}, g)$$

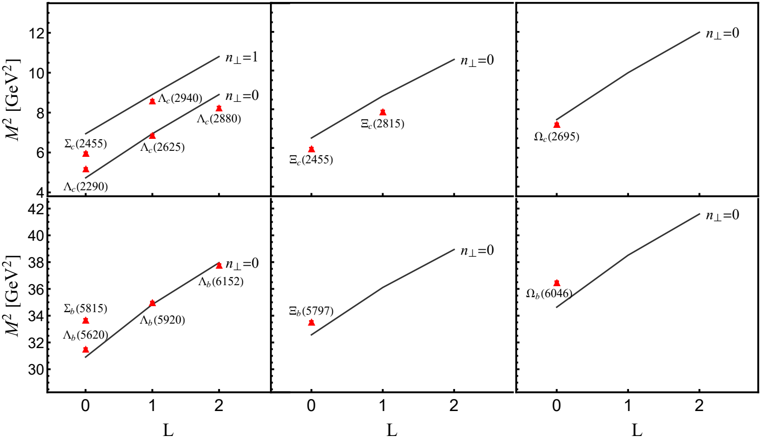
- Parity : $P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T}$
- Charge conjugation : $C = (-1)^{n_\parallel + L_M + S_M} = (-1)^{n_\parallel + L_T + S_T - 1}$
- We find that $n_\parallel \geq n_\perp + L$ (underlying link between the holographic Schrödinger Equation and the ‘t Hooft Equation)

[2] M. Neilson, S.J. Brodsky, Phys. Rev. D 97, 114001 (2018)

Light baryons

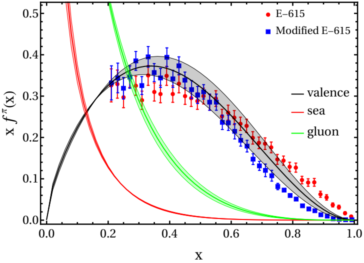


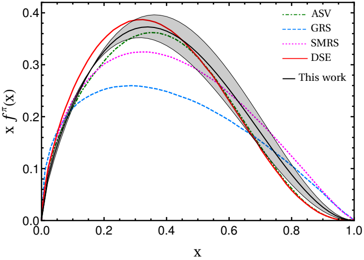
Heavy-Light baryons



[1] Particle Data Group (2020)

Application of LFWFs : Pion PDFs





LF wavefunction $\Rightarrow$ Initial PDFs $\Rightarrow$ Scale evolution $(0.240 \pm 0.024 \to 16\ \mathrm{GeV}^2)$

- Pion valence PDF falls off as $(1-x)^{1.9}$, favoring a slightly slower falloff compared to the $(1-x)^2$ predicted by perturbative QCD and DSEs.

Valence moments	$\langle x \rangle$ 1.69	$\langle x \rangle$ 4	$\langle x^2 \rangle$ 1.69	$\langle x^2 \rangle$ 4
This work	0.271	0.245	0.121	0.103
JAM (2018)	0.268	0.245	0.127	0.108
BLFQ-NJL (2019)	0.271	0.245	0.124	0.106
BSE (2018)	0.268	0.240	0.125	—
ETM (2018) [Lattice]	—	0.207(11)	—	0.163(33)

- Gluons: {36.8, 39.8, 42.7}% of pion momentum at {1.69, 4, 10, 16} GeV^2.

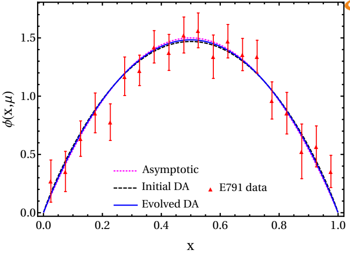
[1] JAM: PRL 121 (2018), BLFQ-NJL: PRL 122 (2019), BSE: PRL 124 (2020), ETM: PRD 99 (2019).

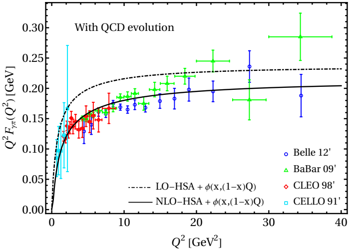
Distribution Amplitude and Transition Form Factor

$\pi \to \gamma^* \gamma$ TFF:[1]

$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \mathrm{d}x\, T_H(x, Q^2)\, \phi(x, \bar{x}Q)$$

- DA evolution: ERBL evol. (Gegenbauer basis) Ruiz, *et. al.* PRD 66, (2002)
- Our DA is close to Asymptotic DA
- Our prediction agrees well with data reported by Belle Collaboration.
- It deviates from the rapid growth of the large Q^2 data reported by BaBar Collaboration.





Decay constant f_π:

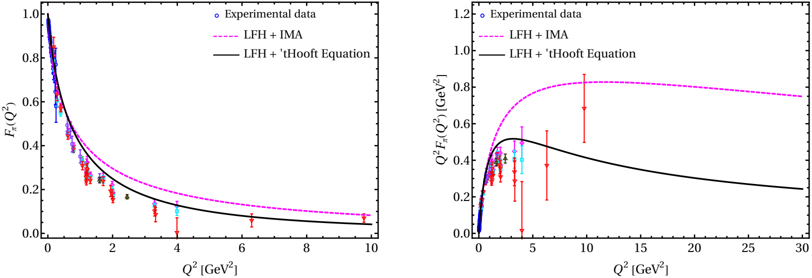
LFH + 't Hooft Equation: 166.4 MeV
Experimental data: 130.2 ± 1.7 MeV

Decay constants for excited pions:

$f_{\pi'} = 3.5$ MeV; $f_{\pi''} = 2.6$ MeV

[1] S. J. Brodsky, F. G. Cao and G. F. de Teramond, Phys. Rev. D 84, 033001 (2011)

Pion Electromagnetic Form Factor



Charge radius:

$$\langle r_c^2 \rangle = -6 \lim_{Q^2 \to 0} \frac{d}{dQ^2} F_{\mathrm{P}}(Q^2).$$

Pion charge radius: 0.69 fm

Experimental data: 0.672 ± 0.008 fm

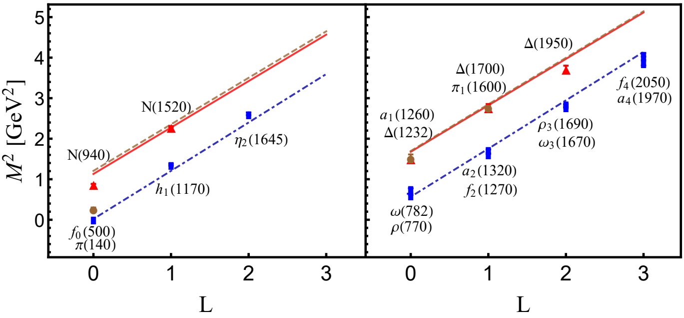
[1] in preparation

Conclusions

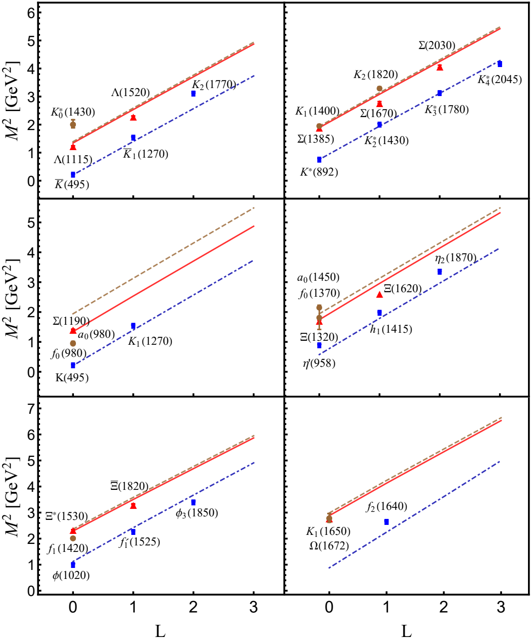
- The 't Hooft Equation is complementary to the holographic Schrödinger Equation in predicting the full hadron spectrum.
- We find that the emerging hadronic scale κ of LFH remains universal across the full spectrum. For heavy-heavy mesons, κ coincides with the 't Hooft coupling as expected for rotational symmetry in nonrelativistic limit.
- The baryons and mesons spectra are in good agreement with the data. For the tetraquark candidates, the agreement is less impressive.
- Wave functions lead to a good description of Pion FF, PDFs, PDA, and transition form factor ($\pi^0 \to \gamma^* \gamma$).

Thank You

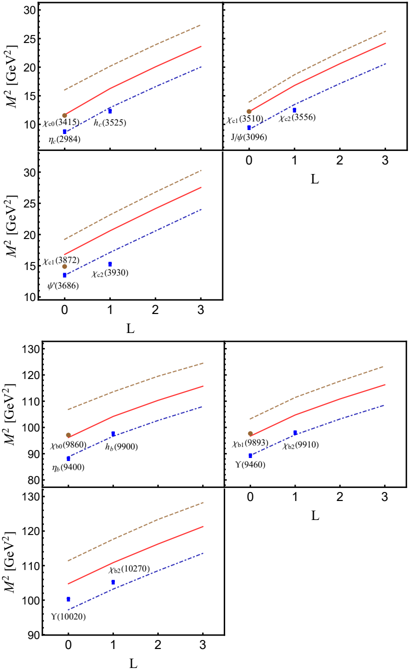
Light super-partners



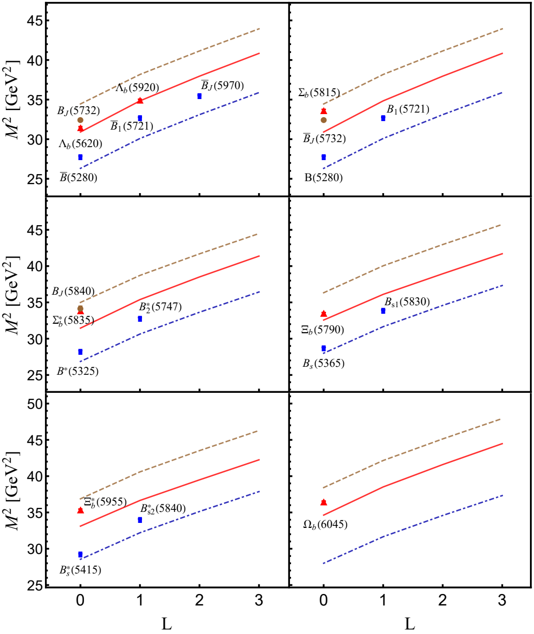
Light-Strange super-partners



Heavy super-partners



Heavy-Light super-partners



Heavy-Light super-partners